Nearly Optimal Parallel Broadcast in the Plain Public Key Model

Ran Gelles (Bar-Ilan University), Christoph Lenzen (CISPA), Julian Loss (CISPA), Sravya Yandamuri (Duke University and Common Prefix)

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Byzantine Broadcast

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Sender

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Receivers

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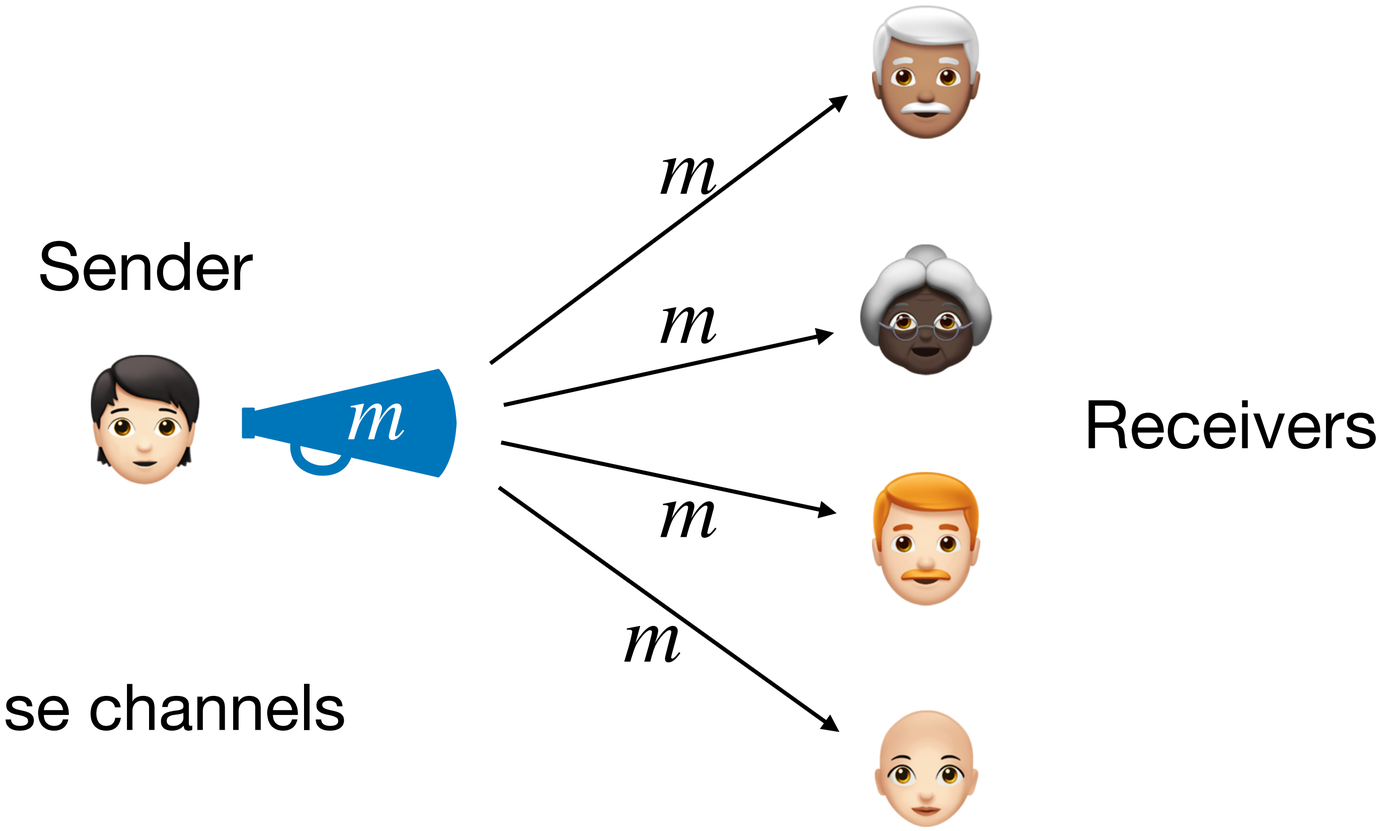

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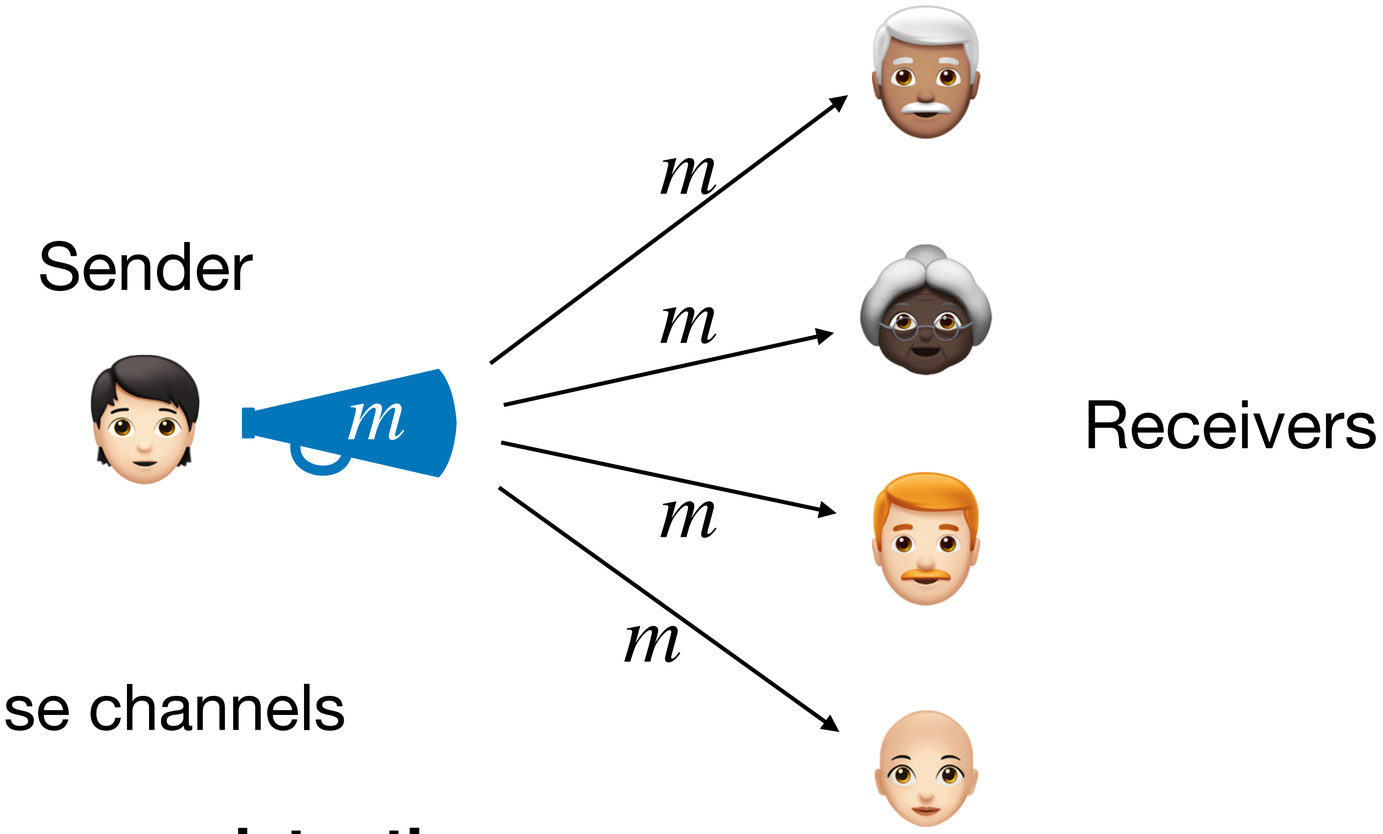
Byzantine Broadcast



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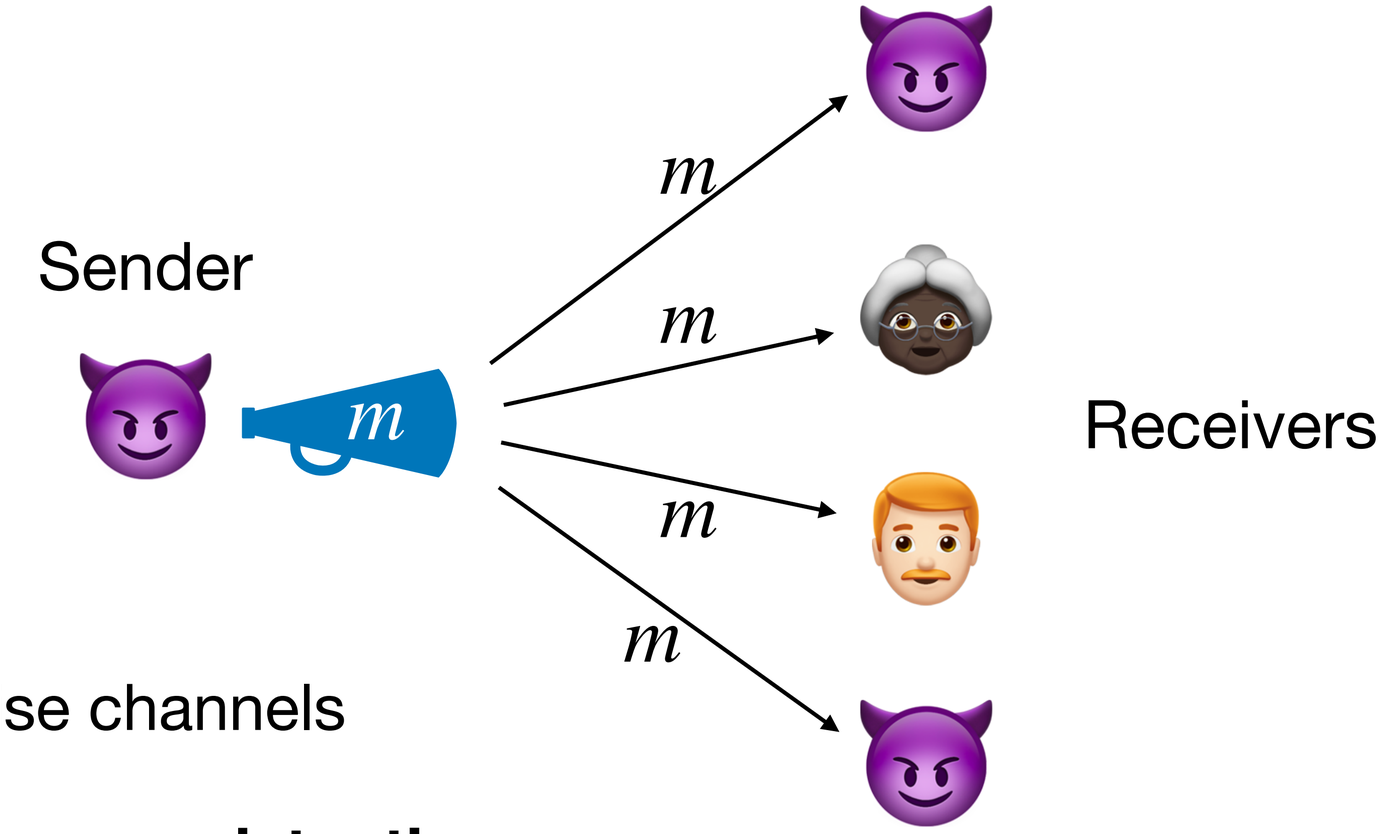

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- **Goal:** Sender distributes m **consistently**
- **Problem: Majority** of $t < n$ parties is malicious

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- Many applications e.g. MPC and Secret Sharing

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- **Our work:** improve communication complexity

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- Shared clock

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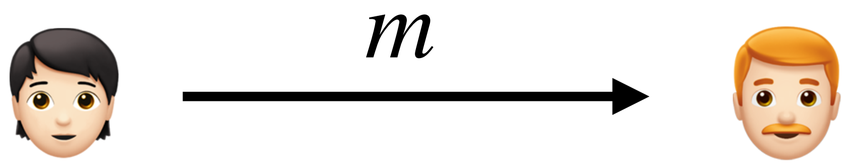

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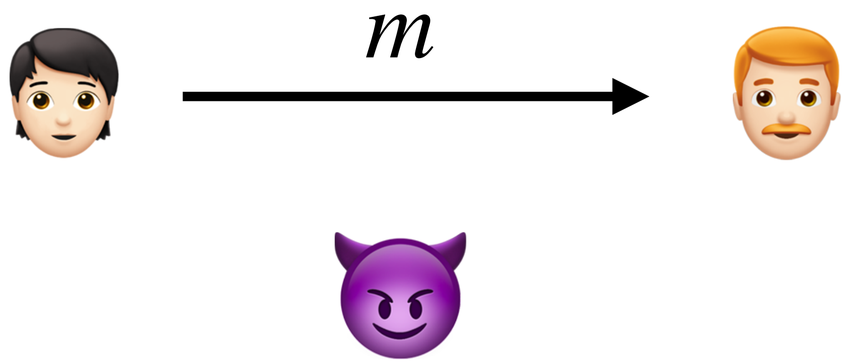

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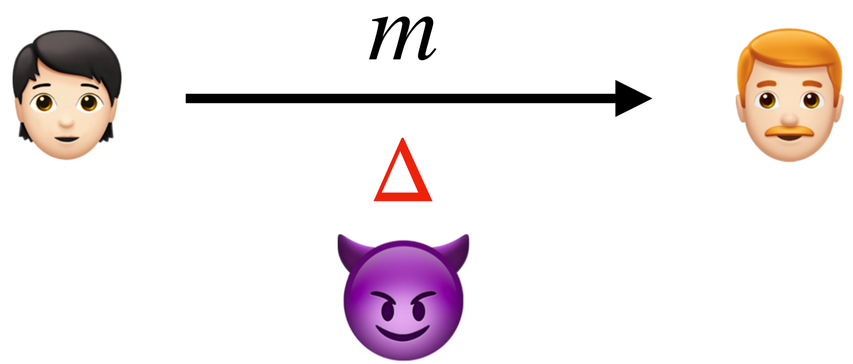

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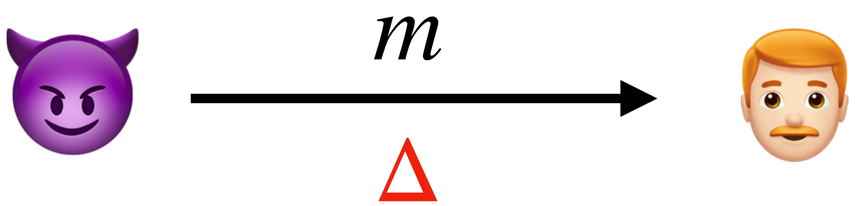

- Shared clock
- Honest messages delivered in Δ time, **no drops**
- Up to $t < n/2$ **malicious** corruptions

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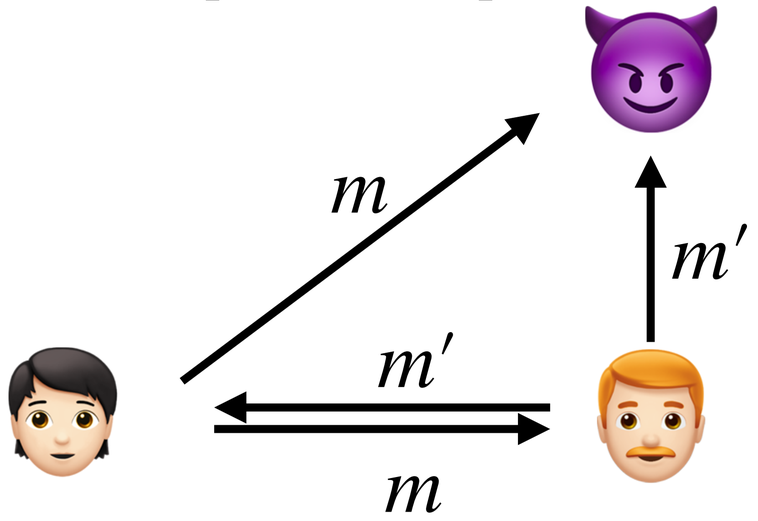

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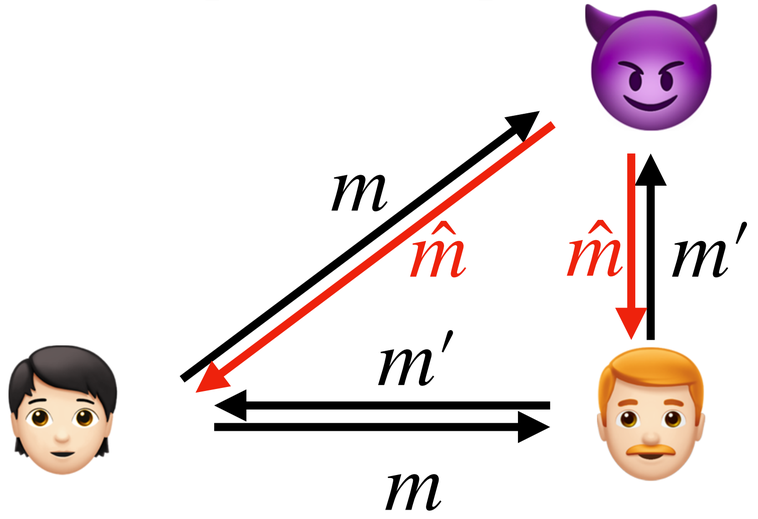

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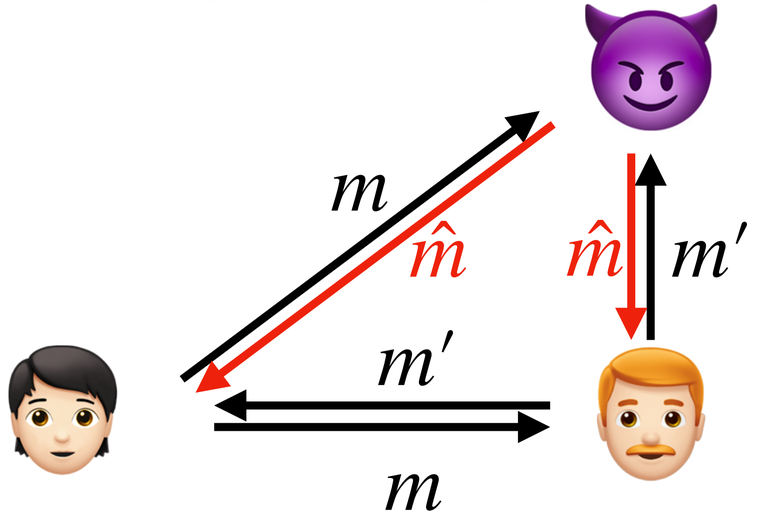

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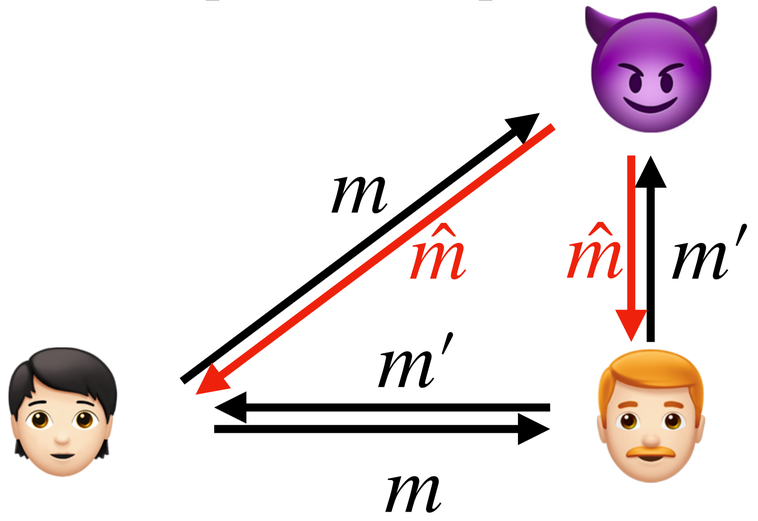

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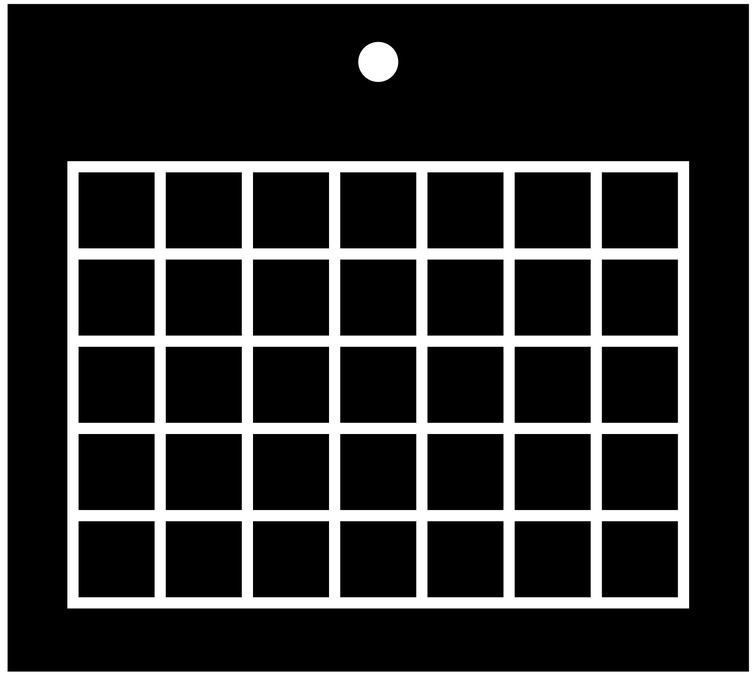
  - Can not corrupt during **atomic send operation**

Dolev-Yao Model



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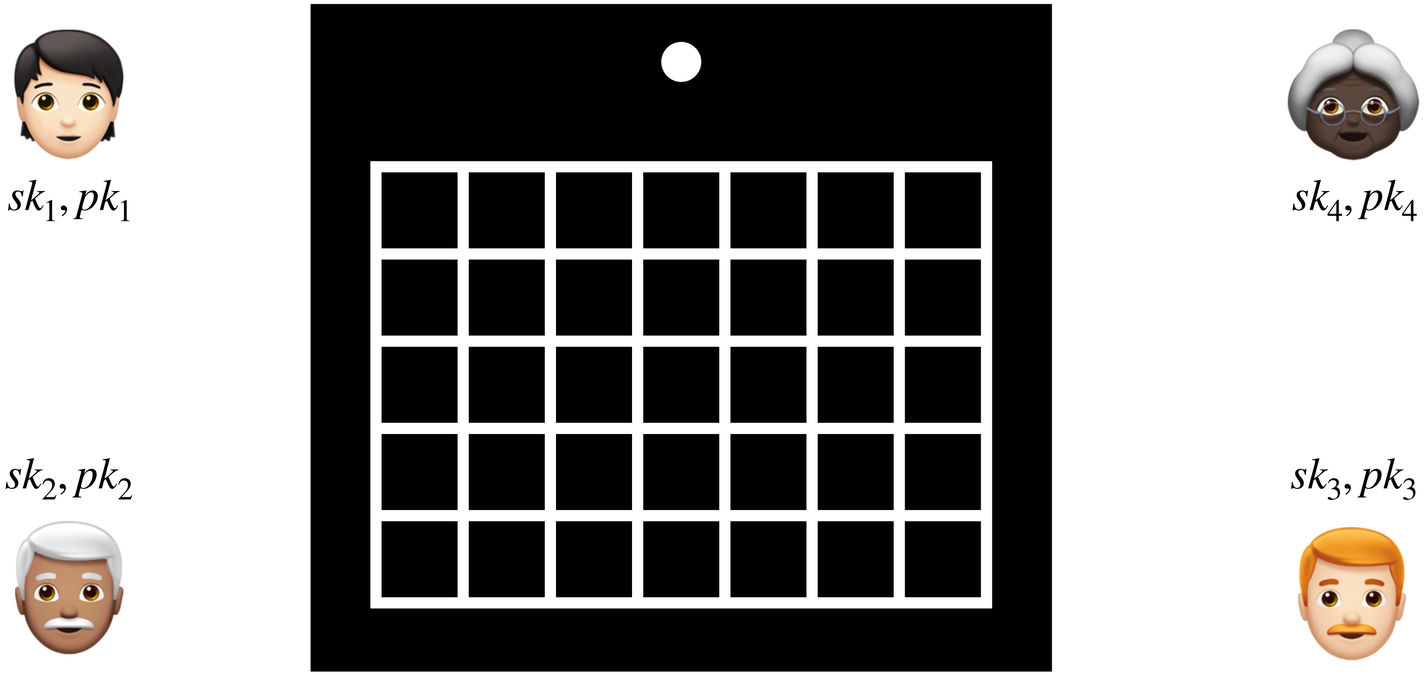

