Sample Efficient Search to Decision for kLIN

Andrej Bogdanov	*University of Ottawa*
Alon Rosen	*Bocconi University*
Kel Zin Tan	*National University of Singapore*

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Search: Recover s from $(A, As + e)$

Decision: Distinguish $(A, As + e)$ and (A, u) where u is random binary vector

↑ Planted

↑ Null

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- Decision problem is a pseudorandom generator (PRG)
- Search to Decision reduction implies a getting PRG from OWF in NC0

Hardness Comparison

Search: Recover s from $(A, As+e)$

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Learning Parity With Noise (LPN)

Dense Matrix $A \leftarrow \mathbb{F}_2^{m\times n}$

Each entry in the matrix is a random bit

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Search: Non-trivial Idea [App16]

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Gap: Improve the sample complexity

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Suppose Decision Algorithm D that has constant advantage

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- [App12] $\tilde{O}(m^3)$, success prob high

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Larger Stretch For PRG

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How to make this happen?

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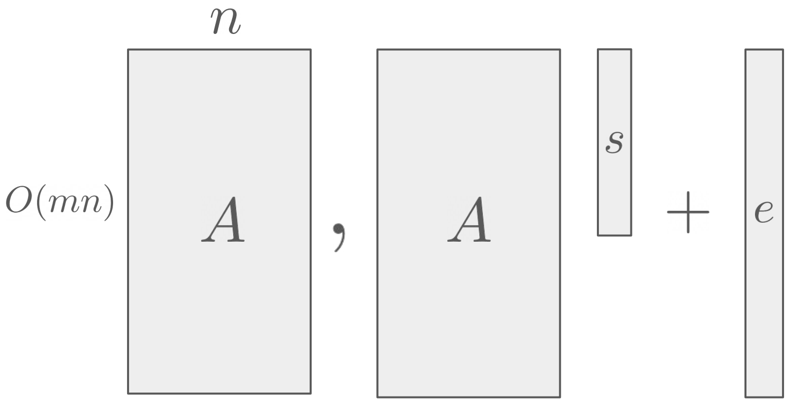

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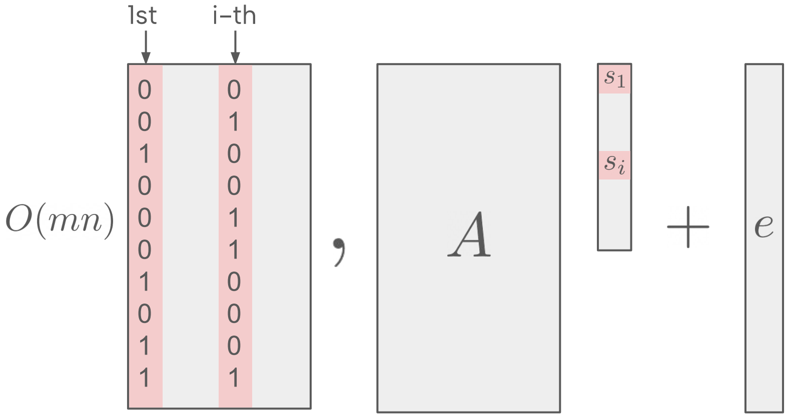

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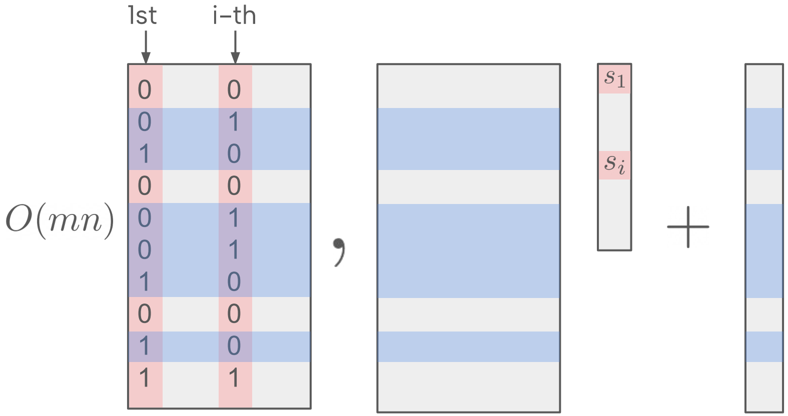

