batching adaptively-sound SNARGs for NP

Lali Devadas
MIT

joint work with Brent Waters (UT Austin and NTT Research) and David J. Wu (UT Austin)

Succinct Non-interactive ARGument (SNARG)

NP language L

common reference string

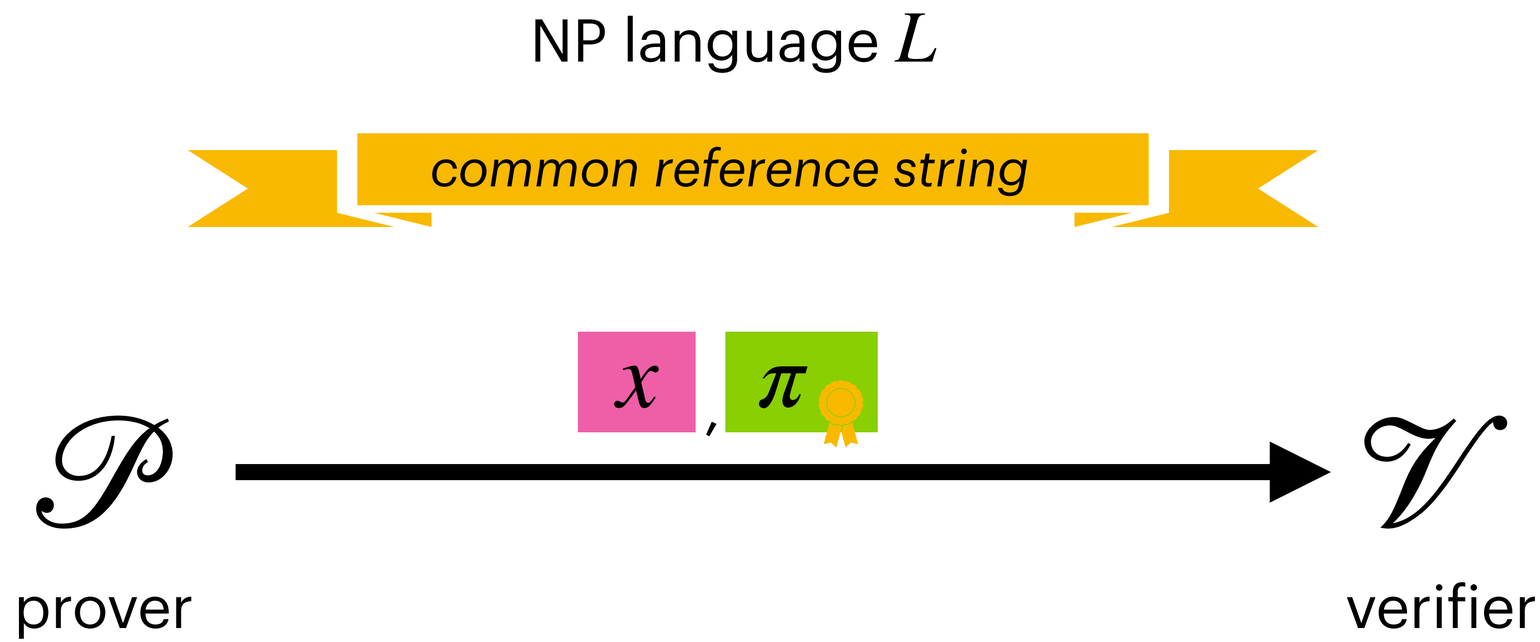
$\mathcal{P}$

prover

$\mathcal{V}$

verifier

Succinct Non-interactive ARGument (SNARG)



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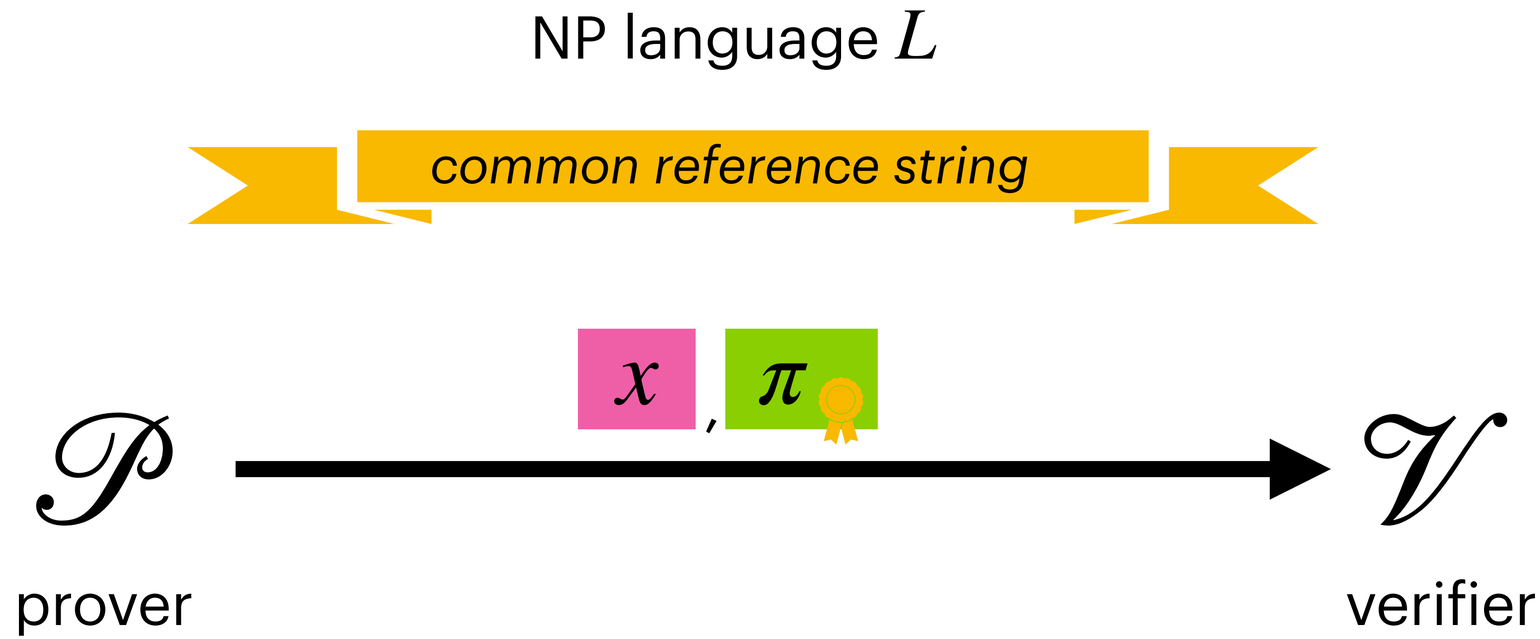

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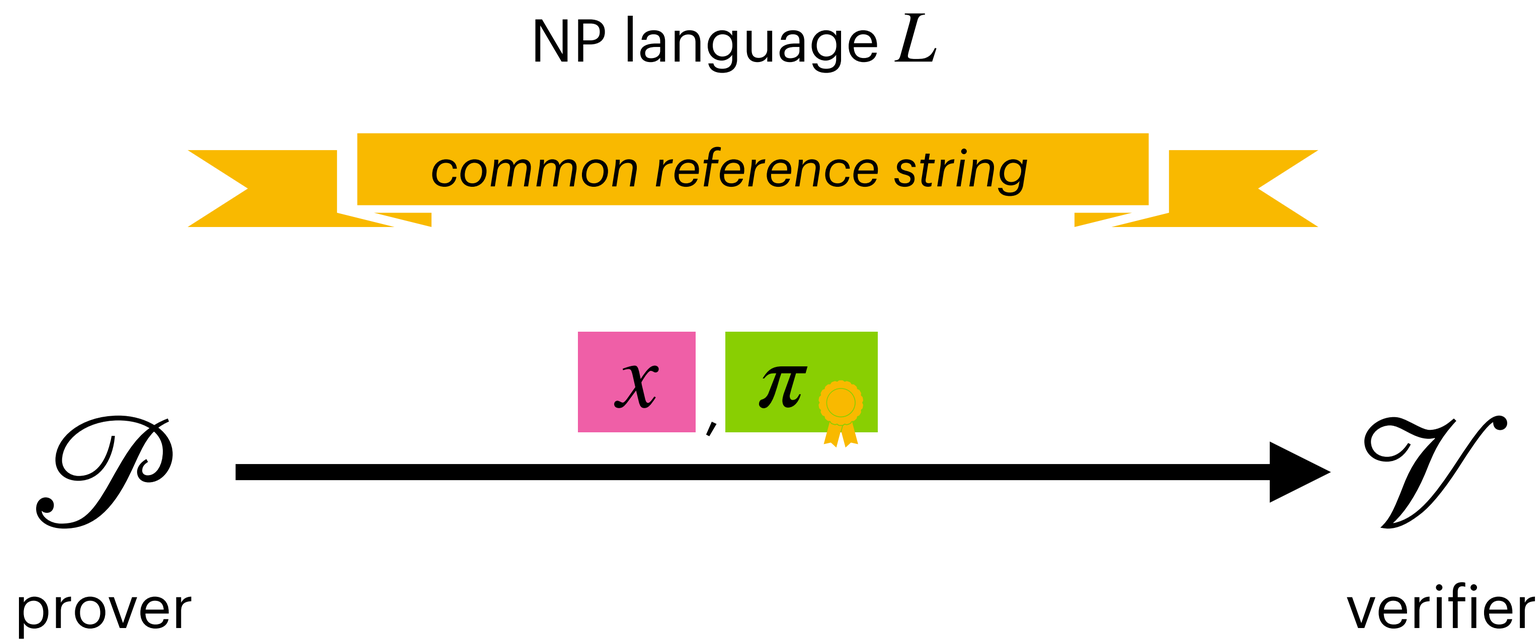