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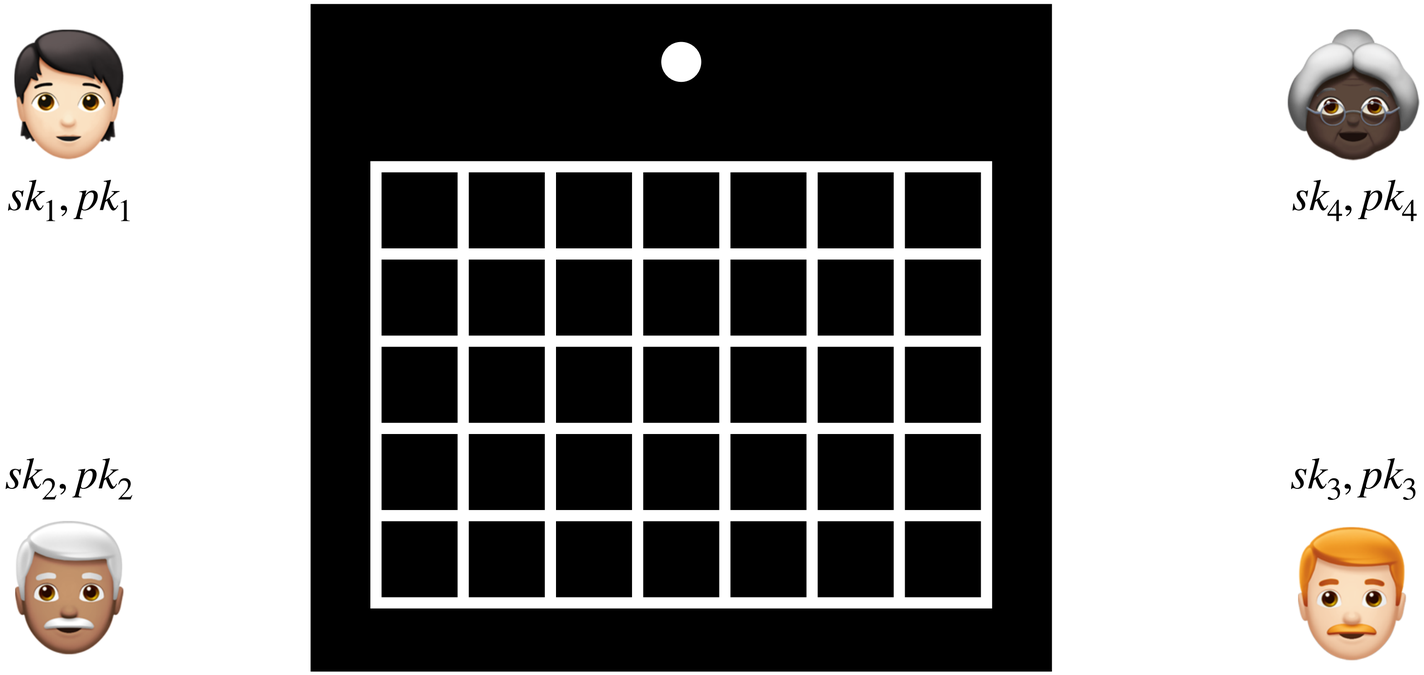

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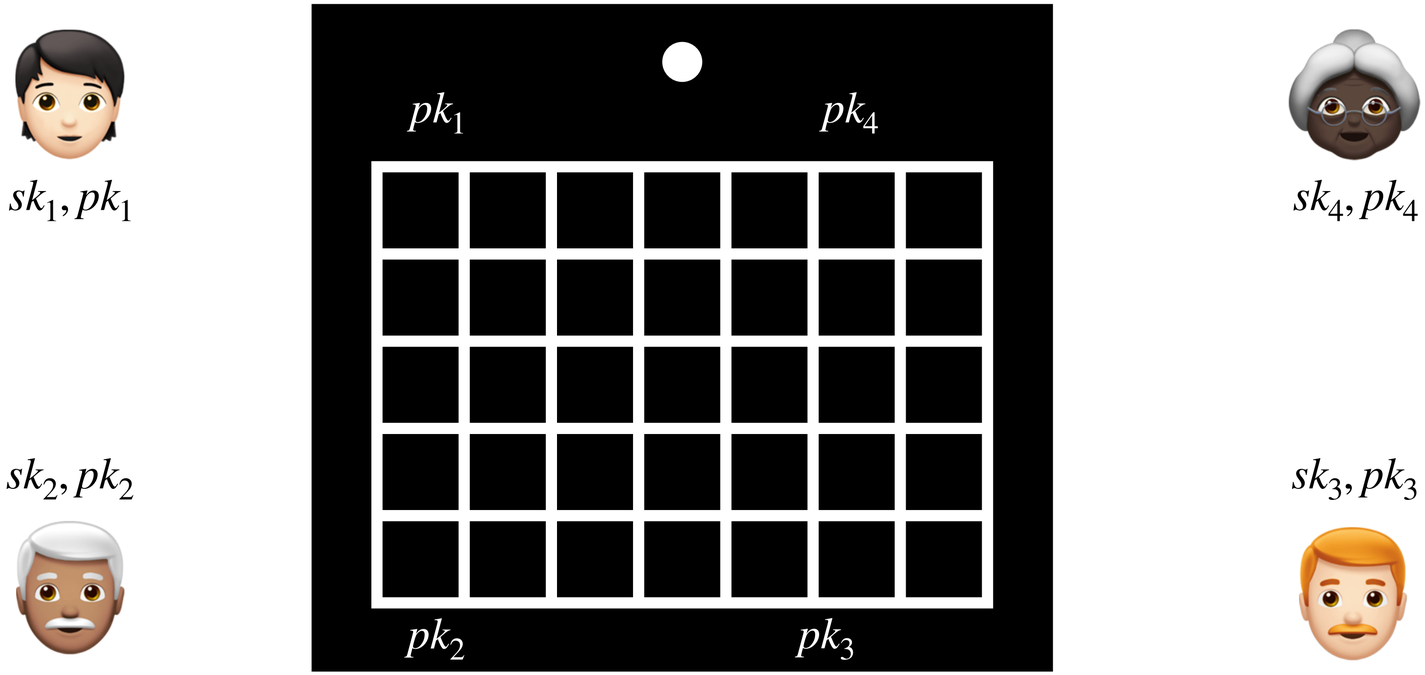

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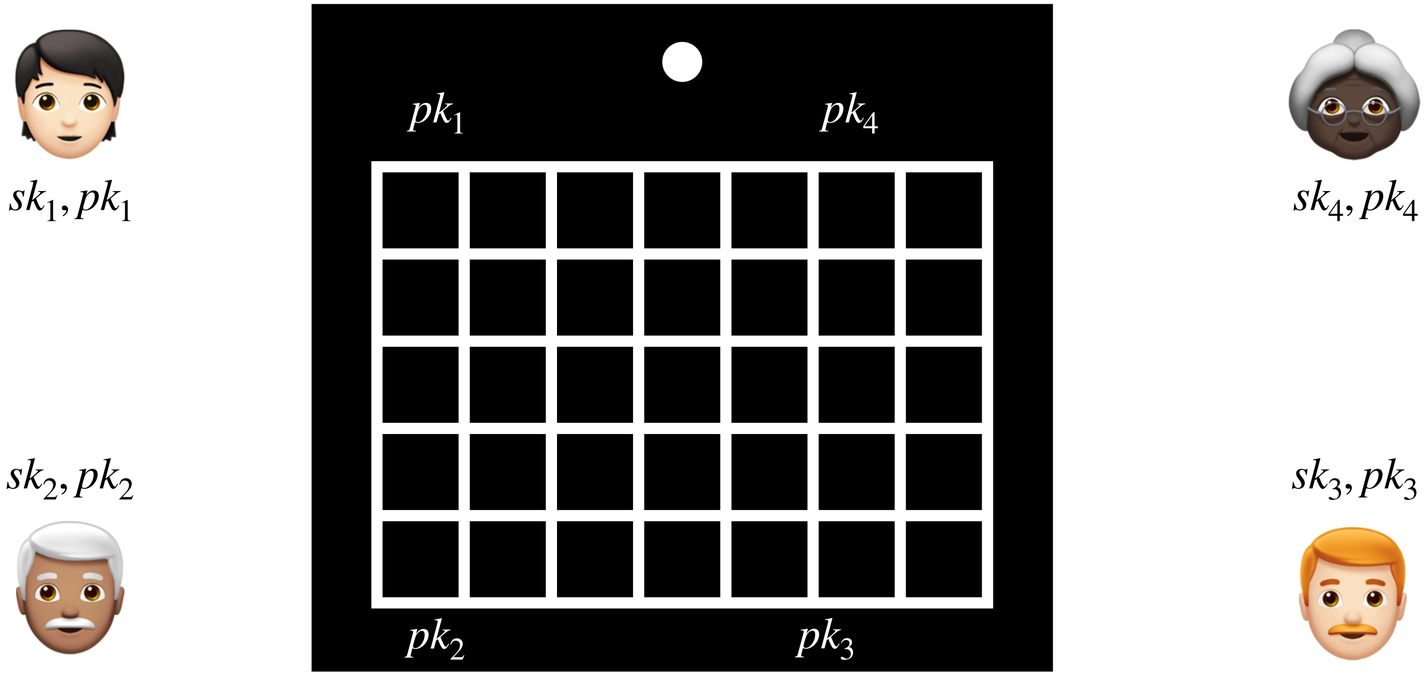

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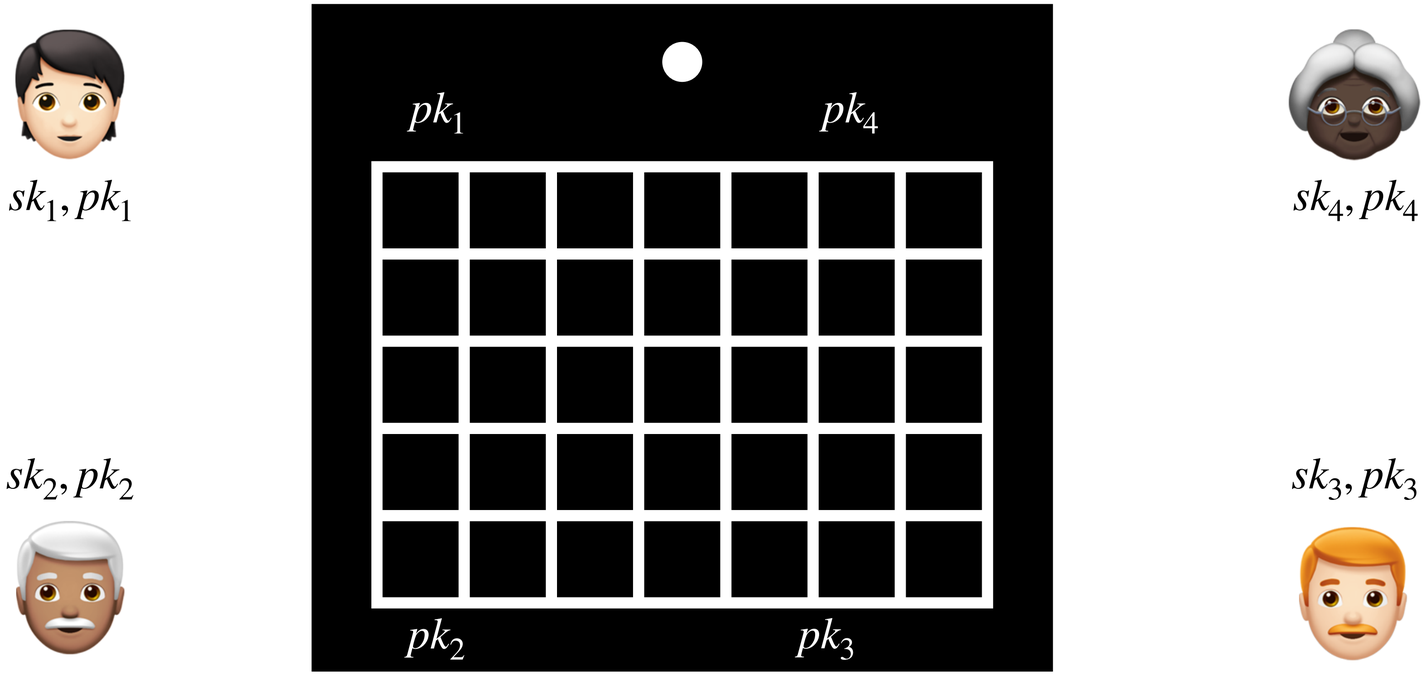

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- Pro: Crypto compatible with DY model is **simple** and **very efficient**
- Con: Harder to construct **asymptotically efficient** protocols

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- λ and κ factors matter in practice!

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- **Communication:** $O(n^2 \cdot \ell \cdot \log(n) + n \cdot \kappa \cdot \log^4(n))$

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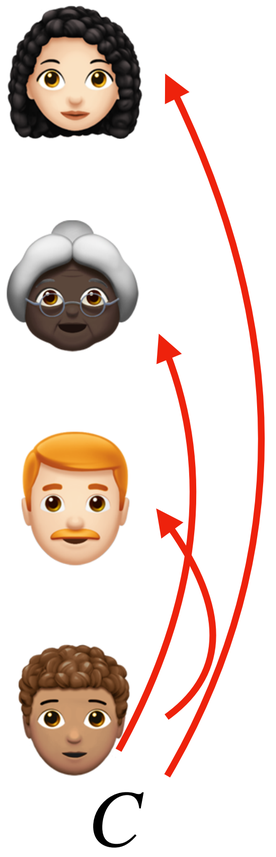

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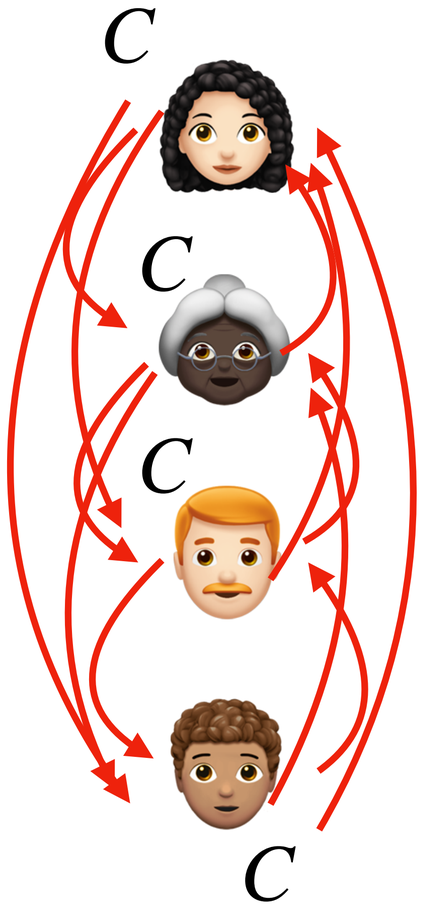

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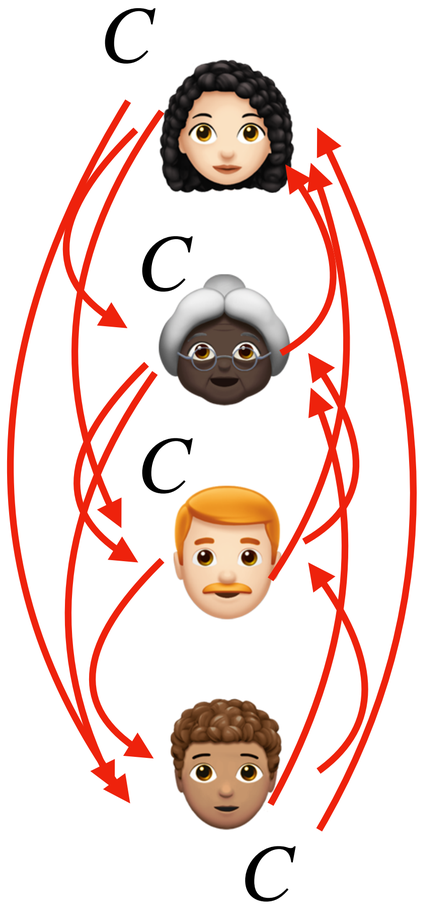

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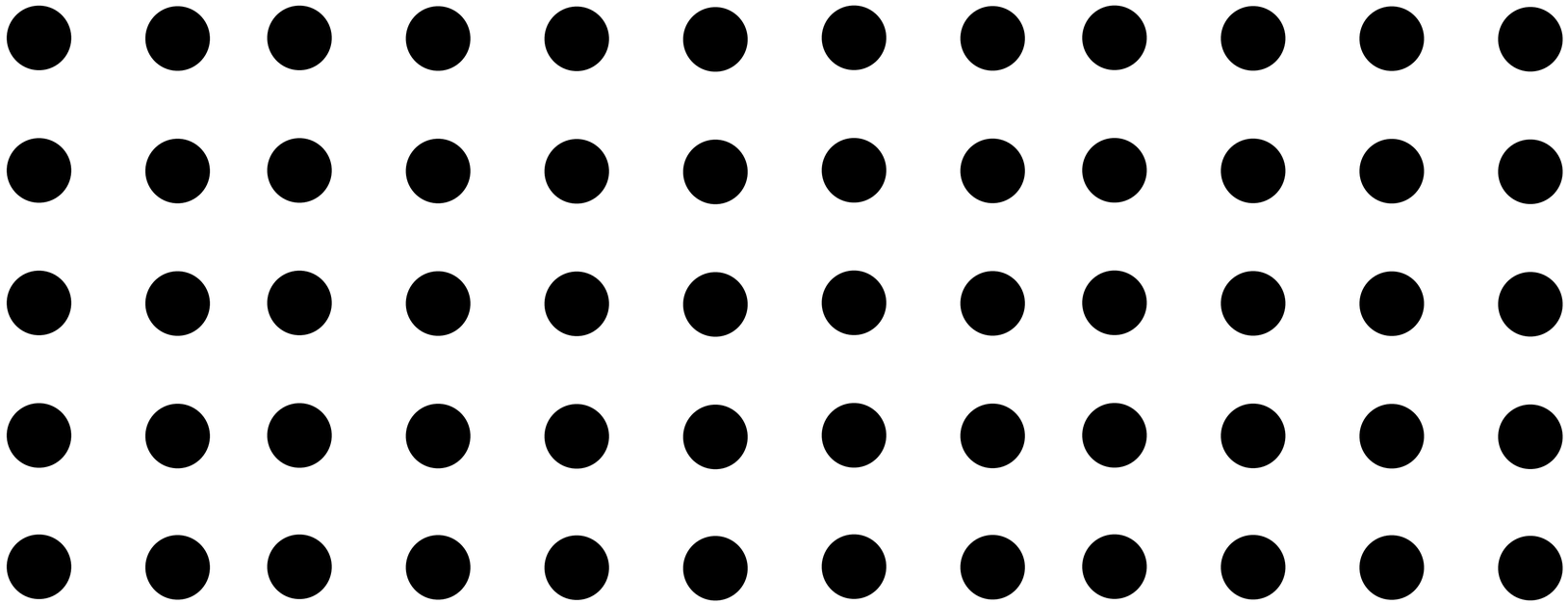
- Naive echoing: $O(n^3 \cdot \kappa)$ bits

Idea: Replace Send-to-All with Gossip



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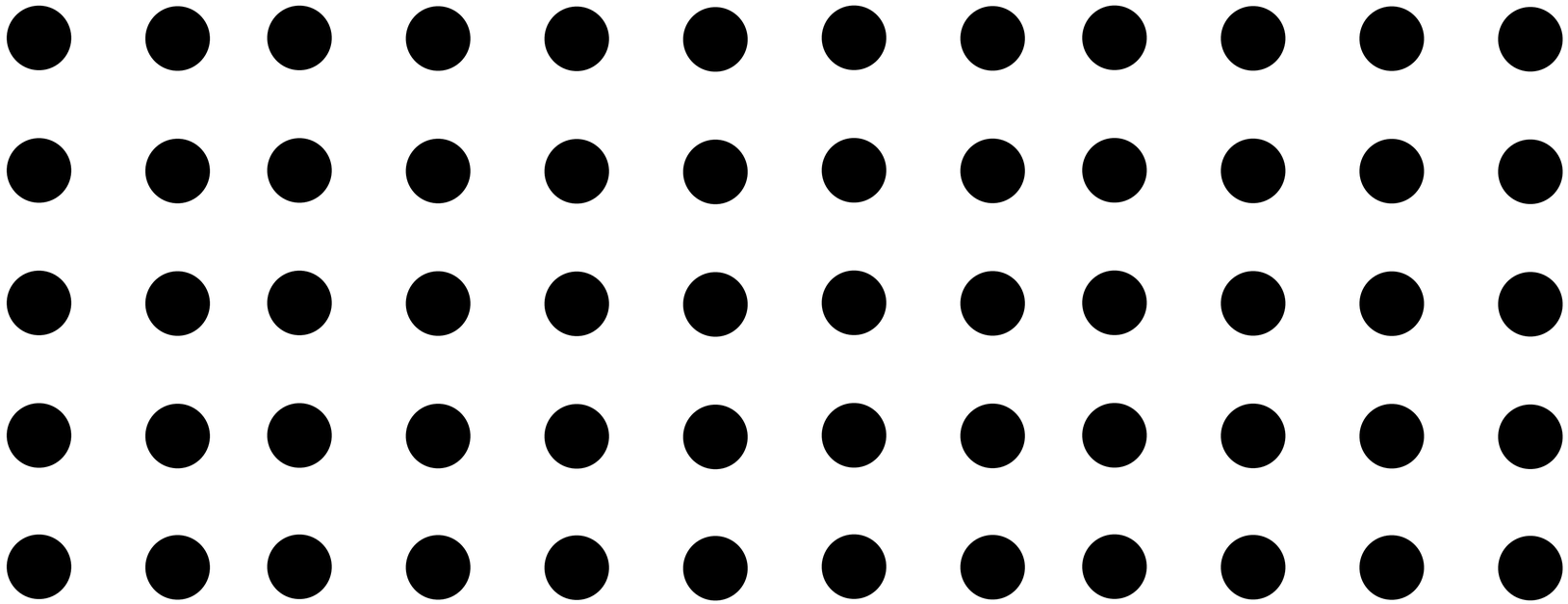

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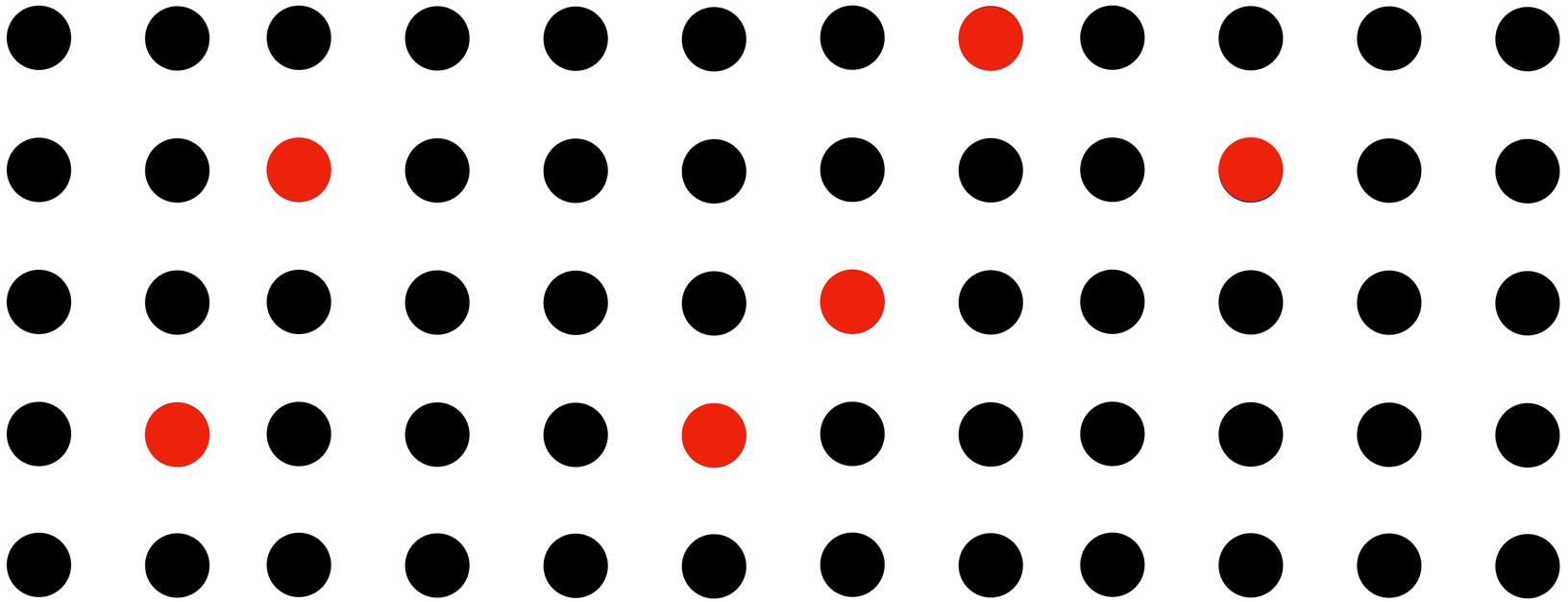