Collect the rows with distinct value at 1 and i

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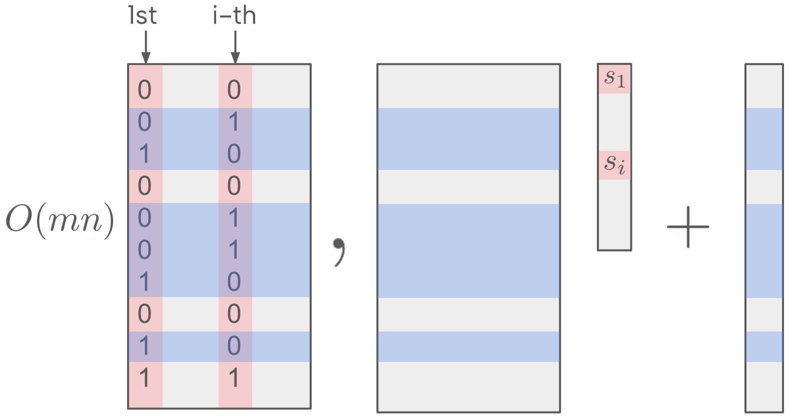

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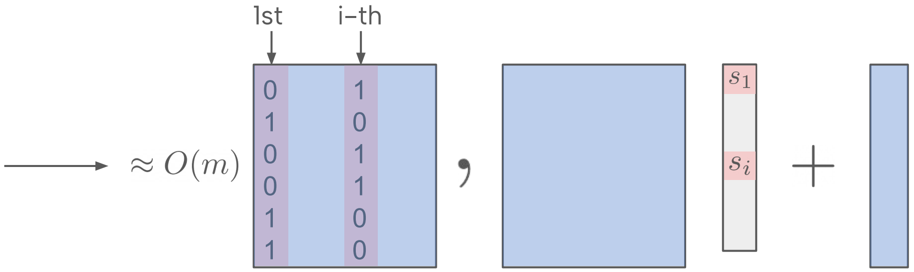

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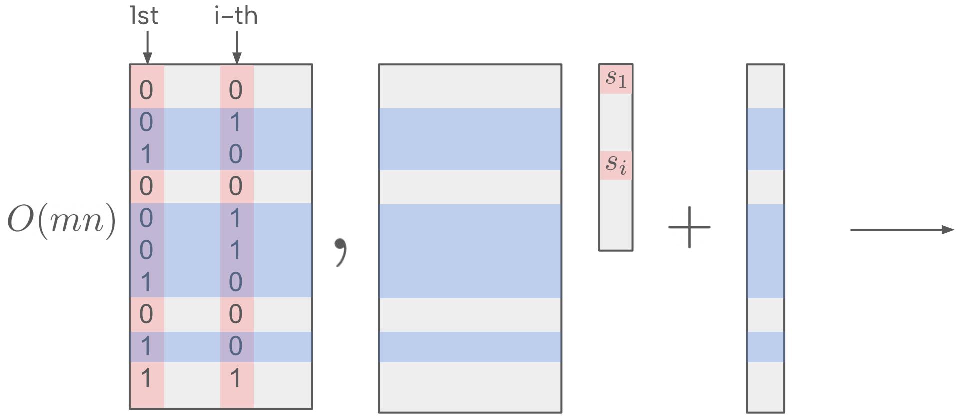

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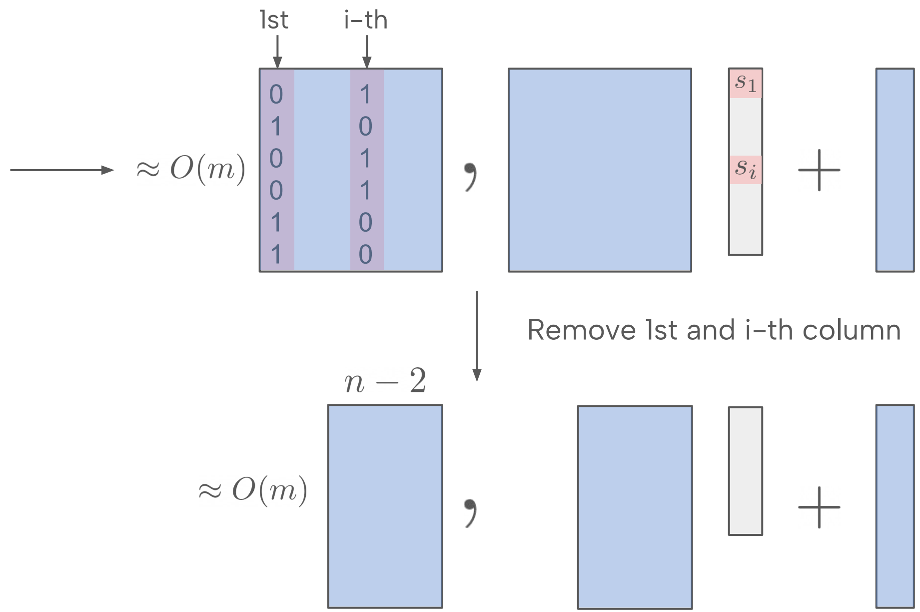

