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Real-Valued Somewhat-Pseudorandom Unitaries

Zvika Brakerski, **Nir Magrafta**

https://arxiv.org/abs/2403.16704

Quantum Pseudorandomness

JLS 2018

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Pseudo**R**andom **S**tate (PRS)

- A keyed family $\{\varphi_k\}_k$ of states
- Efficiently generatable
- Computationally indistinguishable from Haar random state:

$$\left| \Pr_{k \leftarrow \mathcal{K}} \left[\mathscr{A}\left(\left| \varphi_k \right\rangle^{\otimes m} \right) = 1 \right] - \Pr_{\psi \leftarrow Haar} \left[\mathscr{A}\left(\left| \psi \right\rangle^{\otimes m} \right) = 1 \right] \right| \leq negl$$

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Pseudo**R**andom **U**nitary (PRU)

- A keyed family $\{U_k\}_k$ of unitaries
- Efficiently generatable given k
- Computationally indistinguishable from Haar random unitary:

$$\left| \Pr_{k \leftarrow \mathscr{K}} \left[\mathscr{A}^{U_k}\left(1^\kappa \right) = 1 \right] - \Pr_{U \leftarrow Haar} \left[\mathscr{A}^{U}\left(1^\kappa \right) = 1 \right] \right| \leq negl$$

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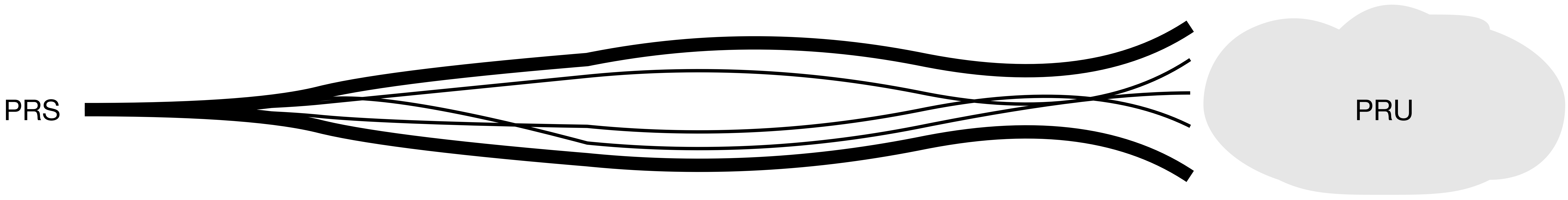
- Quantum chaos, quantum gravity

A Plethora of Pseudorandom Objects



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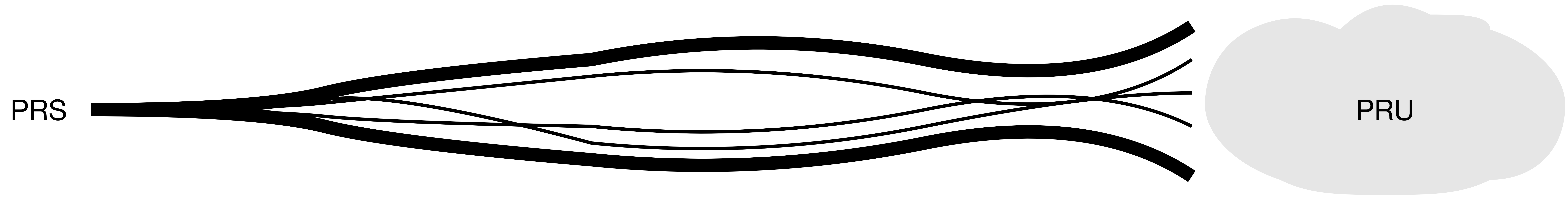

- More complex then states
- Output is harder to model than states
- Variable input

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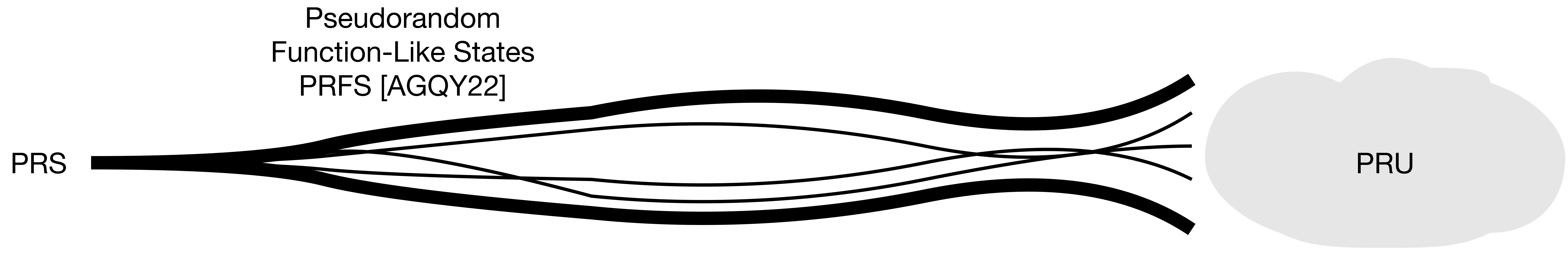

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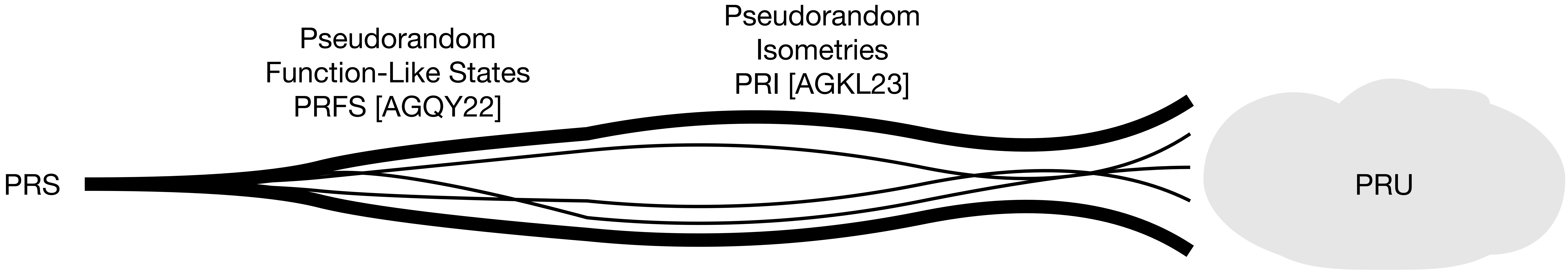

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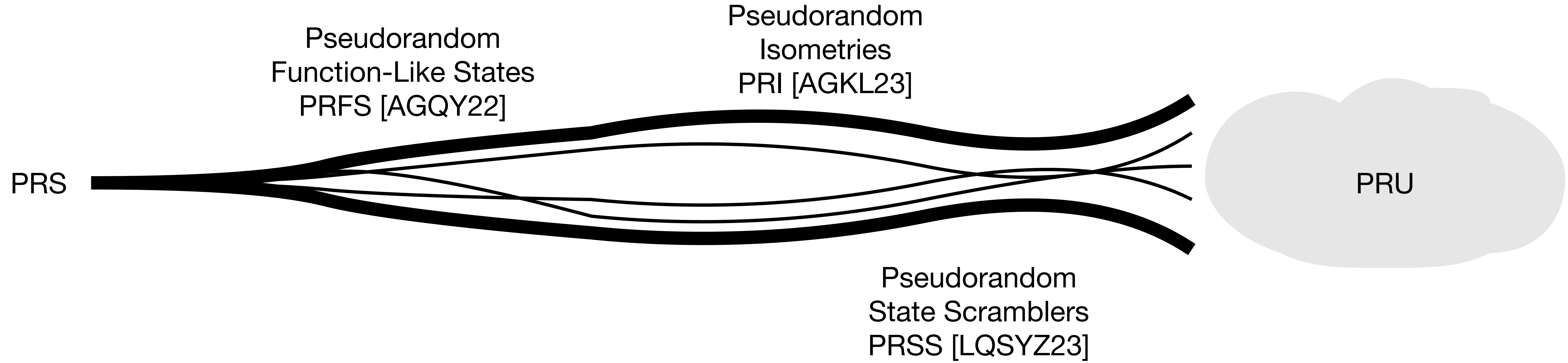

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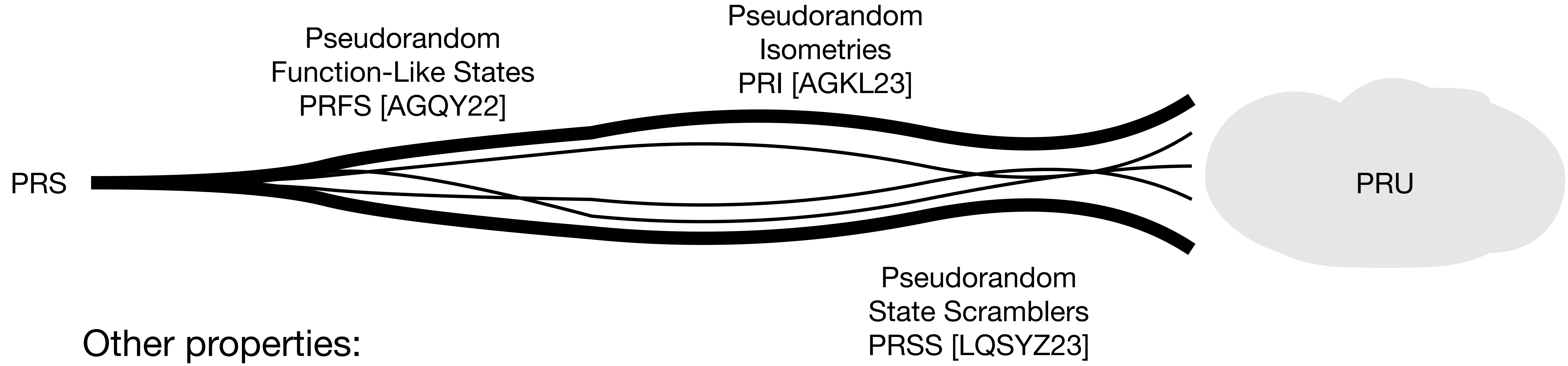

Other properties:

- Binary ± phase [BS19], Subset PRS [GB23, JMW23]
- Scalable PRS [BS20]
- PRS with pseudo-entanglement [ABFGVZZ22]

Our results:
Non-Adaptive Orthogonal-Inputs Pseudorandom Unitary

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Let $s(n), t(n)$ be polynomials in the number of qubits n, and let $\left\{ \left|\psi^1\right\rangle, \ldots, \left|\psi^s\right\rangle \right\}$ be orthogonal states.

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Then any n.u. QPT distinguisher $\mathscr{A}$ that makes a query of the form $\rho_{in} = \bigotimes_{j\in[s]} \left(\left|\psi^j\right\rangle\left\langle\psi^j\right| \right)^{\otimes t}$

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We construct a family of **real valued** unitaries $\{U_k\}_k$ which satisfy this definition

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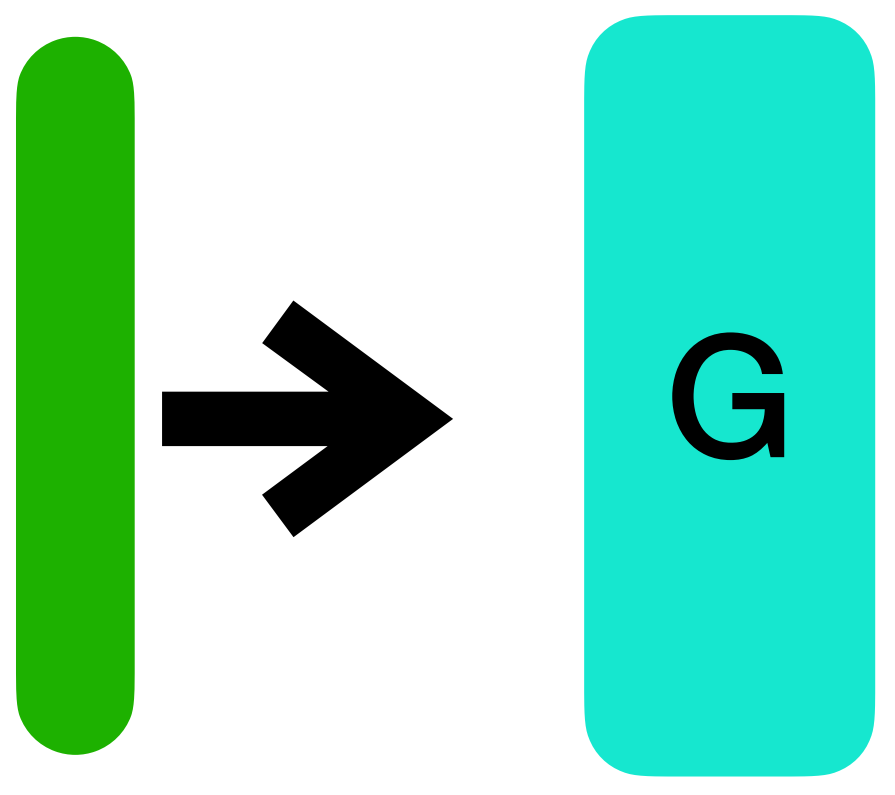
- Randomize the phase



