Key-Homomorphic and Aggregate Verifiable Random Functions

G. Malavolta (Bocconi University)

European Research Council

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Verifiable Random Functions [MRV99]

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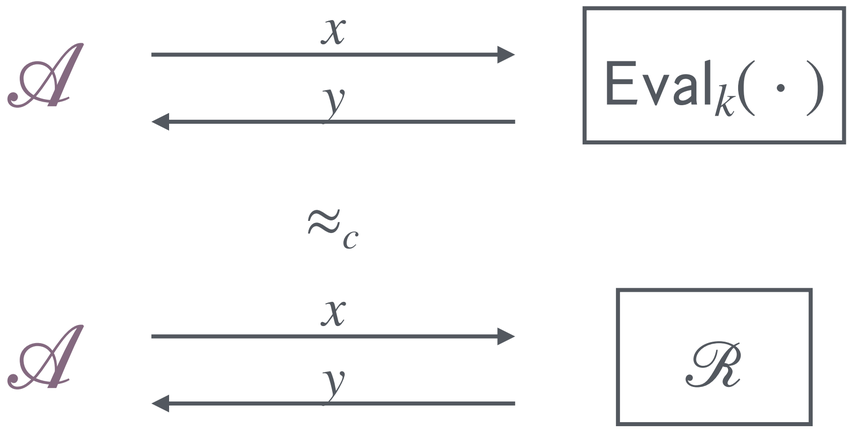

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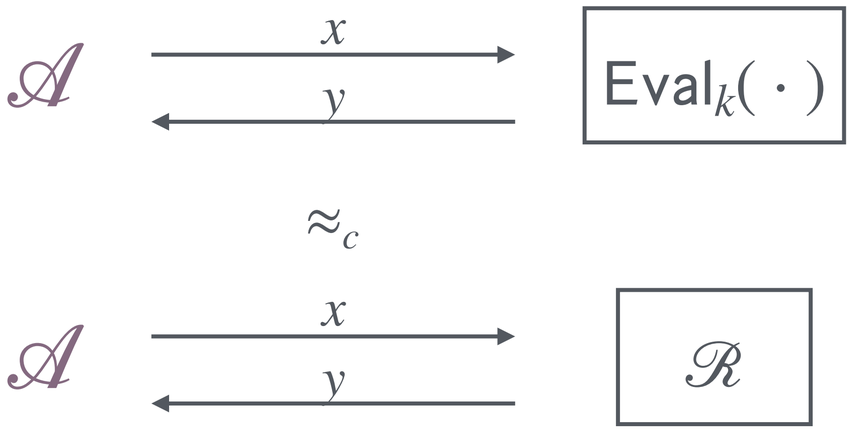


UNIQUENESS

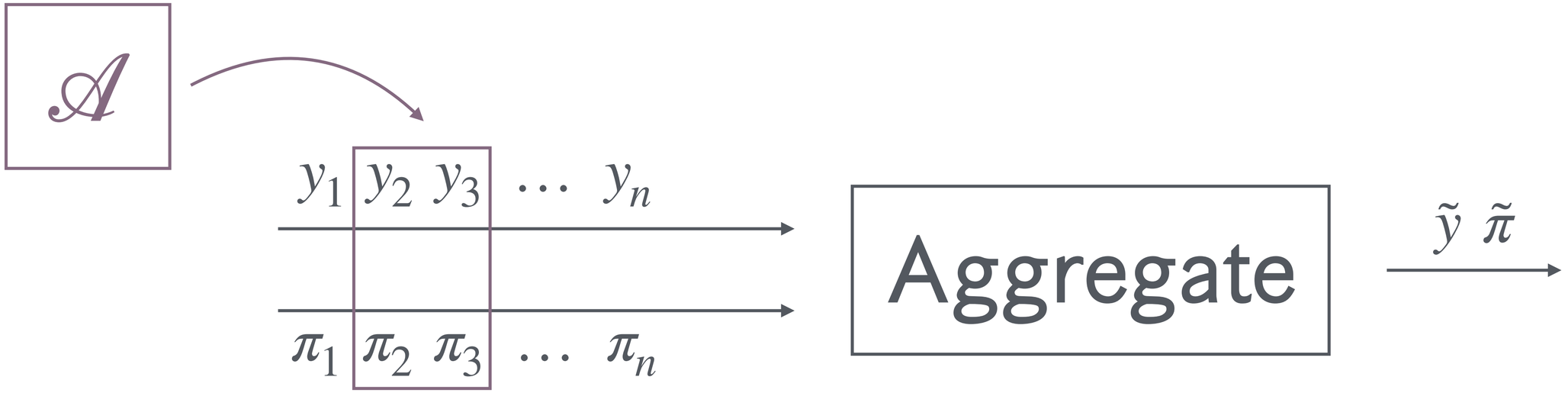
$\mathsf{Prove}_k(x) \to \pi$

$\mathsf{Verify}(x, y, \pi) \to \{0,1\}$

$$\Pr_{\mathcal{A}}\left[\mathsf{Verify}(x^*, y^*, \pi^*) = 1 : y^* \neq \mathsf{Eval}_k(x^*)\right] \approx 0$$

$(x^*, y^*, \pi^*) \leftarrow \mathcal{A}$


$y_1\ y_2\ y_3\ \cdots\ y_n$
$\pi_1\ \pi_2\ \pi_3\ \ldots\ \pi_n$
Aggregate
$\tilde{y}\ \tilde{\pi}$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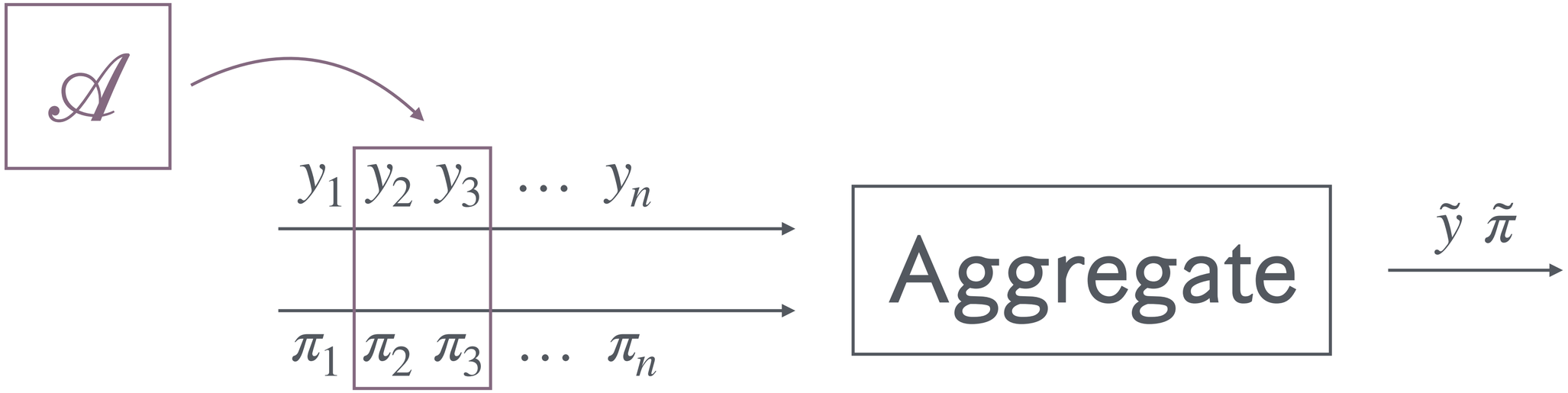
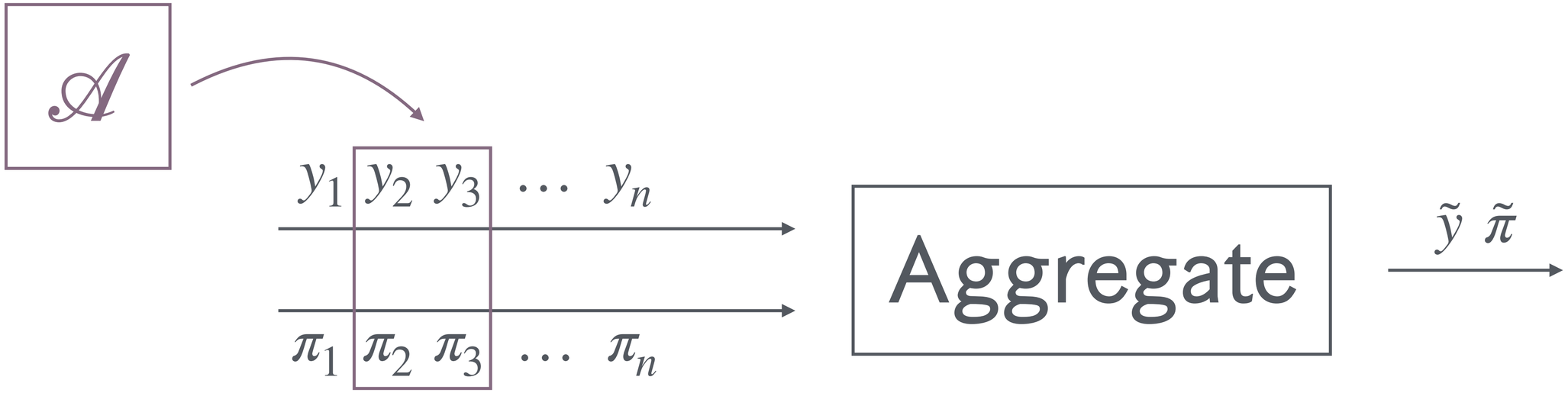


- Well-studied for unpredictable functions (= digital signatures)



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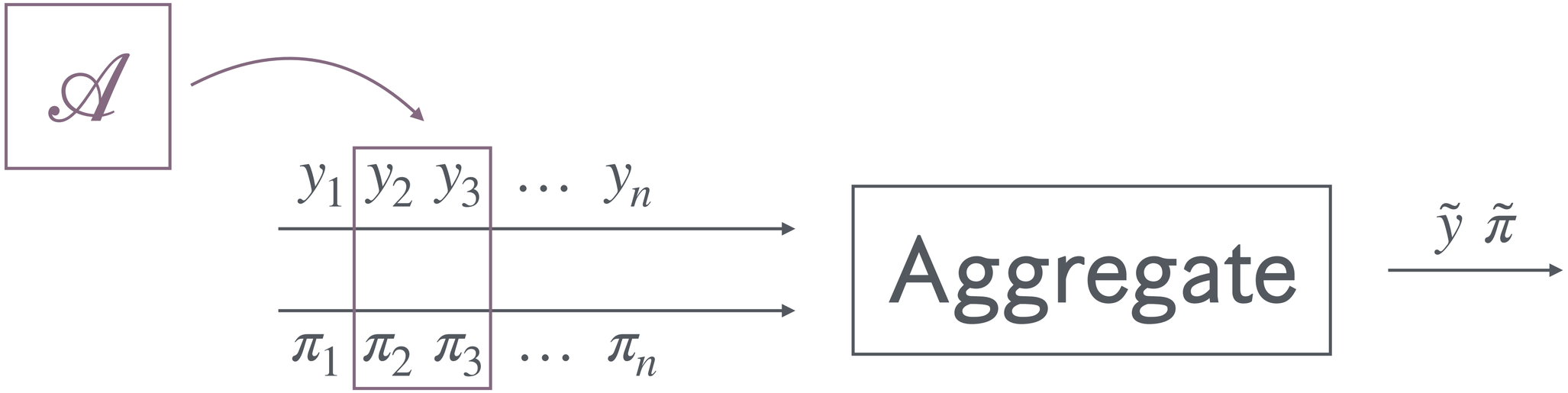

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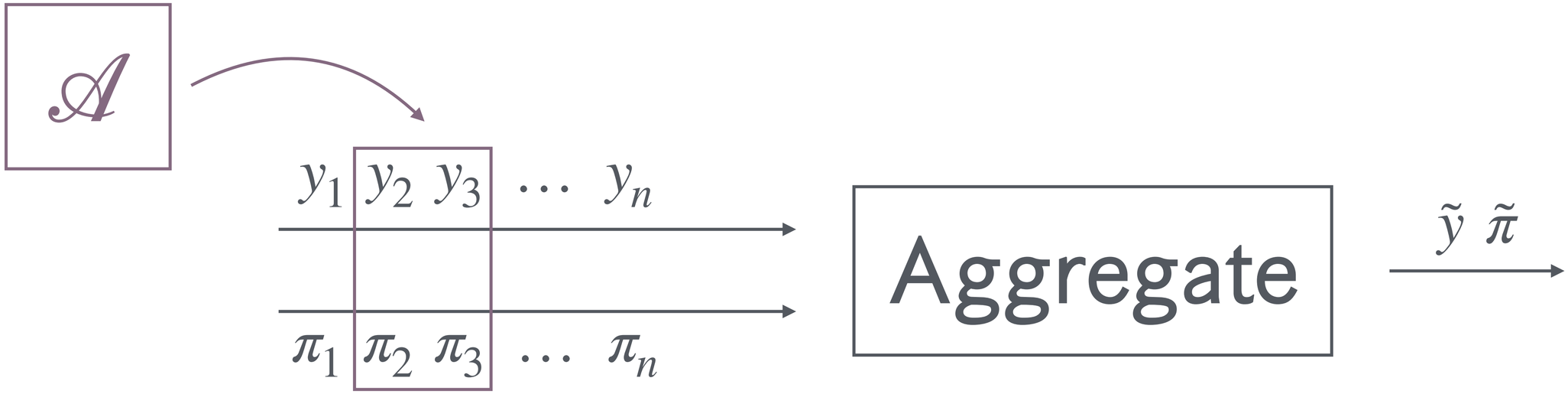

- Well-studied for unpredictable functions (= digital signatures)
- Ideal aggregate signature
- Simple one-round distributed VRF
- Many other conceivable applications

Results

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- Definitions

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  - (Aggregate) Pseudorandomness

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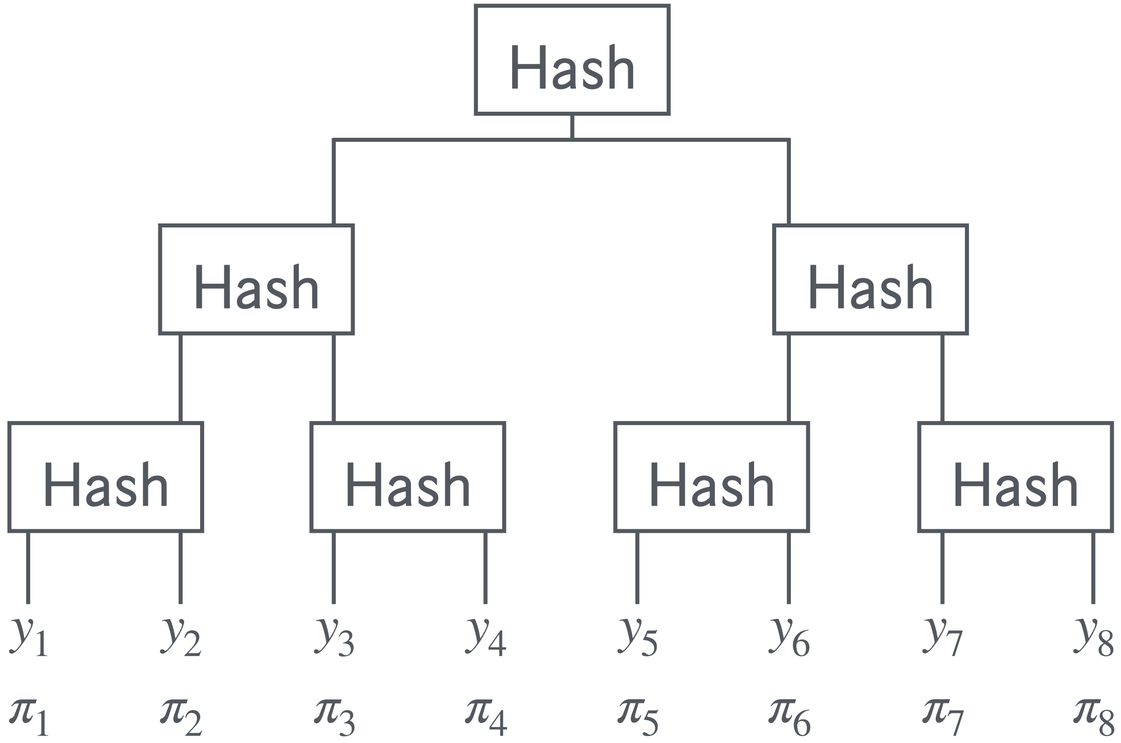
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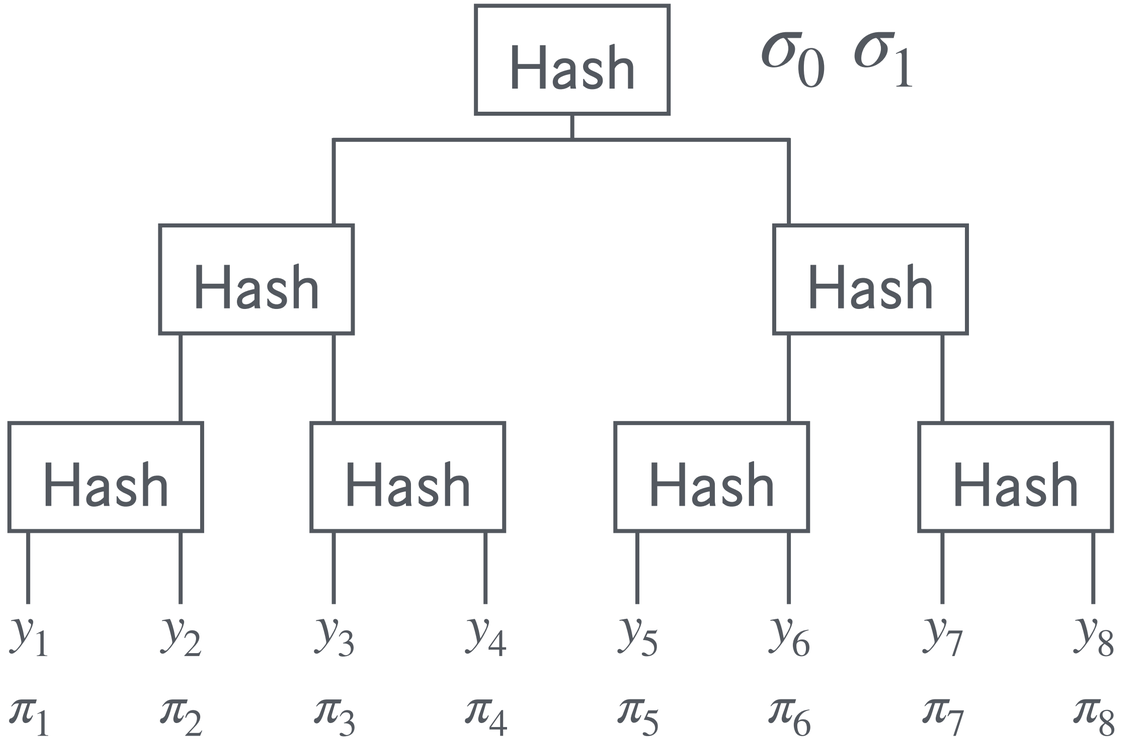
BATCH ARGUMENTS FOR NP