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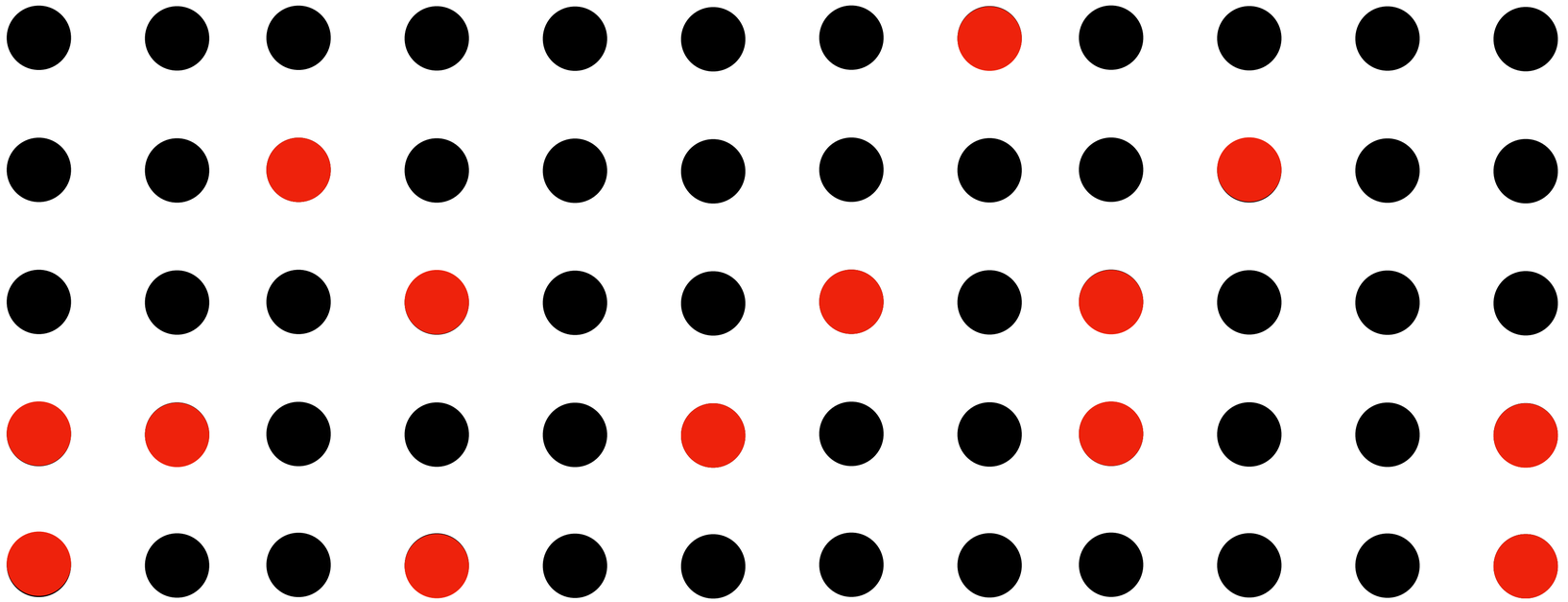

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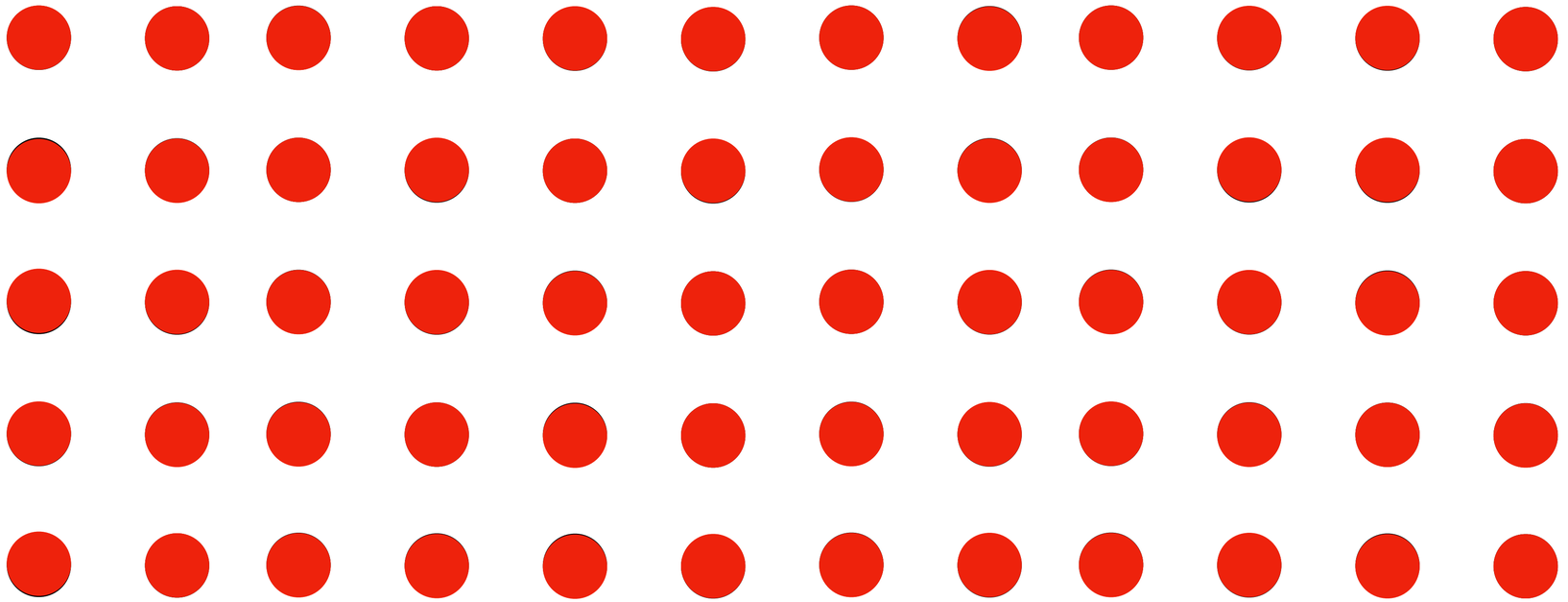

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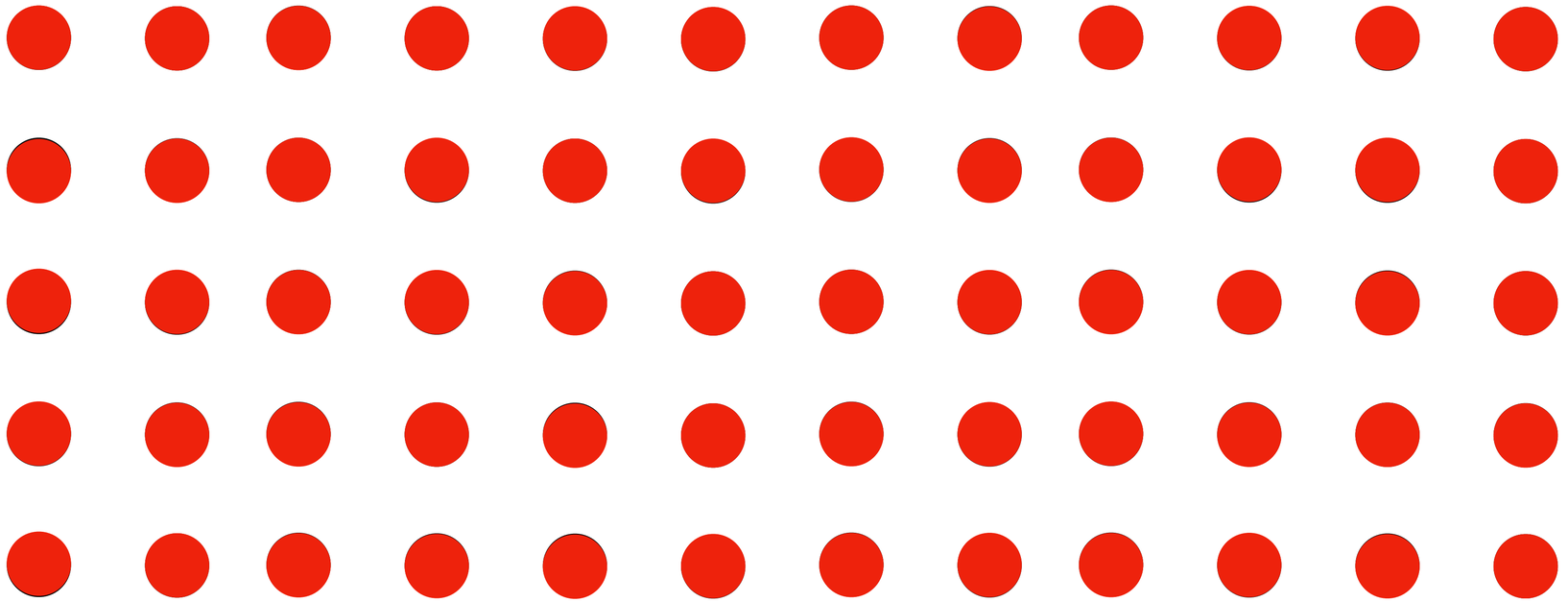

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- Everybody learns the message in $\log(n)$ rounds with $\tilde{O}(n)$ complexity

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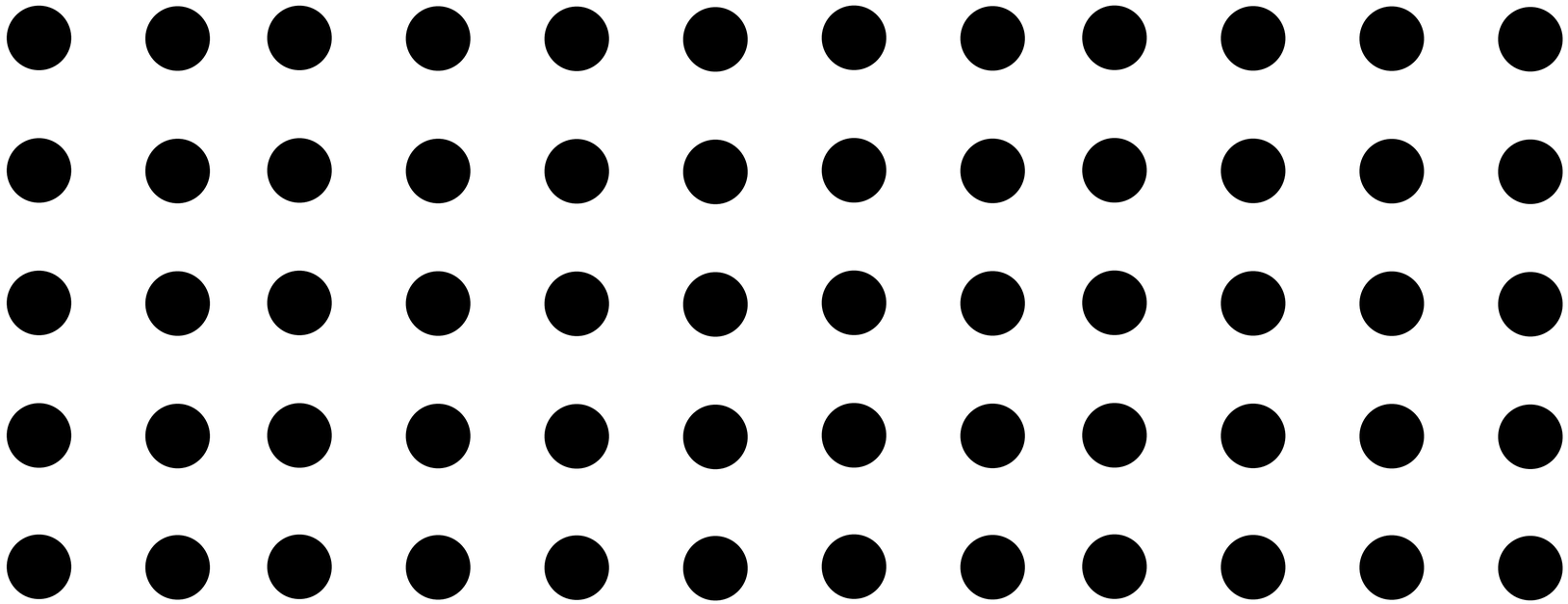

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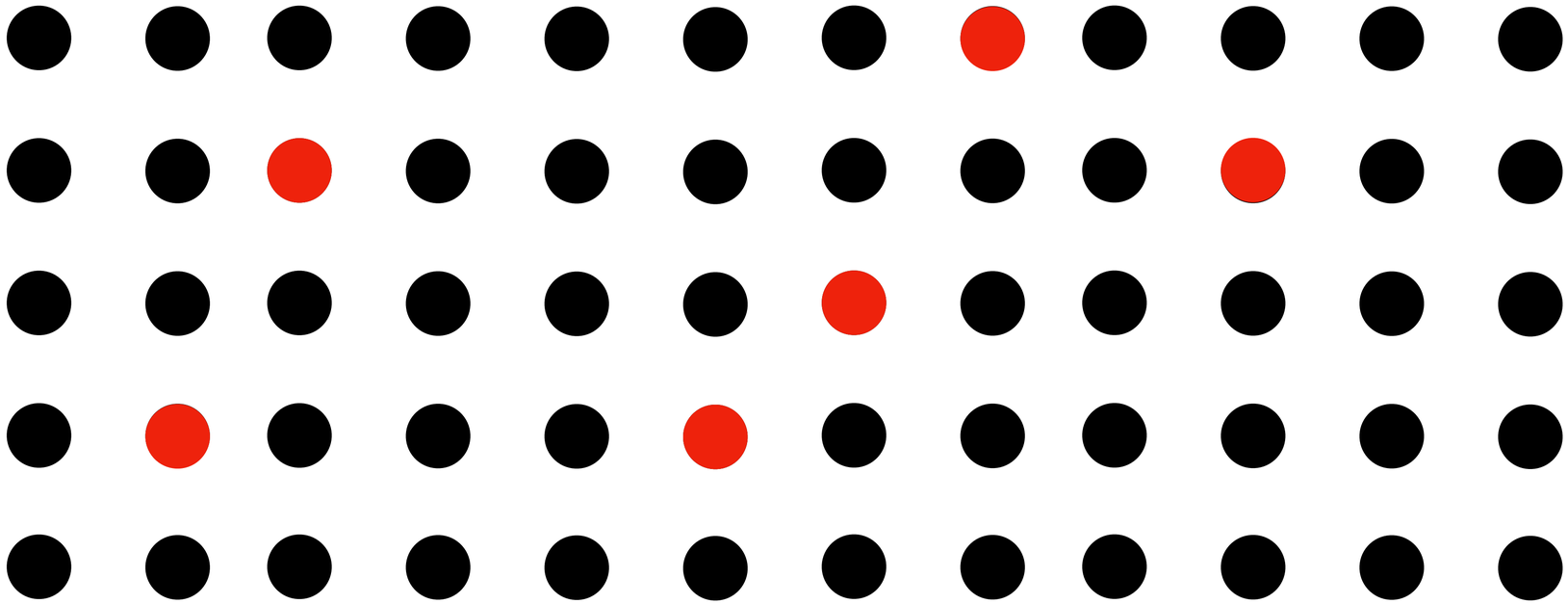

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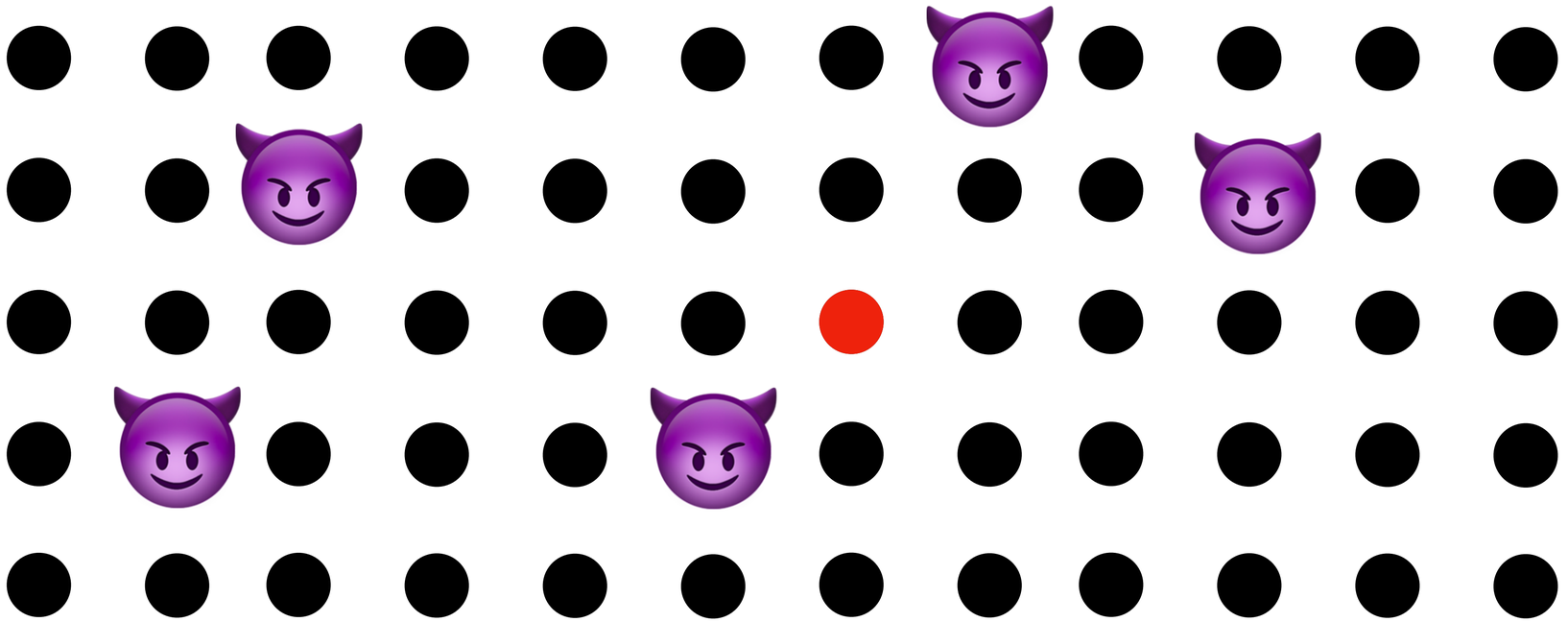

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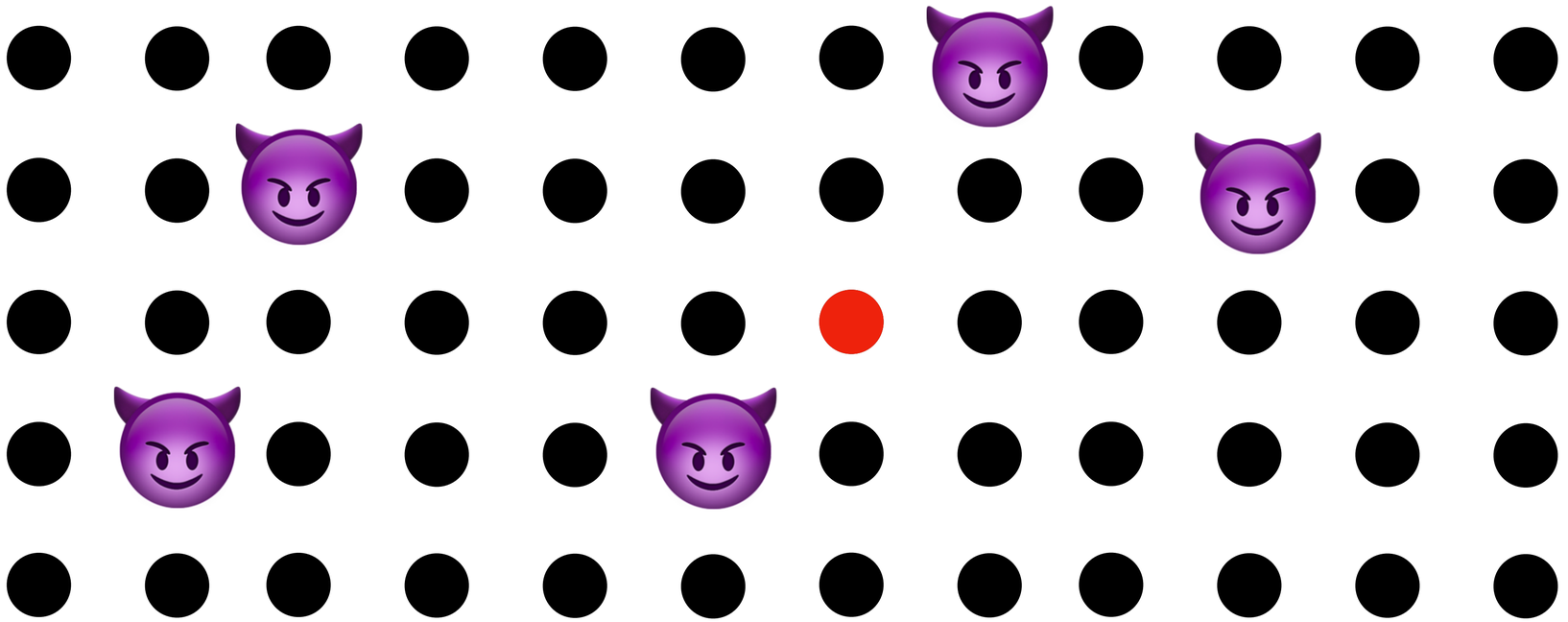

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- Shuts down the process before the message spreads

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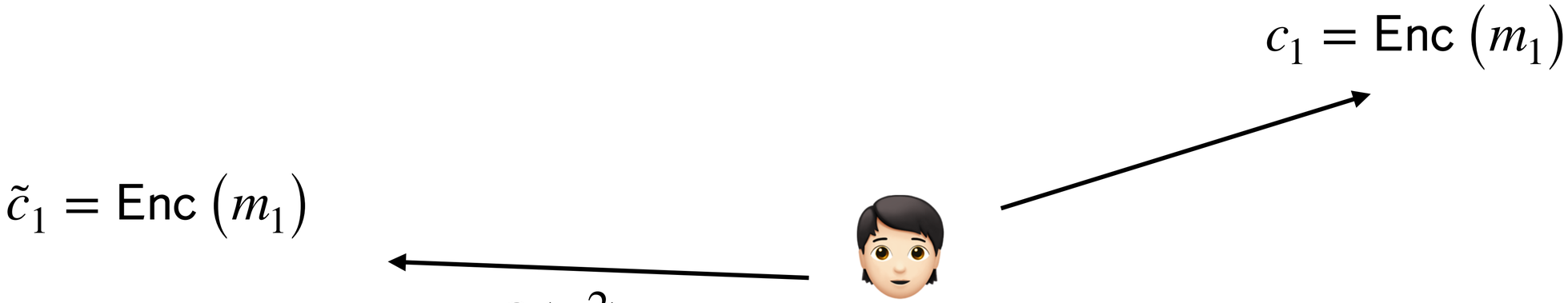

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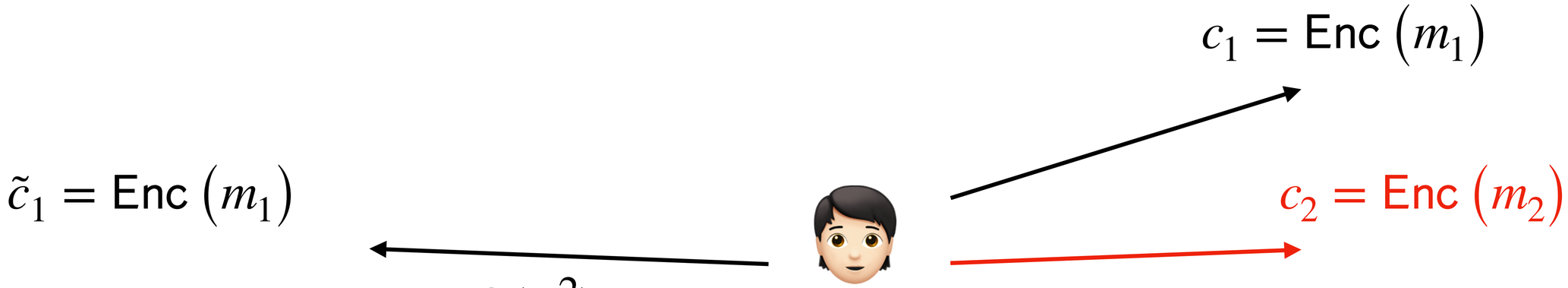

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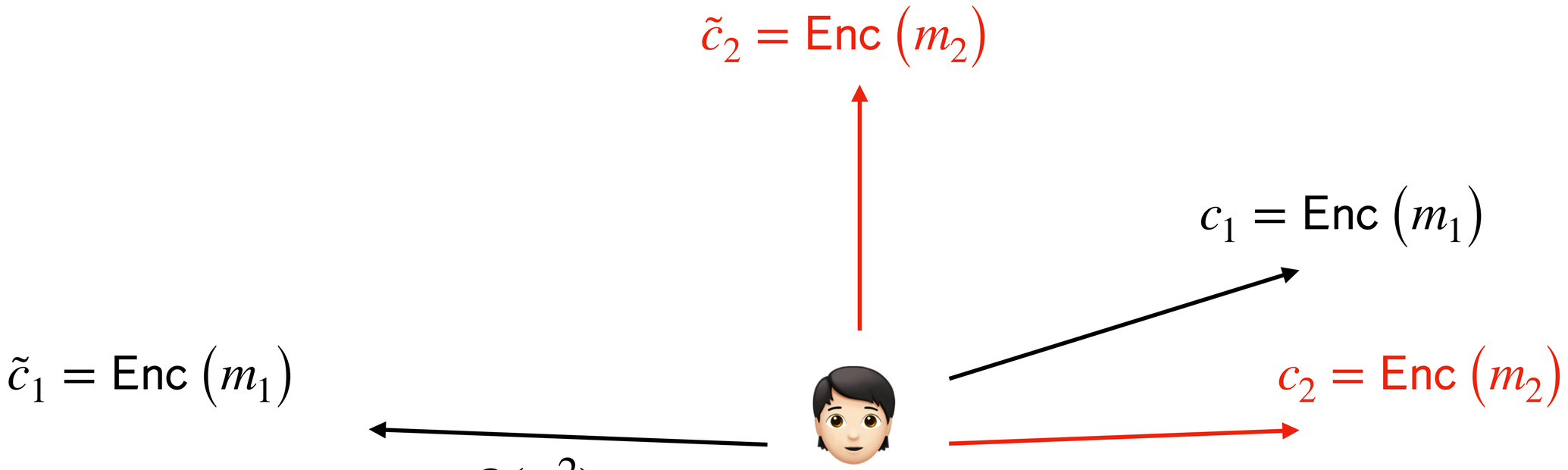

