Cryptography in the Common Haar State Model: Feasibility Results and Separations

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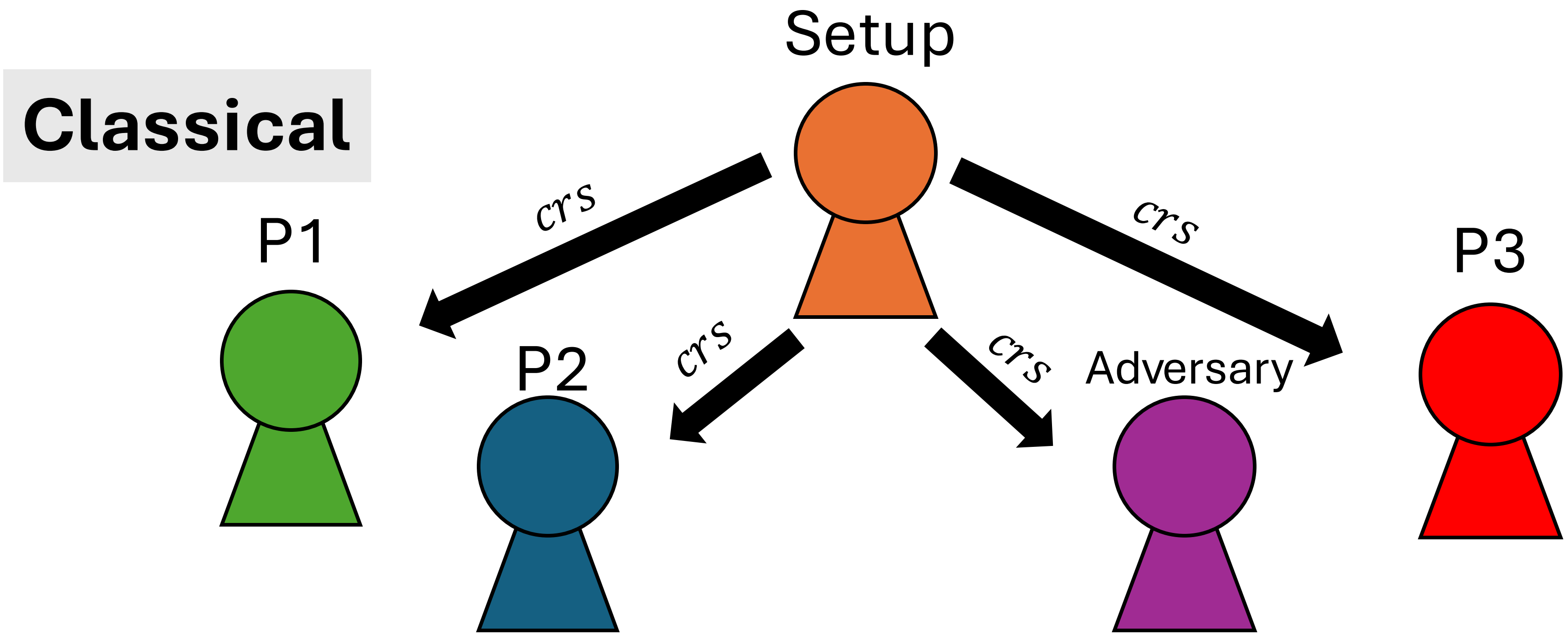
Introduction

