Incrementally Verifiable Computation for NP from Standard Assumptions

based on joint work with






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Motivation: "Multi-Generation" Computation

[Valiant '08]

(Nondeterministic) computation $\mathscr{M}$

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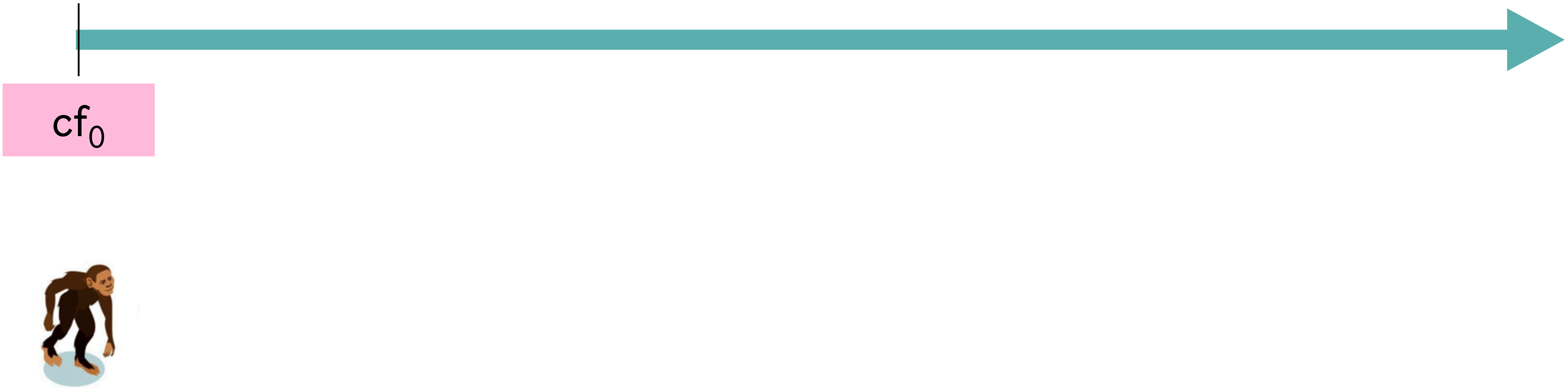

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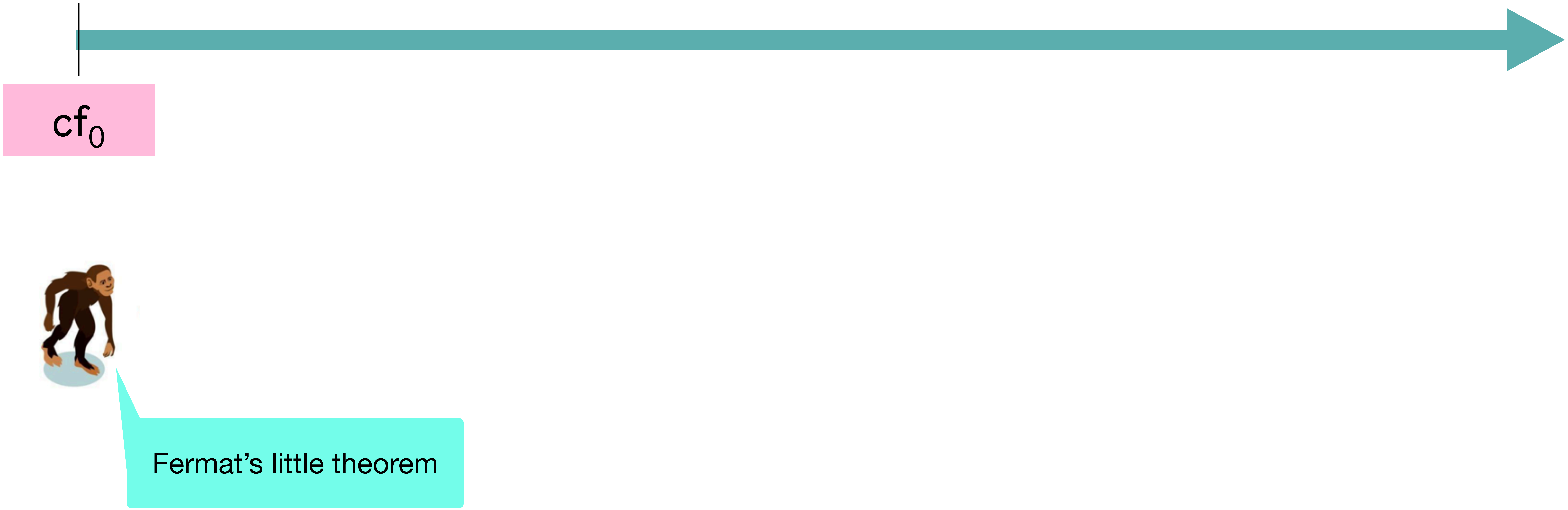

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(Nondeterministic) computation $\mathscr{M}$

cf_0 cf_1

Fermat’s little theorem

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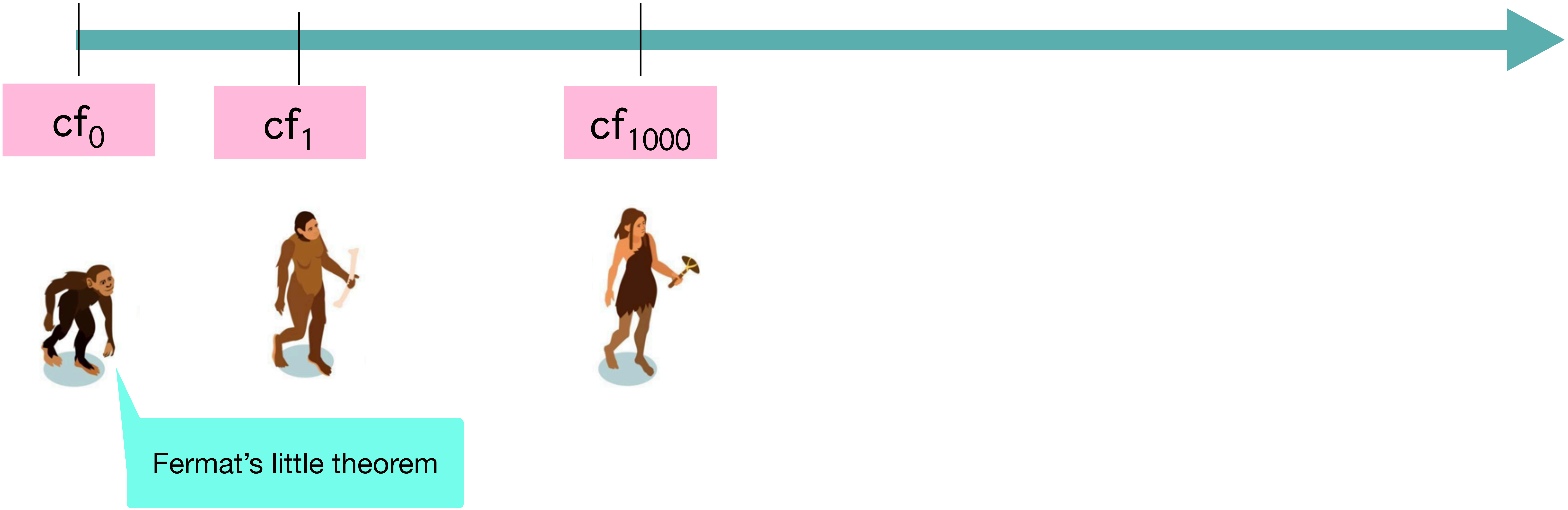

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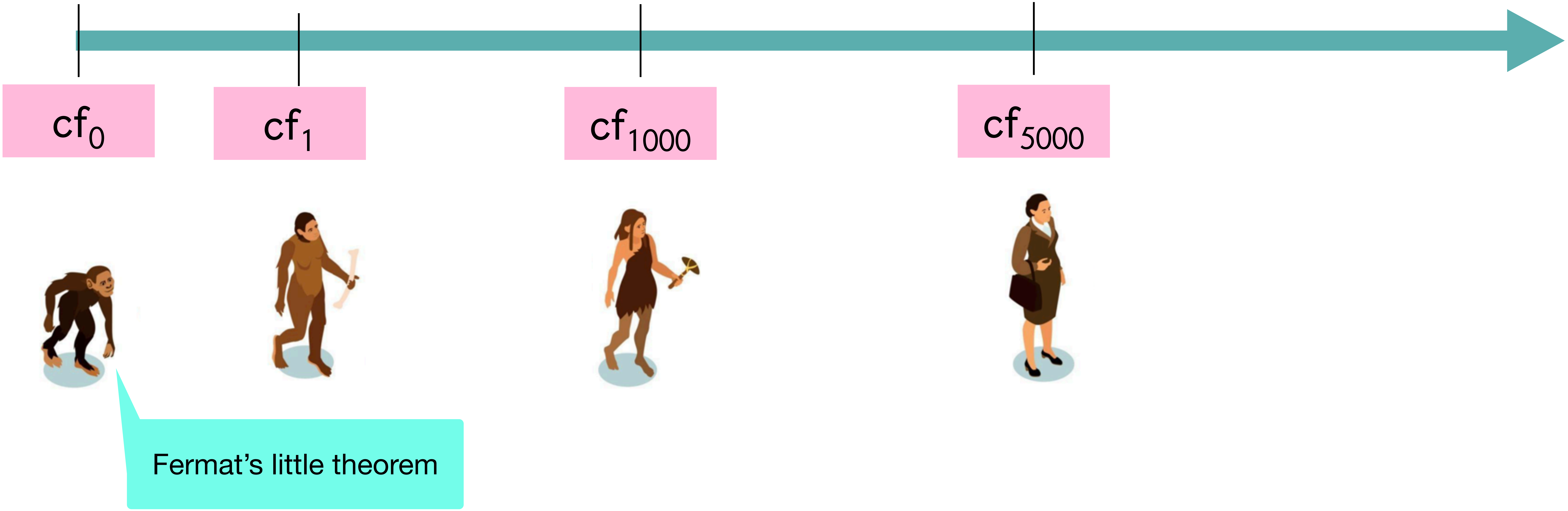

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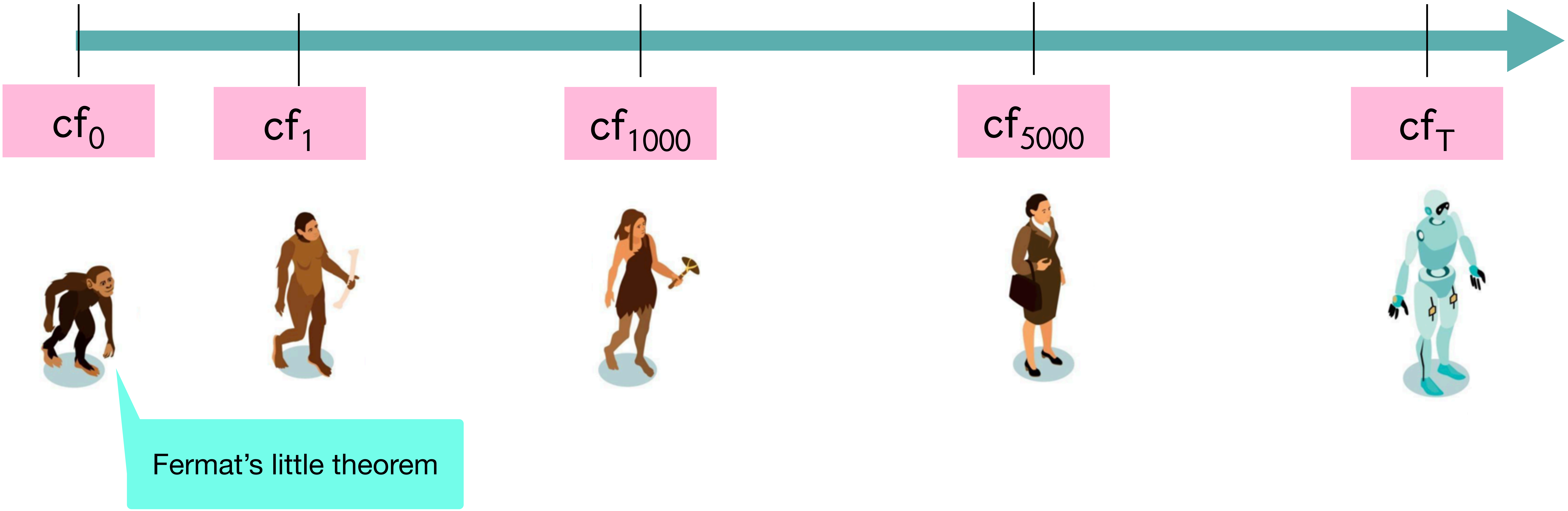