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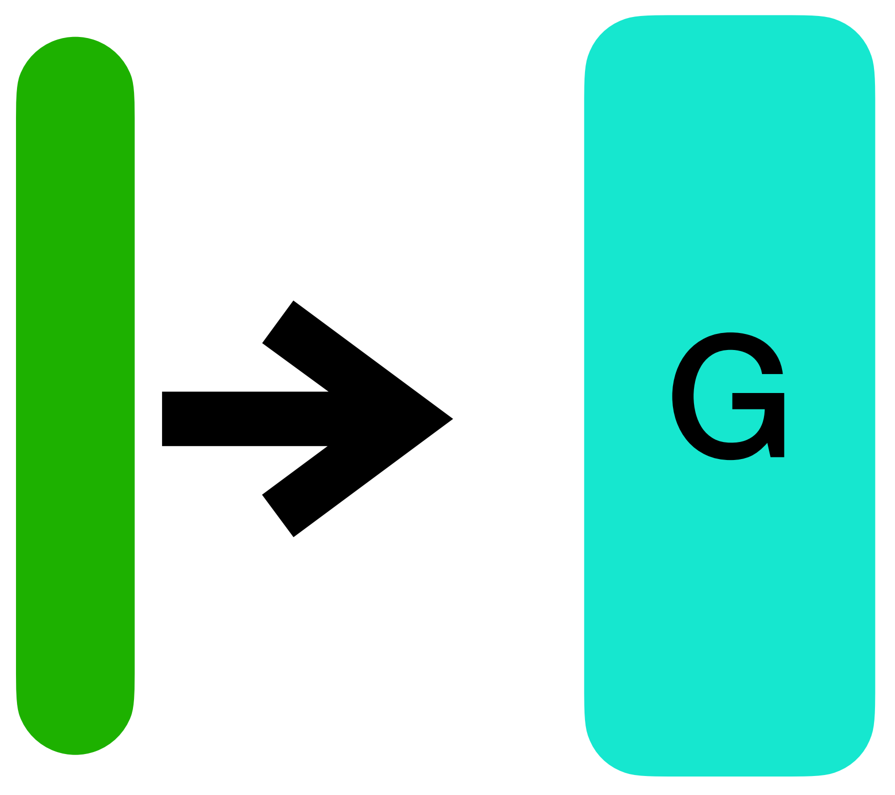

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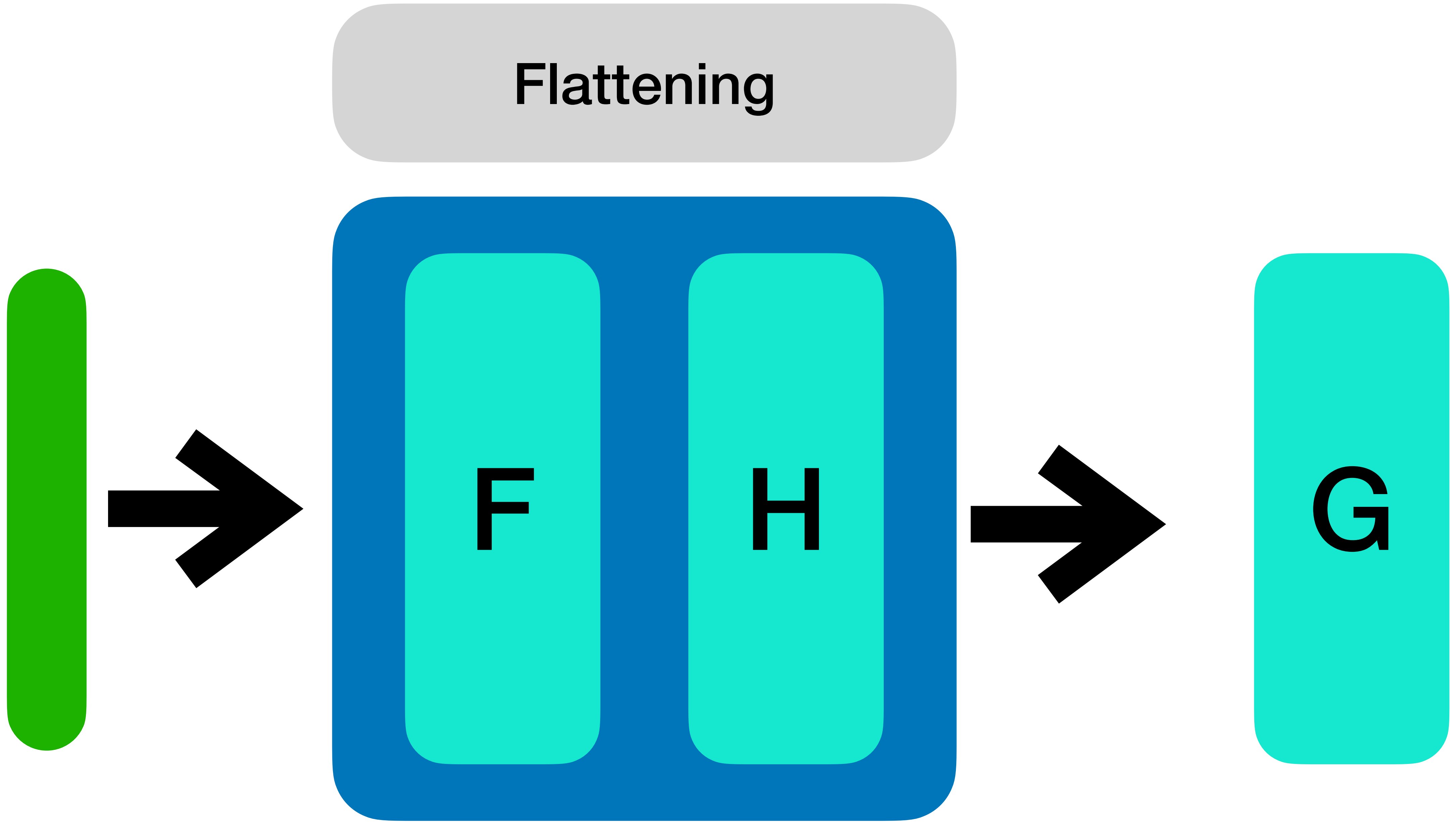

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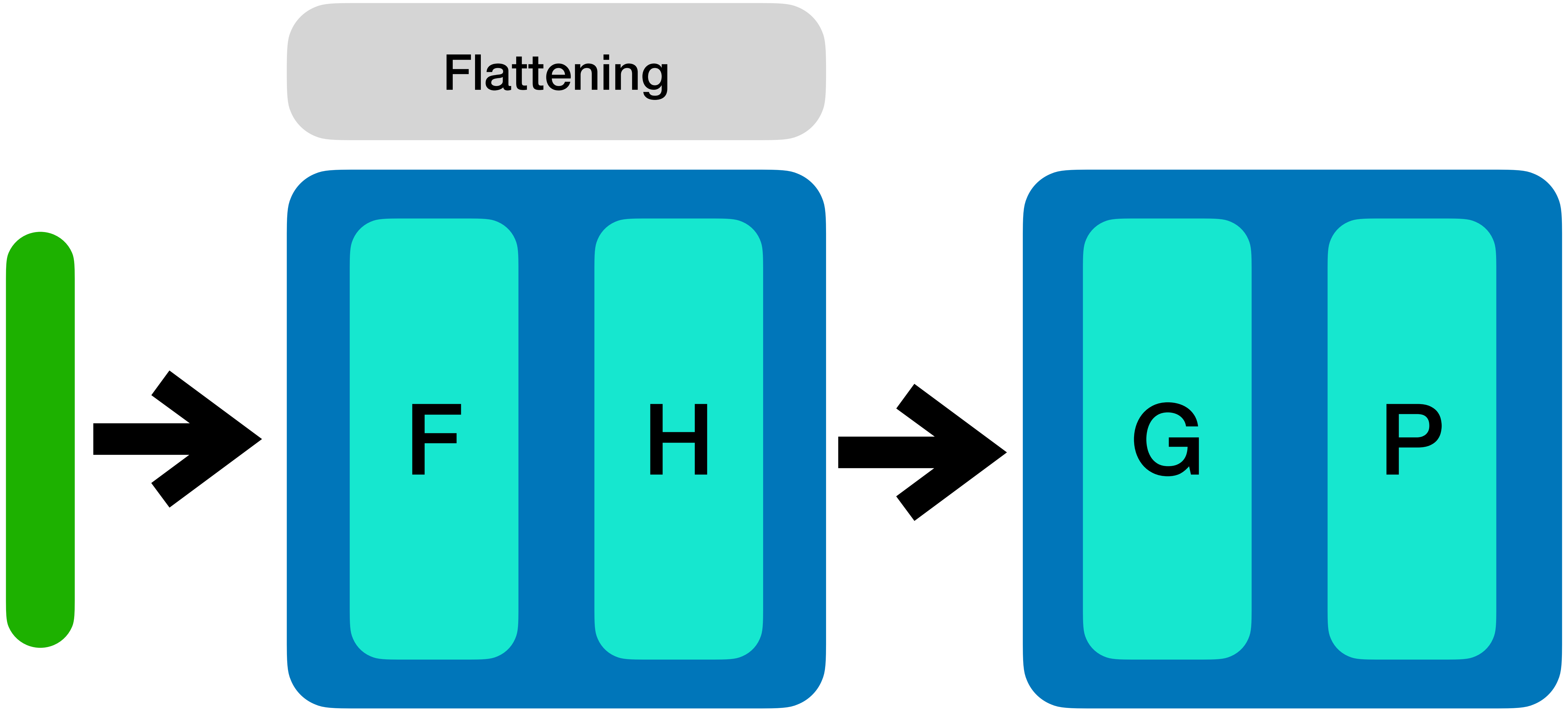

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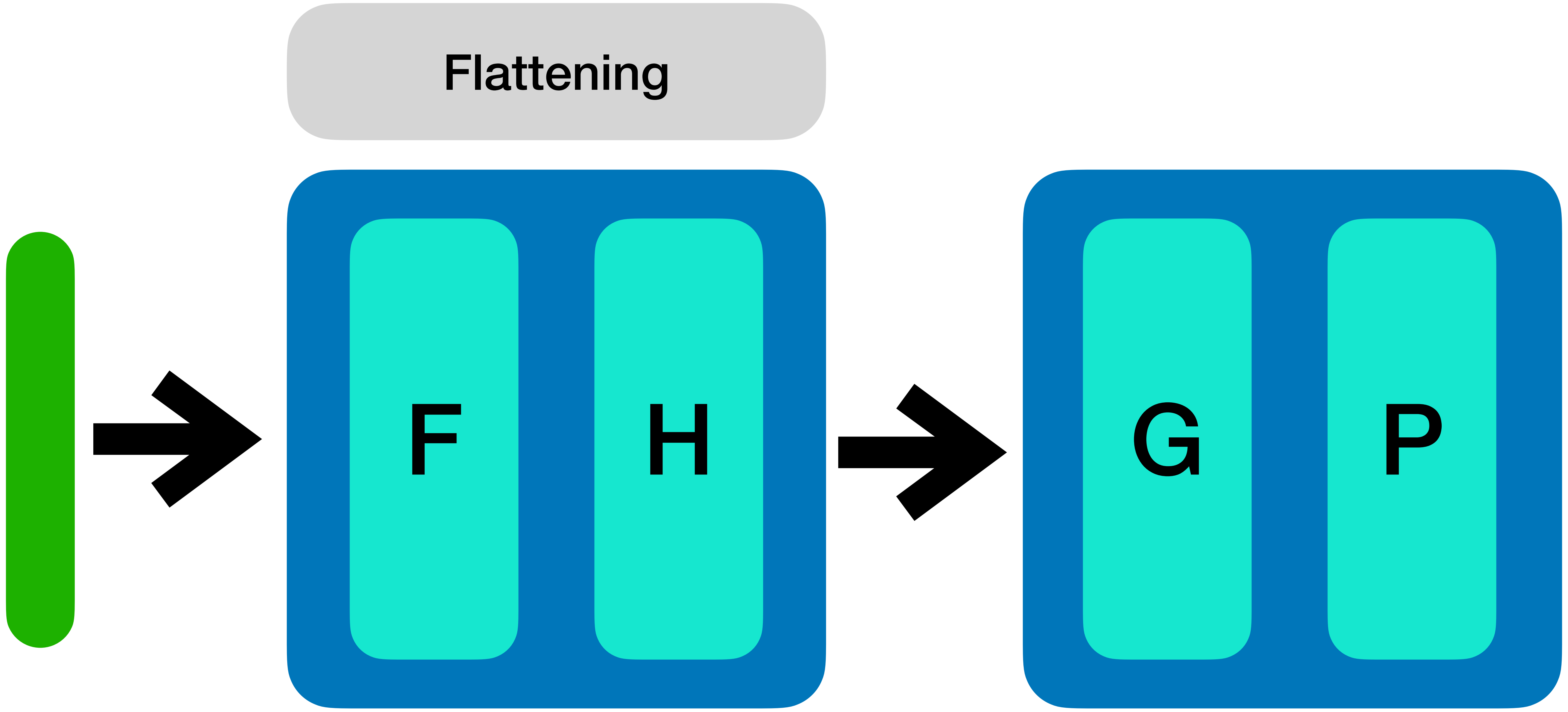
- Flatness follows from concentration bounds