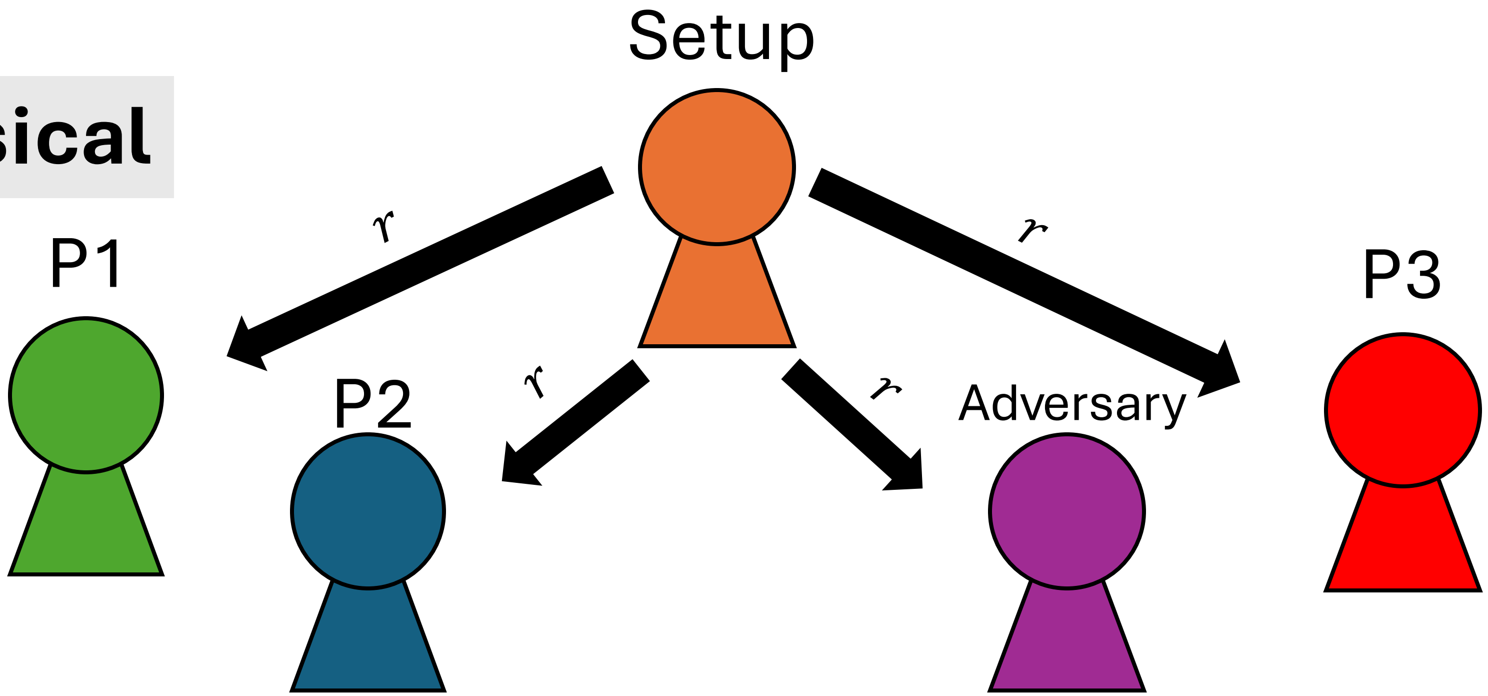
Common Reference String Model



- **Motivation:** Bypass impossibility results in the plain model
- Trusted setup outputs a **reference string** crs to each party, including the adversary
- **Applications:** NIZK, MPC, ...