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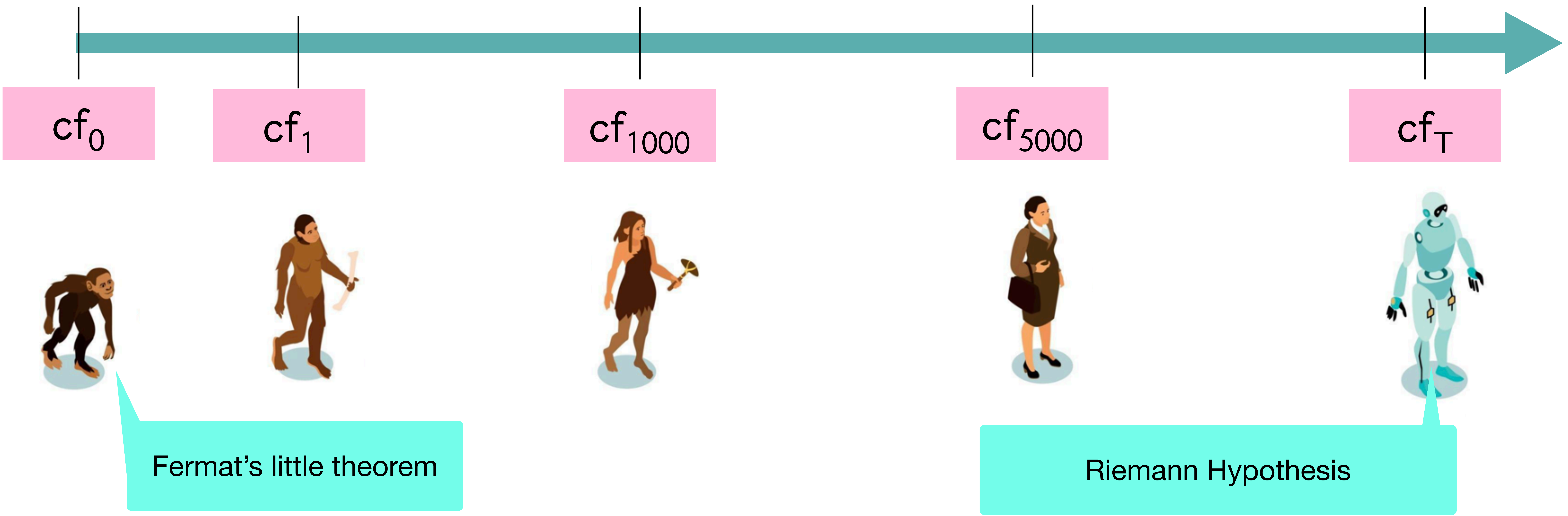

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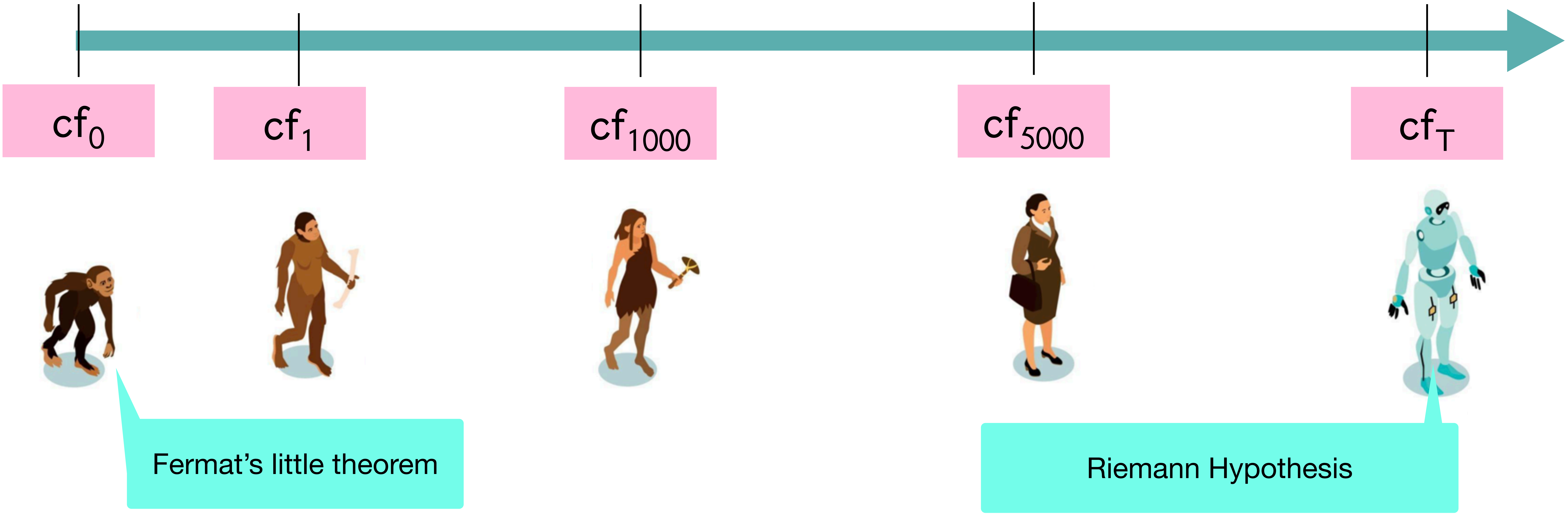

How can we **trust** the validity of the **intermediate** configurations?

Incrementally Verifiable Computation for NP

[Valiant ’08]

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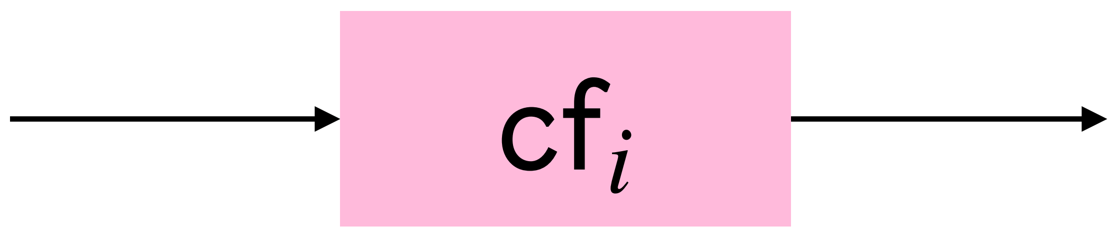

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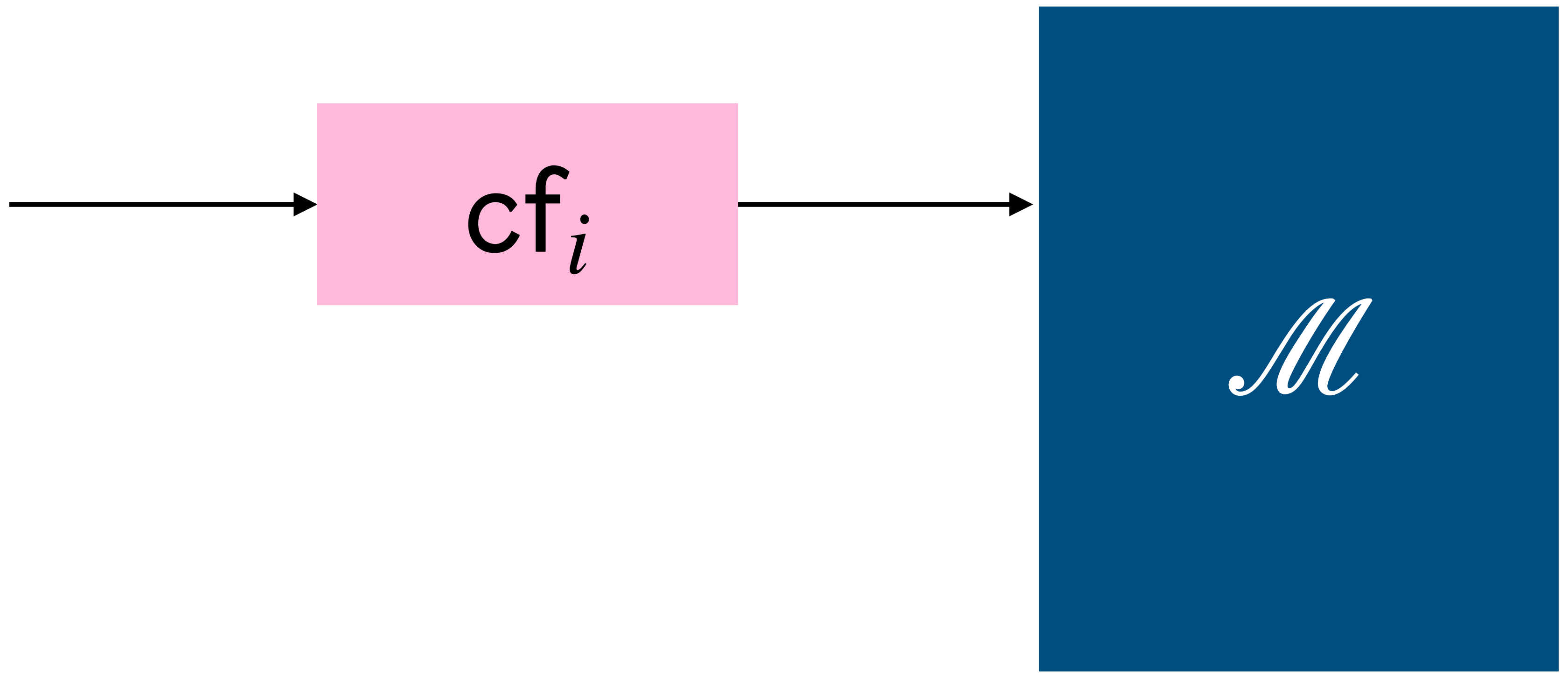

