Homomorphic Secret Sharing with Verifiable Evaluation

TCC 2024

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Johns Hopkins University

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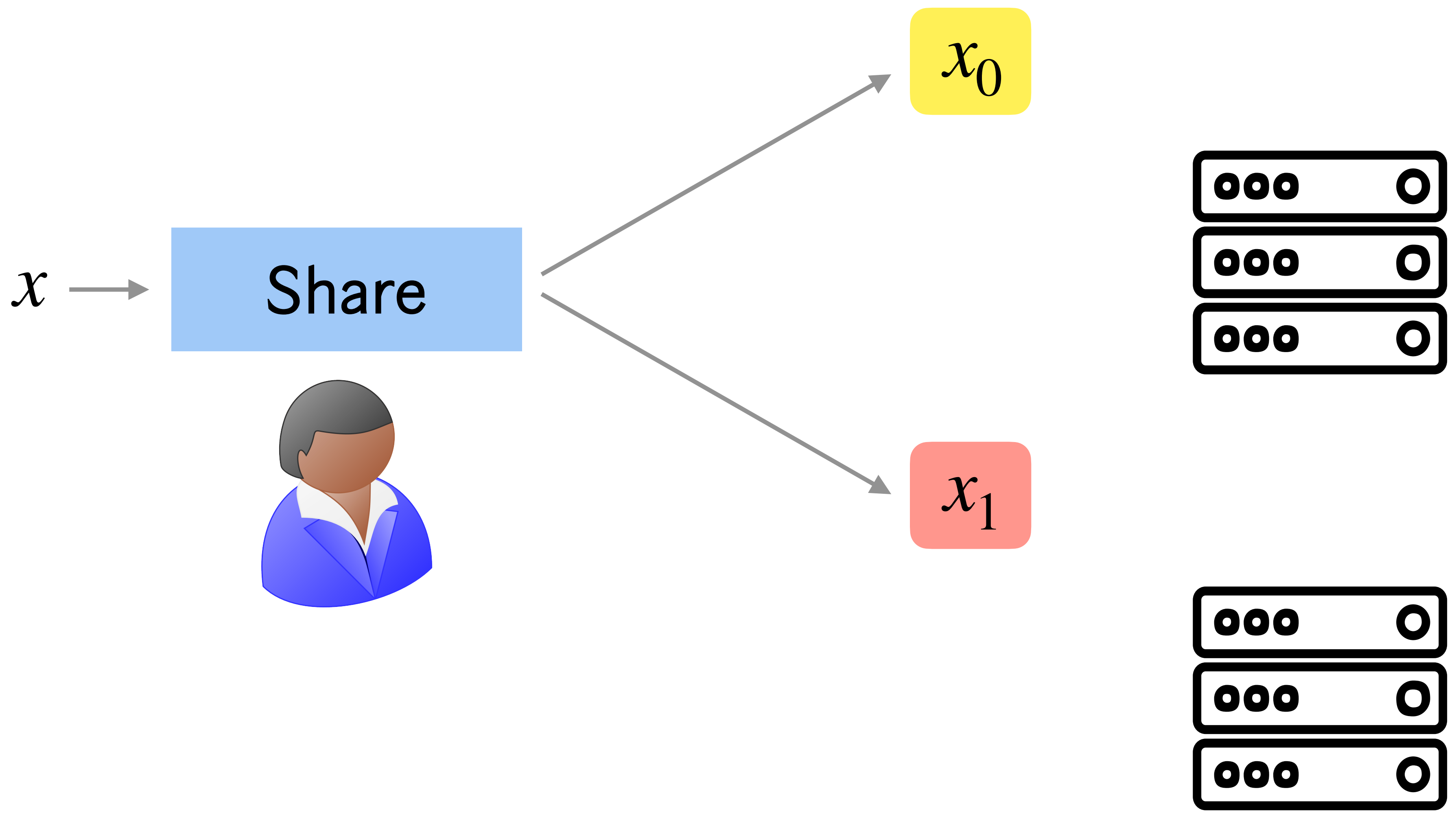
NTT Research
Johns Hopkins University

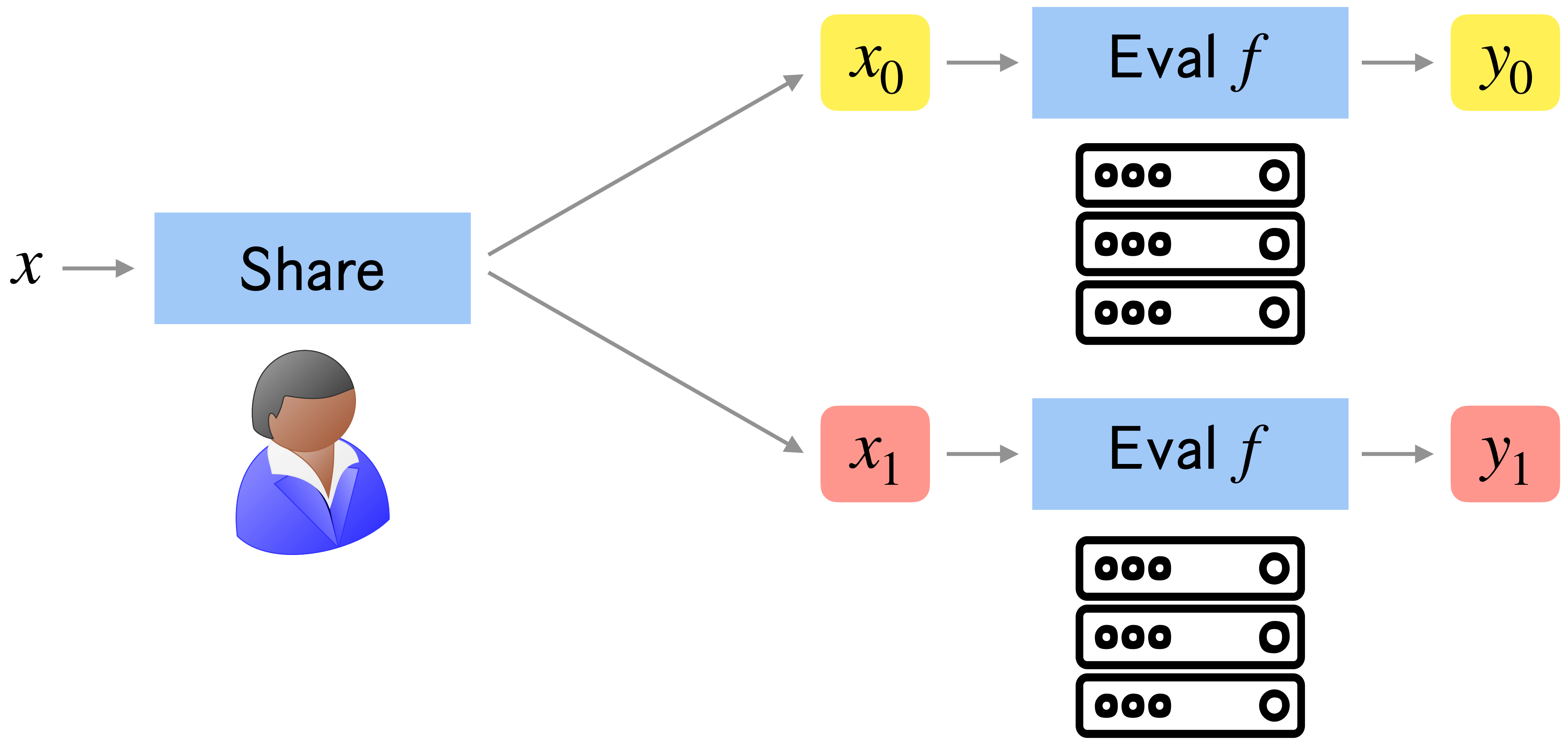
Homomorphic Secret Sharing

[Boyle-Gilboa-Ishai 16]



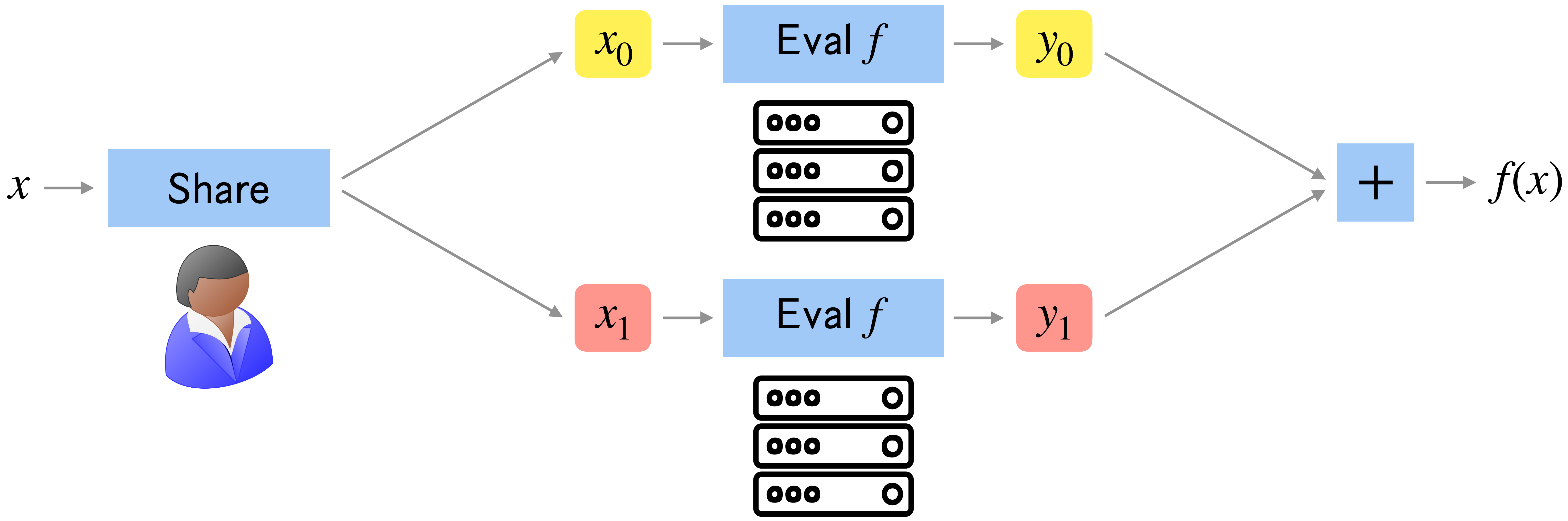
Homomorphic Secret Sharing

[Boyle-Gilboa-Ishai 16]



Homomorphic Secret Sharing

[Boyle-Gilboa-Ishai 16]



Correctness $y_0 + y_1 = f(x)$

Homomorphic Secret Sharing

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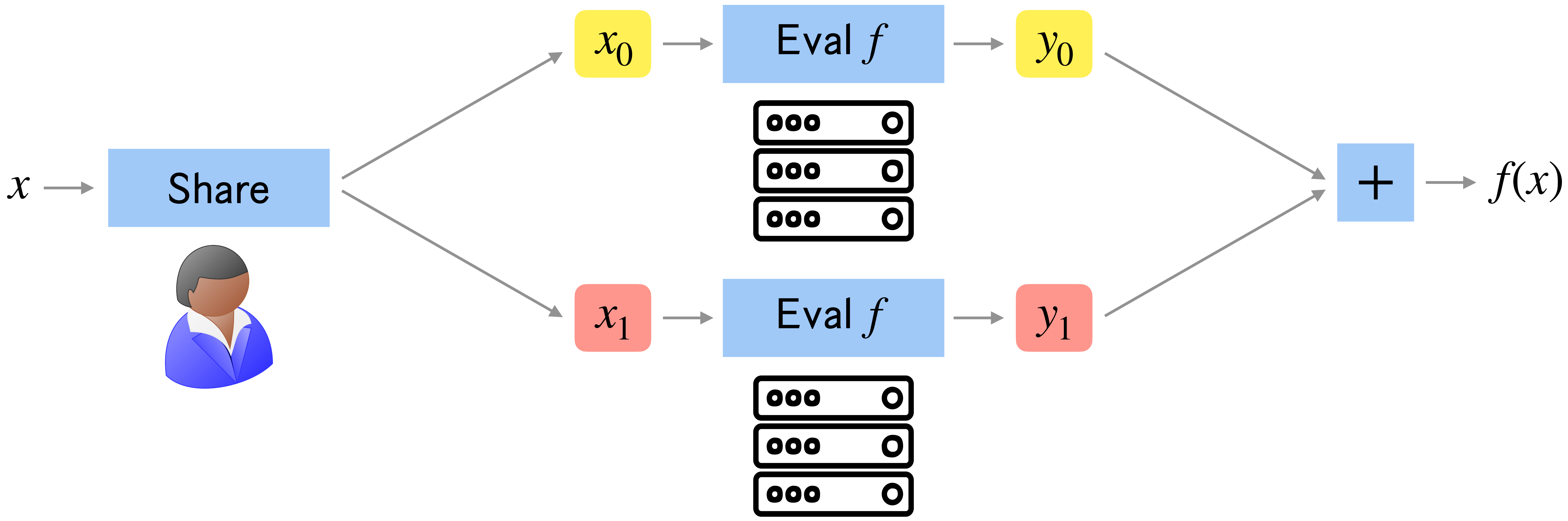

x → Share

x_0 → Eval f → y_0

x_1 → Eval f → y_1

$+$ → $f(x)$

Correctness $y_0 + y_1 = f(x)$

Security x_0 and x_1 hide x

Homomorphic Secret Sharing

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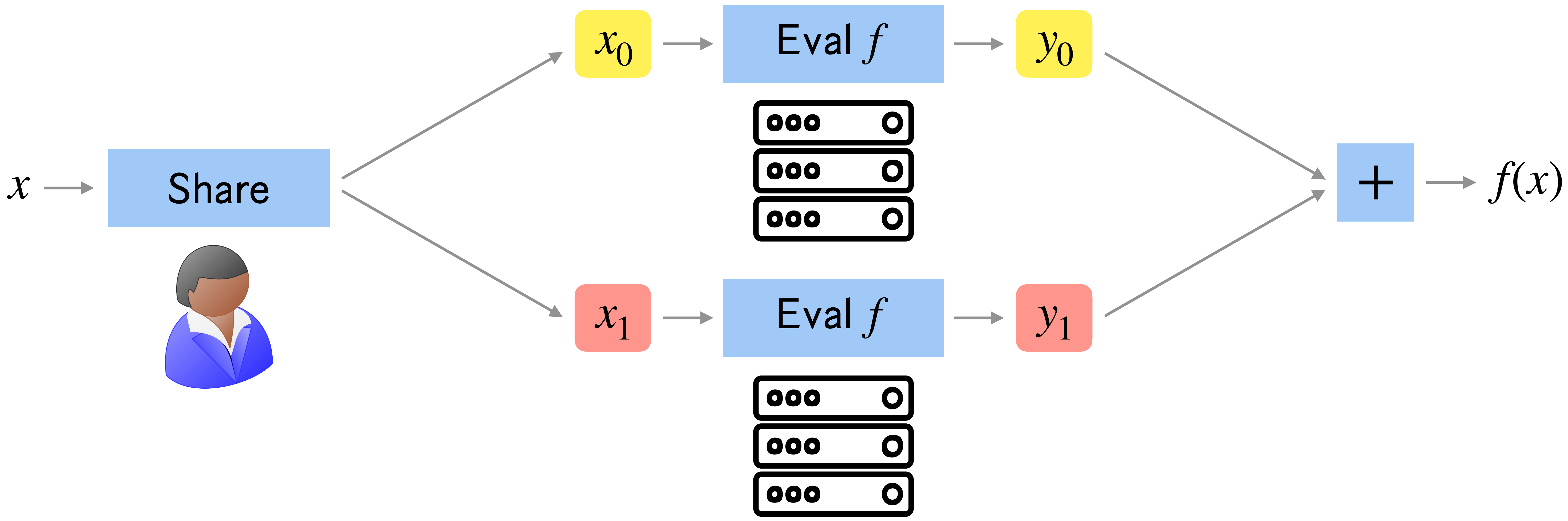
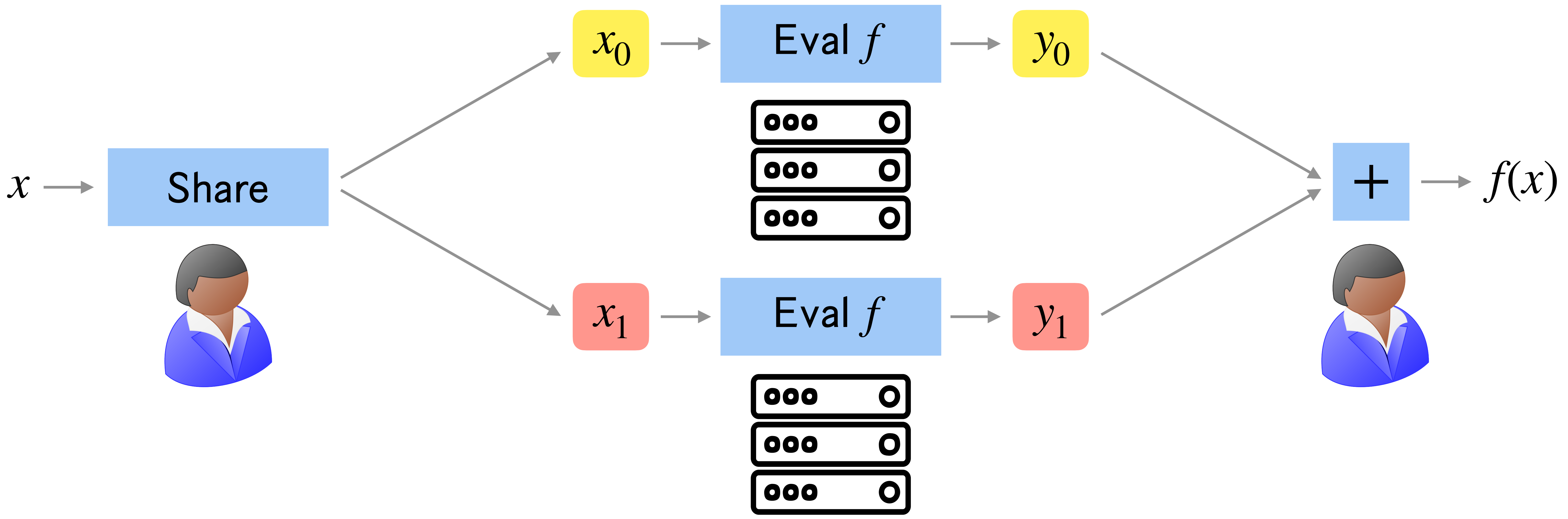


Correctness	$y_0 + y_1 = f(x)$
Security	x_0 and x_1 hide x
Succinctness	y_0 , y_1 same size as $f(x)$

Homomorphic Secret Sharing

[Boyle-Gilboa-Ishai 16]



Enables private delegation of computation

Prior Works

Low degree polynomials

LPN

[Boyle-Couteau-Gilboa-Ishai-Kohl-Scholl 19]

[Couteau-Meyer 21]

Sparse LPN

[Dao-Ishai-Jain-Lin 23]

NC^1

DDH

[Boyle-Gilboa-Ishai 16]

[Boyle-Gilboa-Ishai 17]

[Boyle-Couteau-Gilboa-Ishai-Orrù 17]

DCR

[Fazio-Gennaro-Jafarikhah-Skeith 17]

[Orlandi-Scholl-Yakoubov 21]

[Roy-Singh 21]

Class Groups

[Abram-Damgård-Orlandi-Scholl 22]

LWE

[Boyle-Kohl-Scholl 19]

P/poly

FHE

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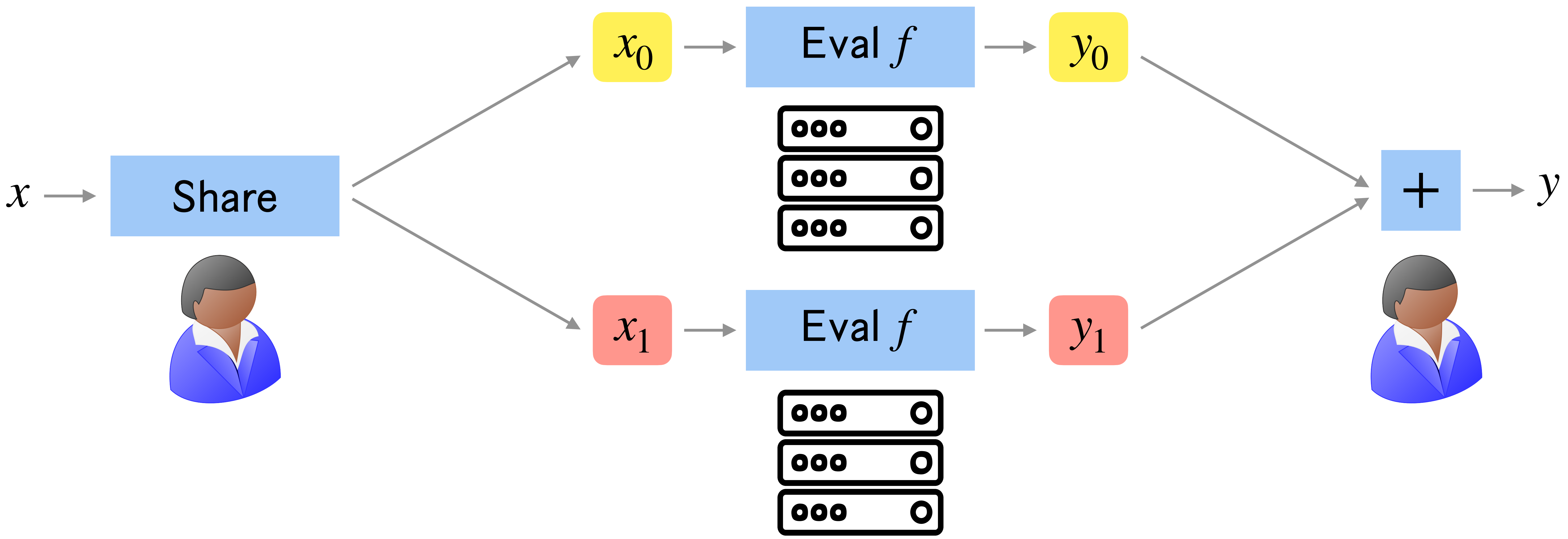
iO + OWF

[Boyle-Gilboa-Ishai 15]

Assume semi-honest servers

HSS with Verifiable Evaluation (ve-HSS)



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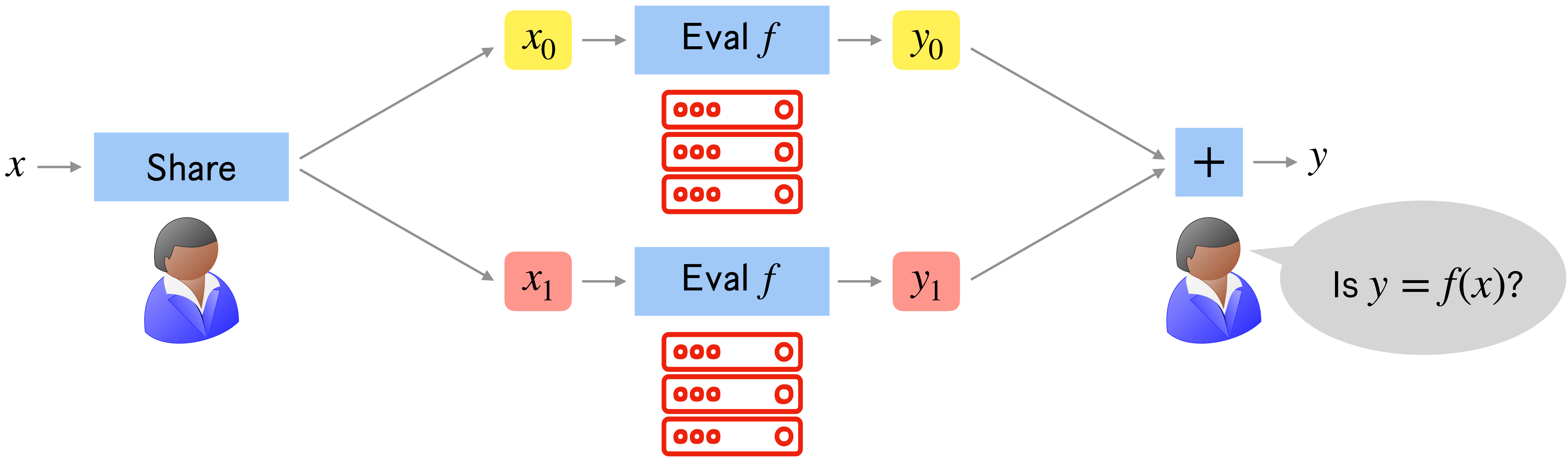

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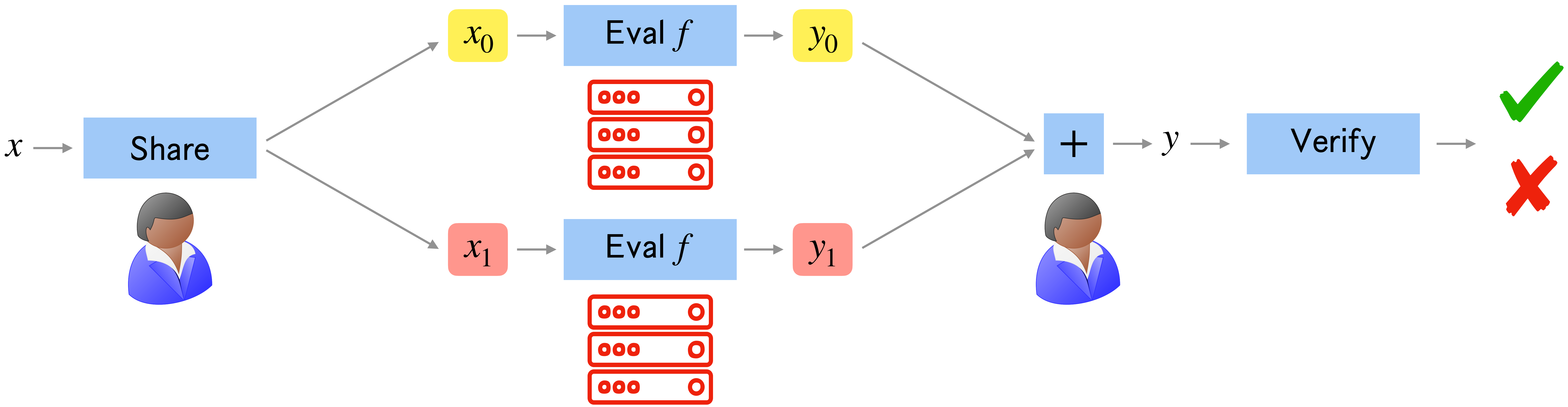