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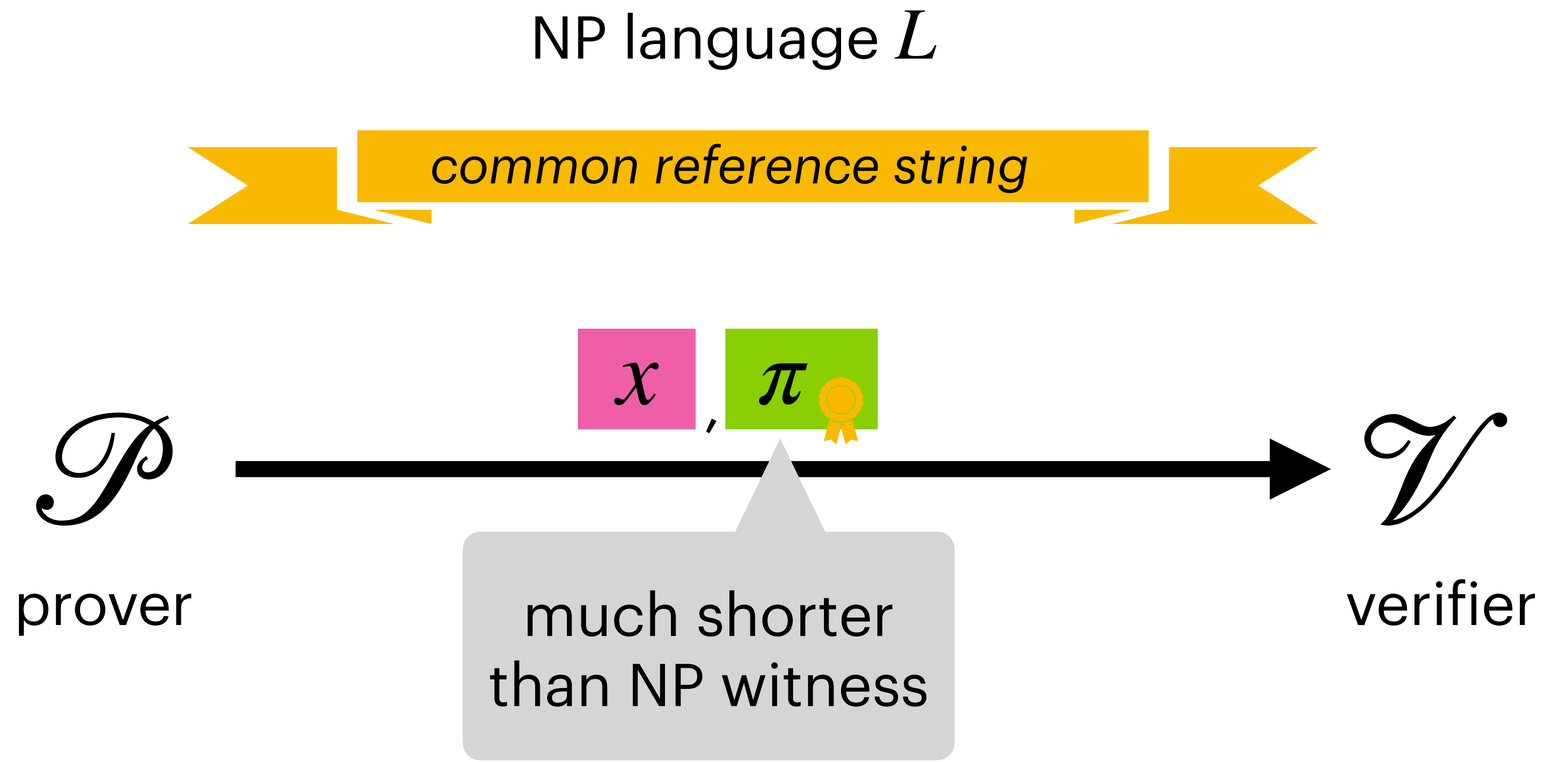

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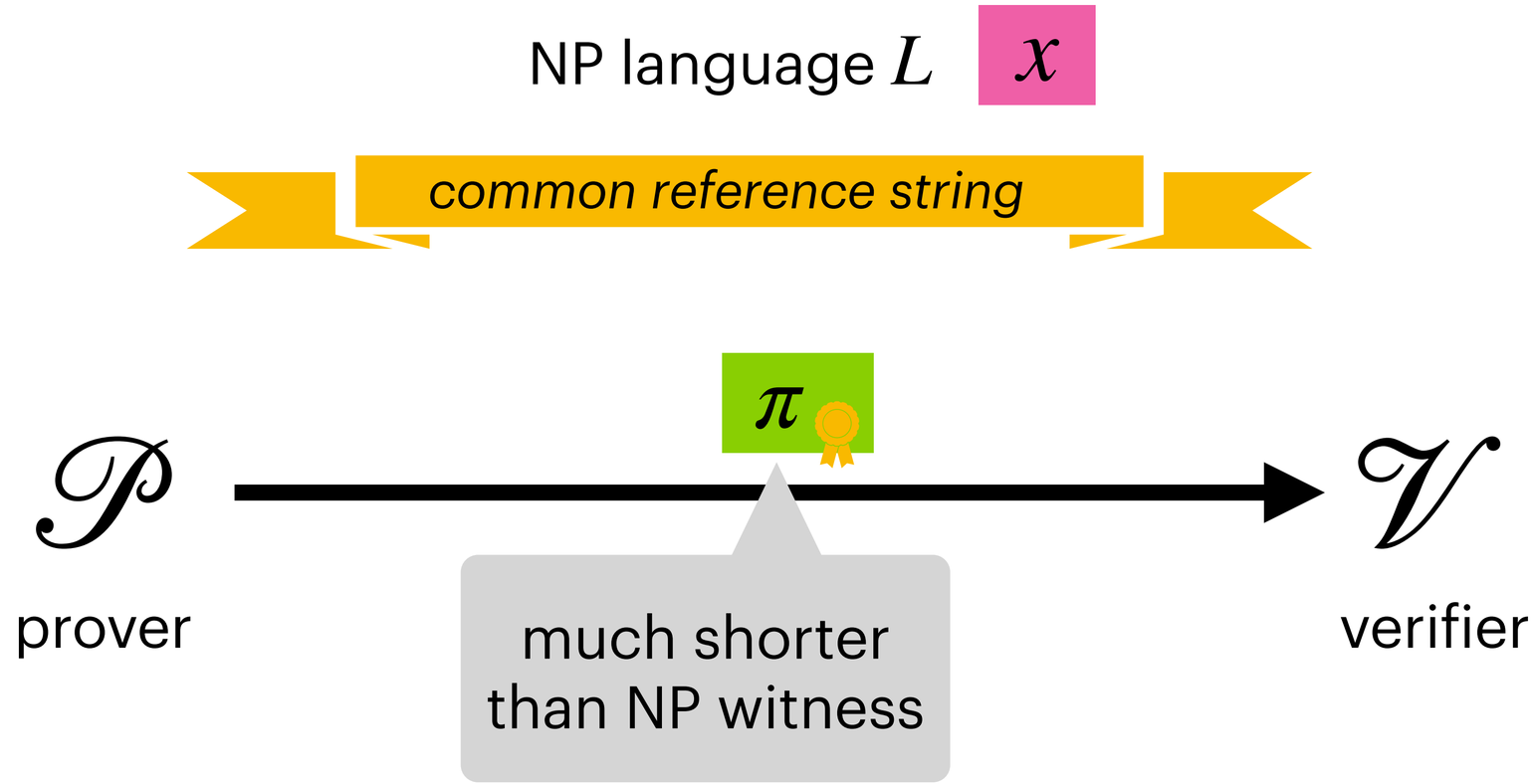

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  - random oracle model [Mic94, Val08, BCS16, BBHR19, CMS19, COS20, CY21, ...]
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what about adaptive soundness?

Gentry-Wichs separation

(informal) theorem. there does not exist an adaptively-sound SNARG for NP with a black-box security reduction to falsifiable assumptions.