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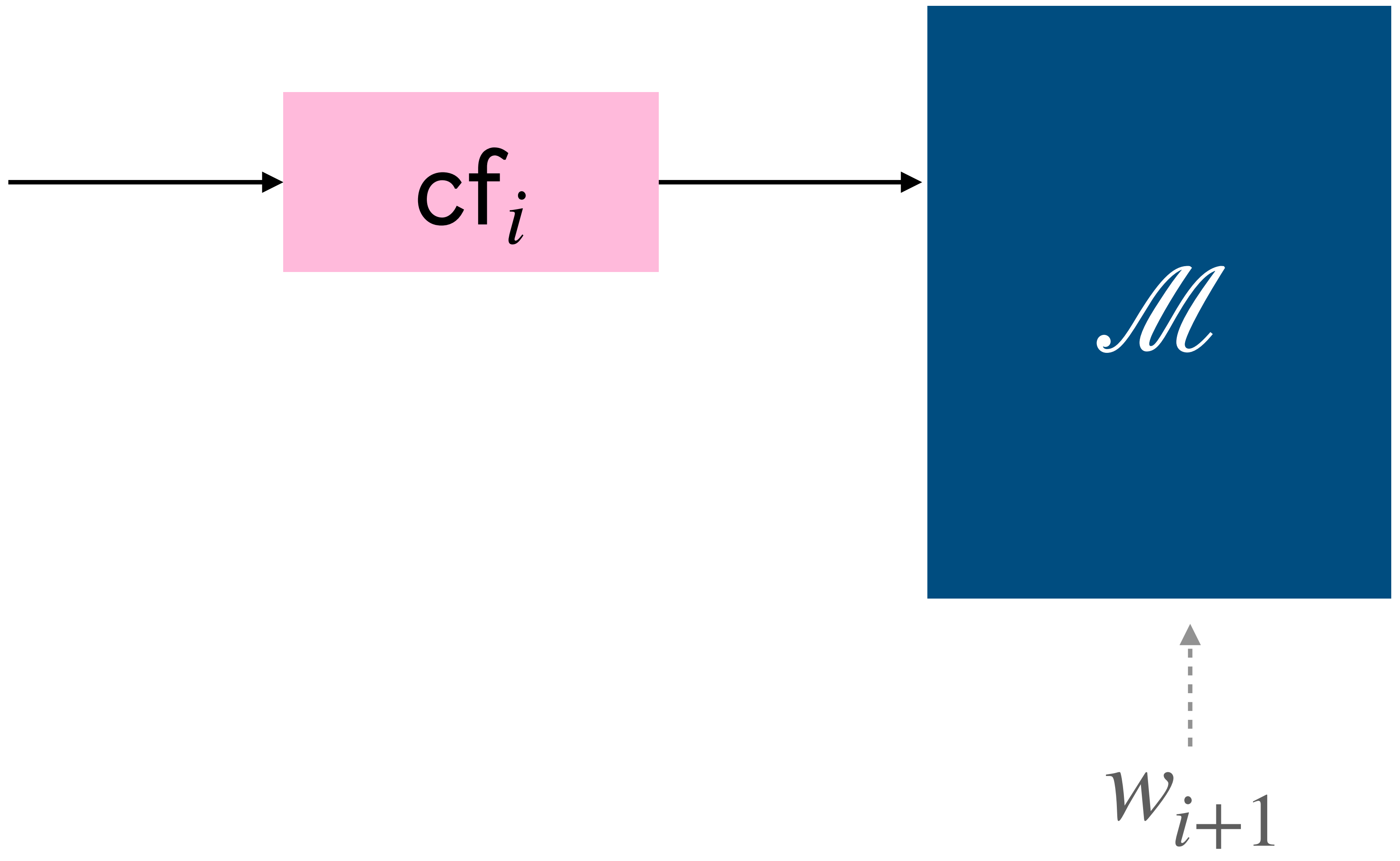

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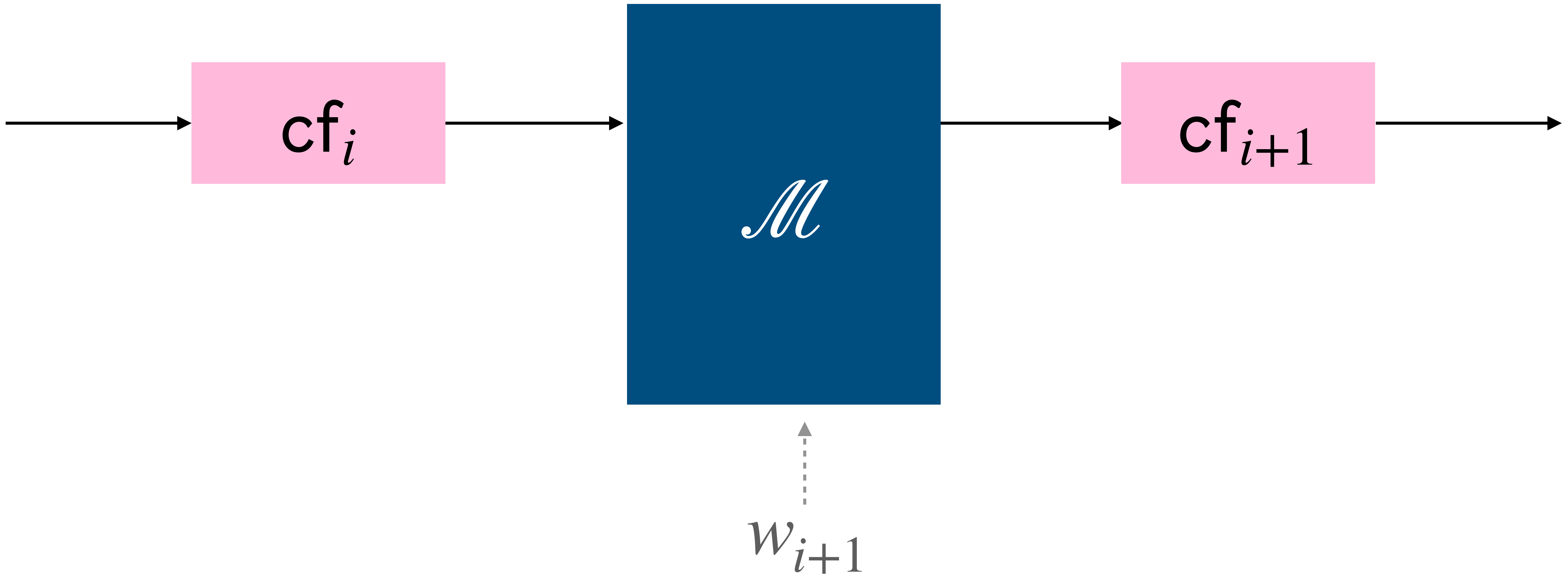

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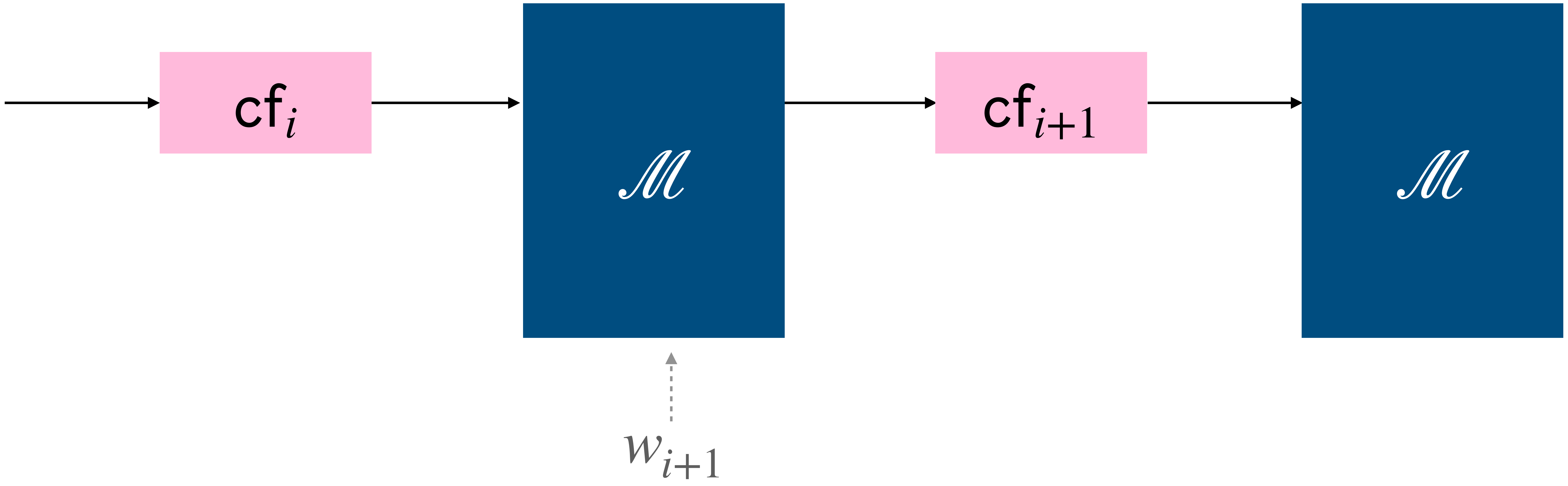

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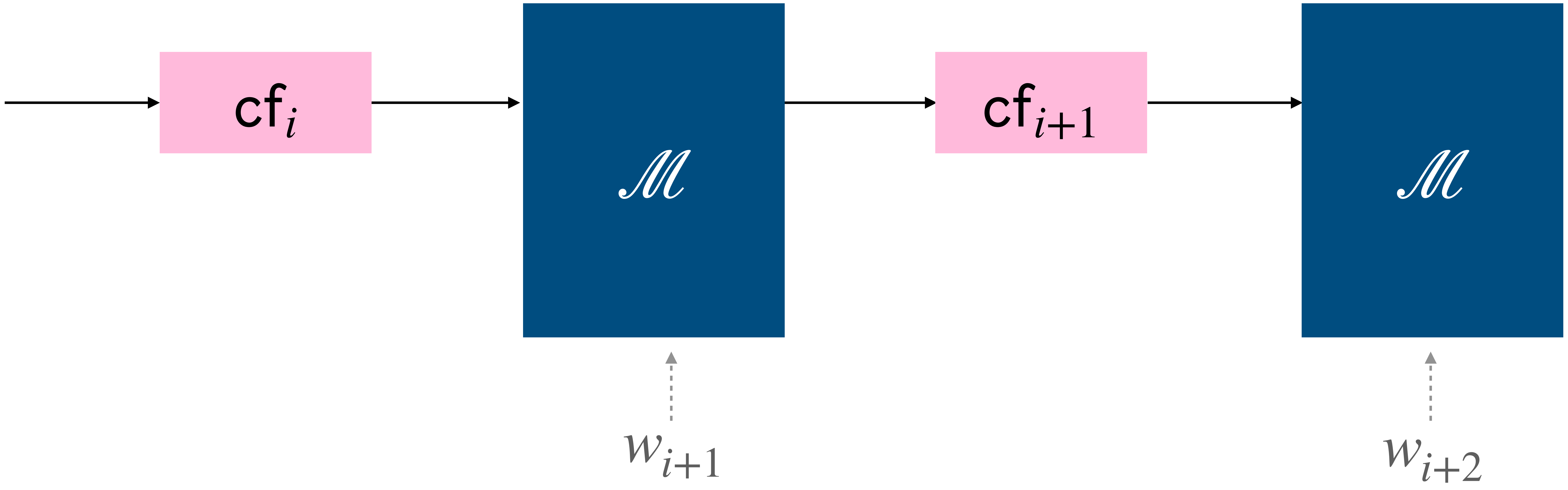

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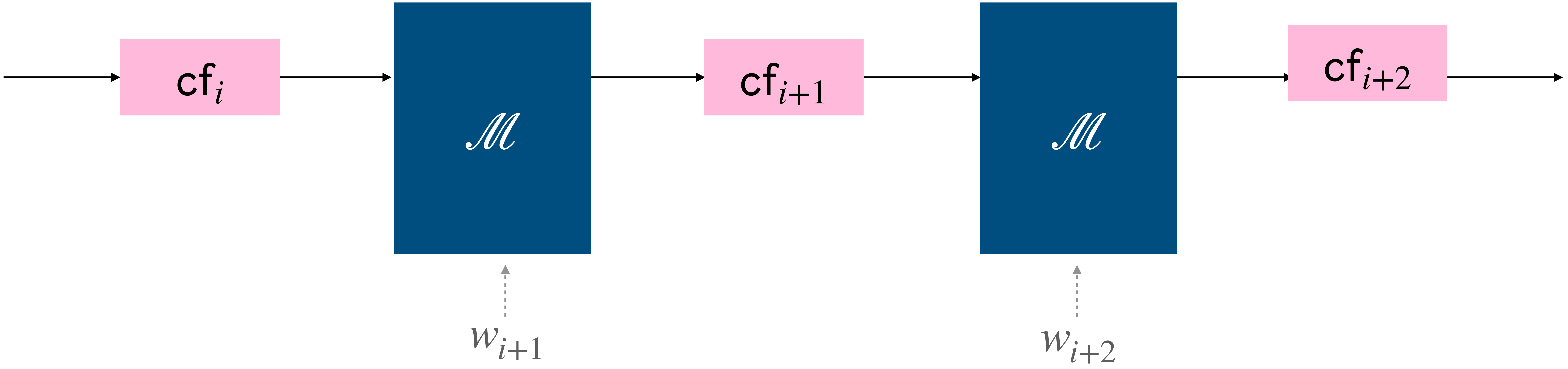