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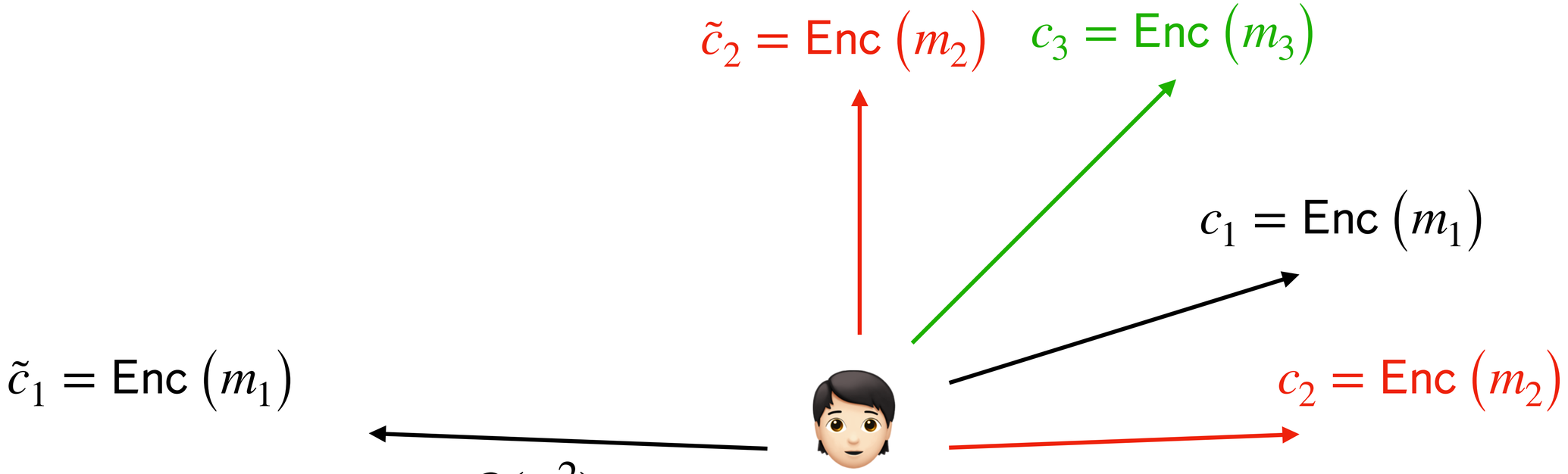

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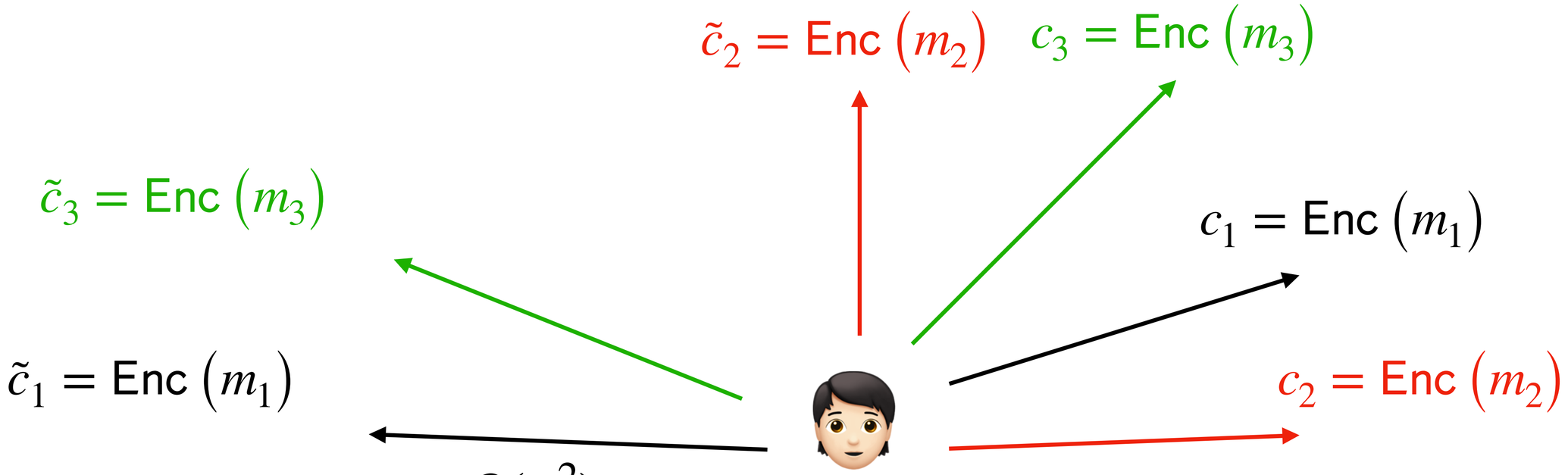

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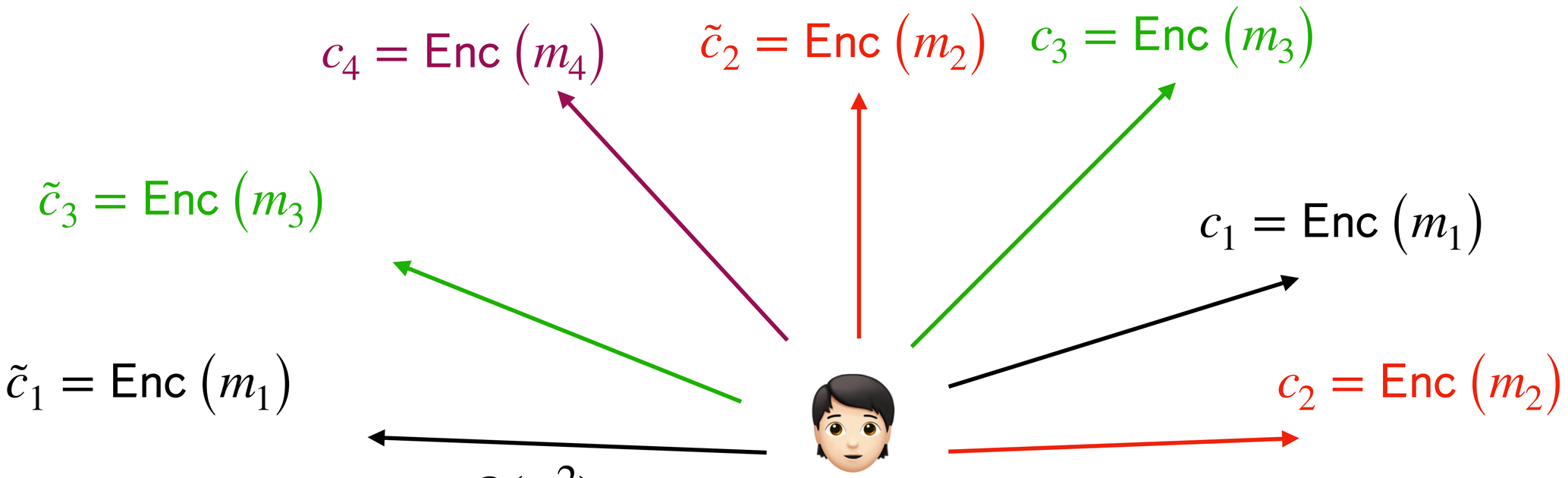

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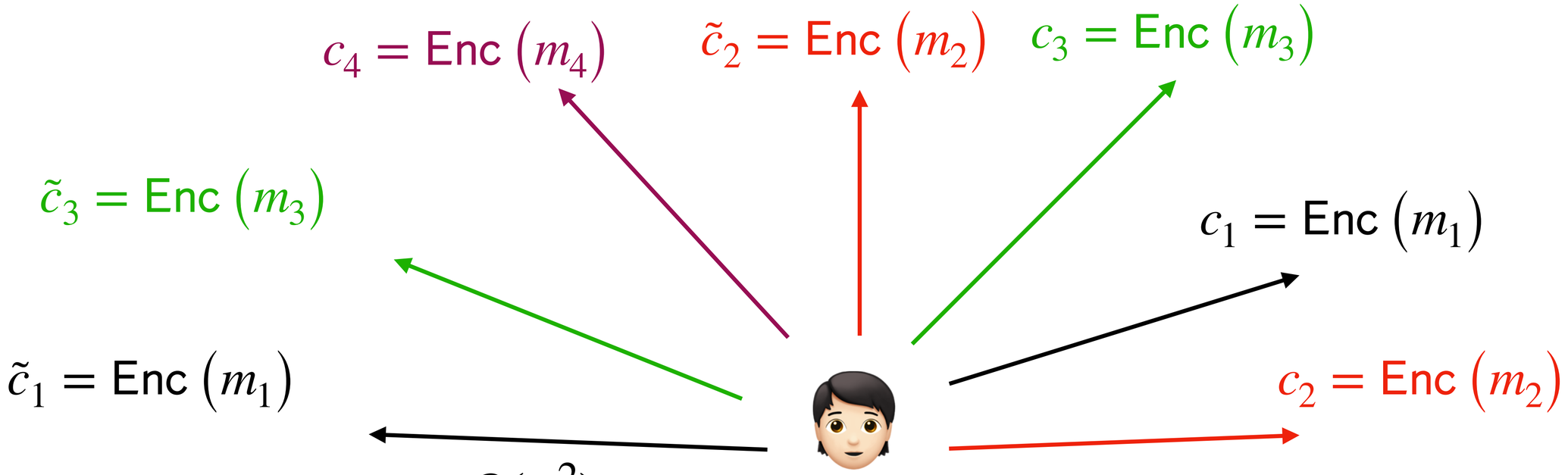

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- Send **same sized ciphertext** to **every party** in **every gossip round**

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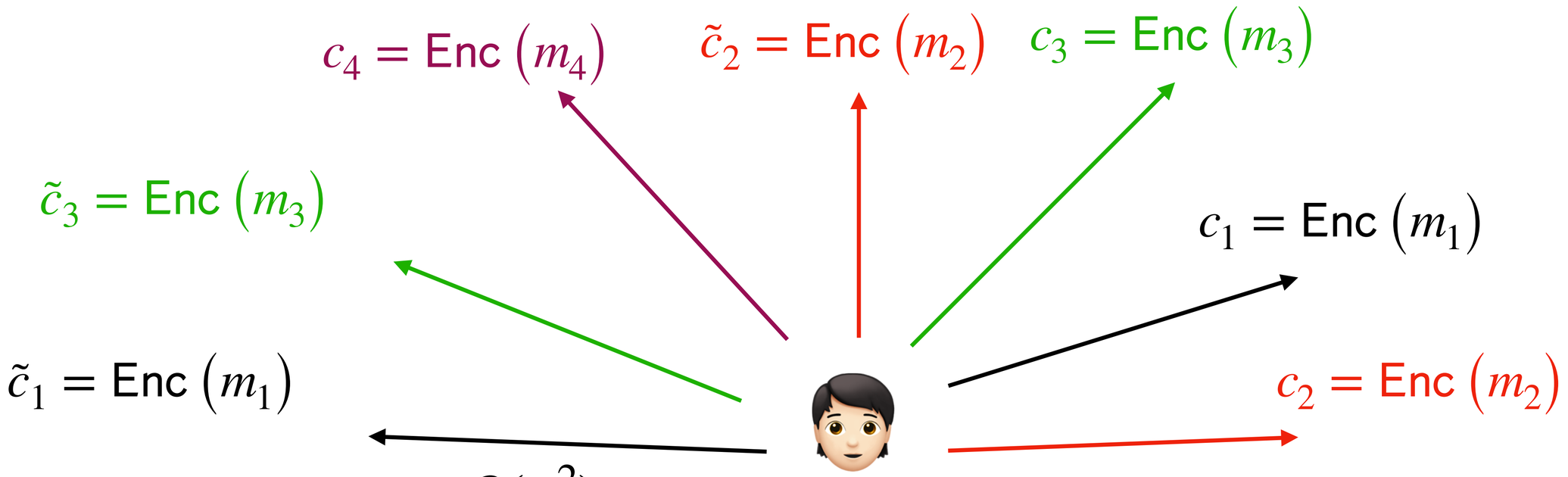

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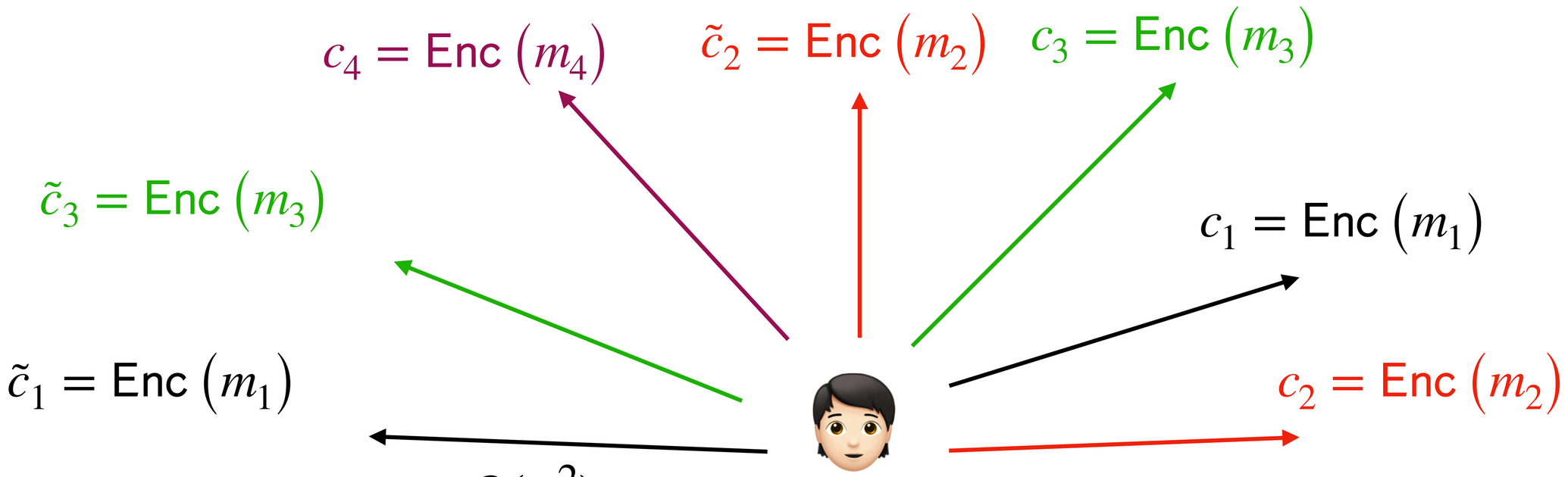
- Idea: run n gossip instances in parallel (one per sender)
- Send **same sized ciphertext** to **every party** in **every gossip round**
- Instances provide **cover traffic** for each other

Key idea: Cover Traffic (Tsimos et al. CRYPTO 22)



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- Idea: run n gossip instances in parallel (one per sender)
- Send **same sized ciphertext** to **every party** in **every gossip round**
- Instances provide **cover traffic** for each other
- **Still costs $O(n^2\kappa\lambda)$ bits**

Two-Step Approach

Two-Step Approach

- Step 1: Reduce PBC to Binary Consensus

Two-Step Approach

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- **Step 2:** Binary consensus in $O(n^2 \cdot \kappa)$ bits using novel DY-based gossip

Pull-Based Gossip

$\sigma_2(m)$

$\sigma_4(m')$
$\sigma_3(m')$

$\sigma_1(m)$

$\sigma_2(m)$
$\sigma_3(m')$

Pull-Based Gossip

- Ask for (pull) signatures from other parties

Pull-Based Gossip

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- Ask for (pull) signatures from other parties
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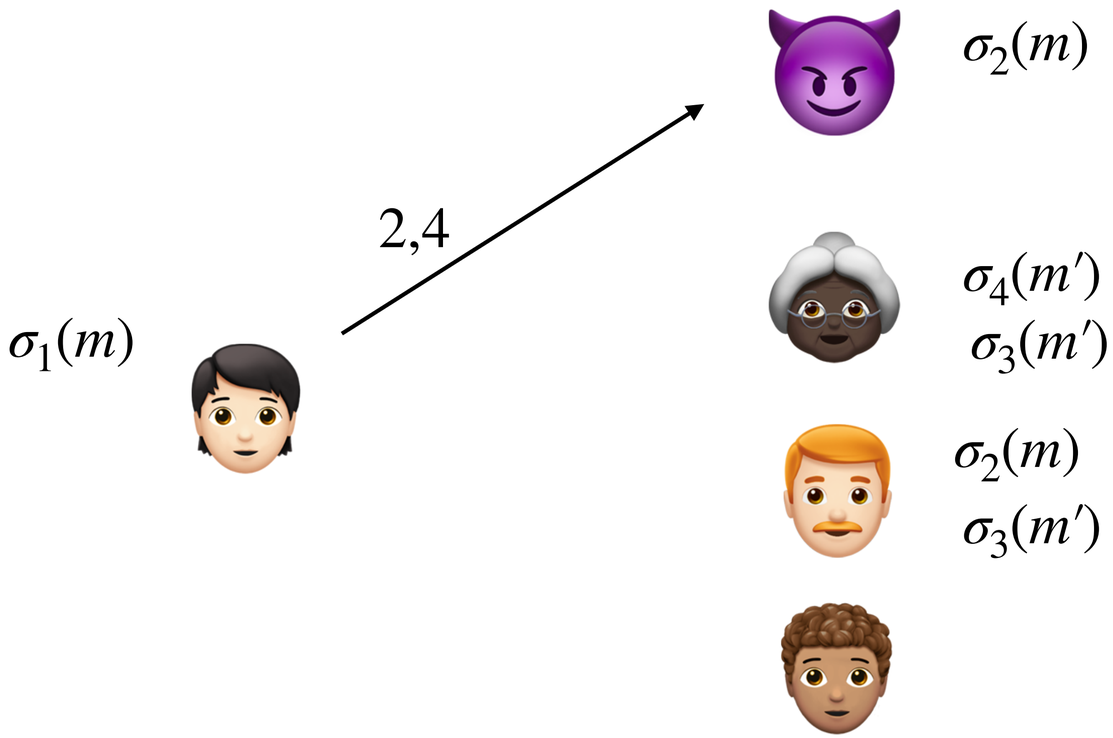
Pull-Based Gossip



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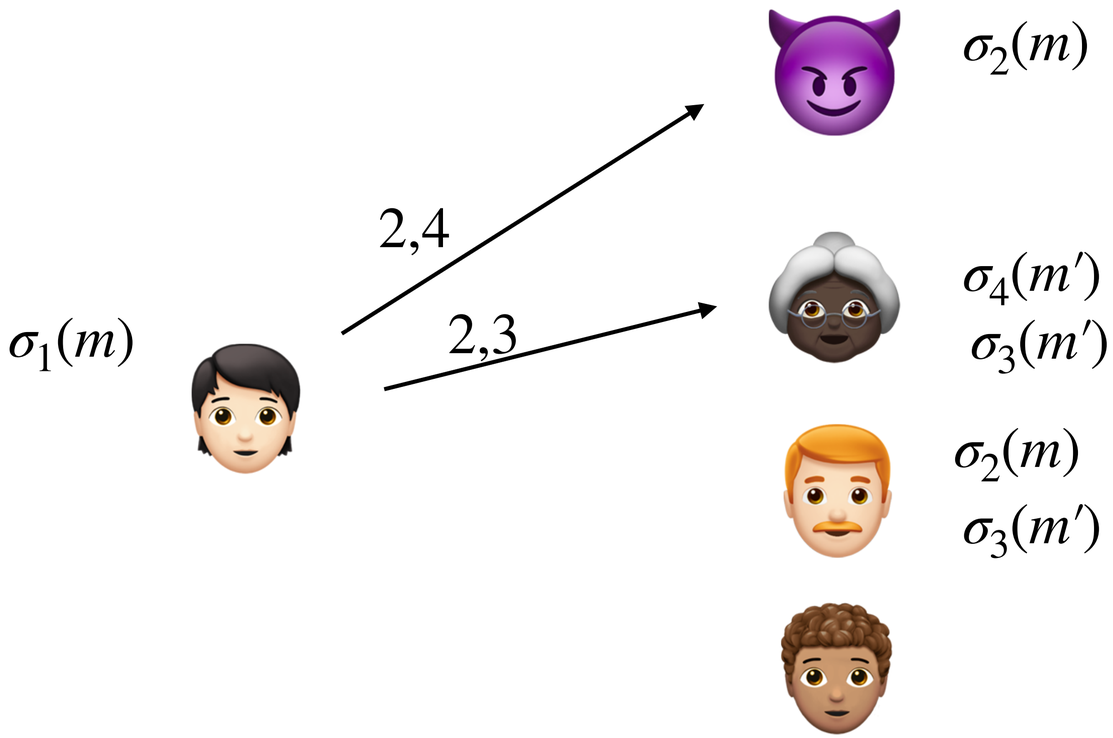

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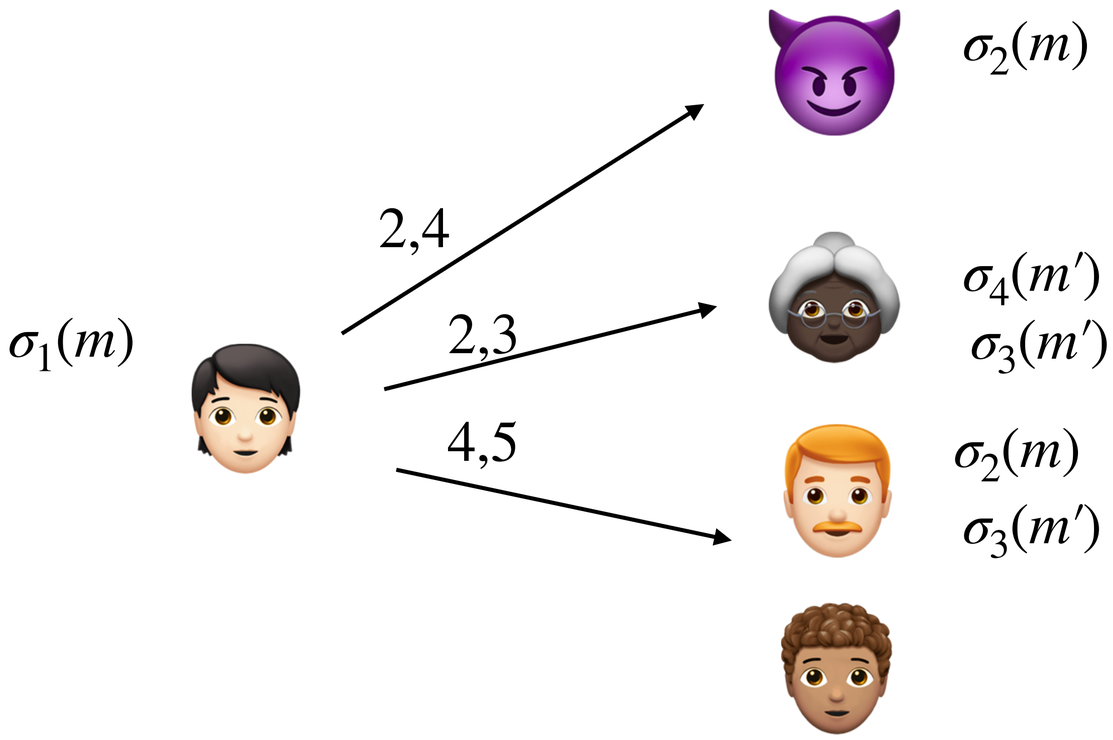

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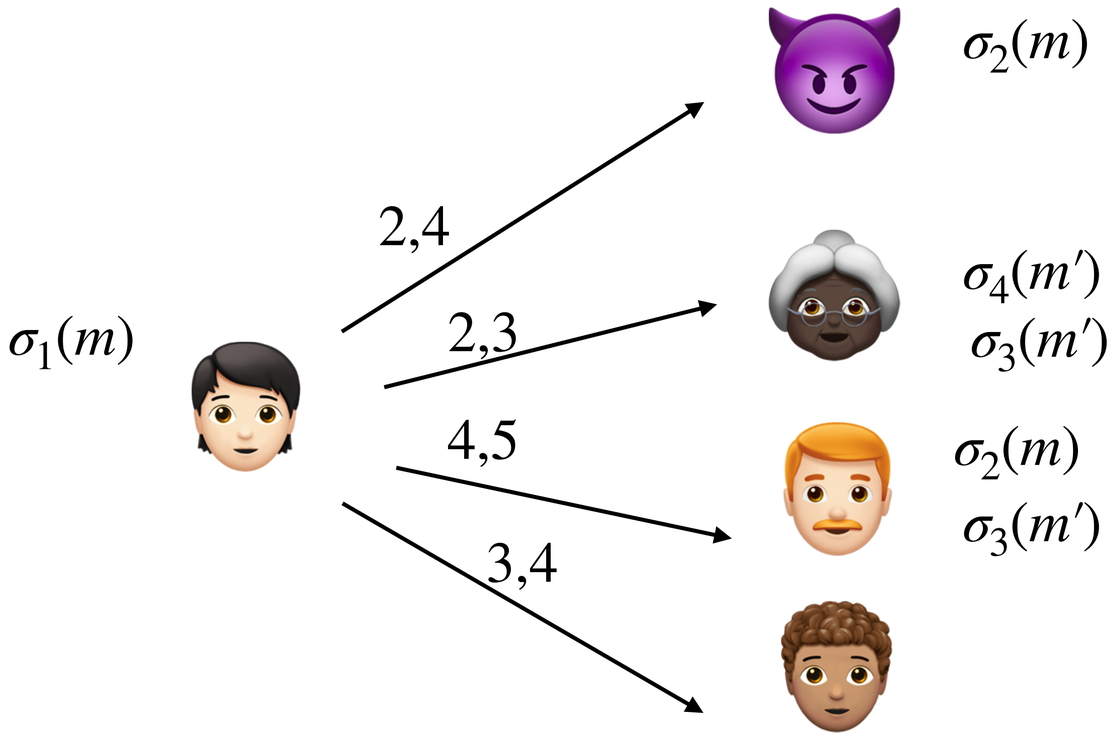