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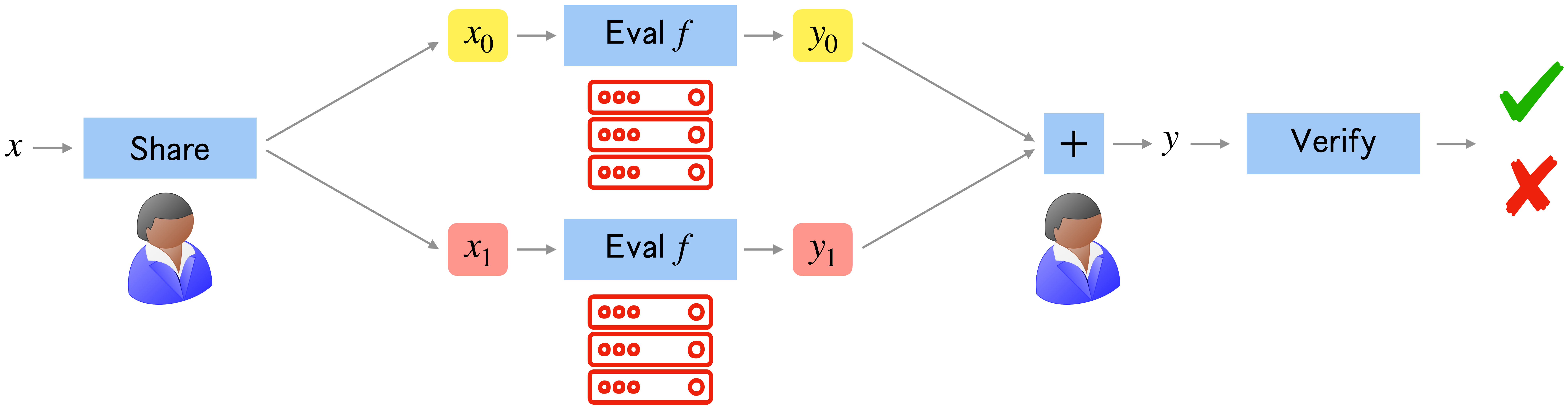

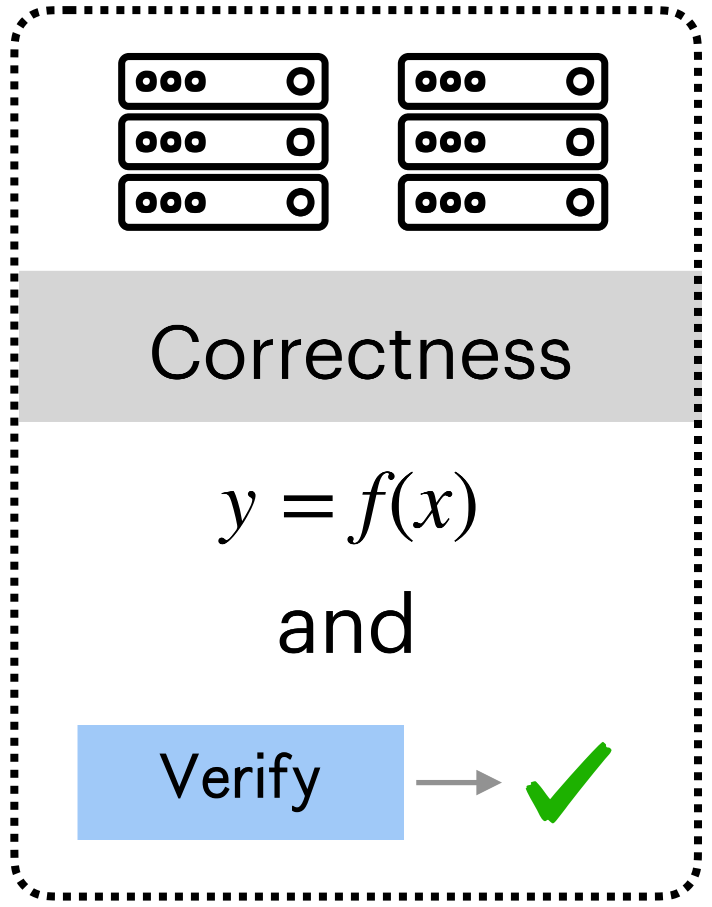

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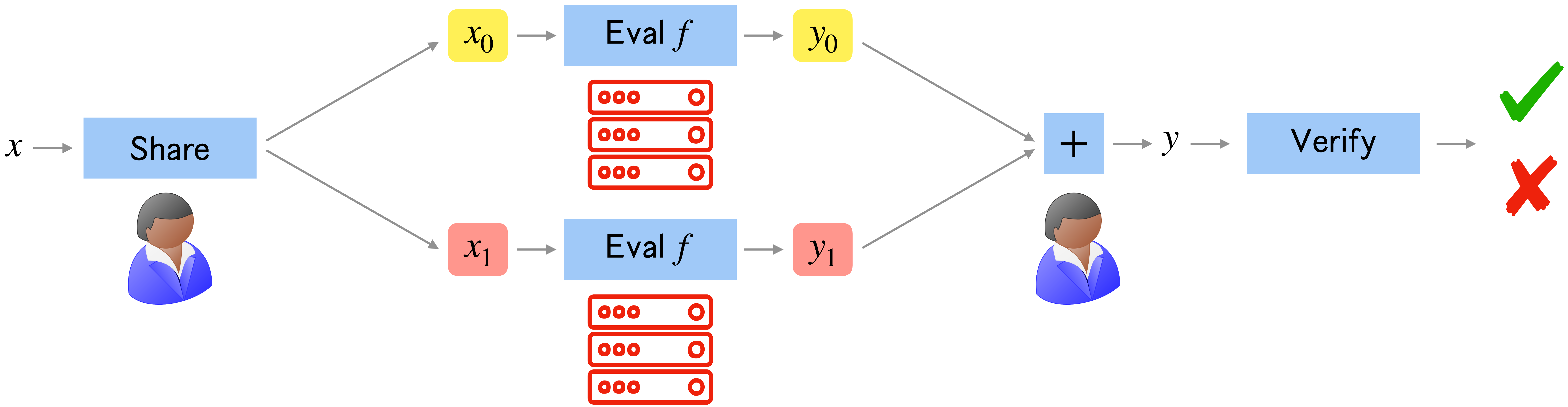


x → Share

x_0 → Eval f → y_0

x_1 → Eval f → y_1

$+$ → y → Verify → ✓ ✗

Correctness

$y = f(x)$

and

Verify → ✓

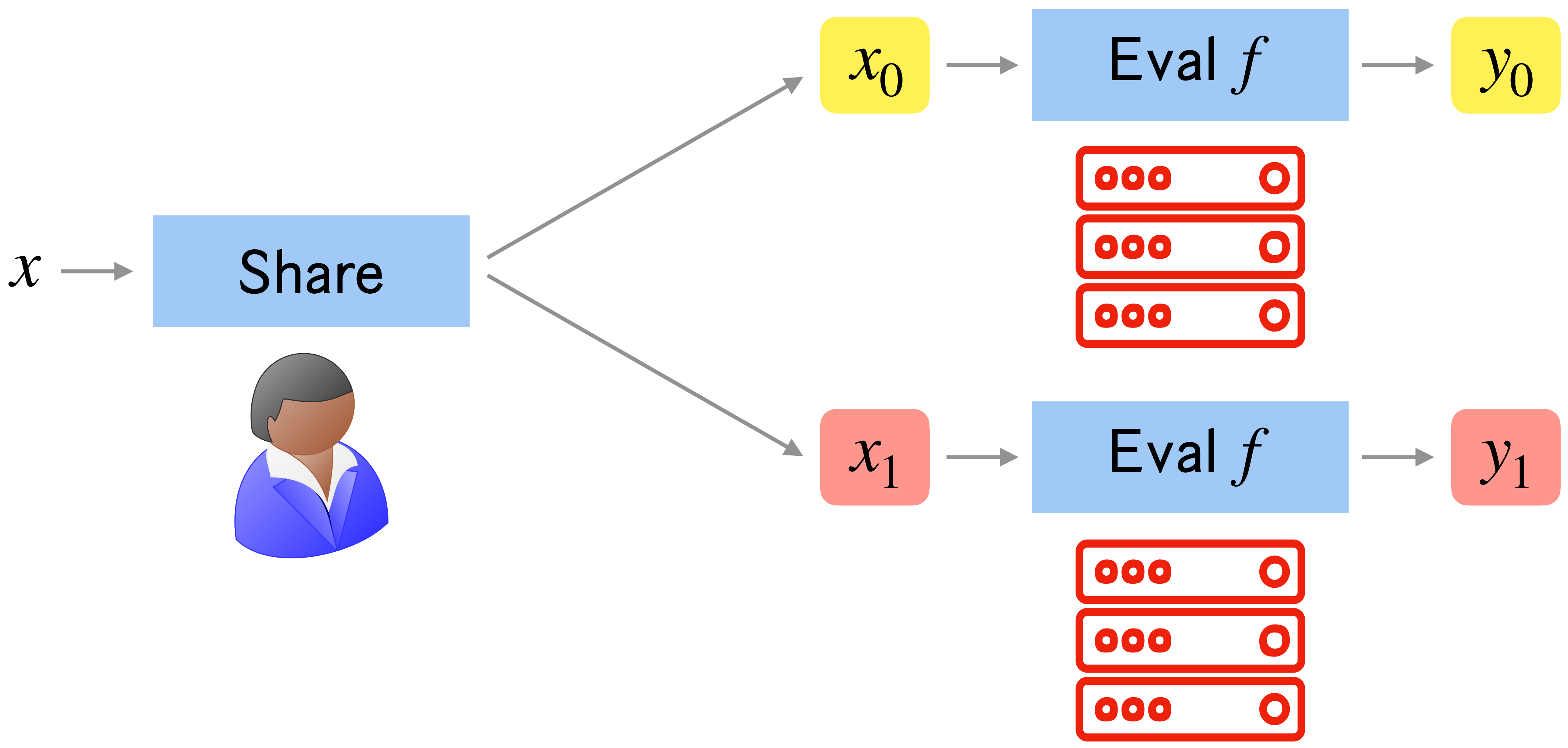
Local Soundness

Verify → ✓

$\Downarrow$

$y = f(x)$

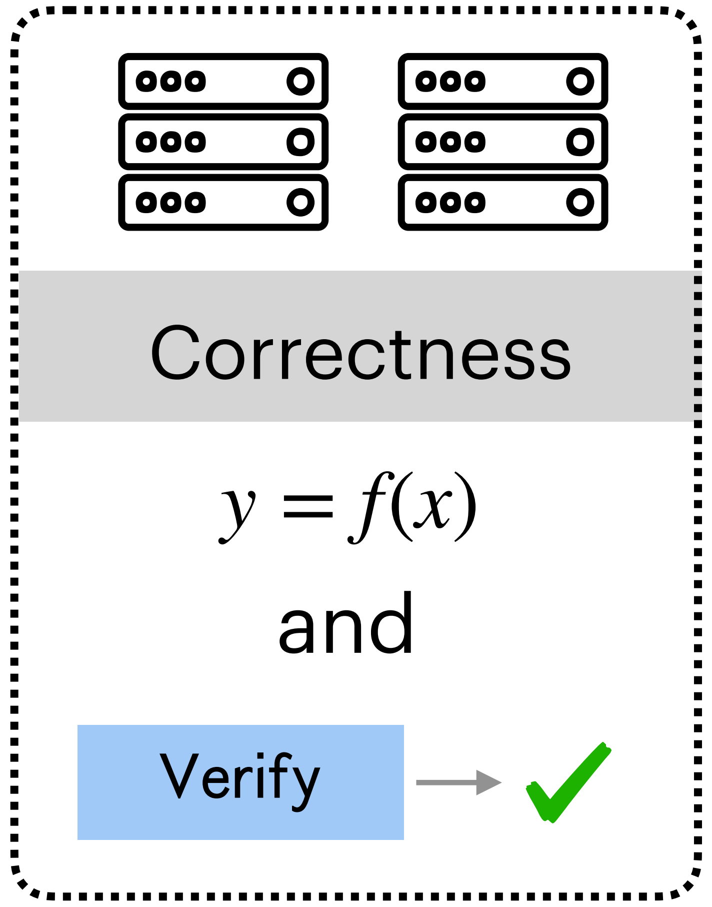
HSS with Verifiable Evaluation (ve-HSS)



$x \rightarrow$ Share

$x_0 \rightarrow$ Eval f $\rightarrow y_0$

$x_1 \rightarrow$ Eval f $\rightarrow y_1$



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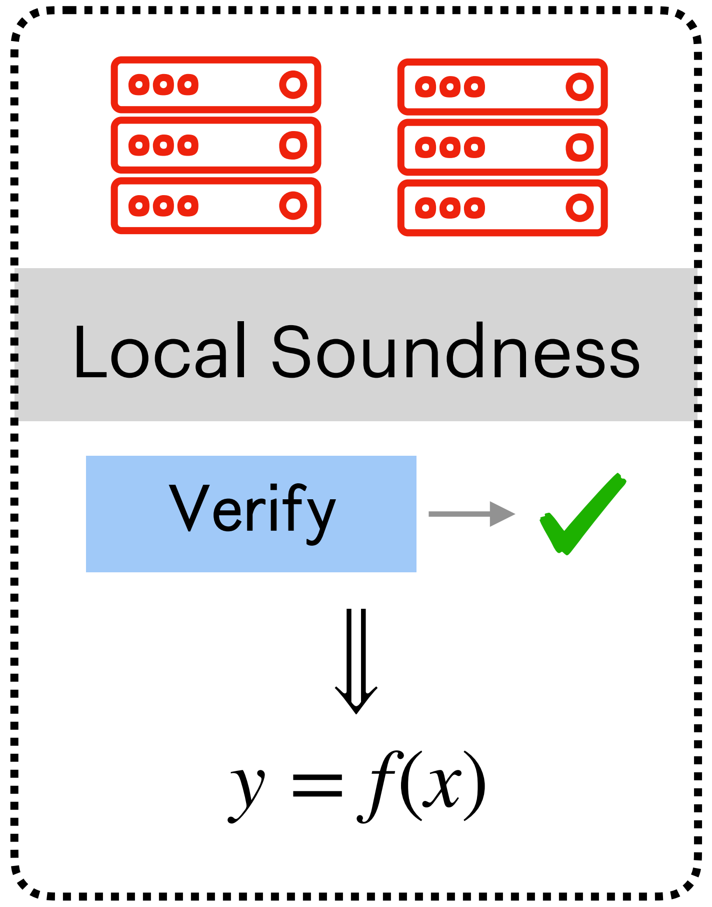
$y = f(x)$

and

Verify → ✓



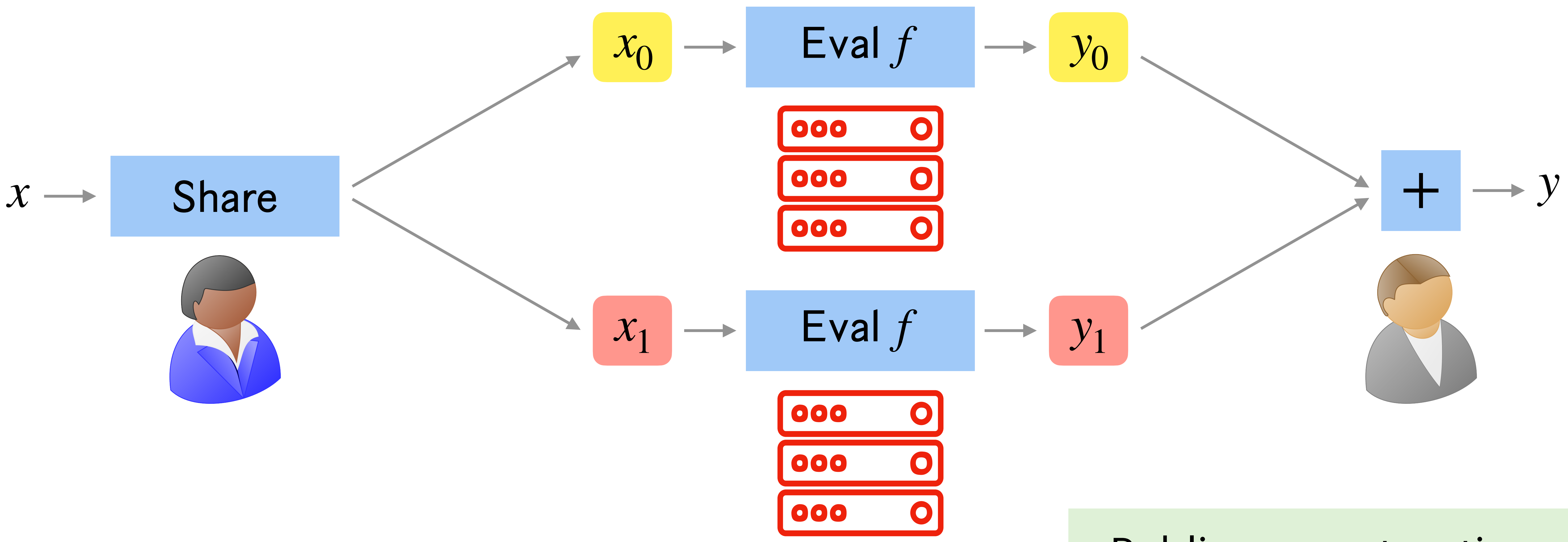
Local Soundness

Verify → ✓

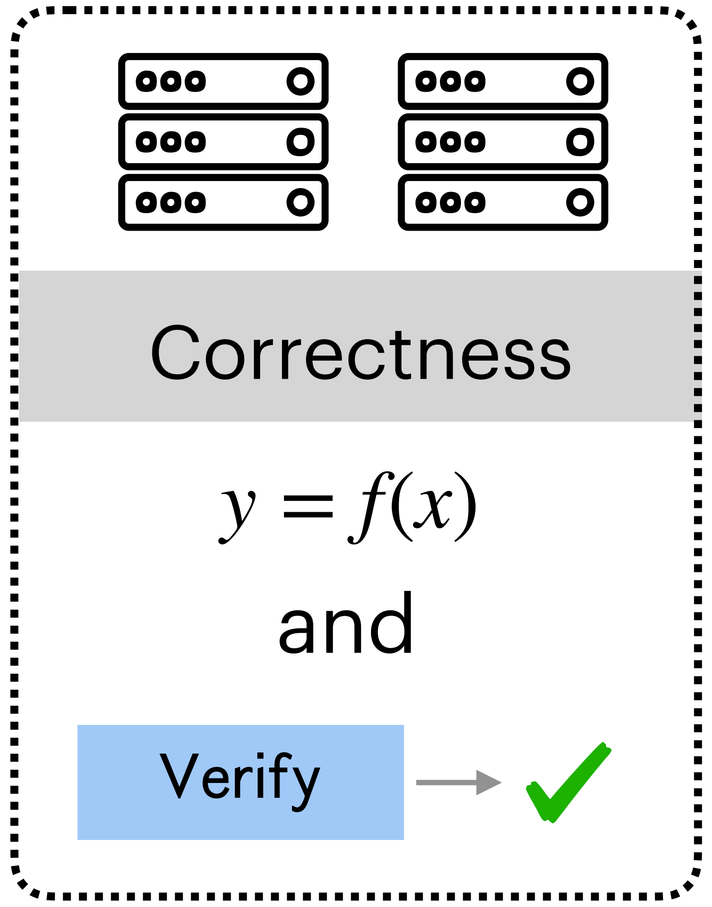
$\Downarrow$

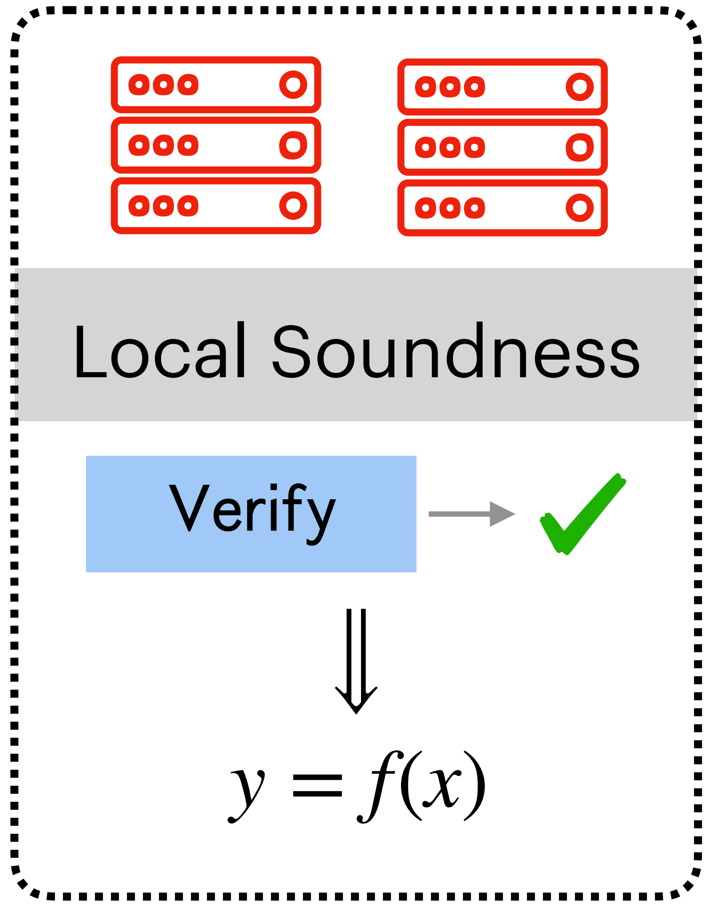
$y = f(x)$

HSS with Verifiable Evaluation (ve-HSS)

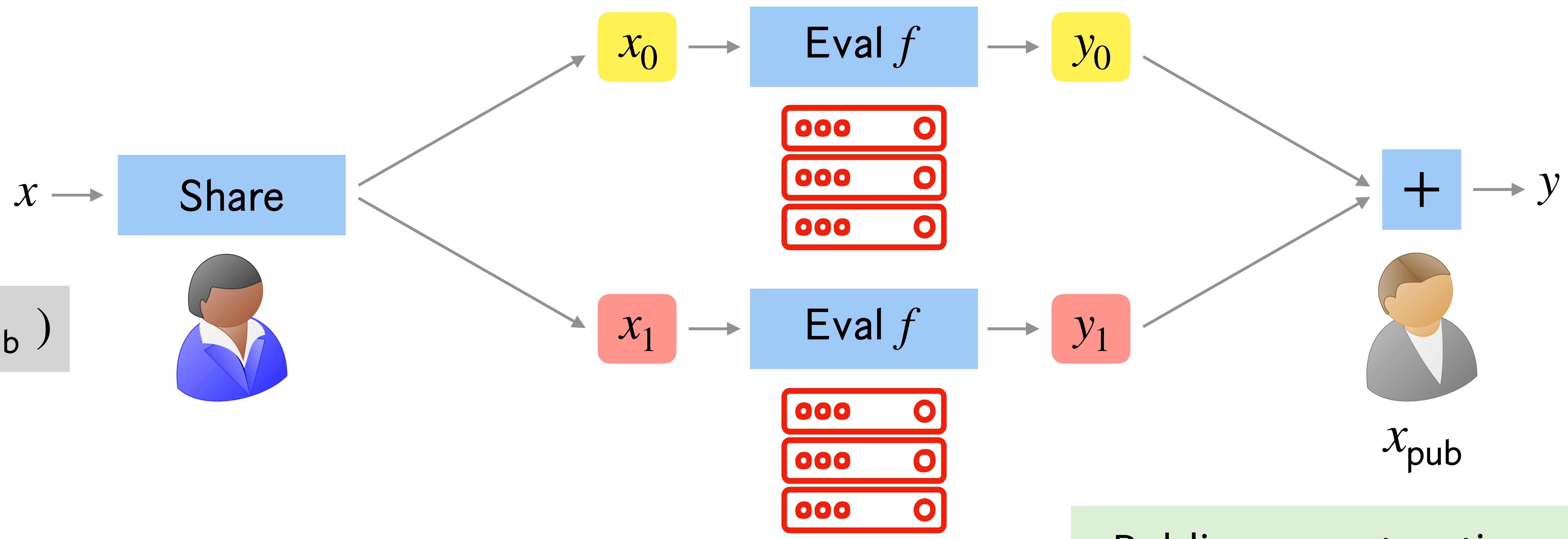


Public reconstruction of output



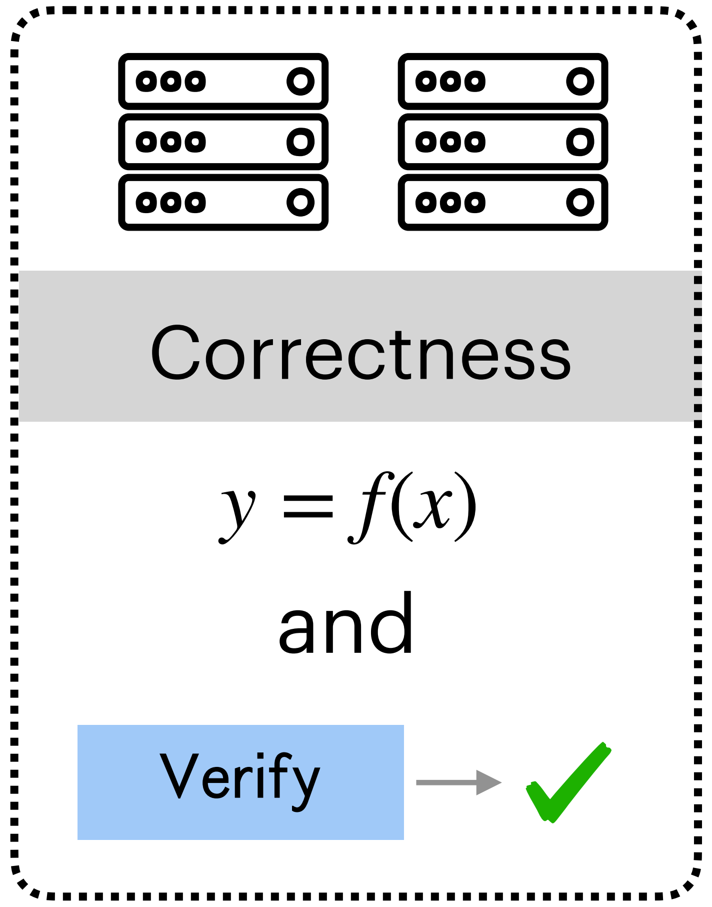


HSS with Verifiable Evaluation (ve-HSS)