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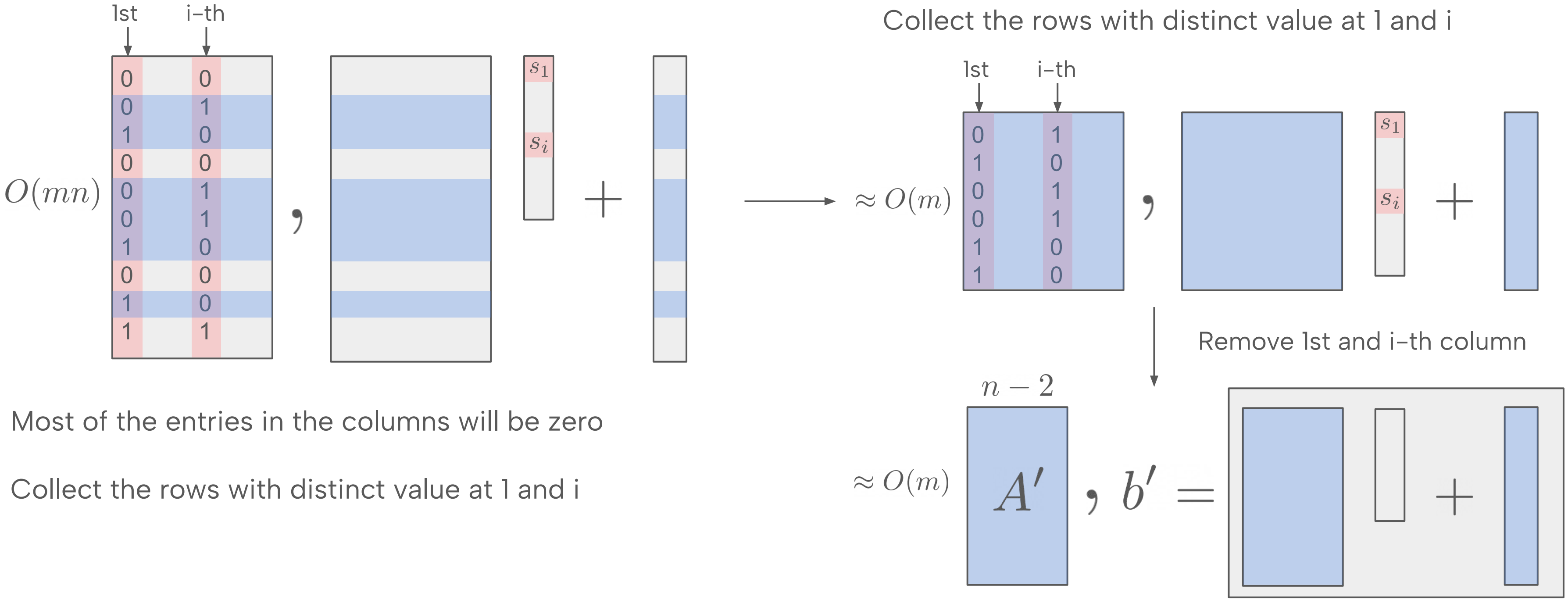