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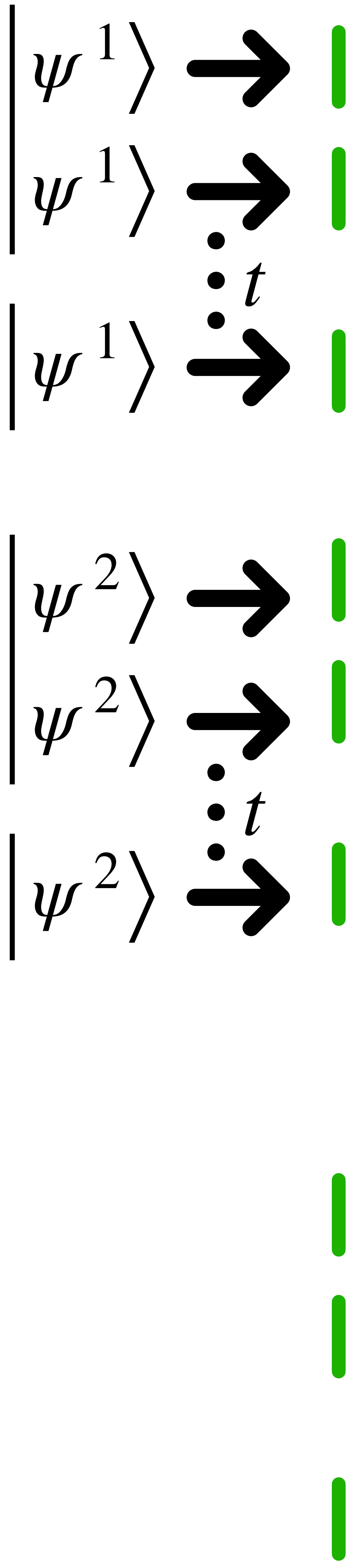

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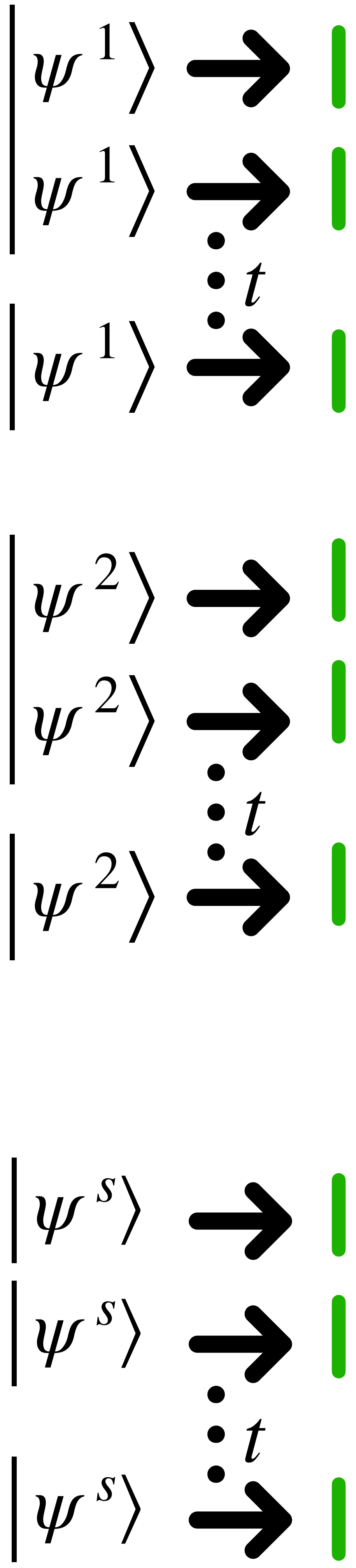

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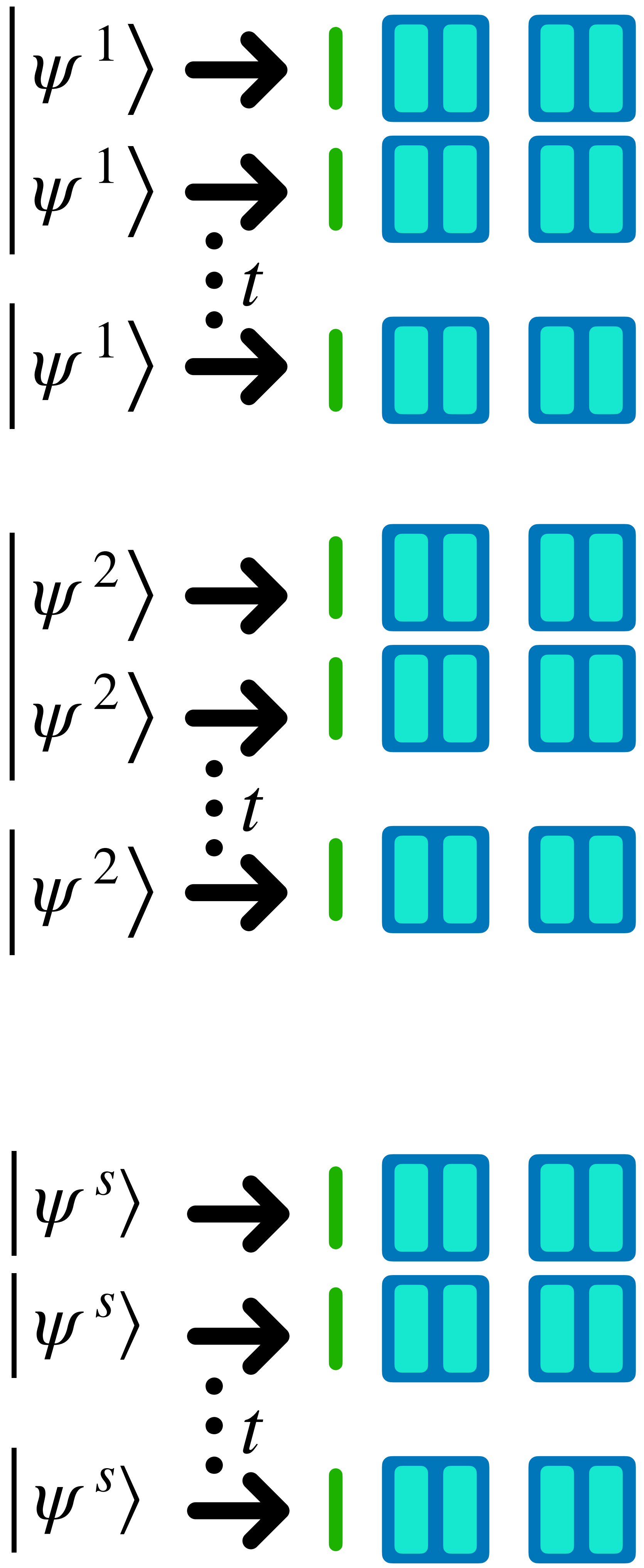

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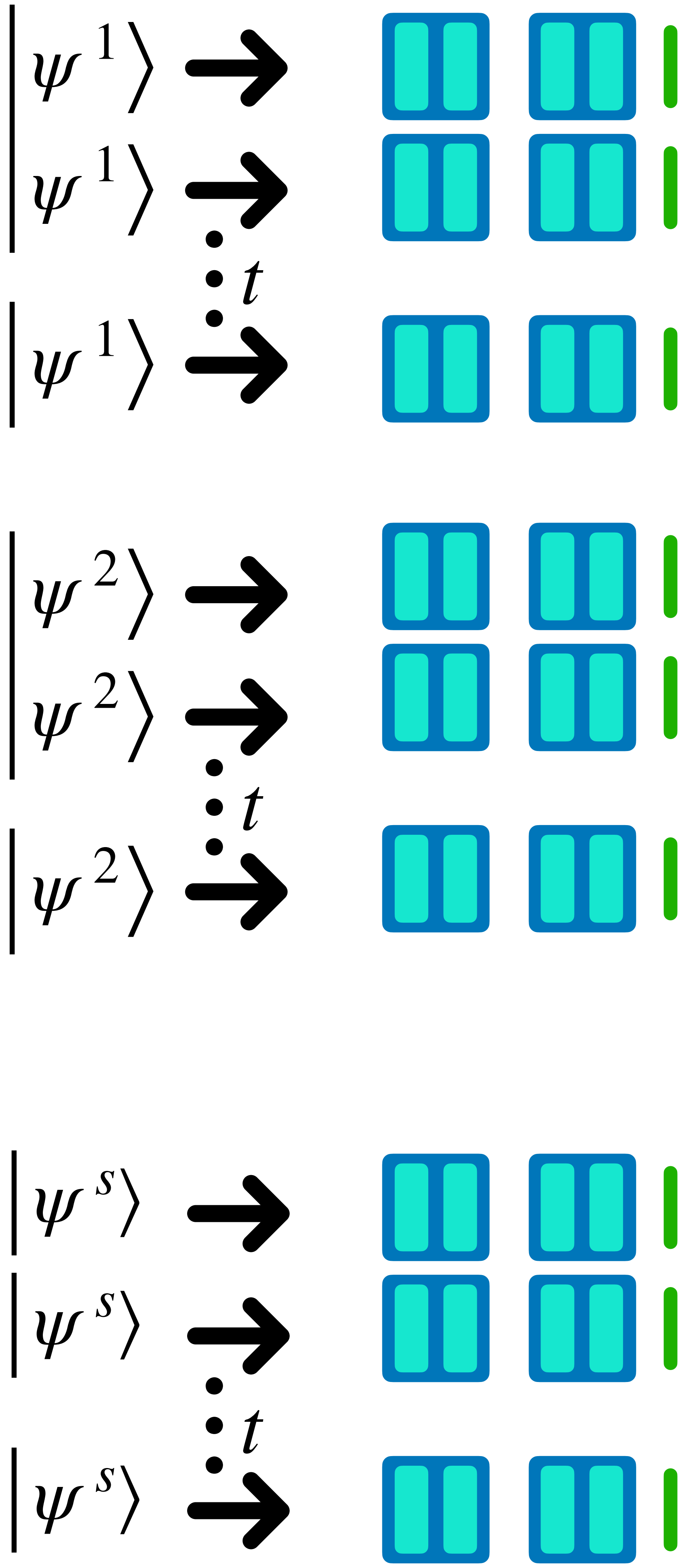