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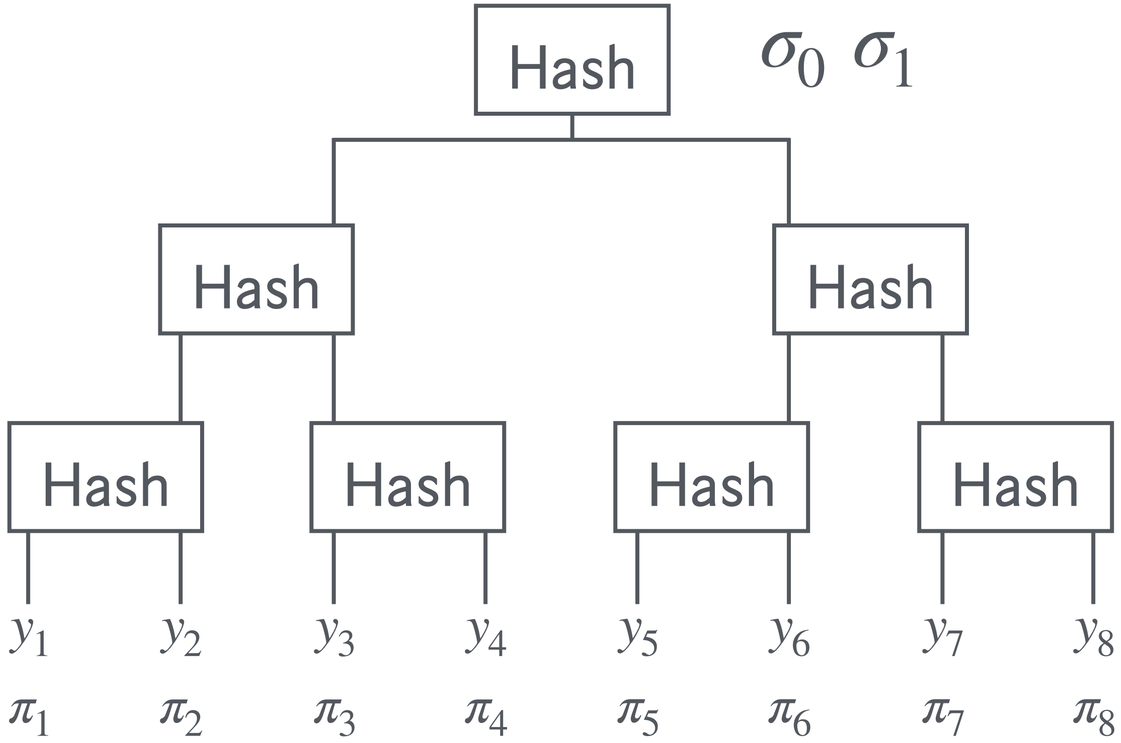

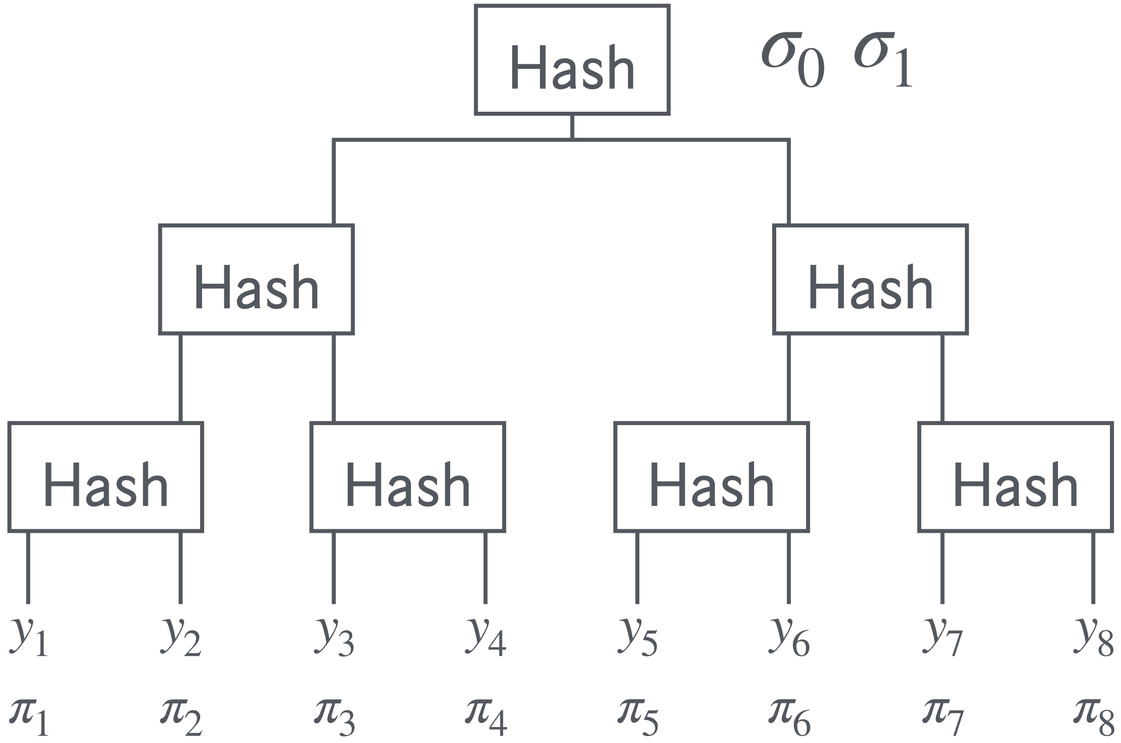

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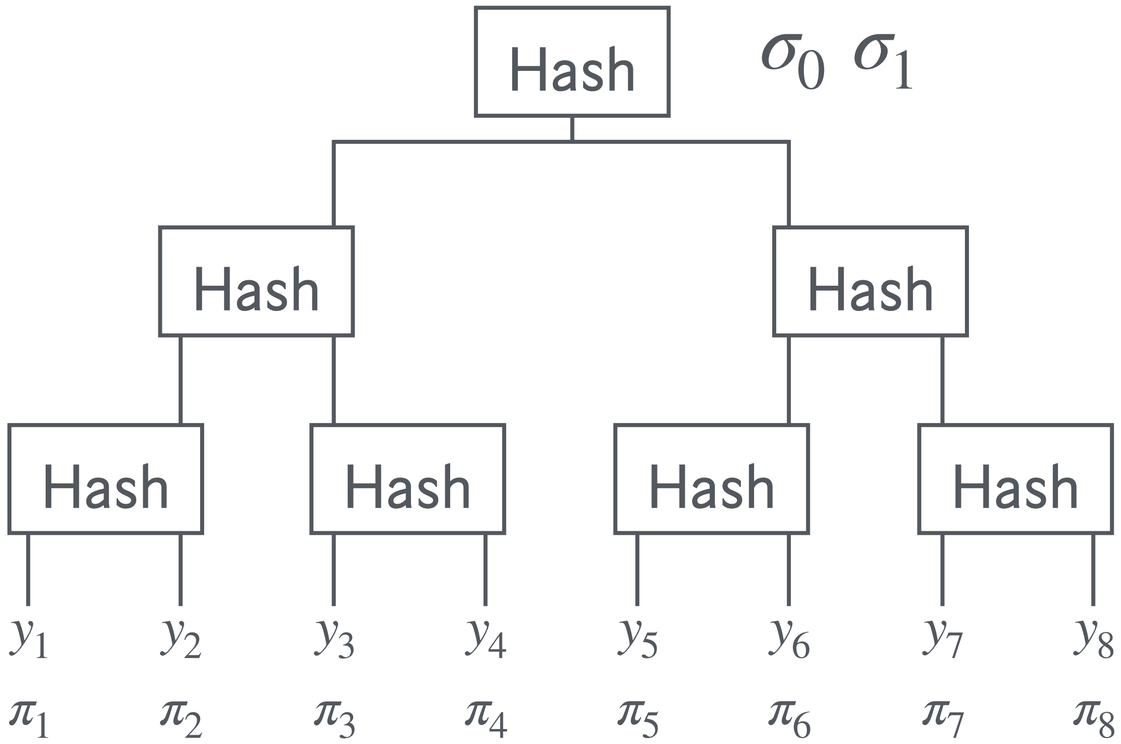

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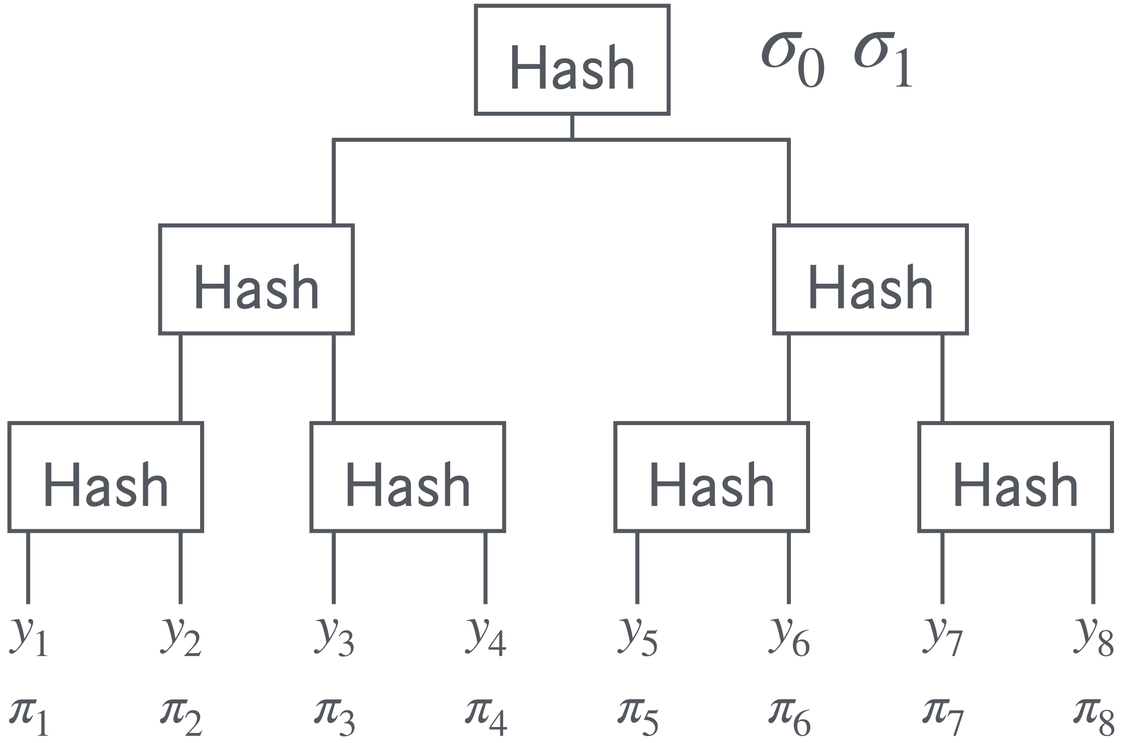

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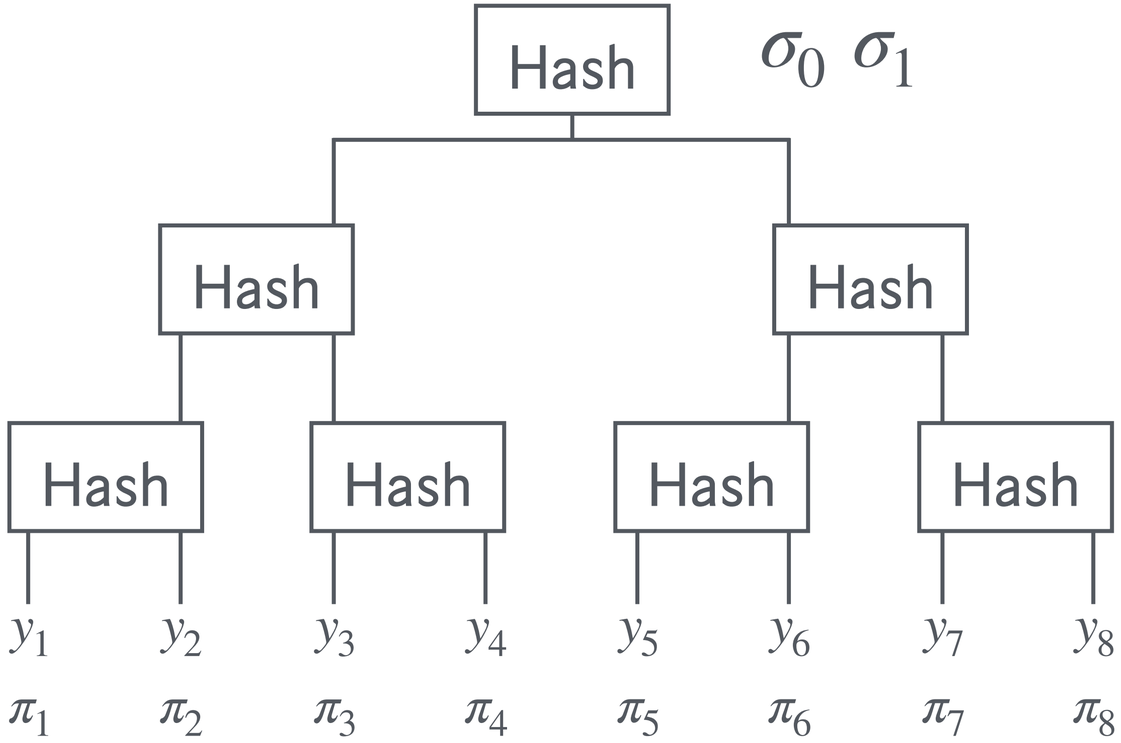

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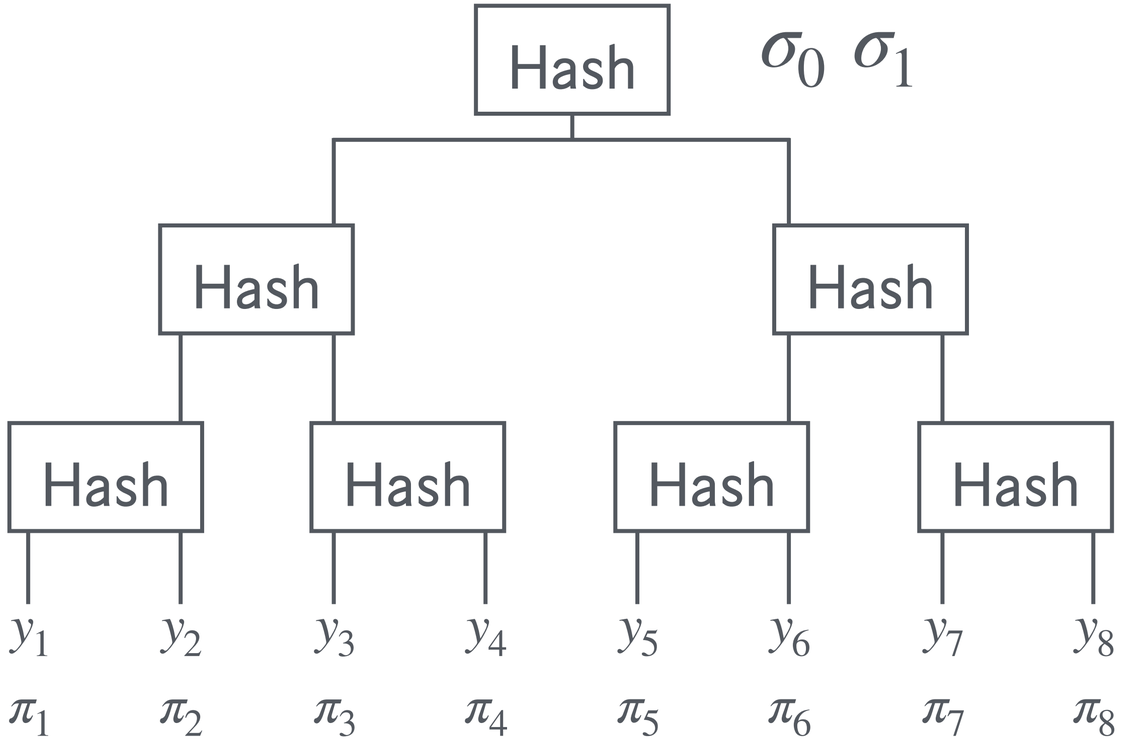