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- does **not** rule out reductions which run in enough time to decide the NP language!
- **one interpretation**: the CRS for a language decidable in time T must at least grow with $\mathsf{polylog}(T)$ to keep the proof succinct

adaptively-sound SNARGs for NP

from falsifiable assumptions

iO = indistinguishability obfuscation
OWF = one-way functions

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- [WW24a]: adaptively-sound SNARG for NP from subexponentially-secure iO+OWF and rerandomizable OWF (e.g., from discrete log / factoring)

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in all the above, the CRS grows with the size of the NP verification circuit

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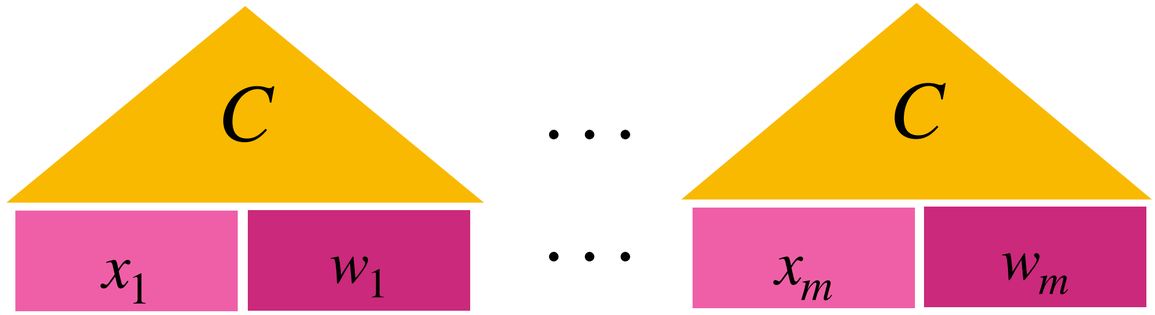

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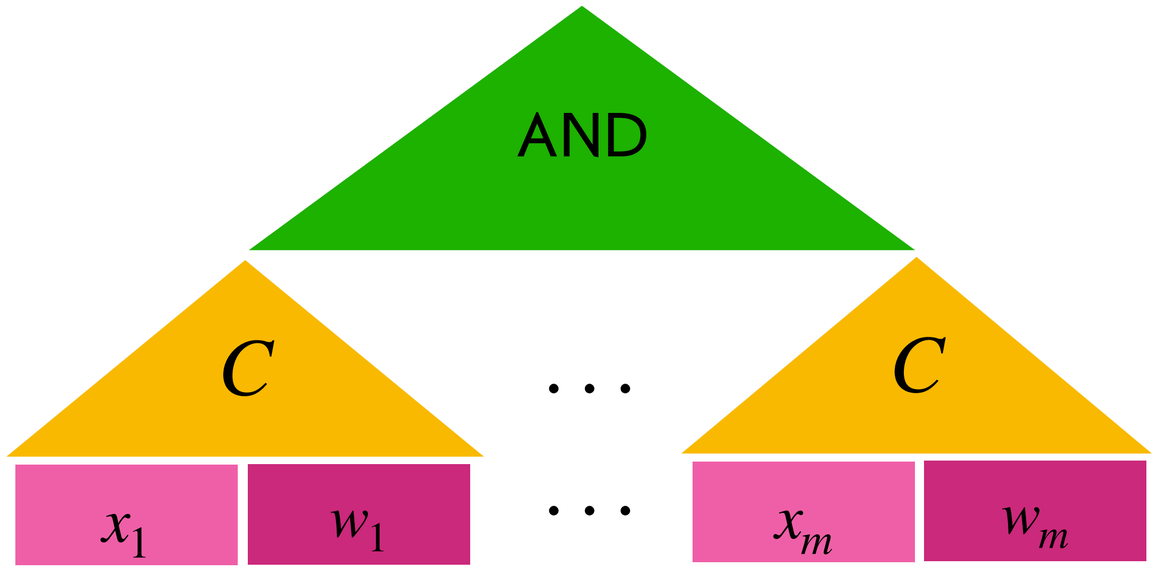

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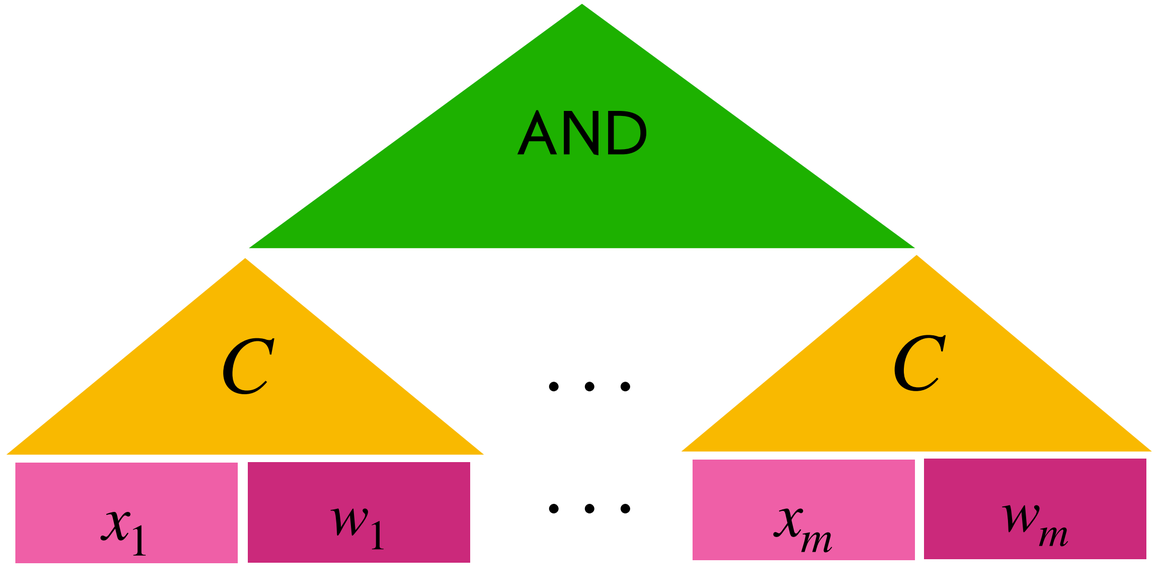

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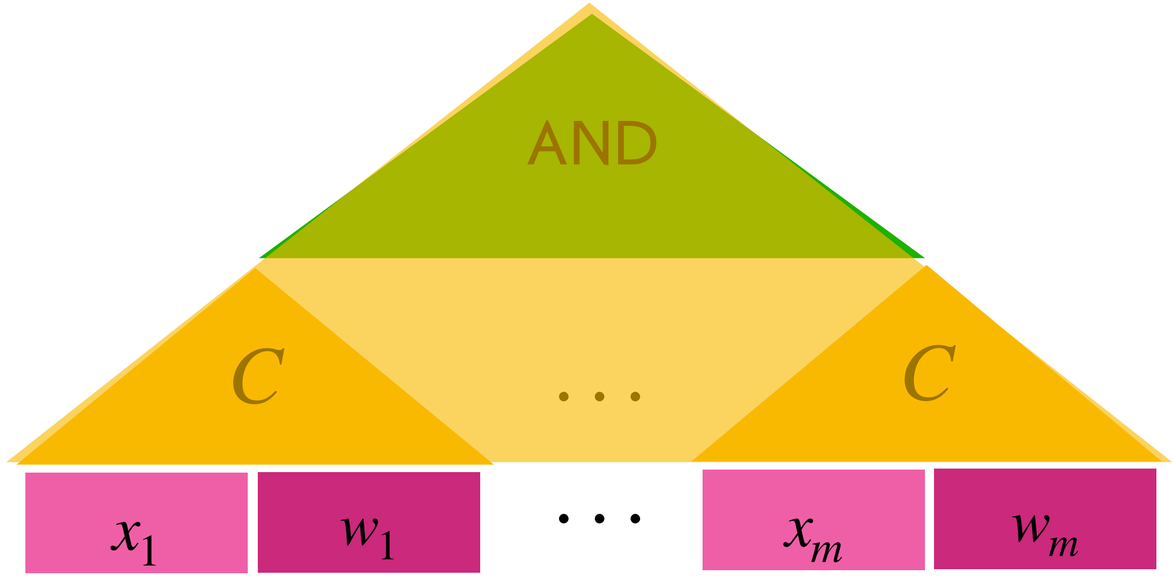

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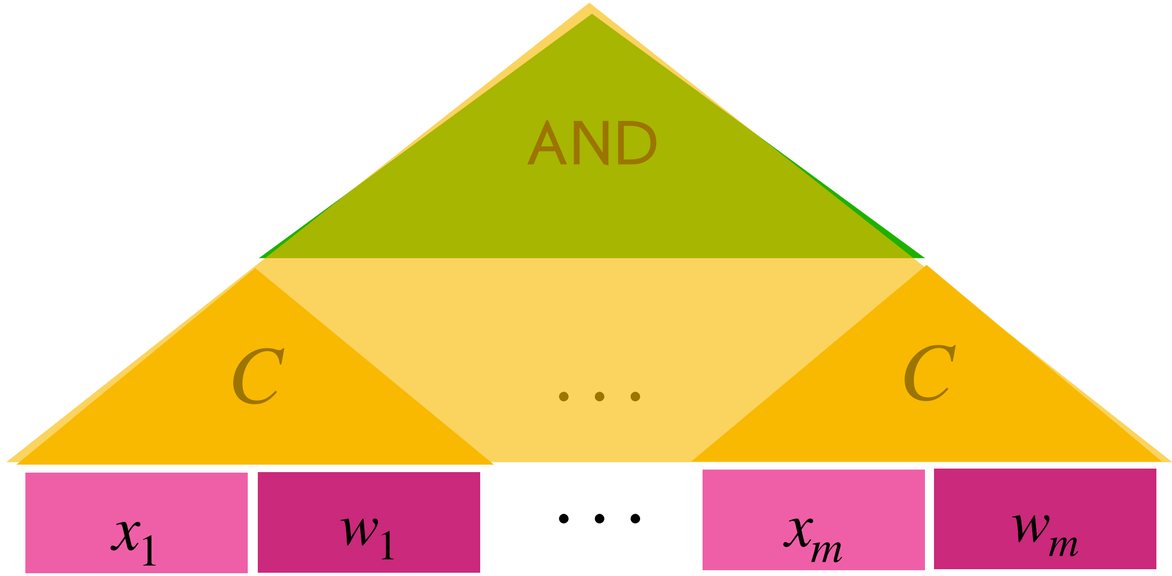

so if we use *any* existing adaptively-sound SNARG for batch-L, the CRS size will grow with $|C| \cdot m$...