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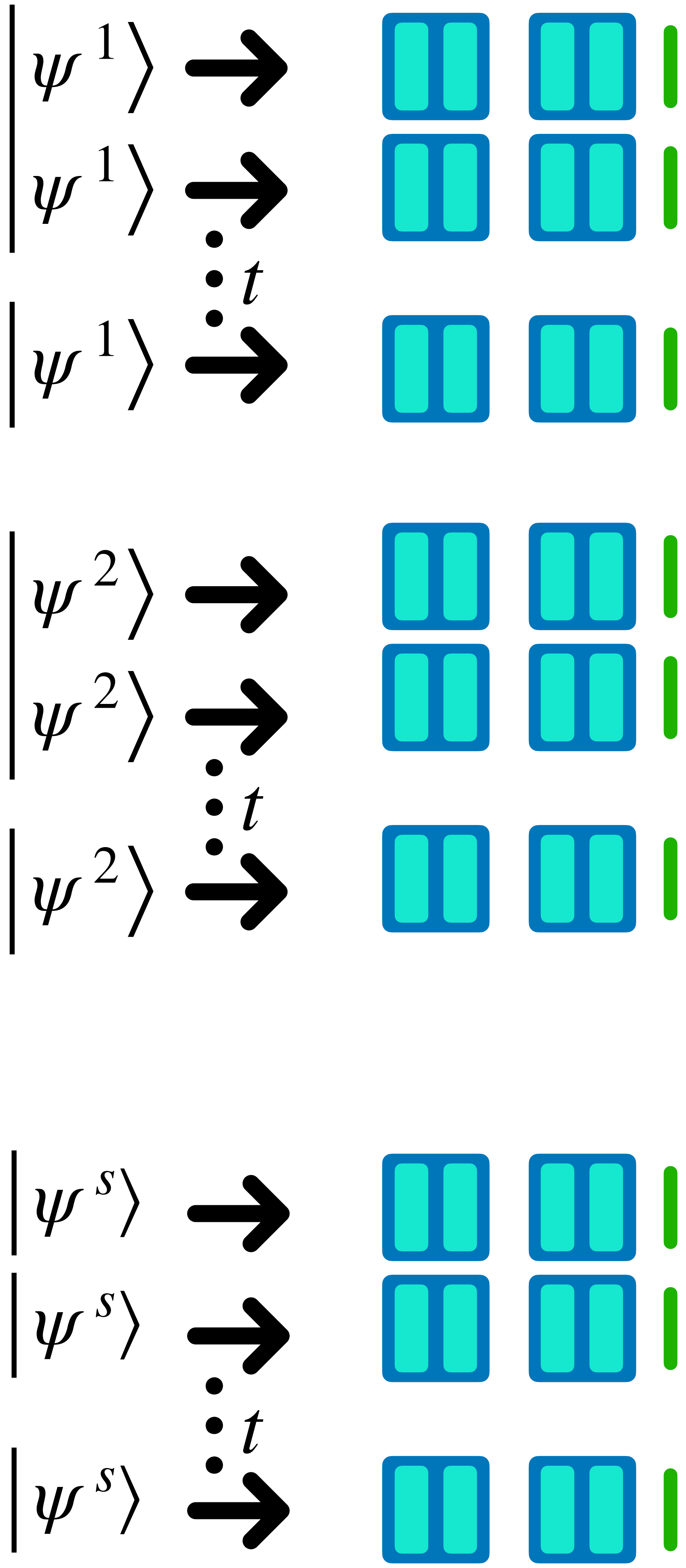

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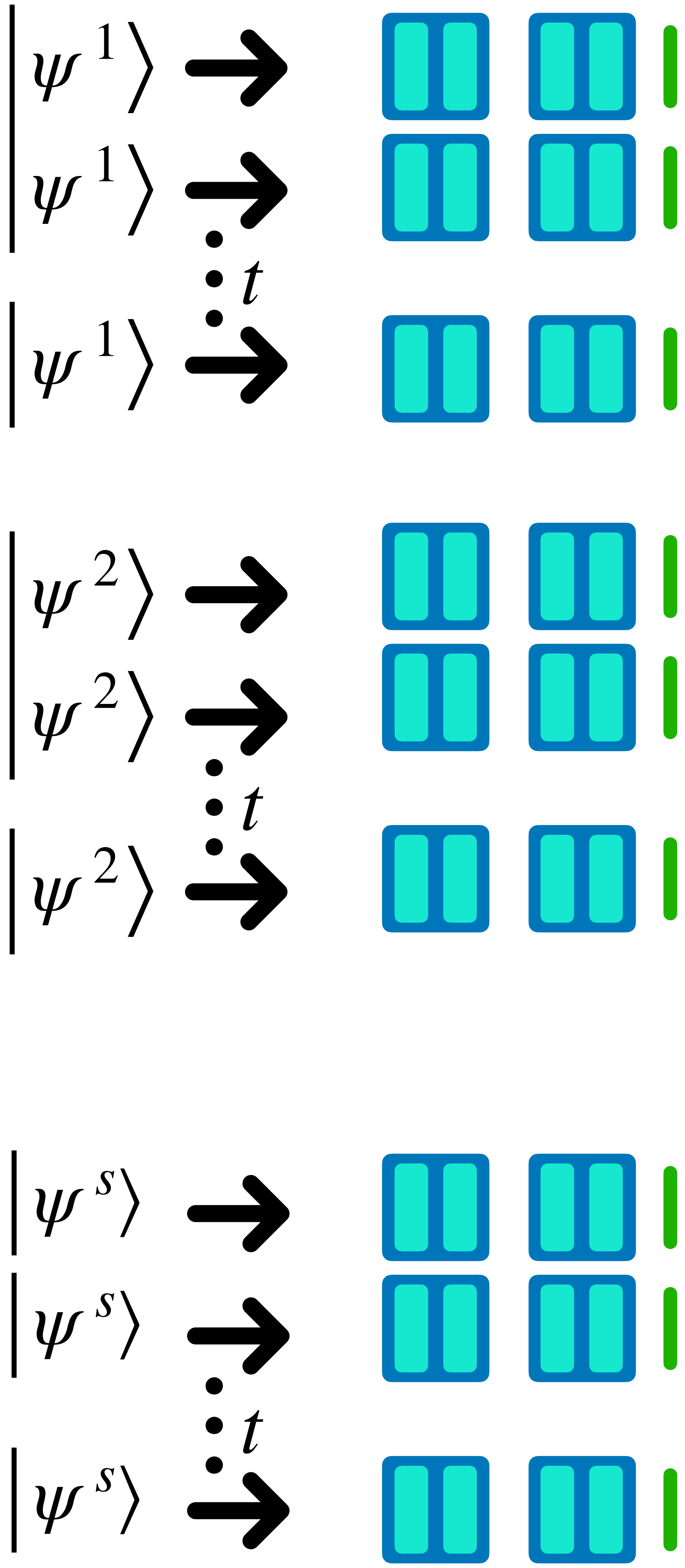

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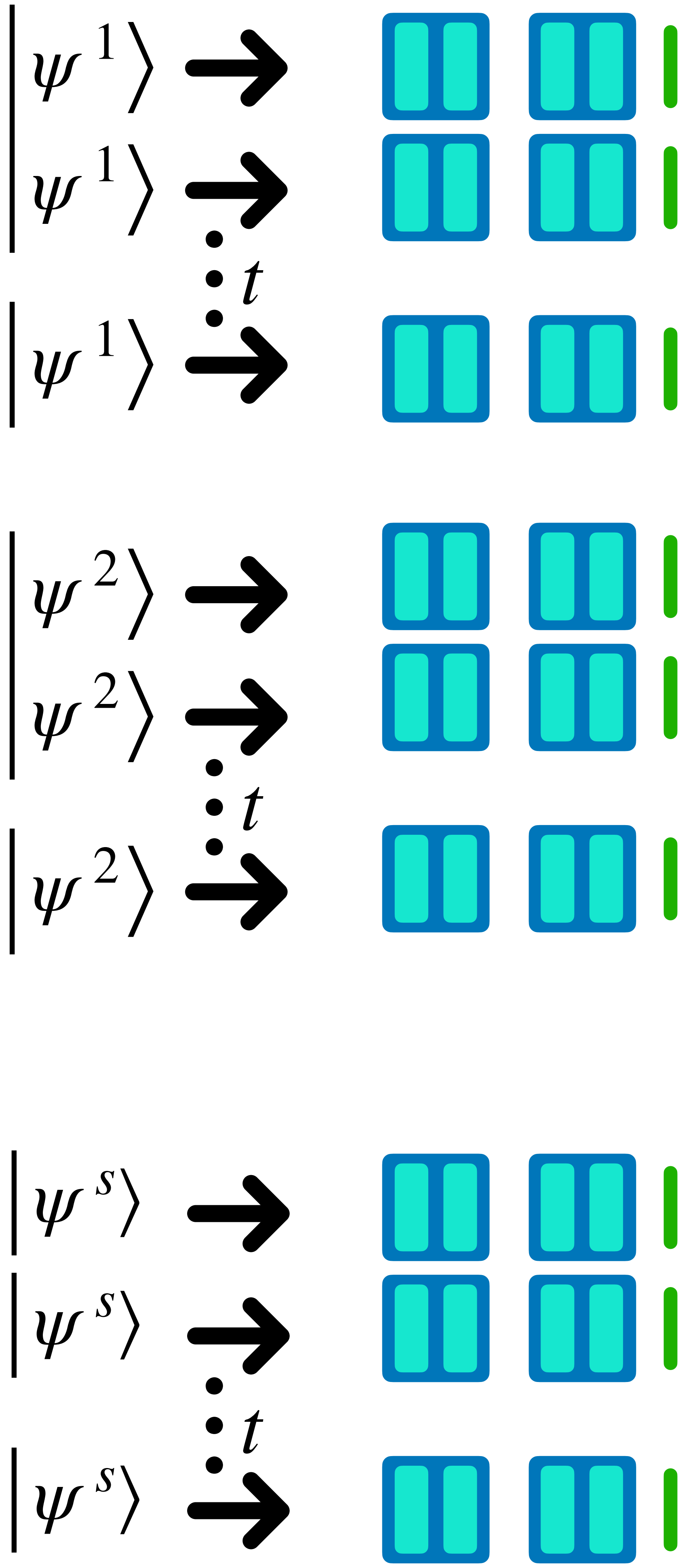

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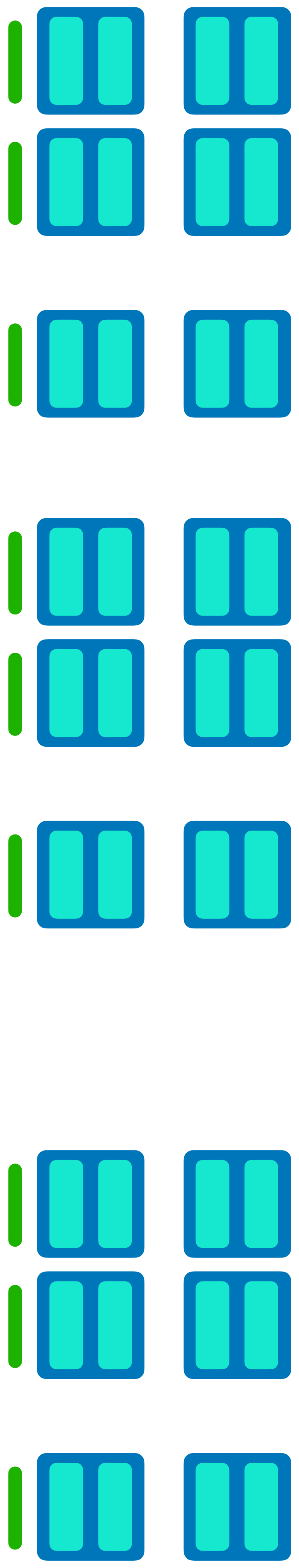