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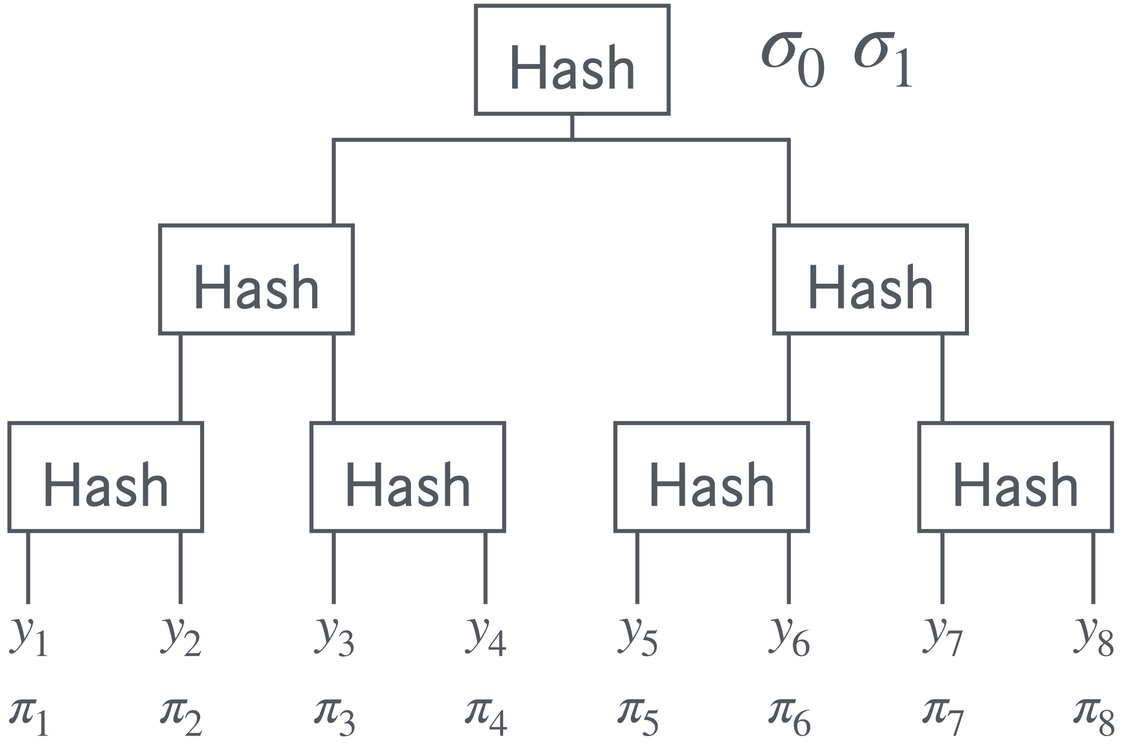

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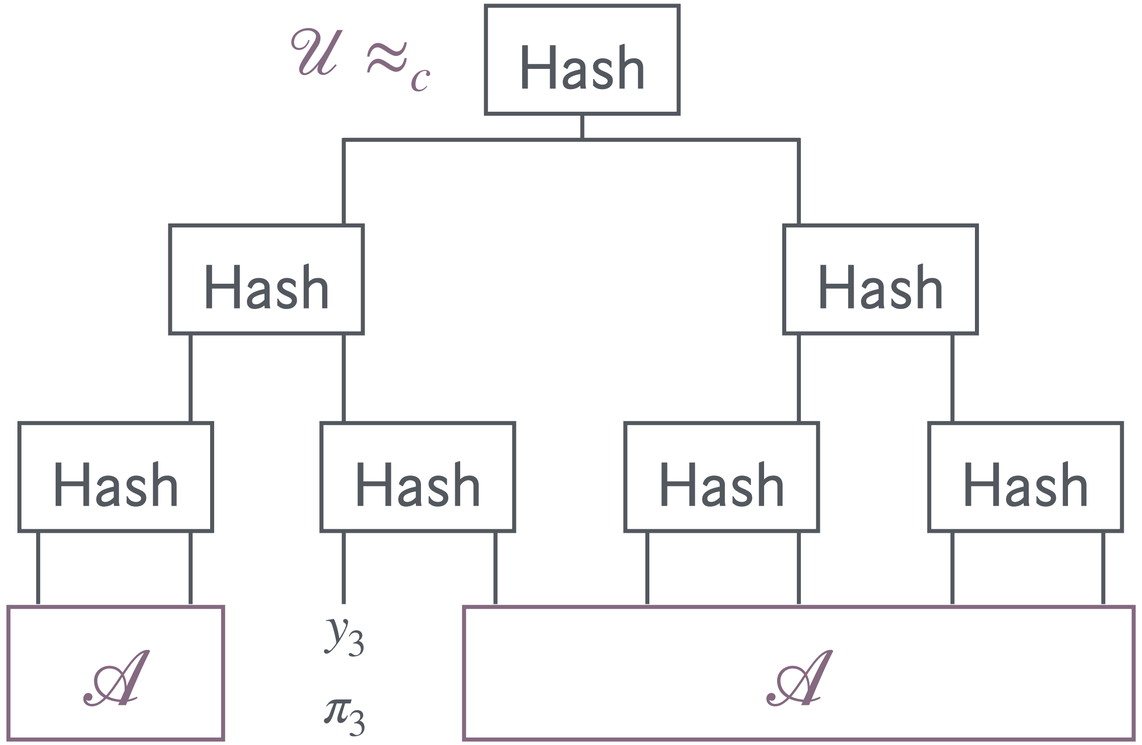
  2. Index-hiding

(AGGREGATE) PSEUDORANDOMNESS



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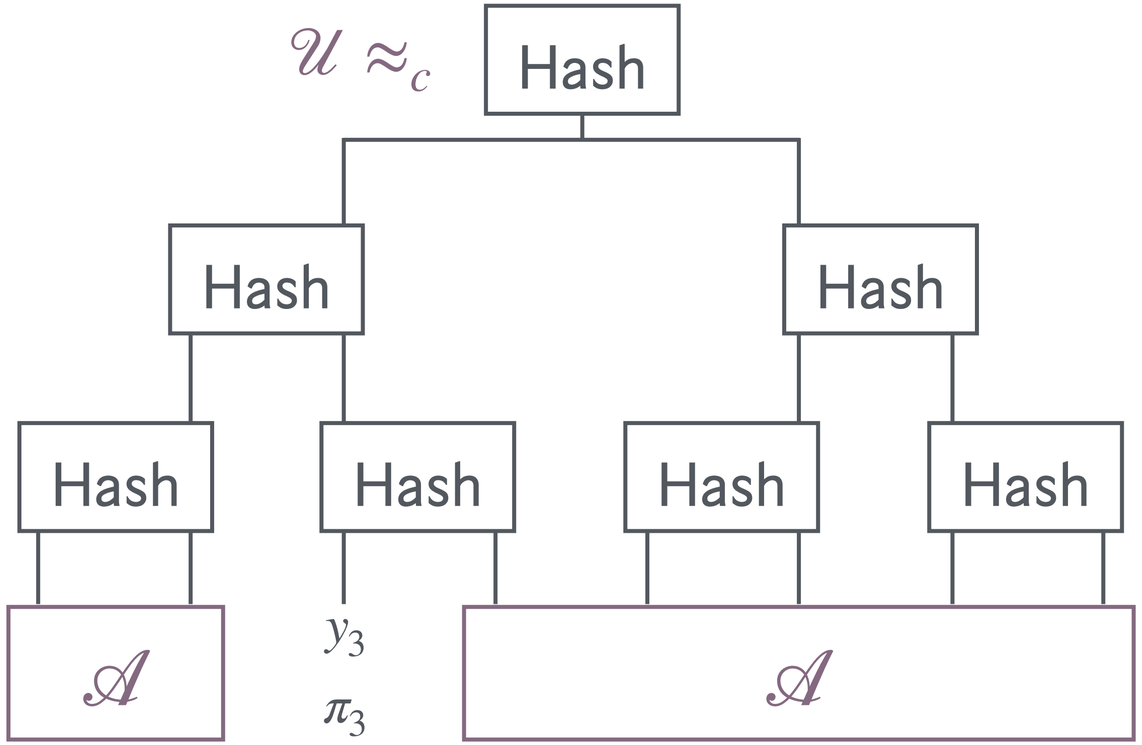

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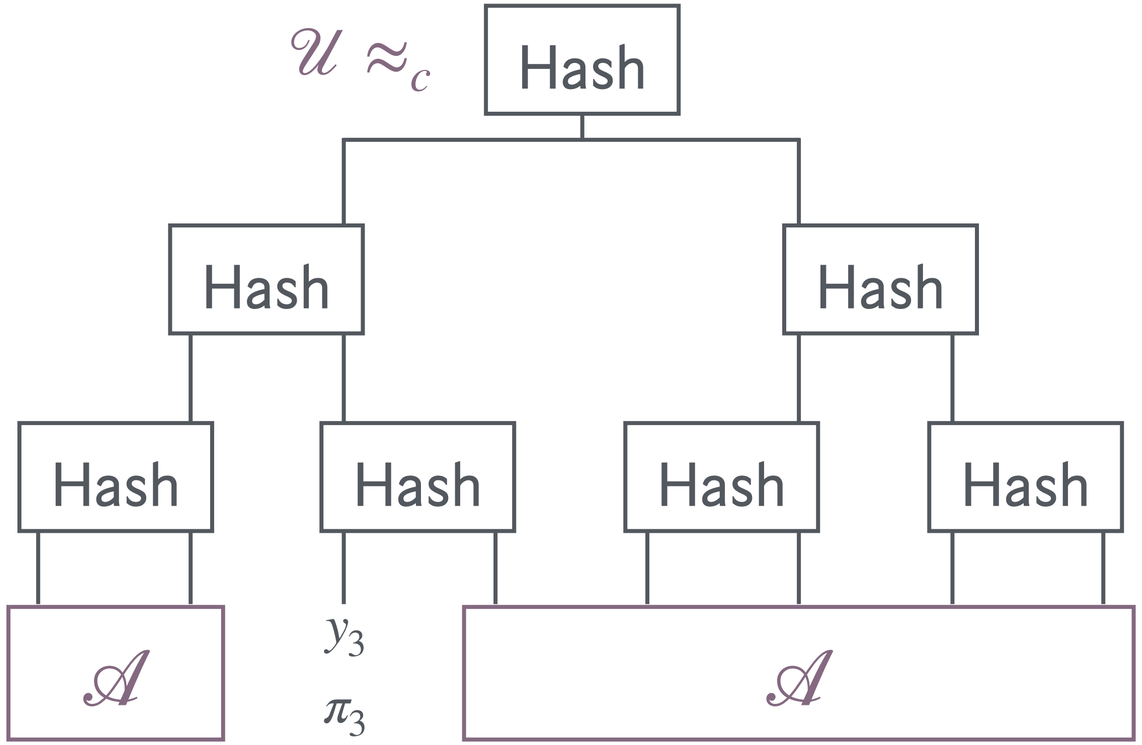

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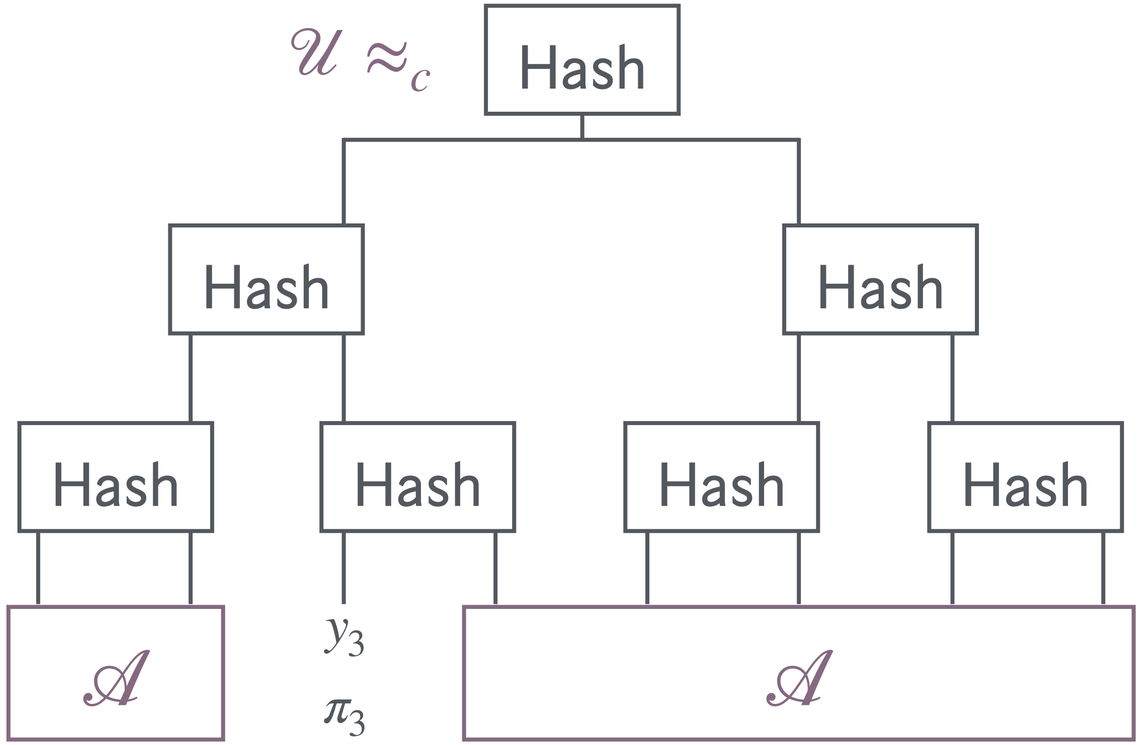

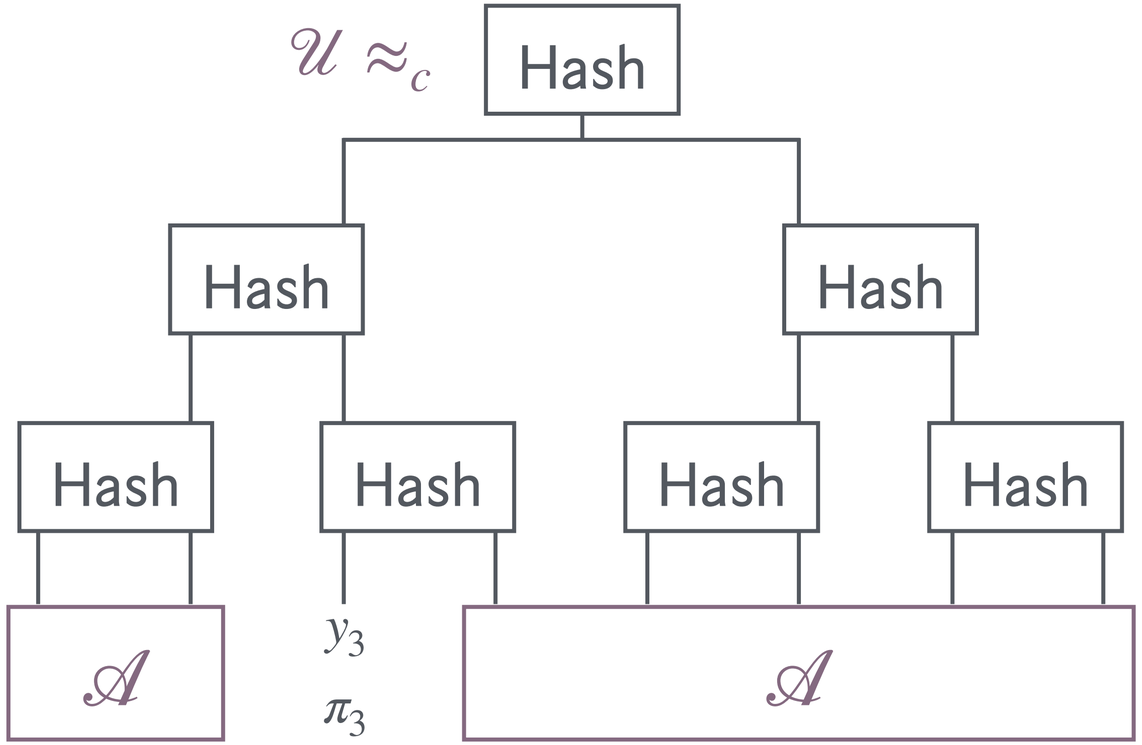

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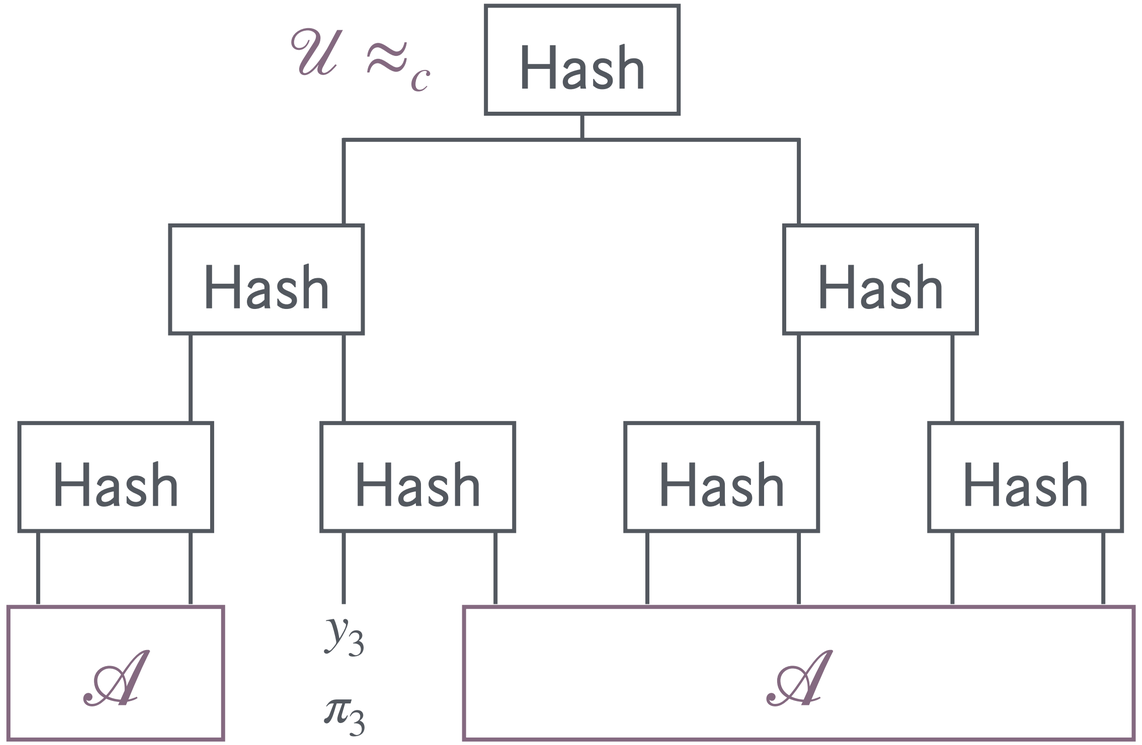