$x = (x_{\text{priv}} \; , \; x_{\text{pub}} \;)$

Public reconstruction of output

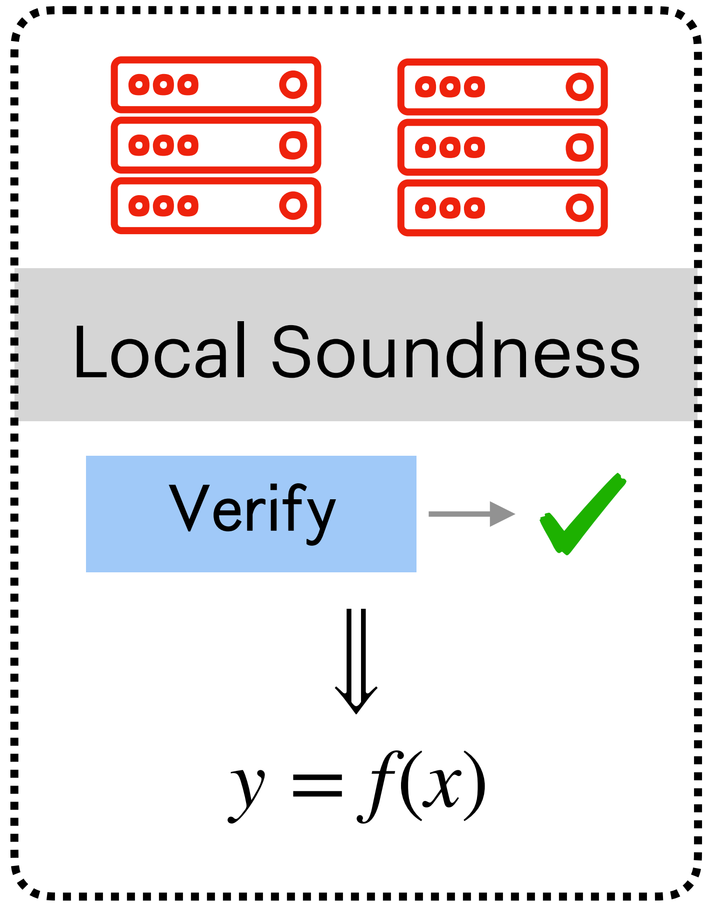


Correctness

$y = f(x)$

and

Verify → ✓



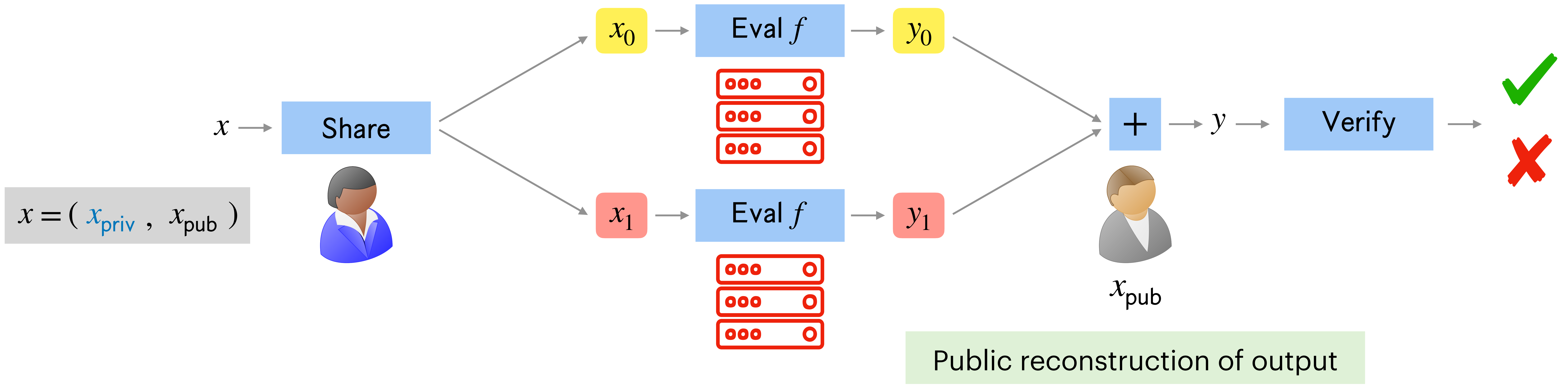
Local Soundness

Verify → ✓

$\Downarrow$

$y = f(x)$

HSS with Verifiable Evaluation (ve-HSS)



$x \rightarrow$ Share

$x = (x_{\text{priv}} , x_{\text{pub}})$

$x_0 \rightarrow$ Eval f $\rightarrow y_0$

$x_1 \rightarrow$ Eval f $\rightarrow y_1$

$+ \rightarrow y \rightarrow$ Verify $\rightarrow$ ✓ ✗

x_{pub}

Public reconstruction of output

Correctness

$y = f(x)$

and

Verify → ✓

Local Soundness

Verify → ✓

$\Downarrow$

$y = f(x)$

Public Soundness

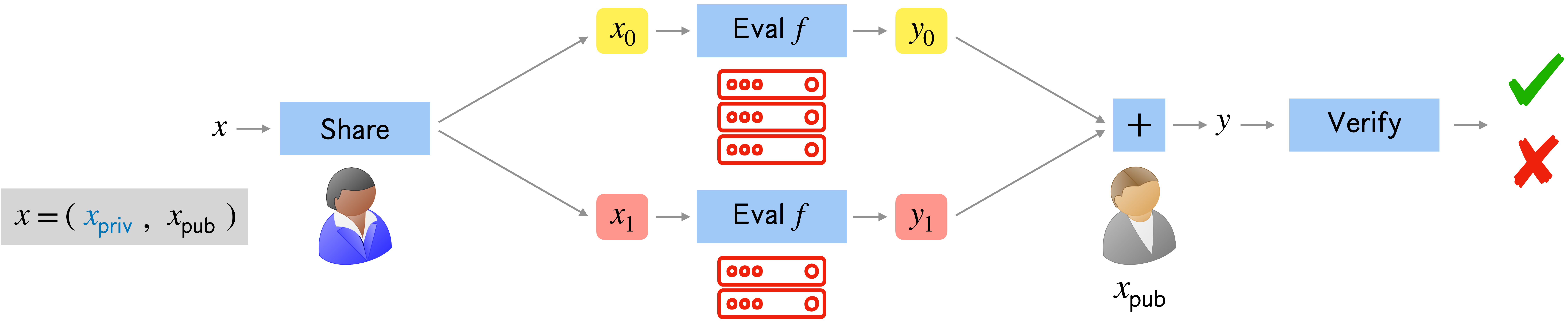
Verify → ✓

$\Downarrow$

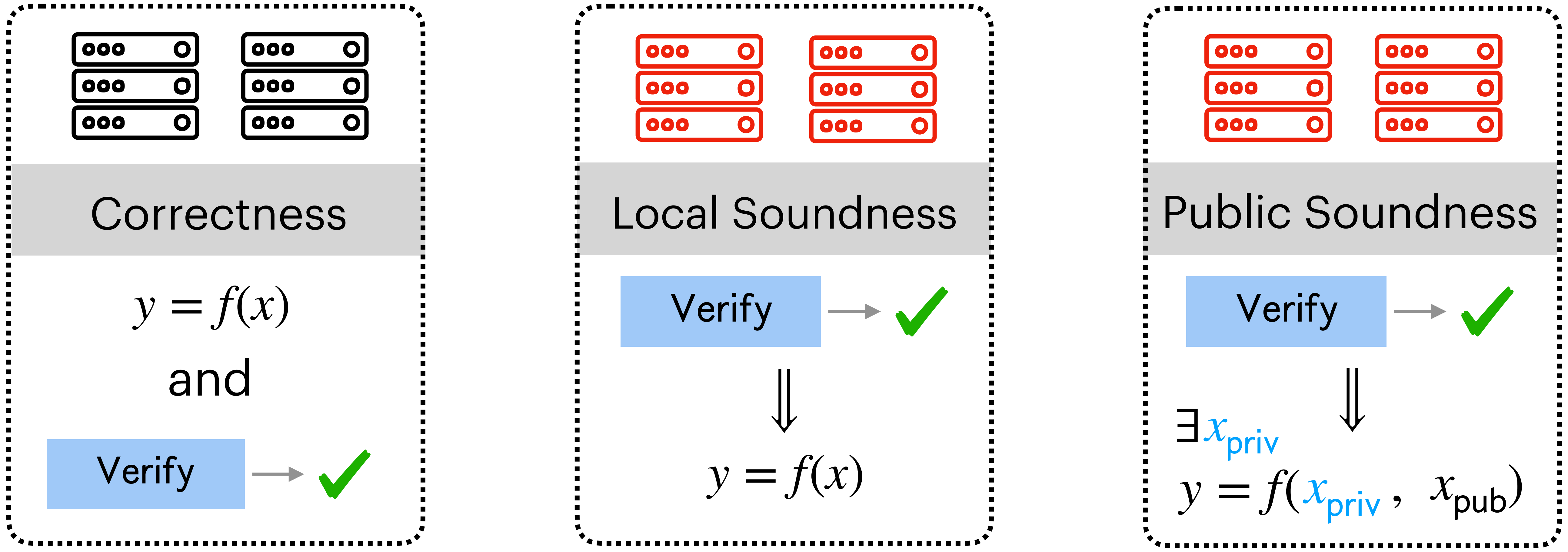
$\exists x_{\text{priv}}$

$y = f(x_{\text{priv}} , x_{\text{pub}})$

HSS with Verifiable Evaluation (ve-HSS)



Public reconstruction of output



Correctness

$y = f(x)$

and

Verify → ✓

Local Soundness

Verify → ✓

$\Downarrow$

$y = f(x)$

Public Soundness

Verify → ✓

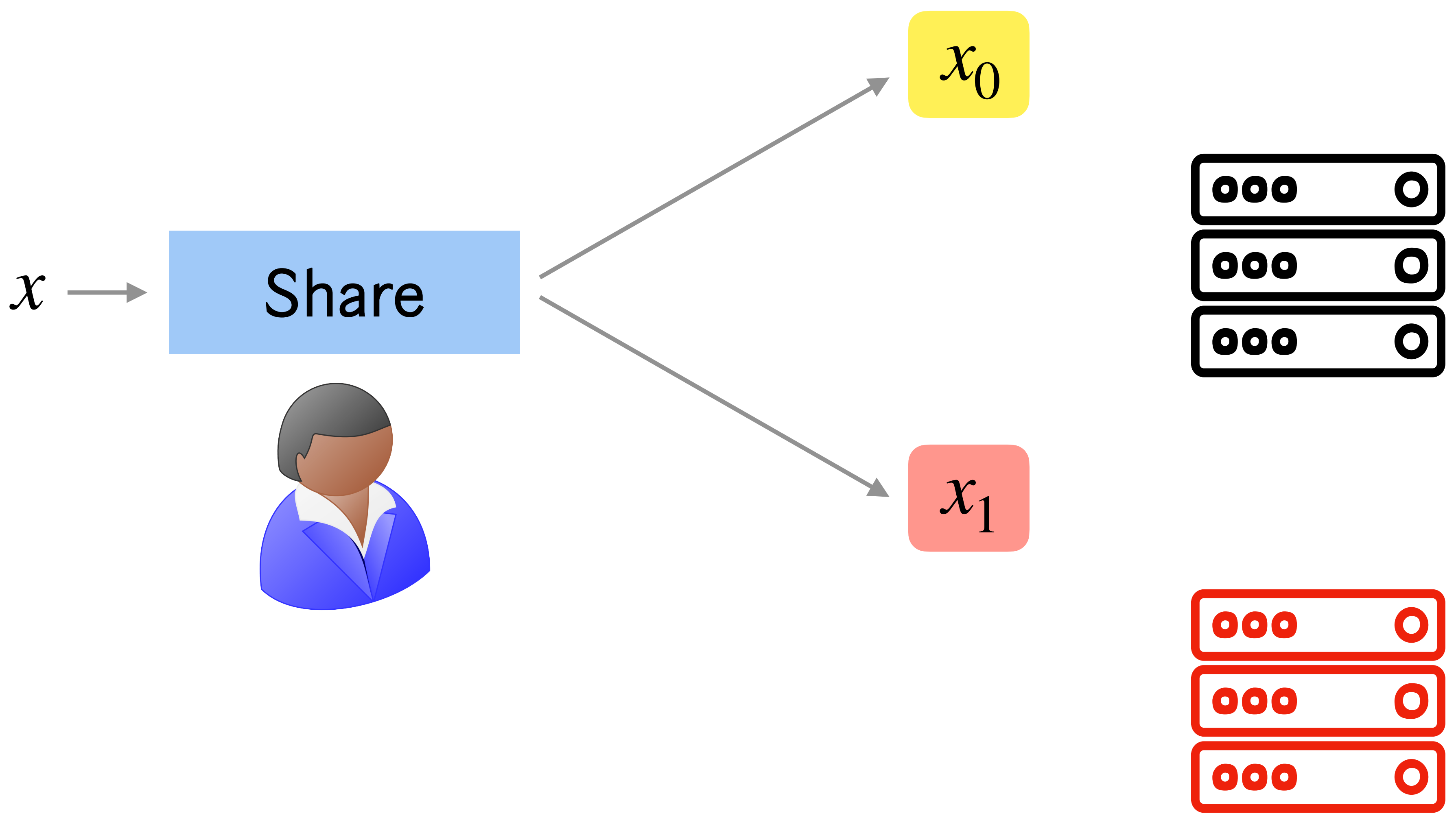
$\Downarrow$

$\exists x_{\text{priv}}$

$y = f(x_{\text{priv}}, x_{\text{pub}})$

Privacy requires at least one honest server

HSS with Verifiable Evaluation (ve-HSS)



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Verify → ✓

$\Downarrow$

$y = f(x)$

Public Soundness

Verify → ✓

$\Downarrow$

$\exists x_{\text{priv}}$

$y = f(x_{\text{priv}}, \; x_{\text{pub}})$

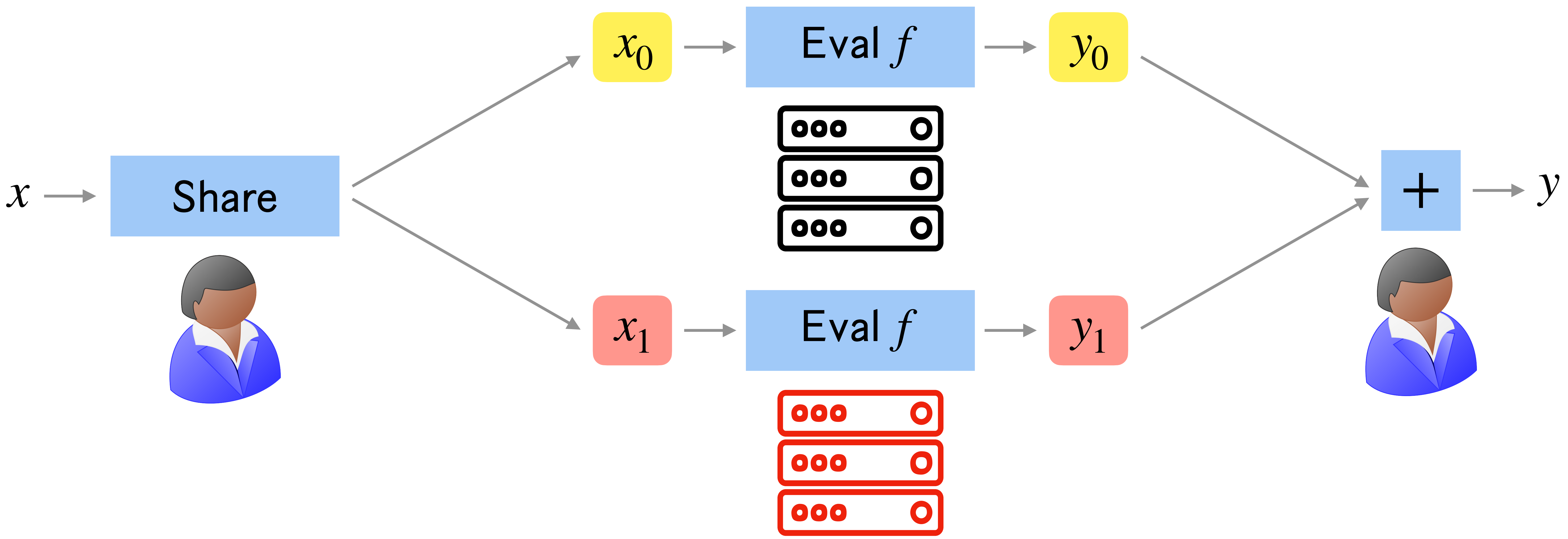
Privacy

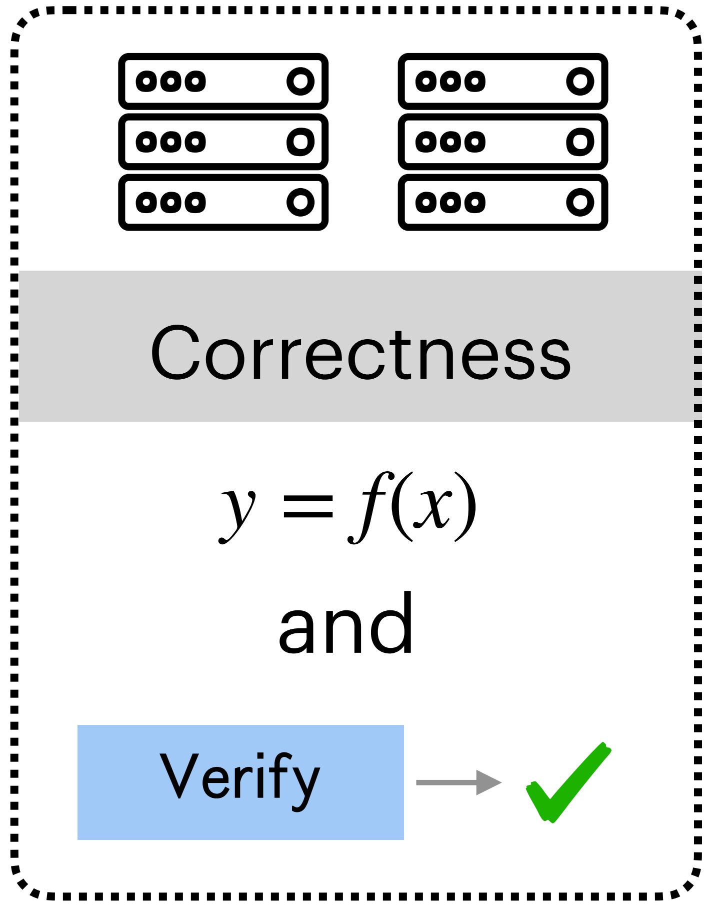
x_1 y_0

Verify → ✓ ✗

simulated from $f(x)$

HSS with Verifiable Evaluation (ve-HSS)





Correctness

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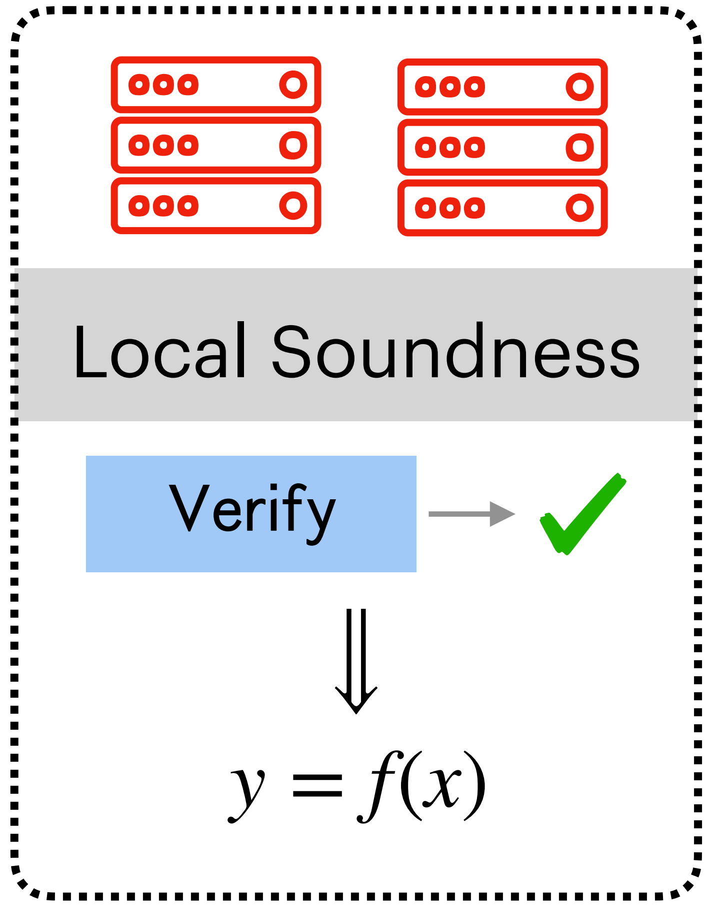
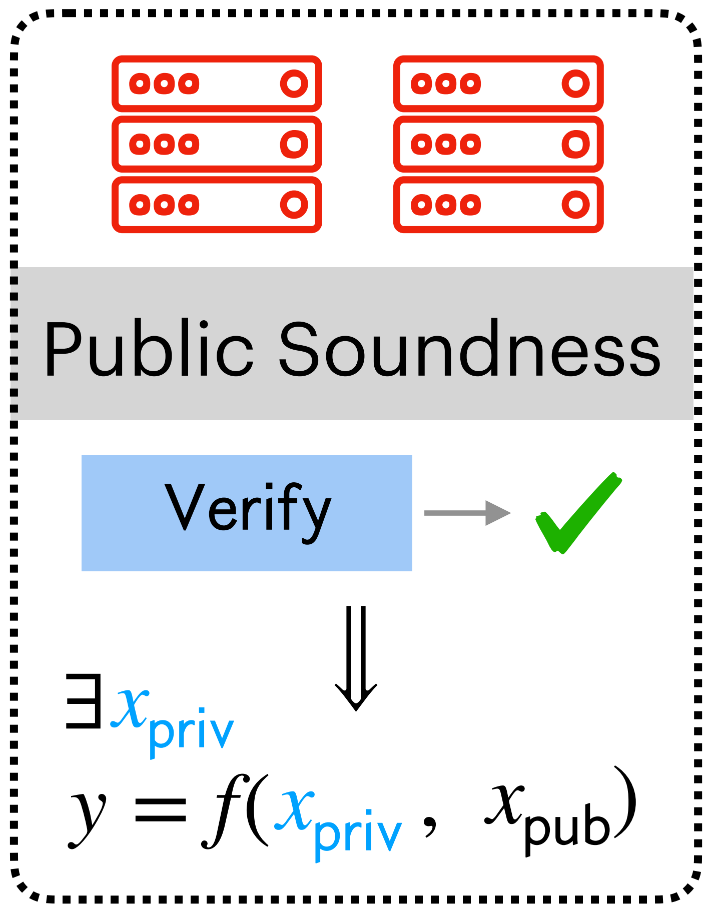


Local Soundness

Verify → ✓

$\Downarrow$

$y = f(x)$

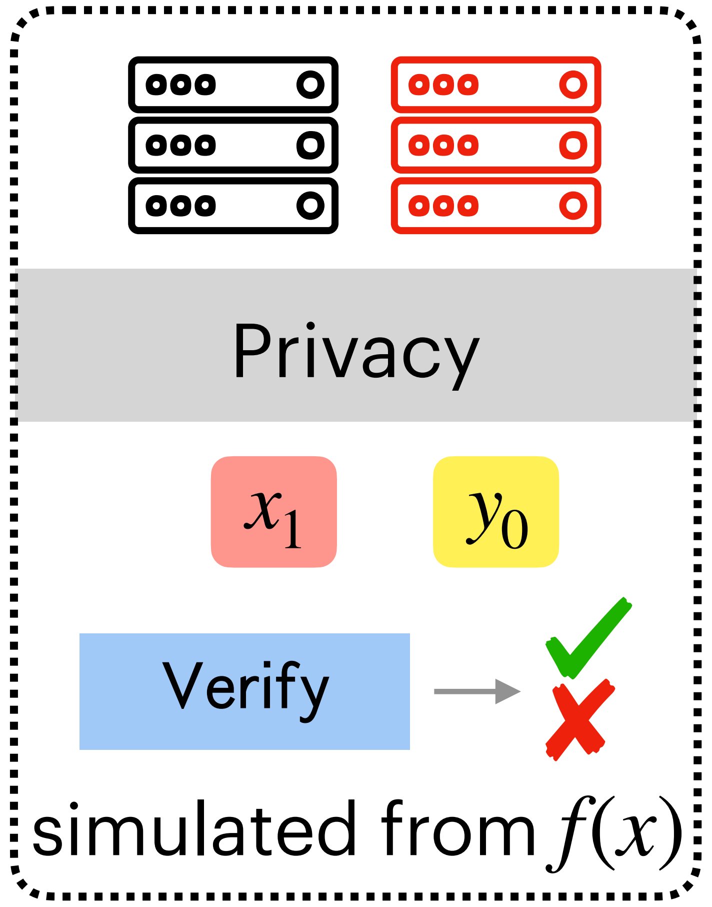


Public Soundness

Verify → ✓

$\exists x_{\text{priv}}$ $\Downarrow$

$y = f(x_{\text{priv}}, x_{\text{pub}})$



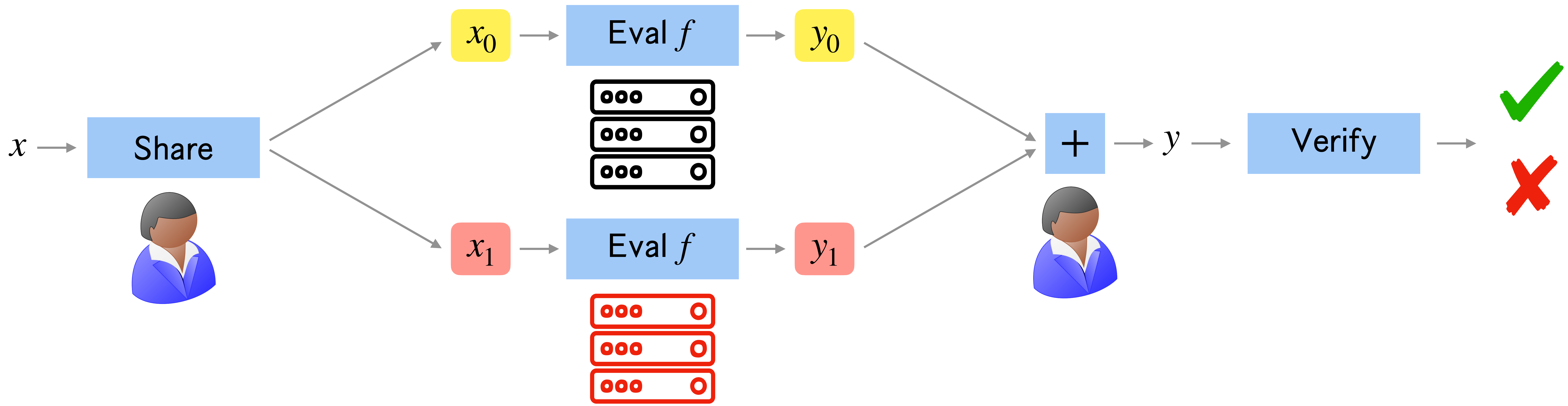
Privacy

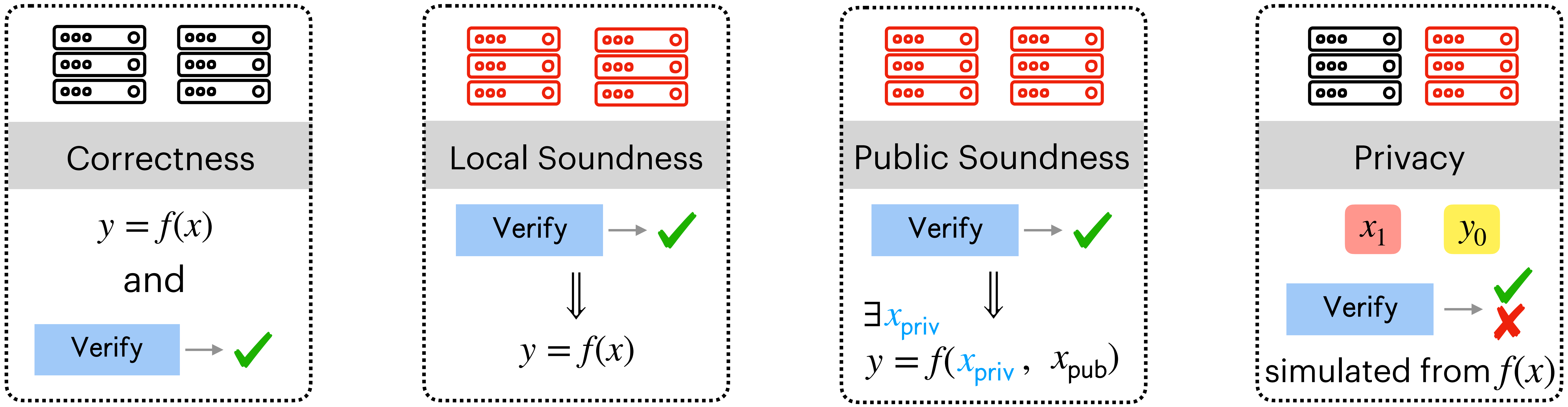
x_1 y_0

Verify → ✓ ✗

simulated from $f(x)$

HSS with Verifiable Evaluation (ve-HSS)