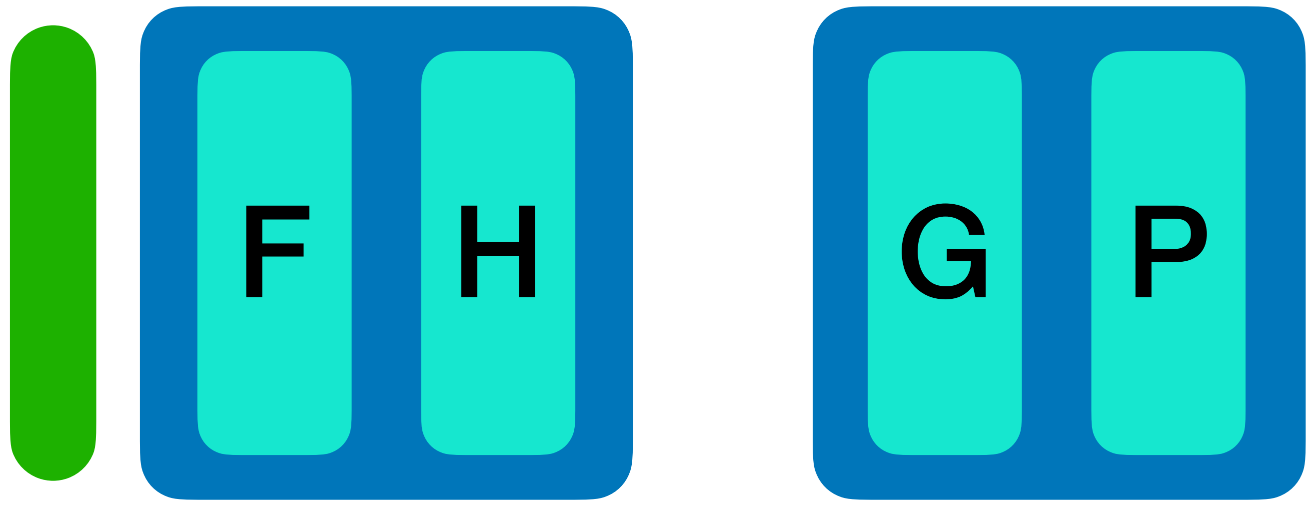

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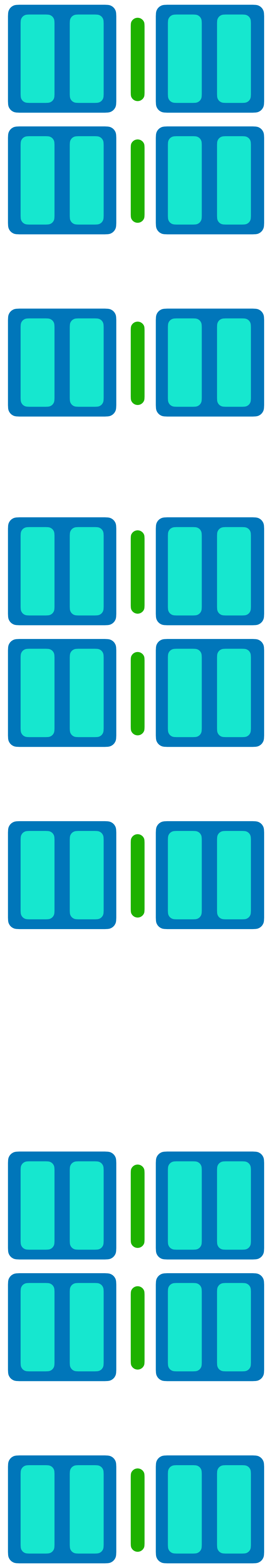

$$\alpha_{\vec{z},\vec{z}'} \left| \vec{z} \right\rangle \left\langle \vec{z}' \right|$$

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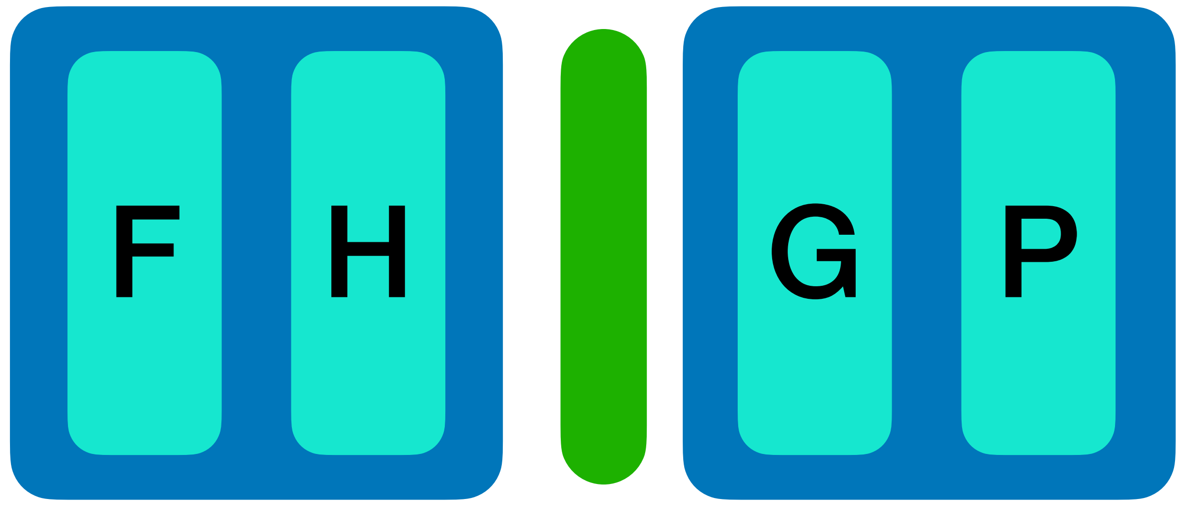

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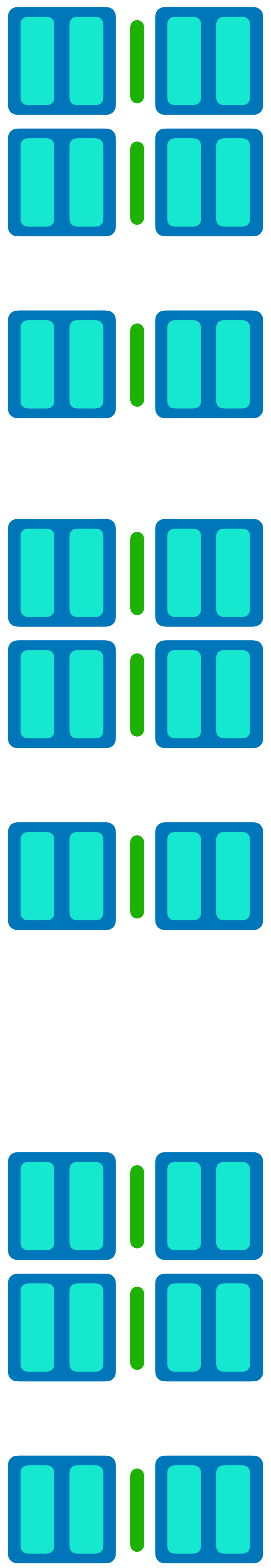

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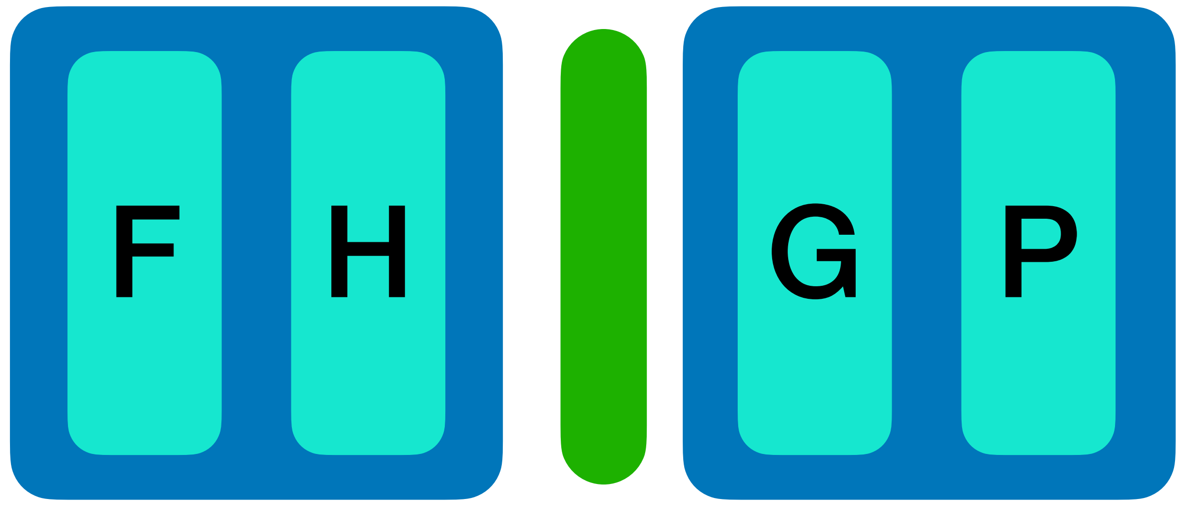

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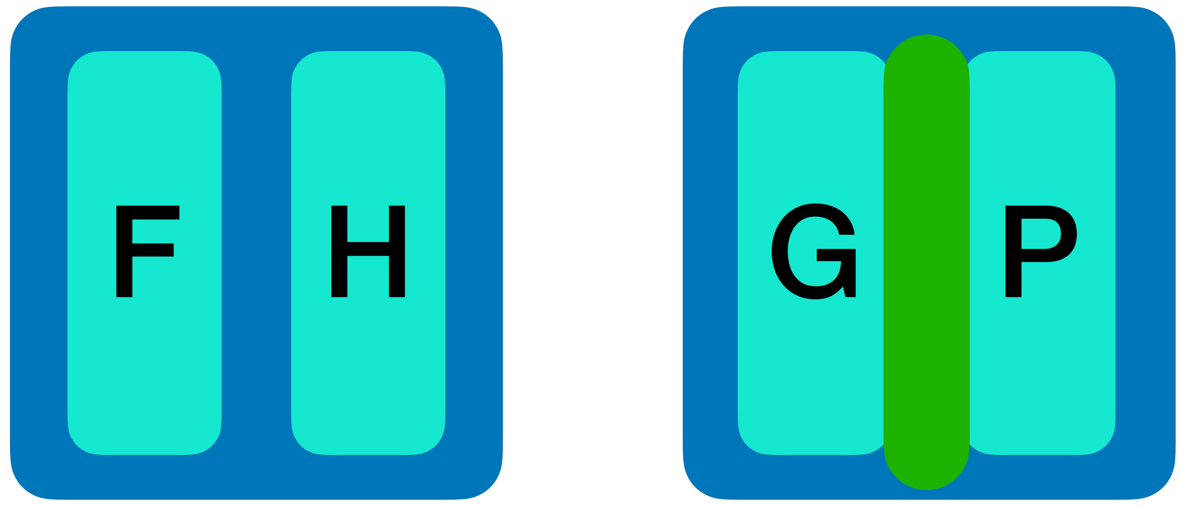

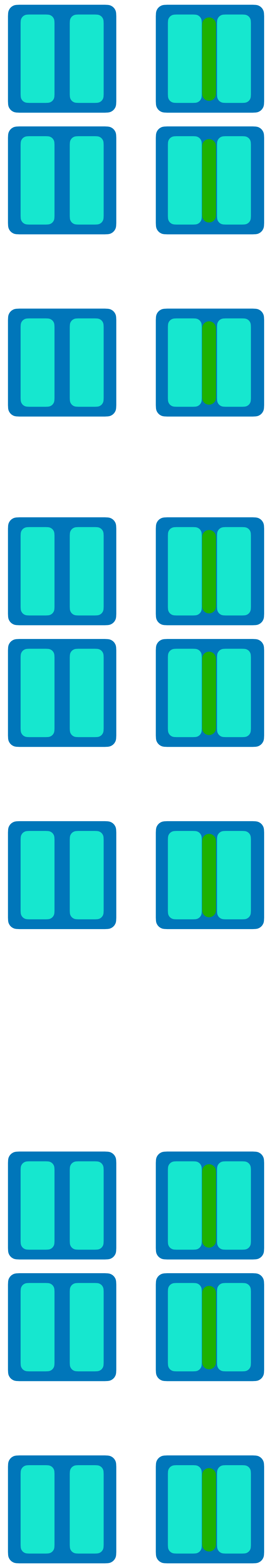

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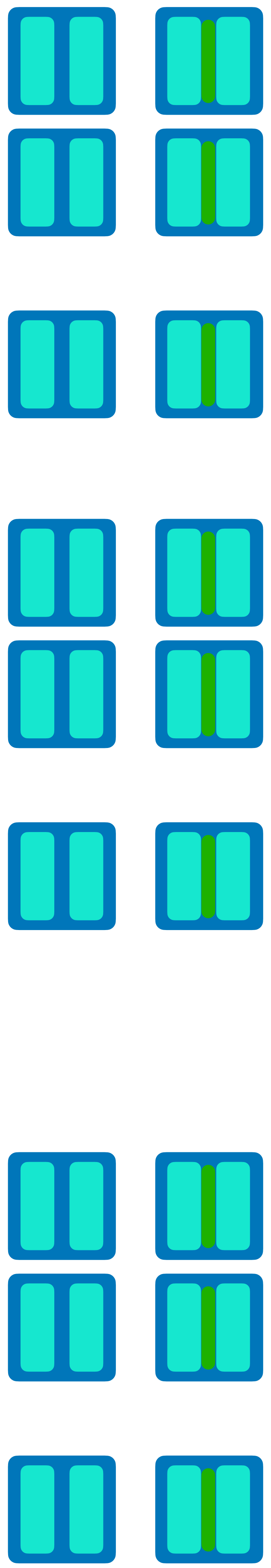

