

RUHR-UNIVERSITÄT BOCHUM

A Tale of Snakes and Horses: Amplifying Correlation Power Analysis on Quadratic Maps

Conference on Cryptographic Hardware and Embedded Systems 2024

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CASA
CYBER SECURITY IN THE AGE
OF LARGE-SCALE ADVERSARIES

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Introduction

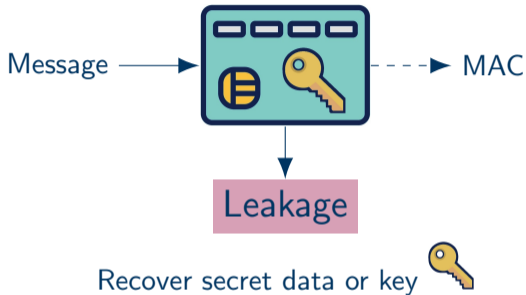
Correlation Power Analysis

Permutation-based Algorithms with a Quadratic S-box

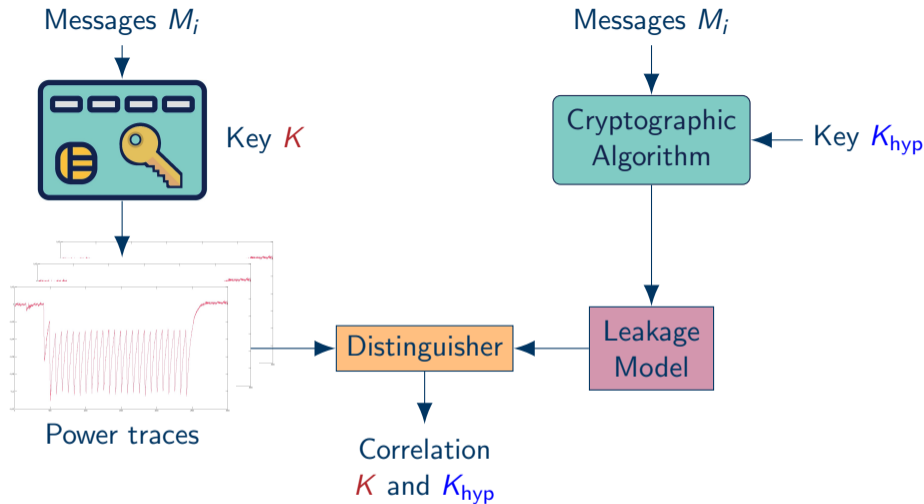
Combined CPA or *Snake Attack*

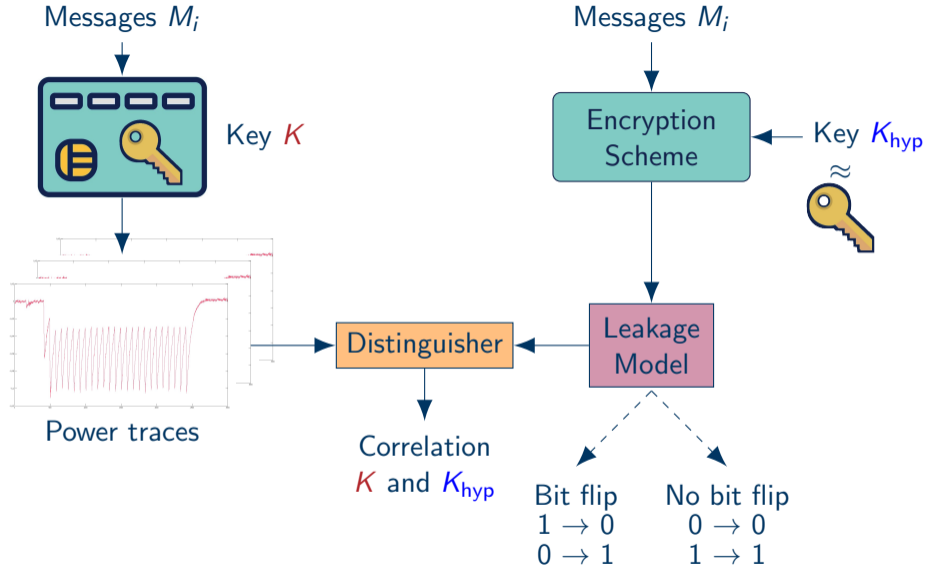
Practical Evaluation

Correlation Power Analysis - An Overview

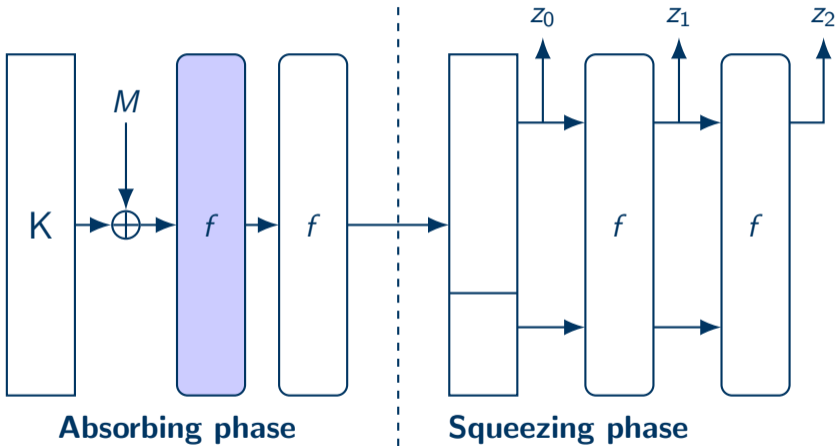


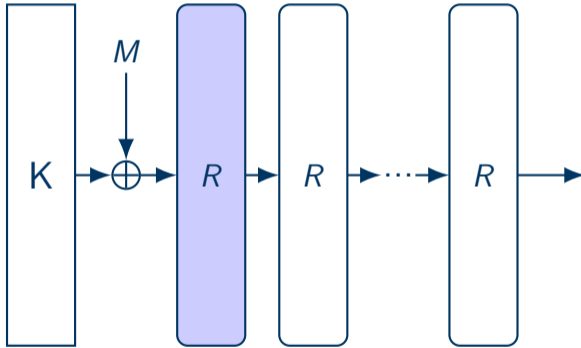
- ▶ Power consumption varies according to activity of device components
- ▶ **Correlation Power Analysis (CPA)**: statistical analysis of power consumption measurements (traces)