Correctness

$y = f(x)$

and

Verify → ✓

Local Soundness

Verify → ✓

$\Downarrow$

$y = f(x)$

Public Soundness

Verify → ✓

$\Downarrow$

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Privacy

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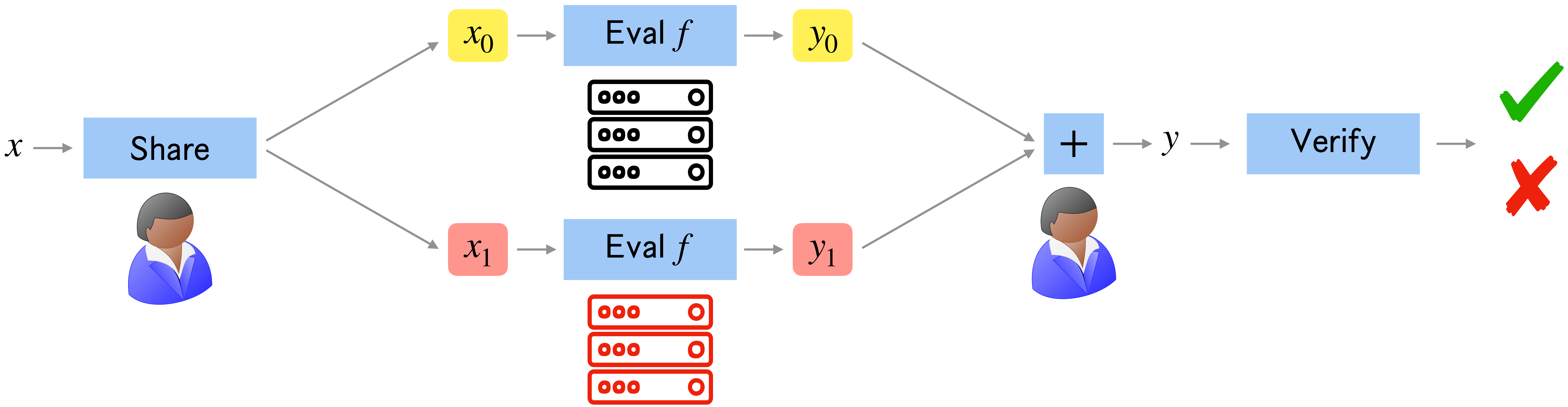
HSS with Verifiable Evaluation (ve-HSS)



x → Share

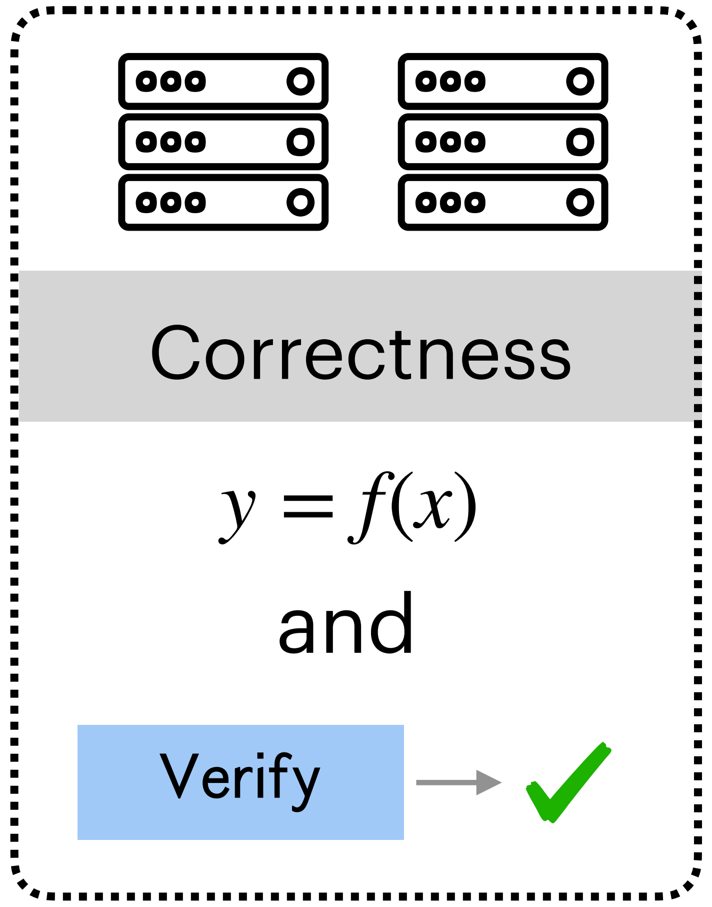
x_0 → Eval f → y_0

x_1 → Eval f → y_1

+ → y → Verify →

✓

✗



Correctness

$y = f(x)$

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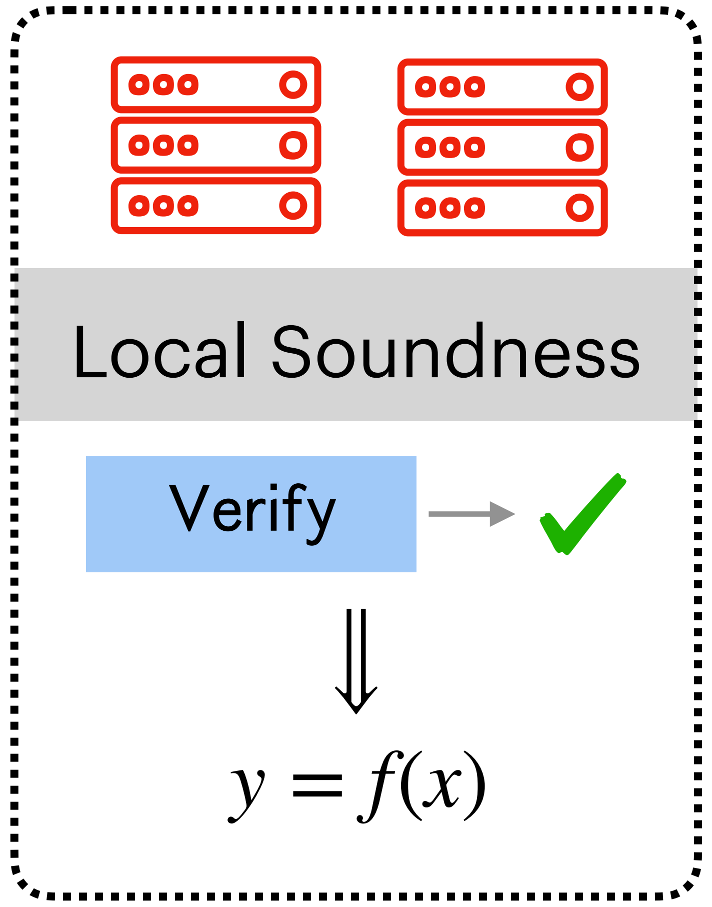
Verify → ✓

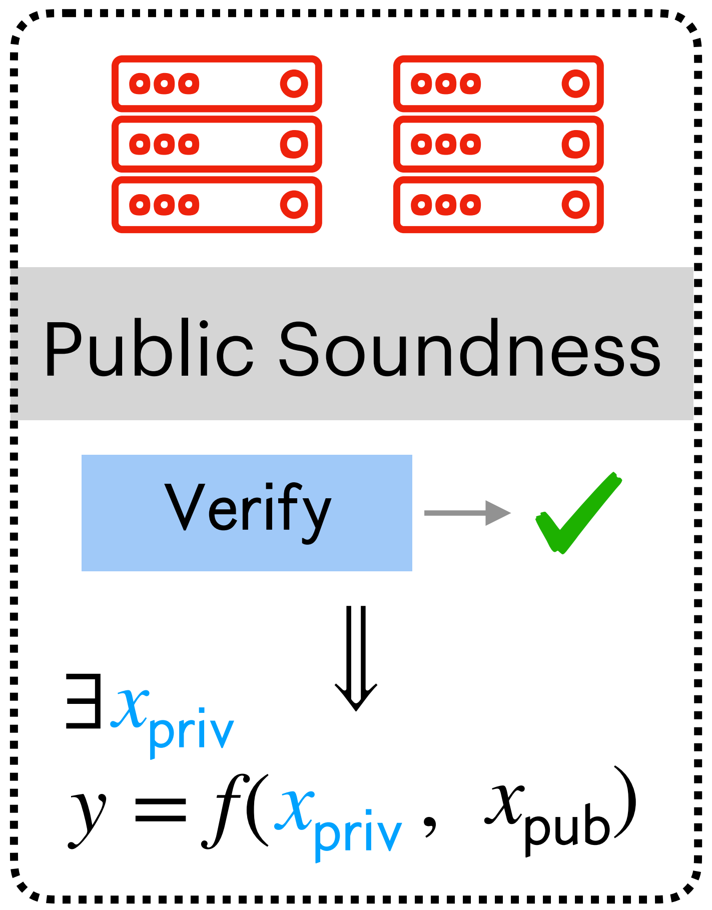


Local Soundness

Verify → ✓

$\Downarrow$

$y = f(x)$

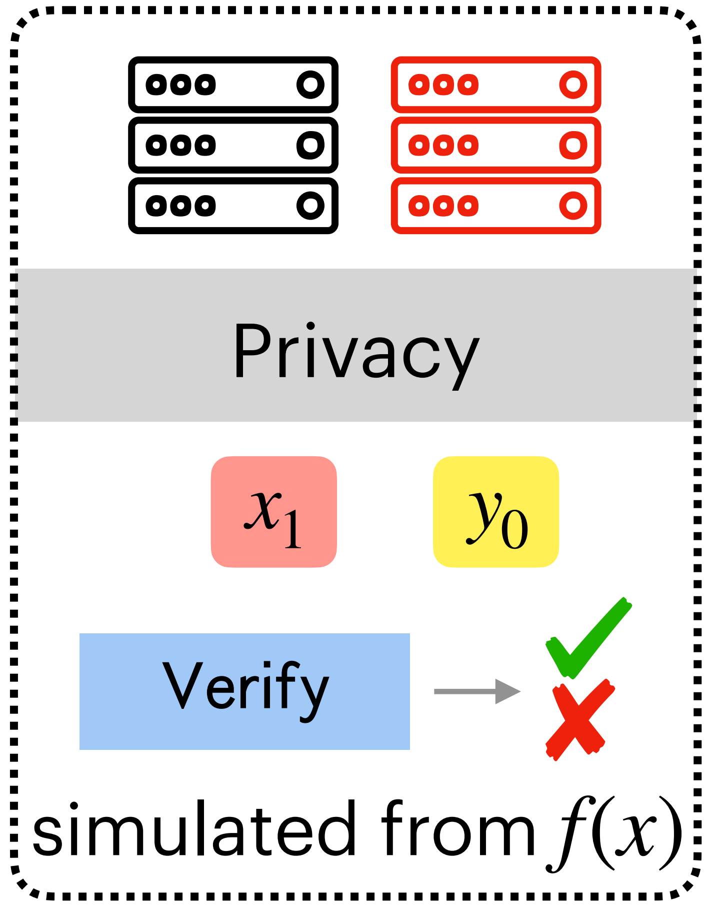


Public Soundness

Verify → ✓

$\exists x_{\mathrm{priv}}$ $\Downarrow$

$y = f(x_{\mathrm{priv}}, \ x_{\mathrm{pub}})$

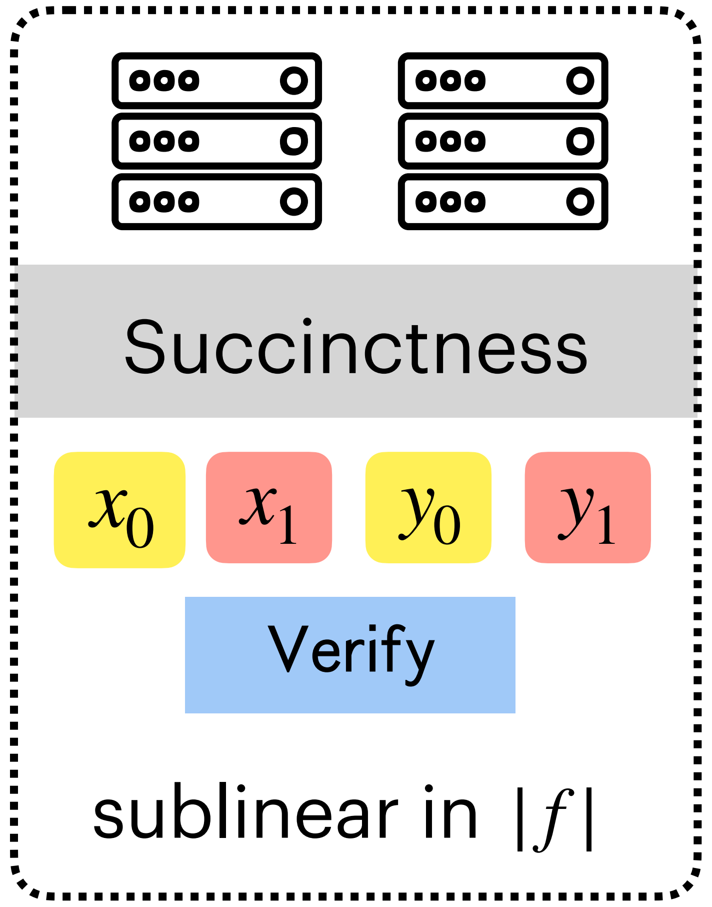


Privacy

x_1 y_0

Verify → ✓ ✗

simulated from $f(x)$



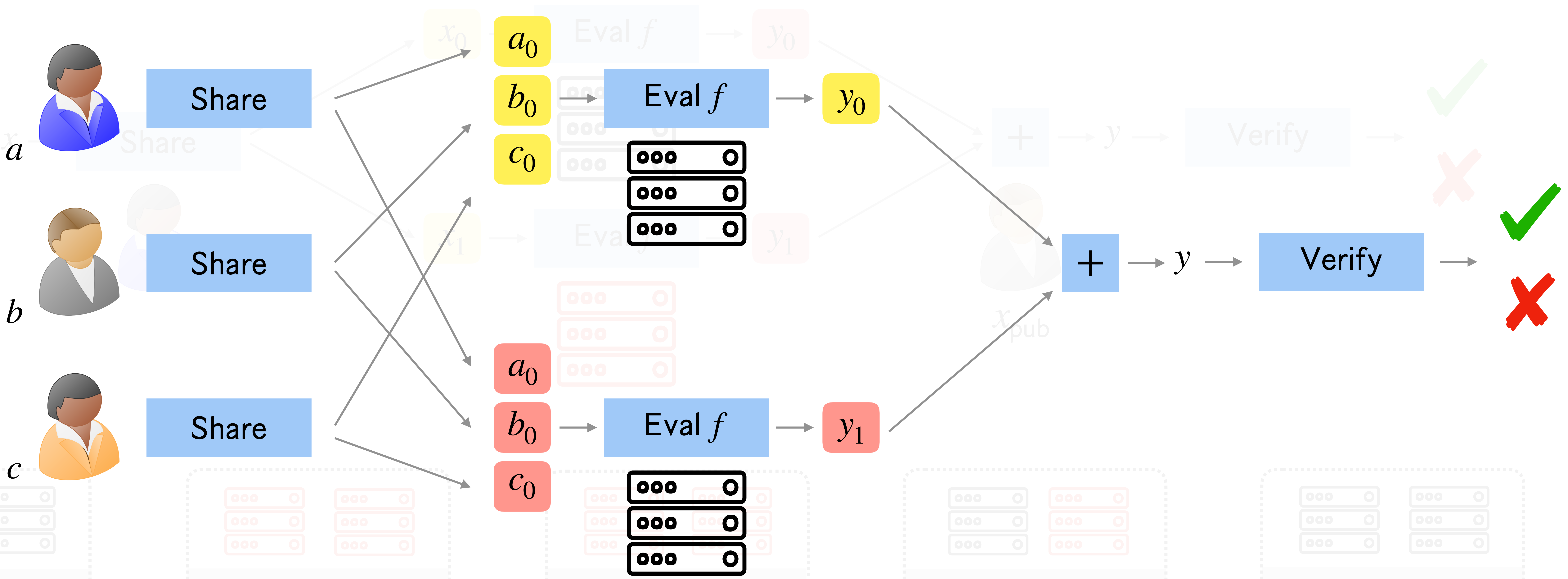
Succinctness

x_0 x_1 y_0 y_1

Verify

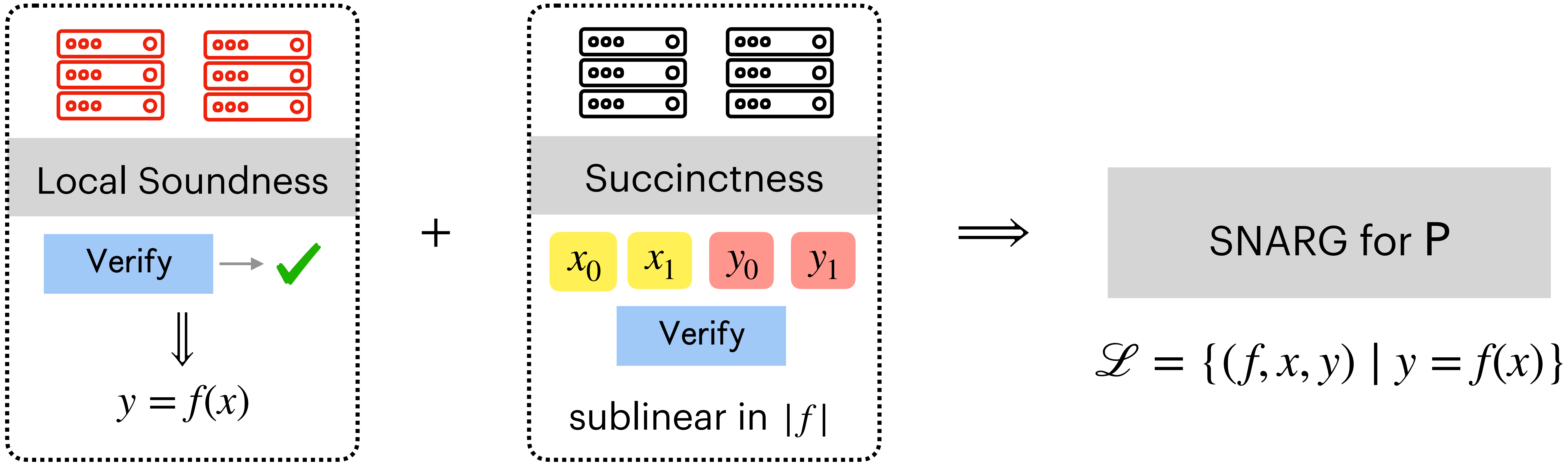
sublinear in $|f|$

HSS with Verifiable Evaluation (ve-HSS)



Multi-client HSS with verifiable evaluation

Implications of HSS with Verifiable Evaluation



Local Soundness

Verify → ✓

$\Downarrow$

$y = f(x)$

+

Succinctness

x_0 x_1 y_0 y_1

Verify

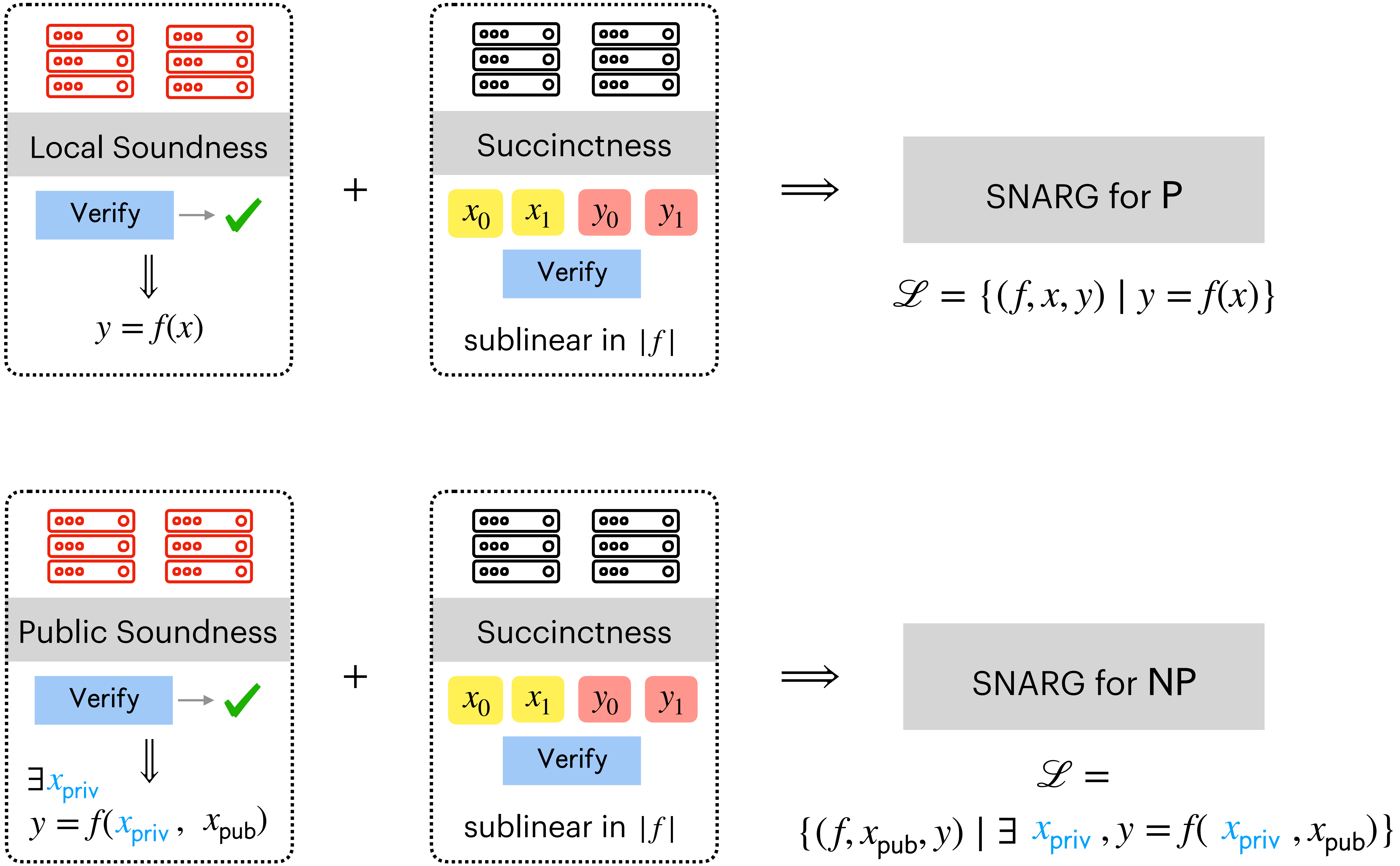
sublinear in $|f|$

$\Longrightarrow$

SNARG for P

$$\mathscr{L} = \{(f, x, y) \mid y = f(x)\}$$

Implications of HSS with Verifiable Evaluation



Local Soundness

Verify → ✓

$\Downarrow$

$y = f(x)$

+

Succinctness

x_0 x_1 y_0 y_1

Verify

sublinear in $|f|$

$\Longrightarrow$

SNARG for P

$\mathscr{L} = \{(f, x, y) \mid y = f(x)\}$

Public Soundness

Verify → ✓

$\Downarrow$

$\exists x_{\text{priv}}$

$y = f(x_{\text{priv}}, x_{\text{pub}})$

+

Succinctness

x_0 x_1 y_0 y_1

Verify

sublinear in $|f|$

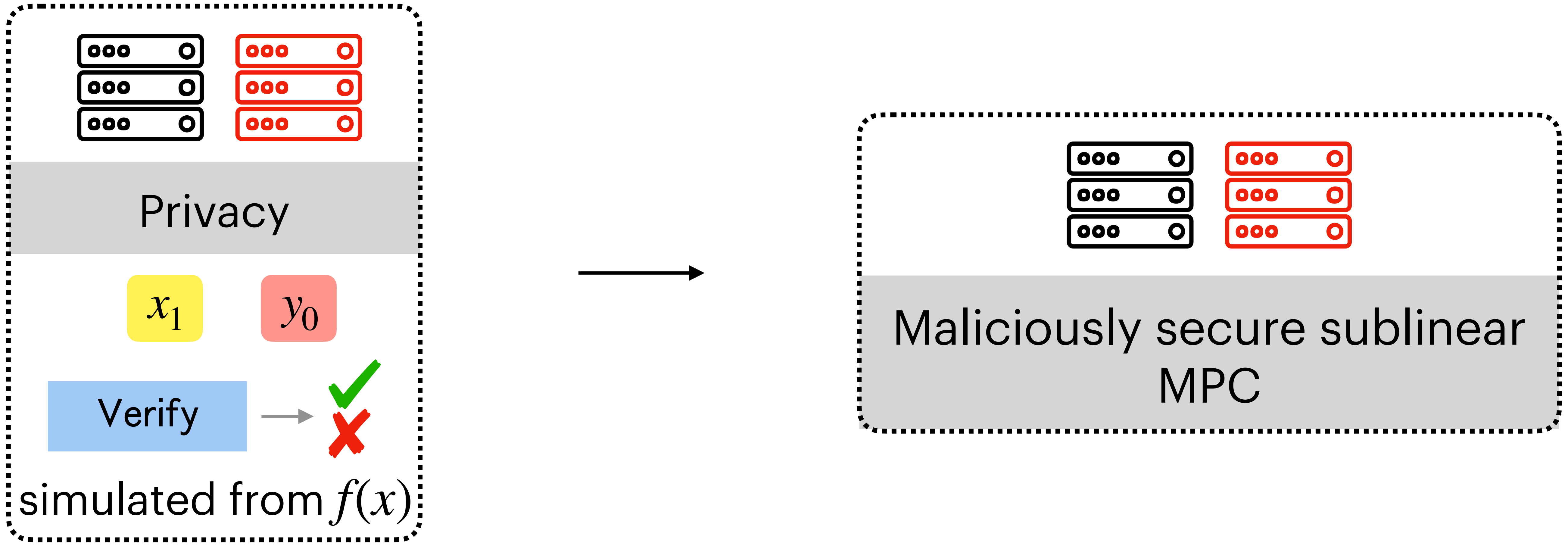
$\Longrightarrow$

SNARG for NP

$\mathscr{L} =$

$\{(f, x_{\text{pub}}, y) \mid \exists\ x_{\text{priv}}, y = f(x_{\text{priv}}, x_{\text{pub}})\}$