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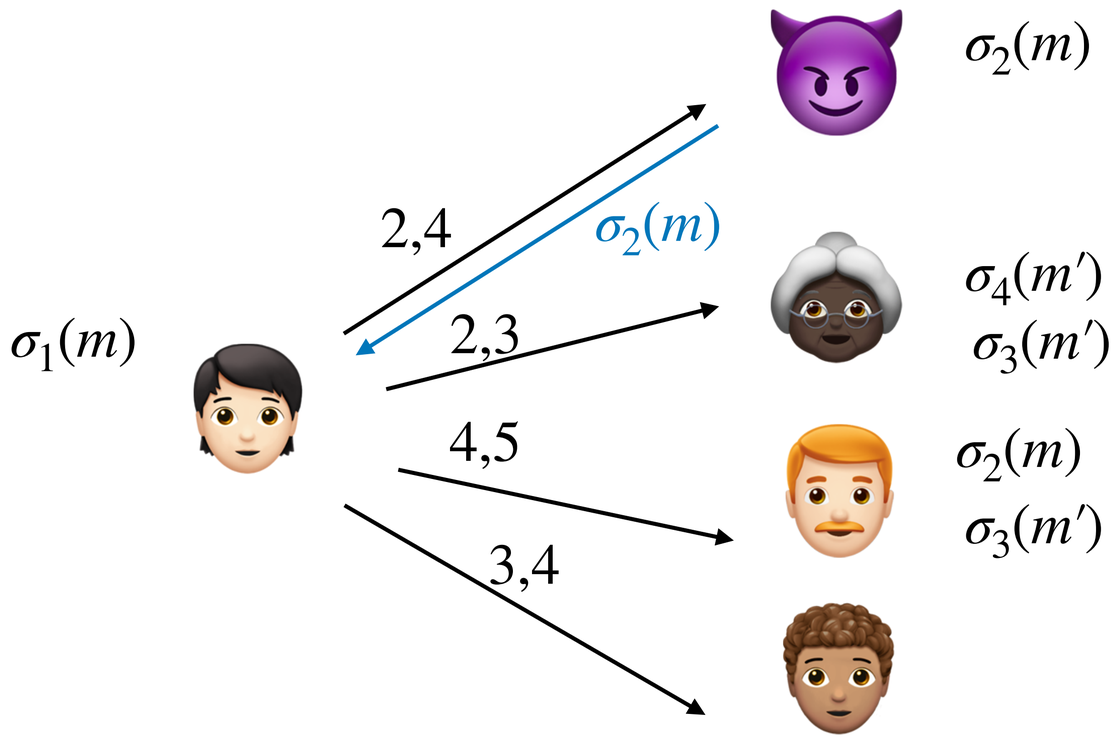

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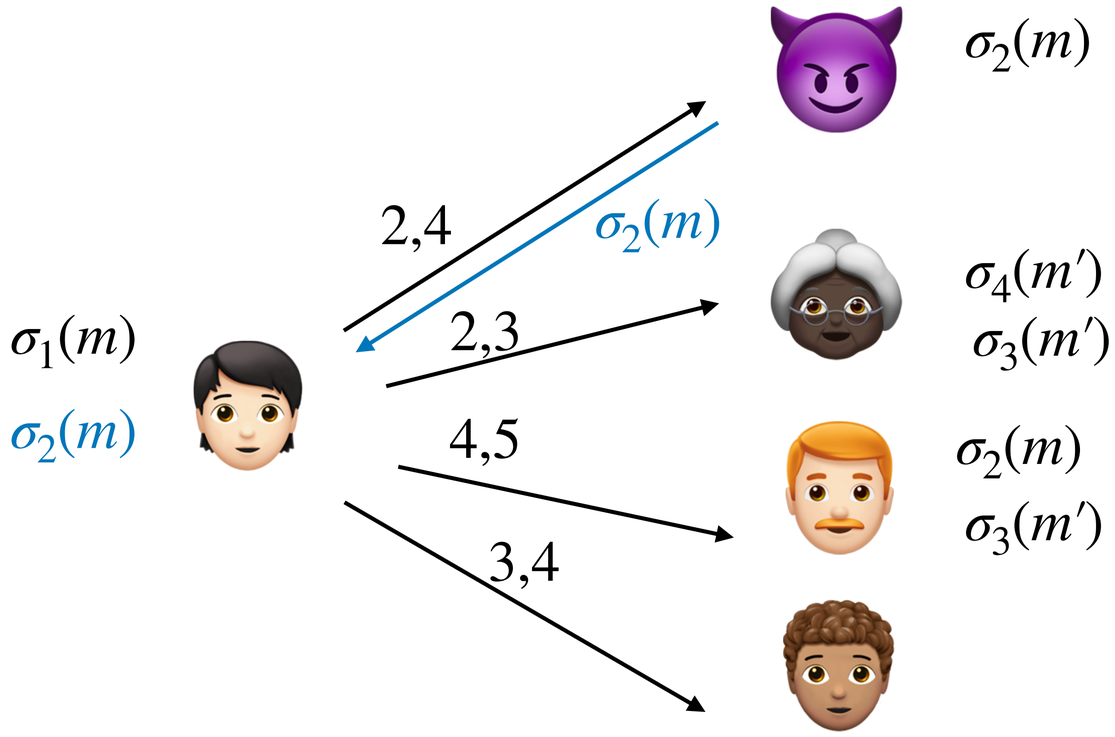
Pull-Based Gossip



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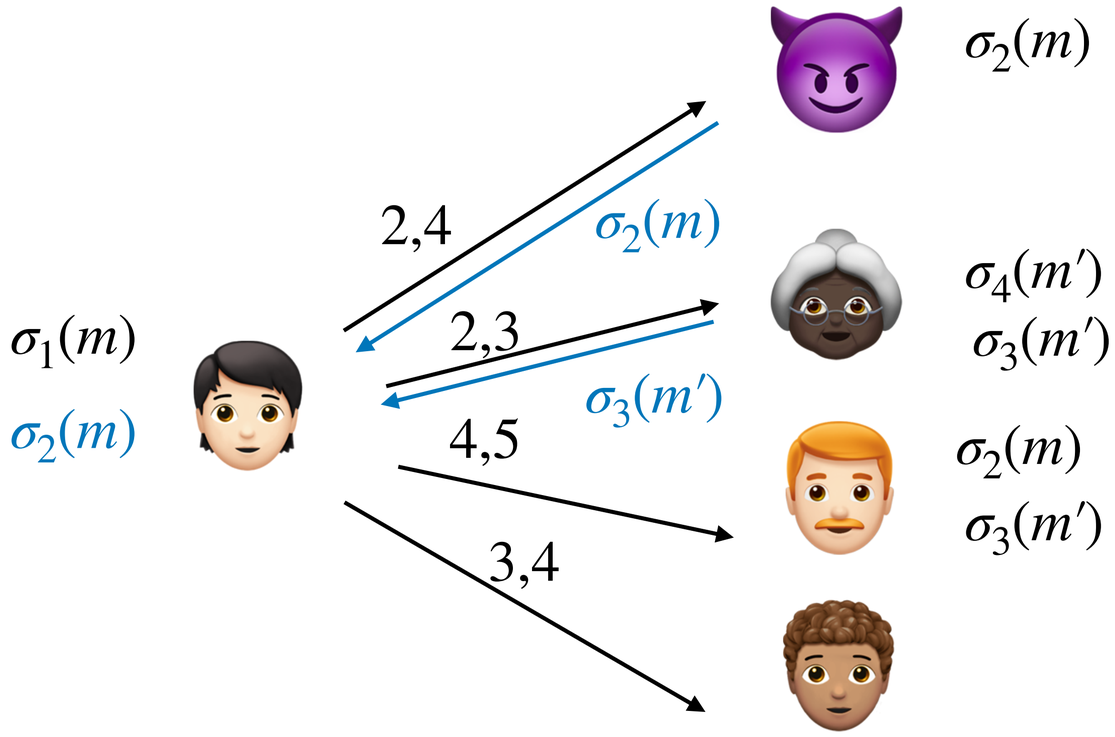

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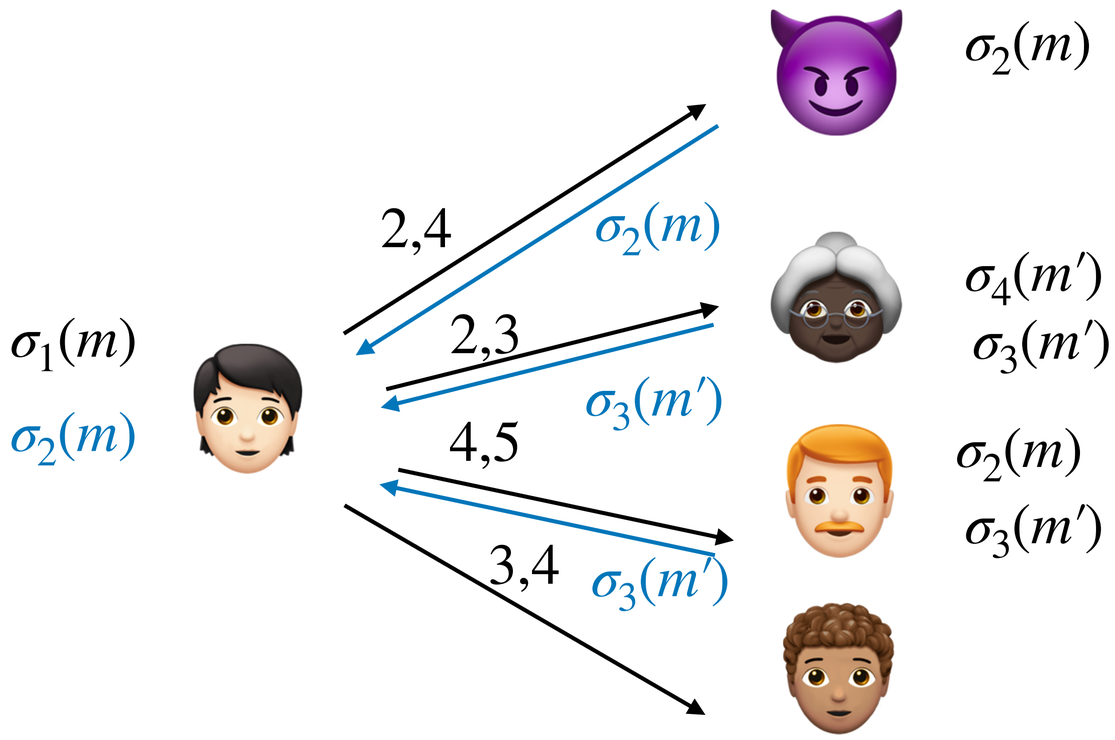

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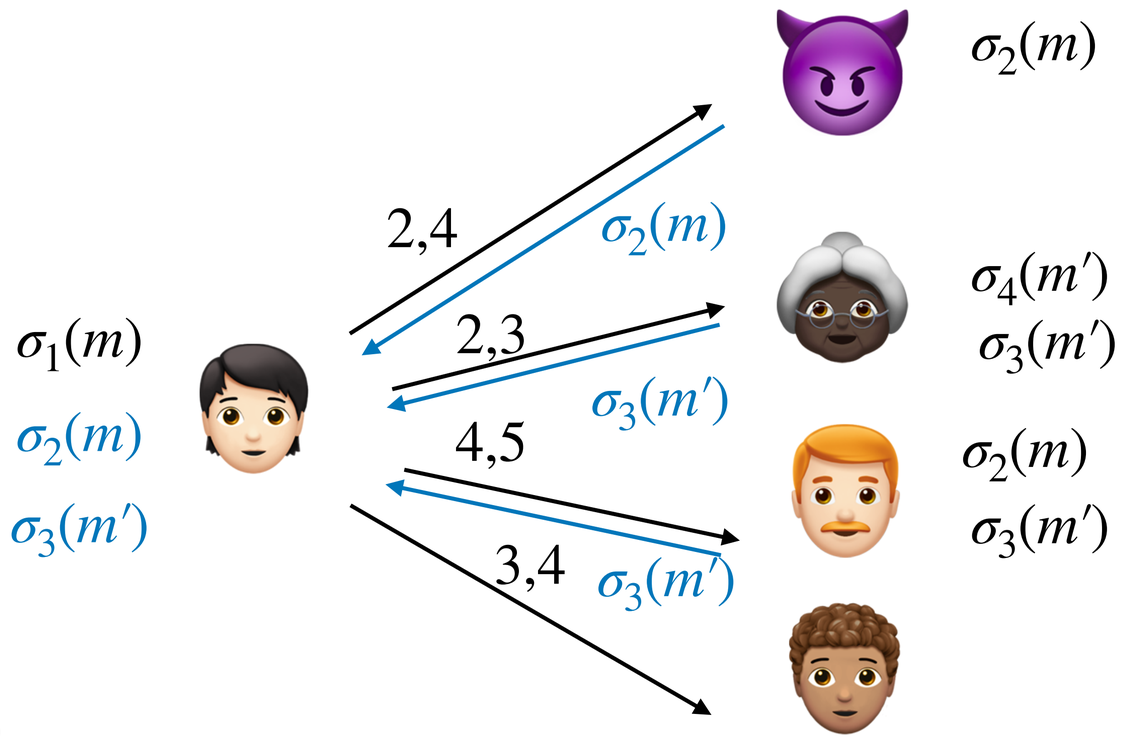

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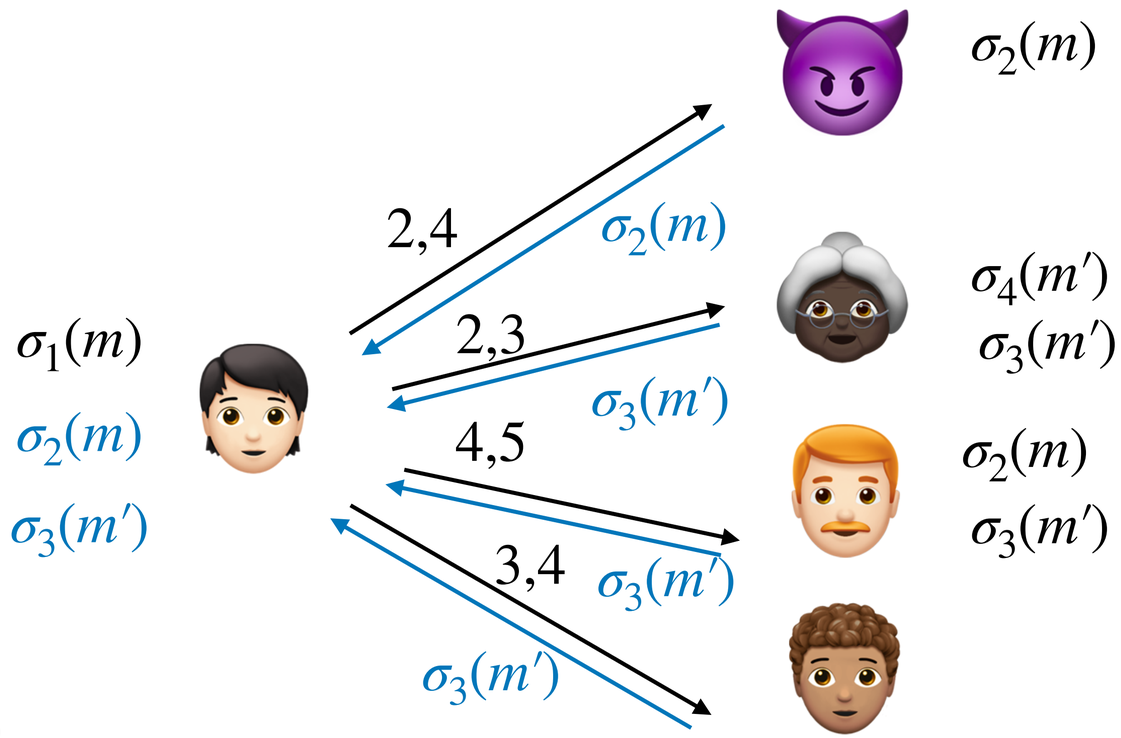

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Issue with Naive Pulling

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$C' = \{\sigma_2(m'), \sigma_3(m'), \sigma_5(m')\}$

Issue with Naive Pulling

- Naive attempt: pull $O(1)$ missing indices

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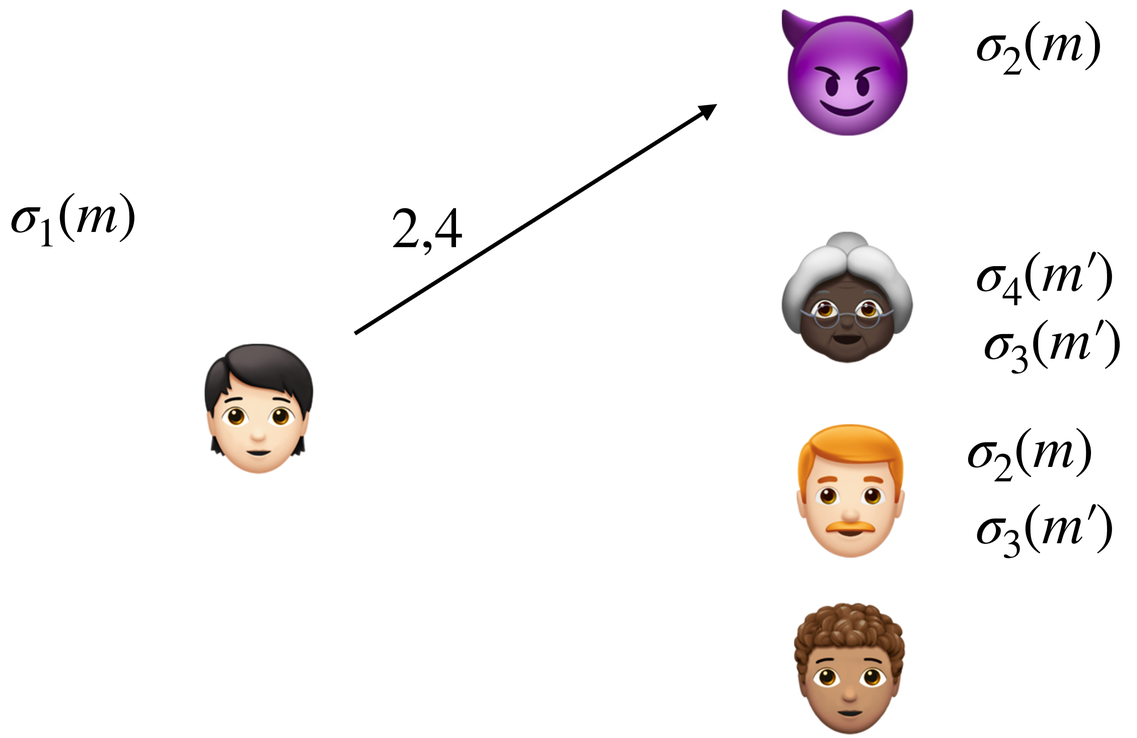

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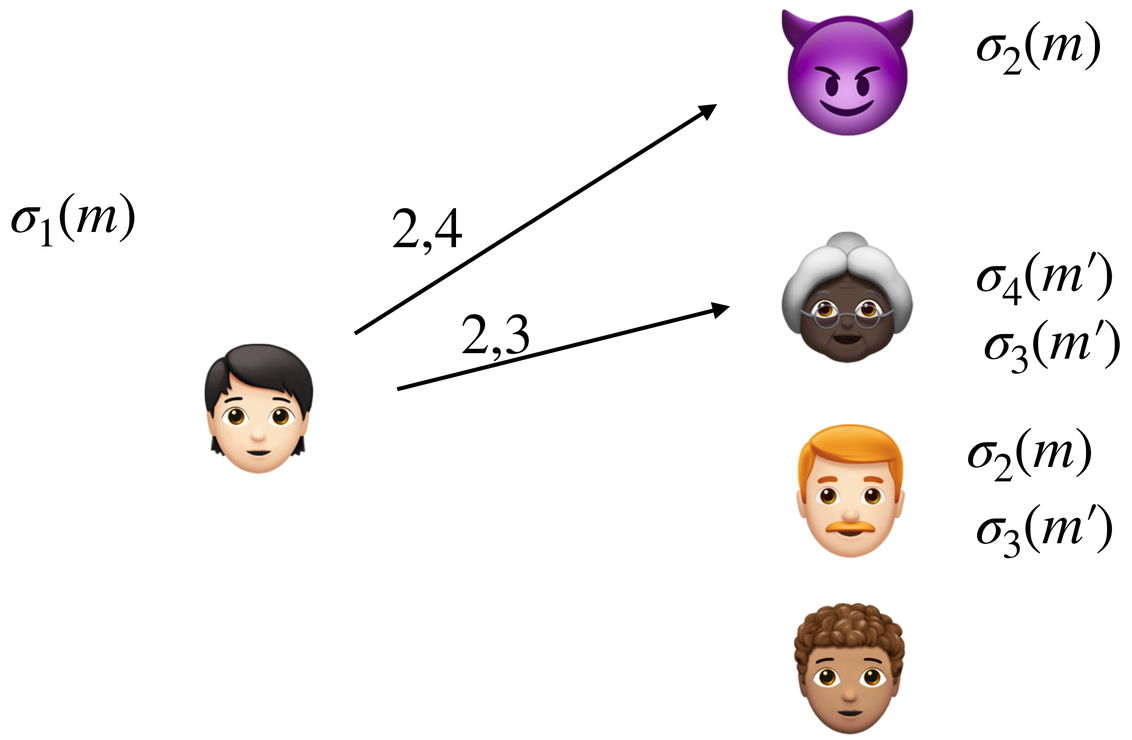

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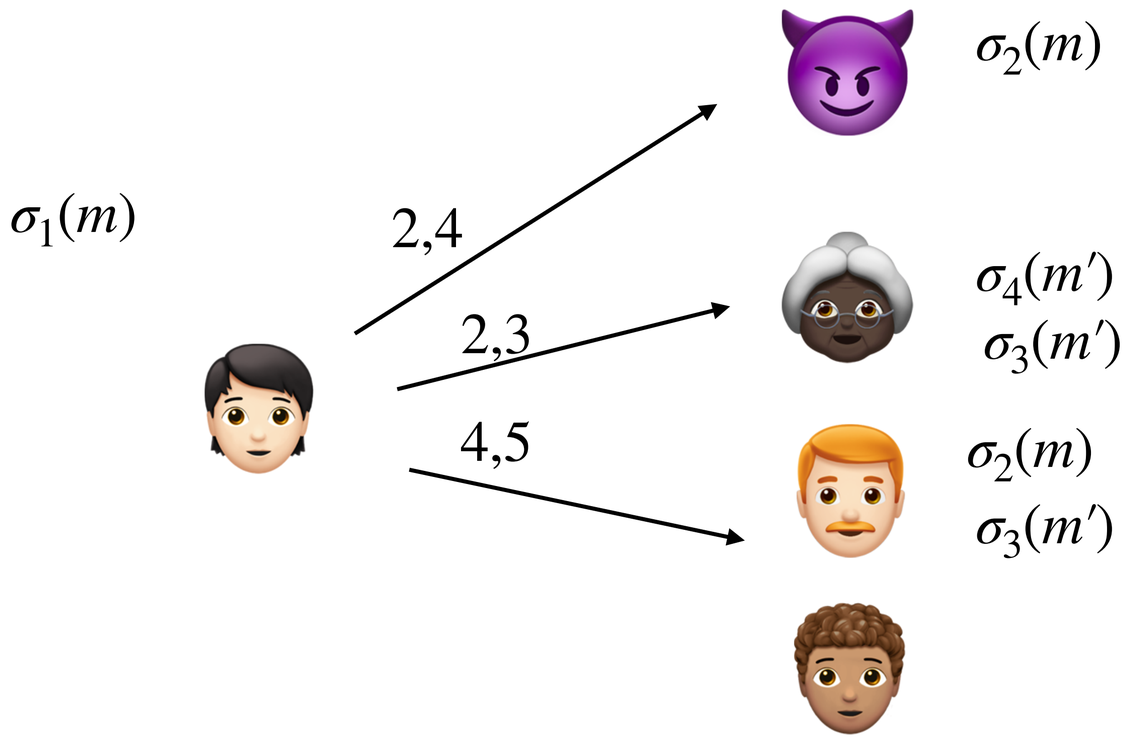