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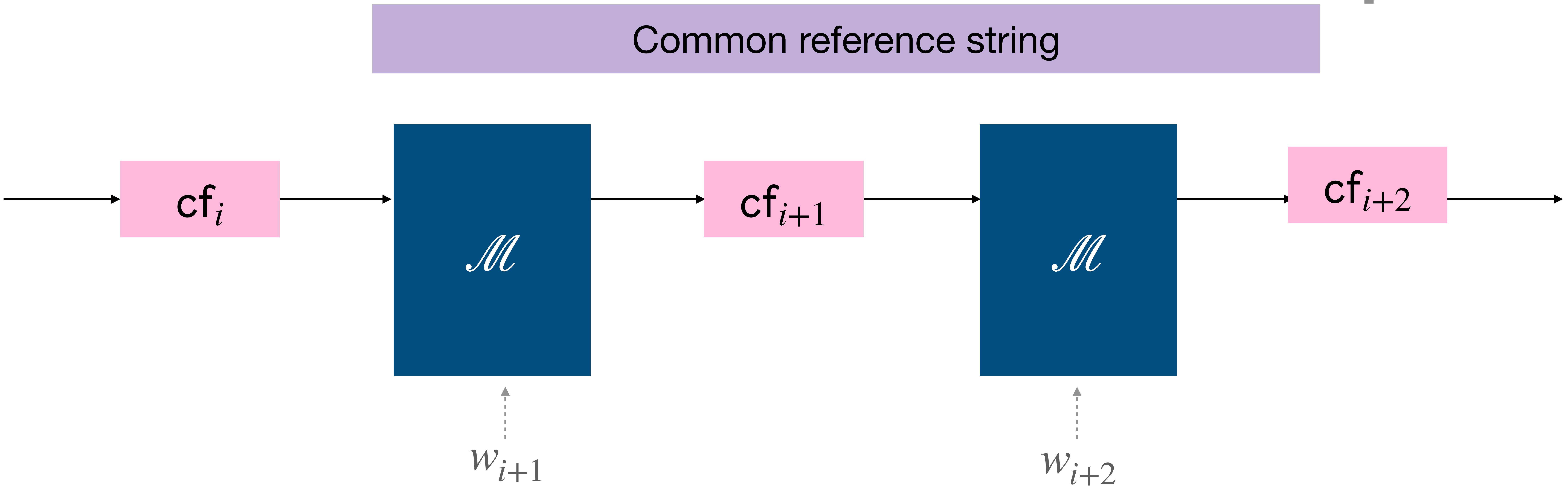

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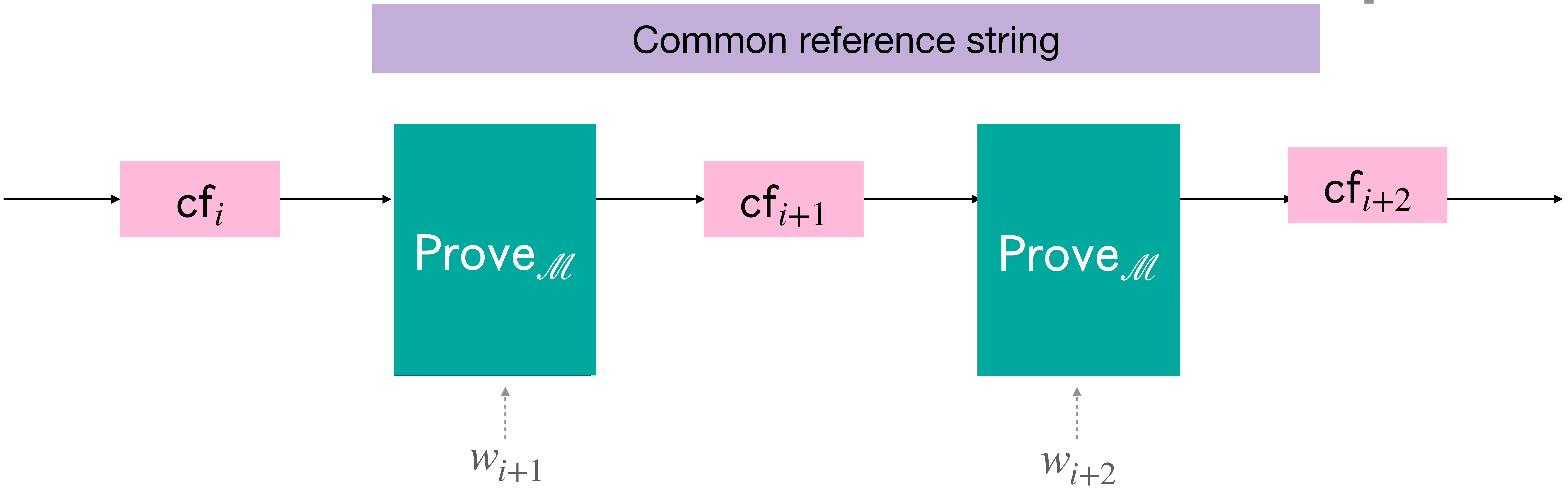

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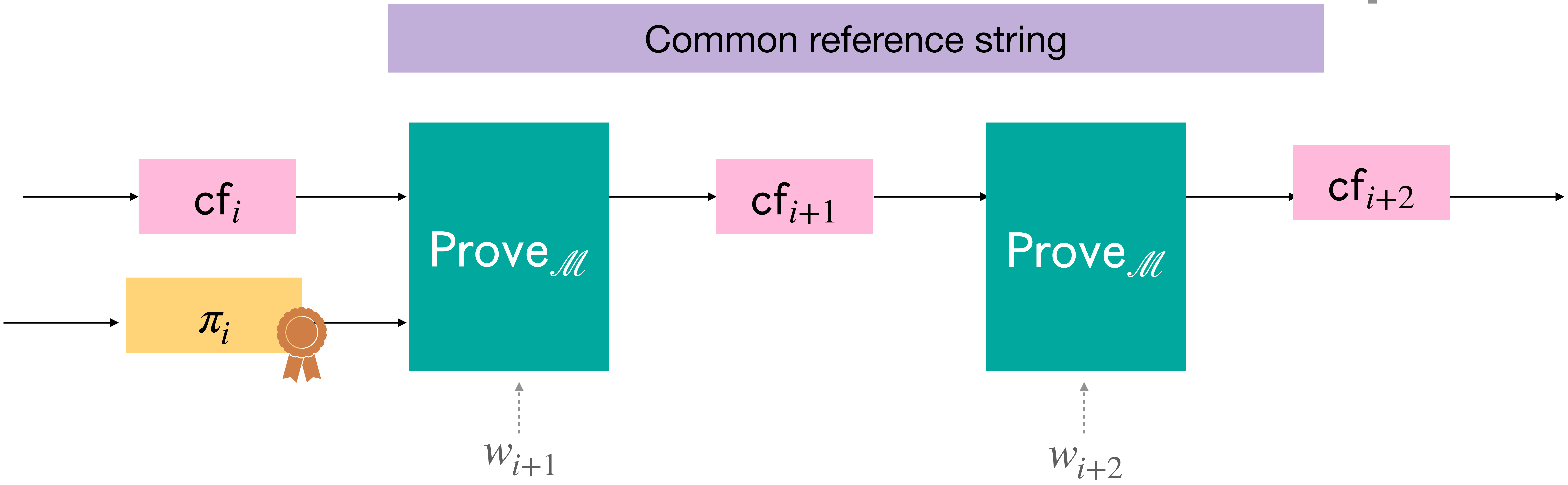

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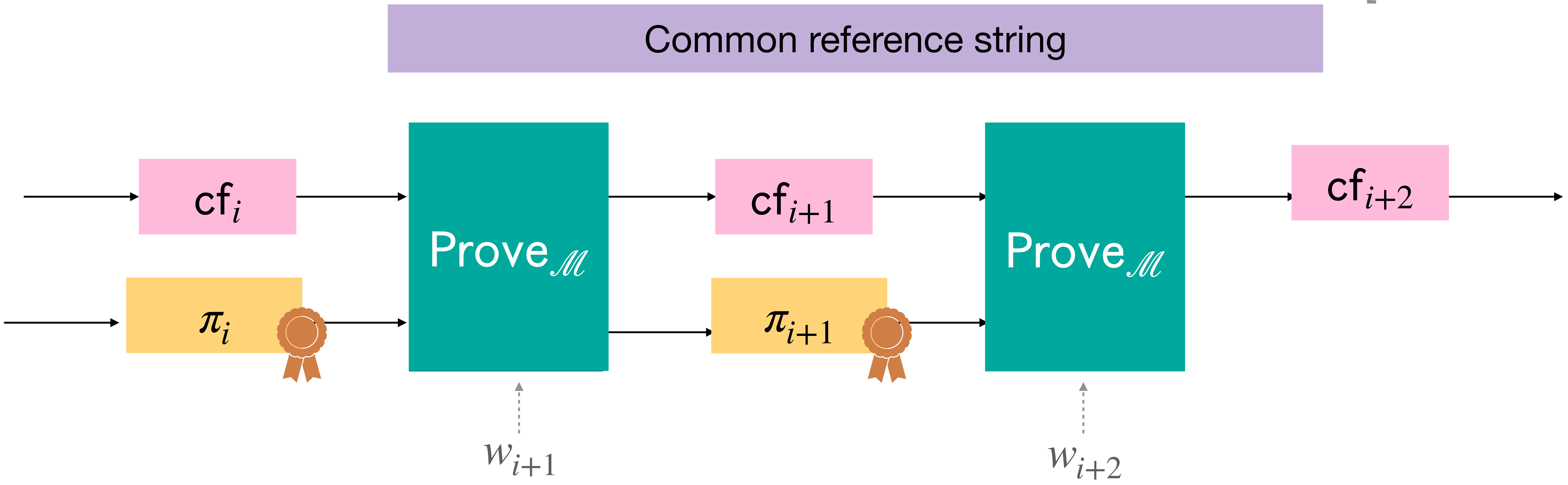

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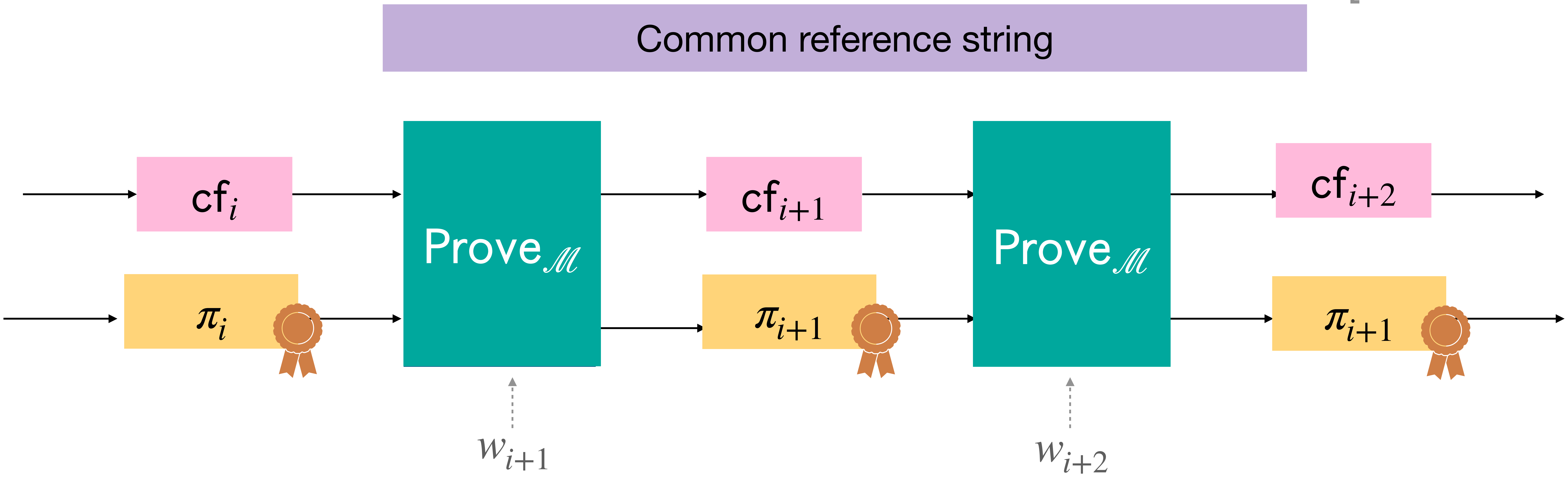