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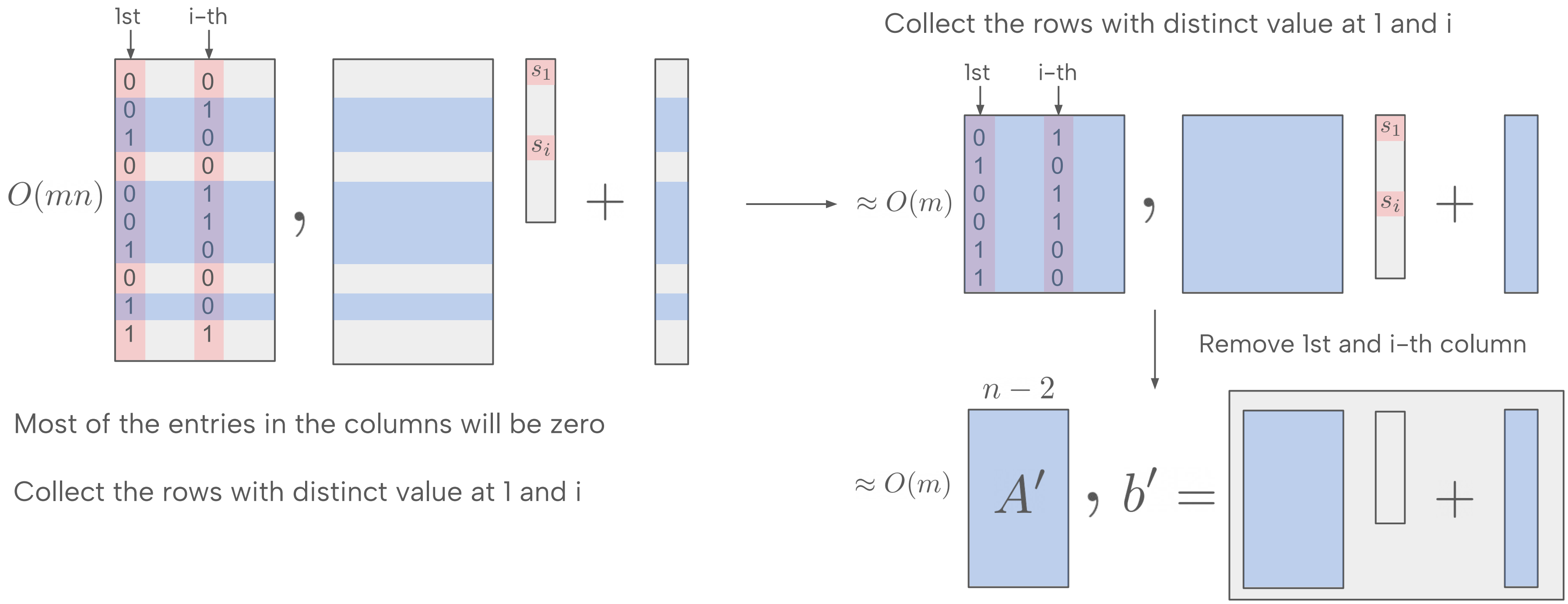

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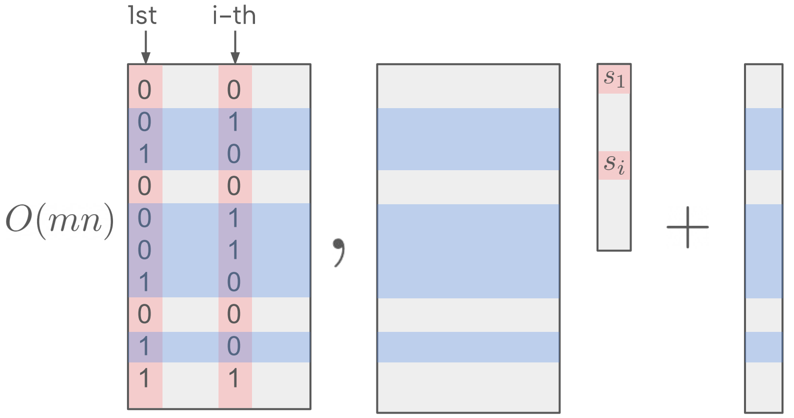
(k-1)Lin with (n-2) variables