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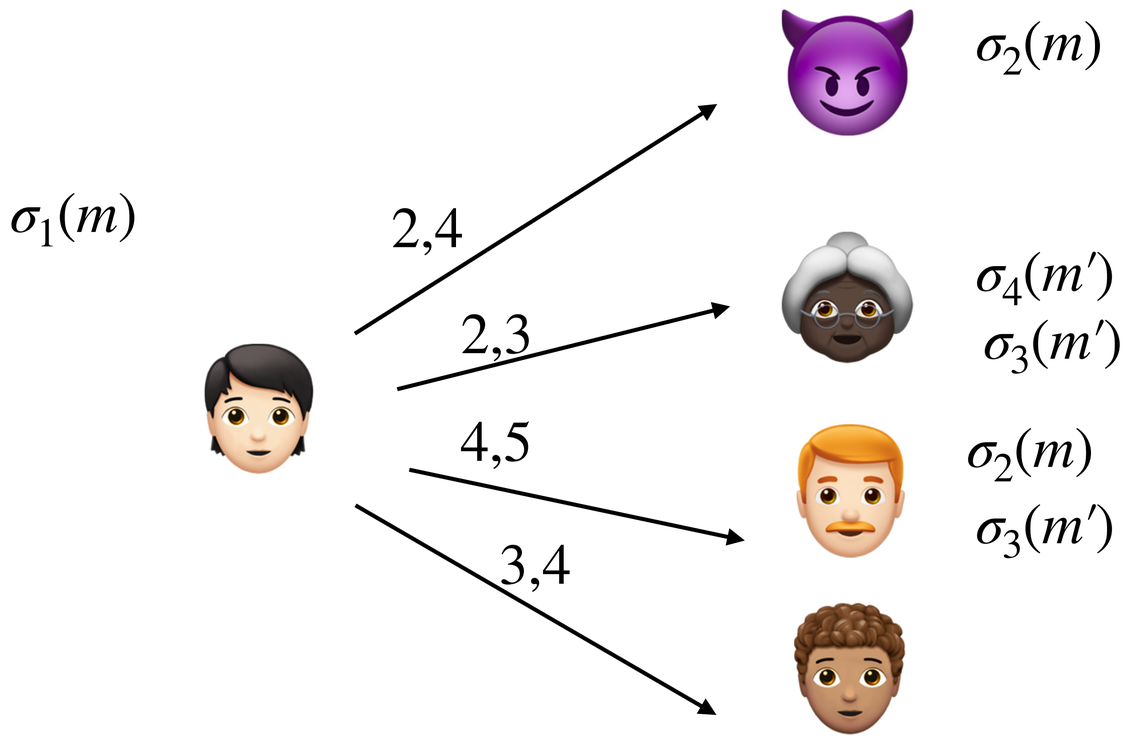

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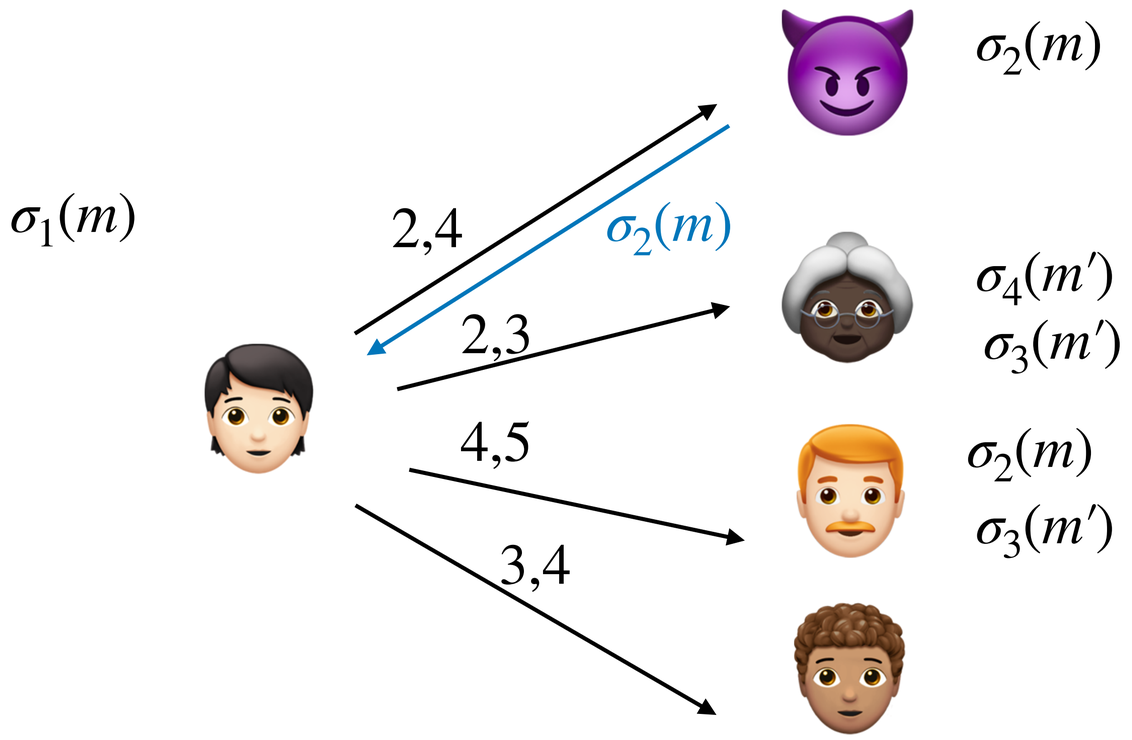

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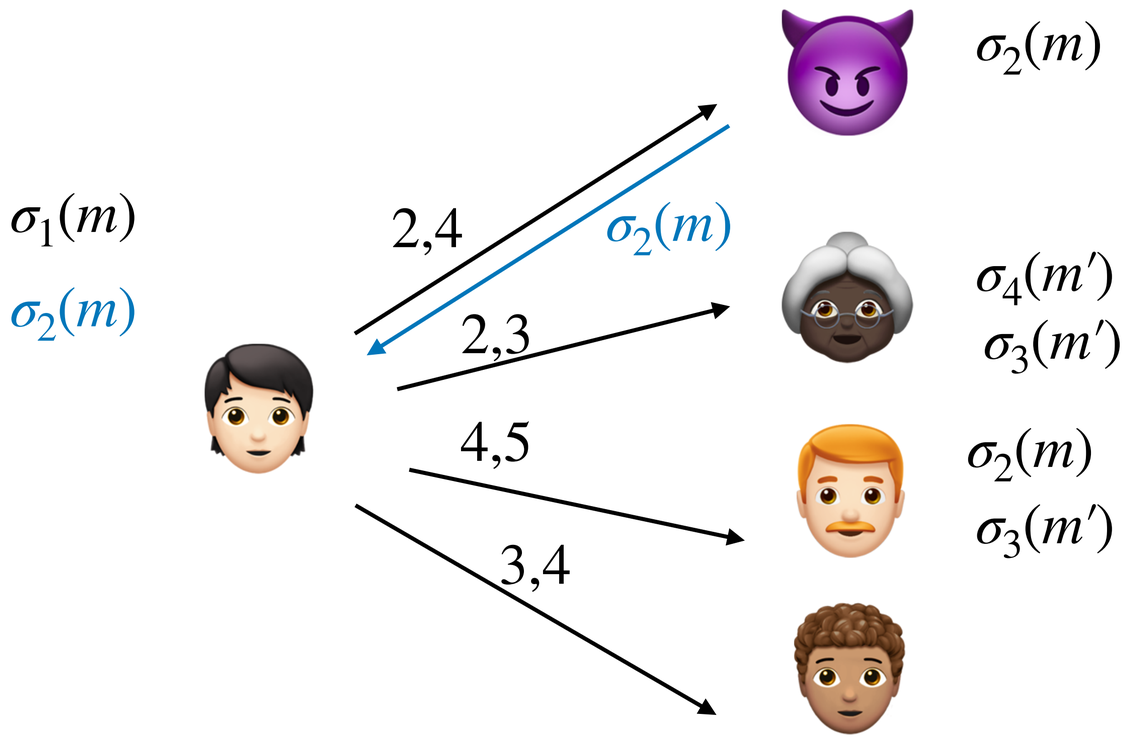

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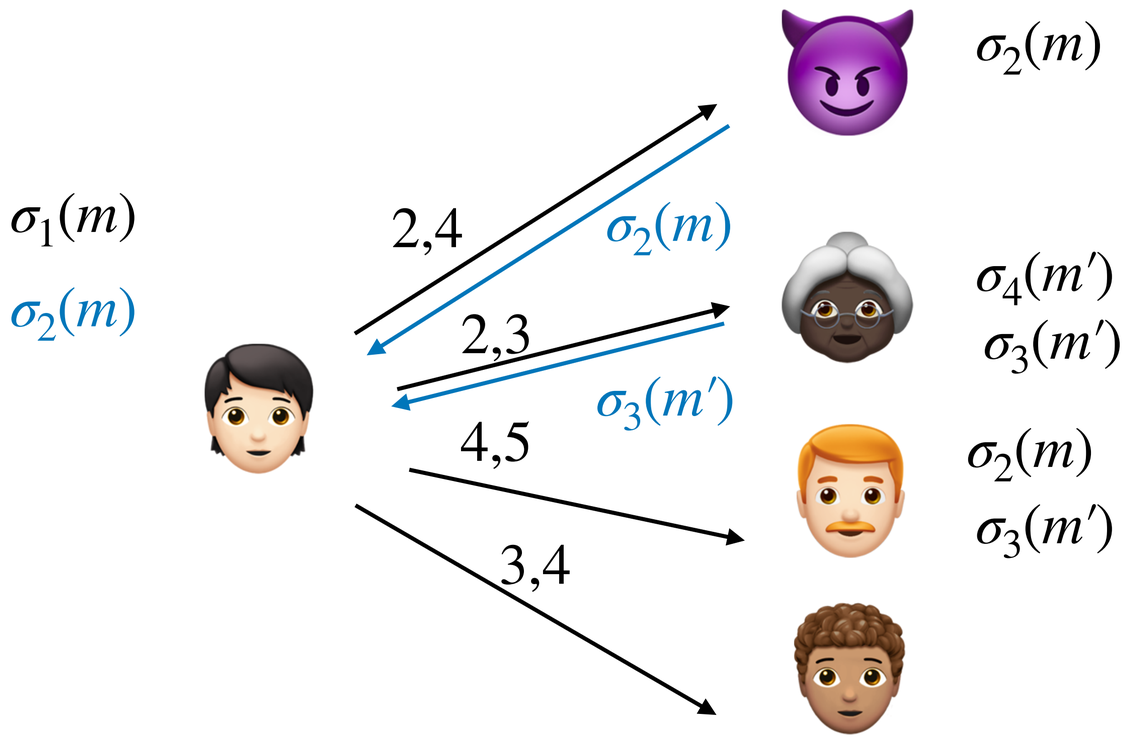

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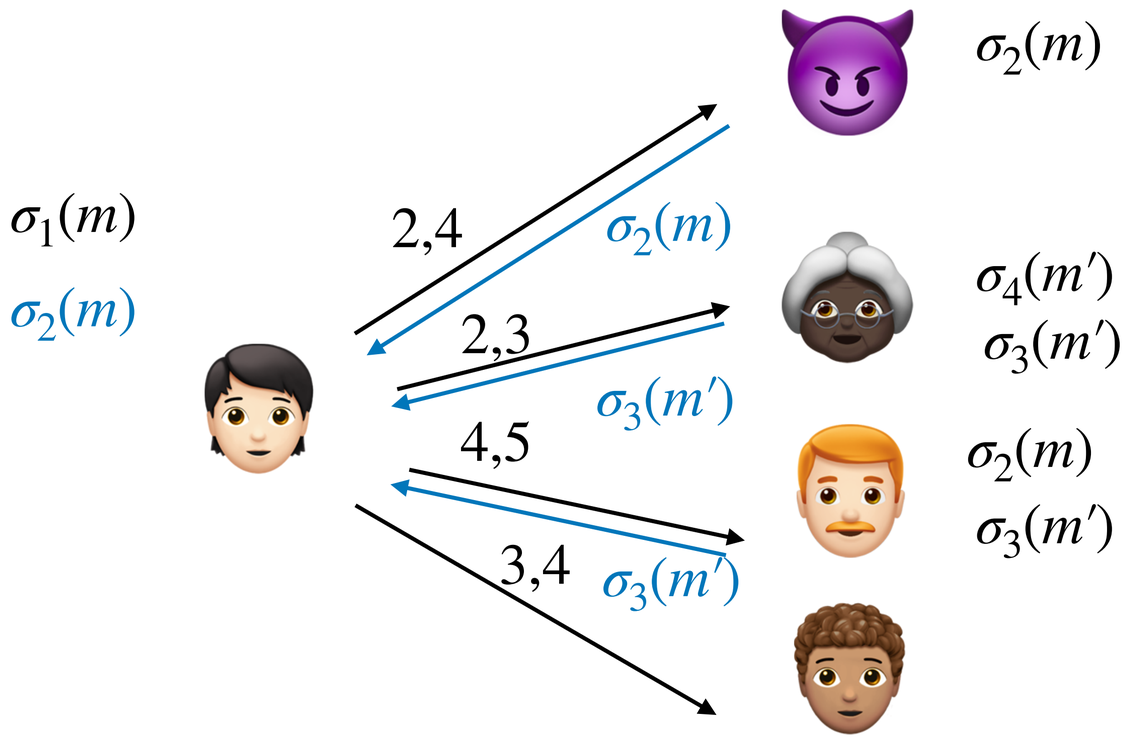

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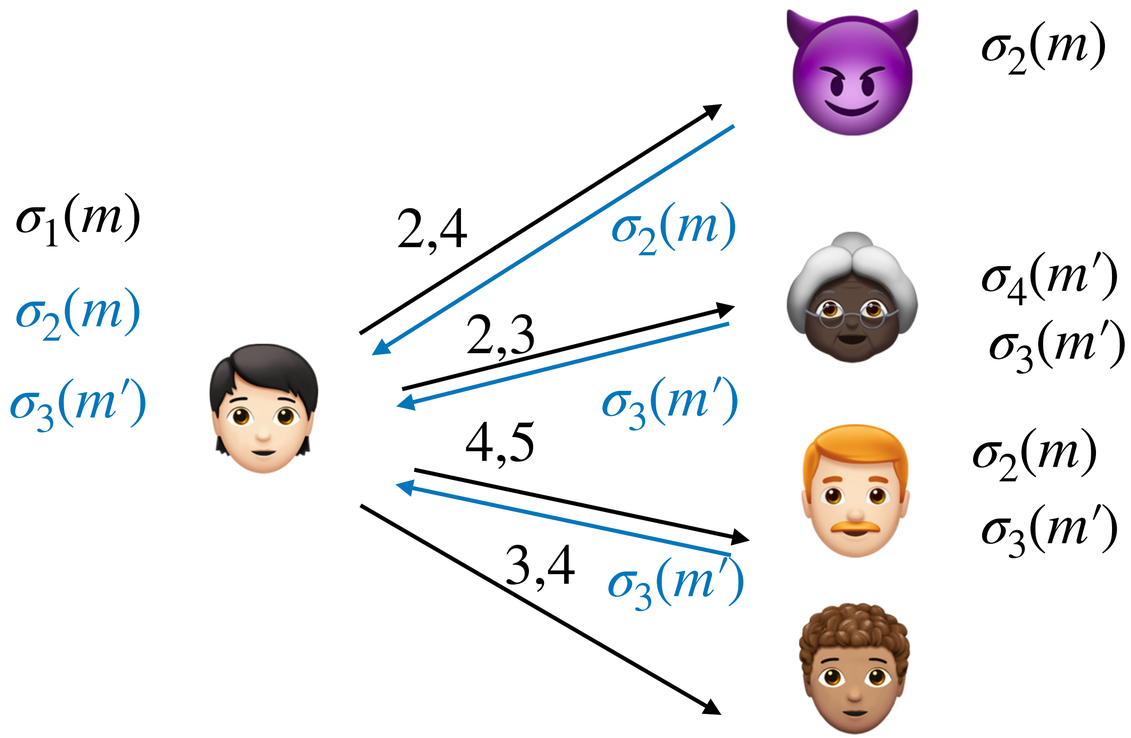

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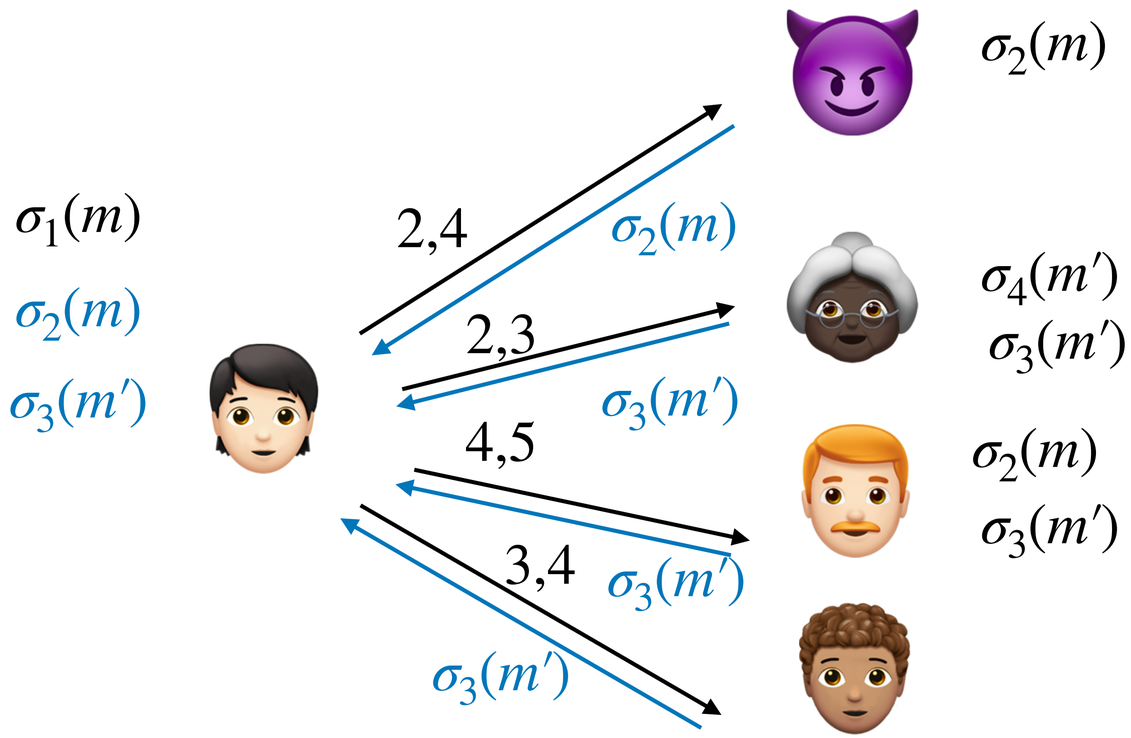

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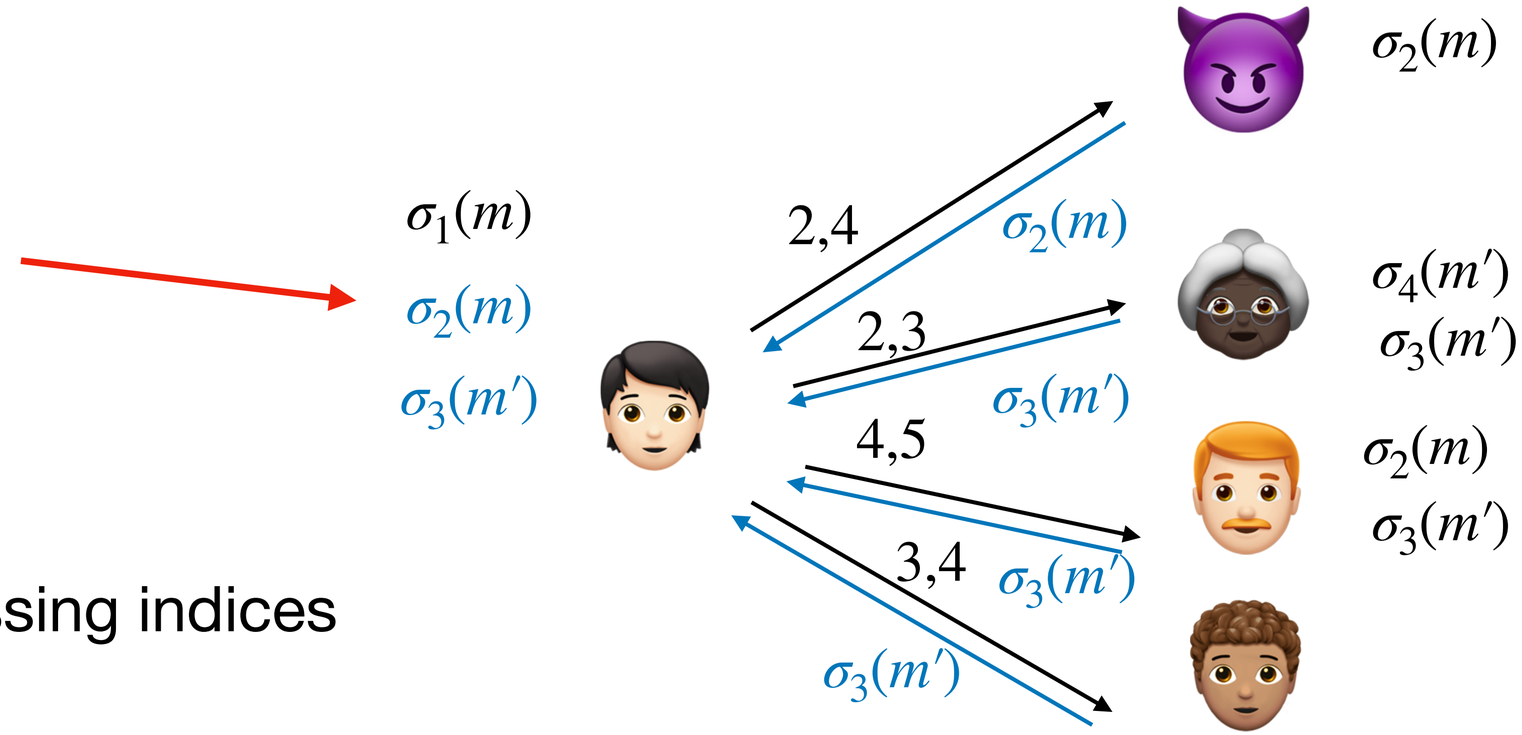

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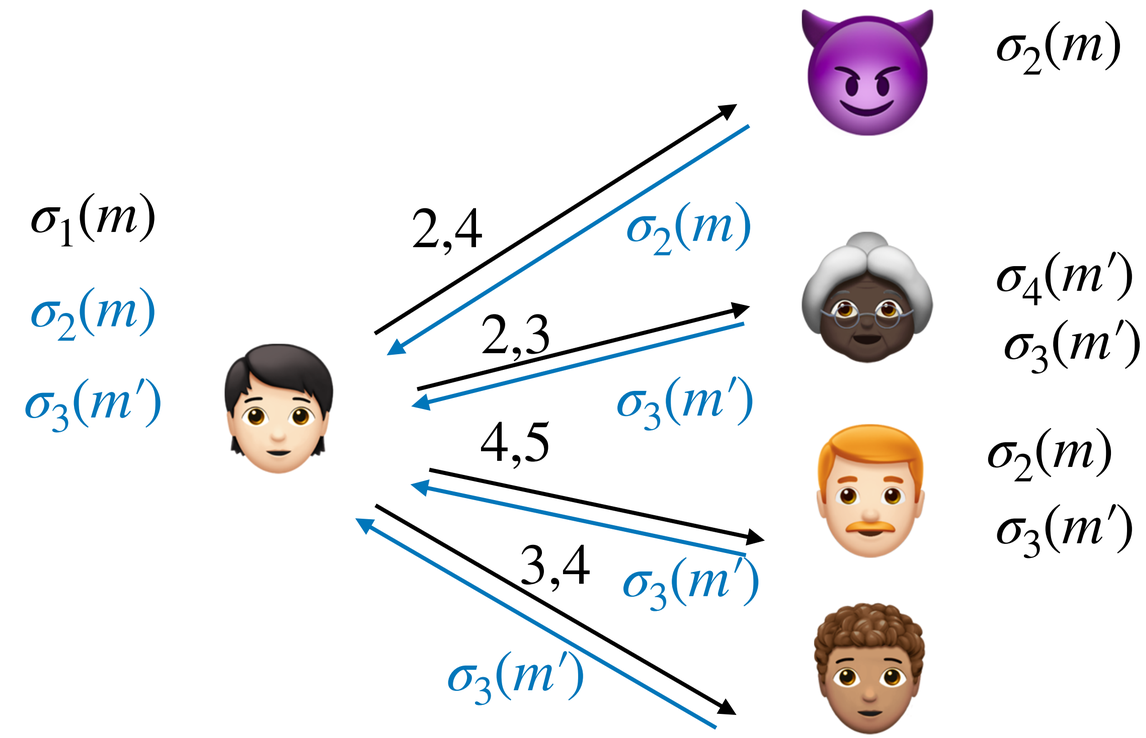