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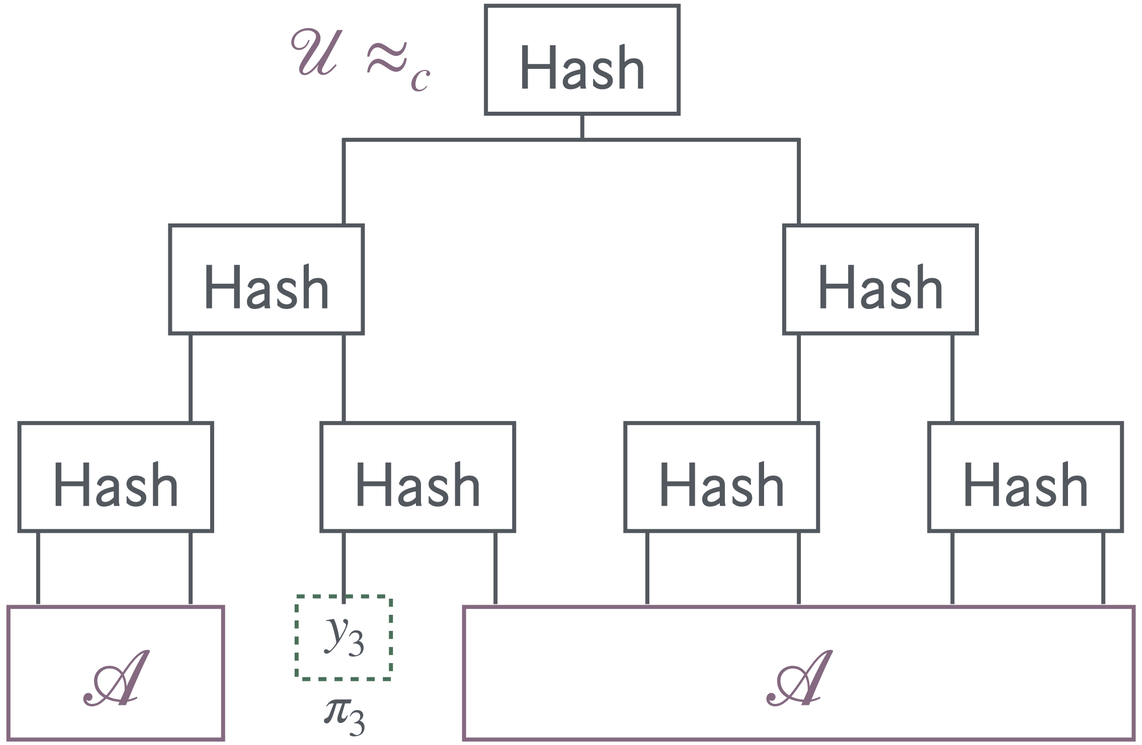

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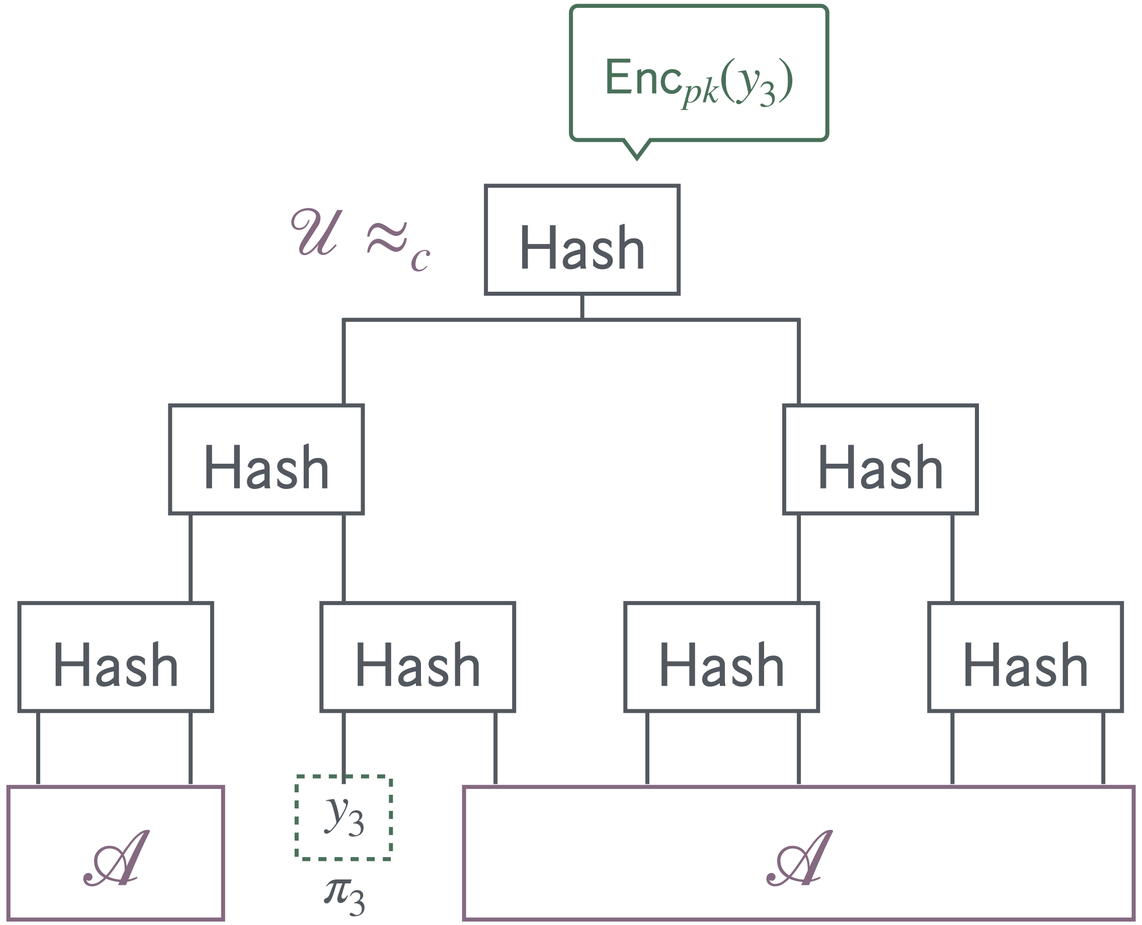

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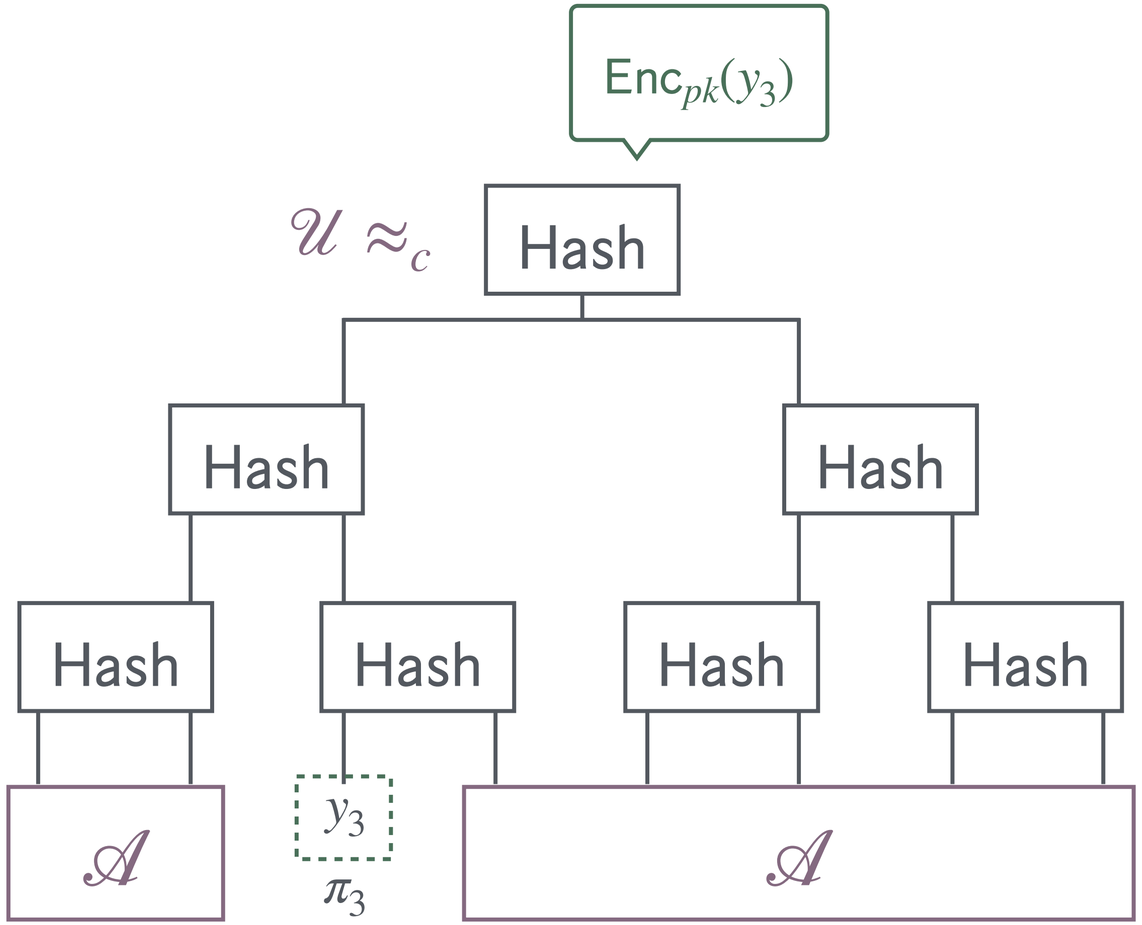

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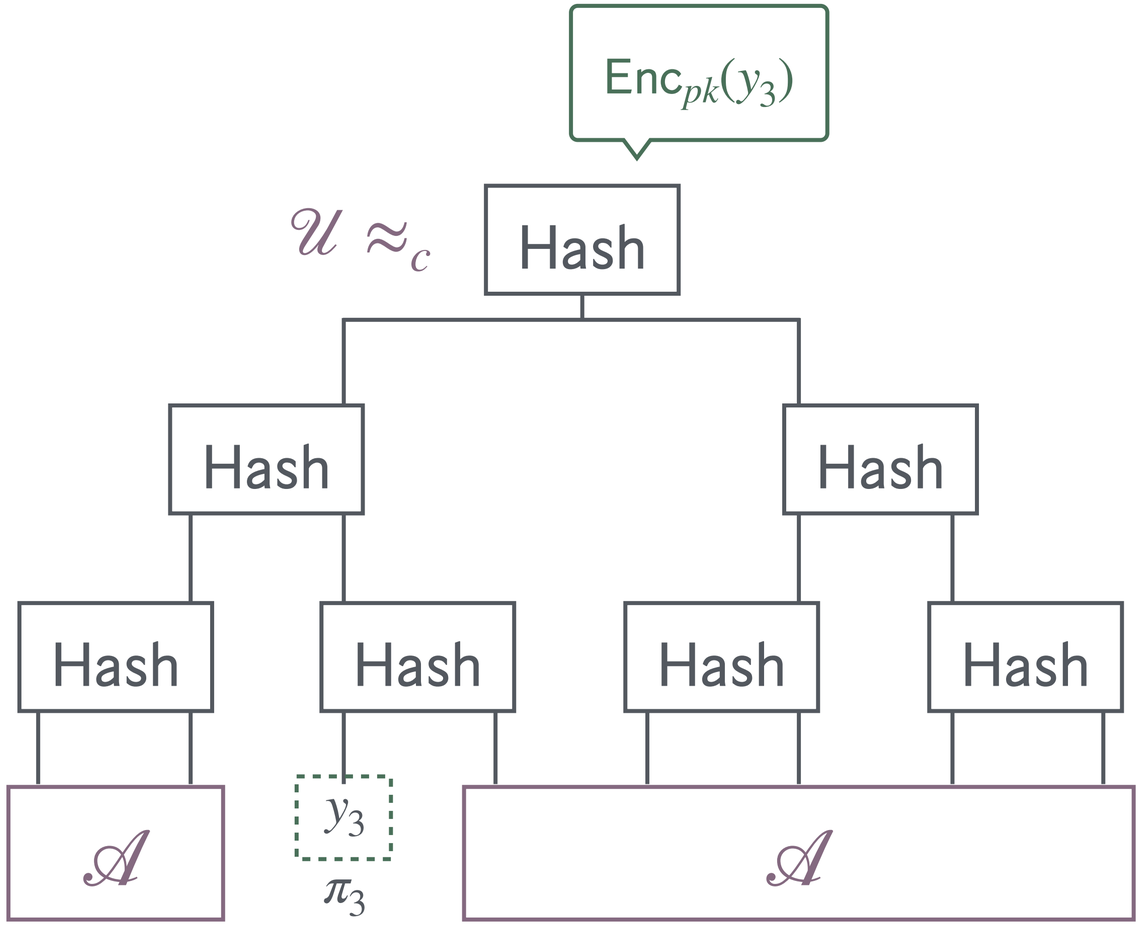

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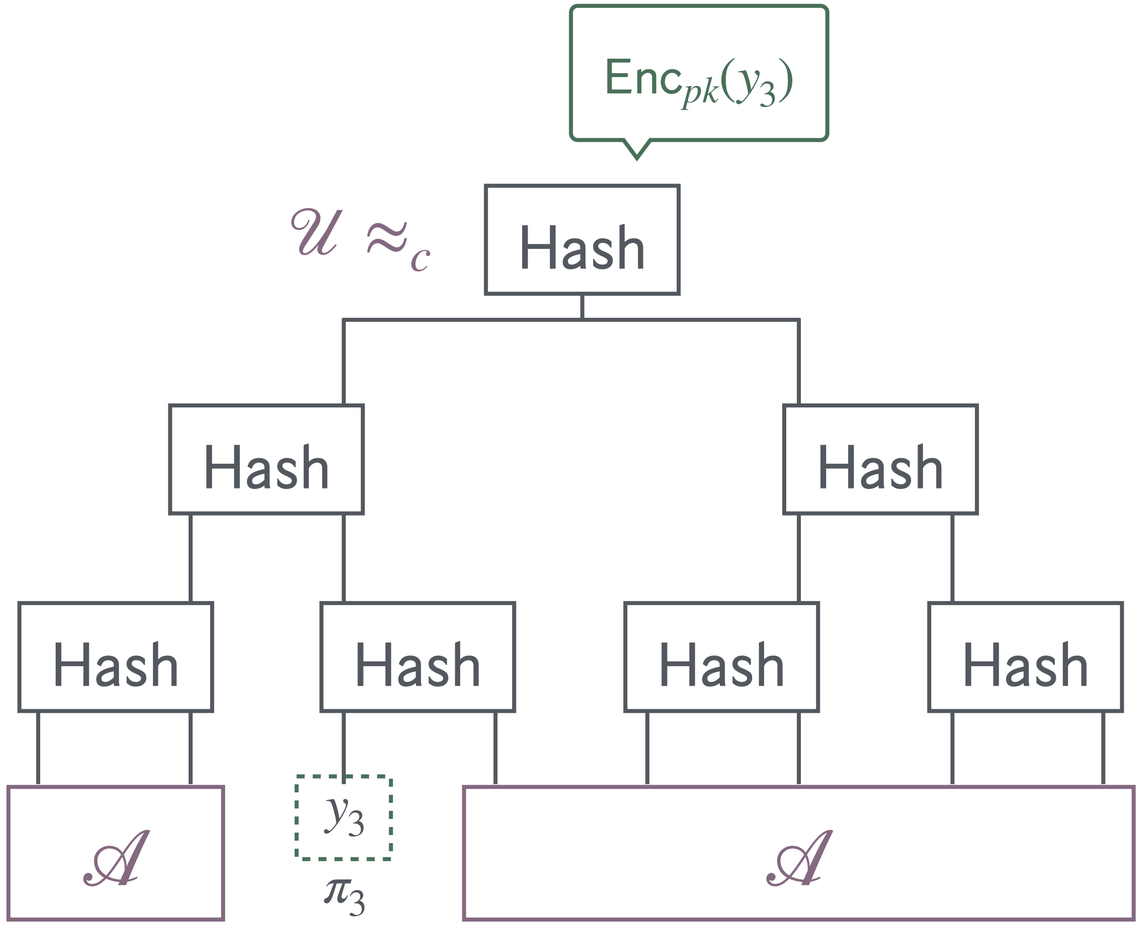