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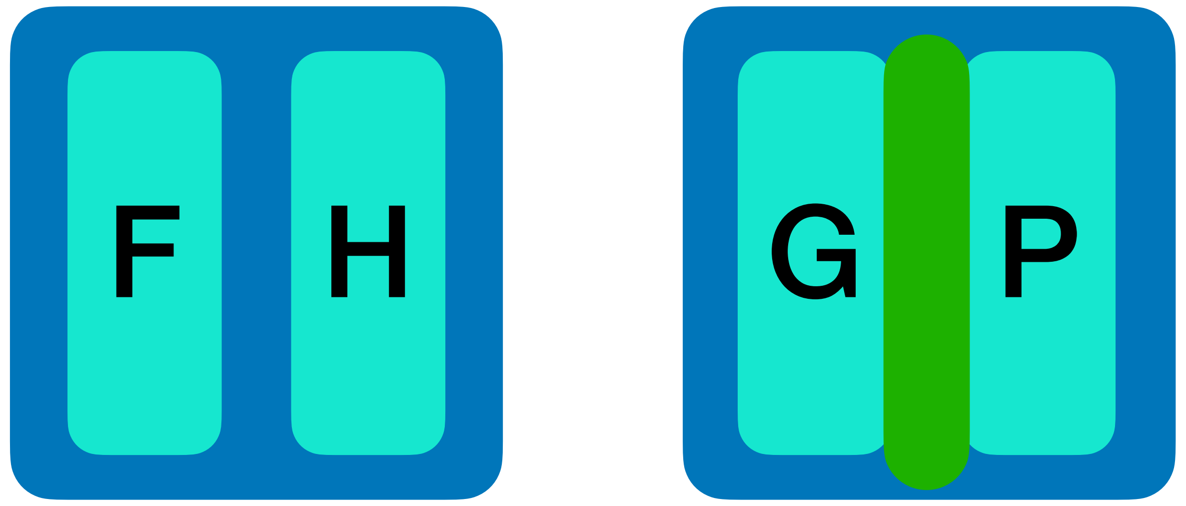

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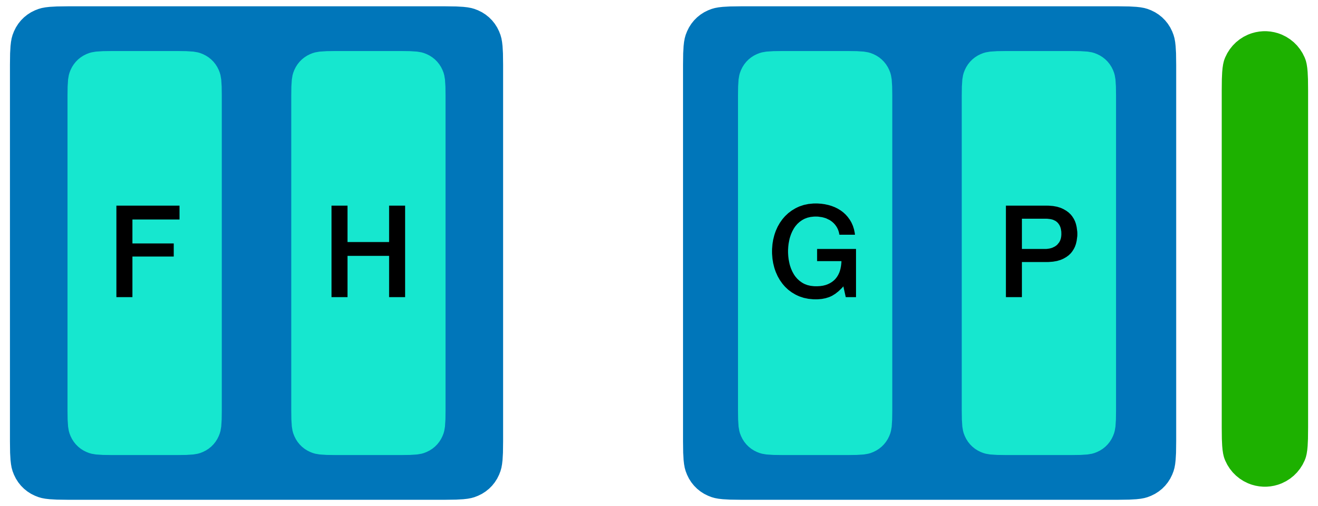

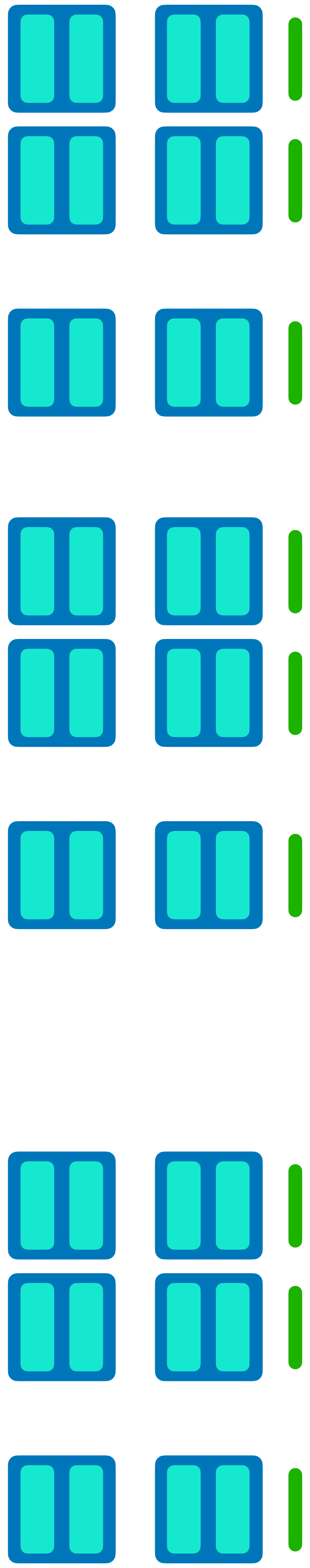

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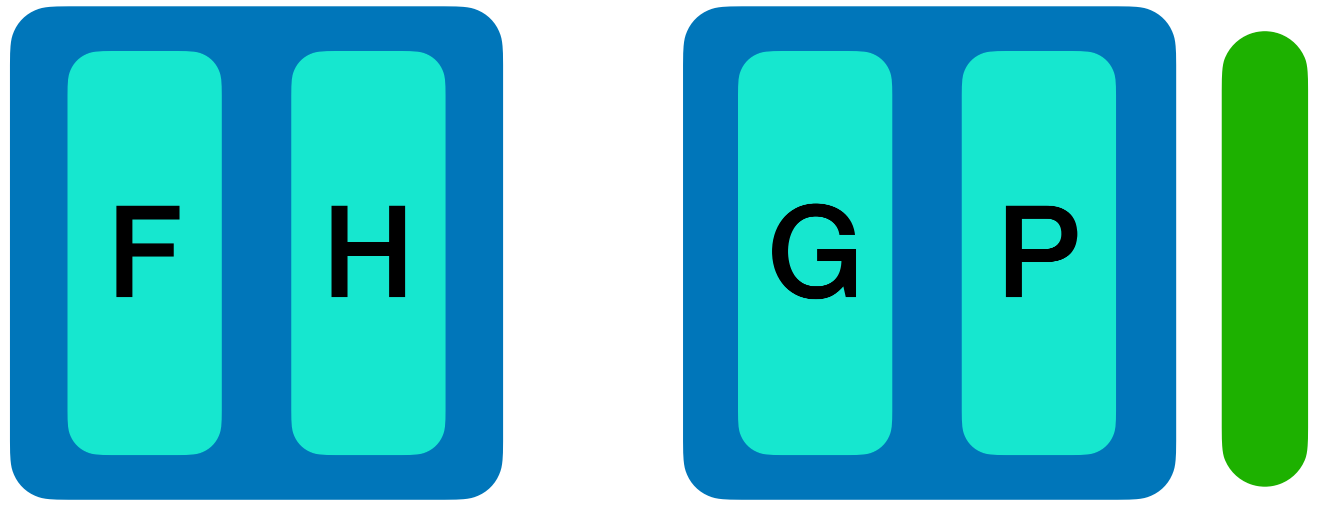

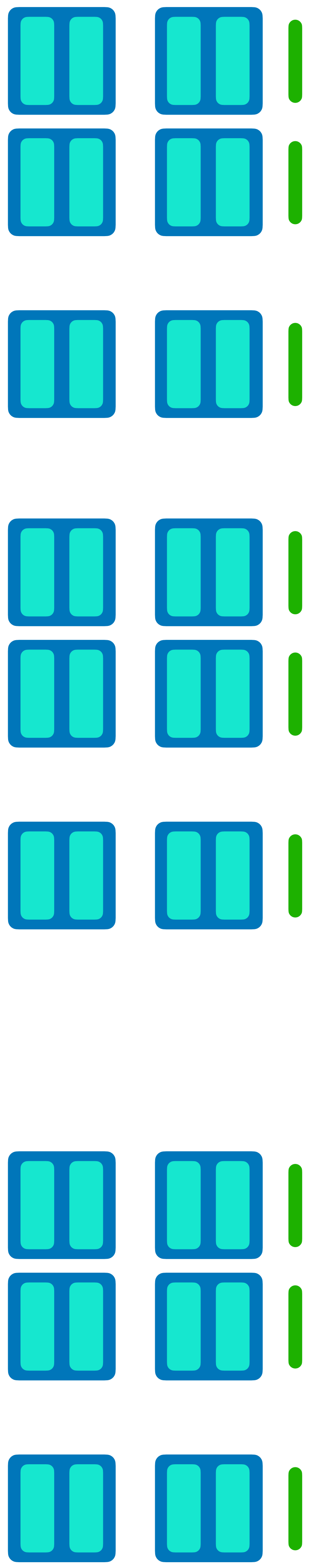

$\alpha_{\vec{z},\vec{z}'}\left|\vec{z}\right\rangle\left\langle\vec{z}'\right|$ $\qquad \vec{z}, \overrightarrow{z'} \in U_{st}$

$\alpha_{\vec{z},\vec{z}'} \cdot (-1)^{g(z_1)+\cdots+g(z_{st})+g(z_1')+\cdots+g(z_{st}')}\left|\vec{z}\right\rangle\left\langle\vec{z}'\right|$

$\alpha_{\vec{z},\sigma}\left|\vec{z}\right\rangle\left\langle\sigma(\vec{z})\right|$ $\qquad \sigma \in S_{st}$

$\nu_\sigma\left|\vec{z}\right\rangle\left\langle\sigma(\vec{z})\right|$

$$\approx \frac{(N-st)!}{N!}\sum_{\vec{z}\in U_{st}\ ,\ \sigma\in S_t^s}\left|\vec{z}\right\rangle\left\langle\sigma(\vec{z})\right|$$

$$\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_{st} \end{bmatrix}, z_i \neq z_j \ for\ i \neq j \qquad S_t^s = \underbrace{S_t \times \cdots \times S_t}_{s\ times}$$

Proof Overview

1. F, H: Get flat vectors
2. $\approx$ Unique entries
3. G: Remove $\left|\vec{z}\right\rangle\left\langle\vec{z}'\right|$ for $\vec{z}, \vec{z}'$ with different entry histogram
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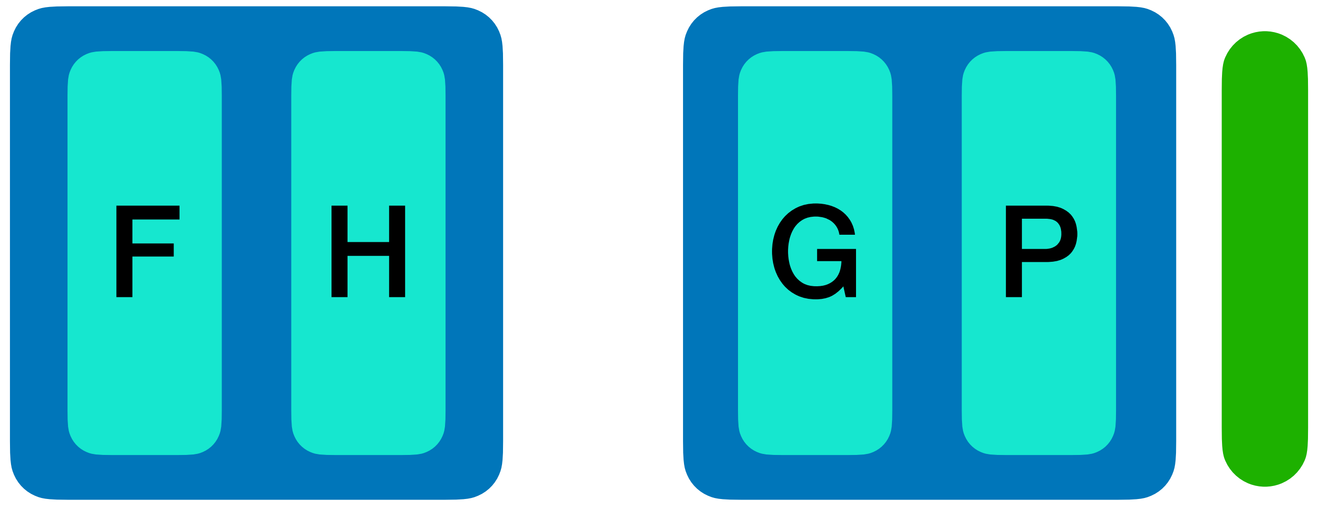