Implications of HSS with Verifiable Evaluation



Privacy

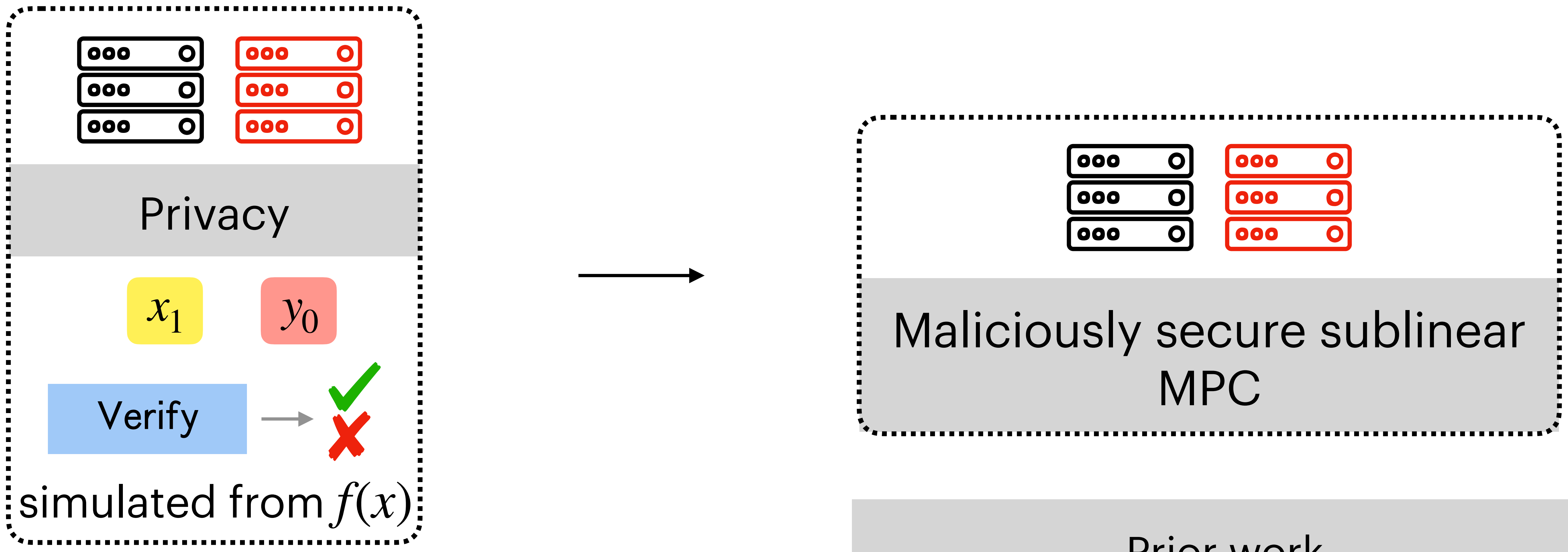
x_1 y_0

Verify → ✓ ✗

simulated from $f(x)$

Maliciously secure sublinear MPC

Implications of HSS with Verifiable Evaluation



Privacy

x_1 y_0

Verify → ✓ ✗

simulated from $f(x)$

Maliciously secure sublinear MPC

Prior work
[Abram-Damgård-Orlandi-Scholl 22]

Our Results

HSS schemes with verifiable evaluation (ve-HSS) that only make black-box use of cryptography

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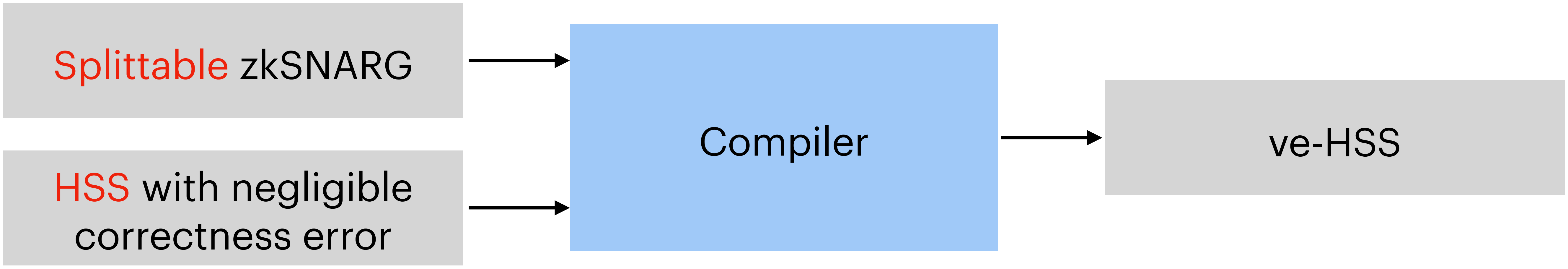
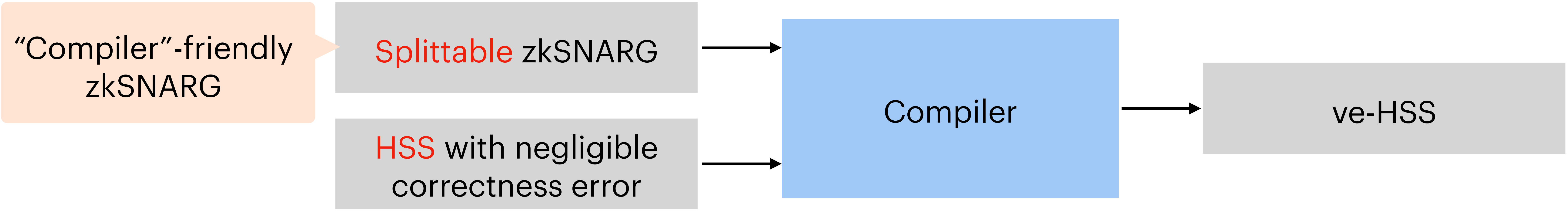


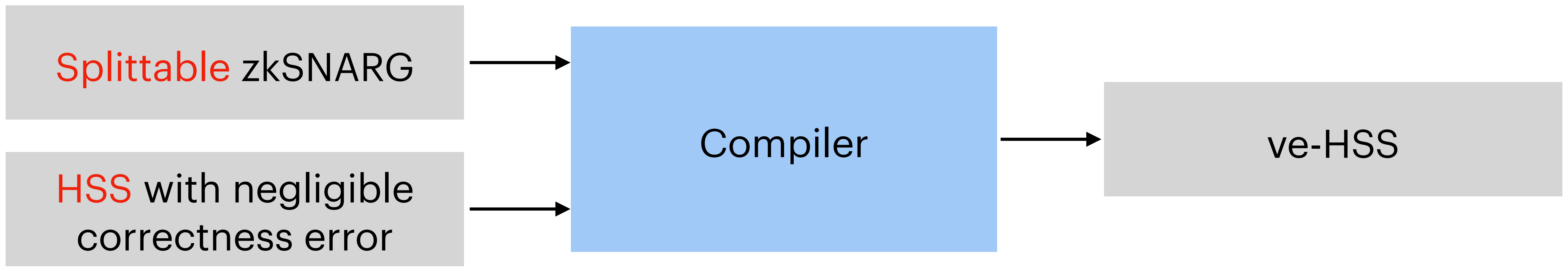
Our Results

HSS schemes with verifiable evaluation (ve-HSS) that only make black-box use of cryptography


"Compiler"-friendly zkSNARG
Splittable zkSNARG
HSS with negligible correctness error
Compiler
ve-HSS

Our Results

HSS schemes with verifiable evaluation (ve-HSS) that only make black-box use of cryptography



	HSS	Function Class
Splittable zkSNARG Generic Bilinear Group Model [Groth 16]	DCR [Orlandi-Scholl-Yakoubov 21] [Roy-Singh 21] Class Groups [Abram-Damgård-Orlandi-Scholl 22] LWE [Boyle-Kohl-Scholl 19]	NC^1
	FHE [Dodis-Halevi-Rothblum-Wichs 16] [Chilloti-Orsini-Scholl-Smart-Leeuwen 22] iO + OWF [Boyle-Gilboa-Ishai 15]	P/poly

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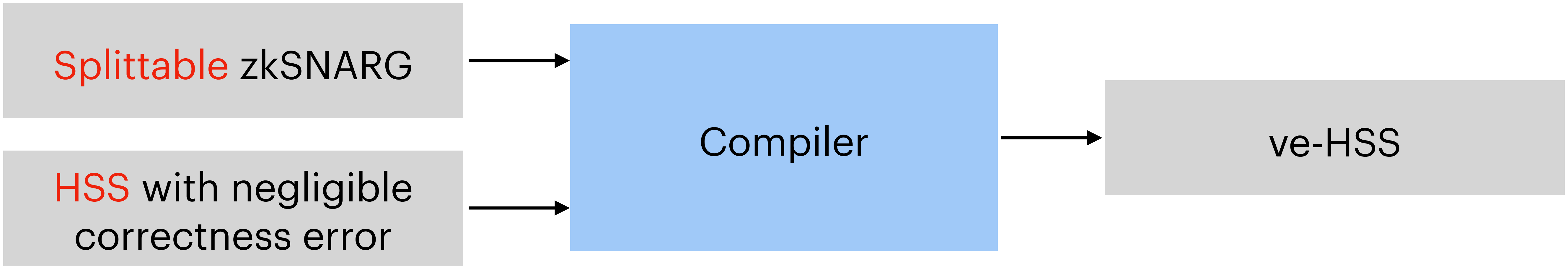


HSS	Function Class
DCR [Orlandi-Scholl-Yakoubov 21] [Roy-Singh 21] Class Groups [Abram-Damgård-Orlandi-Scholl 22] LWE [Boyle-Kohl-Scholl 19]	SIMD NC^1
FHE [Dodis-Halevi-Rothblum-Wichs 16] [Chilloti-Orsini-Scholl-Smart-Leeuwen 22] iO + OWF [Boyle-Gilboa-Ishai 15]	SIMD P/poly

Splittable zkSNARG

Subgroup Decision Assumption

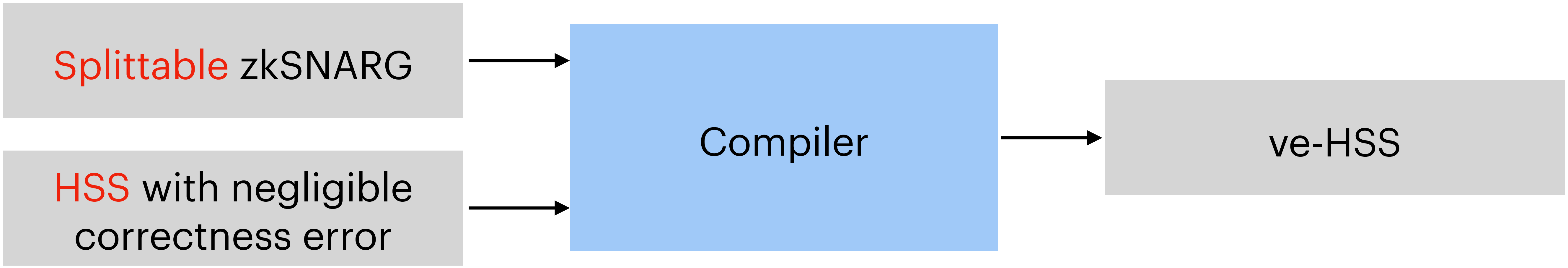
[Waters-Wu 22]

BARG → zkBARG

Black-box in cryptography

Our Results

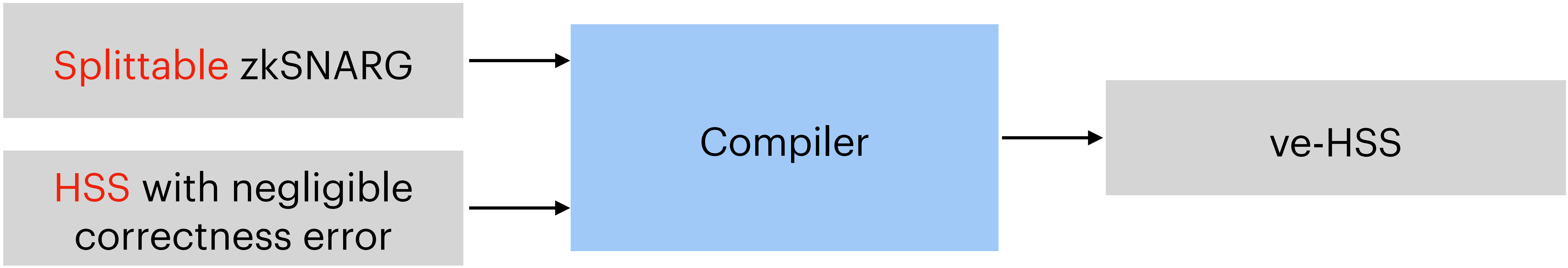
HSS schemes with verifiable evaluation (ve-HSS) that only make black-box use of cryptography



Multi-client HSS schemes with verifiable evaluation that only make black-box use of cryptography

Our Results

HSS schemes with verifiable evaluation (ve-HSS) that only make black-box use of cryptography



Multi-client HSS schemes with verifiable evaluation that only make black-box use of cryptography

Applications

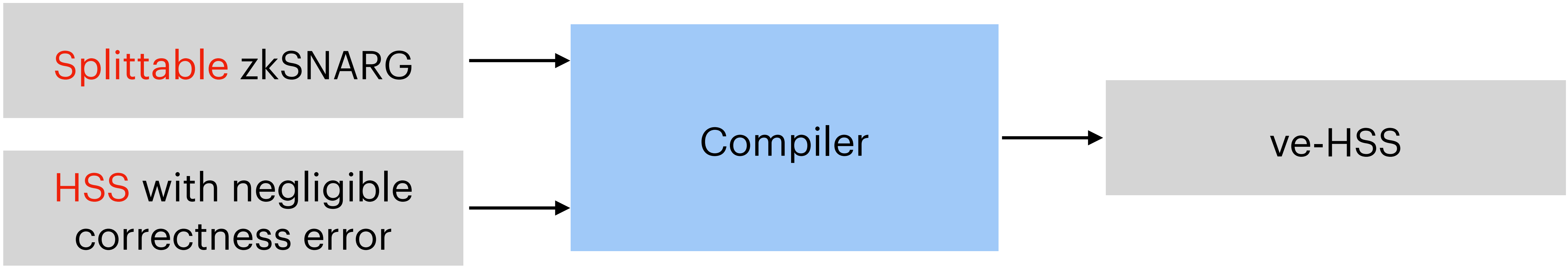
Private and verifiable delegation of computation

Private delegation of zkSNARG computation

Black-box in cryptography!

Our Results

HSS schemes with verifiable evaluation (ve-HSS) that only make black-box use of cryptography



Splittable zkSNARG

HSS with negligible correctness error

Compiler

ve-HSS

Multi-client HSS schemes with verifiable evaluation that only make black-box use of cryptography

Applications

Private and verifiable delegation of computation

Private delegation of zkSNARG computation

Black-box in cryptography!

Strawman Approach

[Goldreich-Micali-Wigderson 87] Paradigm

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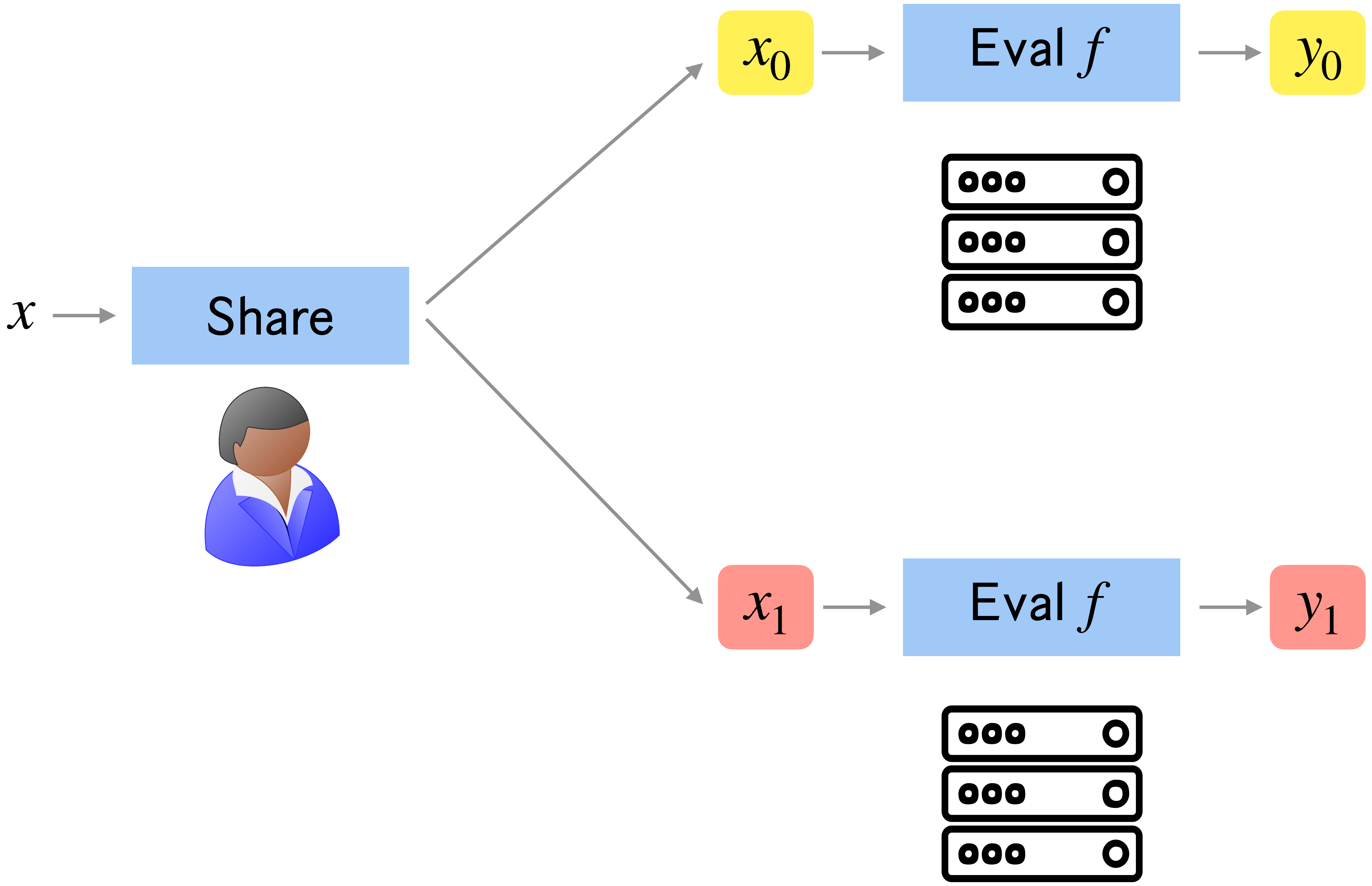
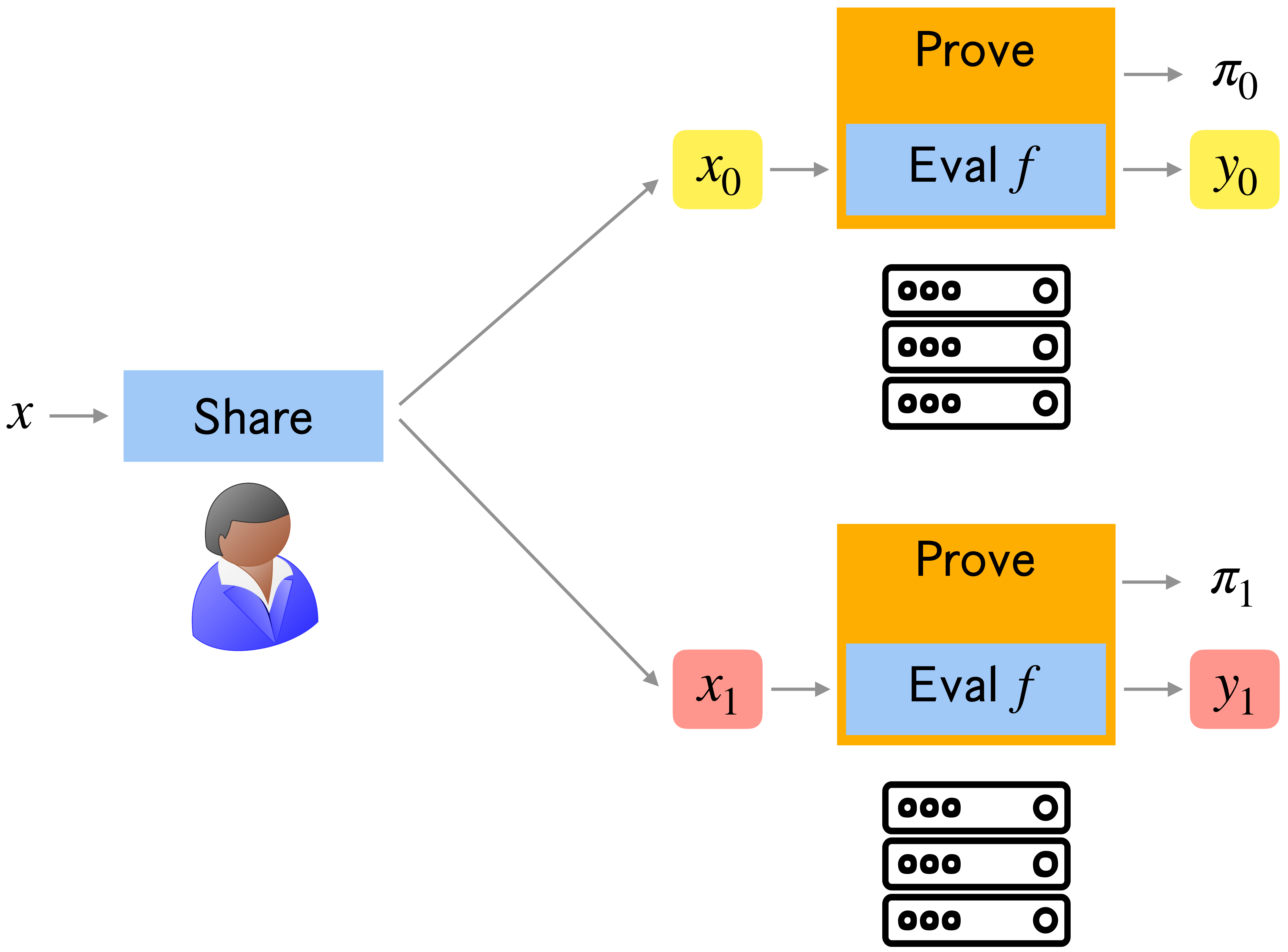


Strawman Approach

[Goldreich-Micali-Wigderson 87] Paradigm



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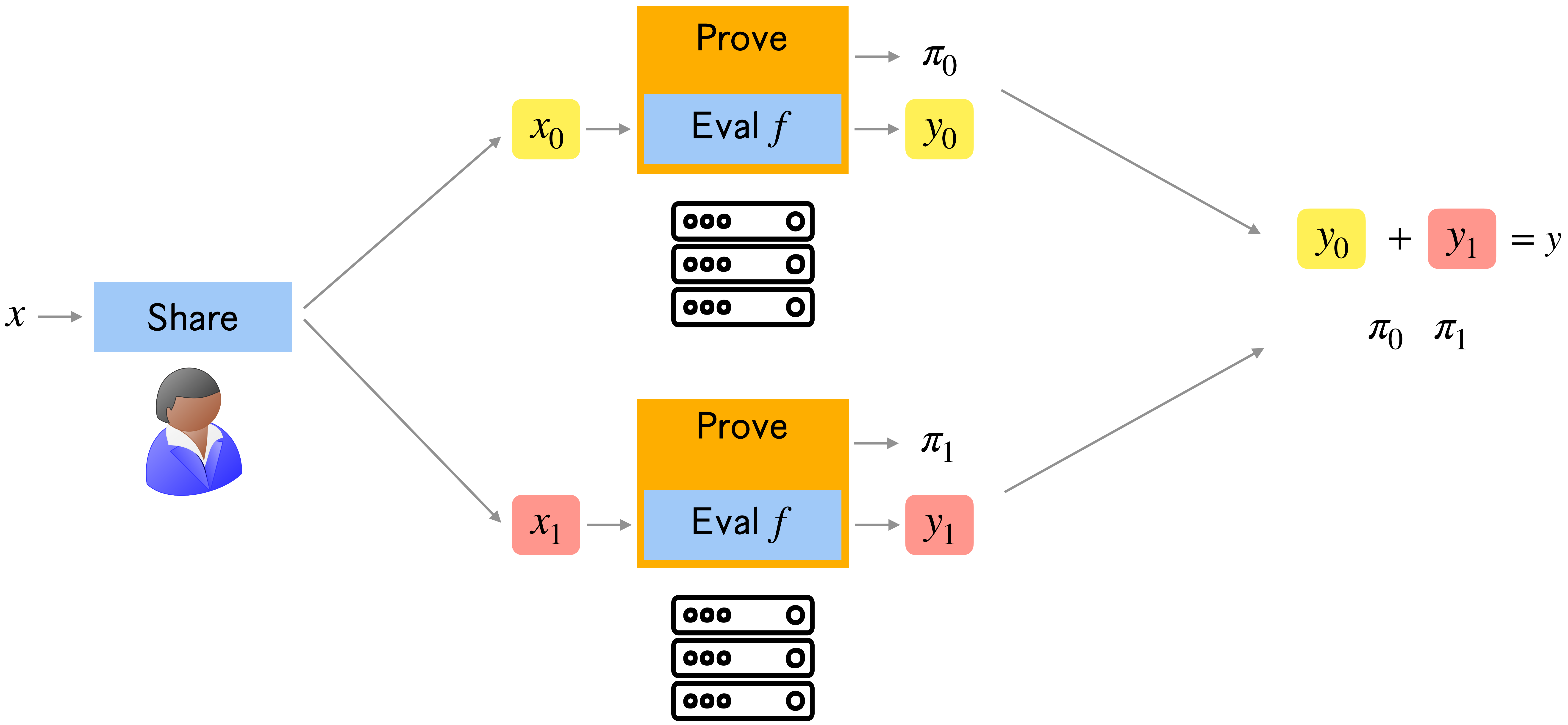