Common Random String Model

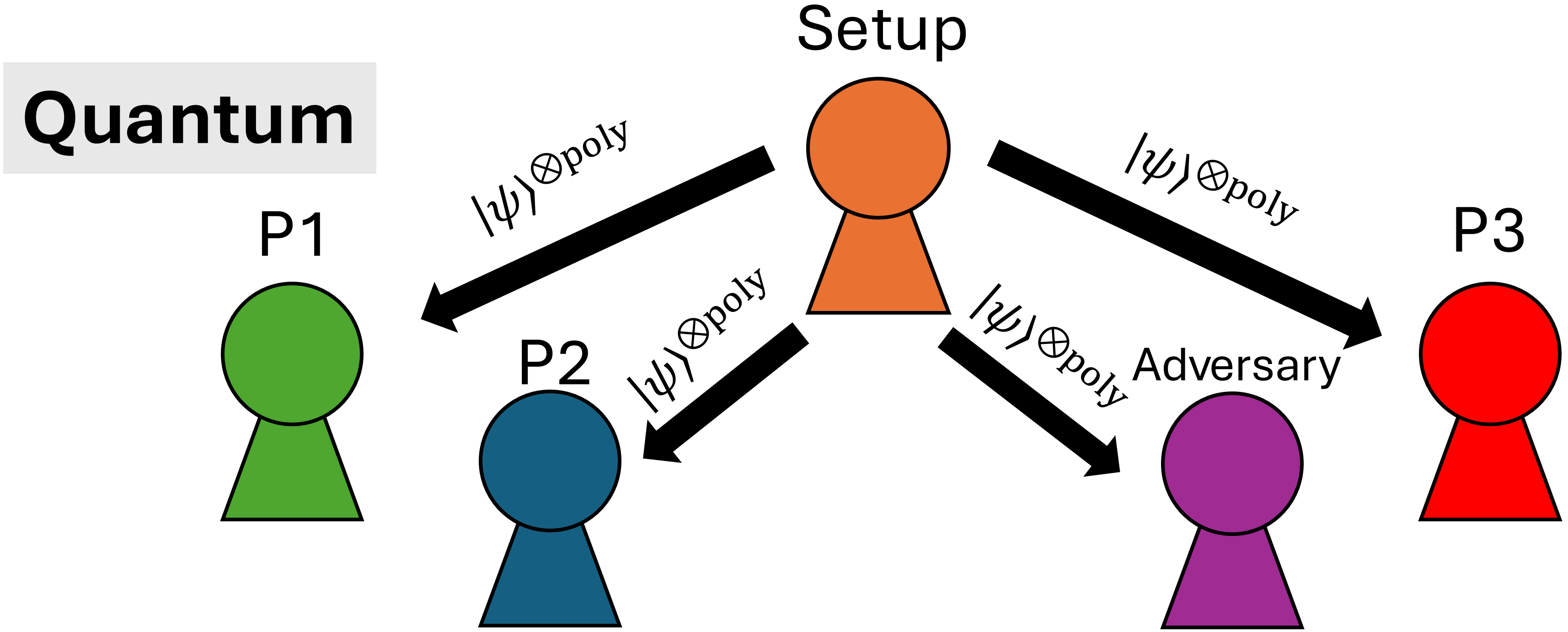
Classical



- Trusted setup outputs a **random string** r to each party, including the adversary
- Lack of structure $\Longrightarrow$ Easier to instantiate (e.g. lottery draw, cloud pattern)
- More desirable than the Common Reference String Model

Could Quantum be Useful?

Common Reference Quantum State Model



- [Morimae-Nehoran-Yamakawa'24] (see also [Qian'24]):

Stat.-hiding & Stat.-binding quantum commitments **exist** in the Common Reference Quantum State Model (quantum analogue of the Common **Reference** String Model)

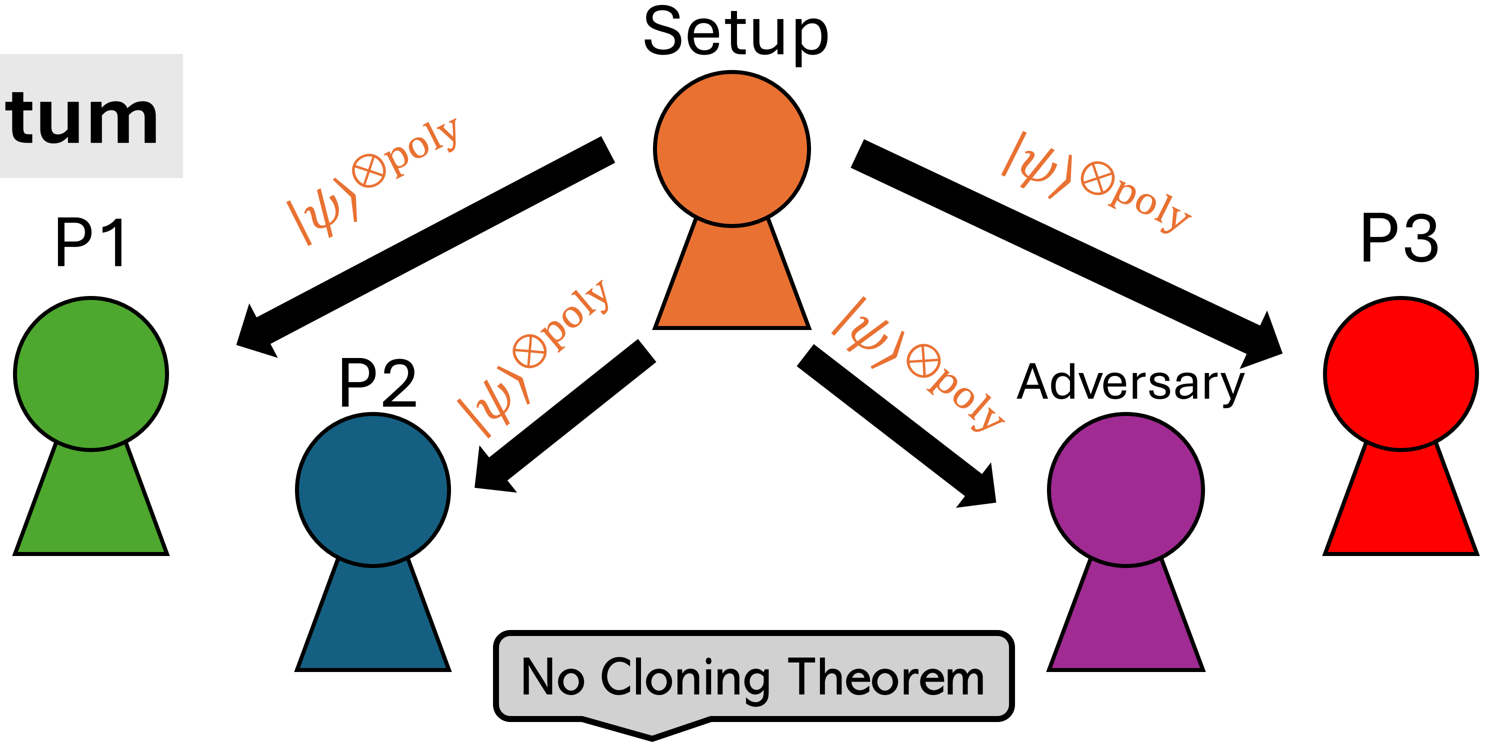
- Impossible in the Common Reference String Model