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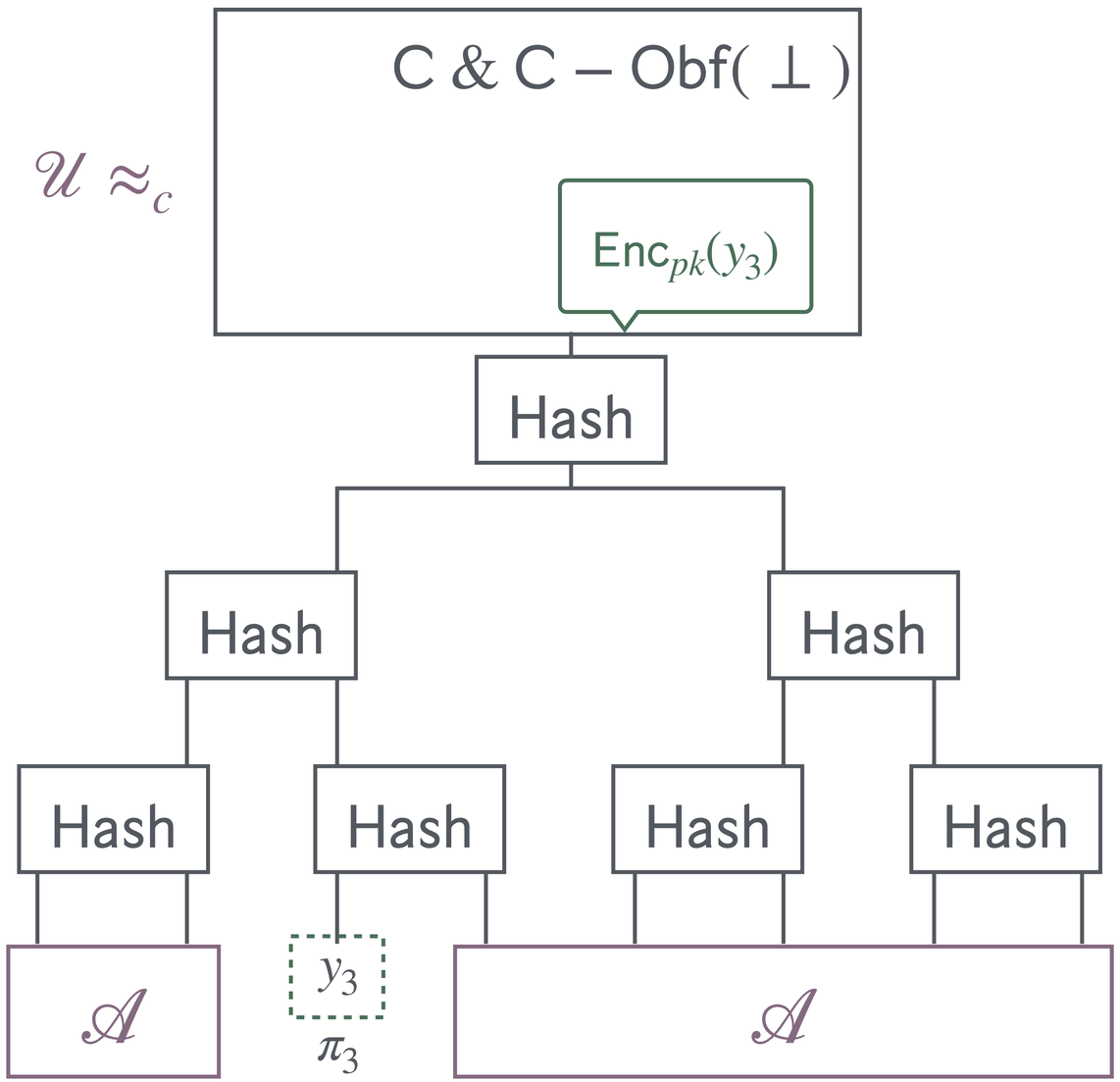

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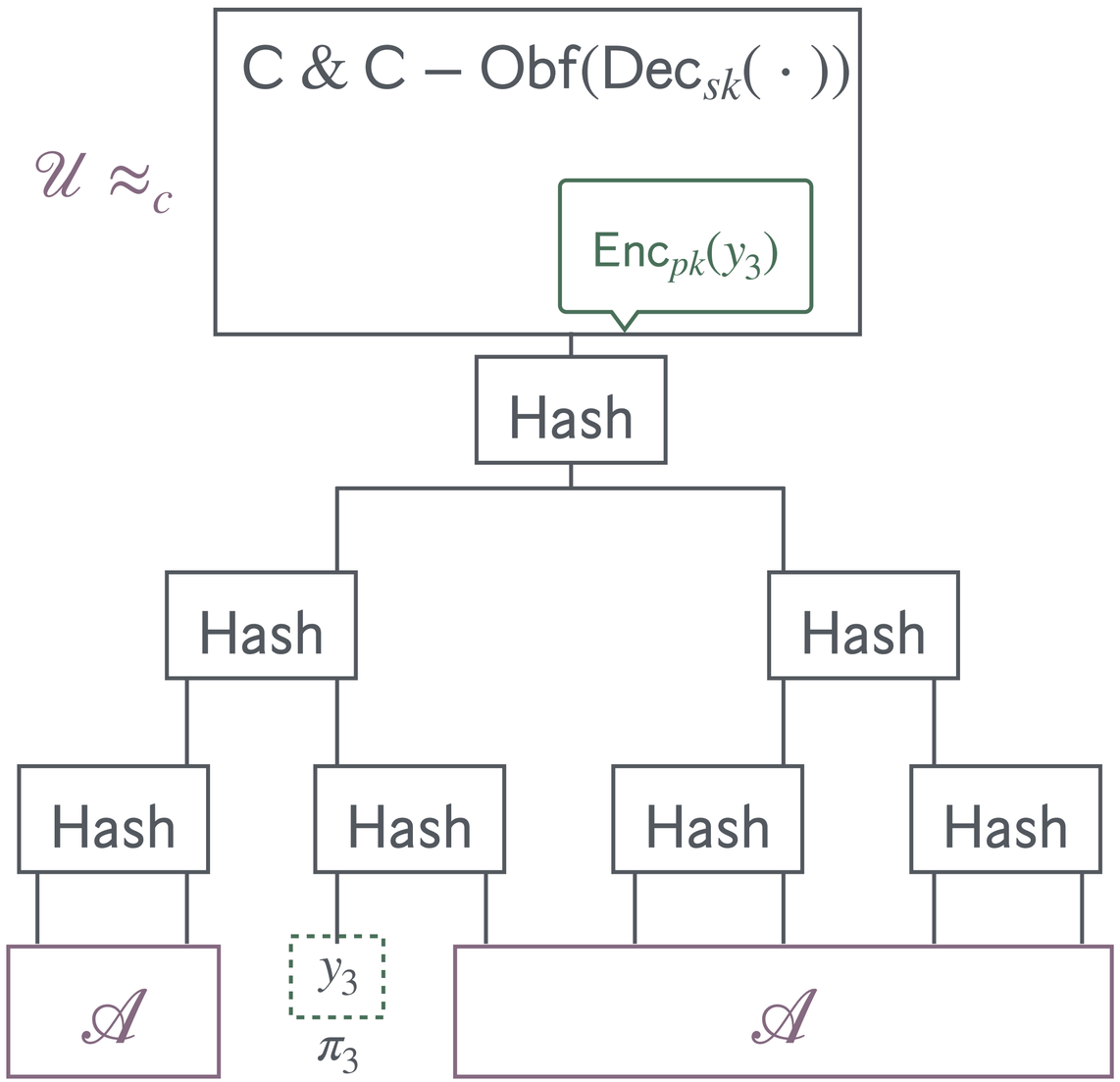

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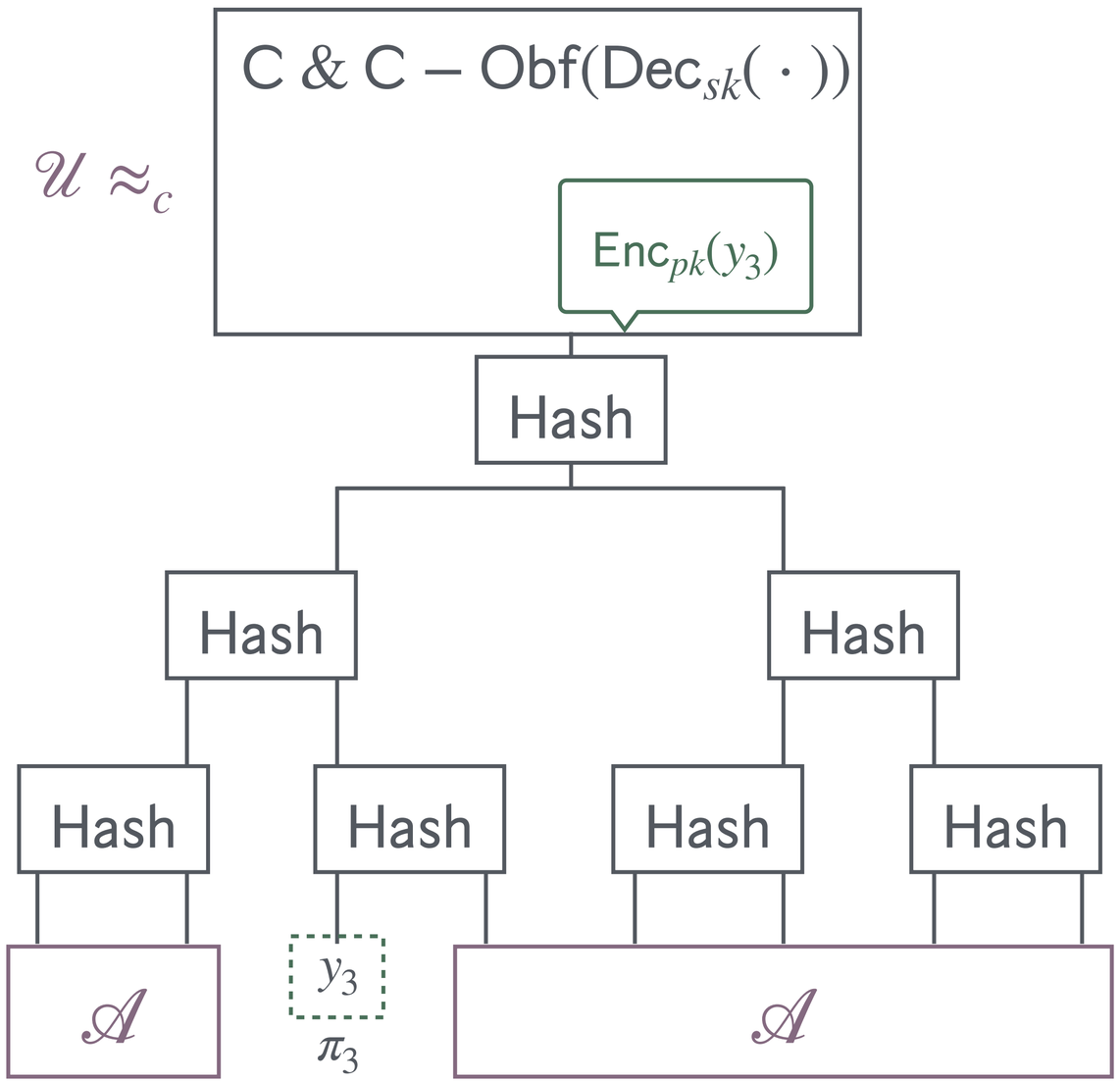

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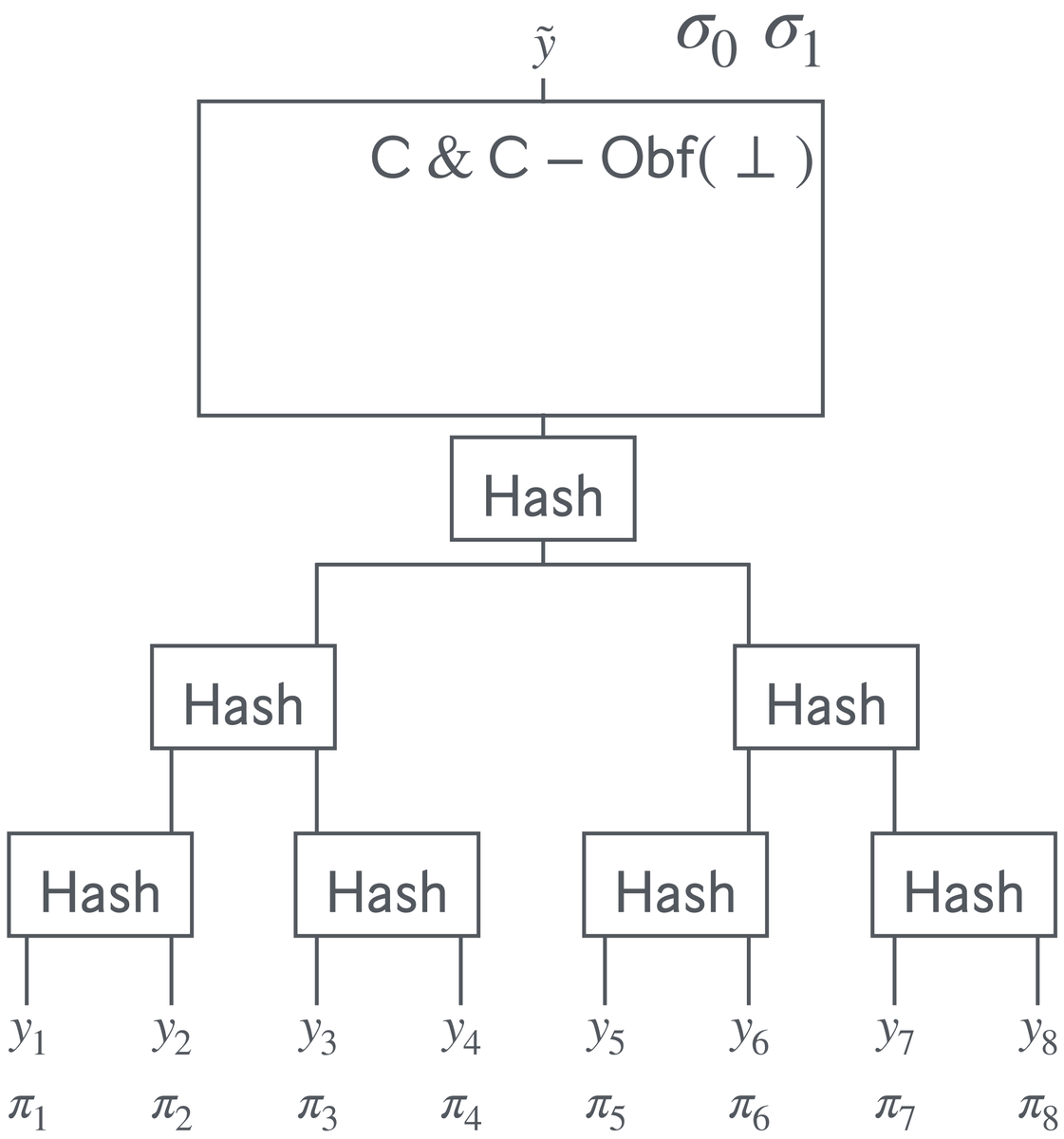
  - Use the randomness extraction property of random matrices



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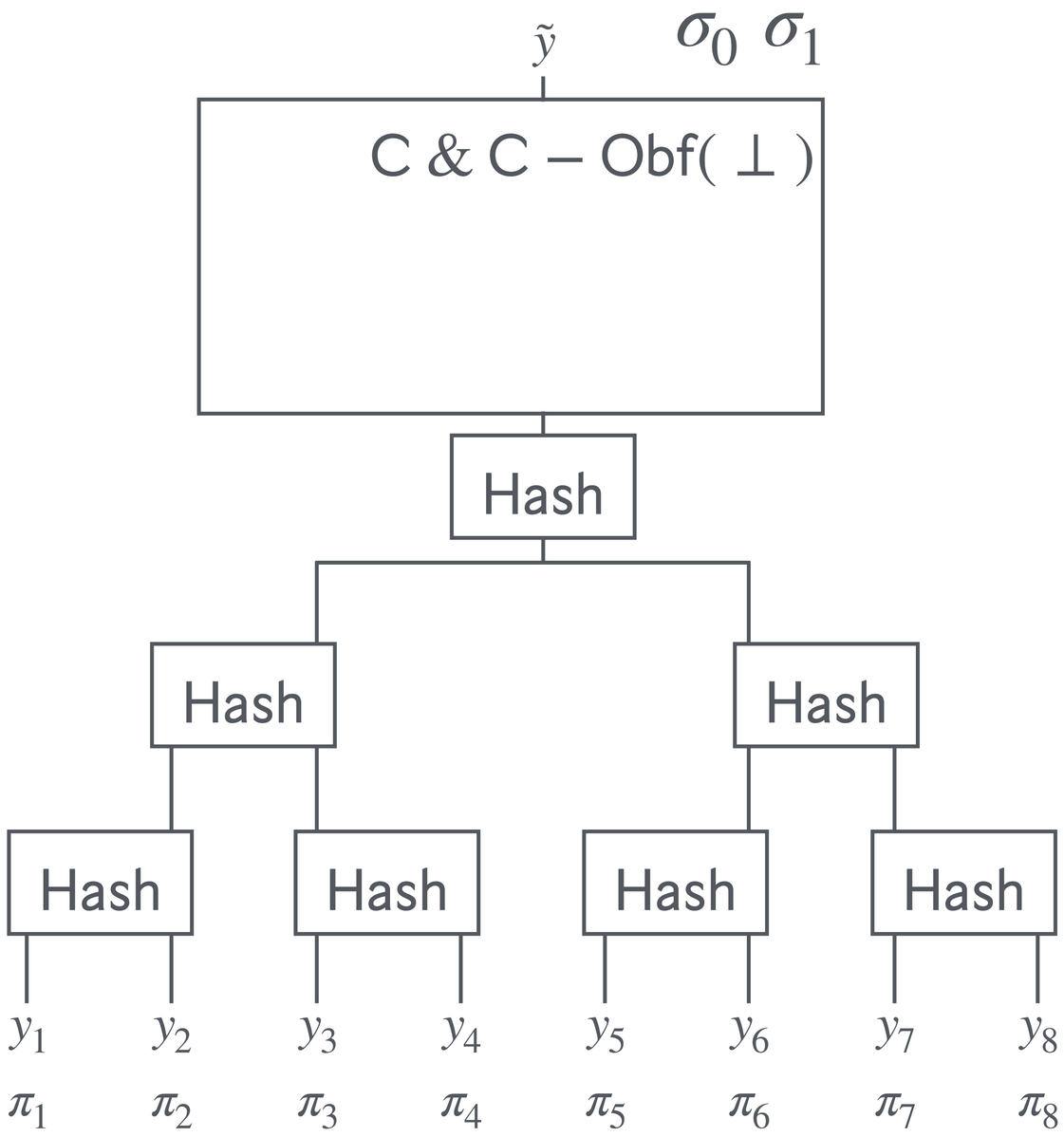