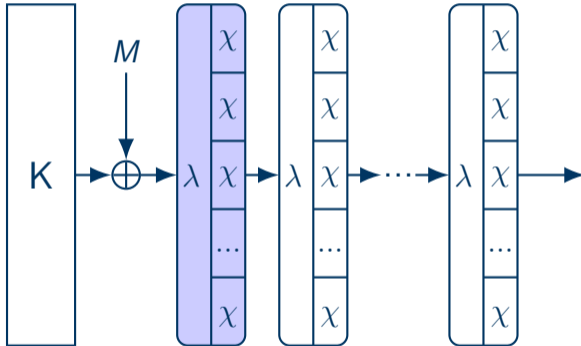


Full State Keyed Sponge-based MAC



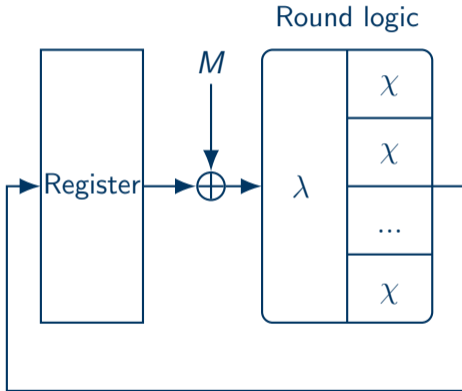


- ▶ A permutation f consists of several rounds R

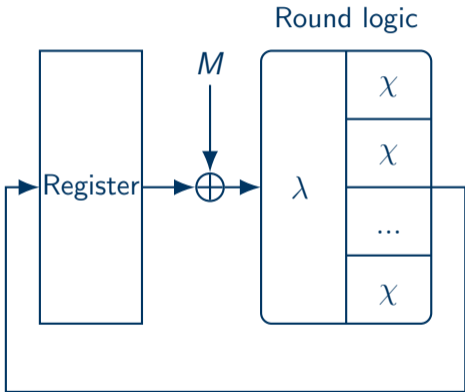


- ▶ A round R consists of a linear layer λ and a non-linear one
- ▶ **Non-linear layer** consists of χ mappings in parallel (S-boxes)

Round-based Hardware Architecture



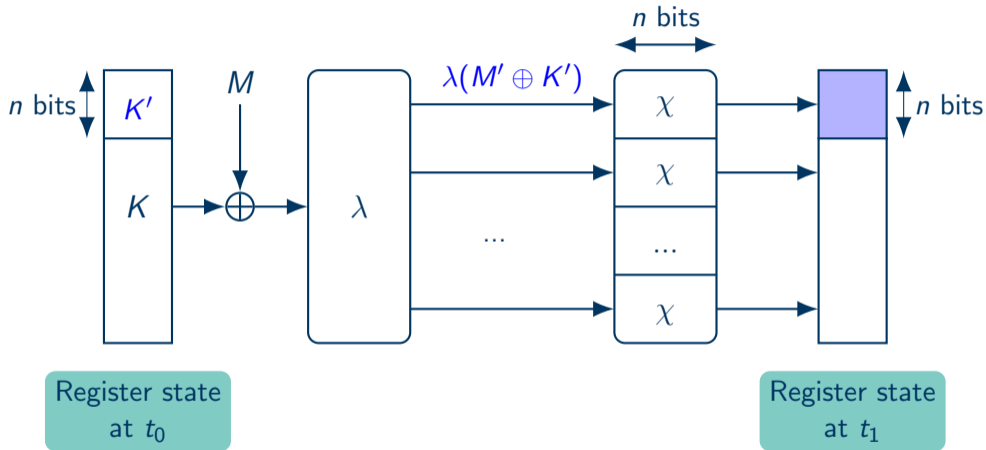
Round-based Hardware Architecture



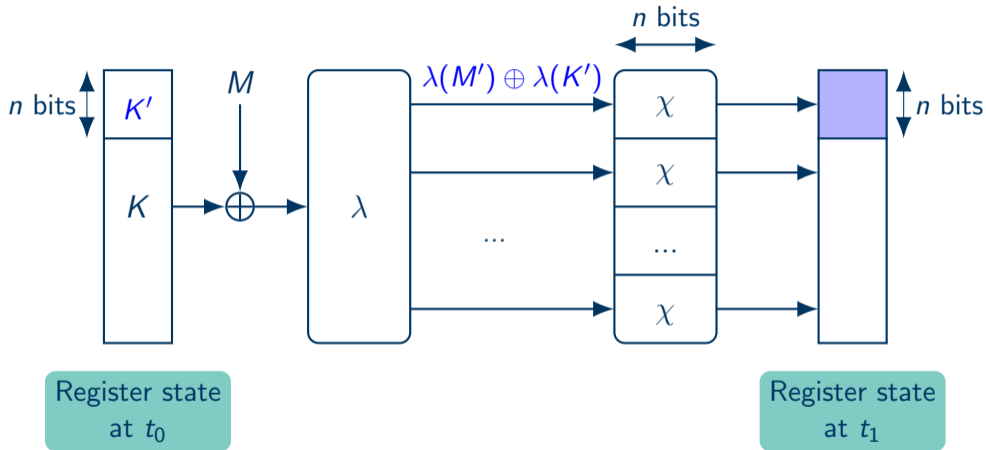
► Leakage model:

- No power consumption from round logic
- Exploit leakage from register
- Quantifying noise from somewhere in the device