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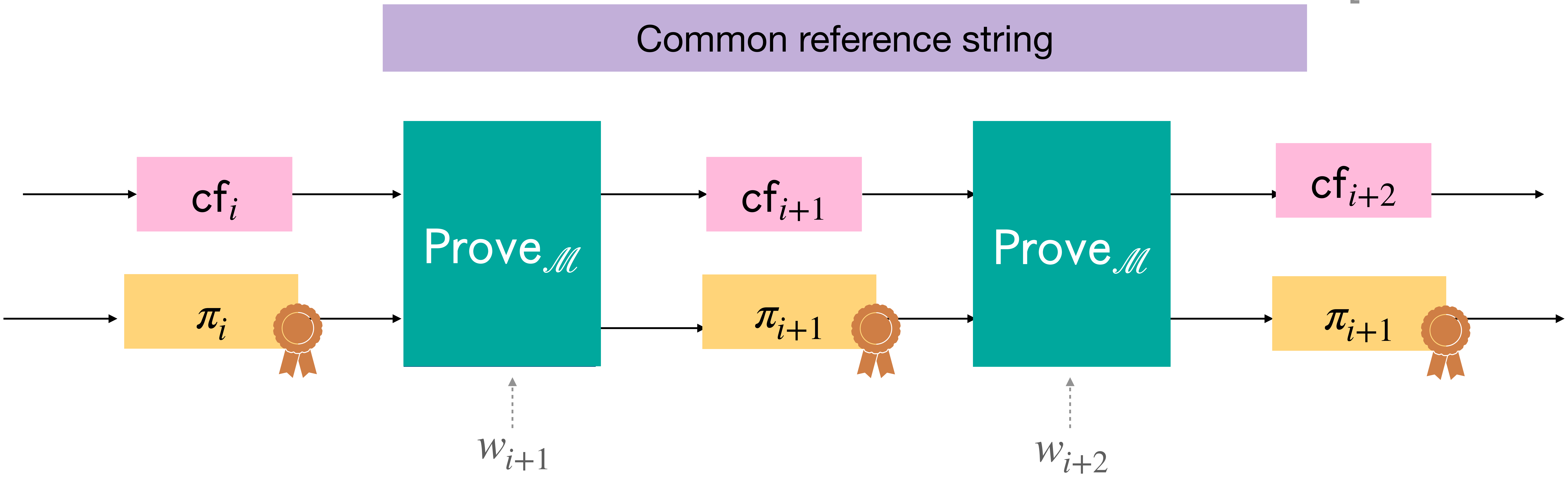

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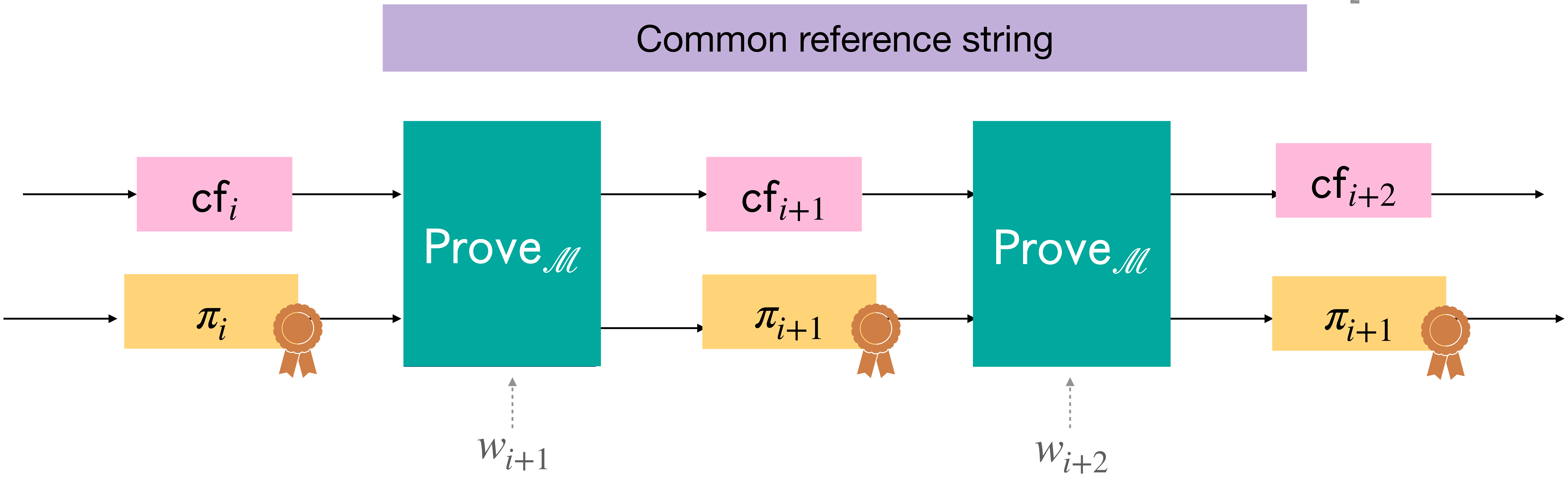

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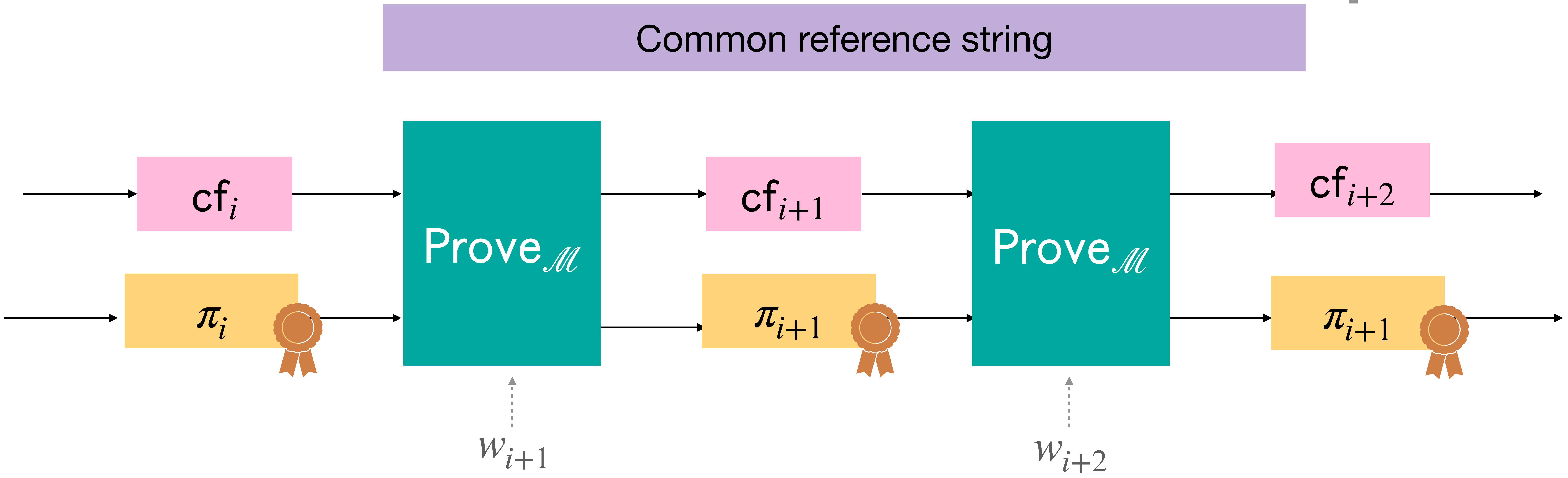

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*We will not consider knowledge soundness since we are focusing on **standard assumptions.**

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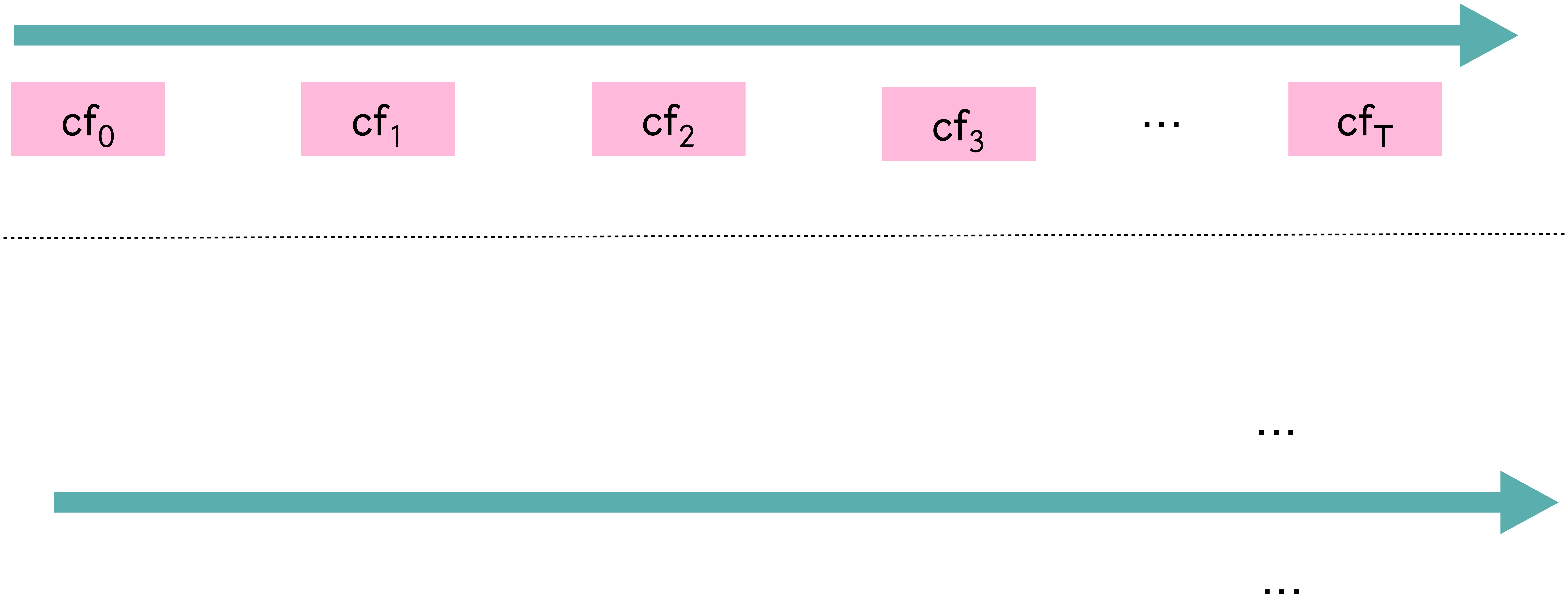