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can we construct an adaptively-sound SNARG for batch-NP where the CRS grows with $|C| \cdot \log m$ instead of $|C| \cdot m$?

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this work: **yes!**

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- another proof of theorem assuming iO+OWF and a homomorphic rerandomizable OWF (instantiated from discrete log) in the paper

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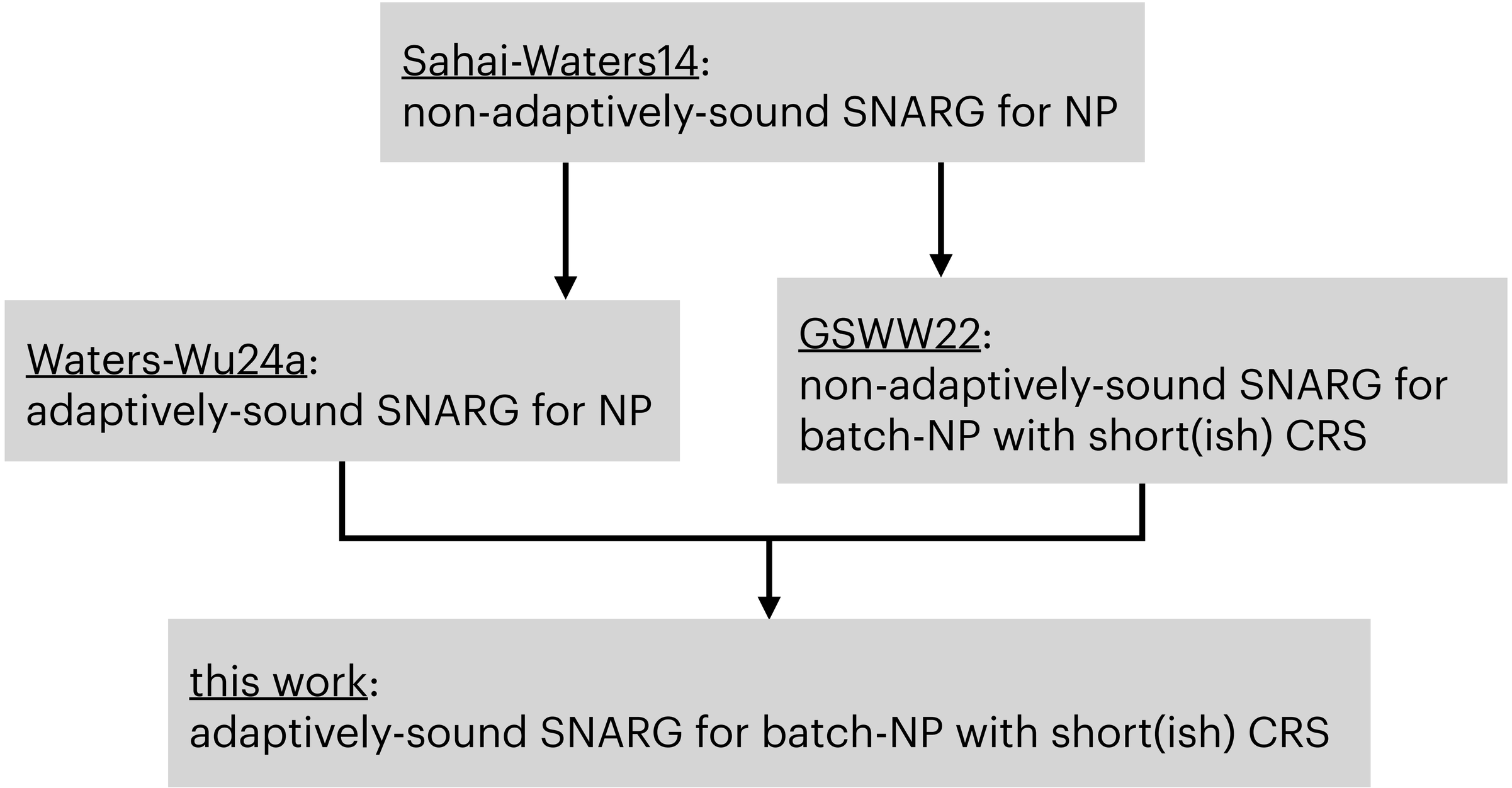
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$\text{Prove}_k(x, w)$:
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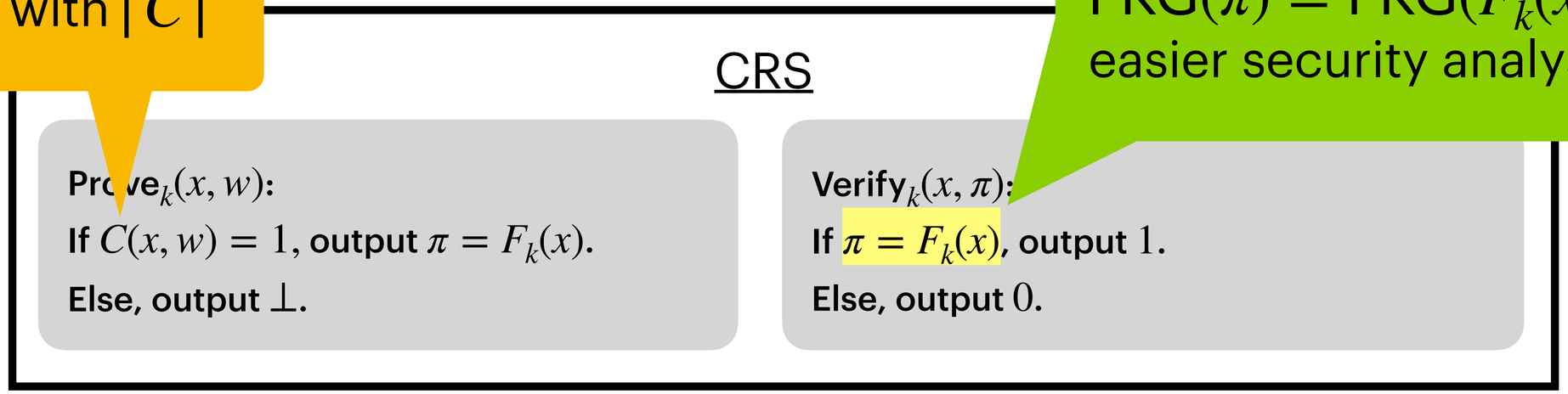
will actually check $\text{PRG}(\pi) = \text{PRG}(F_k(x))$ for easier security analysis



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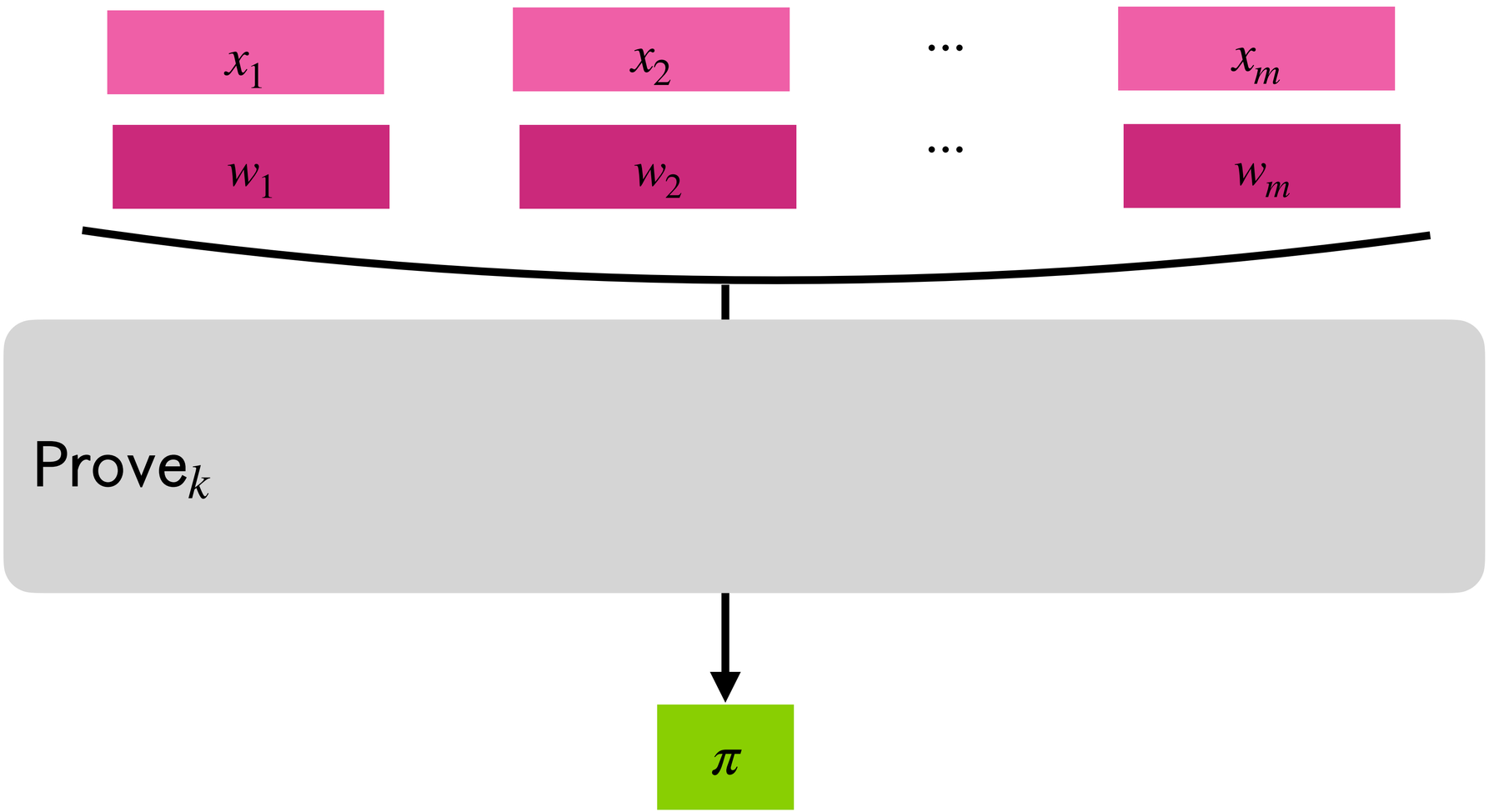