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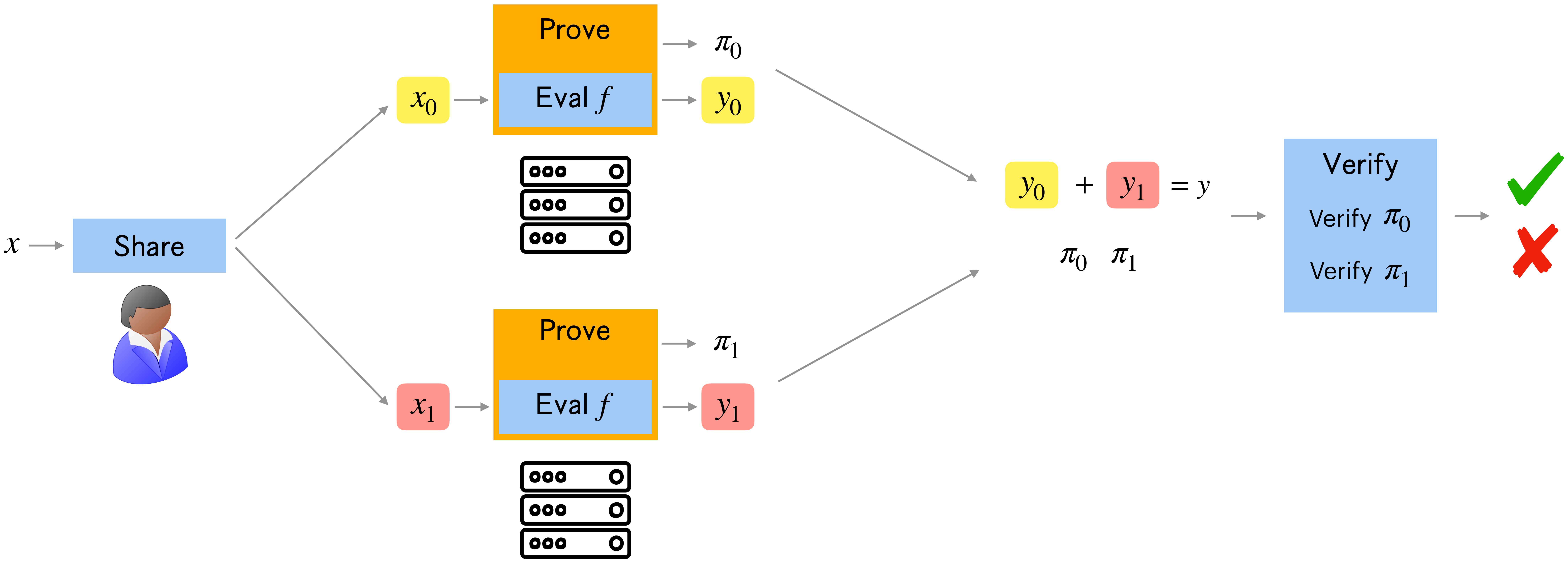

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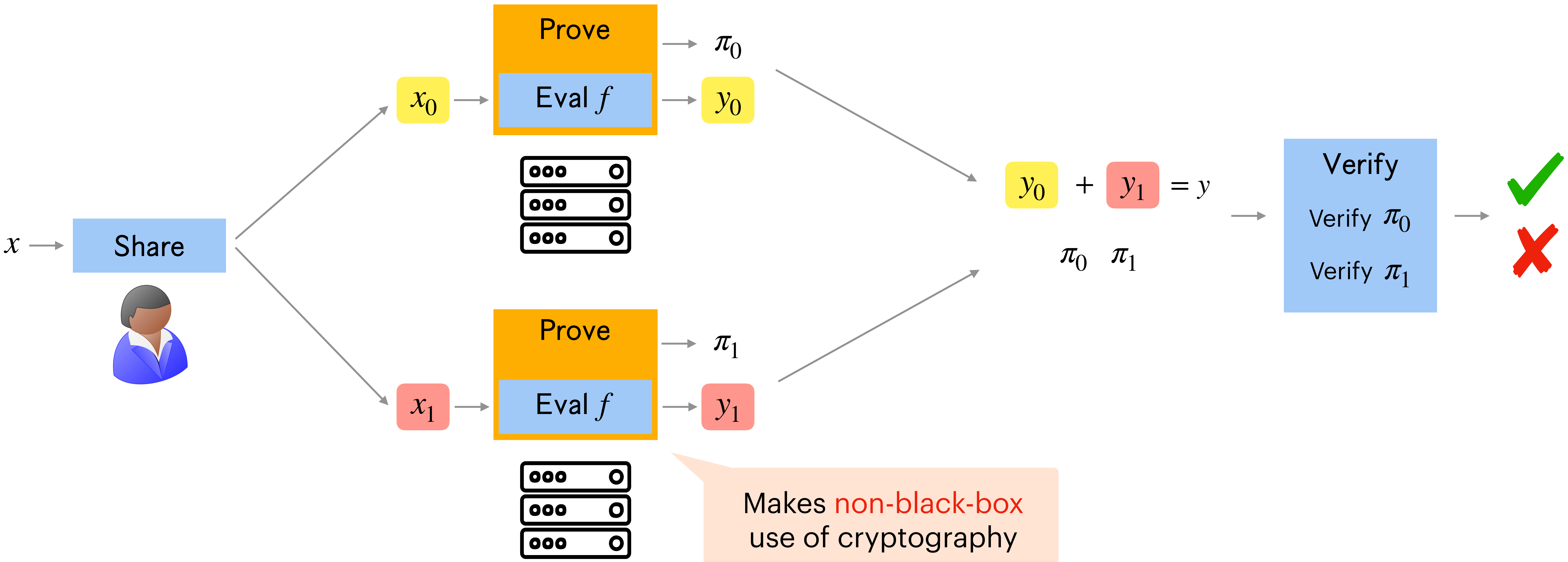
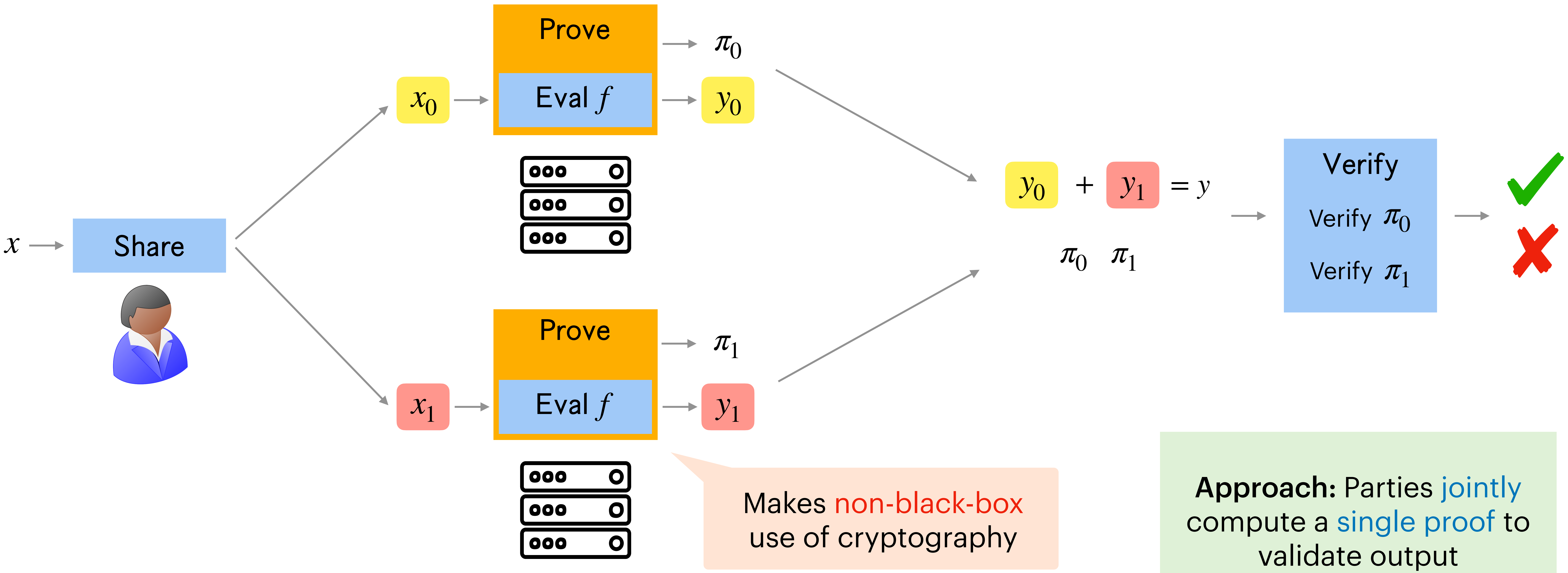


Makes non-black-box use of cryptography

Strawman Approach

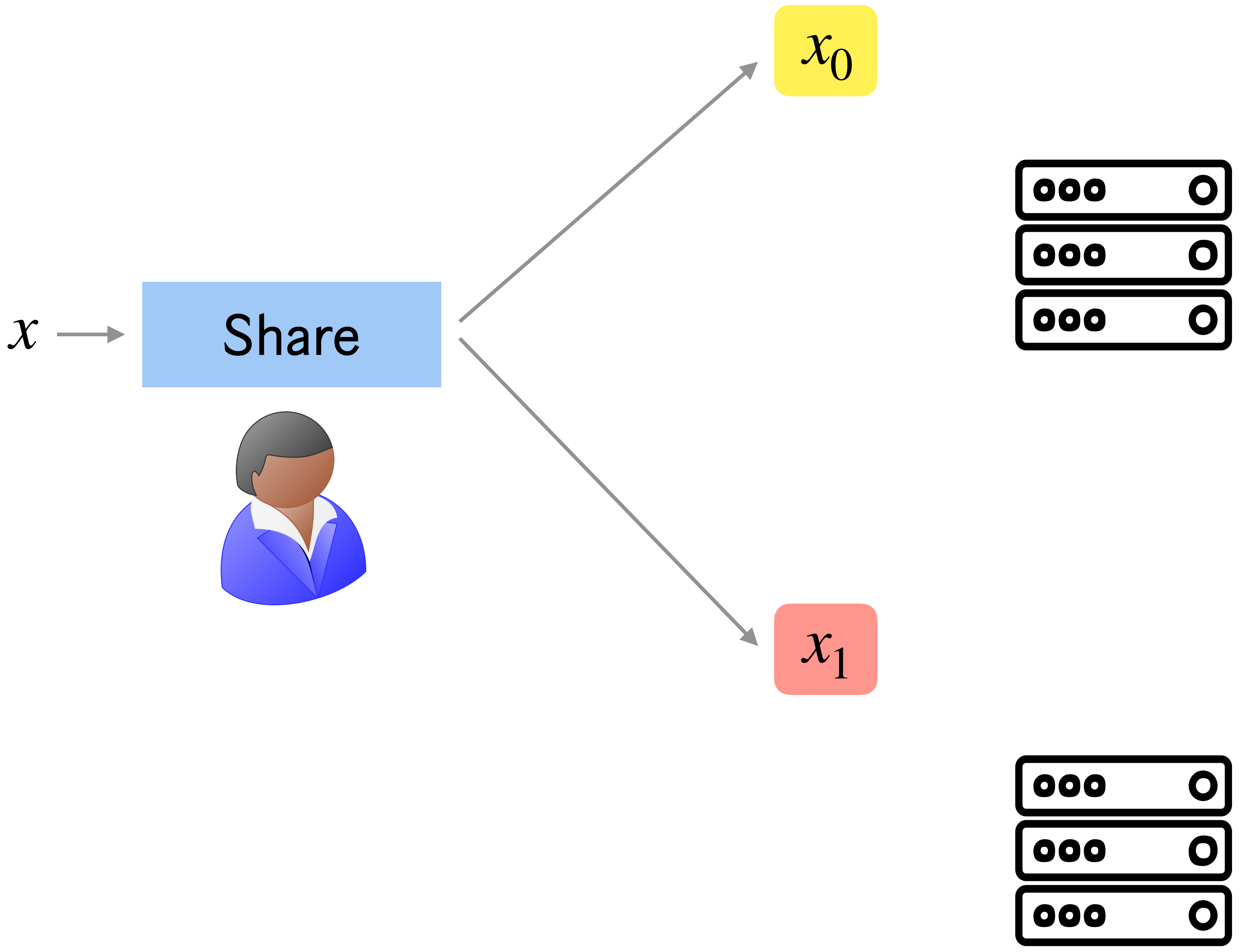
[Goldreich-Micali-Wigderson 87] Paradigm



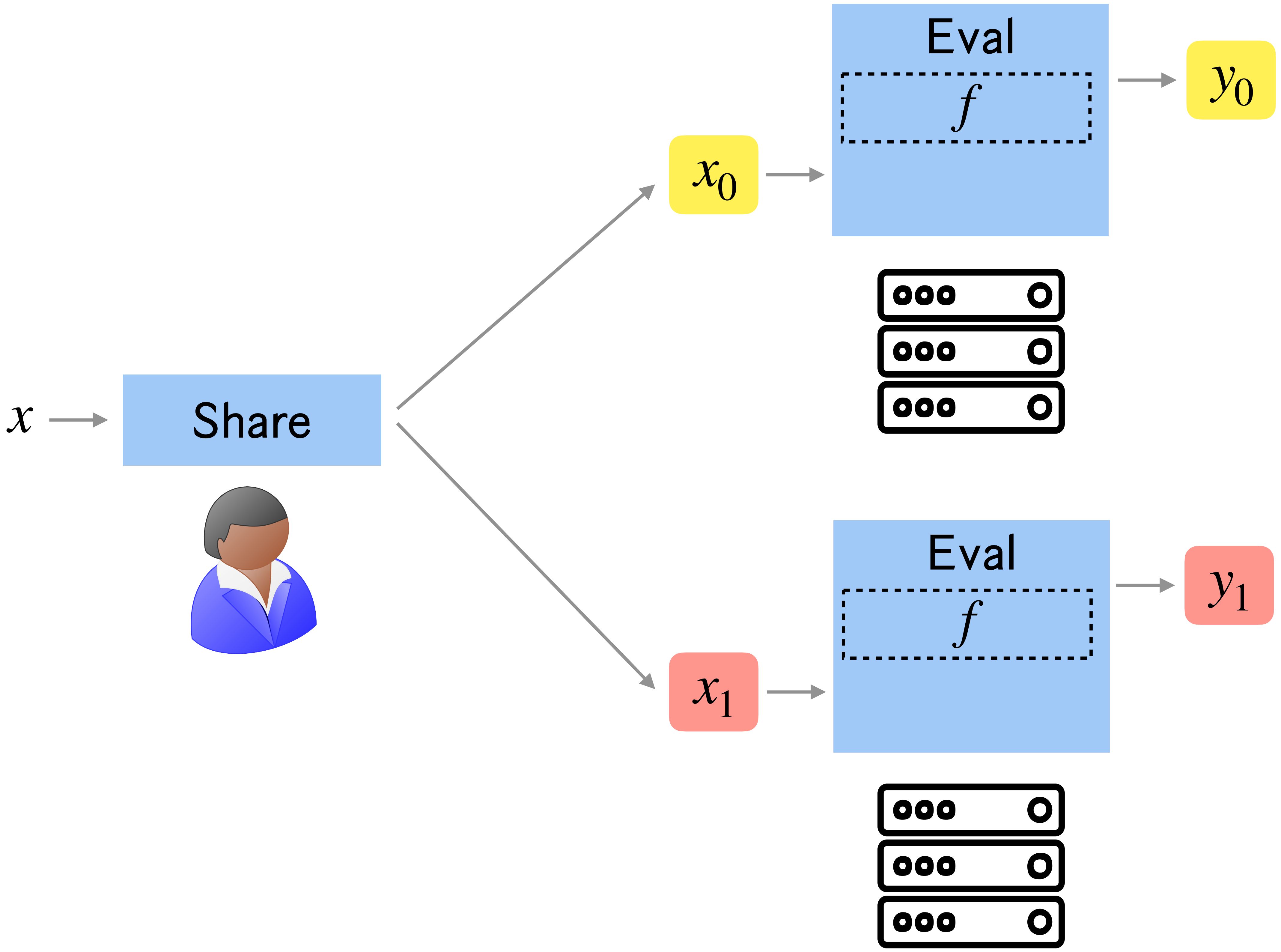
Makes non-black-box use of cryptography

Approach: Parties jointly compute a single proof to validate output

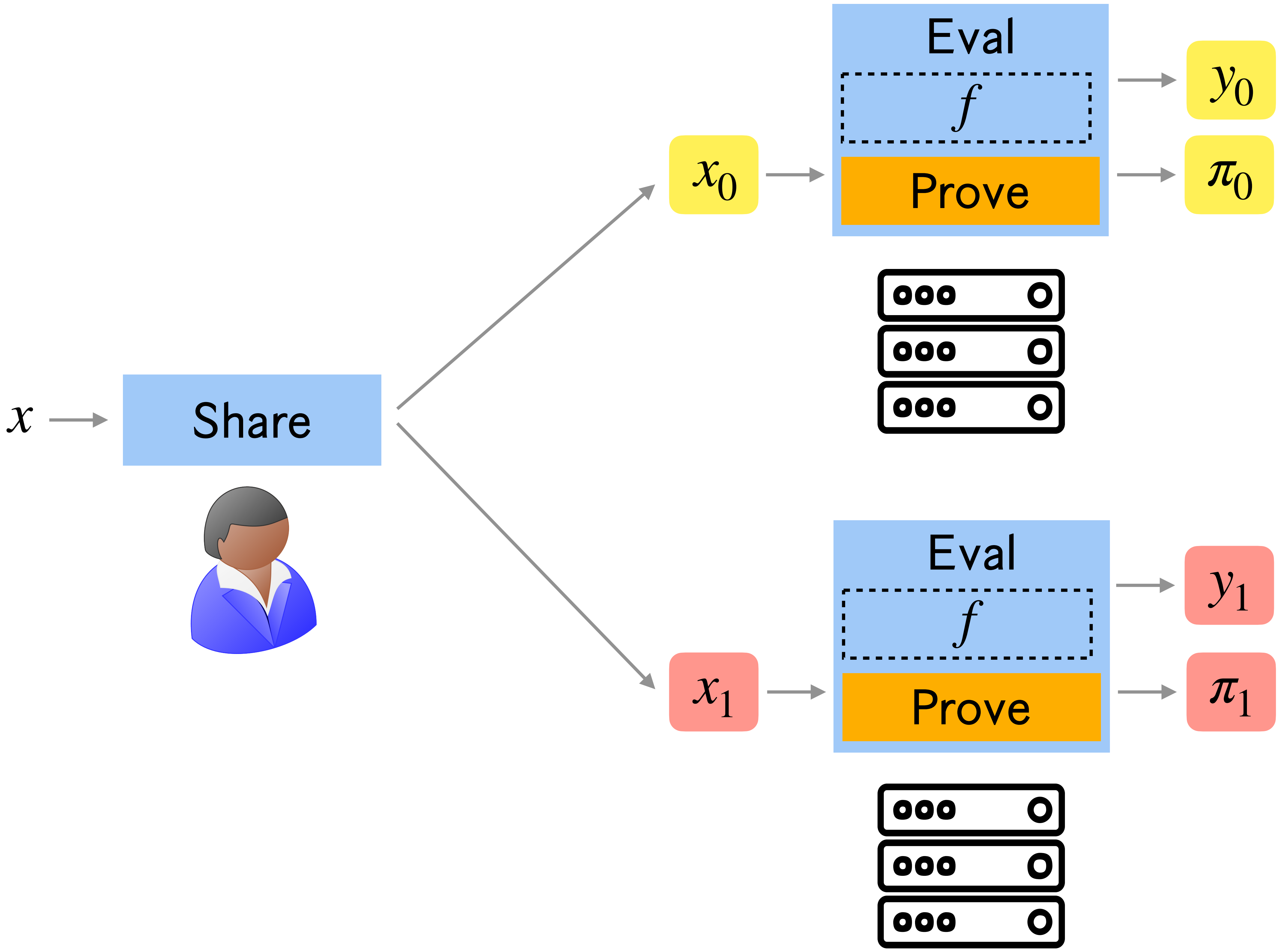
Jointly Computing A Proof



Jointly Computing A Proof



Jointly Computing A Proof



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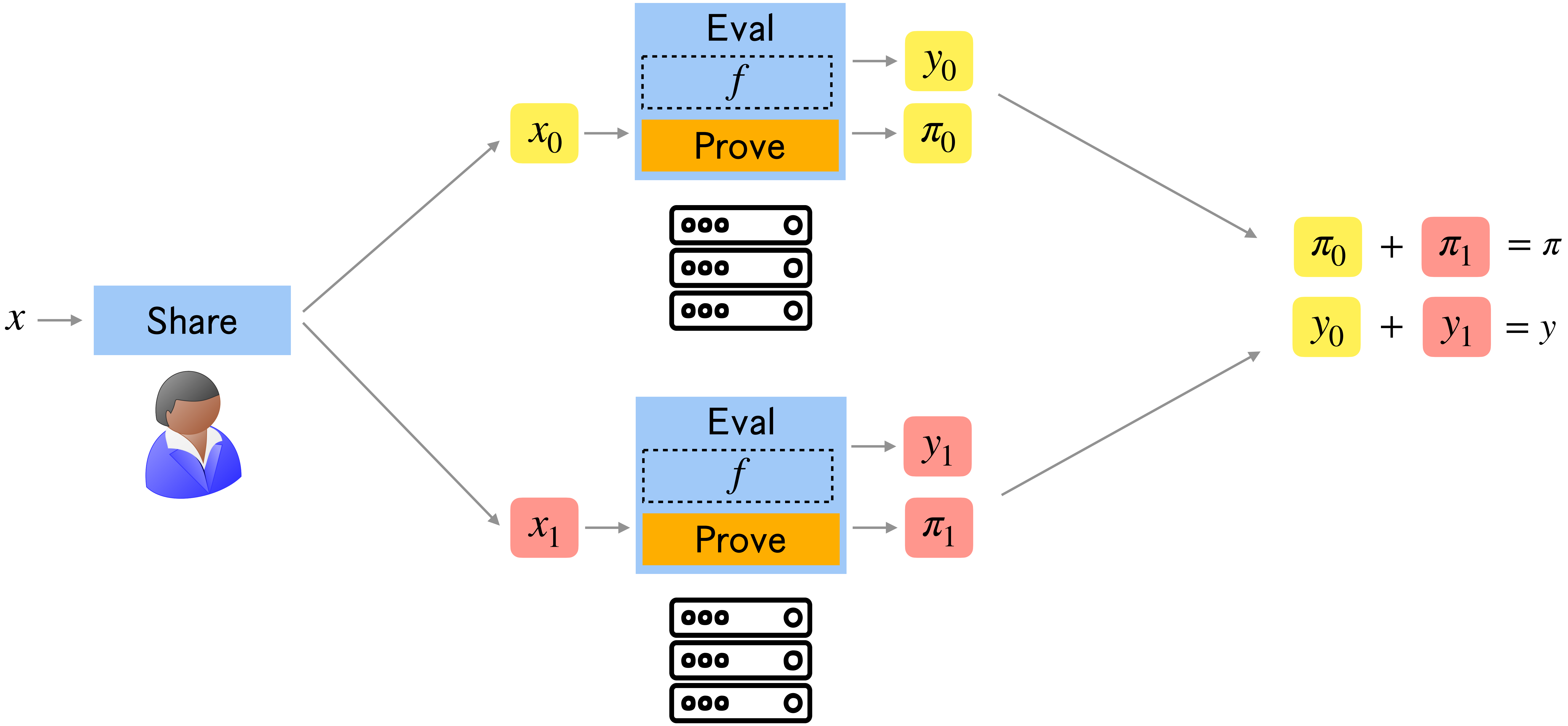

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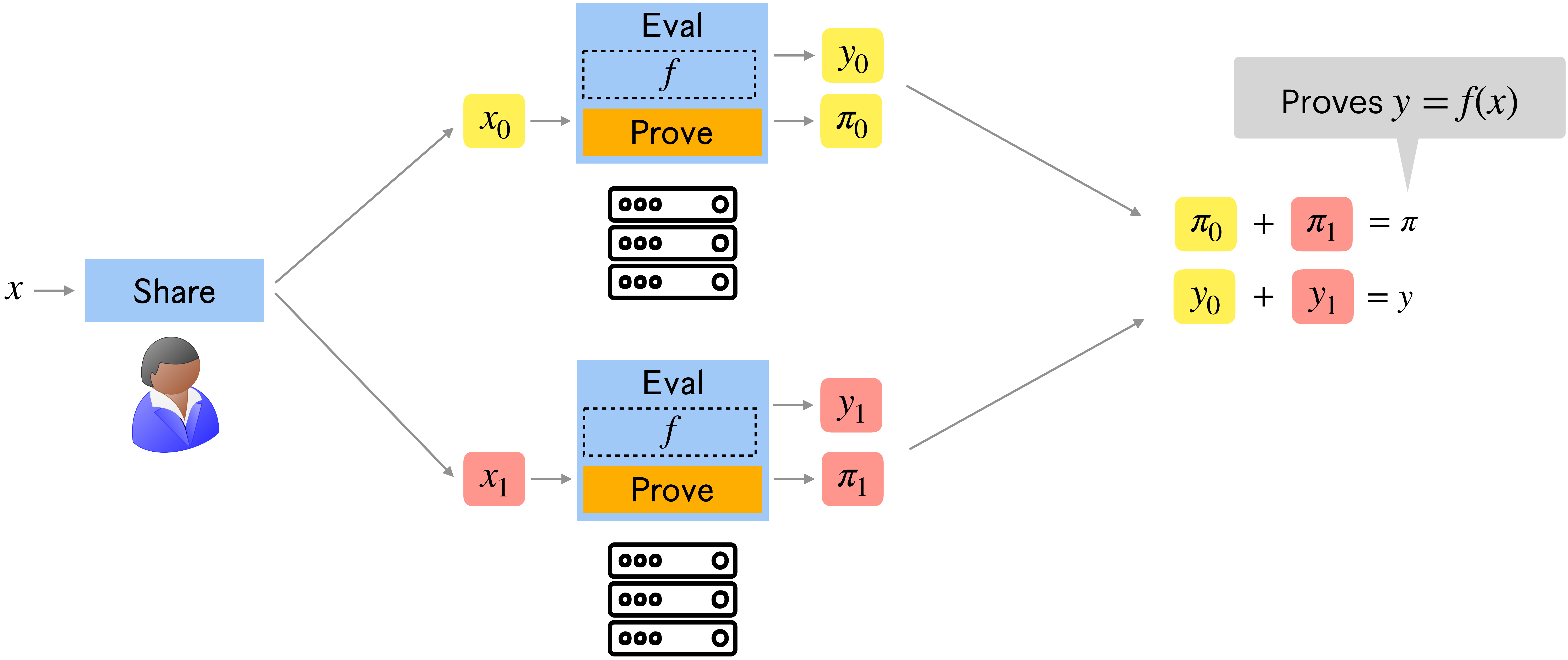

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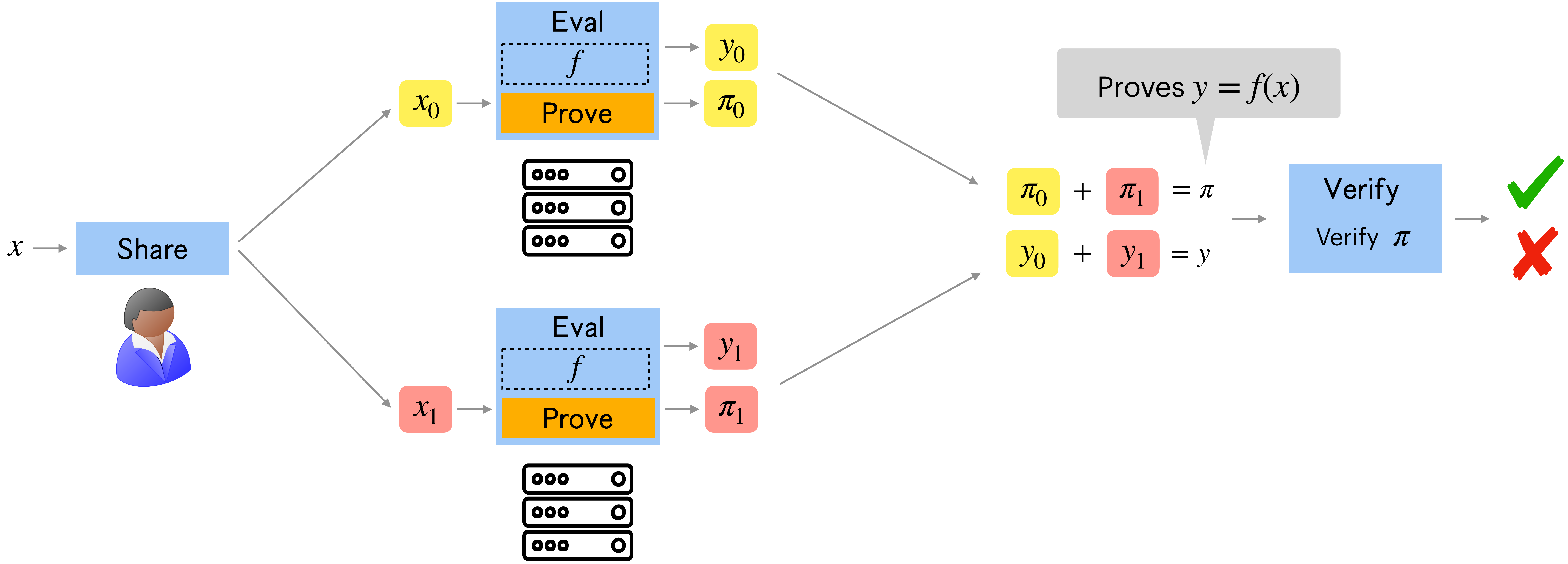