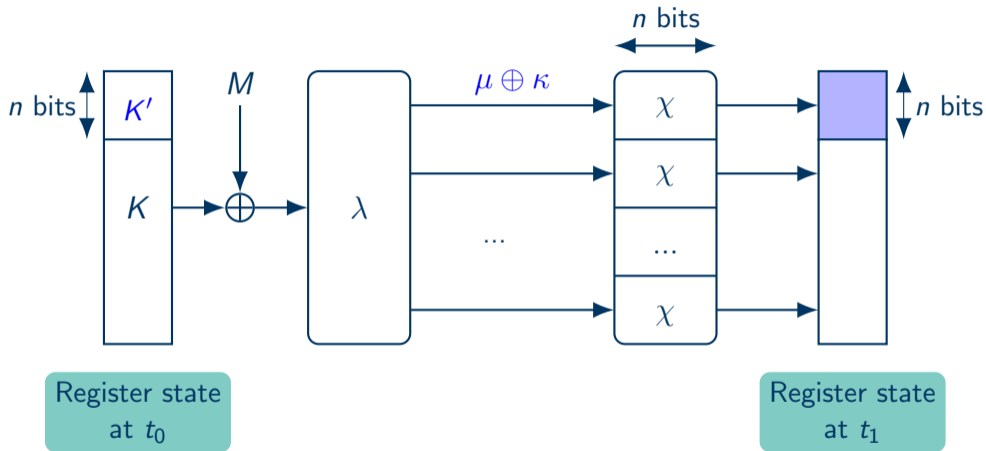
First Round of First Permutation



First Round of First Permutation

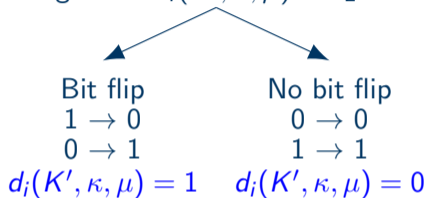


First Round of First Permutation



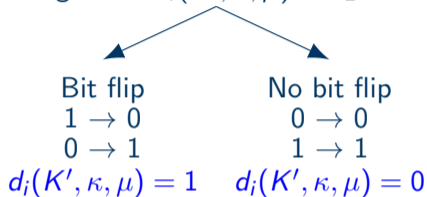
Leakage Model

- Activity of a storage cell: $d_i(K', \kappa, \mu) \in \mathbb{F}_2$ with $i \in \mathbb{Z}/n\mathbb{Z}$

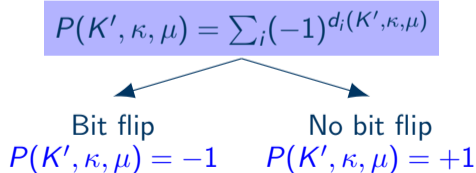


Leakage Model

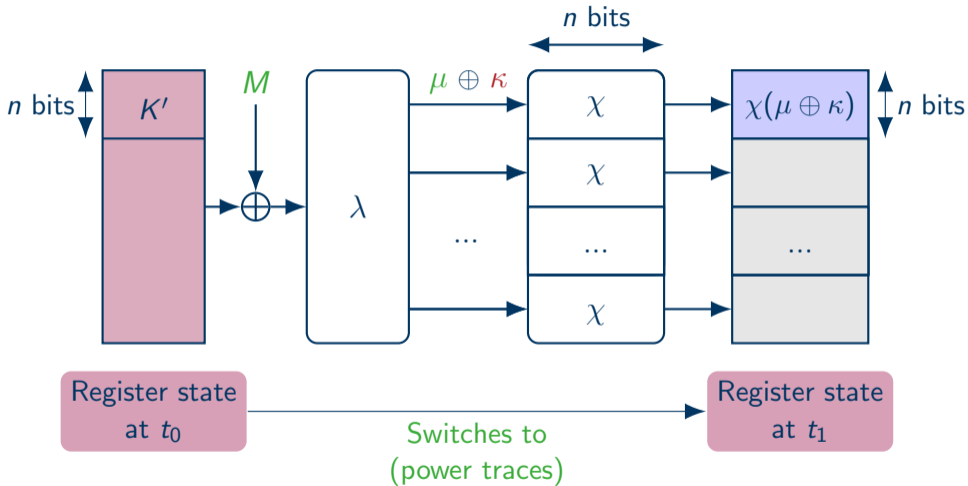
- ▶ Activity of a storage cell: $d_i(K', \kappa, \mu) \in \mathbb{F}_2$ with $i \in \mathbb{Z}/n\mathbb{Z}$



- ▶ Activity of all register bits contributes to **power consumption**:



First n Bits of Register



For the first n bits: $\mu \oplus \kappa = \lambda(M') \oplus \lambda(K') = \lambda(M' \oplus K')$

Refine Leakage Model

- ▶ Activity of register after a round for the first storage cell:

$$d_0(K'_0, \kappa, \mu) = K'_0 \oplus \underbrace{\kappa_0 \oplus \mu_0 \oplus (\kappa_1 \oplus \mu_1 \oplus 1)(\kappa_2 \oplus \mu_2)}_{\text{first bit of } \chi(\mu \oplus \kappa)}$$

Refine Leakage Model

- ▶ Activity of register after a round for the first storage cell:

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- ▶ Power consumption $P = S + R$ of a trace

Signal part

$$S(\mu) = \sum_{i=0}^{n-1} (-1)^{d_i(K_i, \kappa, \mu)}$$

n bits

Noise part

$$R \sim \mathcal{N}(0, \sigma^2)$$

$$\sigma = 1$$

Refine Leakage Model

- ▶ Activity of register after a round for the first storage cell:

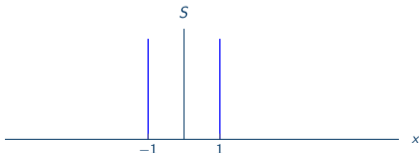
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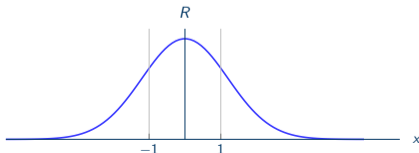
$n = 1$ bit



Noise part

$$R \sim \mathcal{N}(0, \sigma^2)$$

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Refine Leakage Model

- ▶ Activity of register after a round for the first storage cell:

$$d_0(K'_0, \kappa, \mu) = K'_0 \oplus \underbrace{\kappa_0 \oplus \mu_0 \oplus (\kappa_1 \oplus \mu_1 \oplus 1)(\kappa_2 \oplus \mu_2)}_{\text{first bit of } \chi(\mu \oplus \kappa)}$$