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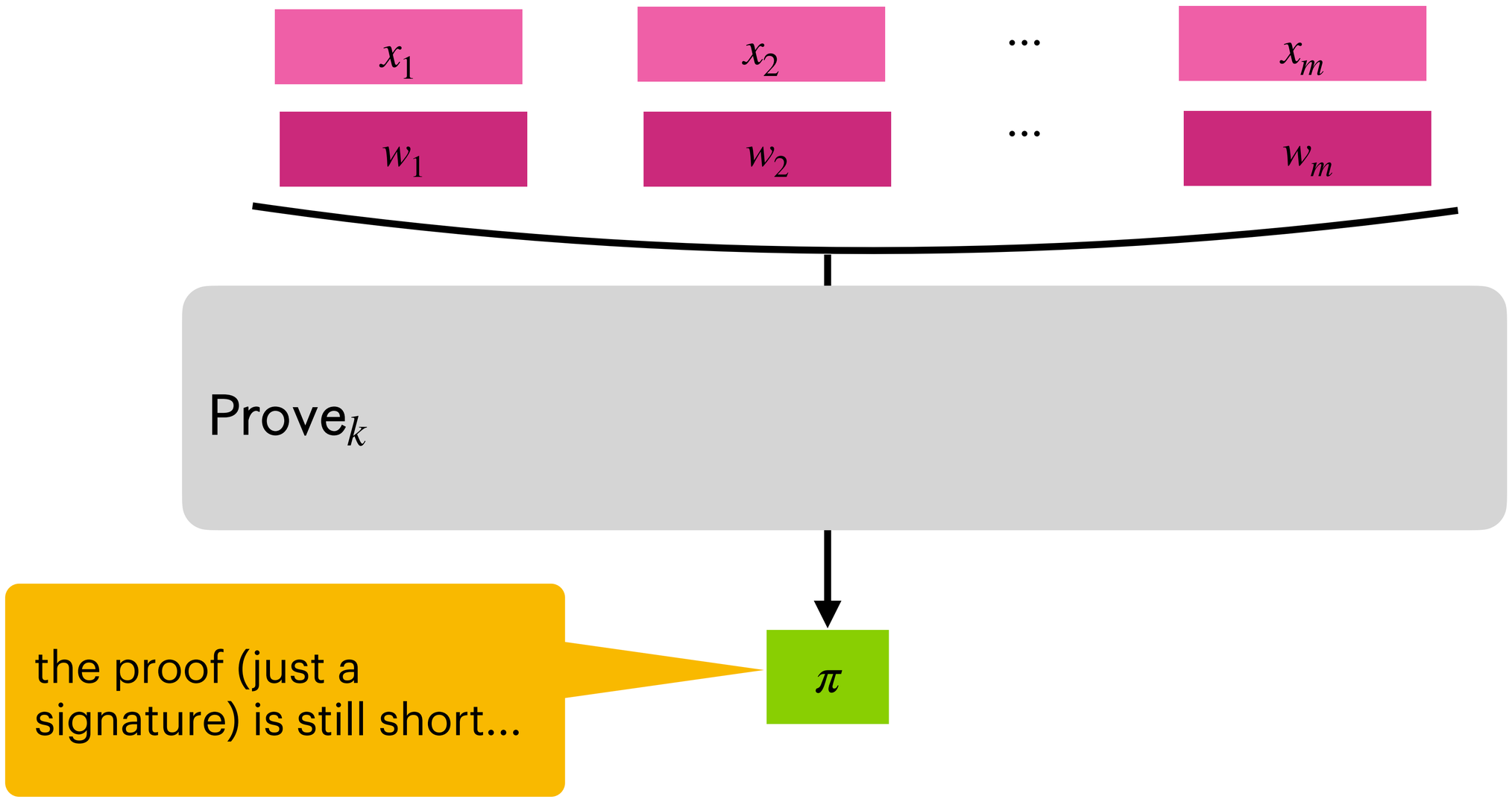

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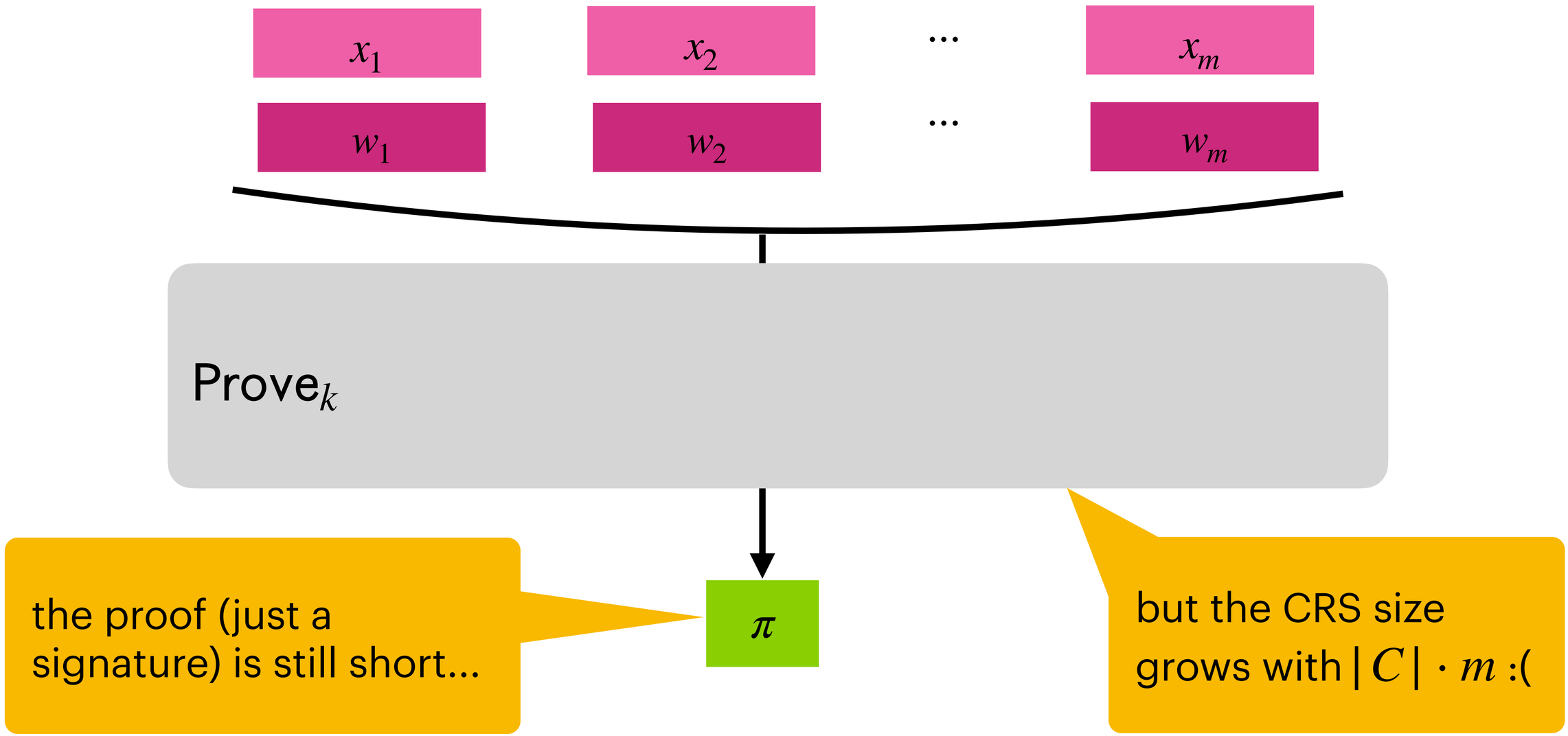