Our work: Common Haar State (CHS) Model

(quantum analogue of the Common **Random** String Model)

Definition: Common Haar State (CHS) Model

Quantum



- Trusted setup outputs **polynomial copies** of a **Haar random state** $|\psi\rangle$ to each party, including the adversary
- An independent and concurrent work by [Chen-Coladangelo-Sattath'24] also introduced the same model

Motivation

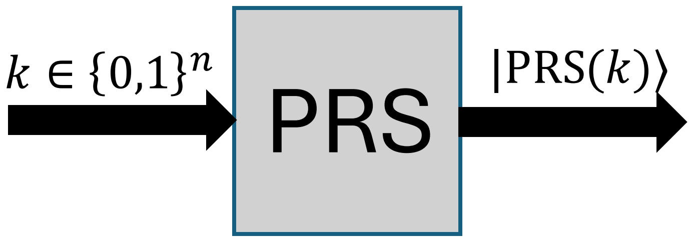
1. Bypassing impossibilities in the plain model
   - Some primitive that requires computational assumptions could be statistically secure in the CHS model

2. Modular approach for designing primitives
   - Instantiate the common Haar state by state designs or pseudorandom states (PRS) in the plain model

3. Black-box separations

Background: Quantum Pseudorandom Primitives

- Pseudorandom States (PRS) Generator:
  - ➢ Defined by [Ji-Liu-Song'18]
  - ➢ Quantum analogue of PRG
  - ➢ Computationally indistinguishable from a Haar state, even when the adversary holds many copies
  - ➢ Stat.-secure, **stretch** PRS is **impossible** in the plain model



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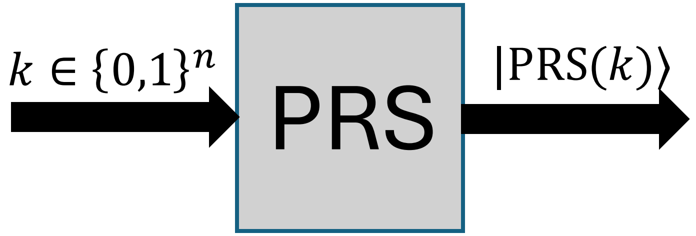

- Pseudorandom Function-Like State (PRFS) Generator:
  - ➢ Defined by [**Ananth**-Qian-Yuen'21]
  - ➢ Quantum analogue of PRF
  - ➢ Computationally indistinguishable from an oracle that outputs an i.i.d. Haar state $|\psi_x\rangle$ on input x

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Under the parameter regime that is impossible to achieve in the plain model

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  - Stronger results + simpler proof compared to [Chen-Coladangelo-Sattath'24]

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- Negative results:
  1. Optimality of our construction:
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Main technical tool:
LOCC Haar Indistinguishability