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Signal part

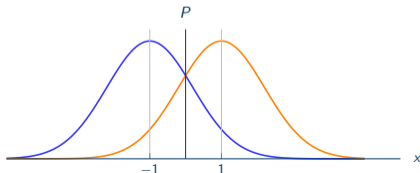
$$S(\mu) = \sum_{i=0}^{n-1} (-1)^{d_i(K_i, \kappa, \mu)}$$

$n = 1$ bit

Noise part

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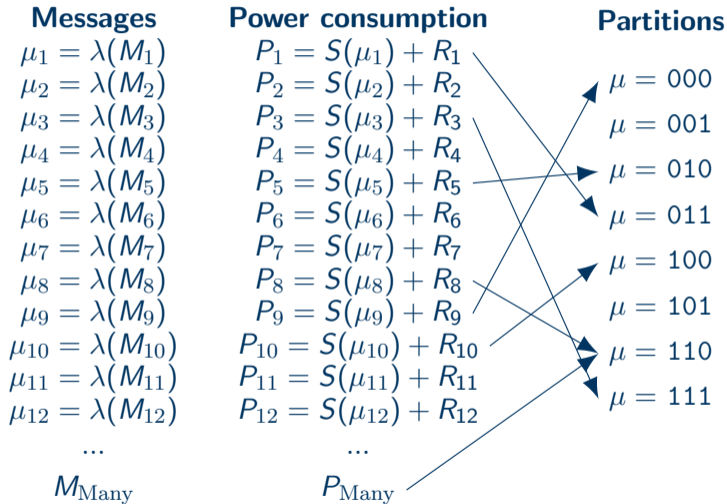
Partitioning Power Traces

Messages	Power consumption	Partitions
$\mu_1 = \lambda(M_1)$	$P_1 = S(\mu_1) + R_1$	
$\mu_2 = \lambda(M_2)$	$P_2 = S(\mu_2) + R_2$	
$\mu_3 = \lambda(M_3)$	$P_3 = S(\mu_3) + R_3$	
$\mu_4 = \lambda(M_4)$	$P_4 = S(\mu_4) + R_4$	
$\mu_5 = \lambda(M_5)$	$P_5 = S(\mu_5) + R_5$	Bit flip
$\mu_6 = \lambda(M_6)$	$P_6 = S(\mu_6) + R_6$	
$\mu_7 = \lambda(M_7)$	$P_7 = S(\mu_7) + R_7$	
$\mu_8 = \lambda(M_8)$	$P_8 = S(\mu_8) + R_8$	
$\mu_9 = \lambda(M_9)$	$P_9 = S(\mu_9) + R_9$	No bit flip
$\mu_{10} = \lambda(M_{10})$	$P_{10} = S(\mu_{10}) + R_{10}$	
$\mu_{11} = \lambda(M_{11})$	$P_{11} = S(\mu_{11}) + R_{11}$	
$\mu_{12} = \lambda(M_{12})$	$P_{12} = S(\mu_{12}) + R_{12}$	
...	...	
M_{Many}	P_{Many}	

Partitioning Power Traces

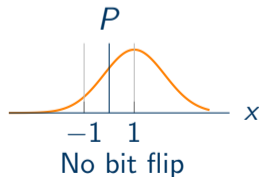
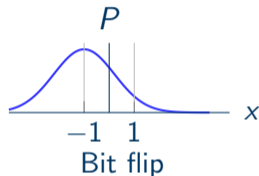
Messages	Power consumption	Partitions
$\mu_1 = \lambda(M_1)$	$P_1 = S(\mu_1) + R_1$	$\mu = 000$
$\mu_2 = \lambda(M_2)$	$P_2 = S(\mu_2) + R_2$	$\mu = 001$
$\mu_3 = \lambda(M_3)$	$P_3 = S(\mu_3) + R_3$	$\mu = 010$
$\mu_4 = \lambda(M_4)$	$P_4 = S(\mu_4) + R_4$	$\mu = 011$
$\mu_5 = \lambda(M_5)$	$P_5 = S(\mu_5) + R_5$	$\mu = 100$
$\mu_6 = \lambda(M_6)$	$P_6 = S(\mu_6) + R_6$	$\mu = 101$
$\mu_7 = \lambda(M_7)$	$P_7 = S(\mu_7) + R_7$	$\mu = 110$
$\mu_8 = \lambda(M_8)$	$P_8 = S(\mu_8) + R_8$	$\mu = 111$
$\mu_9 = \lambda(M_9)$	$P_9 = S(\mu_9) + R_9$	
$\mu_{10} = \lambda(M_{10})$	$P_{10} = S(\mu_{10}) + R_{10}$	
$\mu_{11} = \lambda(M_{11})$	$P_{11} = S(\mu_{11}) + R_{11}$	
$\mu_{12} = \lambda(M_{12})$	$P_{12} = S(\mu_{12}) + R_{12}$	
...	...	
M_{Many}	P_{Many}	

Partitioning Power Traces

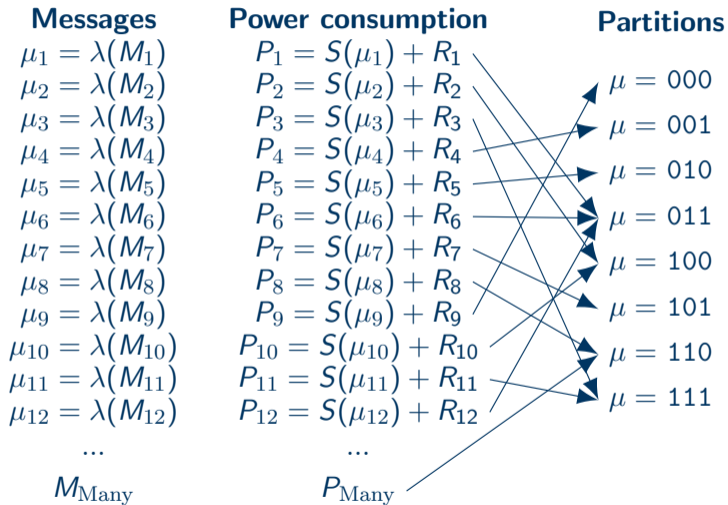


→ Average of each partition

Few traces

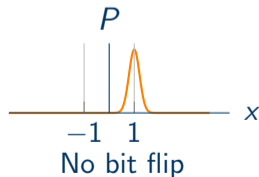
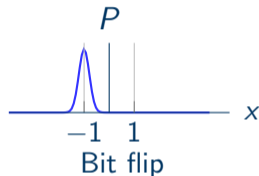