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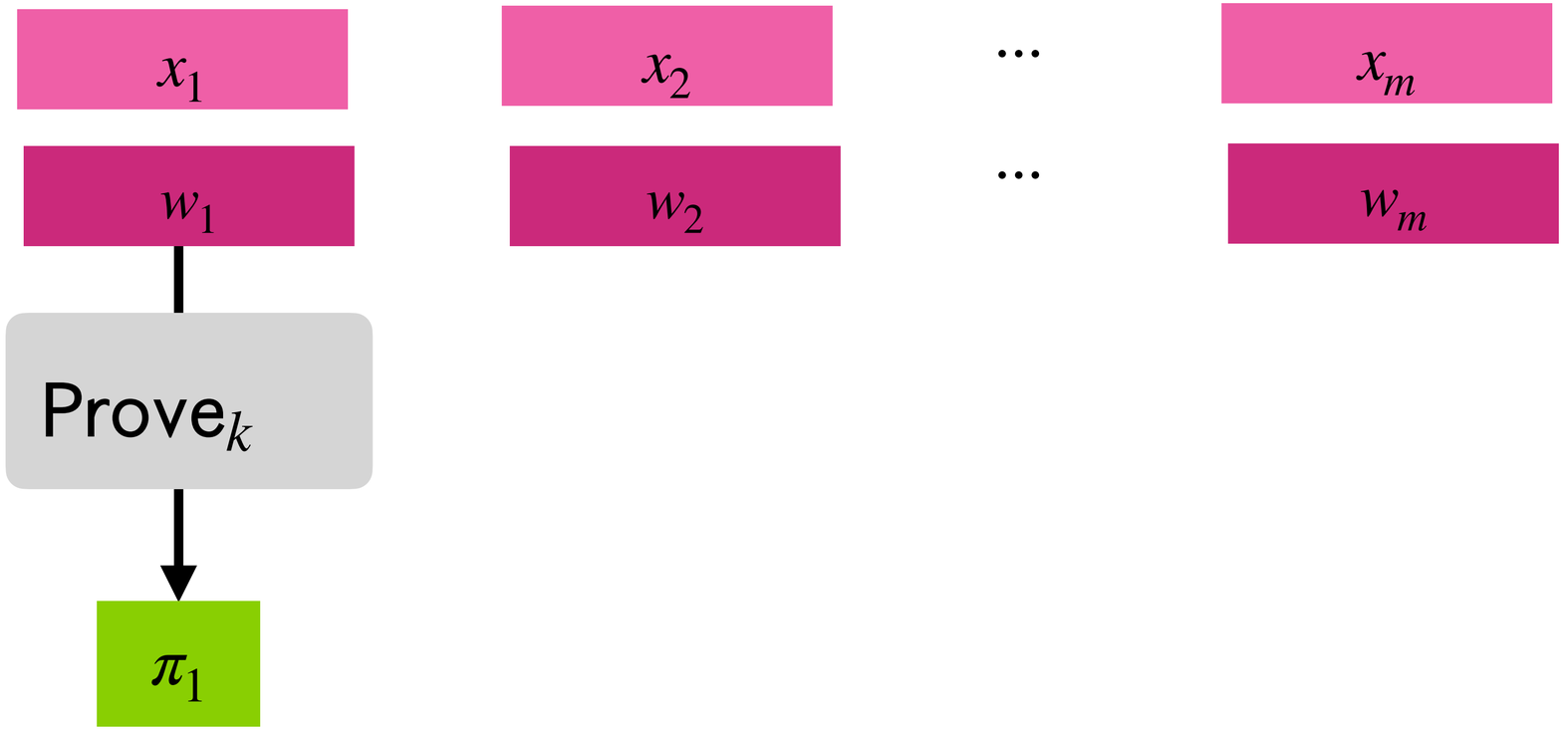

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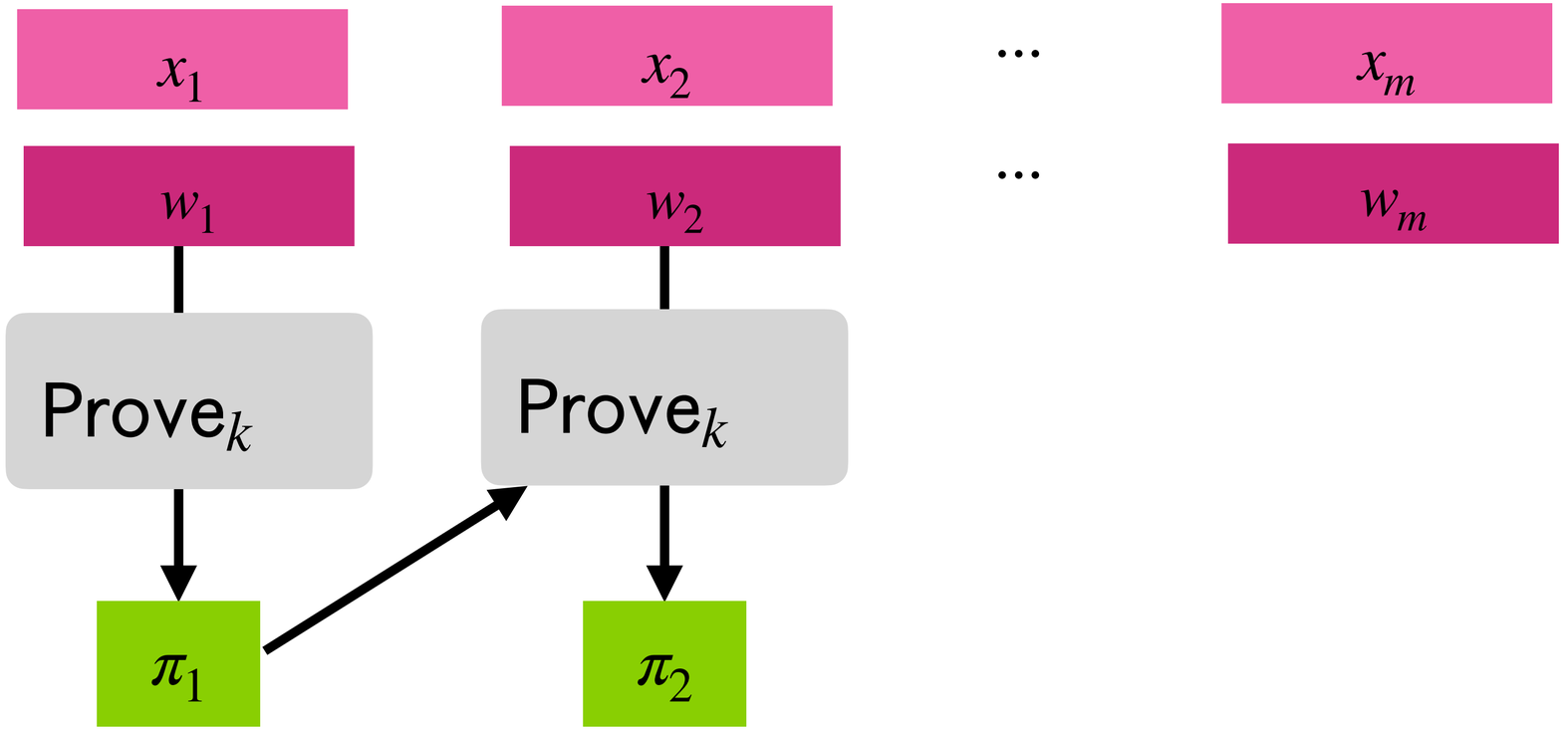