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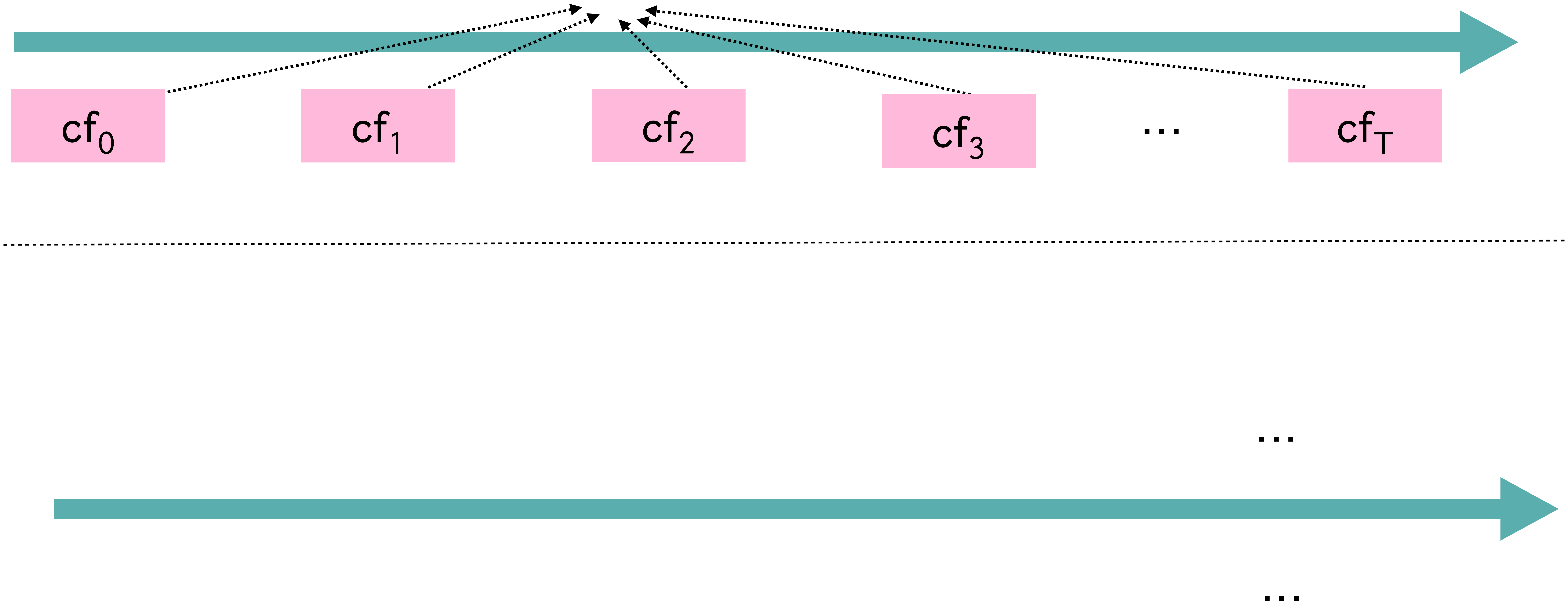

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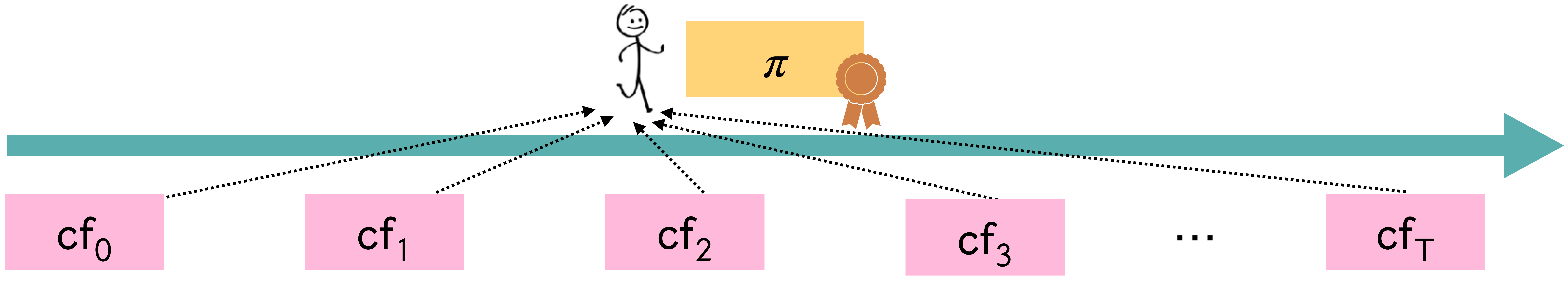

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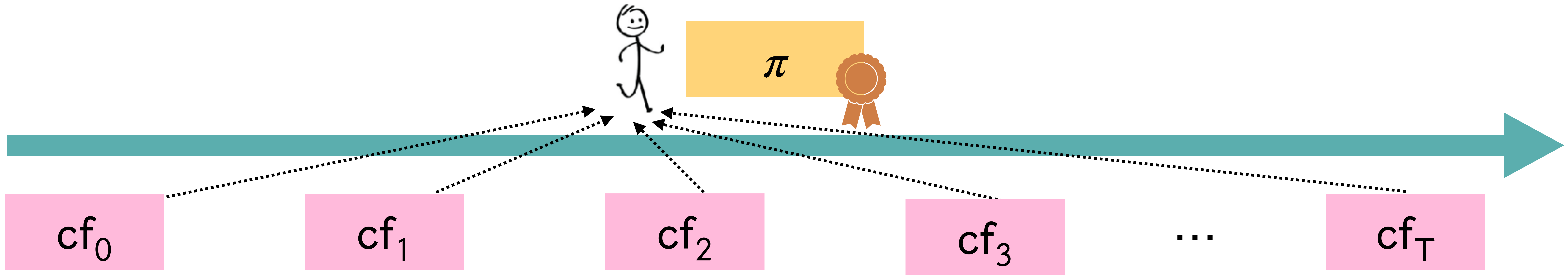

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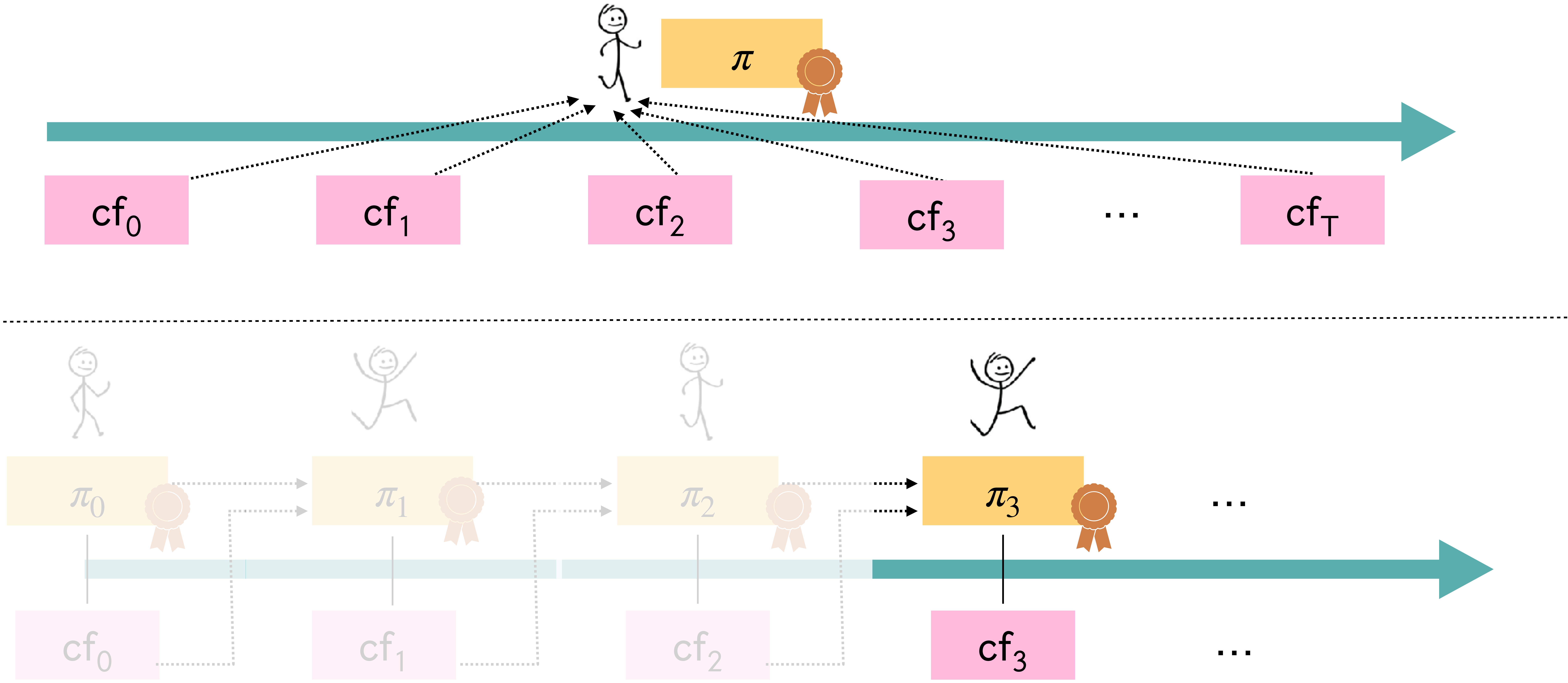

Application: Verifiable Shuffling

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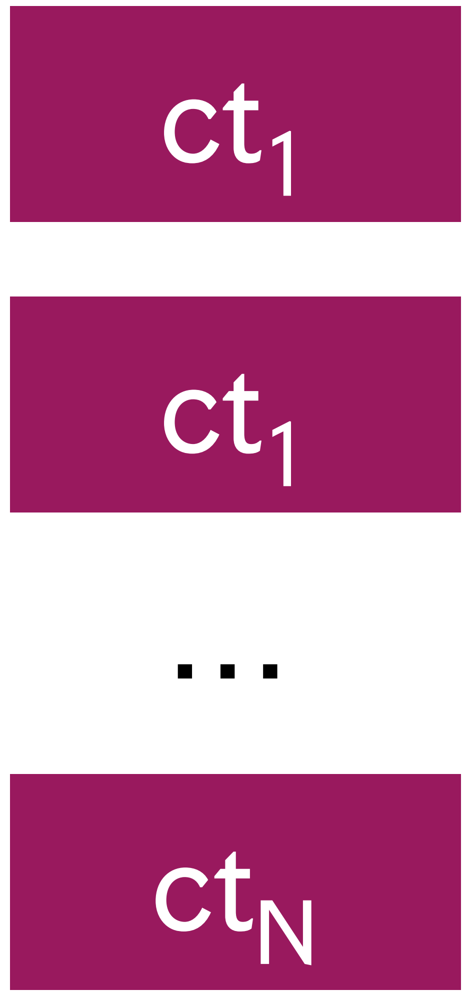

Encrypted votes!