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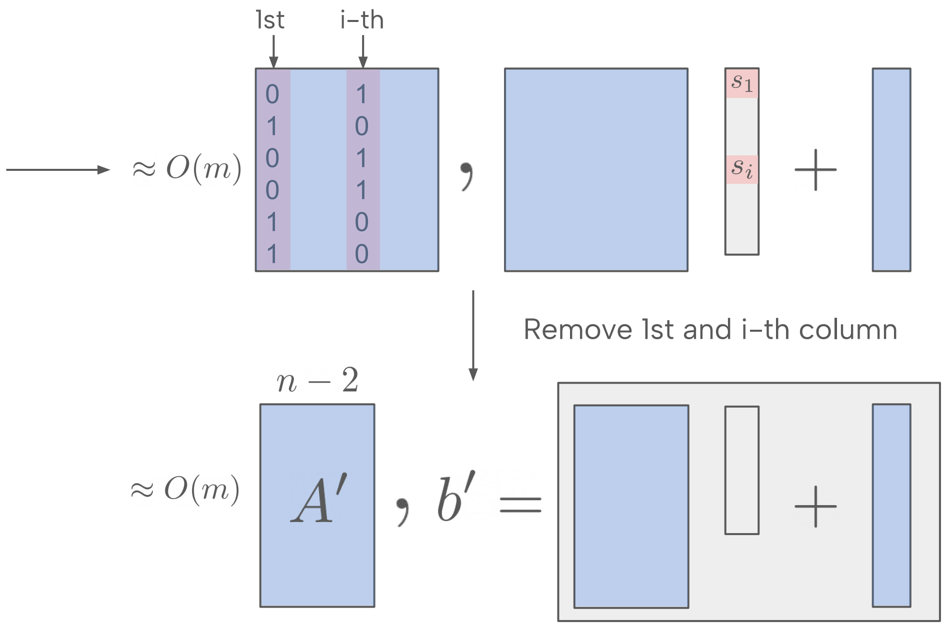

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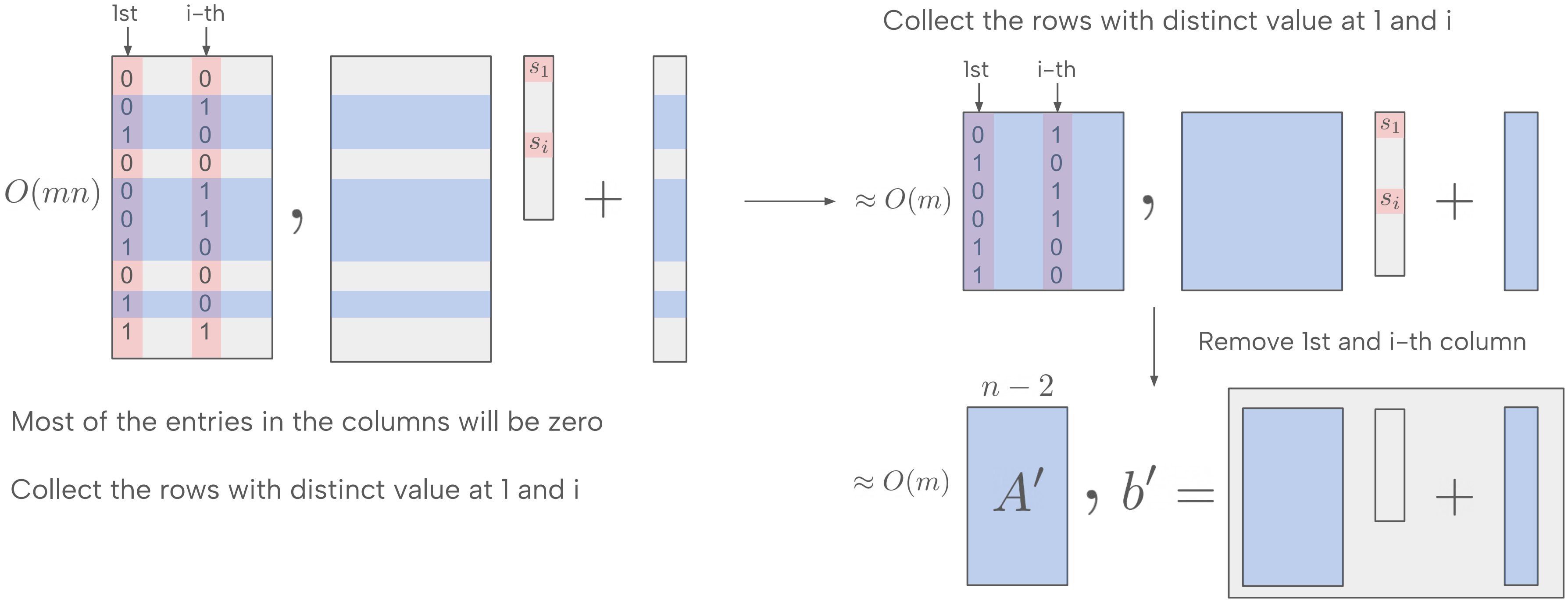