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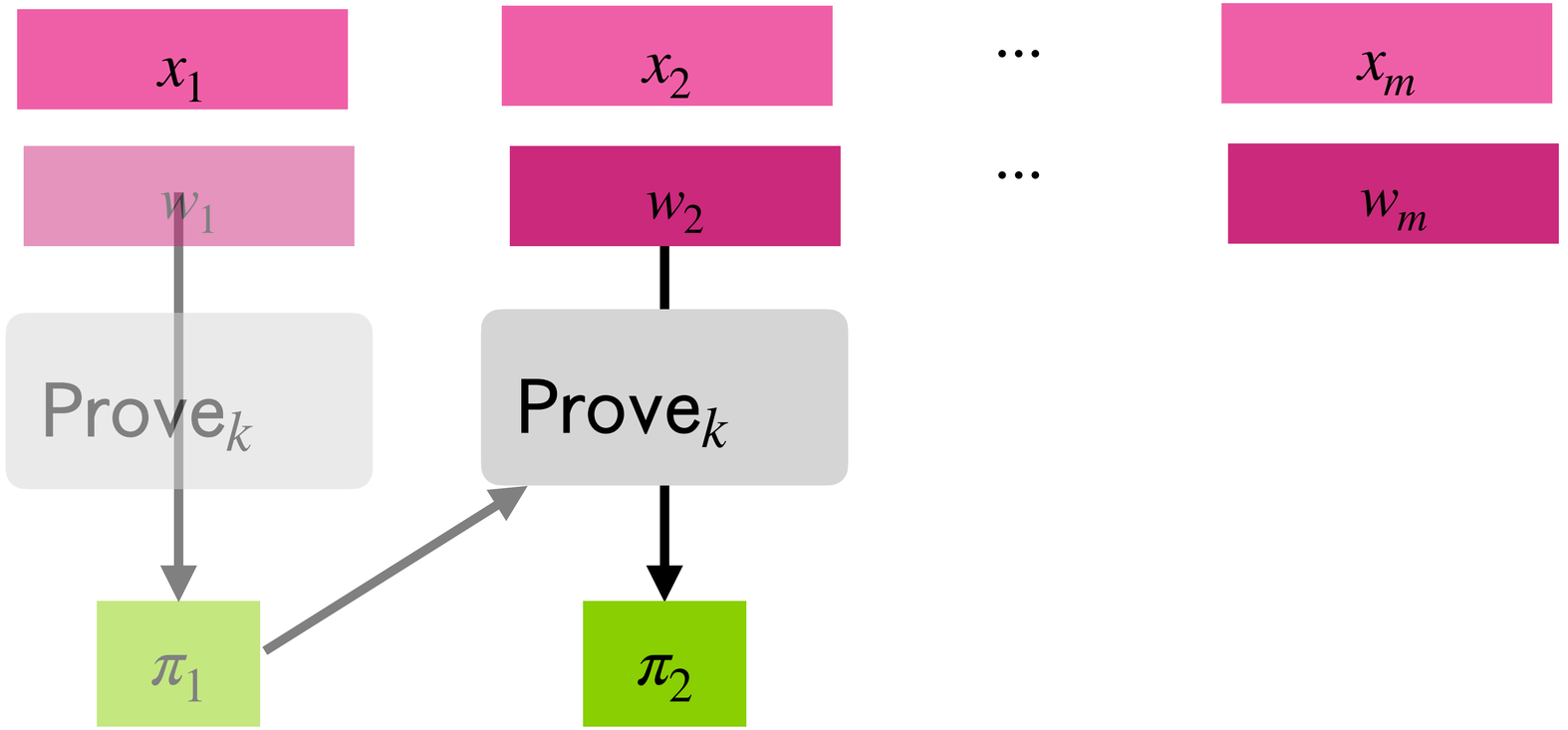

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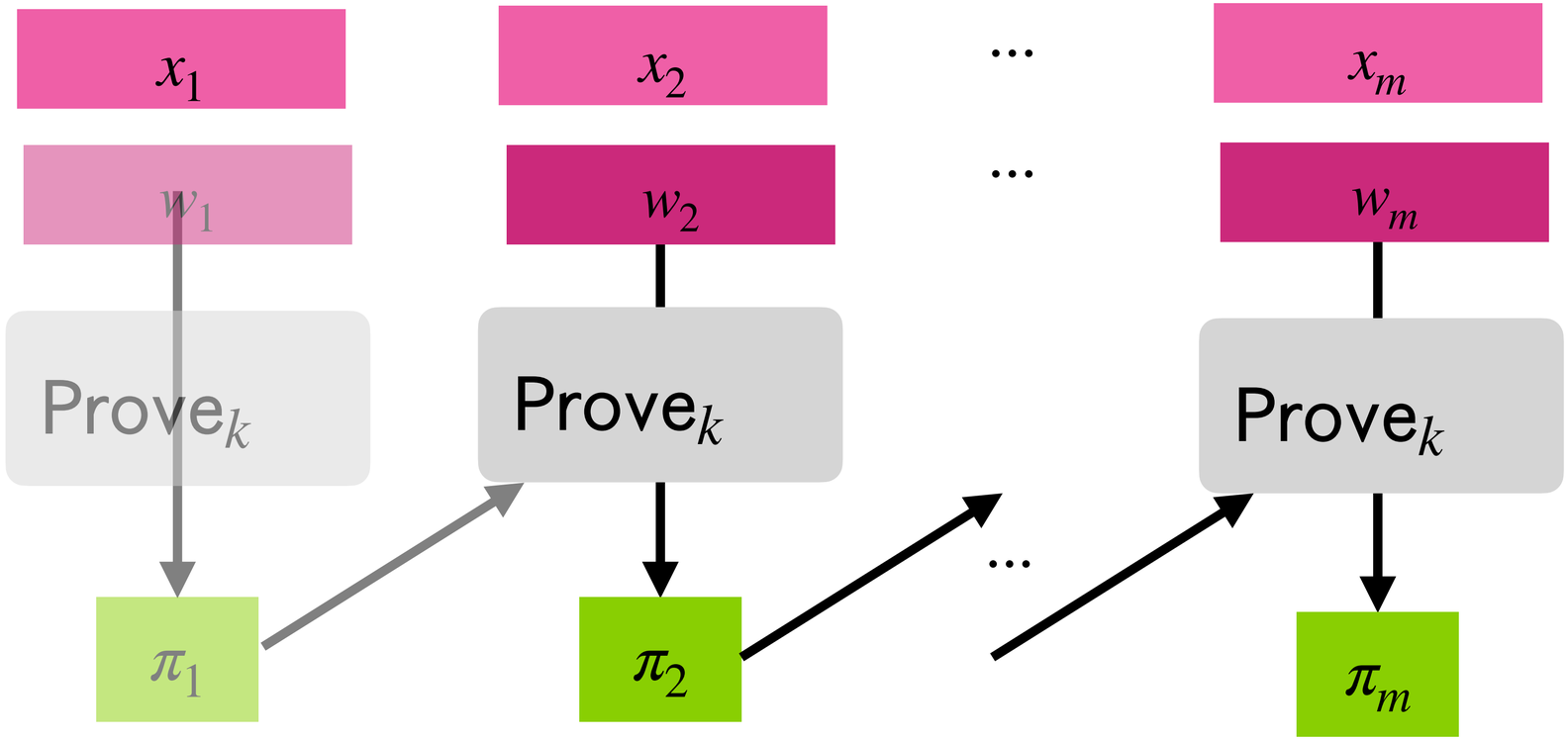

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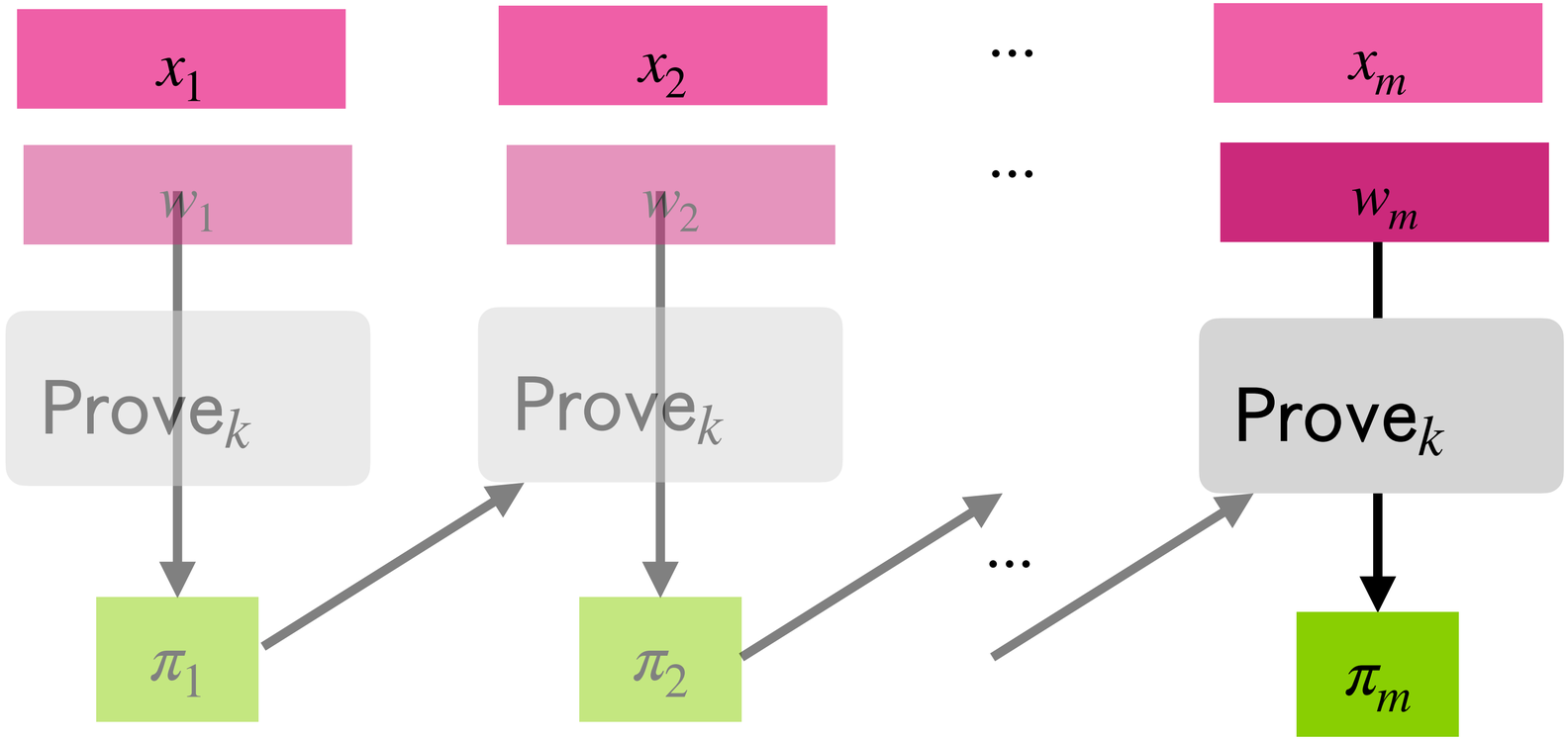