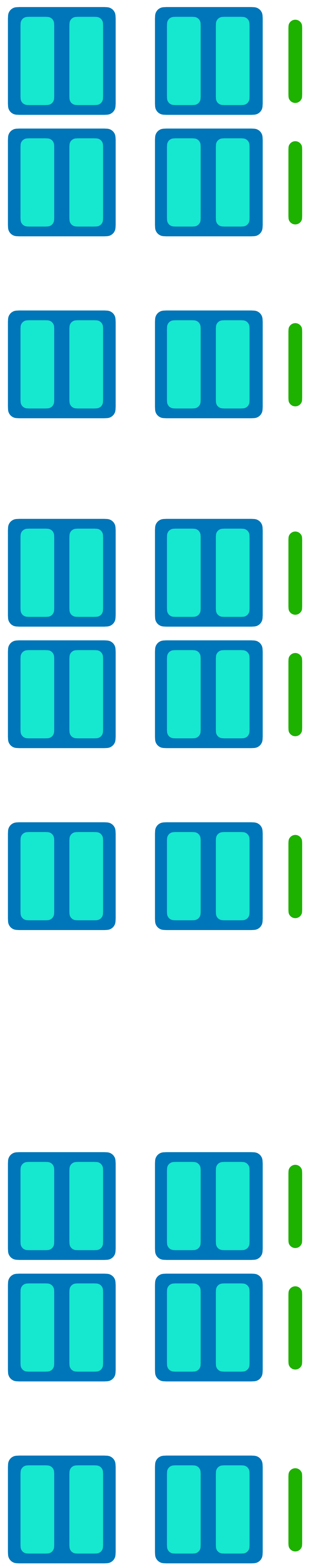

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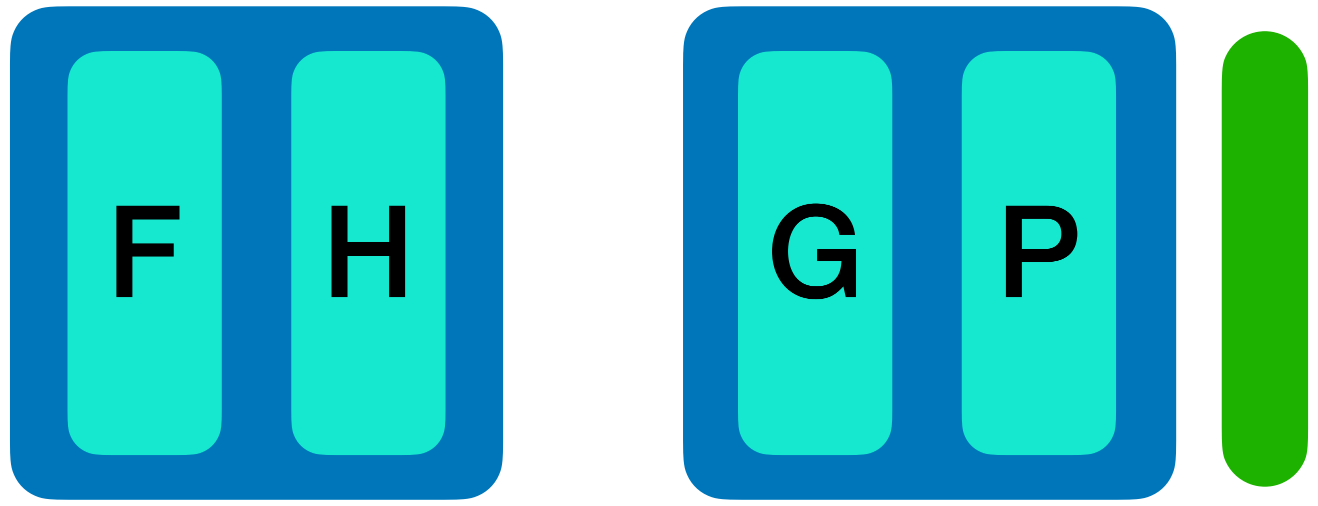

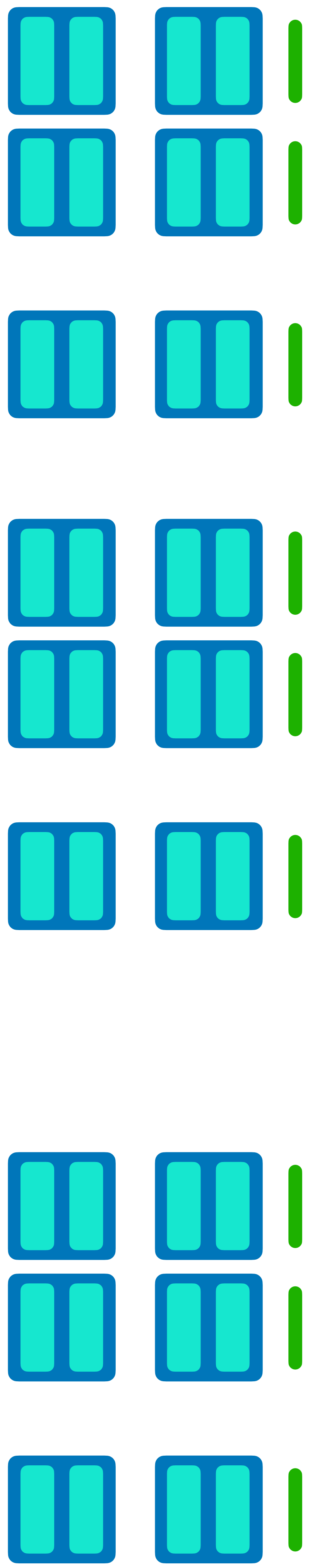

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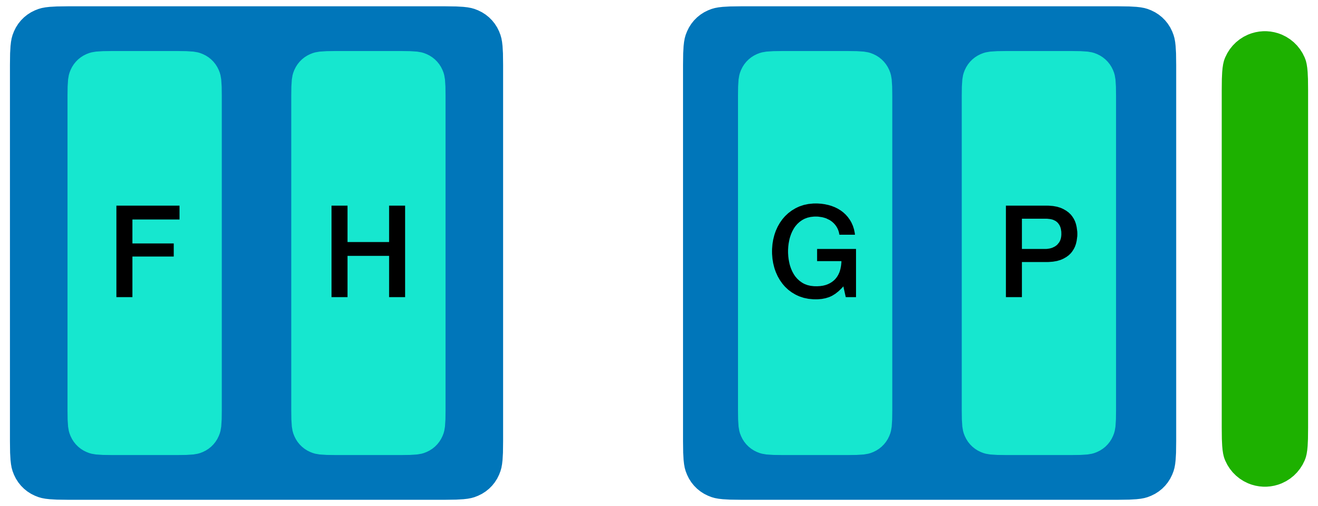

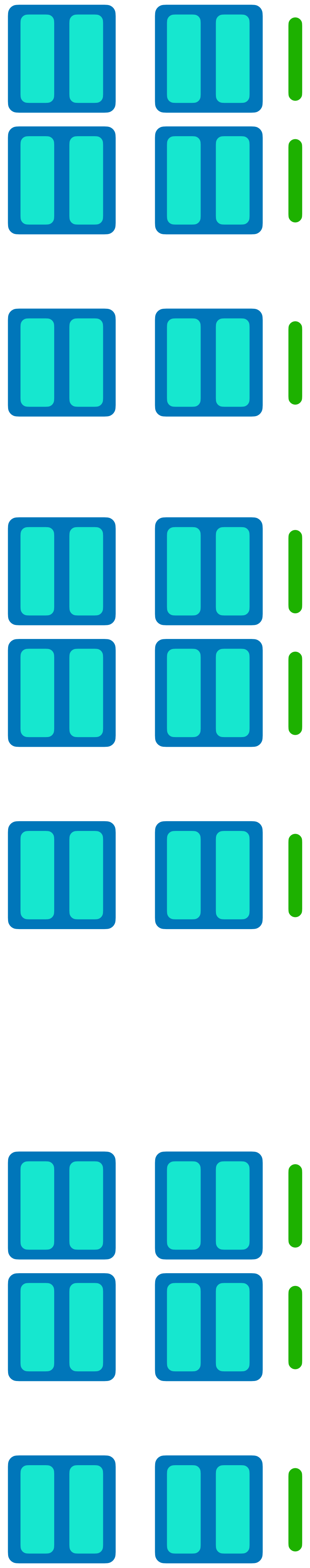

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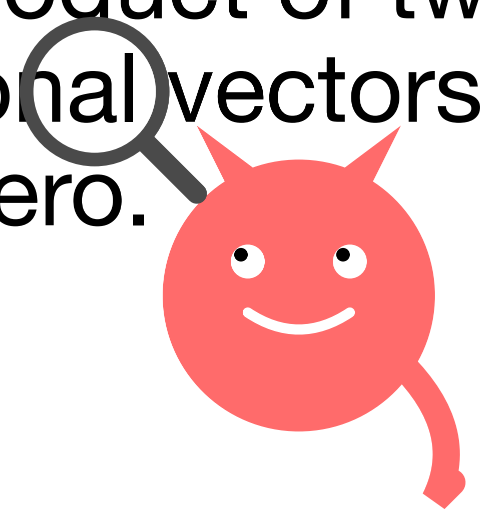

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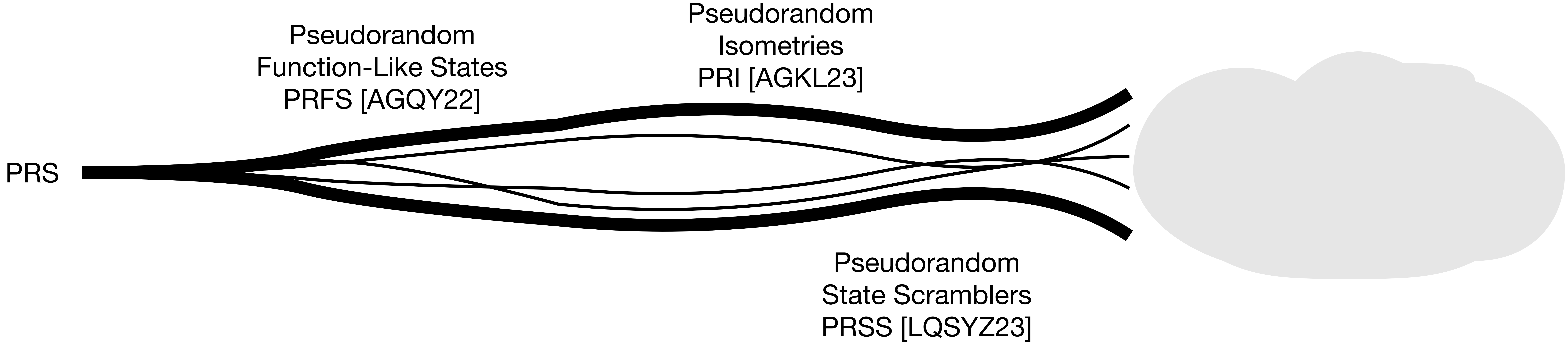
A Plethora of Pseudorandom Objects