Partitioning Power Traces



→ Average of each partition

Many traces



Correlation

▶ Activity $d_0(K'_0, \kappa, \mu) = K'_0 \oplus \kappa_0 \oplus \mu_0 \oplus (\kappa_1 \oplus \mu_1 \oplus 1)(\kappa_2 \oplus \mu_2)$

Correlation

- ▶ Activity $d_0(K'_0, \kappa, \mu) = K'_0 \oplus \kappa_0 \oplus \mu_0 \oplus (\kappa_1 \oplus \mu_1 \oplus 1)(\kappa_2 \oplus \mu_2)$
- ▶ Signal power consumption values S_{ref} for all (κ, μ) possibilities for K'_0

$\mu_0\mu_1\mu_2$	$K'_0 \oplus \kappa_0 \kappa_1\kappa_2$							
	000	001	010	011	100	101	110	111
000	+1	-1	+1	+1	-1	+1	-1	-1
001	-1	+1	+1	+1	+1	-1	-1	-1
010	+1	+1	+1	-1	-1	-1	-1	+1
011	+1	+1	-1	+1	-1	-1	+1	-1
100	-1	+1	-1	-1	+1	-1	+1	+1
101	+1	-1	-1	-1	-1	+1	+1	+1
110	-1	-1	-1	+1	+1	+1	+1	-1
111	-1	-1	+1	-1	+1	+1	-1	+1

Correlation

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$\mu_0\mu_1\mu_2$	$K'_0 \oplus \kappa_0 \kappa_1\kappa_2$							
	000	001	010	011	100	101	110	111
000	+1	-1	+1	+1	-1	+1	-1	-1
001	-1	+1	+1	+1	+1	-1	-1	-1
010	+1	+1	+1	-1	-1	-1	-1	+1
011	+1	+1	-1	+1	-1	-1	+1	-1
100	-1	+1	-1	-1	+1	-1	+1	+1
101	+1	-1	-1	-1	-1	+1	+1	+1
110	-1	-1	-1	+1	+1	+1	+1	-1
111	-1	-1	+1	-1	+1	+1	-1	+1

- ▶ Pearson correlation coefficient $\rho(P, S_{\text{ref}})$: Highest correlation result $\max(\rho(P, S_{\text{ref}}))$

Correlation

- ▶ Activity $d_0(K'_0, \kappa, \mu) = K'_0 \oplus \kappa_0 \oplus \mu_0 \oplus (\kappa_1 \oplus \mu_1 \oplus 1)(\kappa_2 \oplus \mu_2)$
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000	+1	-1	+1	+1	-1	+1	-1	-1
001	-1	+1	+1	+1	+1	-1	-1	-1
010	+1	+1	+1	-1	-1	-1	-1	+1
011	+1	+1	-1	+1	-1	-1	+1	-1
100	-1	+1	-1	-1	+1	-1	+1	+1
101	+1	-1	-1	-1	-1	+1	+1	+1
110	-1	-1	-1	+1	+1	+1	+1	-1
111	-1	-1	+1	-1	+1	+1	-1	+1

- ▶ Pearson correlation coefficient $\rho(P, S_{\text{ref}})$: Highest correlation result $\max(\rho(P, S_{\text{ref}}))$

Correlation

- ▶ Activity $d_0(K'_0, \kappa, \mu) = K'_0 \oplus \kappa_0 \oplus \mu_0 \oplus (\kappa_1 \oplus \mu_1 \oplus 1)(\kappa_2 \oplus \mu_2)$
- ▶ Signal power consumption values S_{ref} for all (κ, μ) possibilities for K'_0

$\mu_0\mu_1\mu_2$	$* \kappa_1\kappa_2$							
	000	001	010	011	100	101	110	111
000	+1	-1	+1	+1	-	-	-	-
001	-1	+1	+1	+1	-	-	-	-
010	+1	+1	+1	-1	-	-	-	-
011	+1	+1	-1	+1	-	-	-	-
100	-1	+1	-1	-1	-	-	-	-
101	+1	-1	-1	-1	-	-	-	-
110	-1	-1	-1	+1	-	-	-	-
111	-1	-1	+1	-1	-	-	-	-

- ▶ Pearson correlation coefficient $\rho(P, S_{\text{ref}})$: $\max(\rho(P, S_{\text{ref}})^2)$ or $\max(|\rho(P, S_{\text{ref}})|)$

Correlation

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	000	001	010	011	100	101	110	111
000	+1	-1	+1	+1	-	-	-	-
001	-1	+1	+1	+1	-	-	-	-
010	+1	+1	+1	-1	-	-	-	-
011	+1	+1	-1	+1	-	-	-	-
100	-1	+1	-1	-1	-	-	-	-
101	+1	-1	-1	-1	-	-	-	-
110	-1	-1	-1	+1	-	-	-	-
111	-1	-1	+1	-1	-	-	-	-

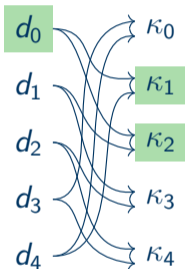
- ▶ Pearson correlation coefficient $\rho(P, S_{\text{ref}})$: $\max(\rho(P, S_{\text{ref}})^2)$ or $\max(|\rho(P, S_{\text{ref}})|)$

Combined CPA or Snake attack: Recovering κ

- Activity function $d_i(K', \kappa, \mu) = K'_i \oplus \kappa_i \oplus \mu_i \oplus (\kappa_{i+1} \oplus \mu_{i+1} \oplus 1)(\kappa_{i+2} \oplus \mu_{i+2})$