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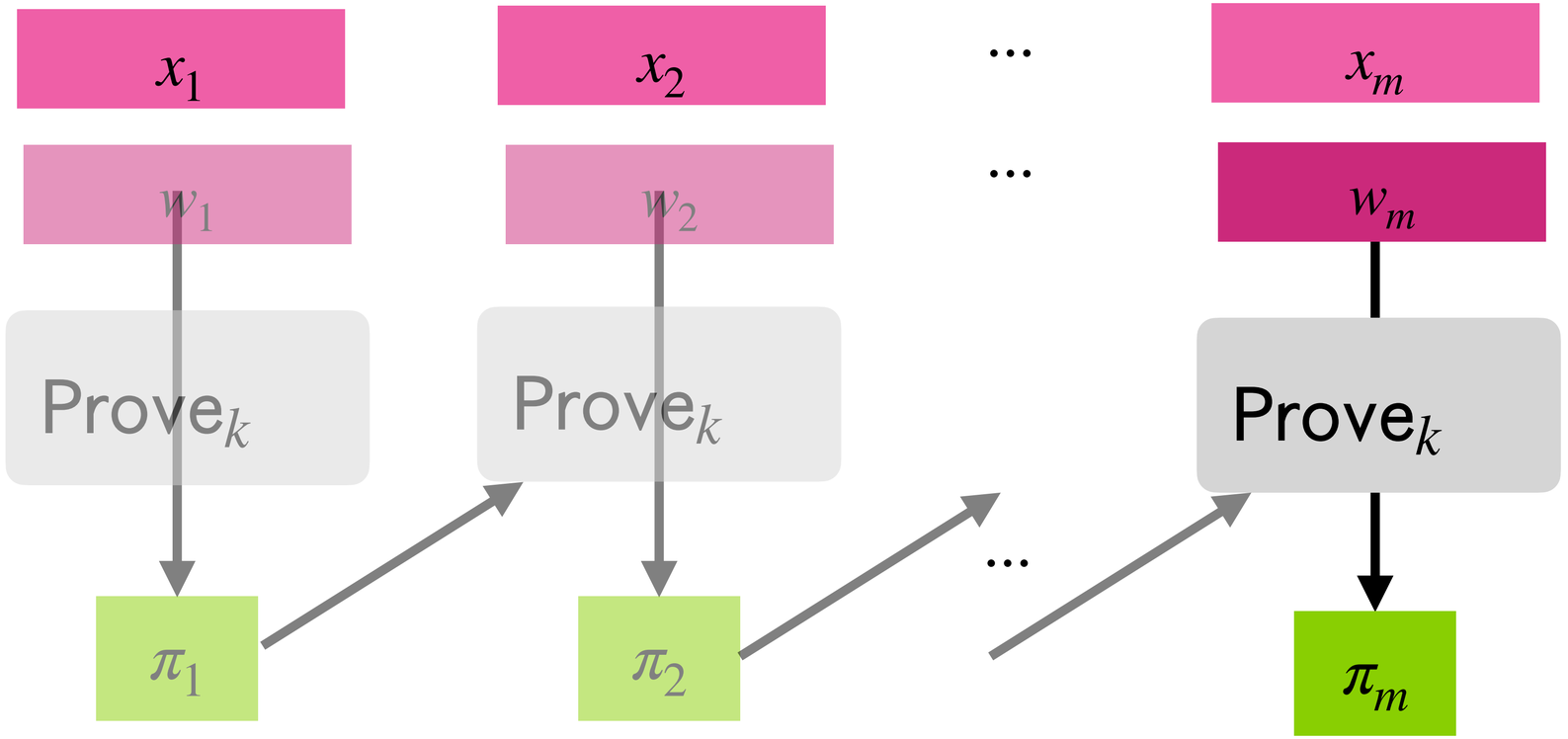

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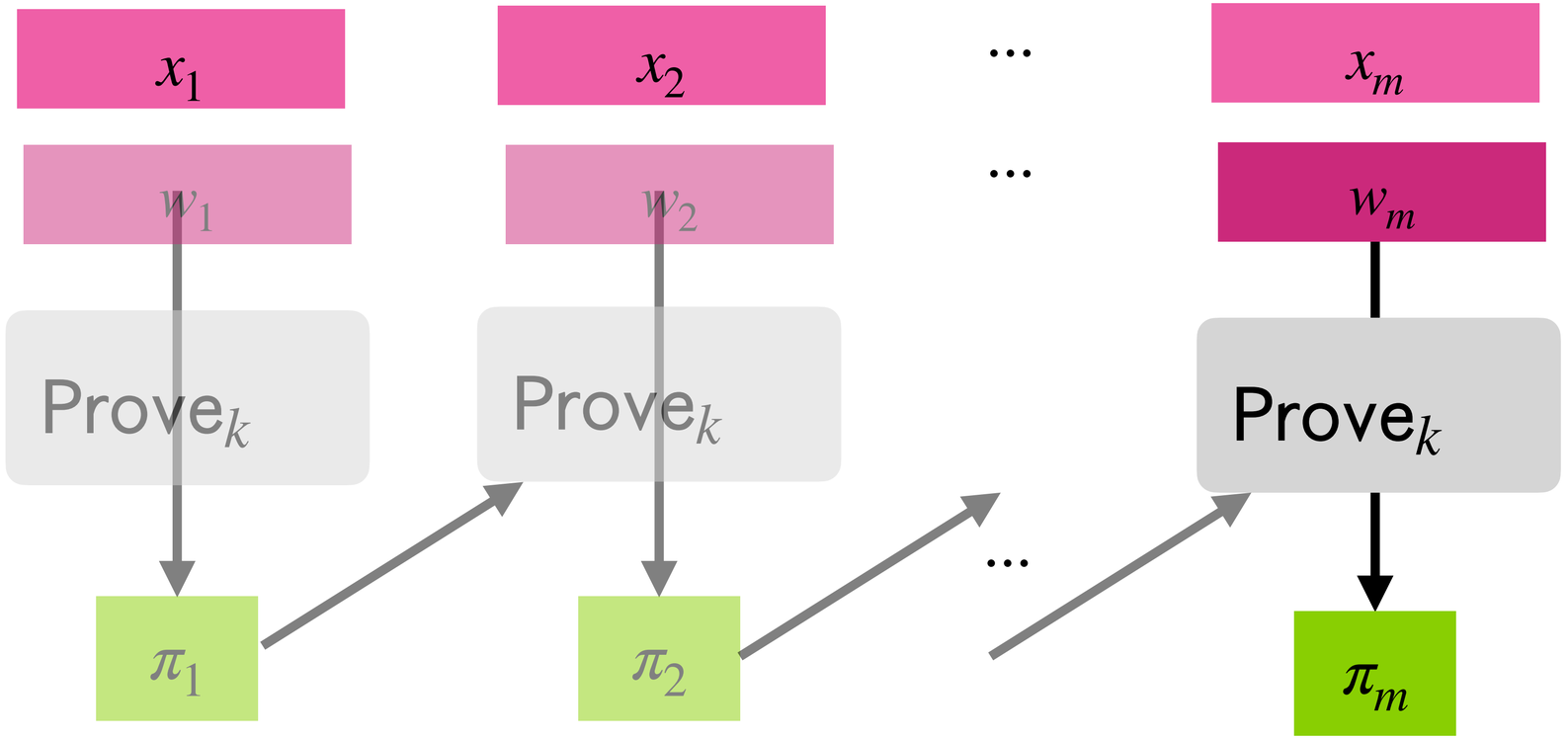

so the obfuscated program in the CRS includes *one* copy of the NP verification circuit C (which the prover runs m times), rather than m copies!

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- **idea from WW24a**: use *exponentially many* hybrids to do an Sahai-Waters-like non-adaptive soundness analysis for every possible statement x_{i*} one-by-one
- we need a **rerandomizable PRG** to make this work with the GSWW batching-via-chaining approach

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$$\hat{y} = (g^{zr}, h^{zr}) = \text{PRG}(zr)$$

this work

theorem. there exists an adaptively-sound SNARG for batch-NP with CRS size poly$(\lambda, |C|, \log m)$ **and proof size** poly(λ).

- assuming iO, OWF, and DDH

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- can we get a short(ish) CRS, i.e., not growing with the statement length, for other languages besides batch-NP?
  - threshold or monotone-policy batch-NP?
- can we build an adaptively-sound SNARG from only subexponentially-secure iO+OWF like in WW24b?
- can we construct rerandomizable PRGs or homomorphic rerandomizable OWFs from other assumptions?

thank you!

full version: eprint.iacr.org/2024/1812

questions?