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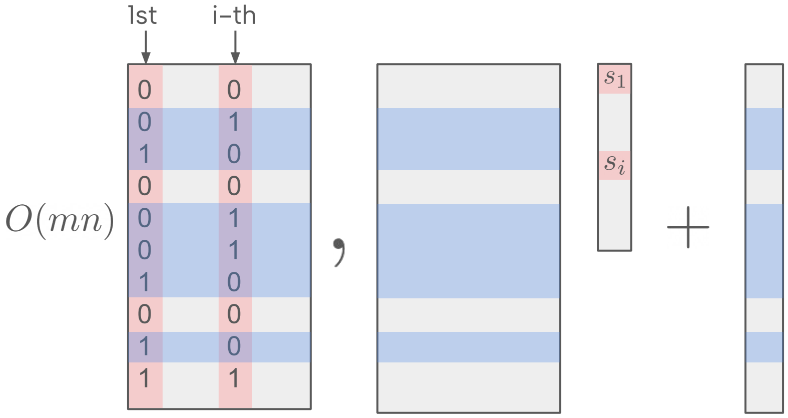

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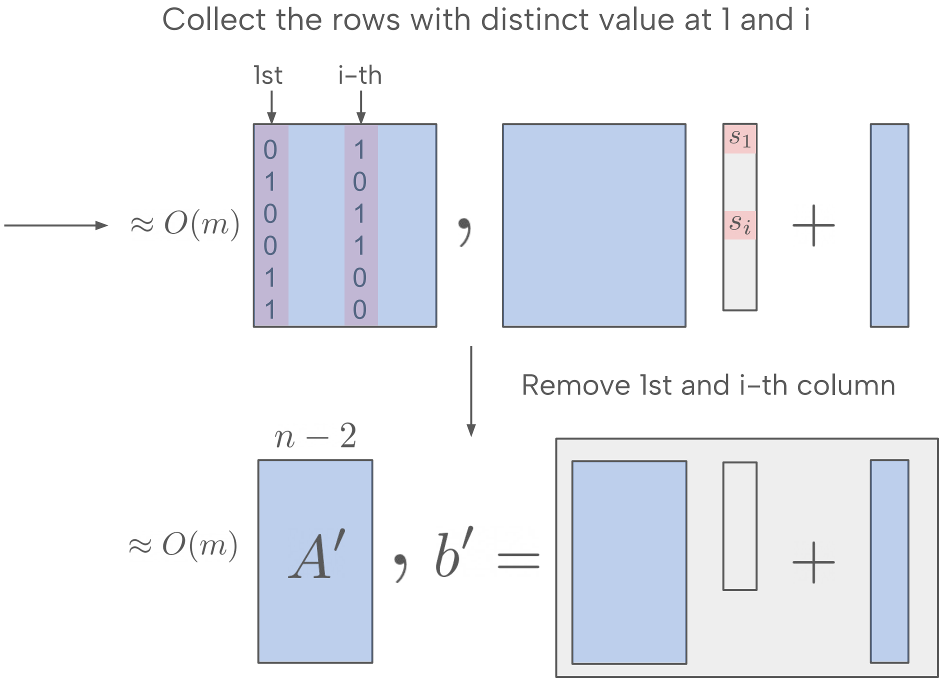