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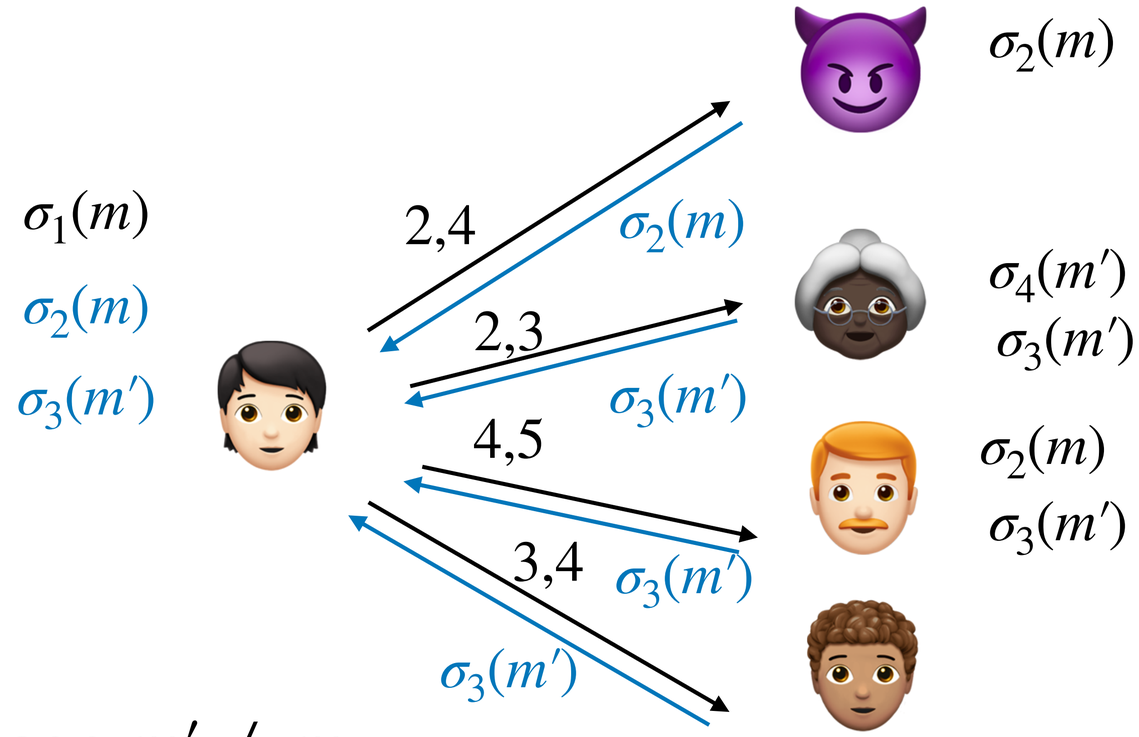

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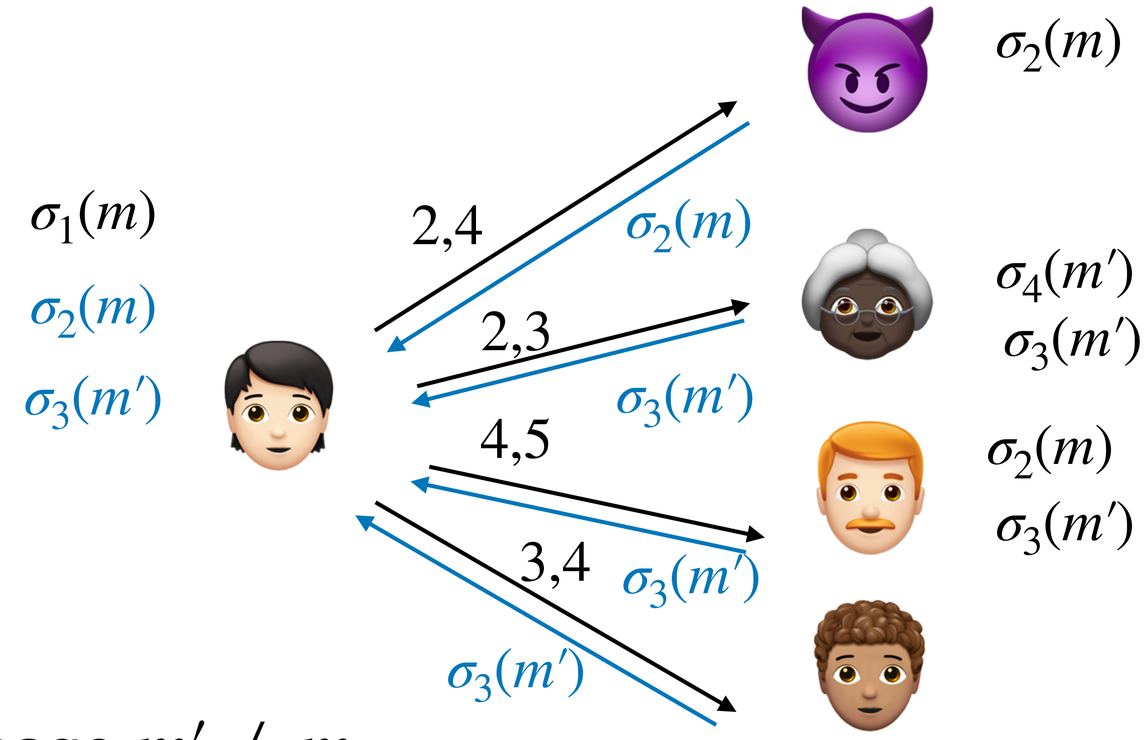

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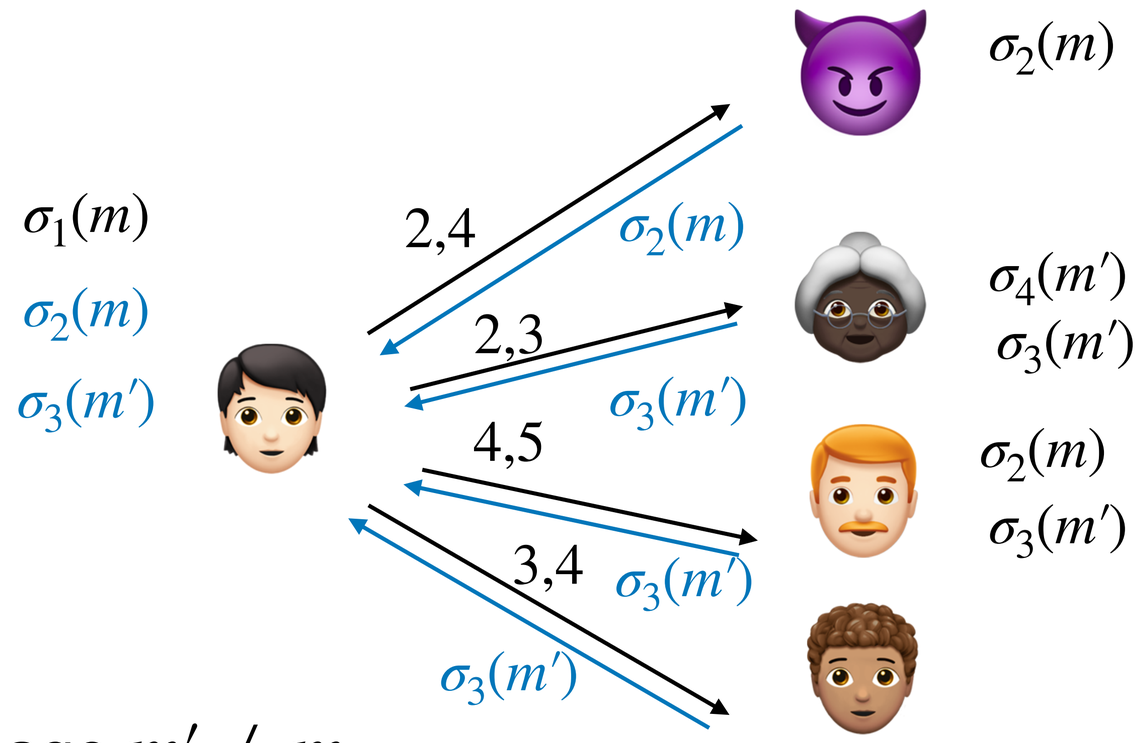


- Naive attempt: pull $O(1)$ missing indices
- **Issue:** Certificate could exist for **any** message $m' \neq m$
- $\sigma_i(m')$ for $m' \neq m$ is never pulled
- Prevents signatures in certificate from spreading

$$C' = \{\sigma_2(m'), \sigma_3(m'), \sigma_5(m')\}$$

Pull-Based Gossip with Testing

P'

P

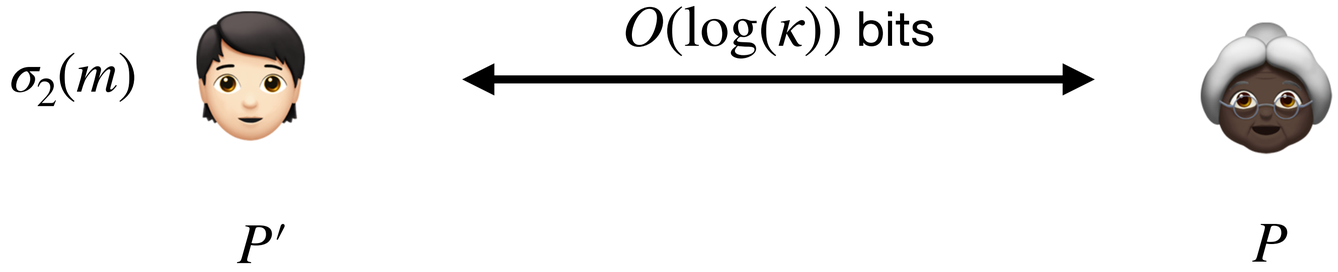
Pull-Based Gossip with Testing

$\sigma_2(m)$ 

P'

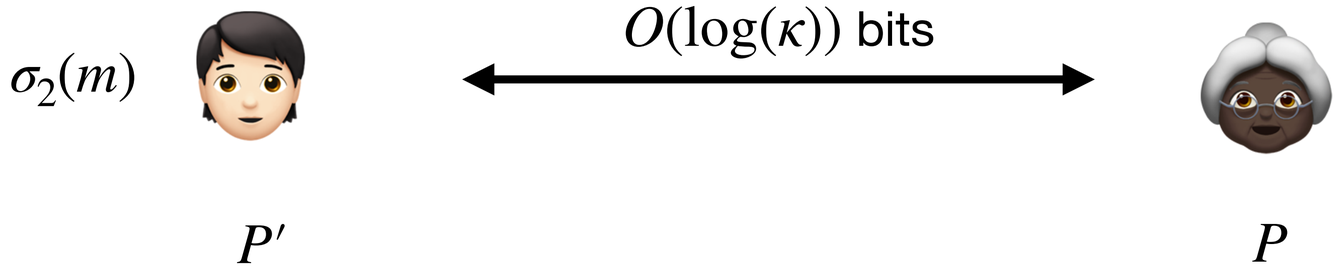
P

Pull-Based Gossip with Testing



- Test for new information before pulling $\Longrightarrow O(\log(\kappa))$ extra bits per index

Pull-Based Gossip with Testing



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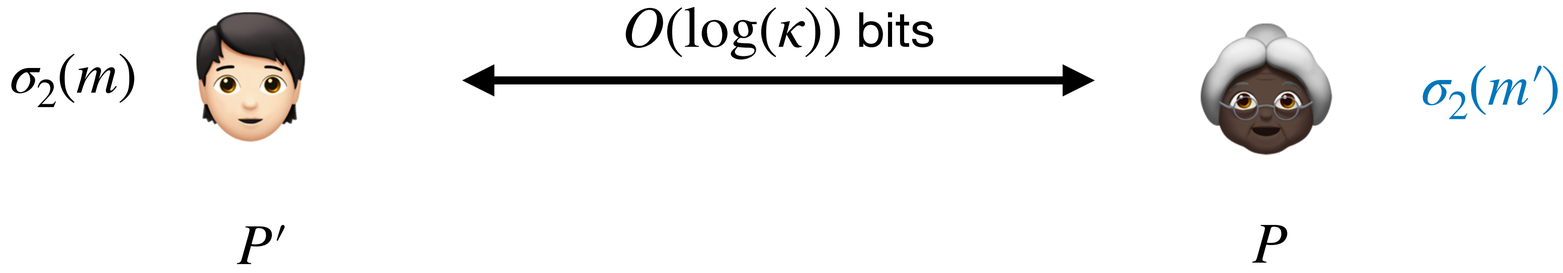

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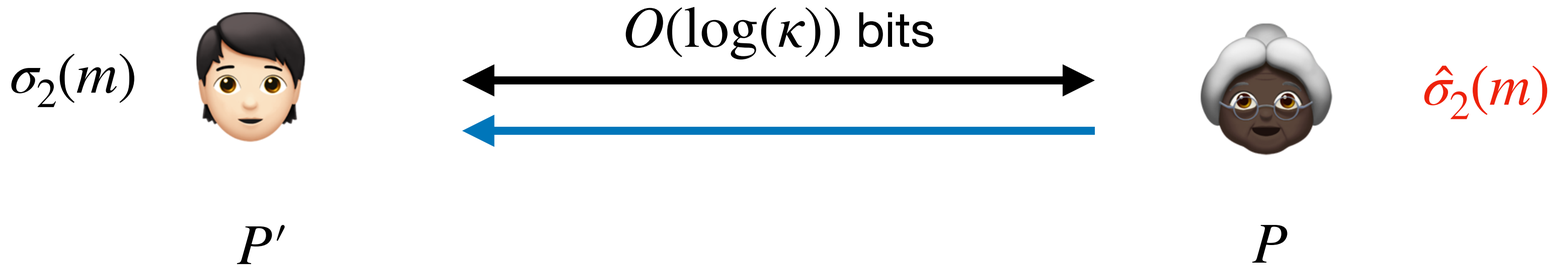

- Test for new information before pulling $\Longrightarrow O(\log(\kappa))$ extra bits per index
- Pull only if test indicates a different signature
- Technical issue: signatures on **same** message can be syntactically different

Issues with Naive Testing

P'

P

Issues with Naive Testing

C'

P'

P

- Incomplete certificate C'; missing **single index**

Issues with Naive Testing

C'

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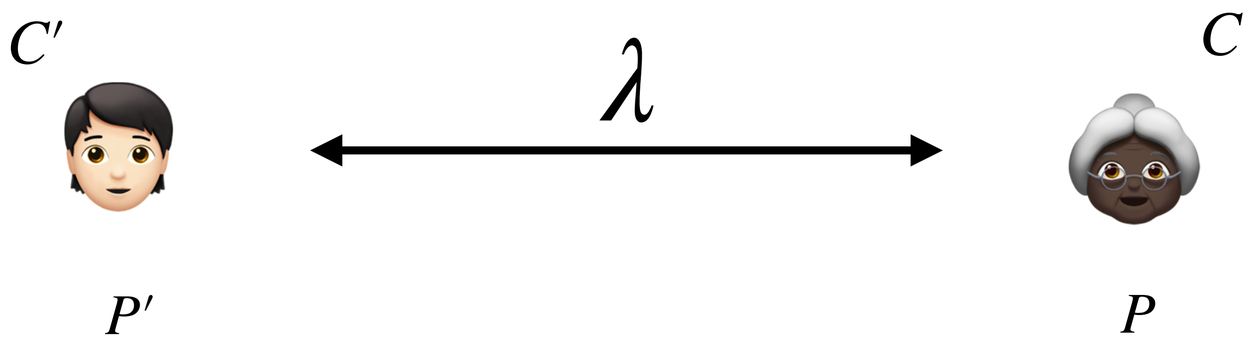

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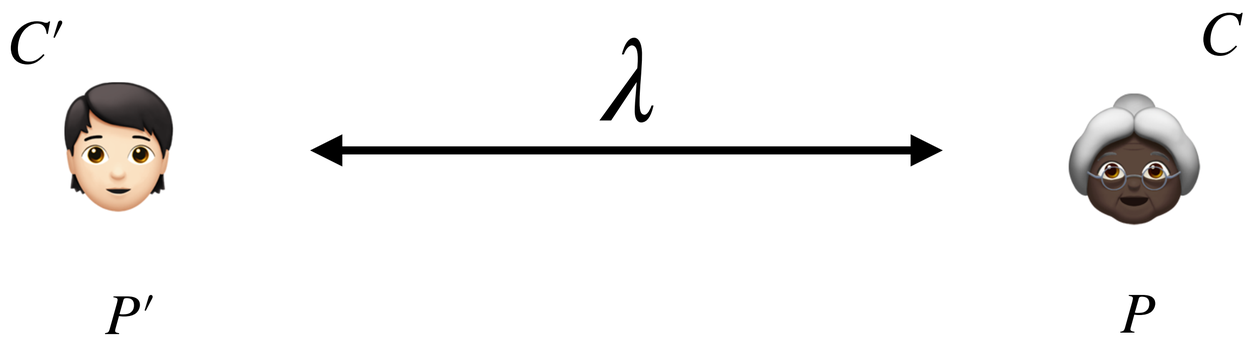

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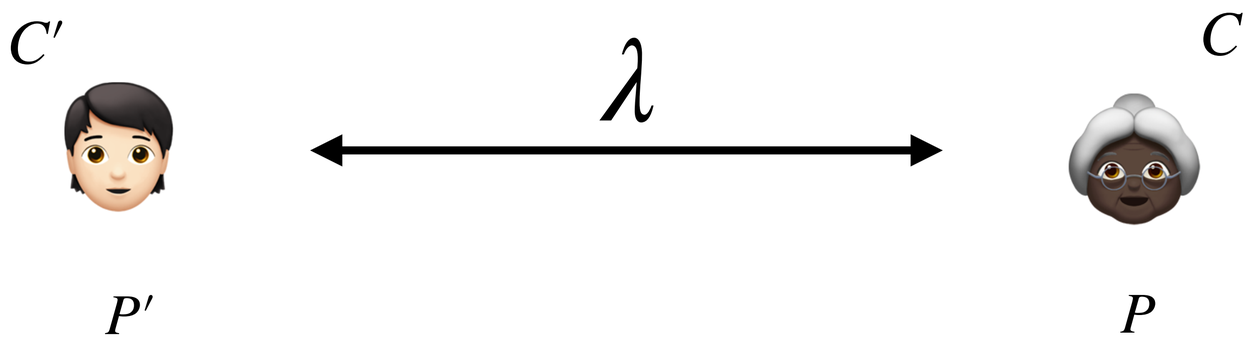

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- All tests positive $\implies$ missing index is pulled with probability $1/n$
- λ rounds to learn missing index with probability $1 - 2^{-\lambda} \implies$ still costs $O(n^2 \cdot \kappa \cdot \lambda)$ bits

Increasing the Probability of Pulling

P_i

P

Increasing the Probability of Pulling

C'

P_i

C

P

Increasing the Probability of Pulling

C'

P_i

C

P

C is a full certificate for m

Increasing the Probability of Pulling

C'

P_i

C

P

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- For each P_i:

Increasing the Probability of Pulling

C'

P_i

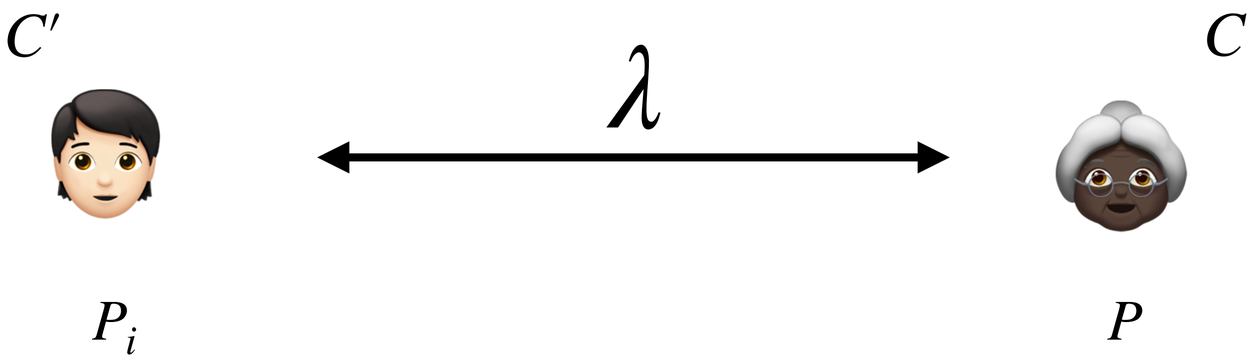
C

P

C is a full certificate for m

- For each P_i:
  - P samples $O(\lambda)$ signatures in C

Increasing the Probability of Pulling



C is a full certificate for m

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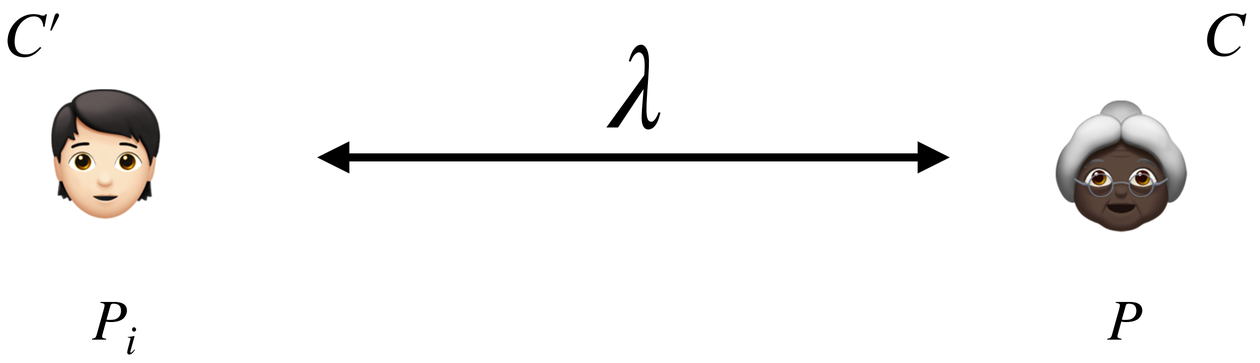
Increasing the Probability of Pulling



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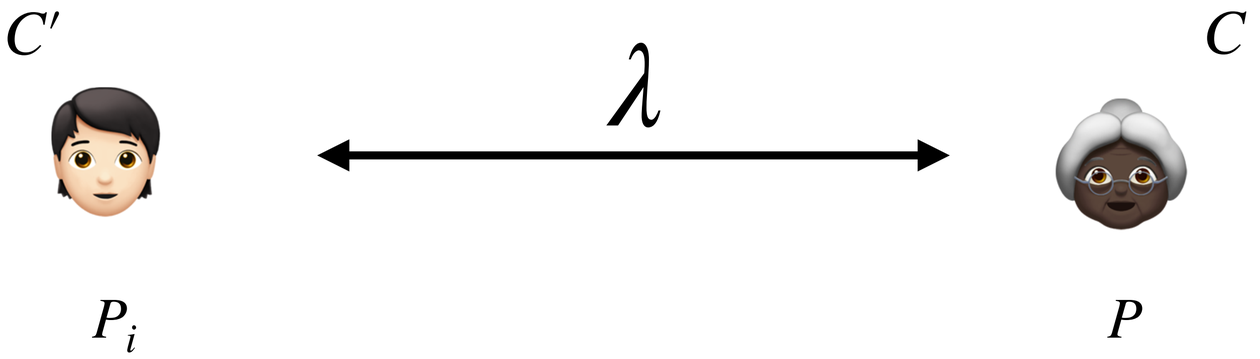