$n = 5$ bits

1 attack



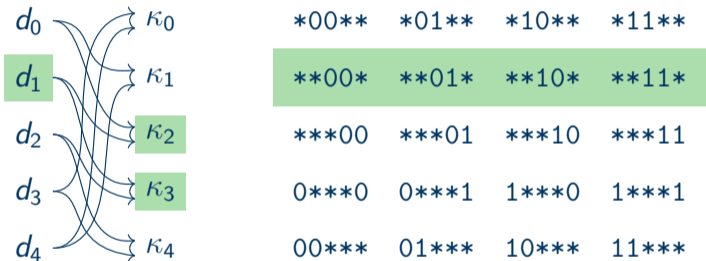
*00**	*01**	*10**	*11**
**00*	**01*	**10*	**11*
***00	***01	***10	***11
0***0	0***1	1***0	1***1
00***	01***	10***	11***

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1 attack

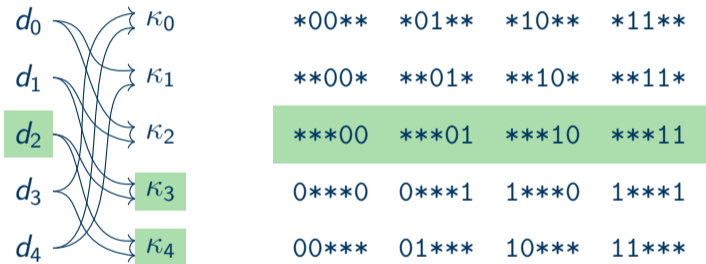


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$n = 5$ bits

1 attack

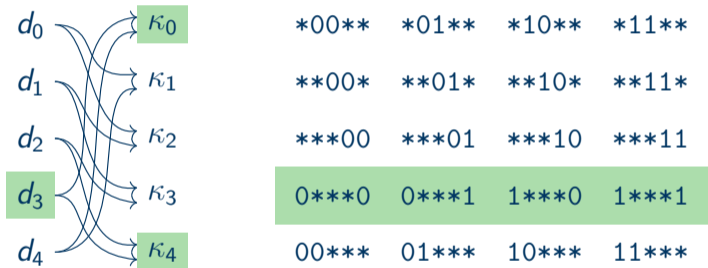


Combined CPA or Snake attack: Recovering κ

- Activity function $d_i(K', \kappa, \mu) = K'_i \oplus \kappa_i \oplus \mu_i \oplus (\kappa_{i+1} \oplus \mu_{i+1} \oplus 1)(\kappa_{i+2} \oplus \mu_{i+2})$

$n = 5$ bits

1 attack

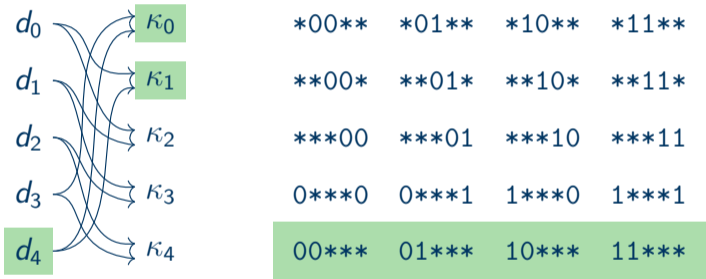


Combined CPA or Snake attack: Recovering κ

► Activity function $d_i(K', \kappa, \mu) = K'_i \oplus \kappa_i \oplus \mu_i \oplus (\kappa_{i+1} \oplus \mu_{i+1} \oplus 1)(\kappa_{i+2} \oplus \mu_{i+2})$

$n = 5$ bits

1 attack



Combined CPA or Snake Attack: Recovering K'

$n = 5$ bits

Correlation result for $i = 0$:	$K'_0 \oplus \kappa_0$	κ_1	κ_2	—	—
Correlation result for $i = 1$:	—	$K'_1 \oplus \kappa_1$	κ_2	κ_3	—
Correlation result for $i = 2$:	—	—	$K'_2 \oplus \kappa_2$	κ_3	κ_4
Correlation result for $i = 3$:	κ_0	—	—	$K'_3 \oplus \kappa_3$	κ_4
Correlation result for $i = 4$:	κ_0	κ_1	—	—	$K'_4 \oplus \kappa_4$

Combined CPA or Snake Attack: Recovering K'

$n = 5$ bits

Correlation result for $i = 0$:	$K'_0 \oplus \kappa_0$	κ_1	κ_2	—	—
Correlation result for $i = 1$:	—	$K'_1 \oplus \kappa_1$	κ_2	κ_3	—
Correlation result for $i = 2$:	—	—	$K'_2 \oplus \kappa_2$	κ_3	κ_4
Correlation result for $i = 3$:	κ_0	—	—	$K'_3 \oplus \kappa_3$	κ_4
Correlation result for $i = 4$:	κ_0	κ_1	—	—	$K'_4 \oplus \kappa_4$

Combined CPA or Snake Attack: Recovering K'

$n = 5$ bits

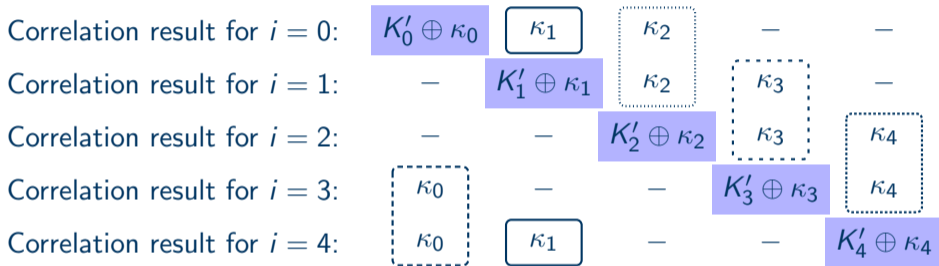
Correlation result for $i = 0$:	$K'_0 \oplus \kappa_0$	κ_1	κ_2	—	—
Correlation result for $i = 1$:	—	$K'_1 \oplus \kappa_1$	κ_2	κ_3	—
Correlation result for $i = 2$:	—	—	$K'_2 \oplus \kappa_2$	κ_3	κ_4
Correlation result for $i = 3$:	κ_0	—	—	$K'_3 \oplus \kappa_3$	κ_4
Correlation result for $i = 4$:	κ_0	κ_1	—	—	$K'_4 \oplus \kappa_4$

► For each bit i : $K'_i = \epsilon_i \oplus \kappa_i$ with $\epsilon_i = 1$ if $\rho(P, S_{ref_i}) < 0$, otherwise $\epsilon_i = 0$.