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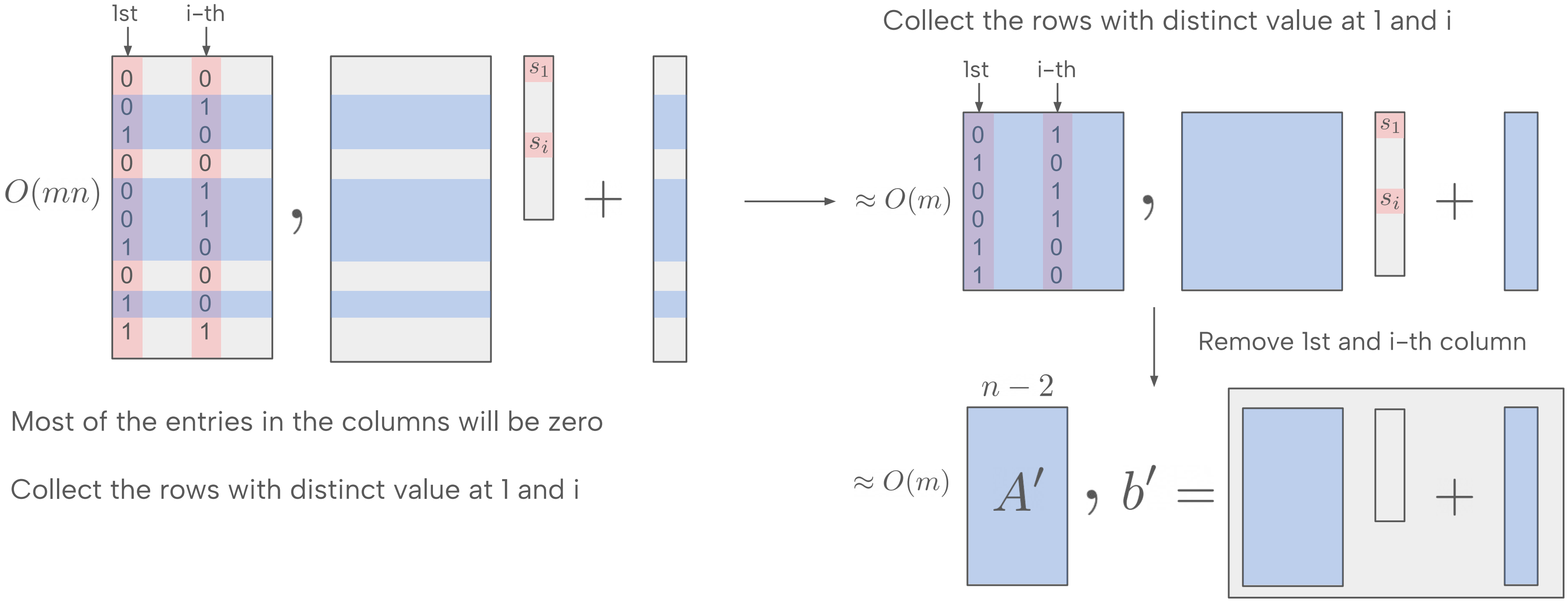

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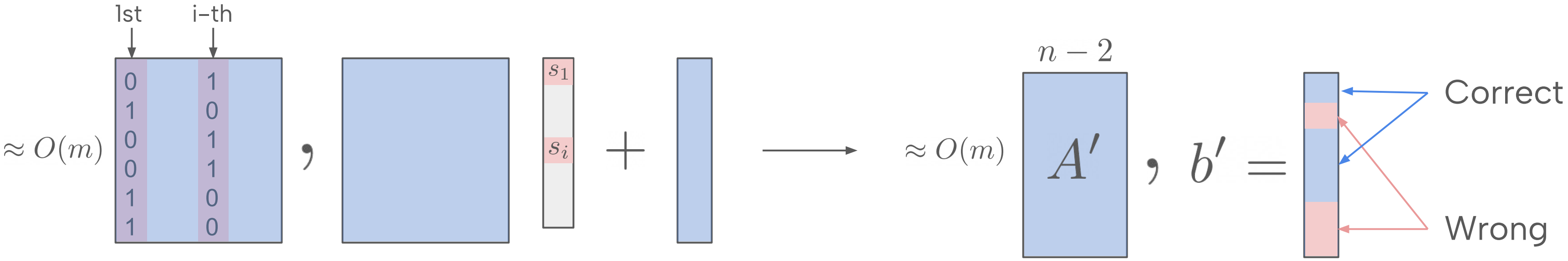

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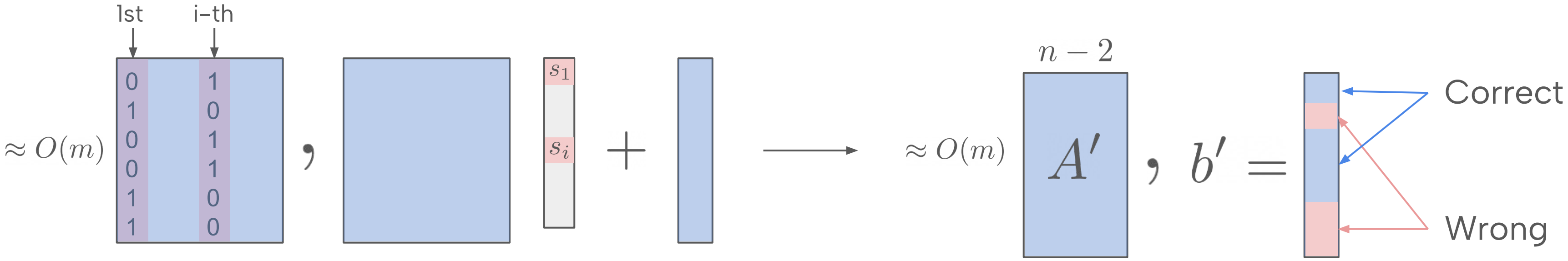