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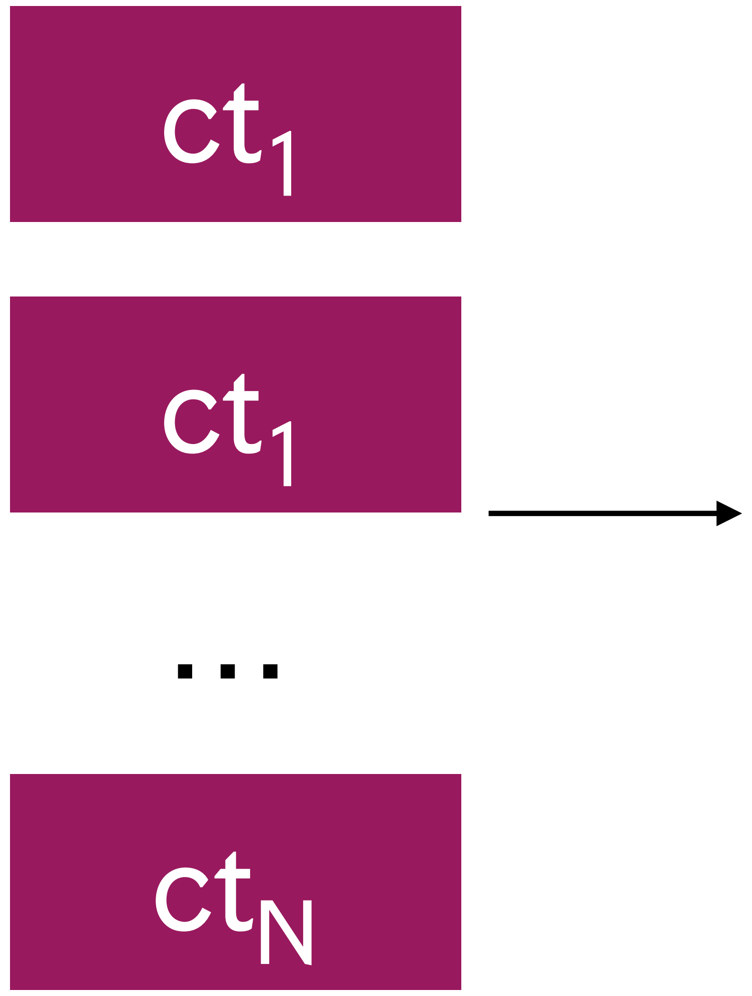

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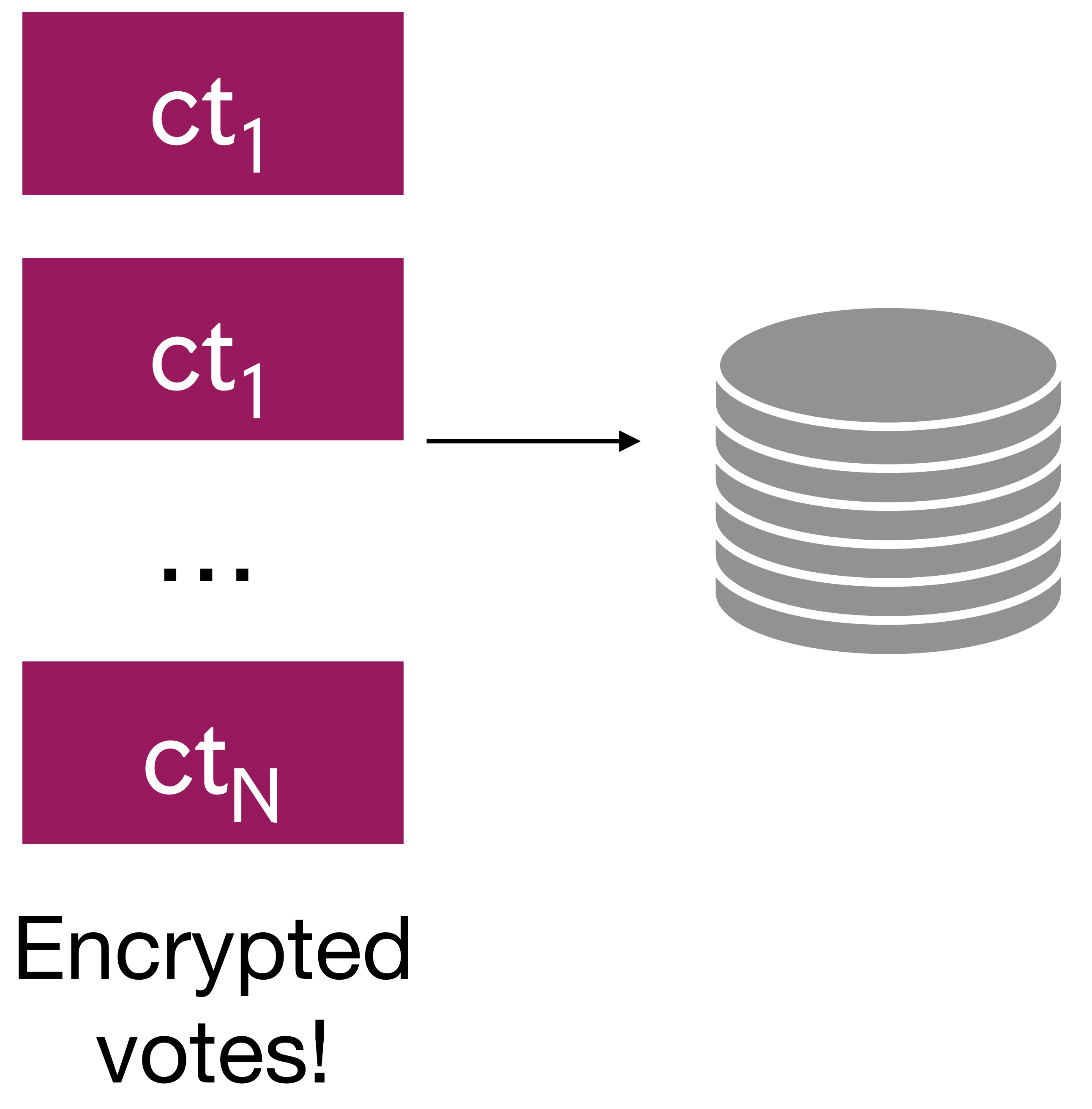

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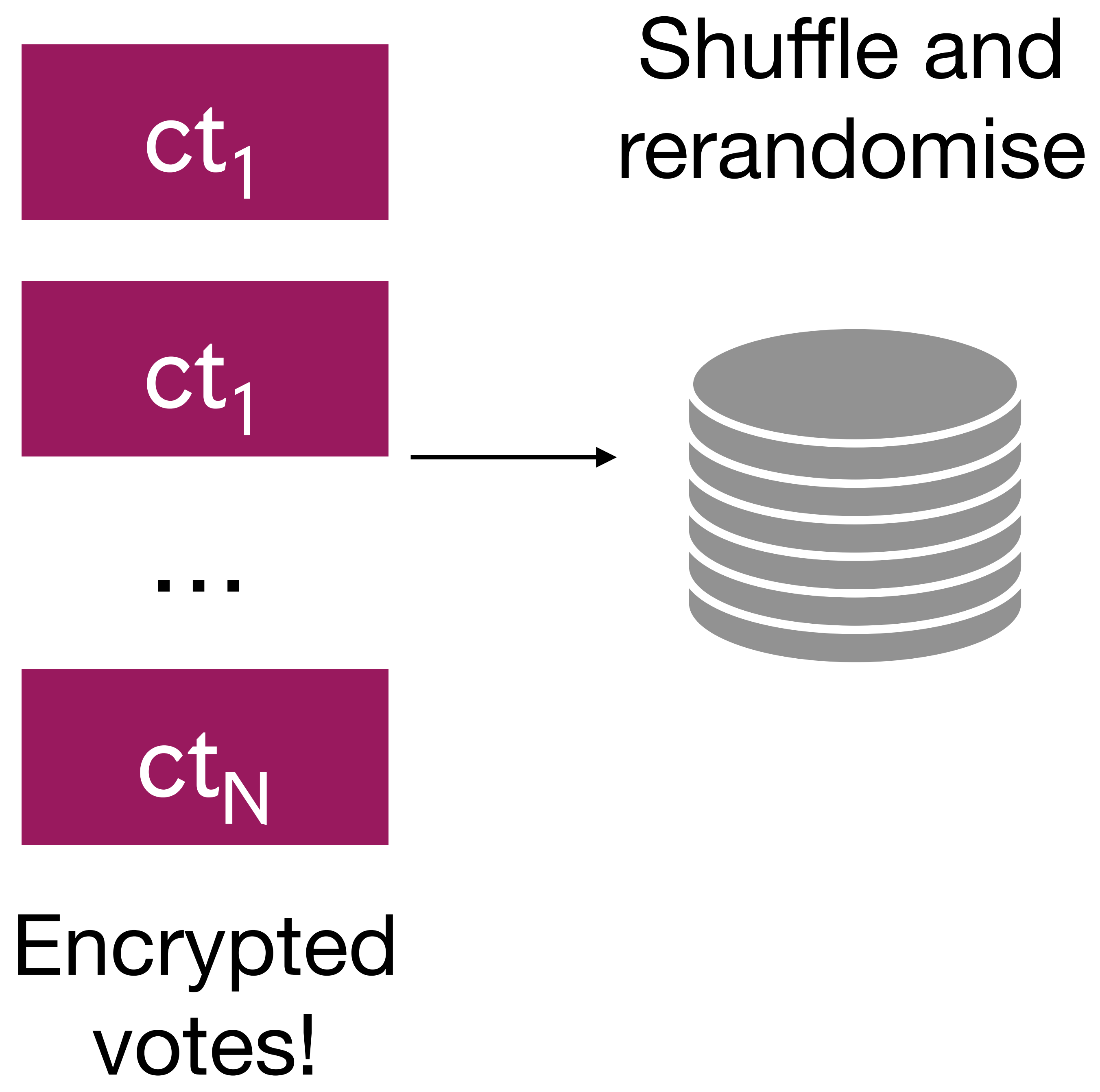

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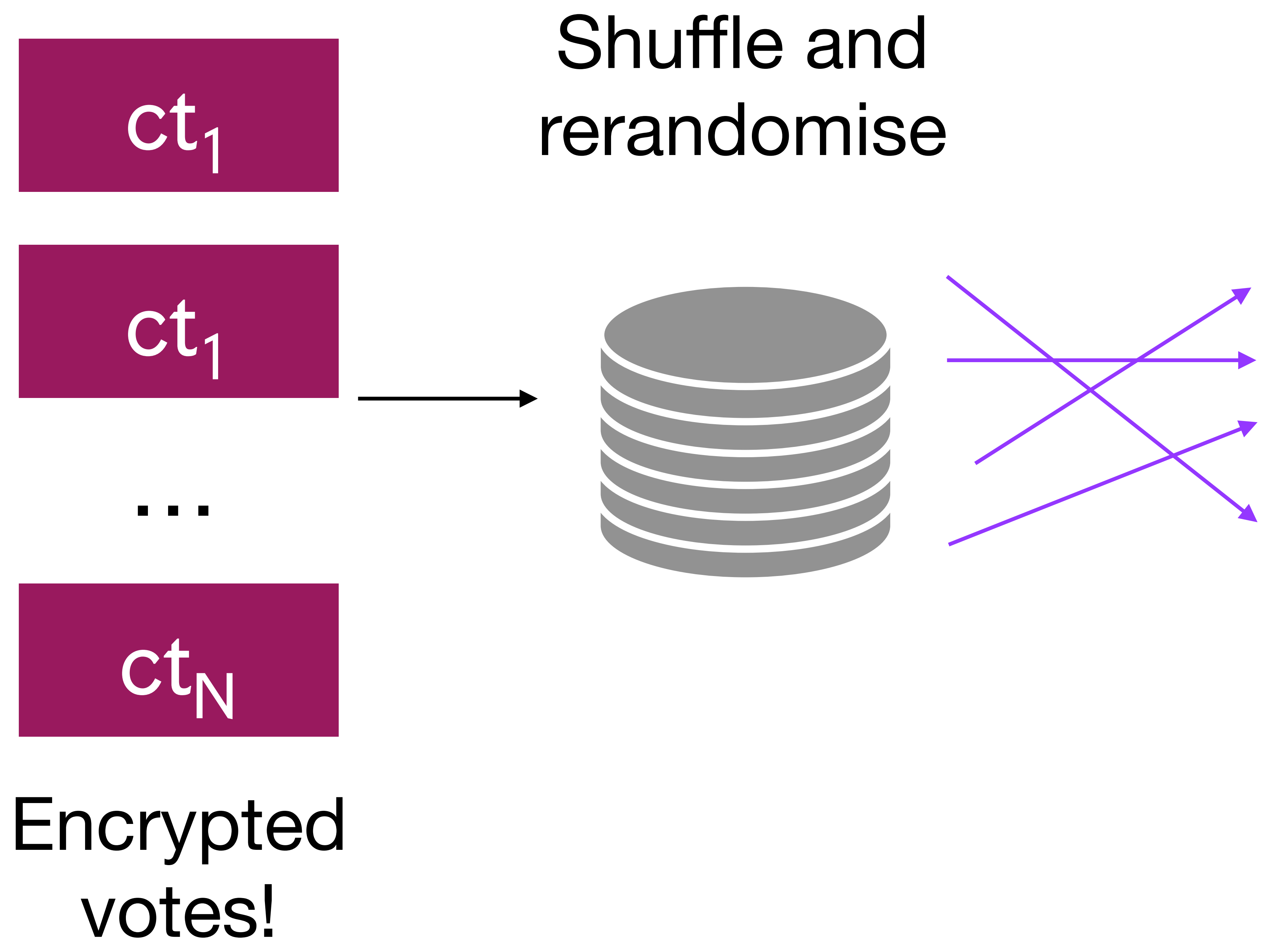