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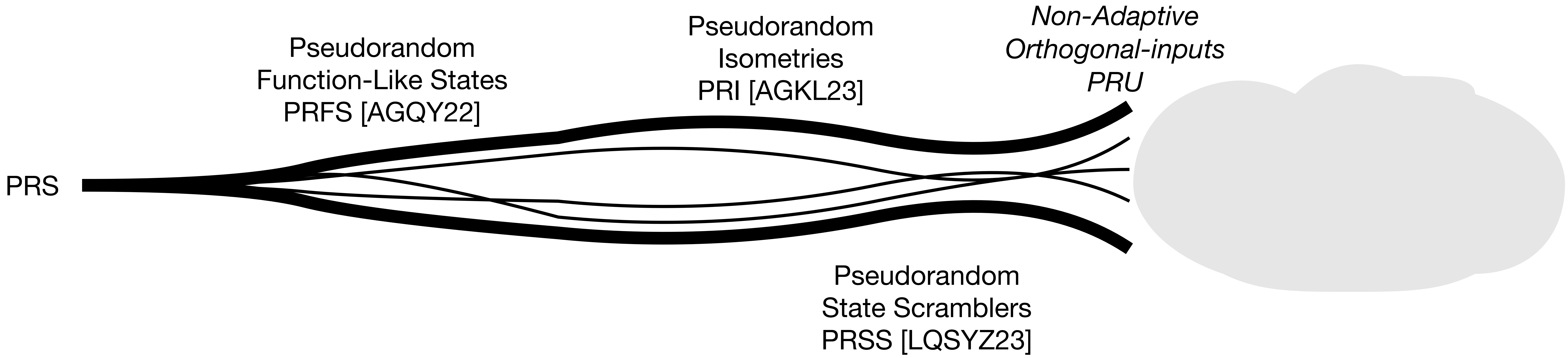

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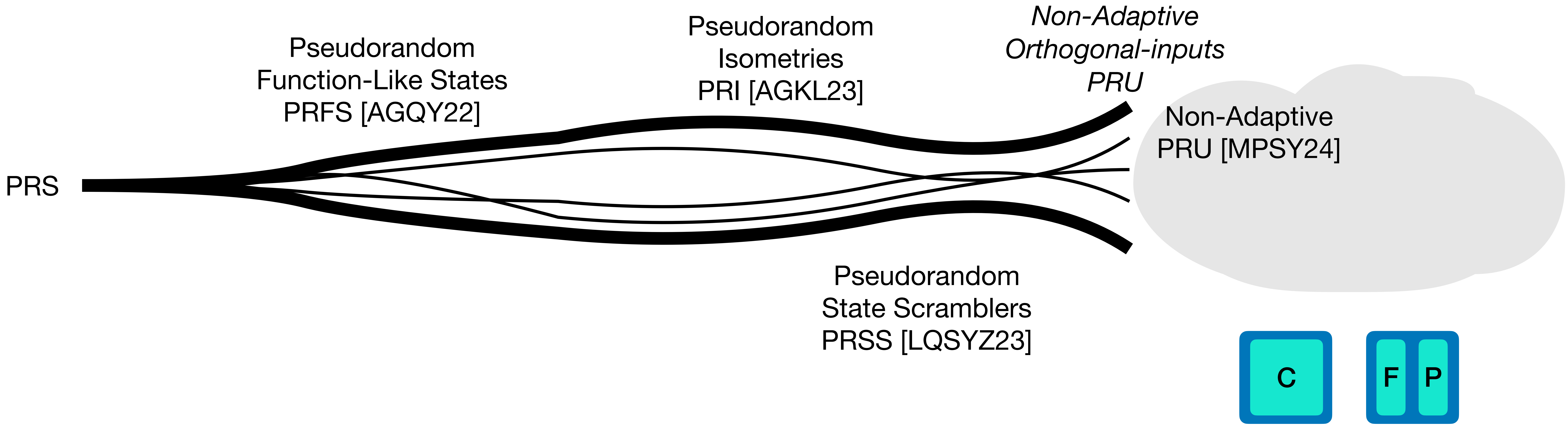

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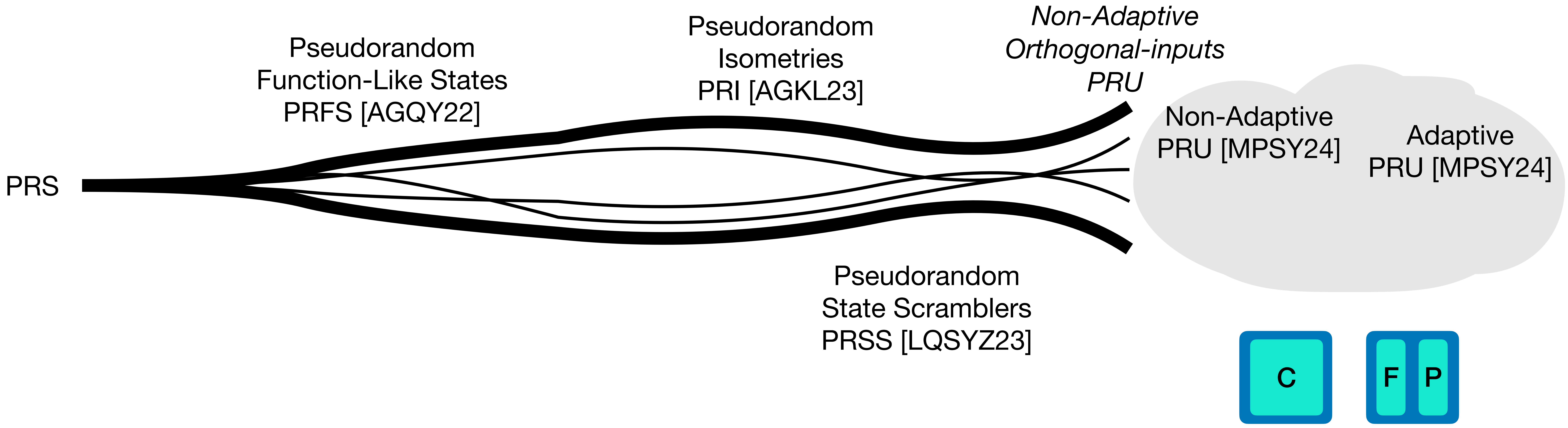

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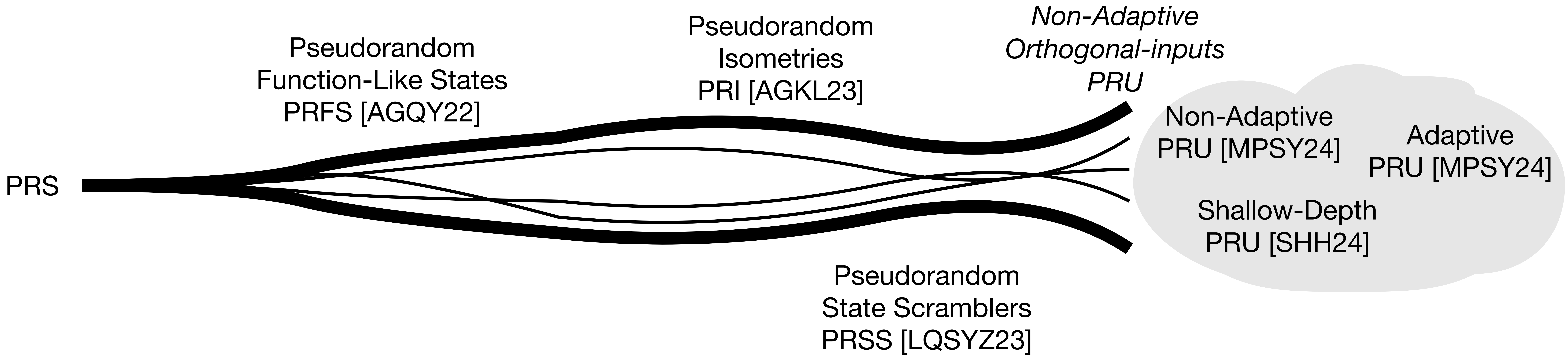

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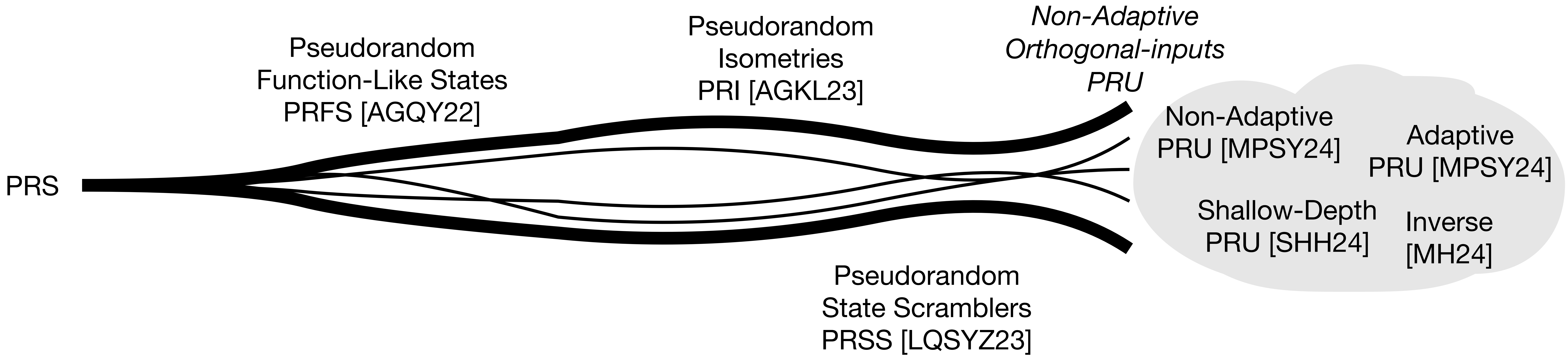

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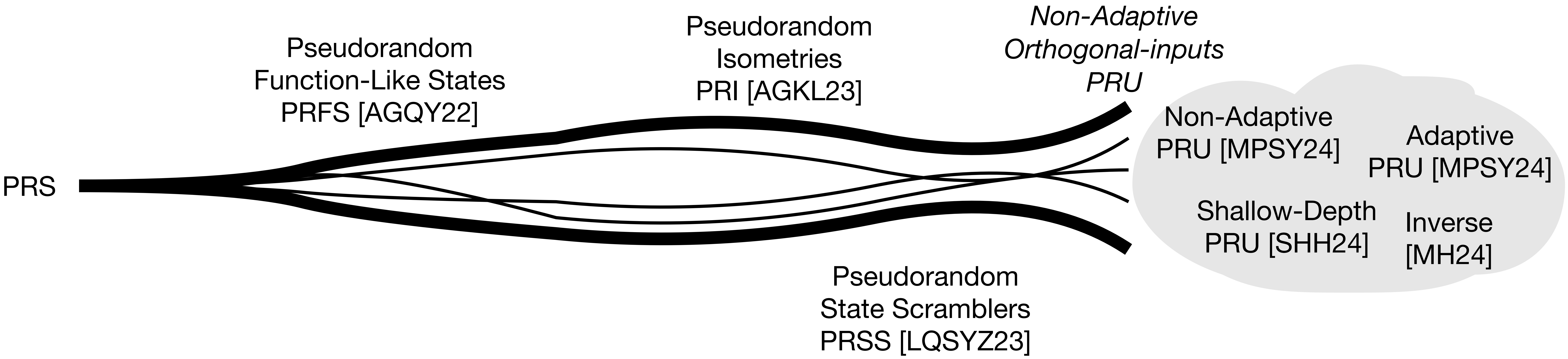

Pseudorandom unitaries are neither real nor sparse nor noise-robust [HBK23]

Future Directions

- Can we prove security for *tensor-product* inputs?
- In what cases is the security definition sufficient, and what does it imply on the need for an imaginary part?