ϵ_i (known) κ_i (guess)

Combined CPA or Snake Attack: Recovering K'

$n = 5$ bits



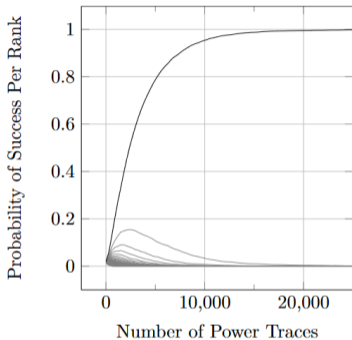
► For each bit i : $K'_i = \epsilon_i \oplus \kappa_i$ with $\epsilon_i = 1$ if $\rho(P, S_{ref_i}) < 0$, otherwise $\epsilon_i = 0$.

ϵ_i (known) κ_i (guess)

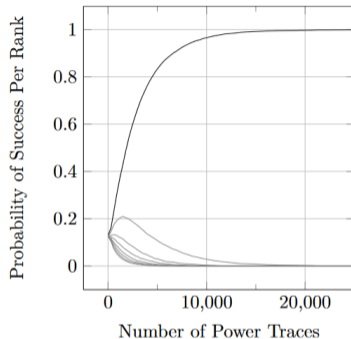
► Reduce computational complexity from 2^{2n} intermediate results to $n2^2 + n$ ones.

Ranked Probabilities of Success for One χ_3 Sequence

$n = 3$ bits



(a) CPA, recovering (K', κ) .

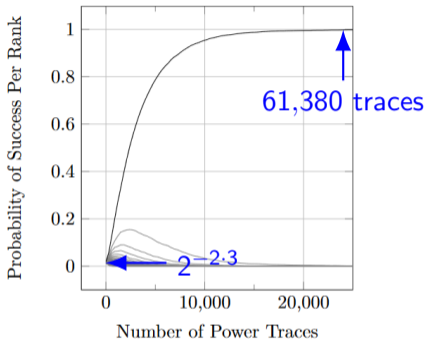


(b) Combined CPA or Snake attack recovering κ . Squared correlation coefficient.

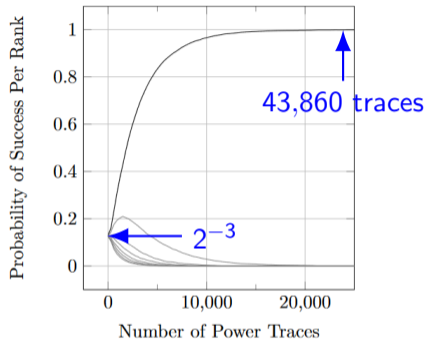
Figure: Ranked success probabilities targeting one χ_3 sequence (Xoodoo).

Ranked Probabilities of Success for One χ_3 Sequence

$n = 3$ bits



(a) CPA, recovering (K', κ) .

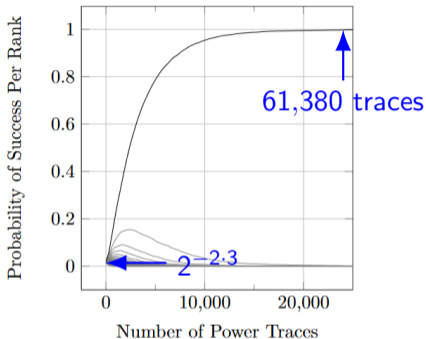


(b) Combined CPA or *Snake* attack recovering κ . Squared correlation coefficient.

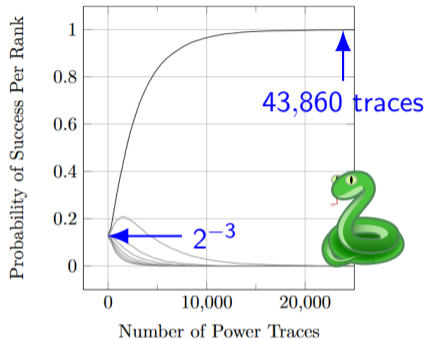
Figure: Ranked success probabilities targeting one χ_3 sequence (Xoodoo).

Ranked Probabilities of Success for One χ_3 Sequence

$n = 3$ bits



(a) CPA, recovering (K', κ) .

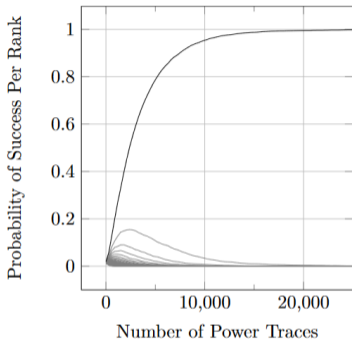


(b) Combined CPA or *Snake* attack recovering κ . Squared correlation coefficient.

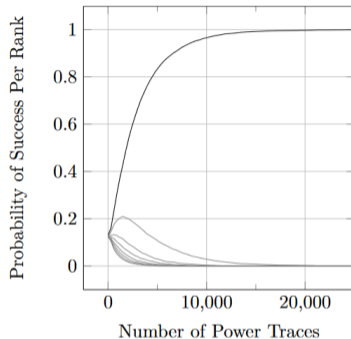
Figure: Ranked success probabilities targeting one χ_3 sequence (Xoodoo).

Ranked Probabilities of Success for One χ_3 Sequence

$n = 3$ bits



(a) CPA, recovering (K', κ) .

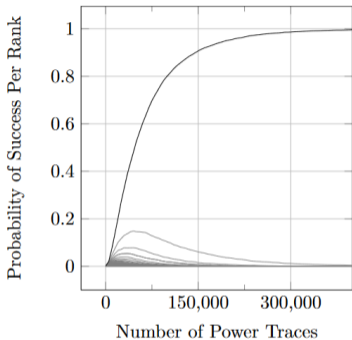


(b) Combined CPA or *Snake* attack recovering κ . Squared correlation coefficient.