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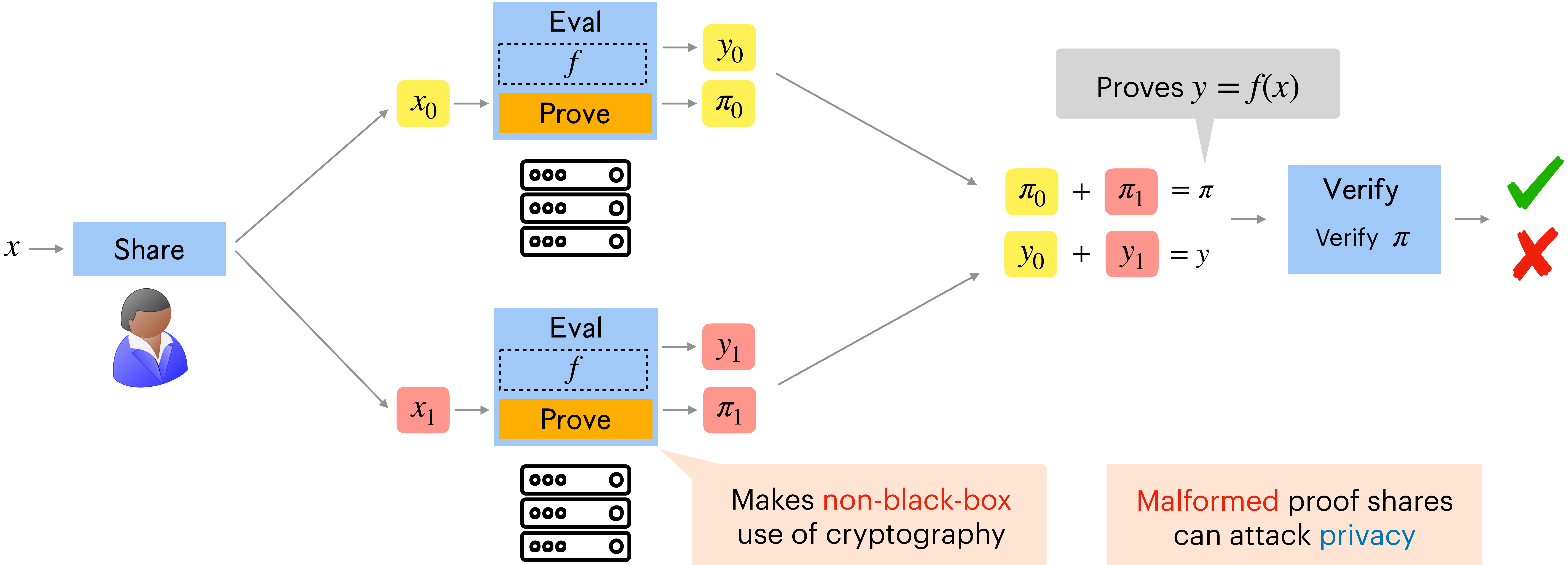
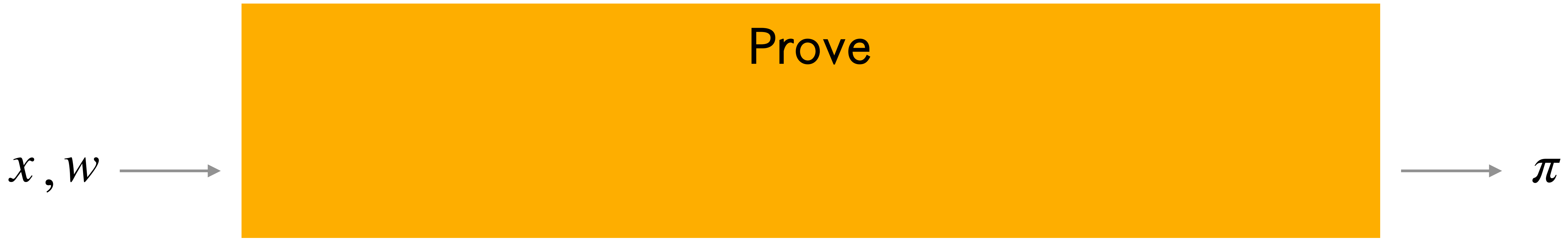


Proves $y = f(x)$

Makes non-black-box use of cryptography

Malformed proof shares can attack privacy

Splittable zkSNARG



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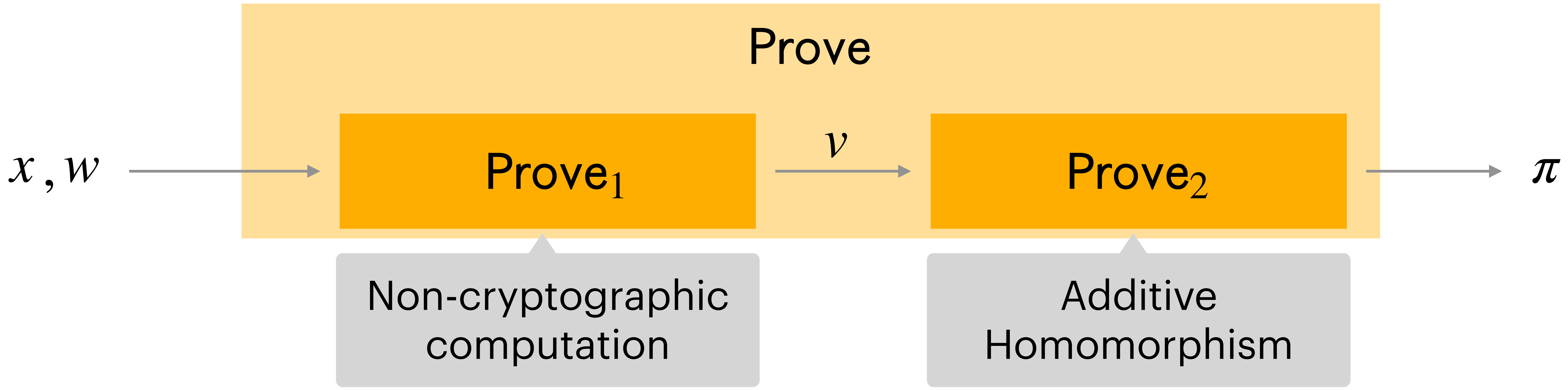

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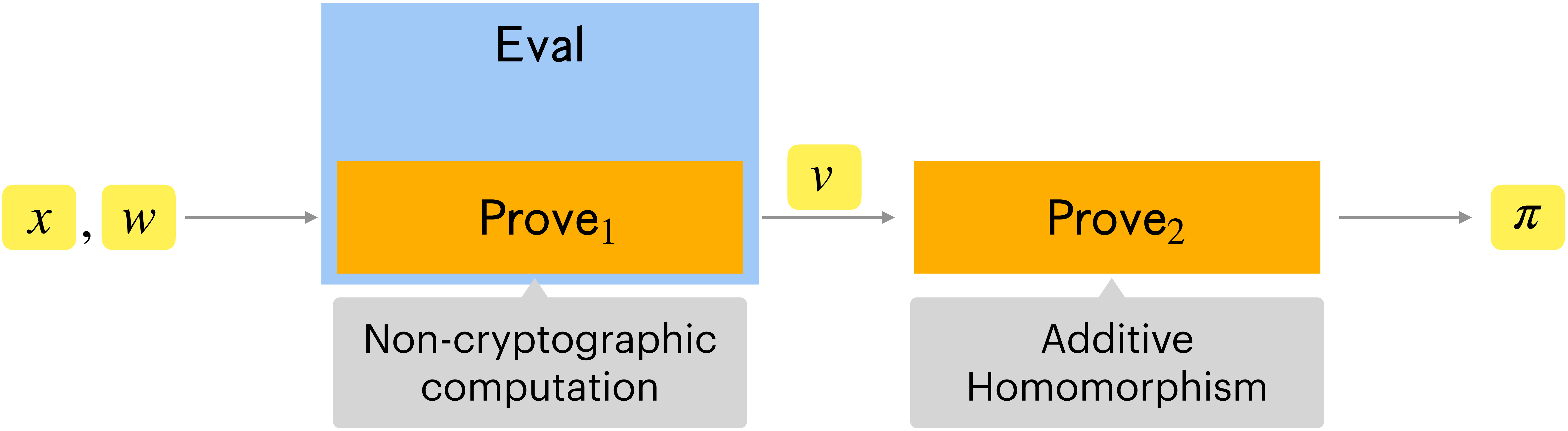

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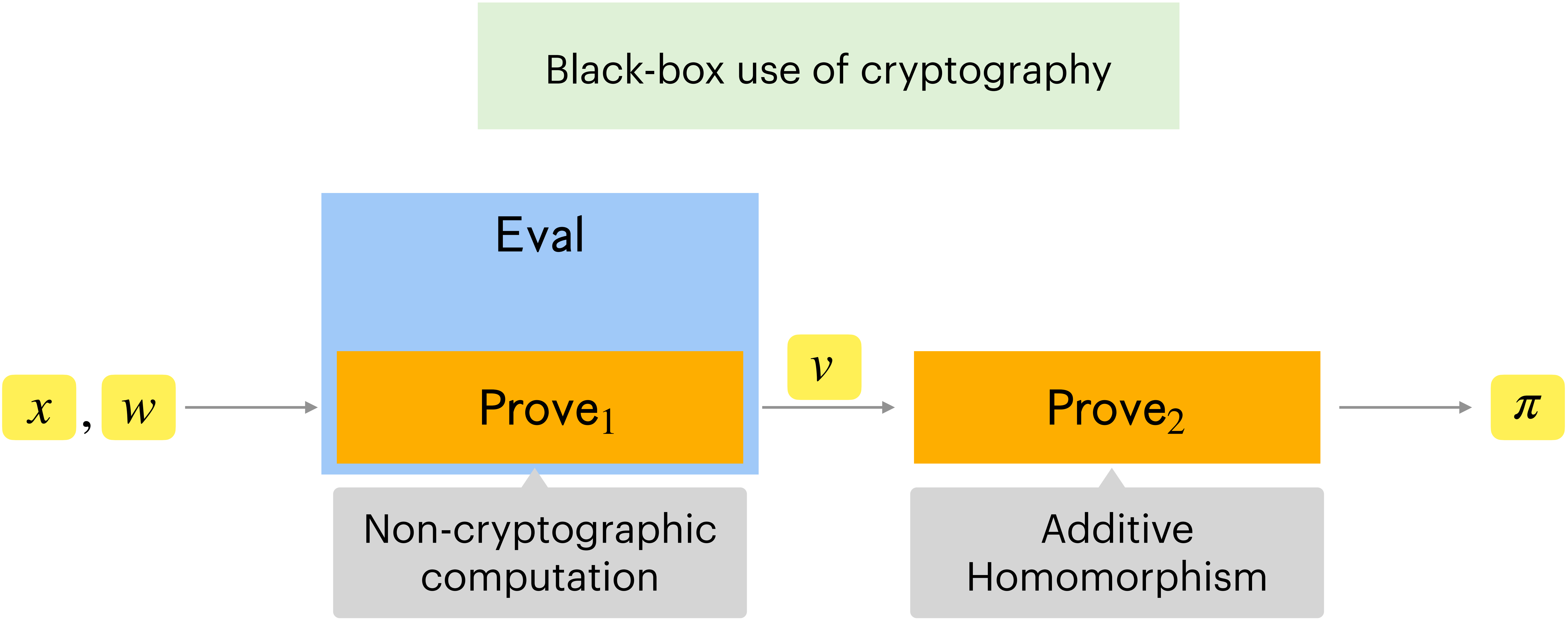
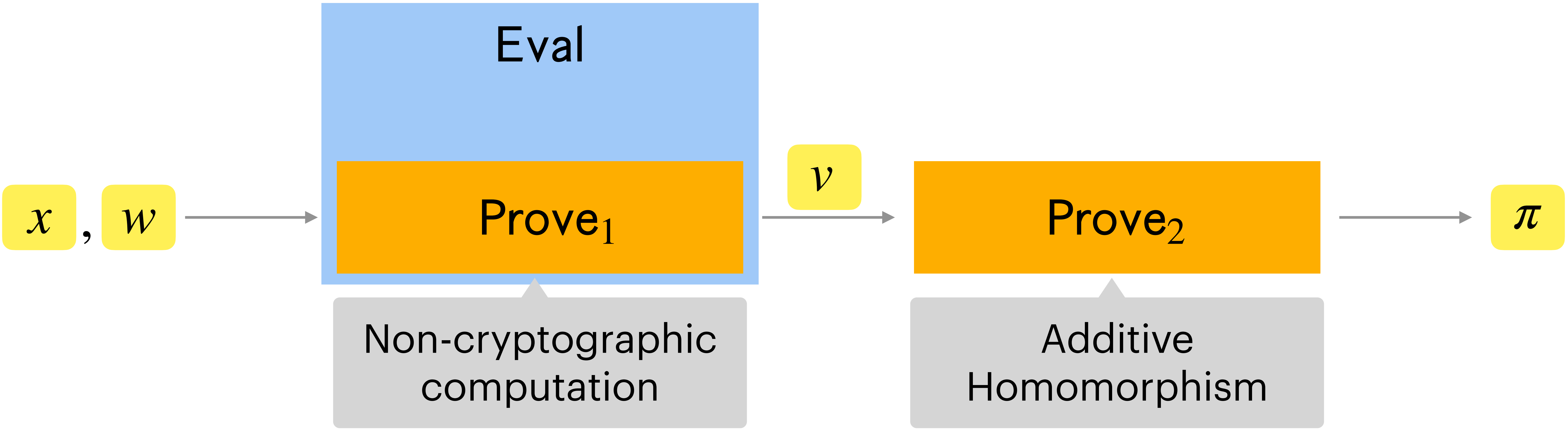


Splittable zkSNARG

Black-box use of cryptography



x, w

Eval

Prove$_1$

v

Prove$_2$

π

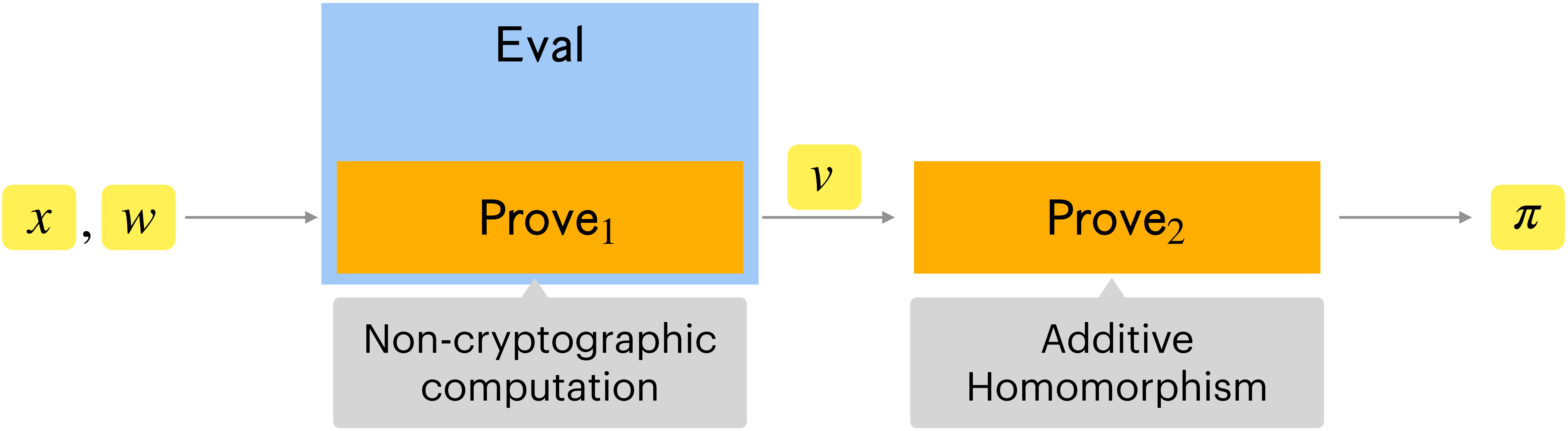
Non-cryptographic computation

Additive Homomorphism

Eval supports a limited class of functions $\Longrightarrow$ Prove$_1$ must be "simple" to compute (low depth circuit)

Splittable zkSNARG

Black-box use of cryptography



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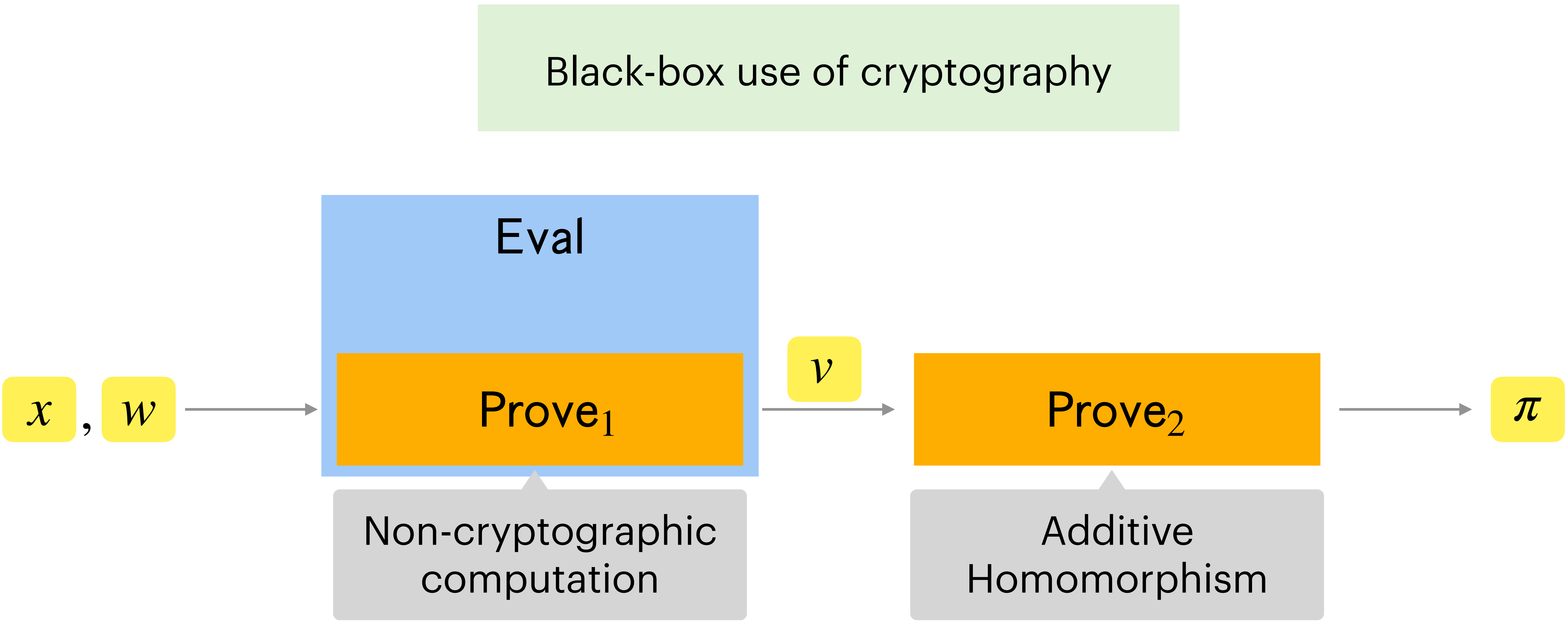
Do such zkSNARGs exist?

Splittable zkSNARG

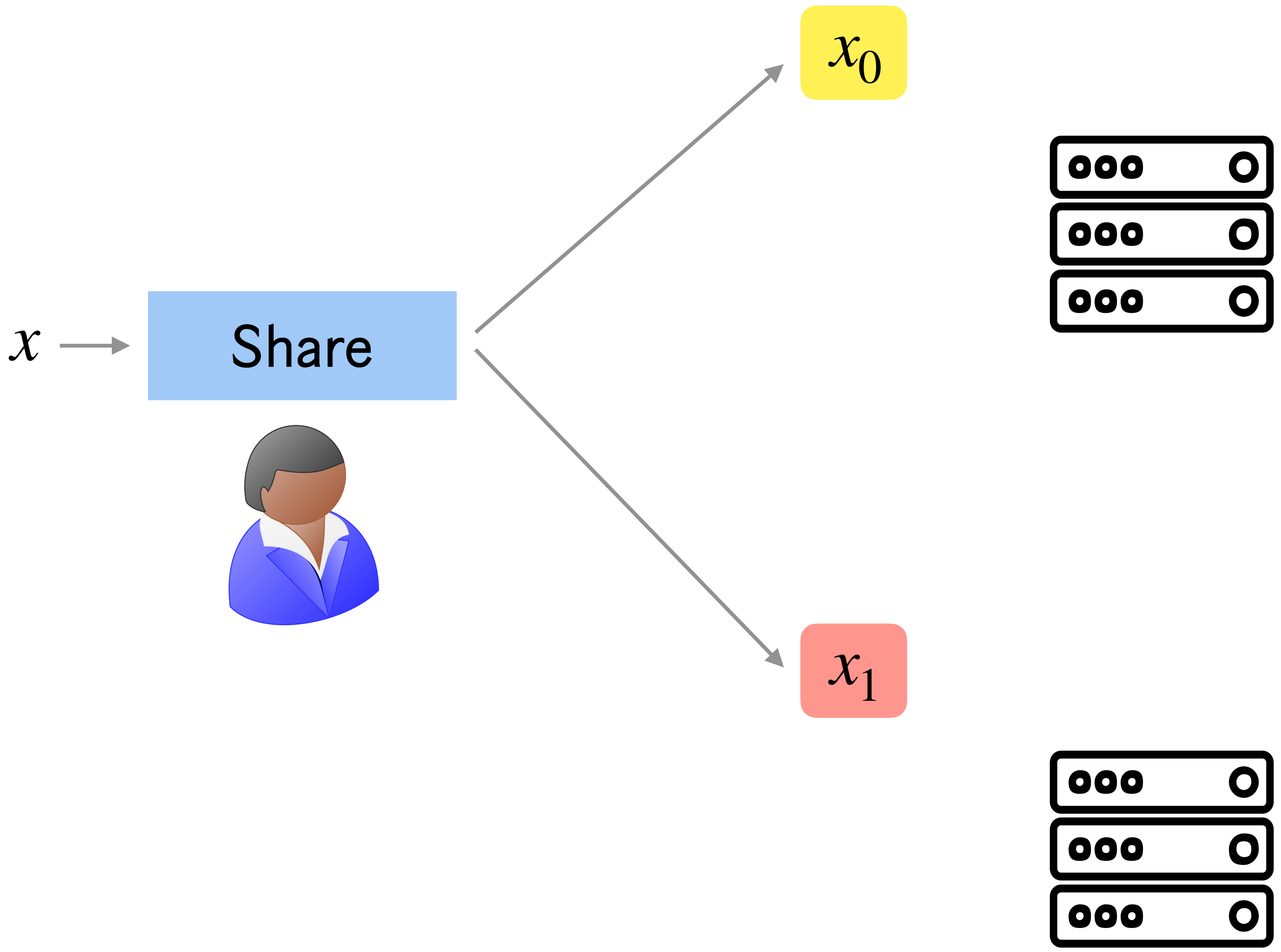


Eval supports a limited class of functions ⟹ Prove$_1$ must be “simple” to compute (low depth circuit)

Do such zkSNARGs exist?