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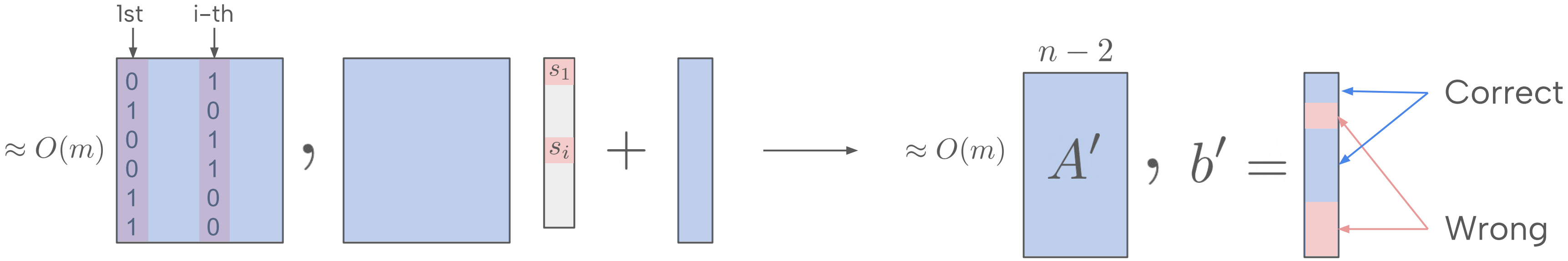
On expectation, half of the rows (equations) are correct, other half is wrong



Claim

If $s_1 \neq s_i$, then (A', b'), $(A', \text{flip}(b'))$ distribute like Null

- If $s_1 = 1, s_i = 0$, (1 … 0 …) row is wrong, (0 … 1 …) row is correct
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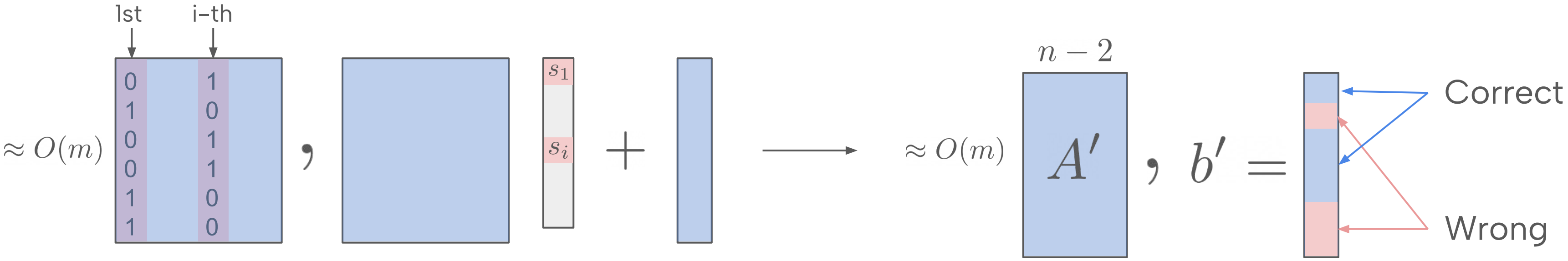
D Does not know the origin of the rows, evaluated vector looks like a random vector



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Error Aware

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2. Substitute in predicted secret
3. Get more accurate secret
4. Repeat

Generalization

Larger Finite Field

Our Technique works for Sparse $\mathbb{F}_q$- LPN

Sample size independent from size of Field, but time complexity affected

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Model kLin as $P(s_{i_1}, s_{i_2}, \ldots, s_{i_k}) = s_{i_1} + s_{i_2} + \cdots + s_{i_k} + e$

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For arbitrary $\mathbb{F}_2$ valued function Q

Assume that Decision Algorithm is secret independent

Concluding Remarks

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Open Question: Improve the reduction by a square root factor

Thank You