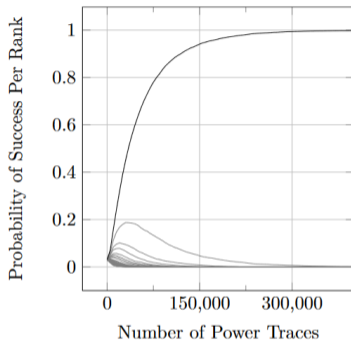
Figure: Ranked success probabilities targeting one χ_3 sequence (Xoodoo).

Ranked Probabilities of Success for One χ_5 Sequence

$n = 5$ bits



(a) CPA, recovering (K', κ) .

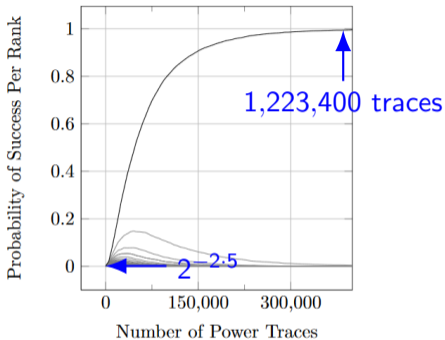


(b) Combined CPA or *Snake* attack recovering κ . Squared correlation coefficient.

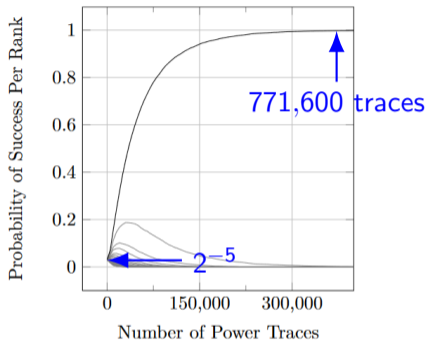
Figure: Ranked success probabilities targeting one χ_5 sequence (Keccak- p).

Ranked Probabilities of Success for One χ_5 Sequence

$n = 5$ bits



(a) CPA, recovering (K', κ) .

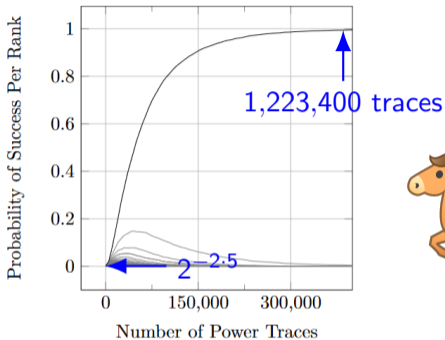


(b) Combined CPA or Snake attack recovering κ . Squared correlation coefficient.

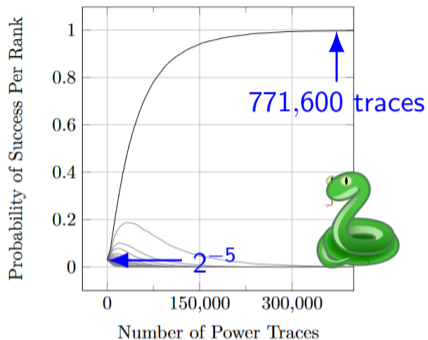
Figure: Ranked success probabilities targeting one χ_5 sequence (Keccak- p).

Ranked Probabilities of Success for One χ_5 Sequence

$n = 5$ bits



(a) CPA, recovering (K', κ) .



(b) Combined CPA or *Snake* attack recovering κ . Squared correlation coefficient.

Figure: Ranked success probabilities targeting one χ_5 sequence (Keccak- p).

Probability of Not Being Correct: CPA vs. Combined CPA (Snake Attack)

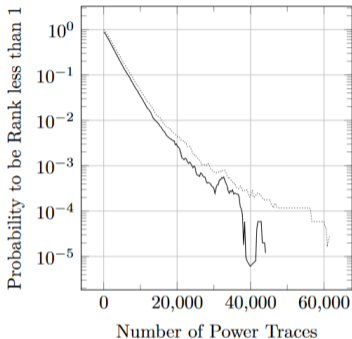
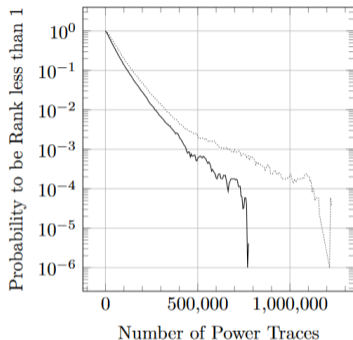
(a) $n = 3$ (b) $n = 5$

Figure: Comparison between CPA (dotted line) and *Snake* attack (solid line) for the probability of the correct hypothesis to *not* be rank 1.

Conclusion



- ▶ Improve the CPA computational complexity from 2^{2n} to $n^2 + n$ intermediate results.
- ▶ **Combined CPA or Snake attack** has a higher probability of success than traditional CPA for the same number of power traces.

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Conclusion



- ▶ Improve the CPA computational complexity from 2^{2n} to $n^2 + n$ intermediate results.
- ▶ **Combined CPA or Snake attack** has a higher probability of success than traditional CPA for the same number of power traces.

Thank you for your attention



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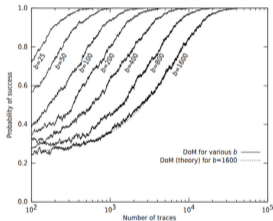


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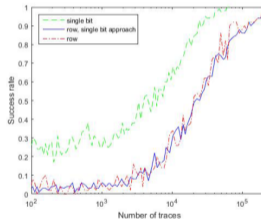
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Previous work

- ▶ Previous work with DPA on one χ_5 row



Bertoni, Daemen, et al. 2012. Power Analysis of Hardware Implementations Protected with Secret Sharing. MICROW'12.



Samwel and Daemen. 2017. DPA on hardware implementations of Ascon and Keyak. CF'17.