Yes!
[Groth16], zkBARG from [Waters-Wu 22]

HSS with Verifiable Evaluation From Splittable zkSNARG



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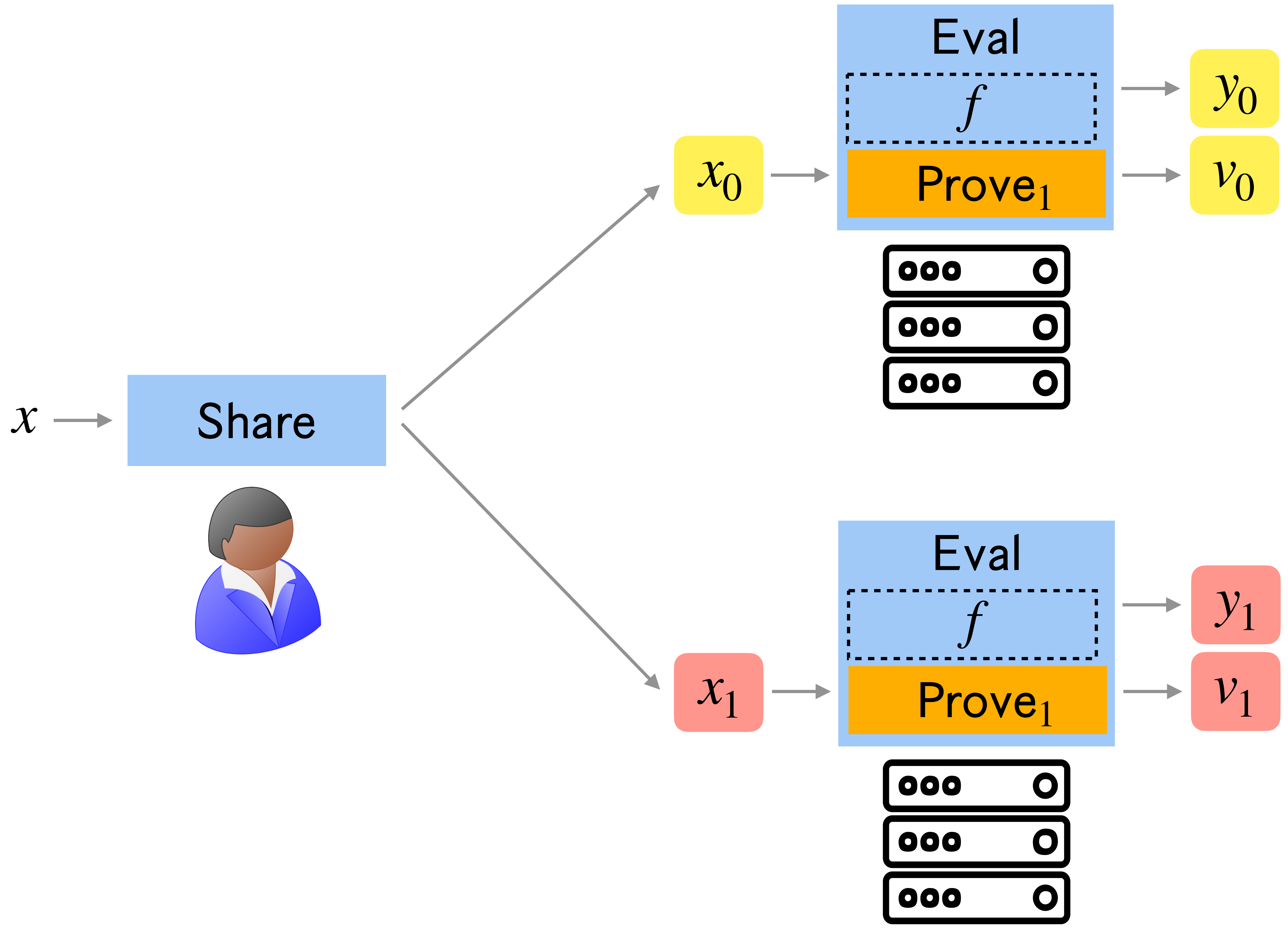

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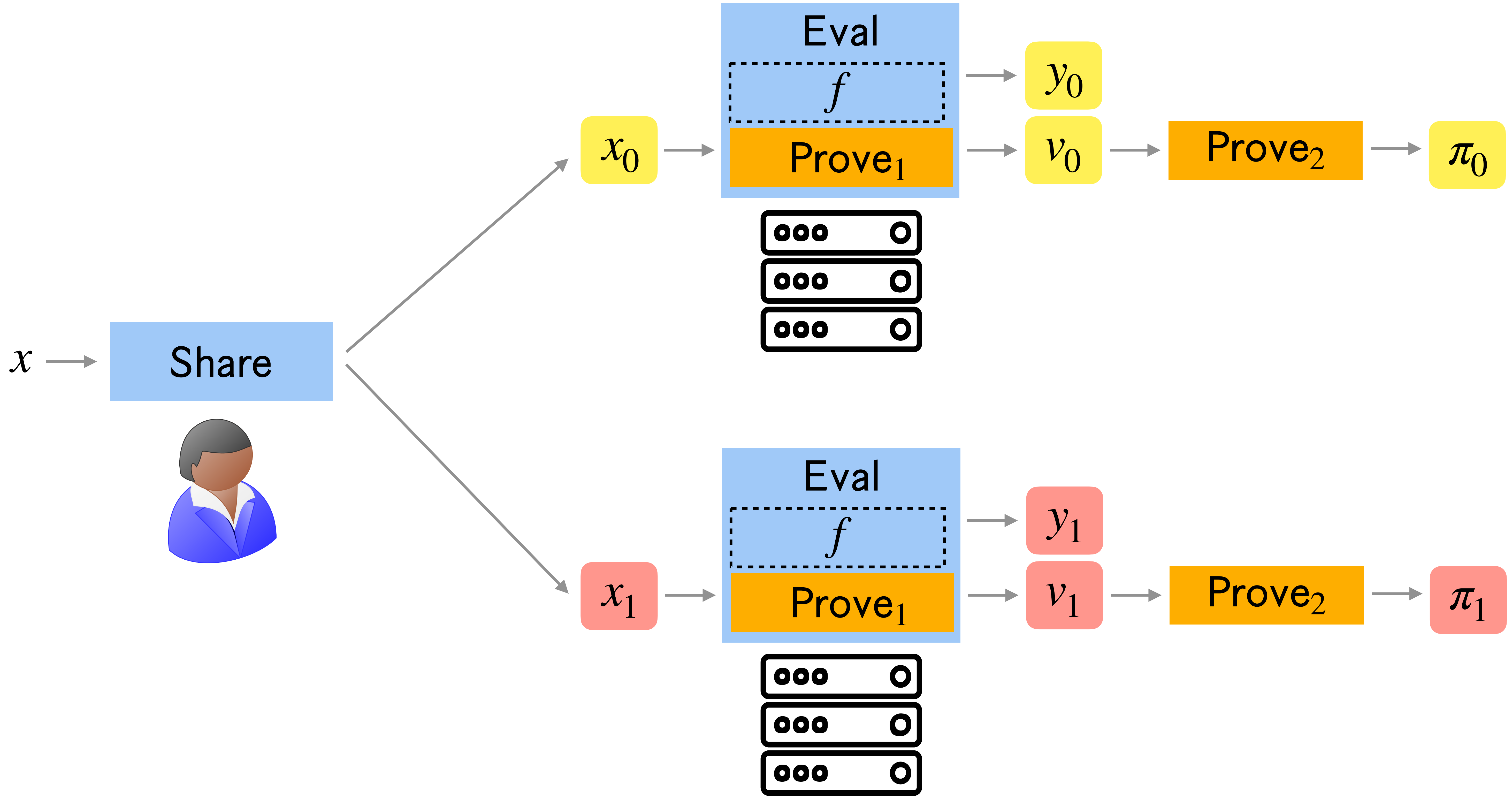

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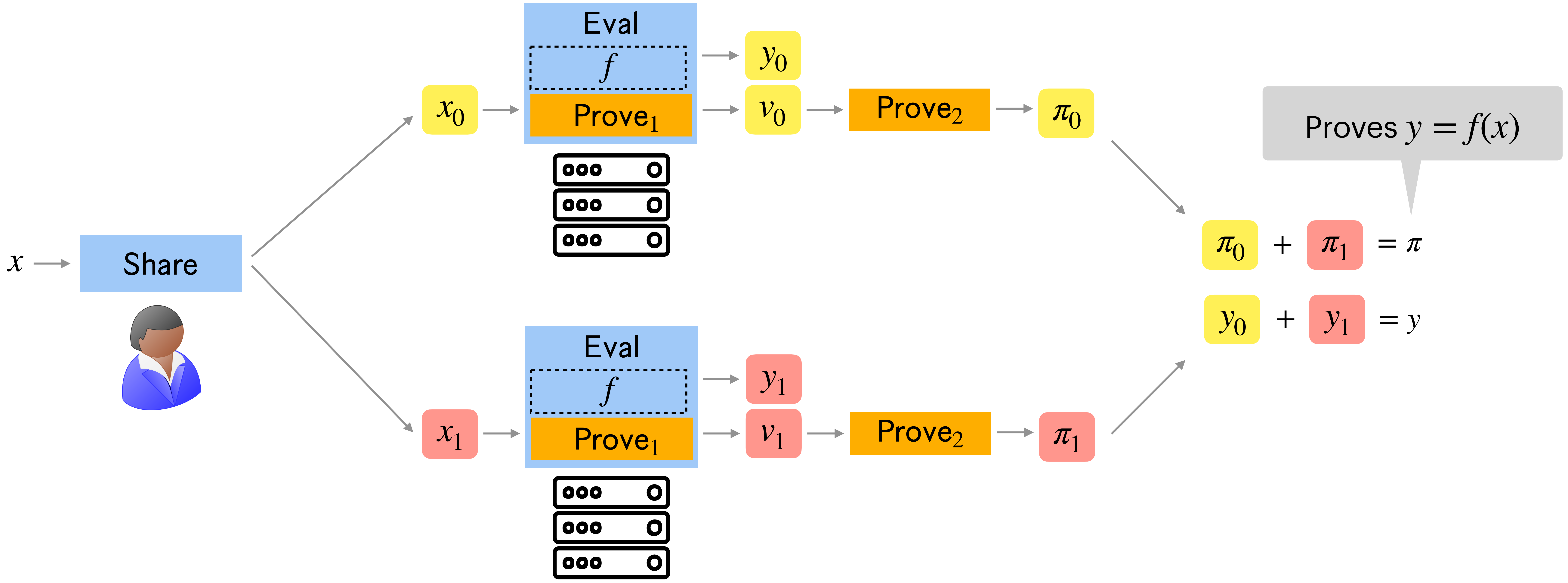

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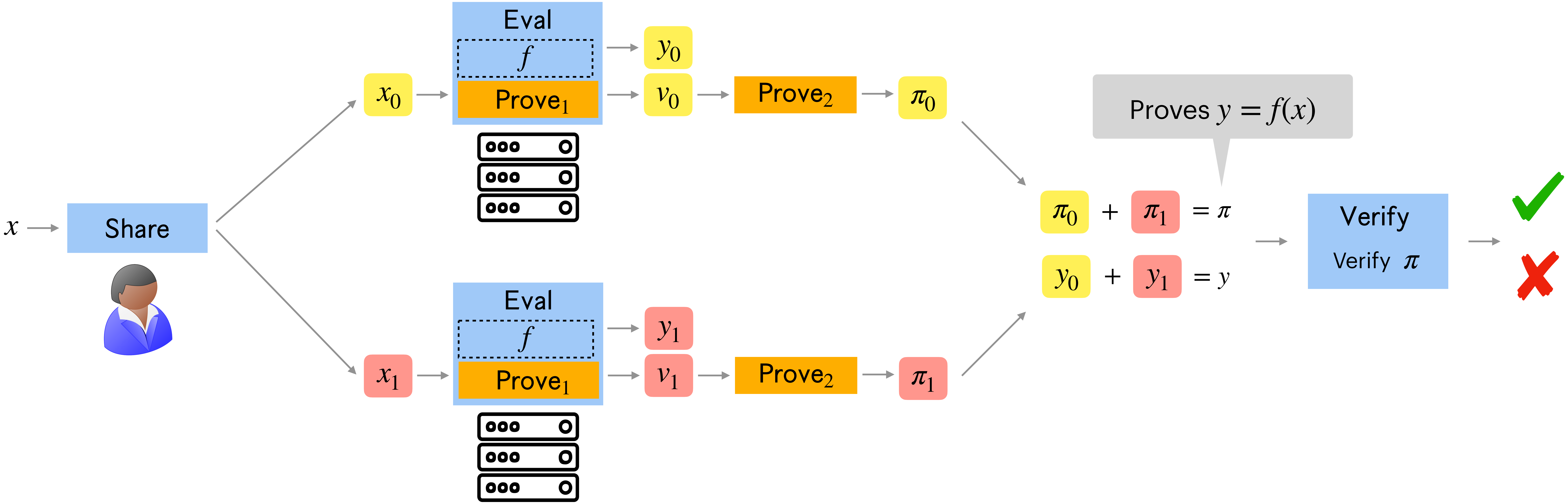

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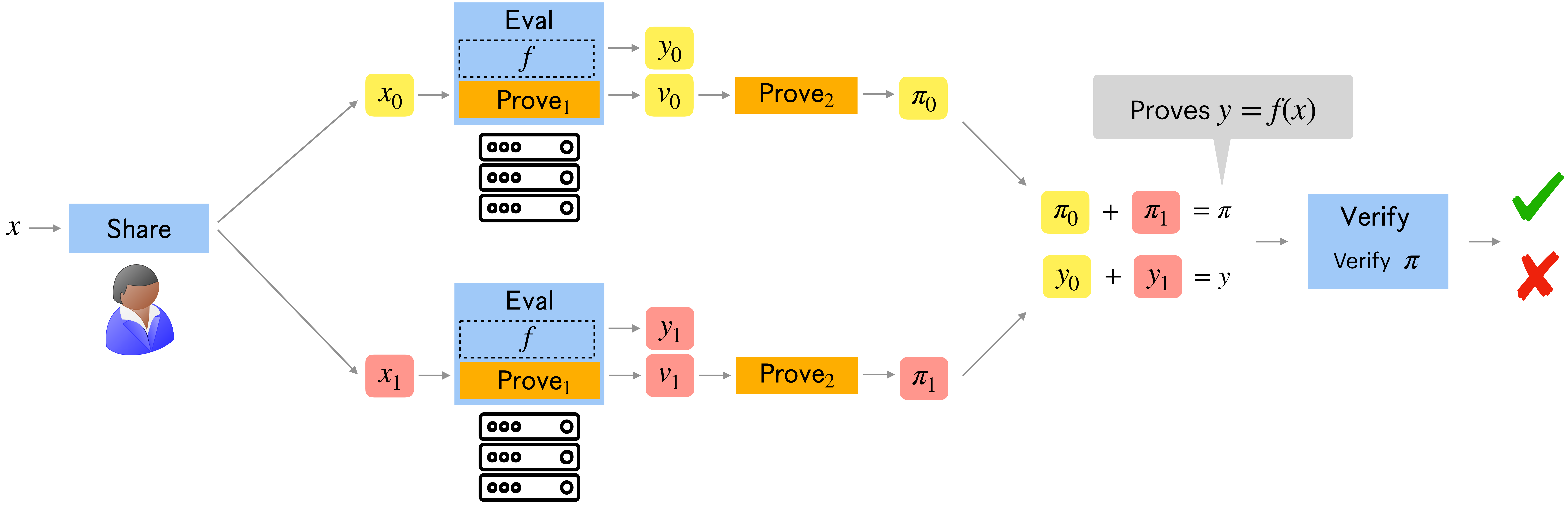
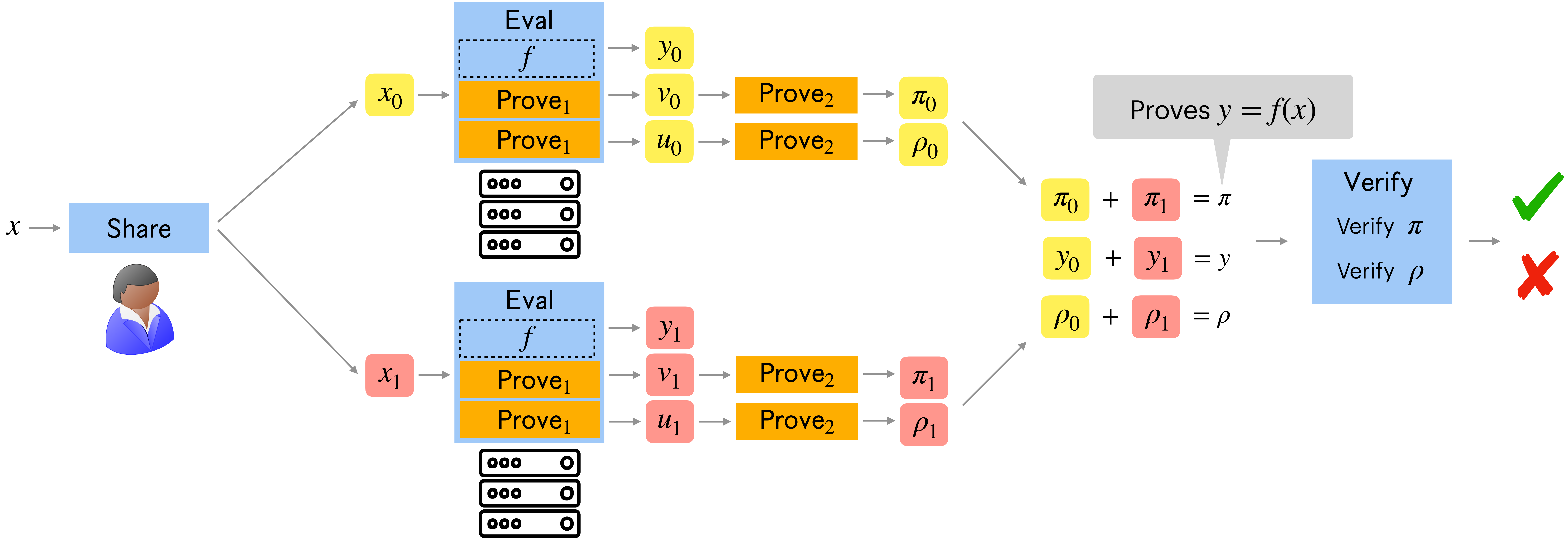


Soundness of $\pi \implies$ Local soundness

HSS with Verifiable Evaluation From Splittable zkSNARG



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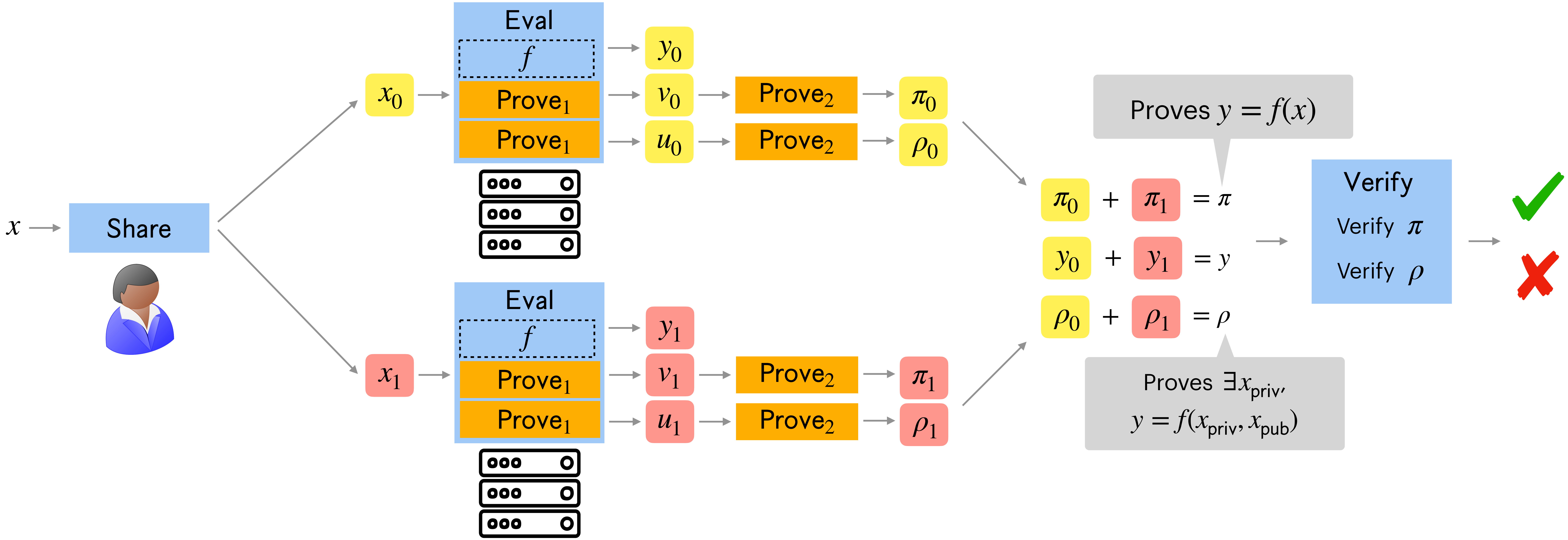

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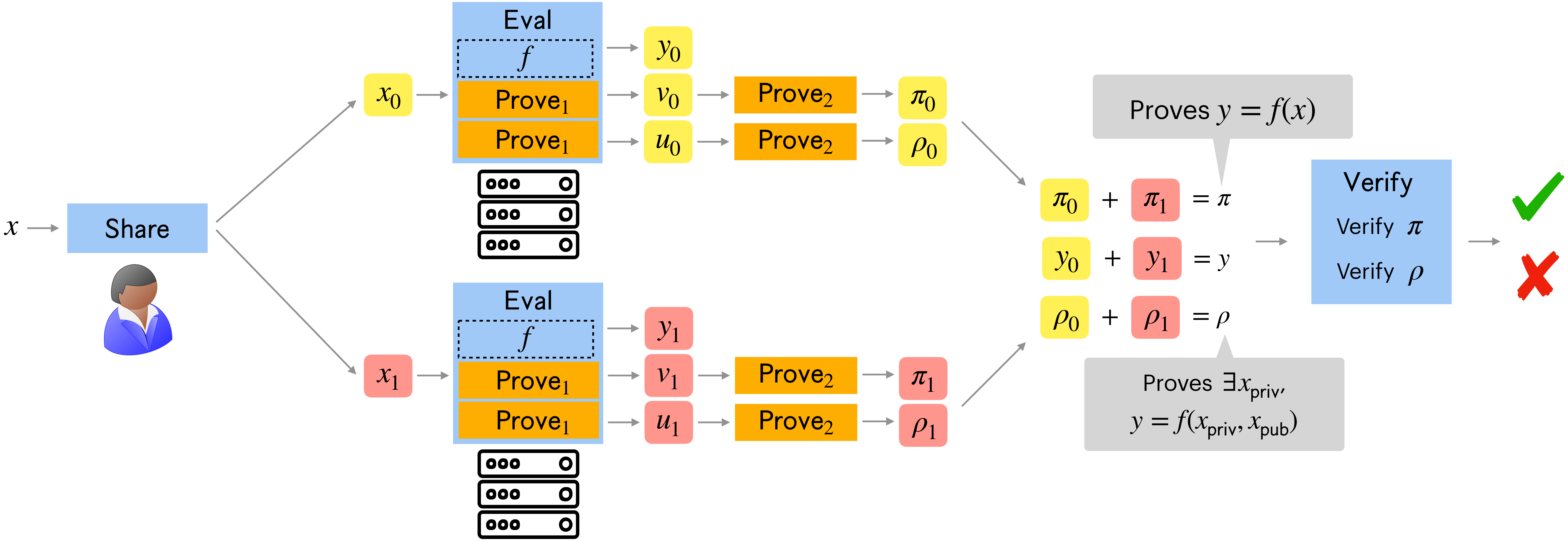
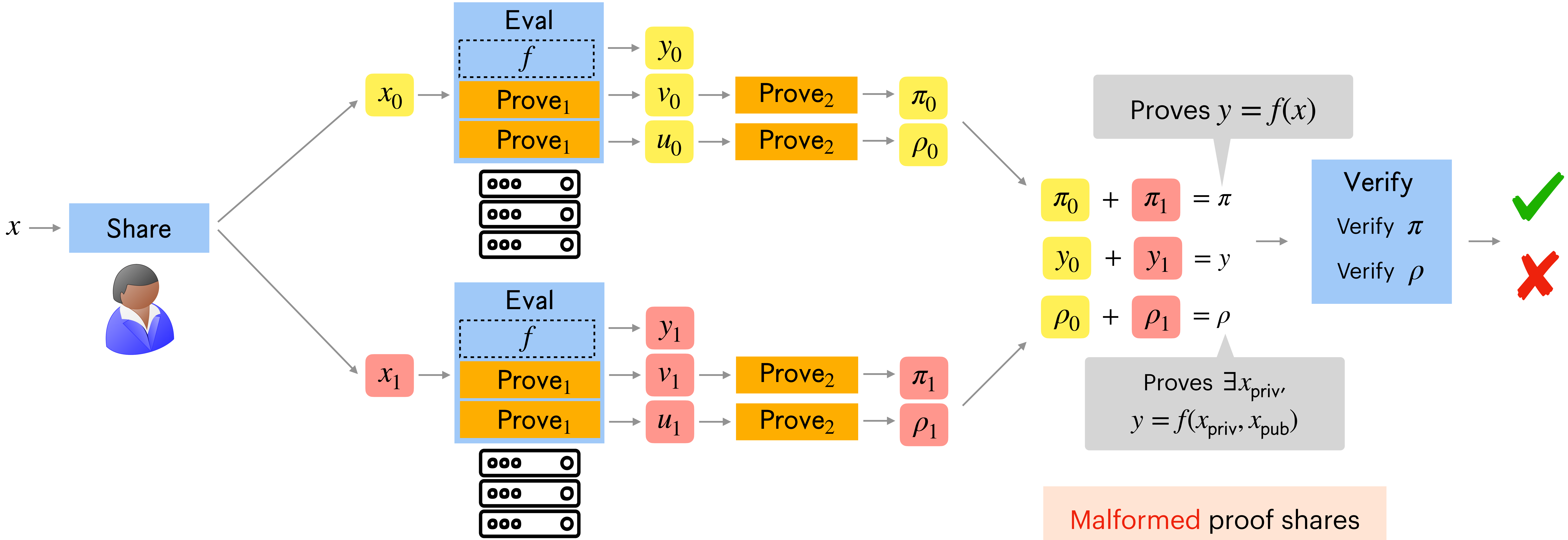


Soundness of $\pi \implies$ Local soundness

Soundness of $\rho \implies$ Public soundness

HSS with Verifiable Evaluation From Splittable zkSNARG

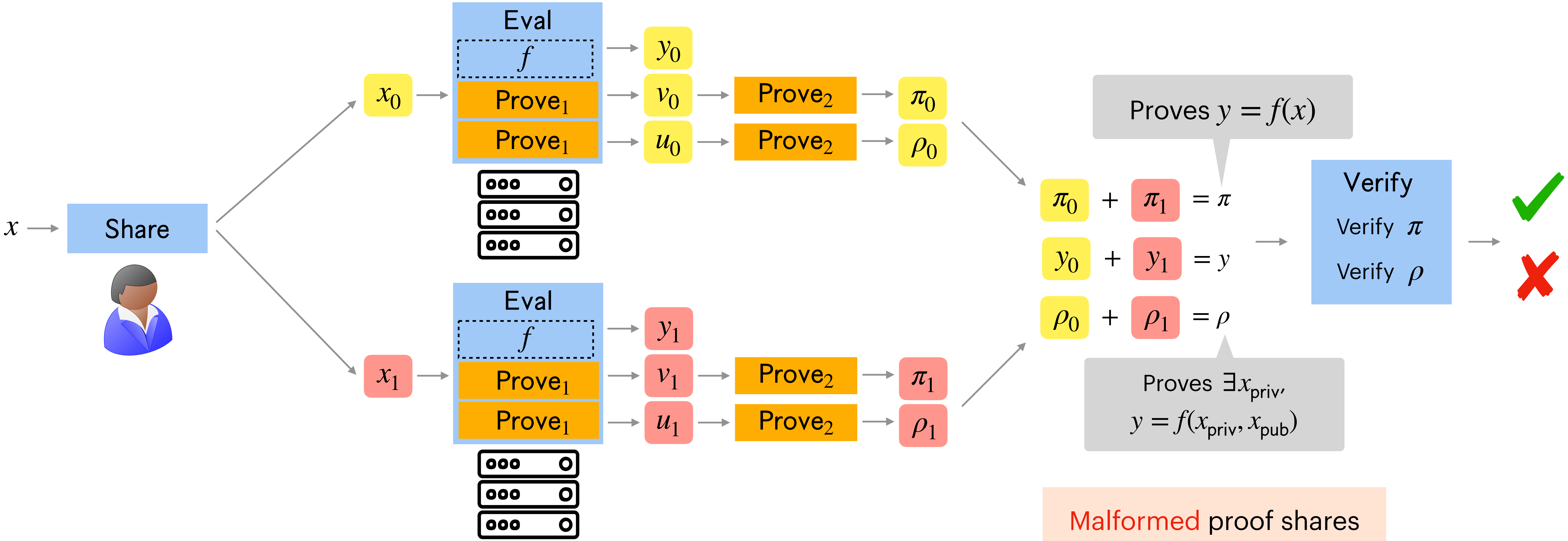


Malformed proof shares can attack privacy

Soundness of $\pi \implies$ Local soundness

Soundness of $\rho \implies$ Public soundness

HSS with Verifiable Evaluation From Splittable zkSNARG



Malformed proof shares can attack privacy

Soundness of $\pi \implies$ Local soundness

Soundness of $\rho \implies$ Public soundness

Robust verification: Malformed proof shares always reconstruct to invalid proofs

Thank You