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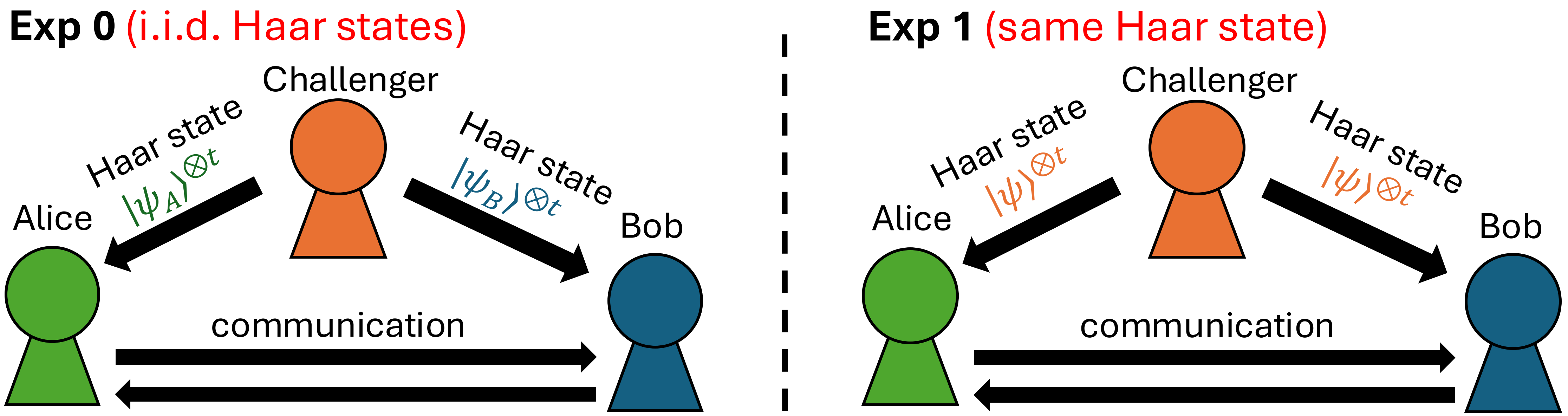
Can two communicating parties w.h.p distinguish i.i.d. vs same Haar states?



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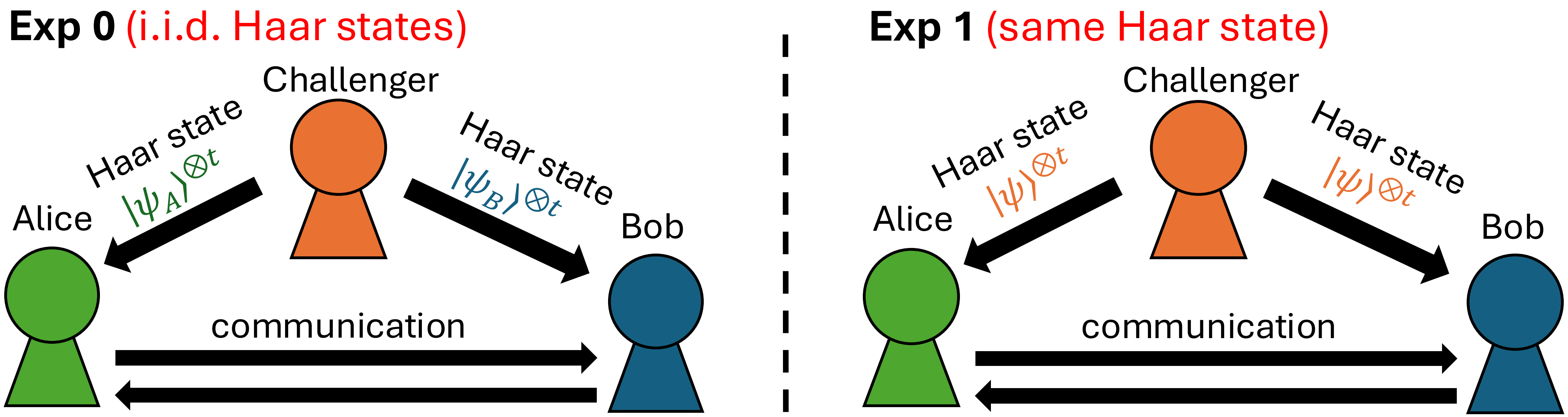