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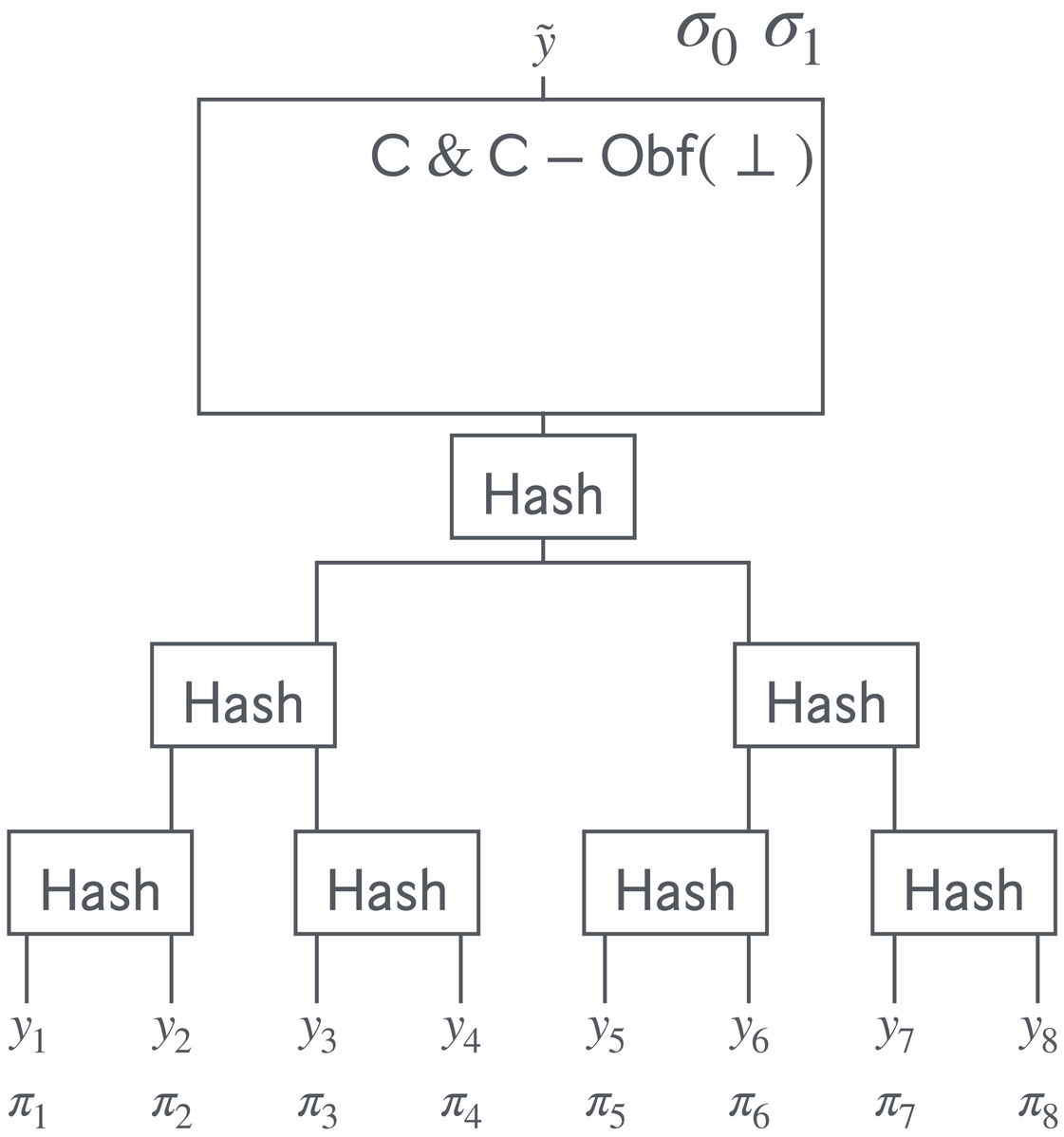

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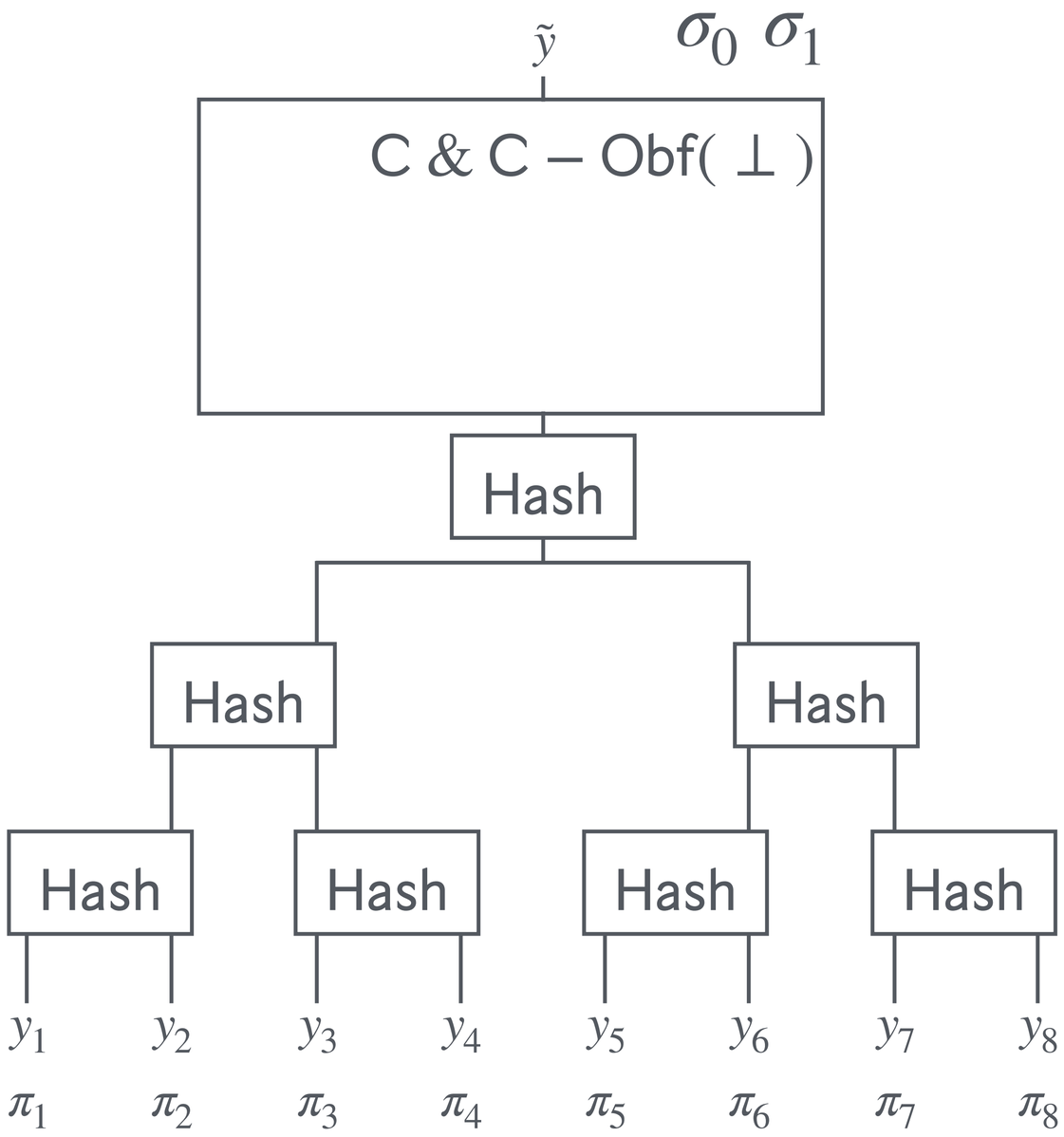

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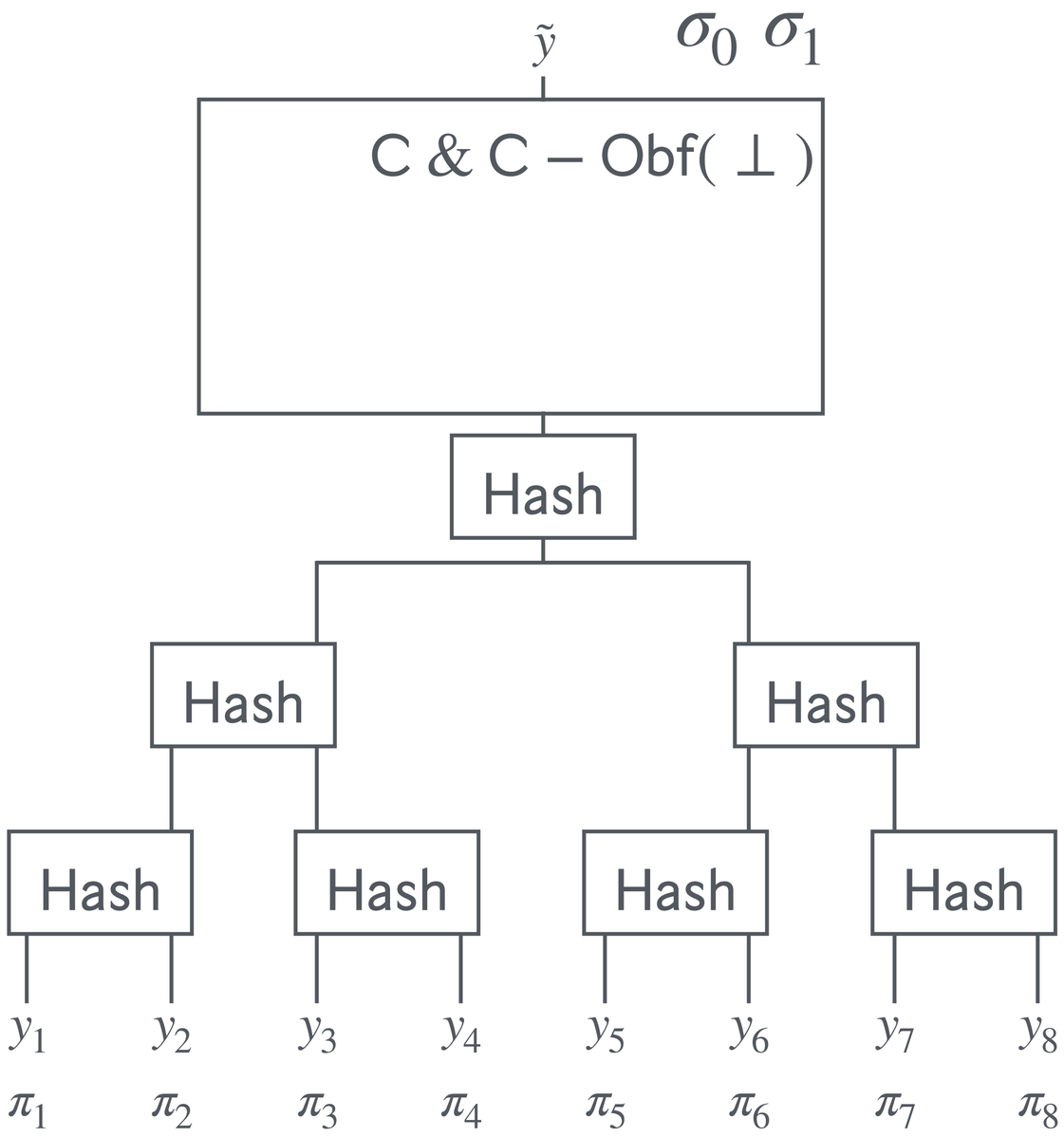

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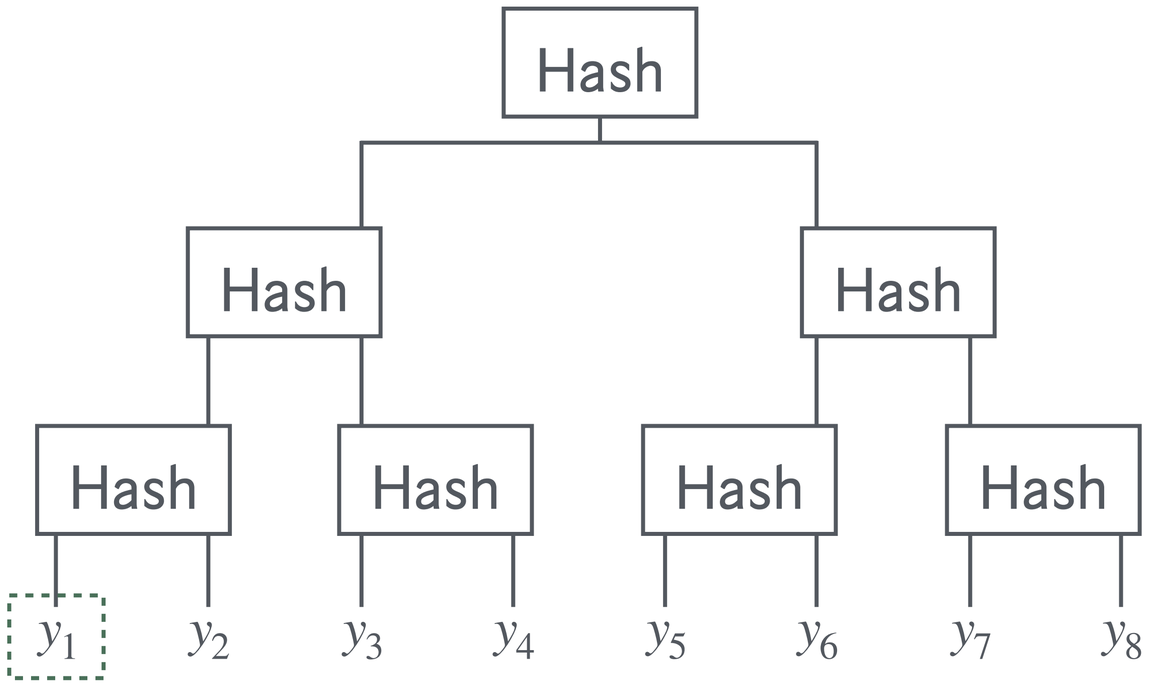
- We want to derive a contradiction against the uniqueness of the (non-aggregate) VRF

HONEST TREE



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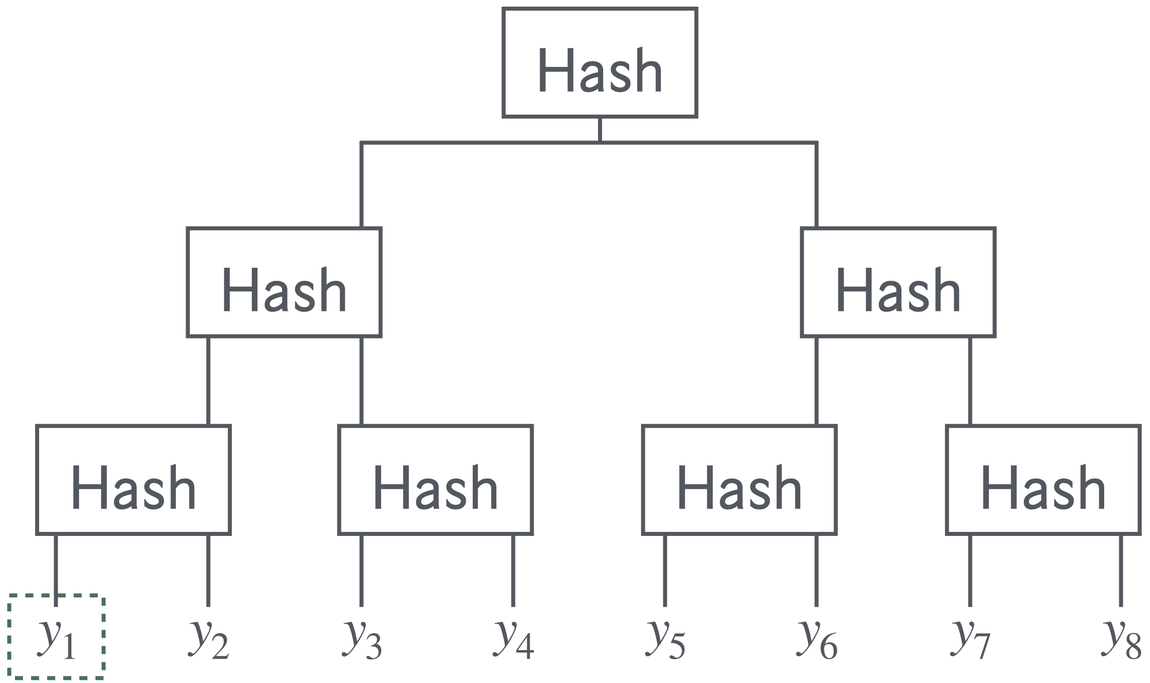

