# The Cost of Maintaining Keys in Dynamic Groups with Applications to Multicast Encryption and Group Messaging

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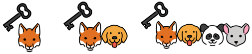
- Examples: Multicast Encryption (ME), Group Messaging.

# Multicast Encryption

- In ME there is a central authority and each user maintains a private state:



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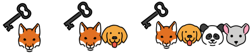

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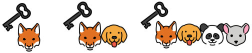

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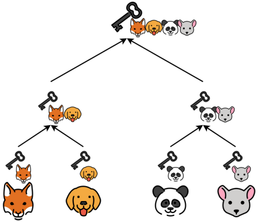

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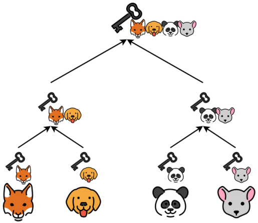

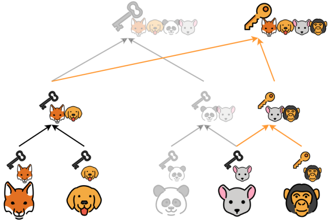

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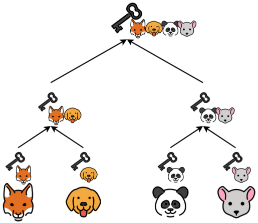

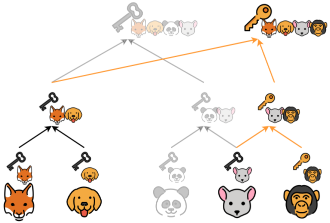

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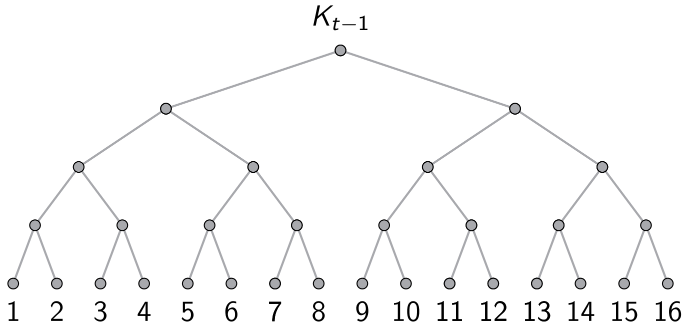
**This Work:** A lower bound for arbitrary $d$ of $\Omega(d \cdot \log_2(n/d))$ (Average Case).

# Combinatorial Model: The Cost Function



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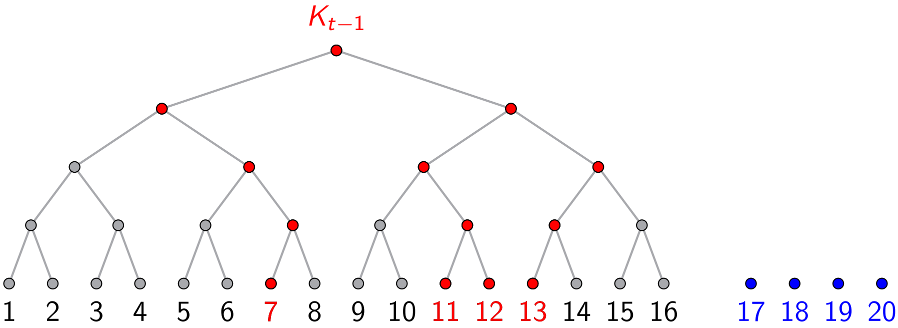

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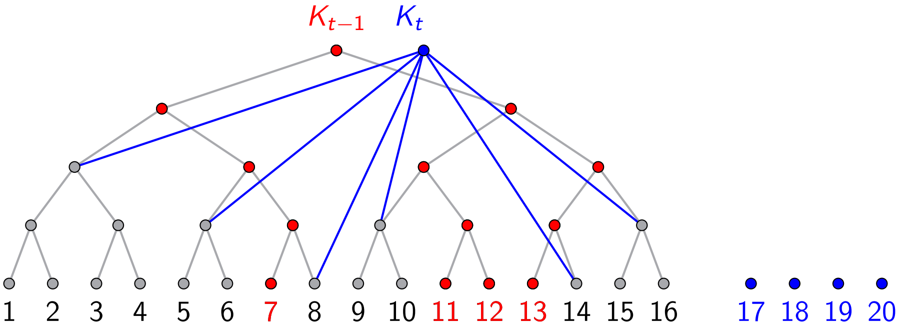

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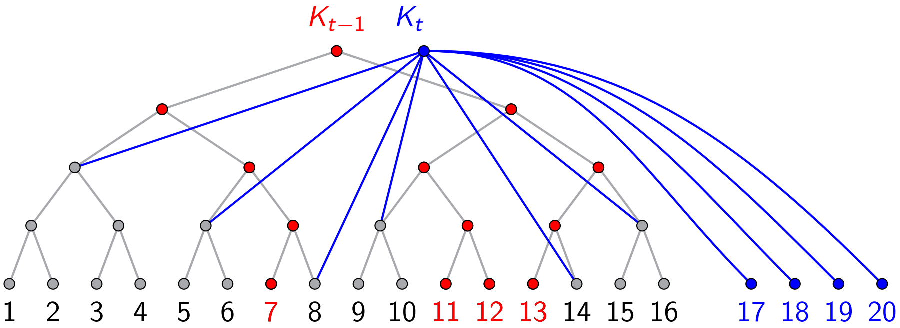


$\mathrm{Cost}(t) =$ red nodes + blue edges

# Combinatorial Lower Bound

**Theorem**

*In every round $t$*

$$\mathbb{E}[\mathrm{Cost}(t)] \geq d \ln\left(\frac{n}{d}\right),$$

*where $d$ denotes the number of users replaced in round $t$ and the set of users removed is sampled uniformly at random in every round.*

$$\mathrm{Cost}(t) = \text{red nodes} + \text{blue edges}$$

Consequence of Bollobás Set Pairs Inequality.

# Lower Bound for Multicast Encryption

**Lemma**

*For any correct and secure ME scheme built using PRGs, PRFs, dual PRFs, symmetric encryption and secret sharing in the symbolic model:* $\sum_{t=0}^{t_{\max}} |\mathtt{M}_t| \geq 1/3 \cdot \sum_{t=0}^{t_{\max}} \mathrm{Cost}(t)$, *where* $|\mathtt{M}_t| =$ *number of messages sent by CA in round* $t$.

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**Theorem**

*Thus it must hold that*

$$\frac{1}{t_{\max}} \mathbb{E}\left[\sum_{t=0}^{t_{\max}} |\mathtt{M}_t|\right] \geq \frac{1}{3} d \ln\left(\frac{n}{d}\right),$$

*where* $d$ *denotes the amount of users replaced per round and the set of users replaced is sampled uniformly at random in every round.*

# Thanks!

https://ia.cr/2024/1097