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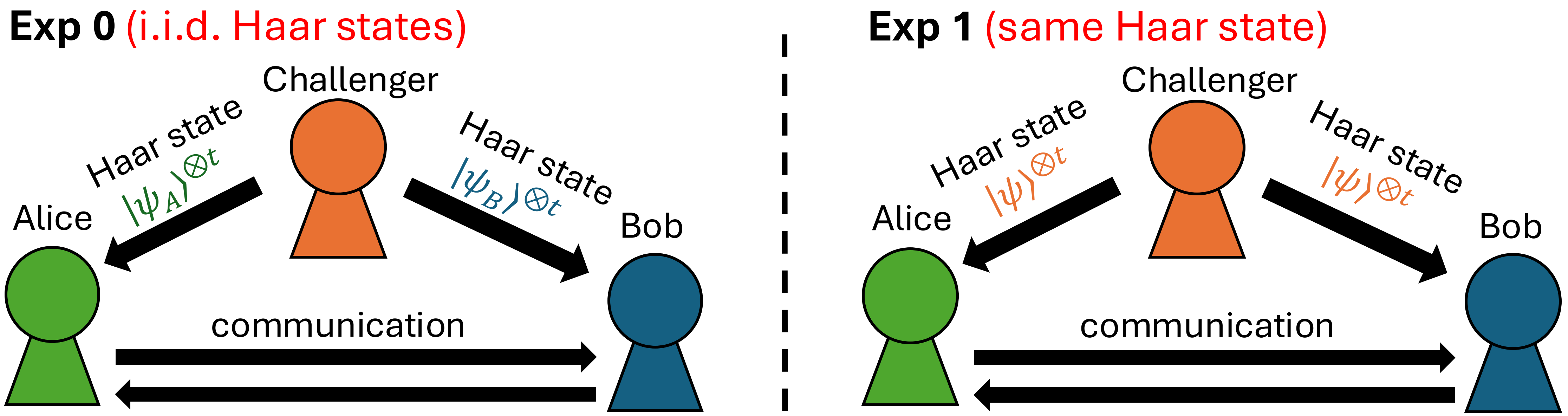

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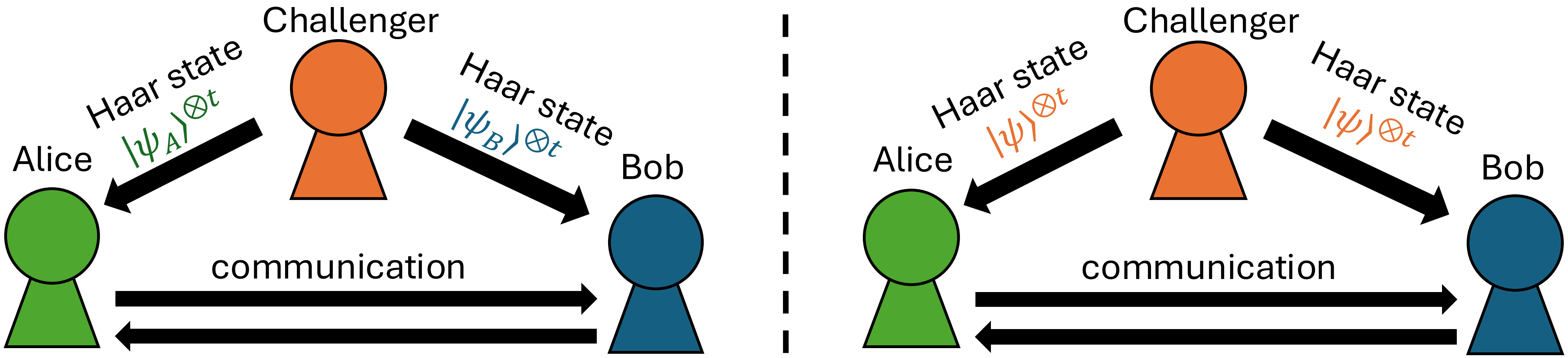
Exp 0 (i.i.d. Haar states) | **Exp 1** (same Haar state)



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- **Our work:** (Alice, Bob)'s distinguishing advantage is $O(t^2/2^n)$
  - Holds for Positive Partial Transpose (PPT) operators, which is a strict superset of LOCC operators
  - The bound is tight: ∃ (Alice, Bob) with advantage $\Omega(t^2/2^n)$

Our Construction of PRS

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Construction:

On key $k \in \{0,1\}^n$ and m-qubit common Haar state $|\psi\rangle$,

$$|\mathrm{PRS}(k)\rangle := (Z^k \otimes \mathrm{id}_{m-n})|\psi\rangle$$

where $Z^k := Z^{k_1} \otimes Z^{k_2} \otimes \cdots \otimes Z^{k_n}$

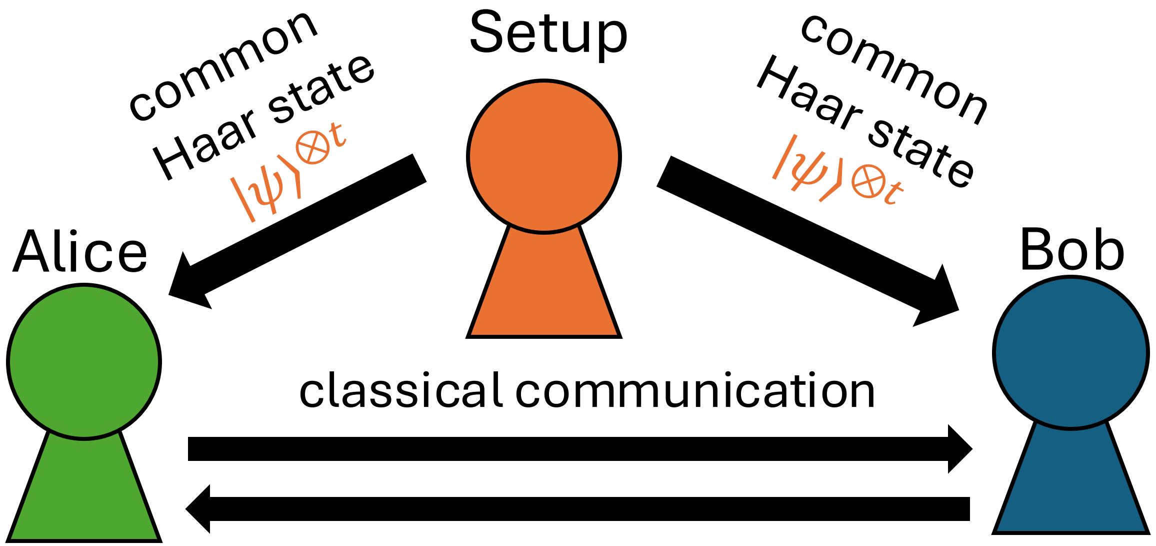
- Efficient generation
- Stretch
- Security: symmetric subspace + combinatorial arguments
- Work for $|\psi\rangle$ of **any** length $\geq$ key length

Impossibility of Interactive QCCC Primitives in the CHS model

A Framework for Proving Impossibilities in CHS model

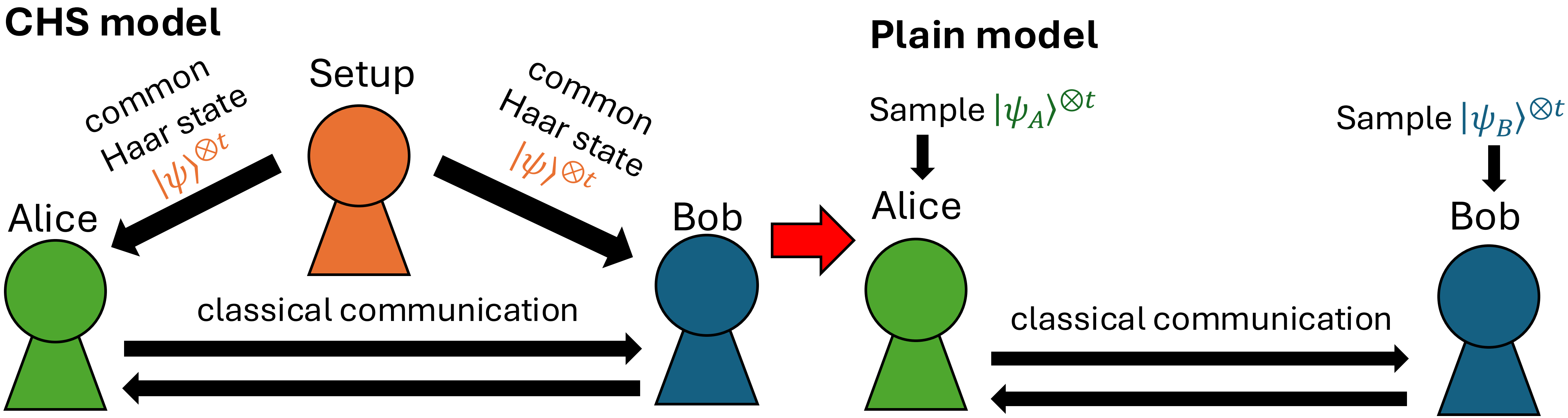
- Some stat.-secure QCCC protocol (e.g. key agreement, commitment) exists in the CHS model