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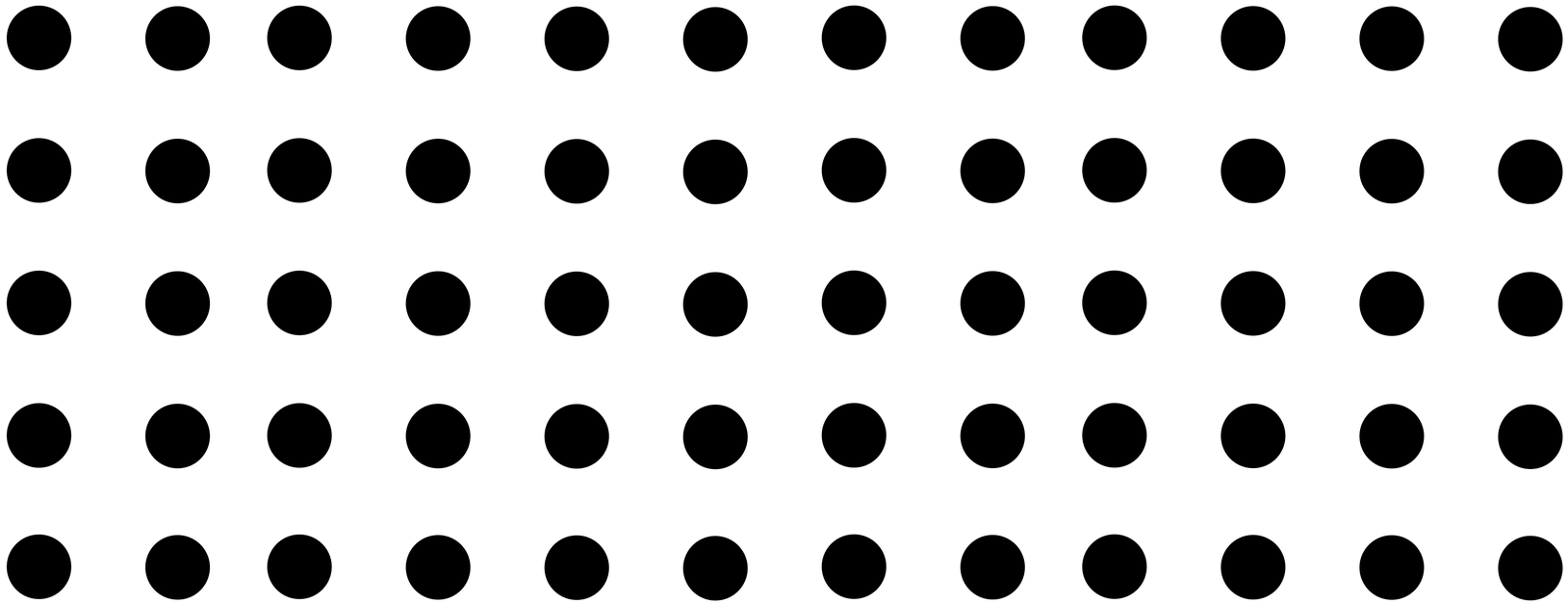
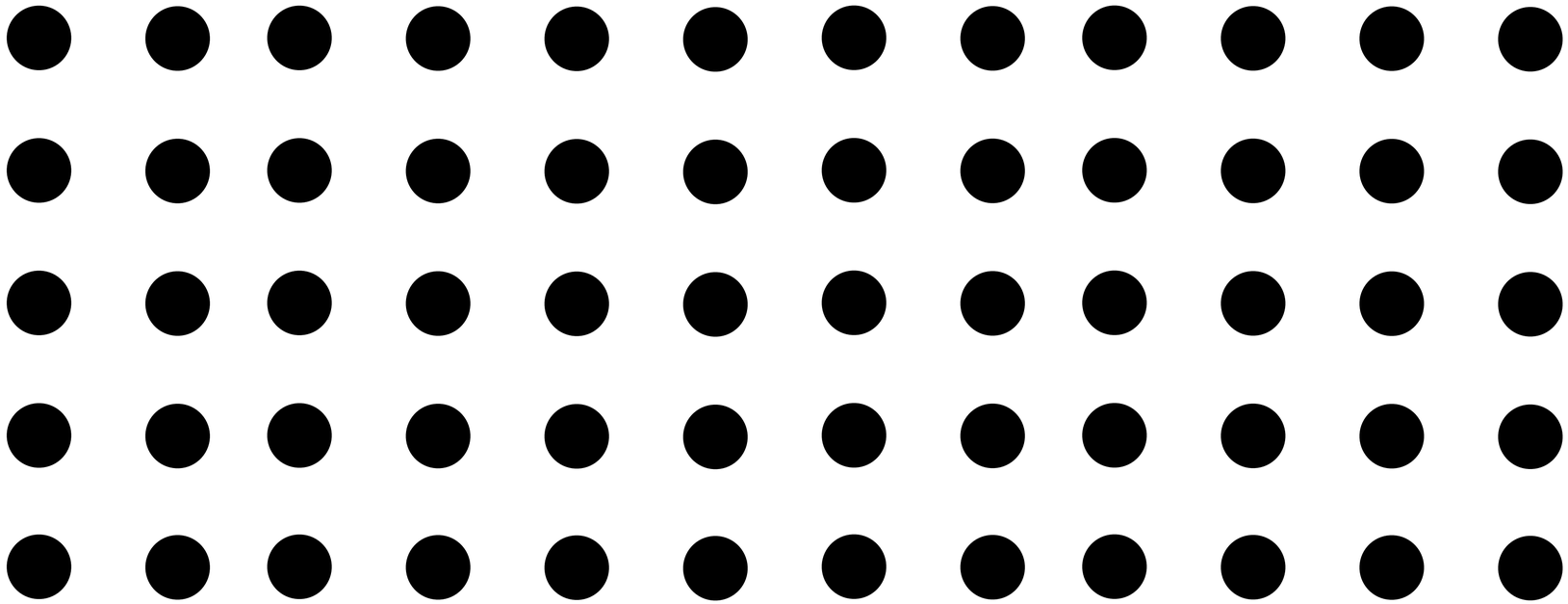
- P_i pulls missing index with probability $O(\lambda/n)$ (improved from naive $O(1/n)$ probability)
- Per round, λ honest parties learn a new signature from P with probability $1 - 2^{-\lambda}$

Alternating Push/Pull Paradigm



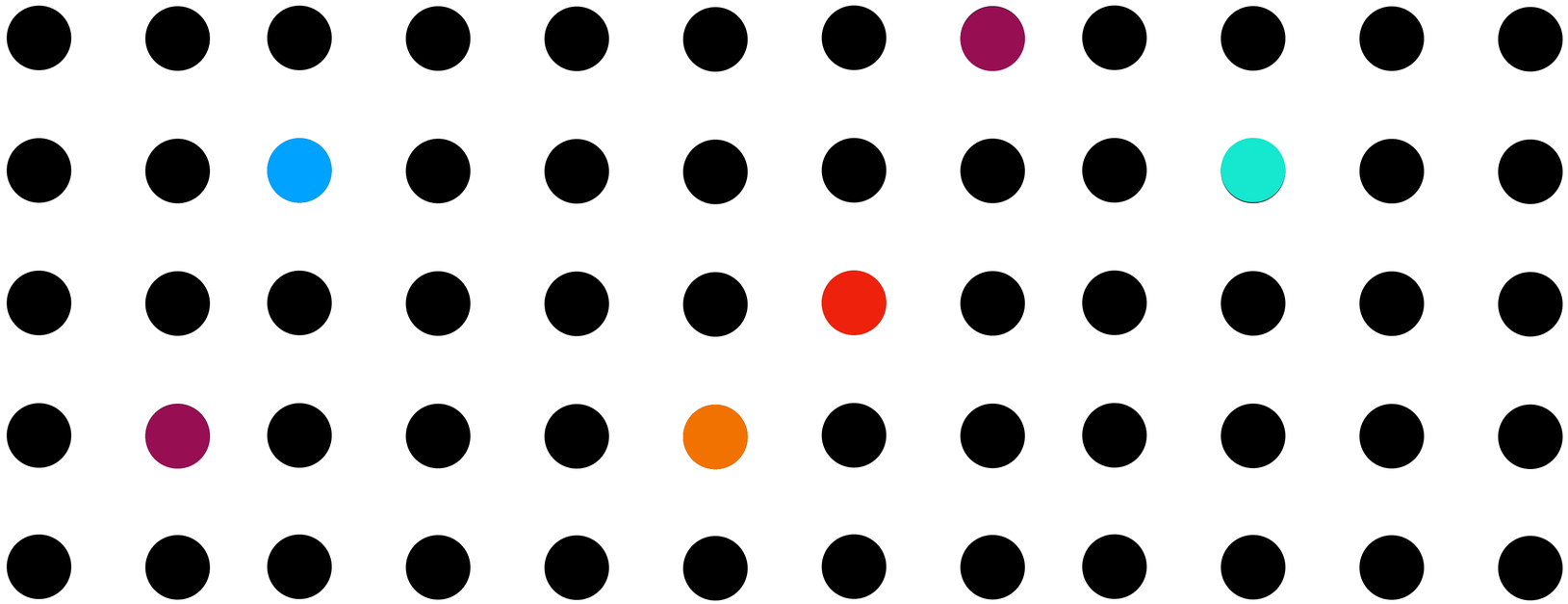
Alternating Push/Pull Paradigm



P

- Perform pulling step on certificate C

Alternating Push/Pull Paradigm



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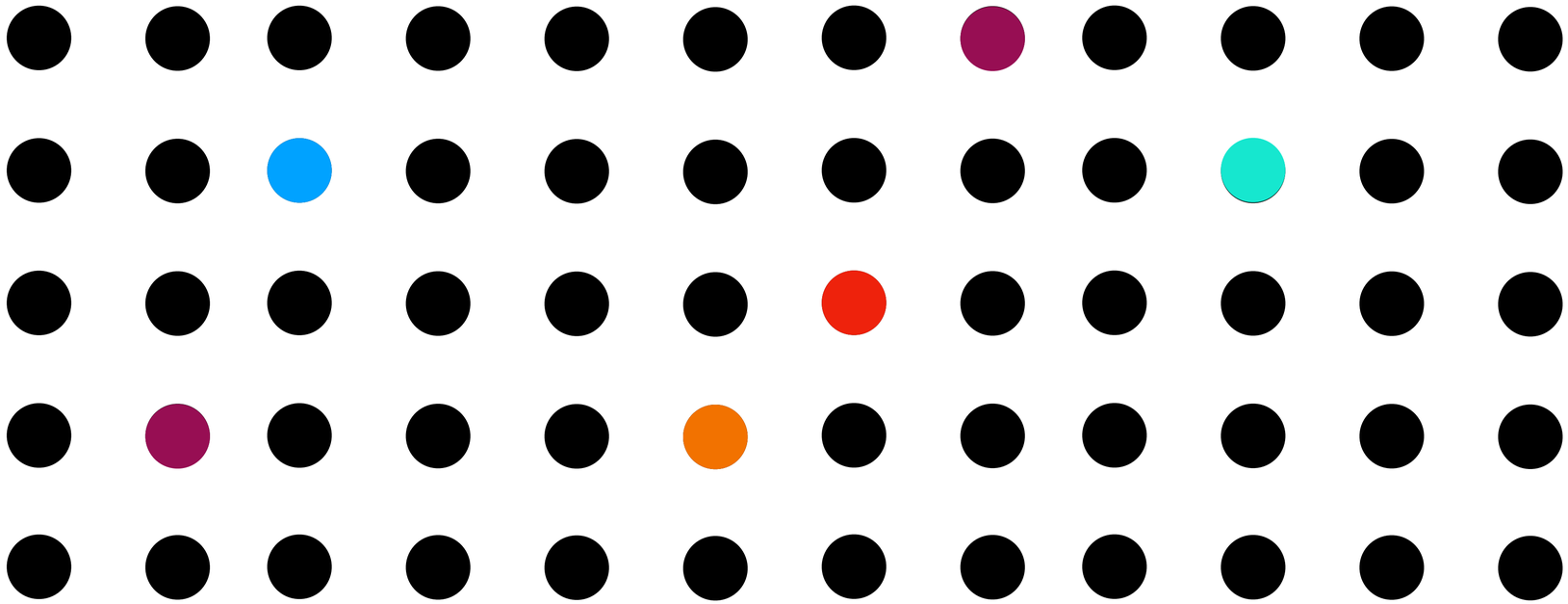

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  - Initially, ensures that $O(n)$ parties each learn $O(1)$ signatures from C with probability $1 - 2^{-n} \geq 1 - 2^{-\lambda}$

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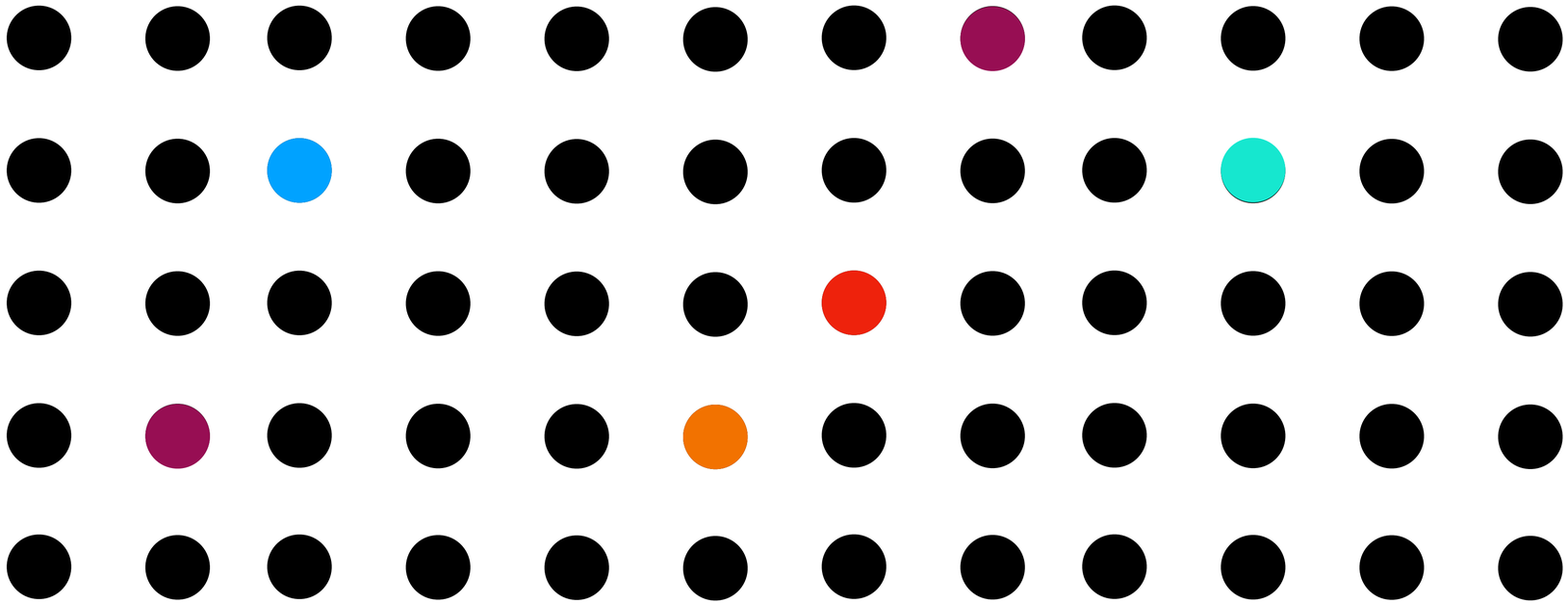

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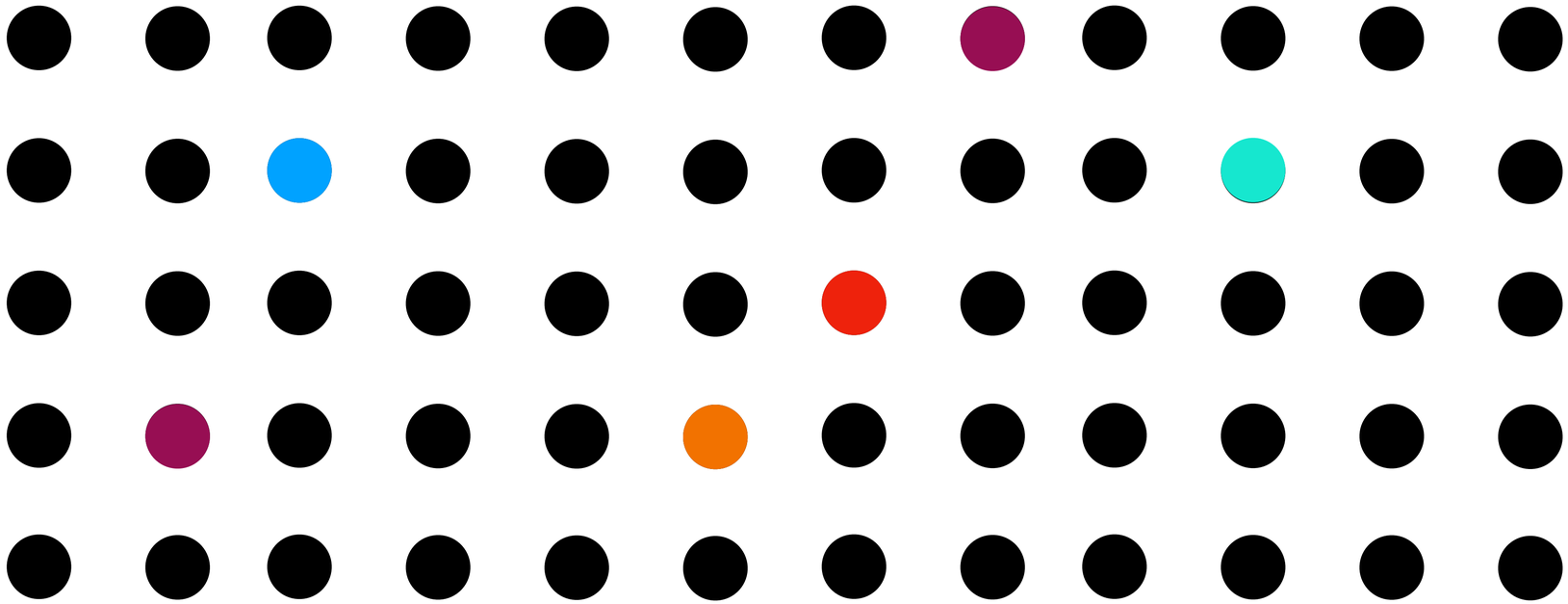

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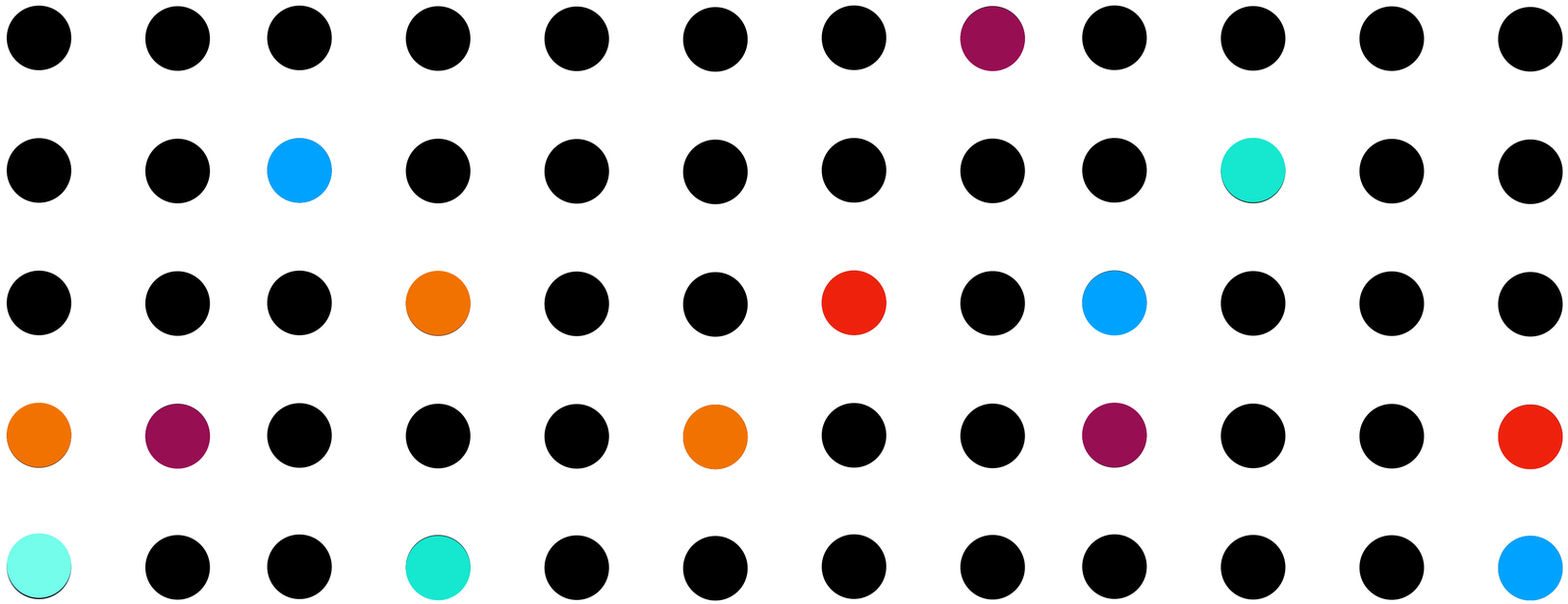
Alternating Push/Pull Paradigm



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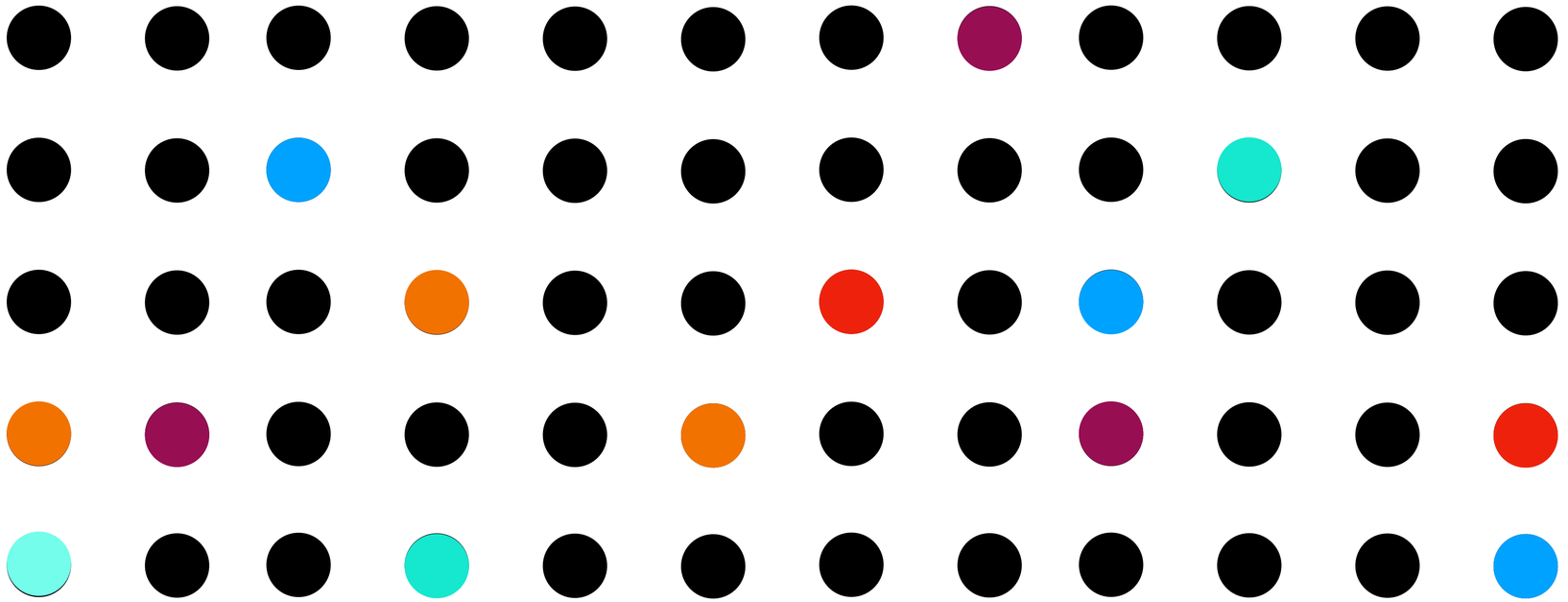

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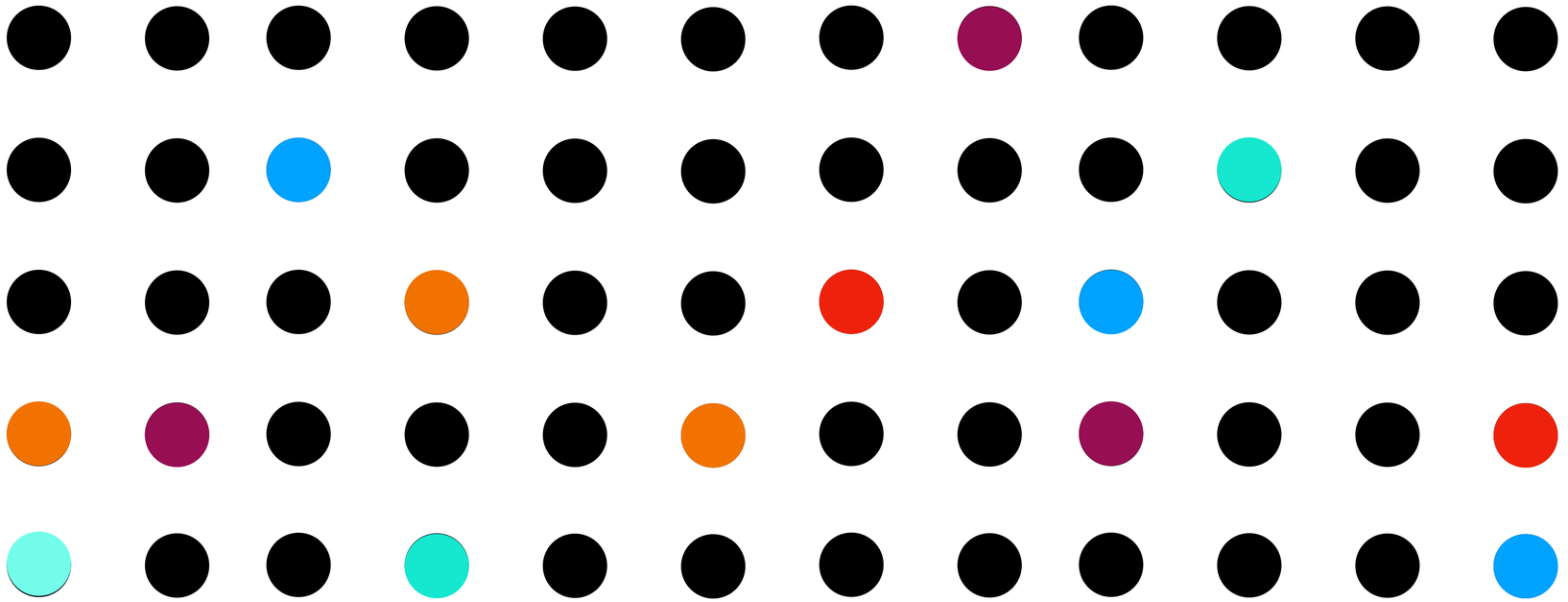

Repeat $\log(n)$ times

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    - Asynchronous common subset in $O(n^2 \cdot \ell)$ in the DY model?