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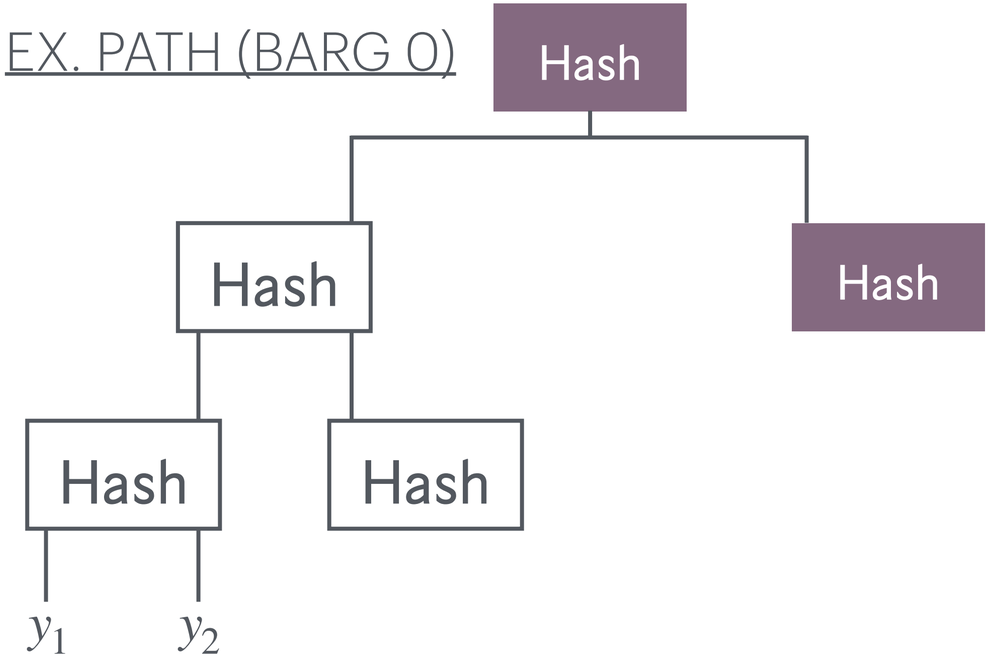

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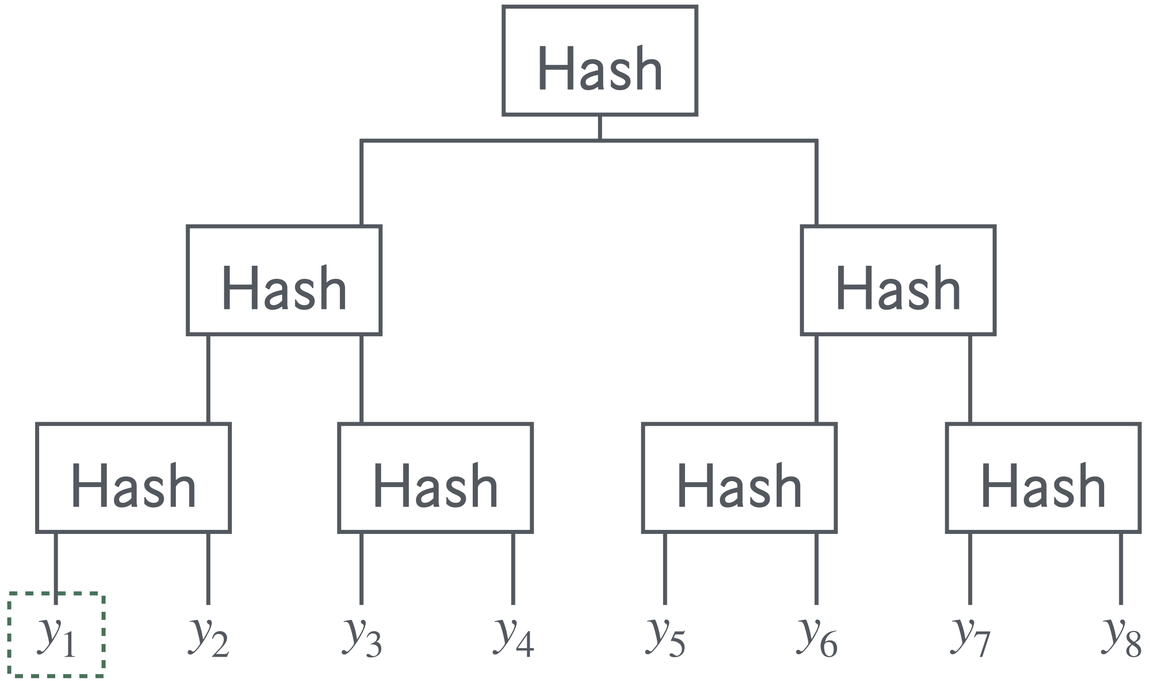

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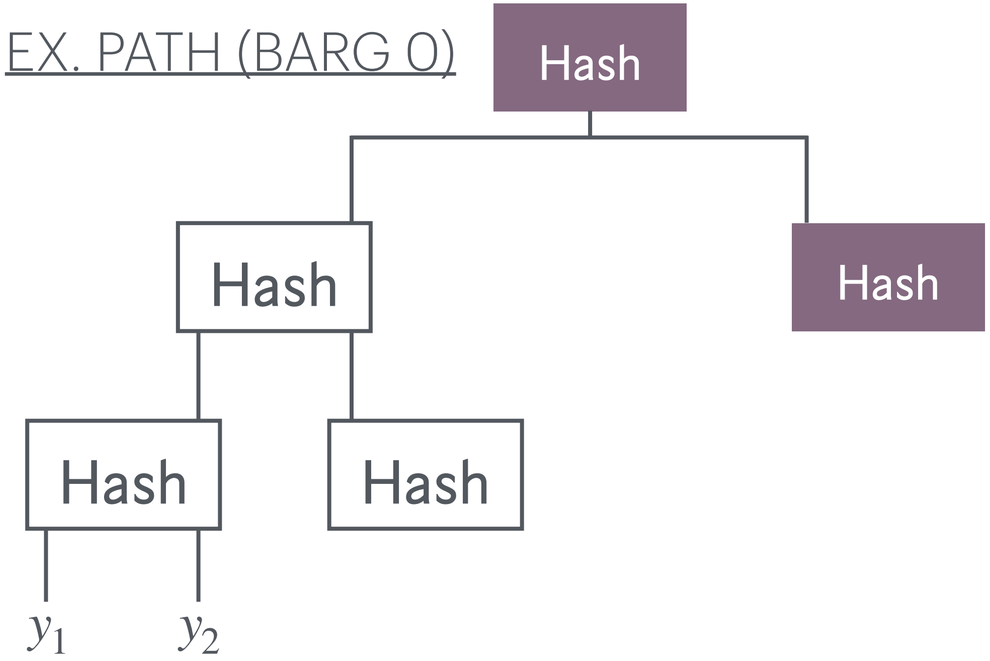

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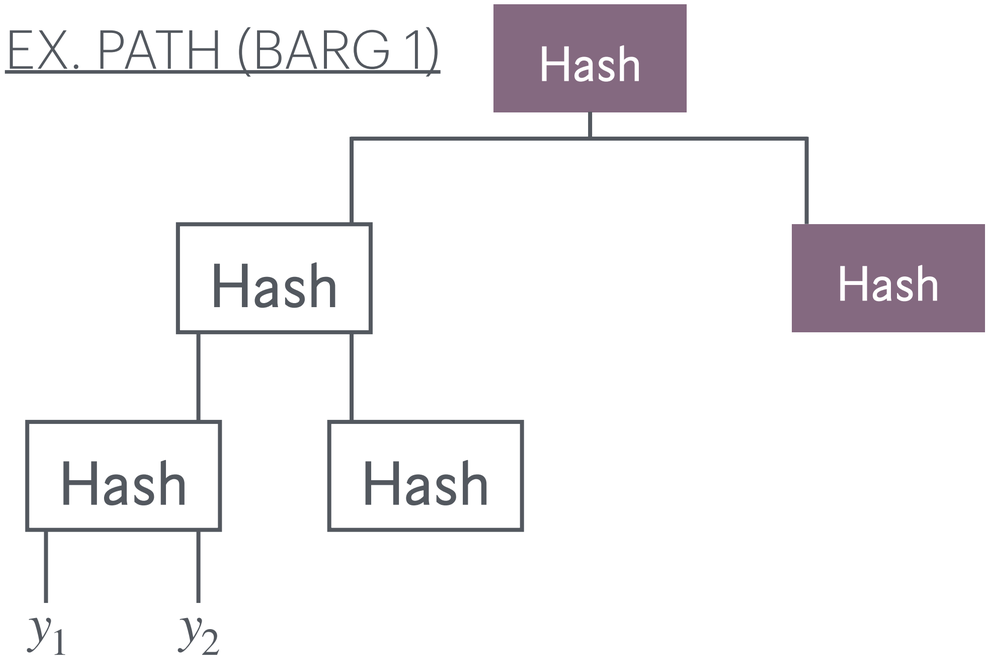
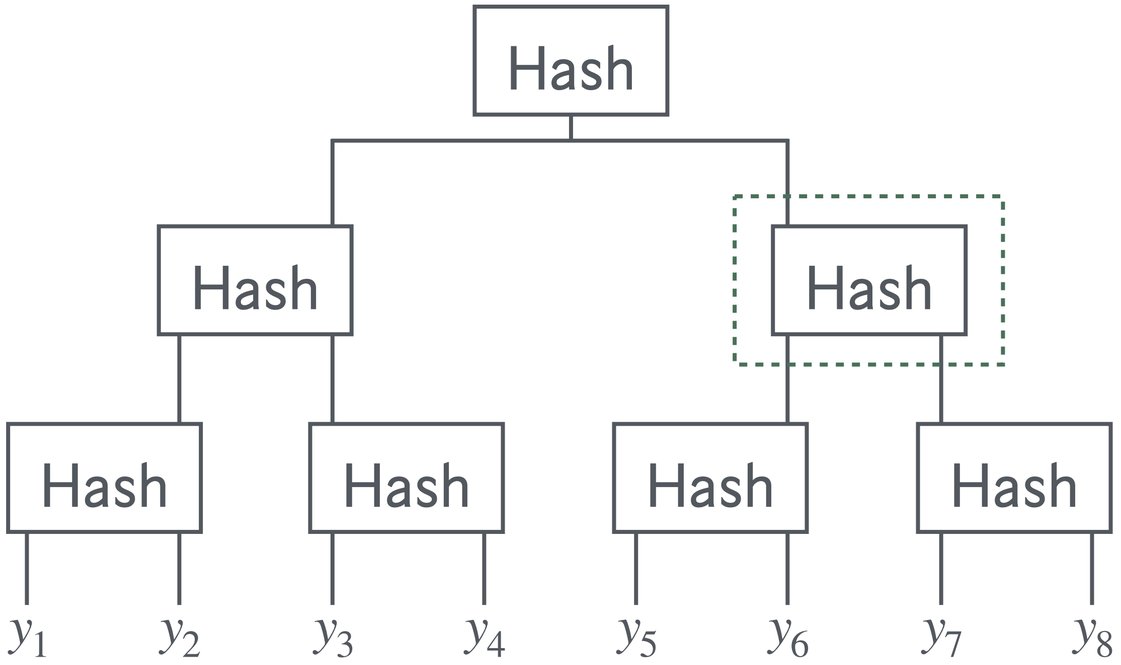
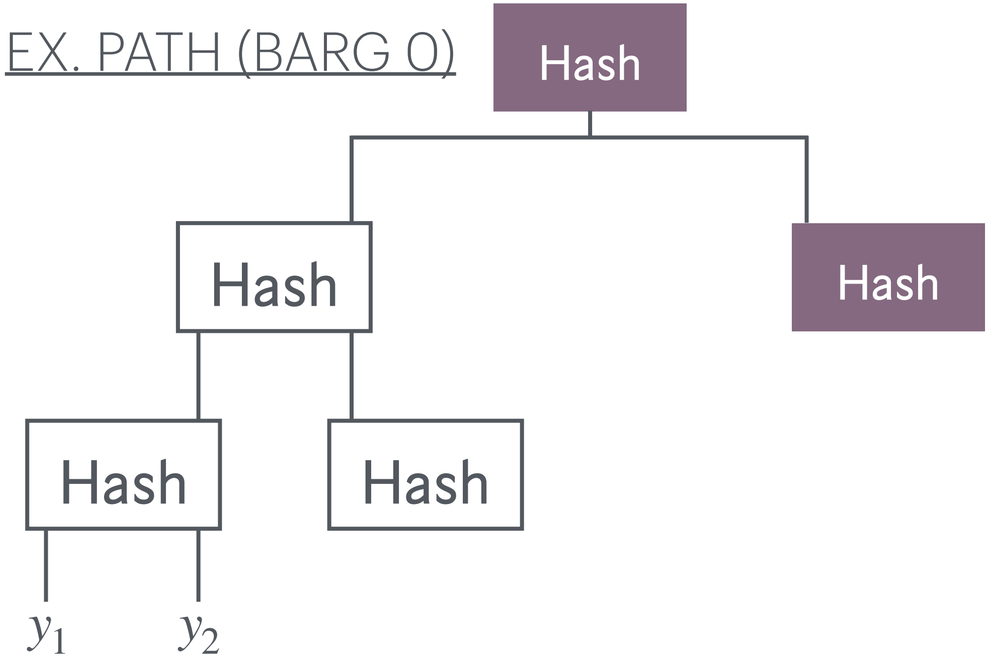


HONEST TREE



EX. PATH (BARG 0)



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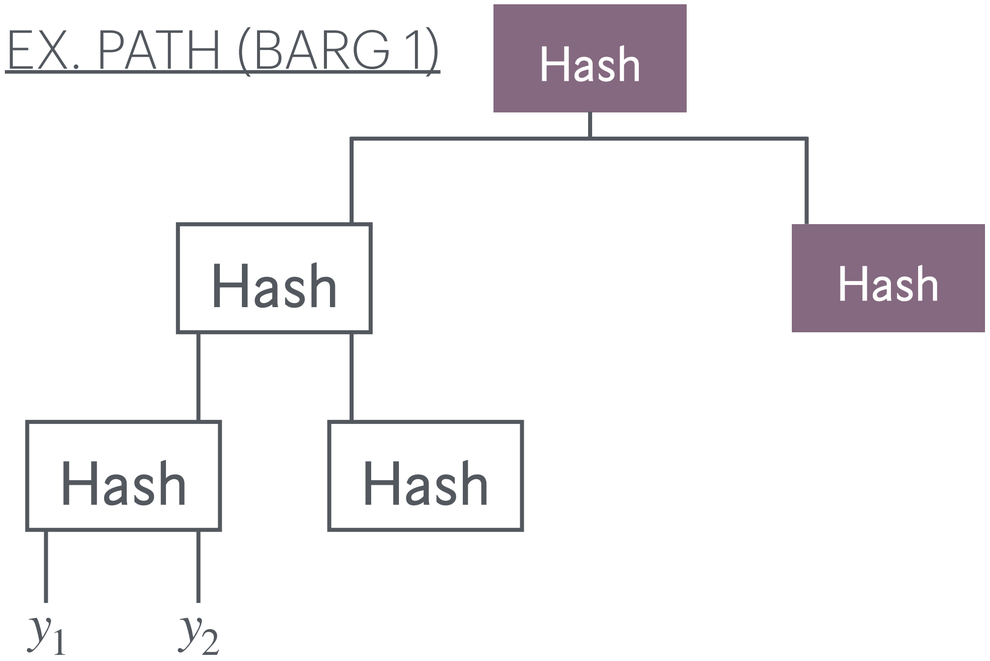

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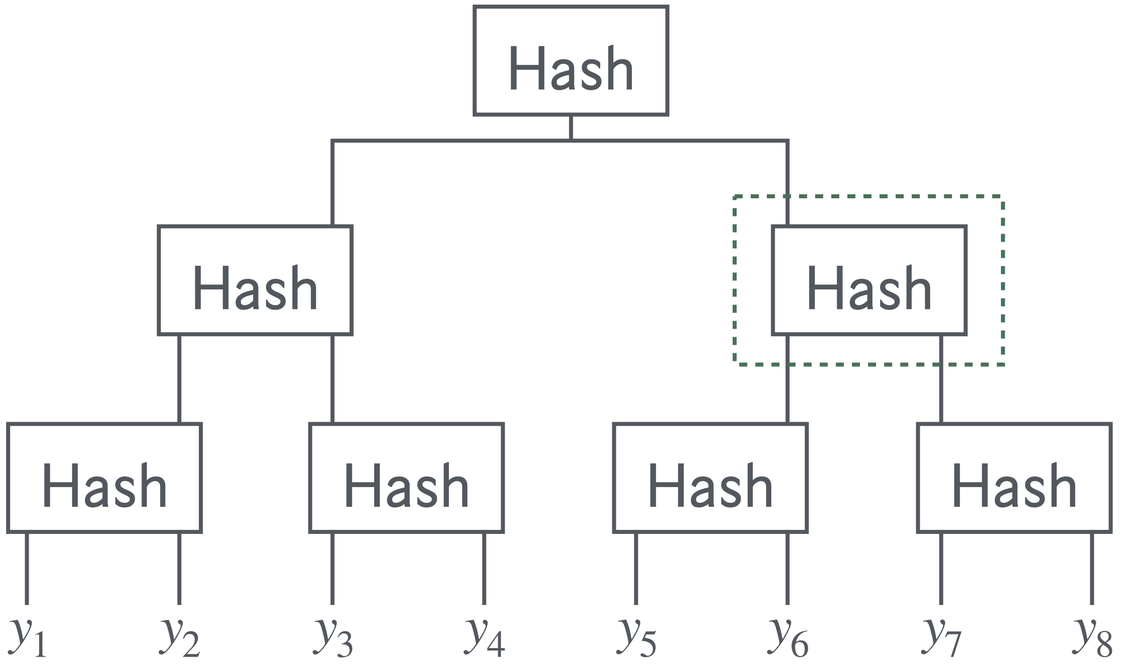