CHS model



A Framework for Proving Impossibilities in CHS model

- Some stat.-secure QCCC protocol (e.g. key agreement, commitment) exists in the CHS model
- Define a new protocol in the **plain model** by replacing $|\psi\rangle$ with $|\psi_A\rangle$ and $|\psi_B\rangle$



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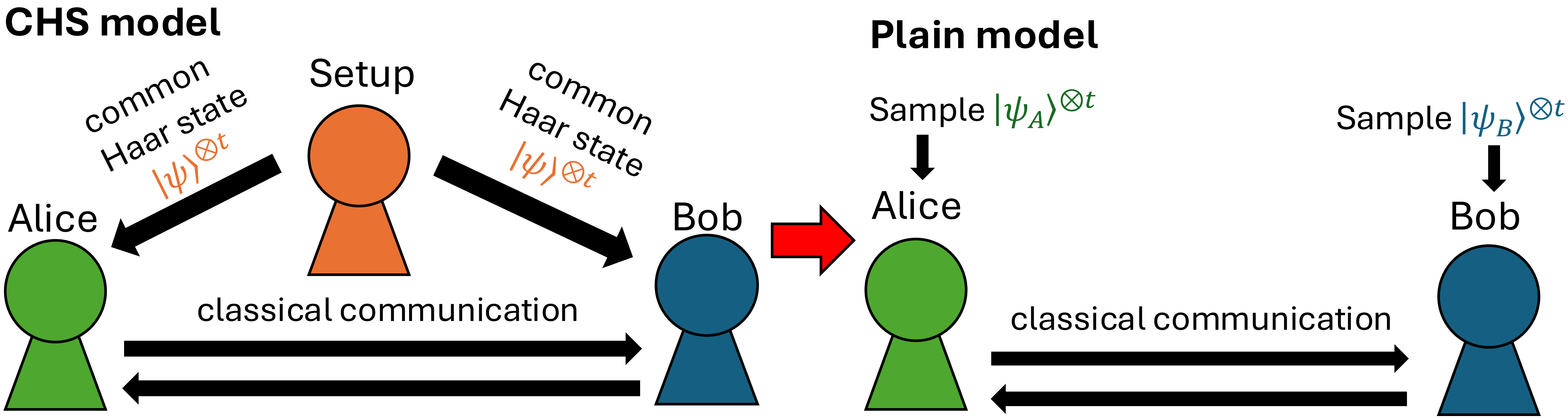
- Some stat.-secure QCCC protocol (e.g. key agreement, commitment) exists in the CHS model
- Define a new protocol in the **plain model** by replacing $|\psi\rangle$ with $|\psi_A\rangle$ and $|\psi_B\rangle$
- By **LOCC Haar indistinguishability**, the new protocol in the plain model remains correct and statistically secure $\Longrightarrow$ Contradiction!



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- PRS with output length $O(\log n)$ implies QCCC commitment [AGQY'22] [ALY'24]

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- PRS with output length $O(\log n)$ implies QCCC commitment [AGQY'22] [ALY'24]
- **Our work:** consider a stronger variant of the CHS model that trivially implies PRFS:
  - Setup prepares a set of i.i.d. Haar states $\{|\psi_{k,x}\rangle\}_{k,x\in\{0,1\}^n}$
  - Party queries on (k, x) classically and gets one copy of $|\psi_{k,x}\rangle$

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  - Party queries on (k, x) classically and gets one copy of $|\psi_{k,x}\rangle$
- Define $|\mathrm{PRFS}(k,x)\rangle := |\psi_{k,x}\rangle$
- Using the same idea to rule out { QCCC key agreement, QCCC commitment } relative to $\{|\psi_{k,x}\rangle\}_{k,x\in\{0,1\}^n}$

Summary

- Common Haar State Model: a quantum analogue of the Common Random String Model
- Some stat.-secure primitives, which are impossible in the plain model, exist in the CHS model
- Separating interactive QCCC primitives from PRFS with super-logarithmic output length

Open Questions & Follow-Up Works

- Quantum Haar Random Oracle Model: Each party has access to a **Haar unitary** oracle
  - ➢Feasibilities & Limitations?
  - ➢Very recent works: [**Ananth**-Bostanci-**Gulati**-**Lin**'24], [Hhan-Yamada'24], …
- LOCC Haar Indistinguishability in the **oracle** setting ? $(A^U, B^U) \approx_{\mathrm{LOCC}} (A^U, B^V)$?

Thanks!