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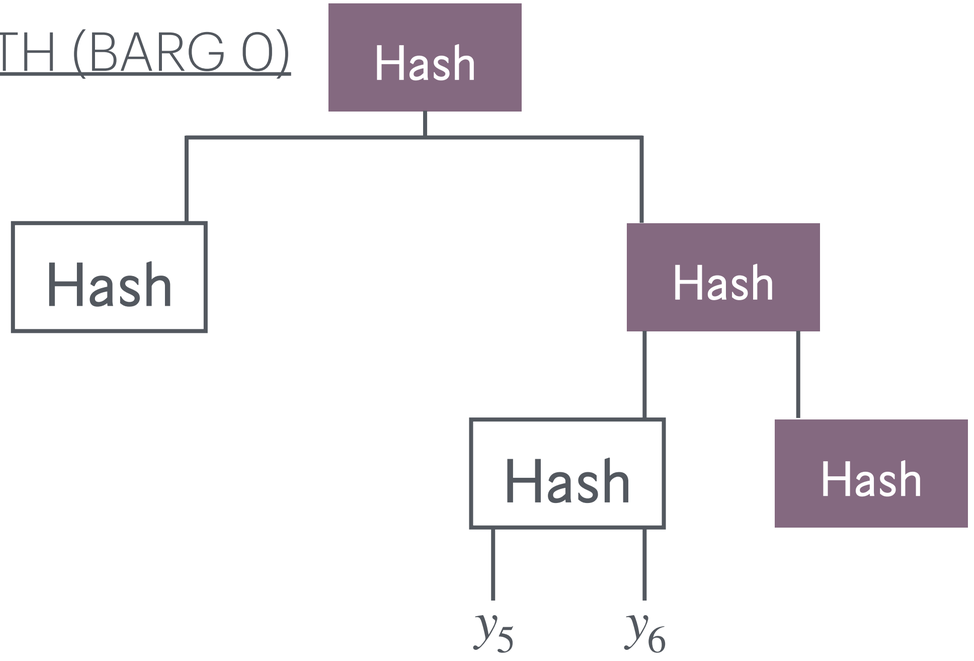

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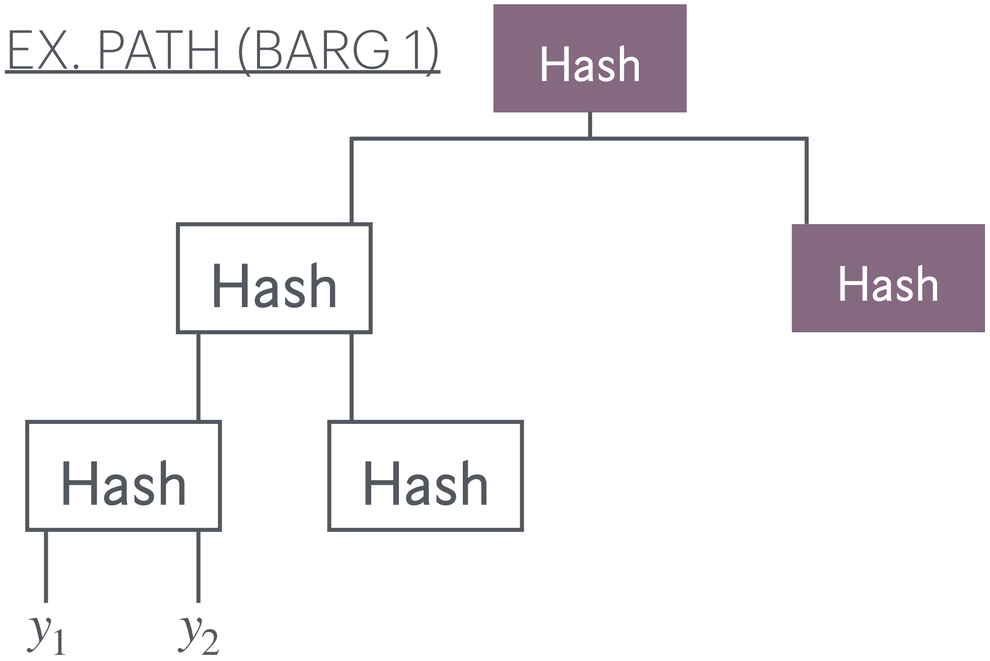

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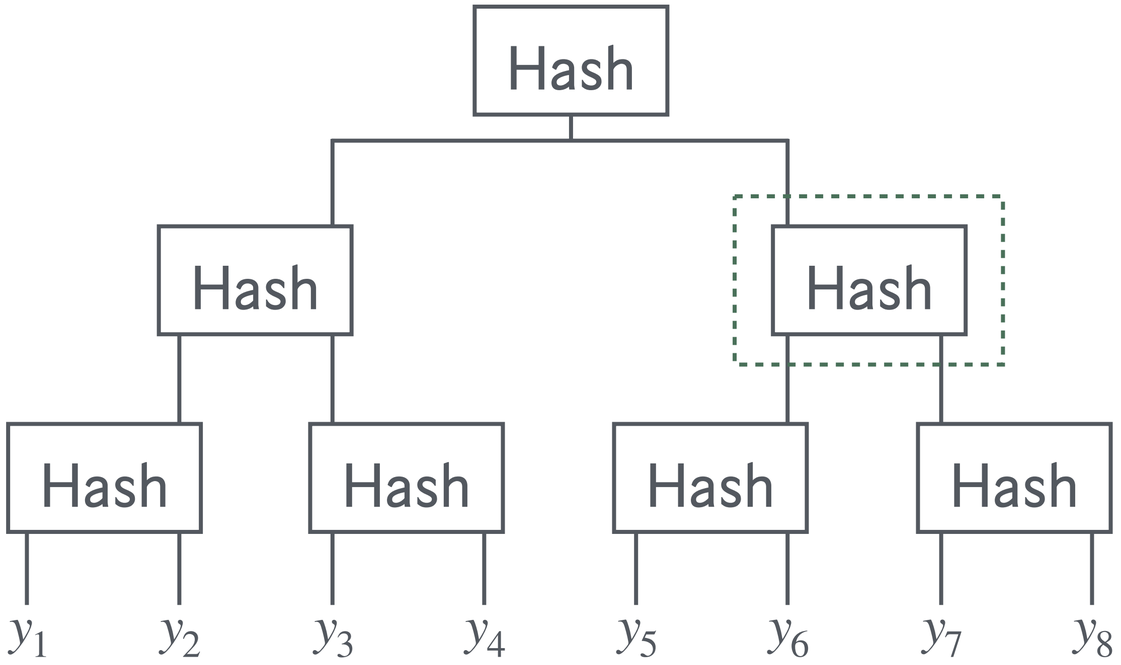

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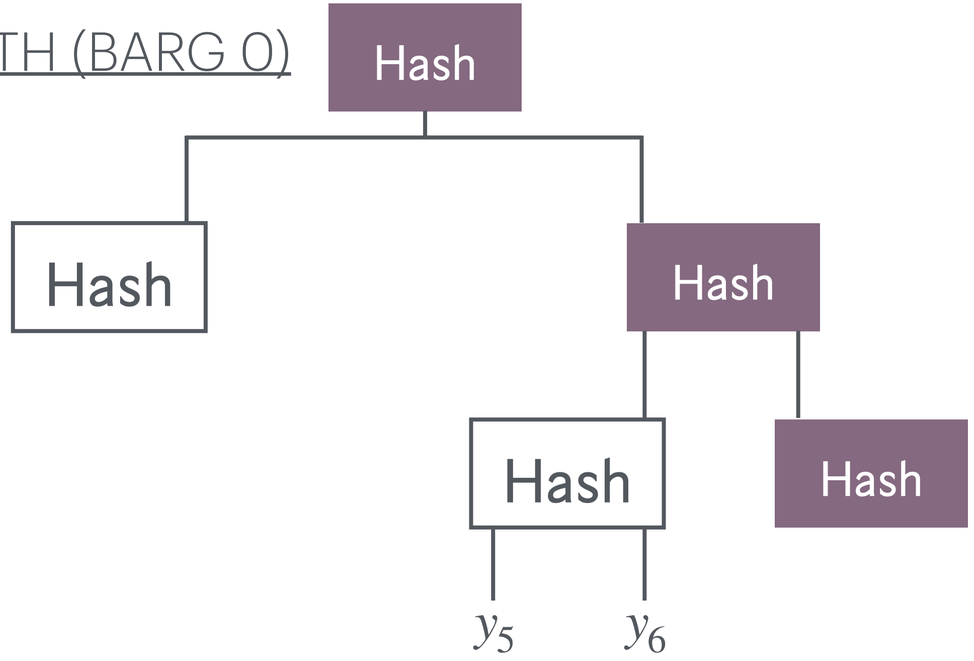

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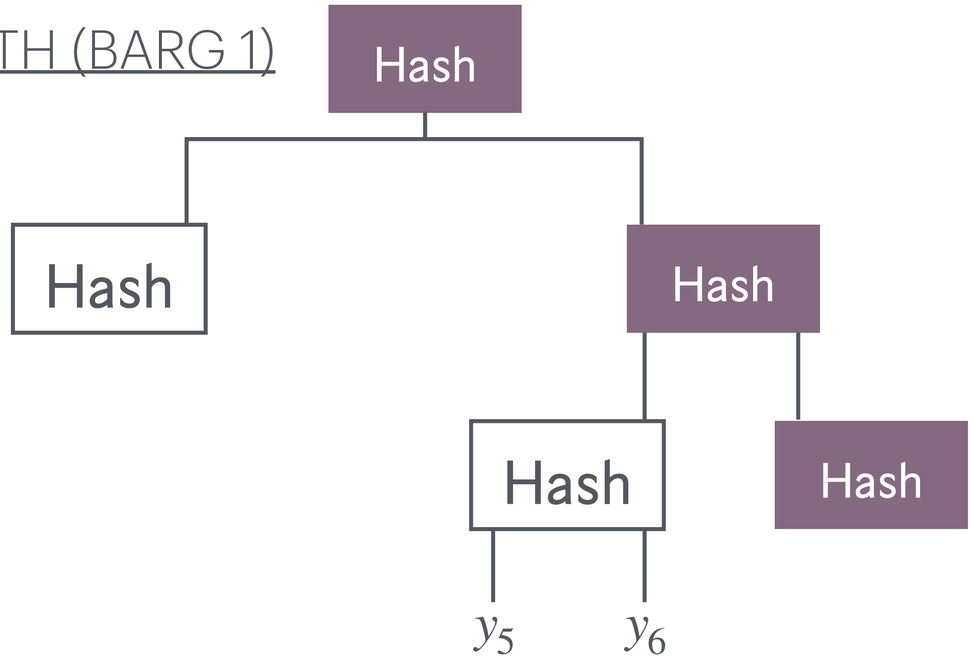

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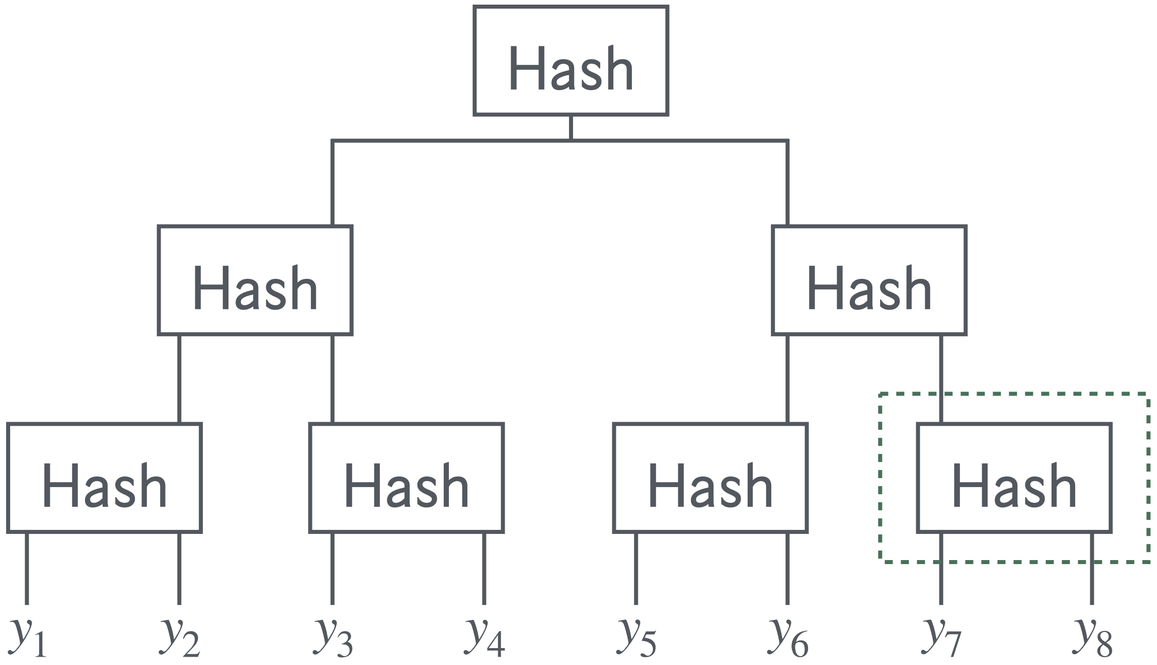

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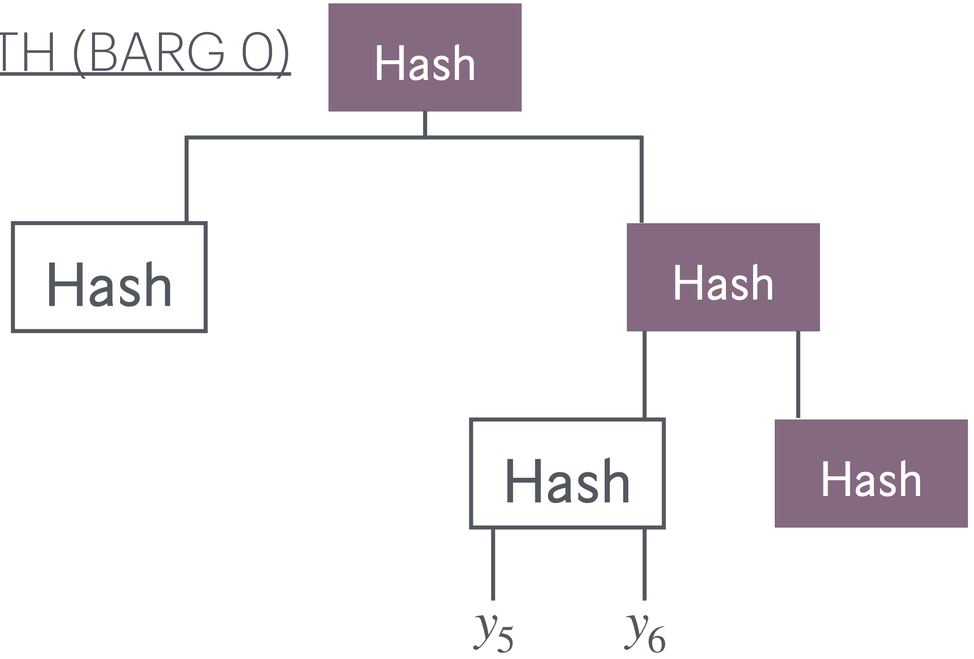
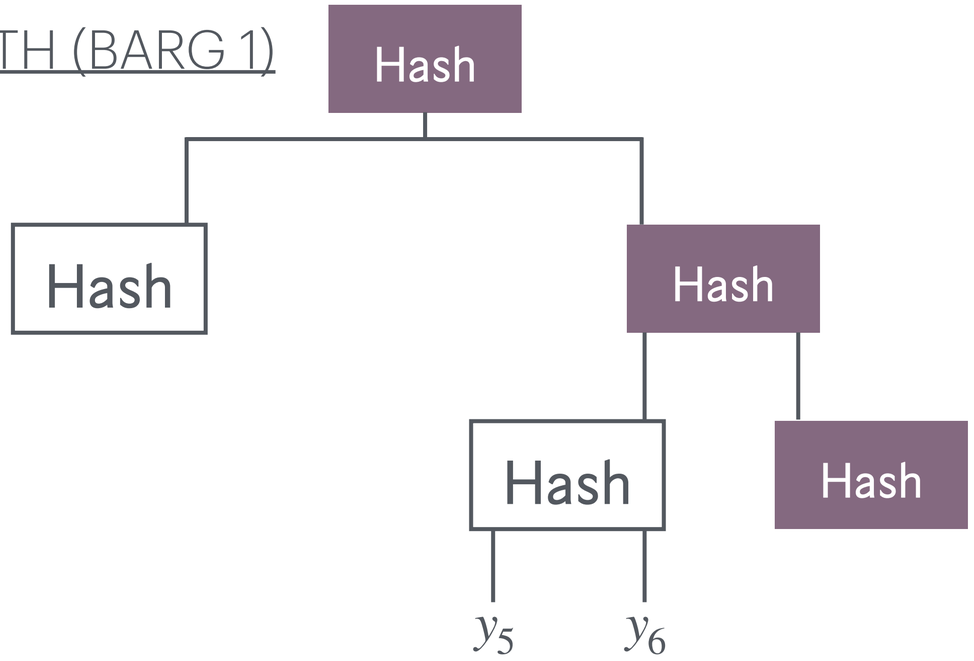


EX. PATH (BARG 1)



Bonus

$$vk = (h, h^k)$$
$$sk = k$$

$$y = e(g, \mathscr{H}(x))^k$$

$$y =^? e(g, \underbrace{\mathscr{H}(x)^k}_{=:\pi}) \quad \& \quad e(h^k, \mathscr{H}(x)) =^? e(h, \pi)$$

Open Problems

Open Problems

- Security with a malicious aggregator?

Open Problems

- Security with a malicious aggregator?
- More applications?

Open Problems

- Security with a malicious aggregator?
- More applications?
- Different computational assumptions?

Open Problems

- Security with a malicious aggregator?
- More applications?
- Different computational assumptions?
- Better concrete efficiency, in the standard model?

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- Security with a malicious aggregator?
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Thank you!