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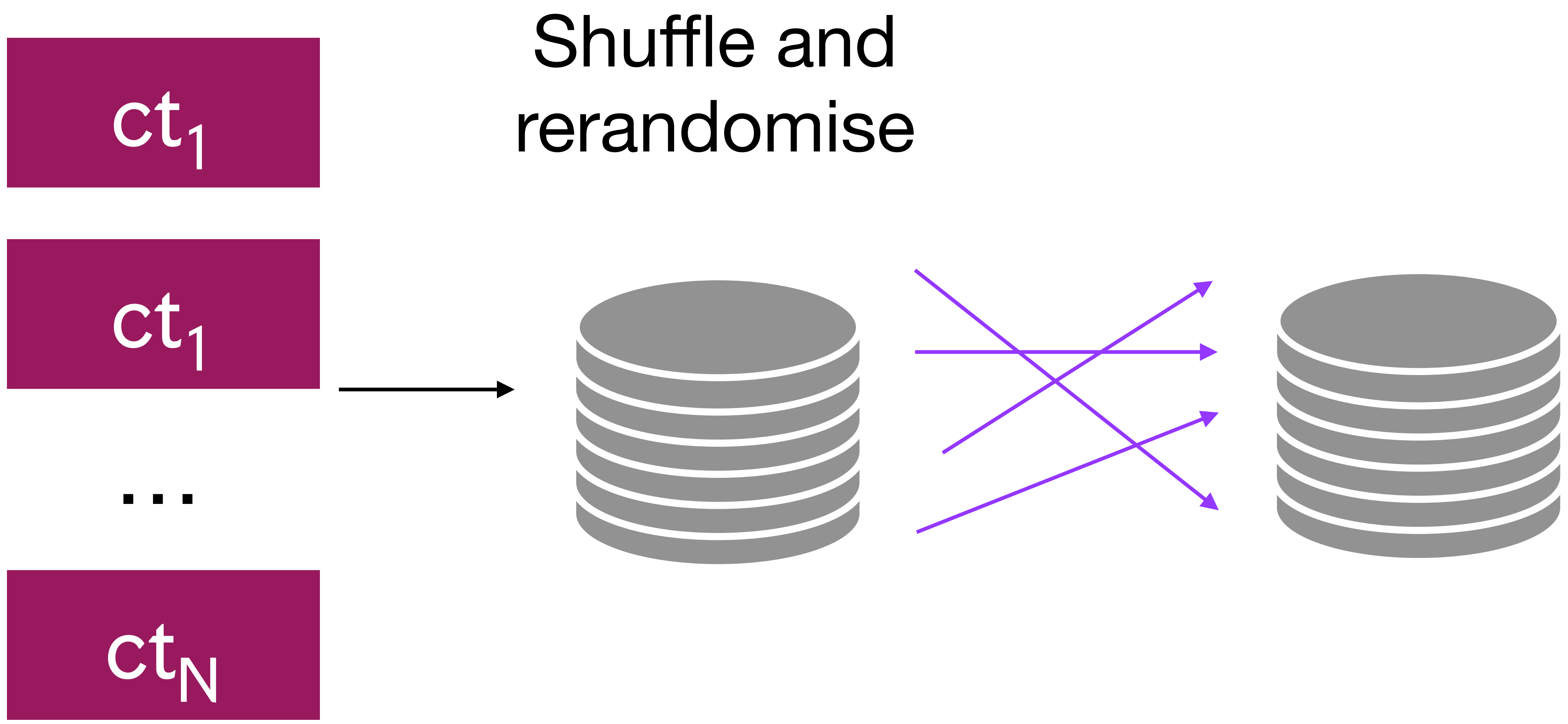

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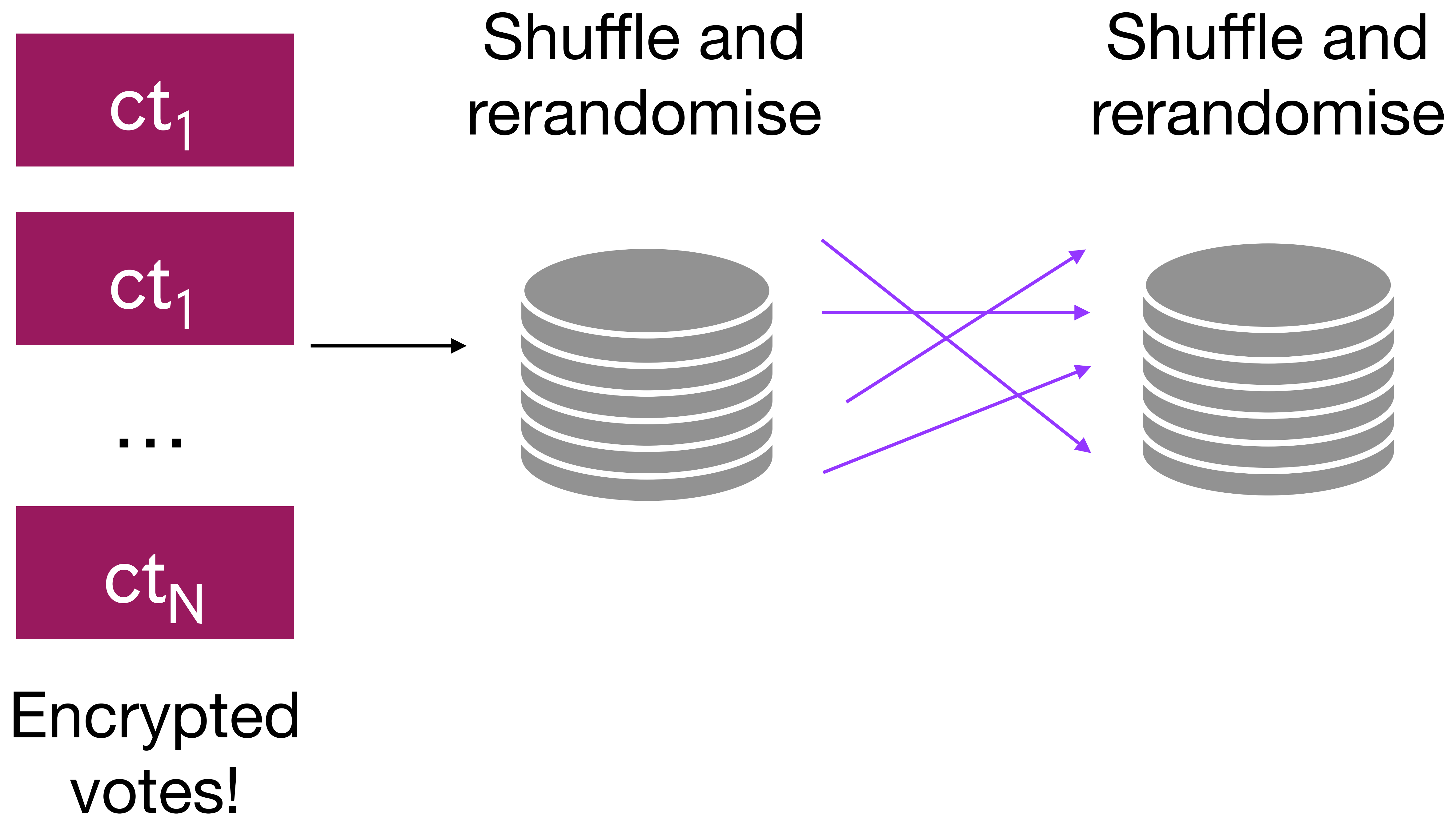

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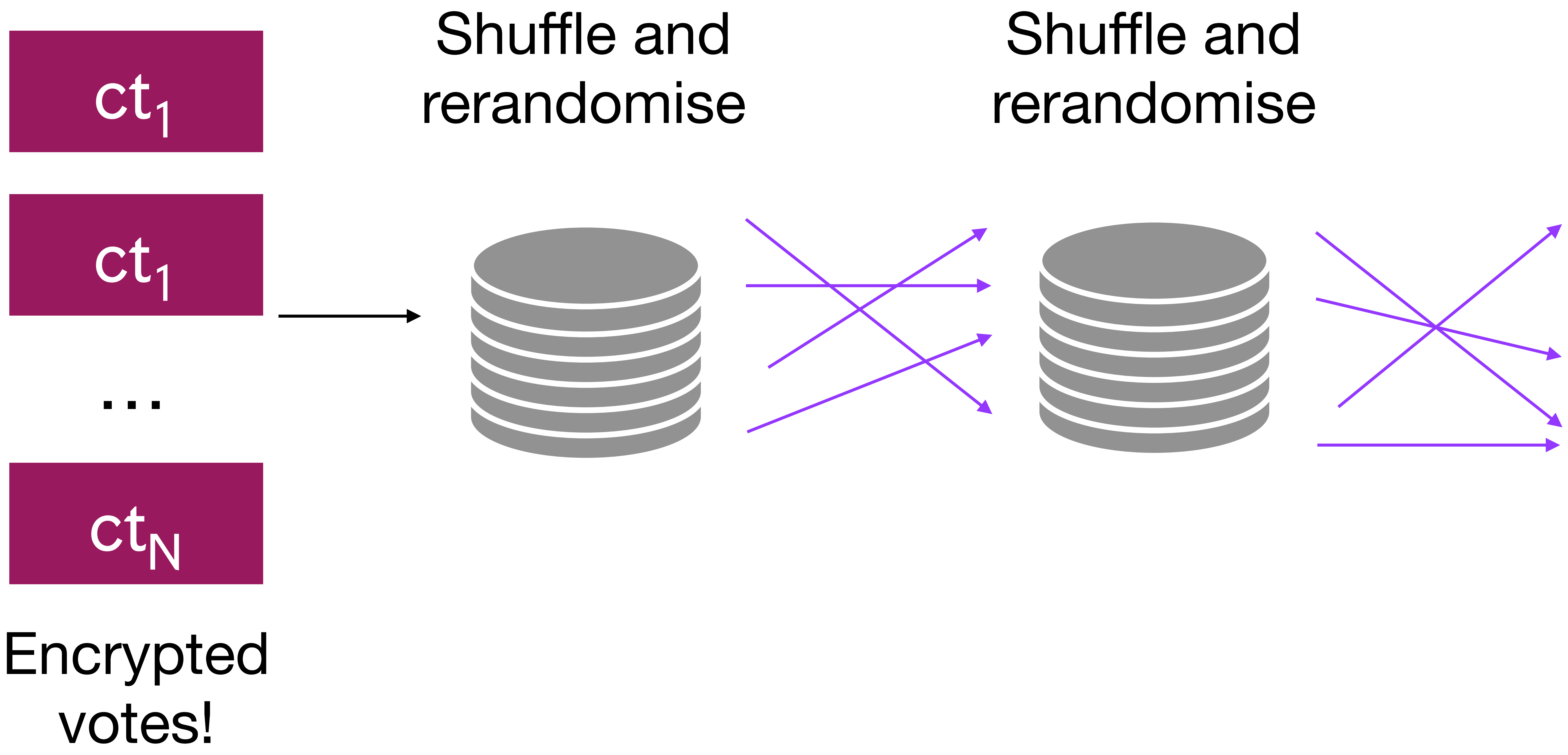

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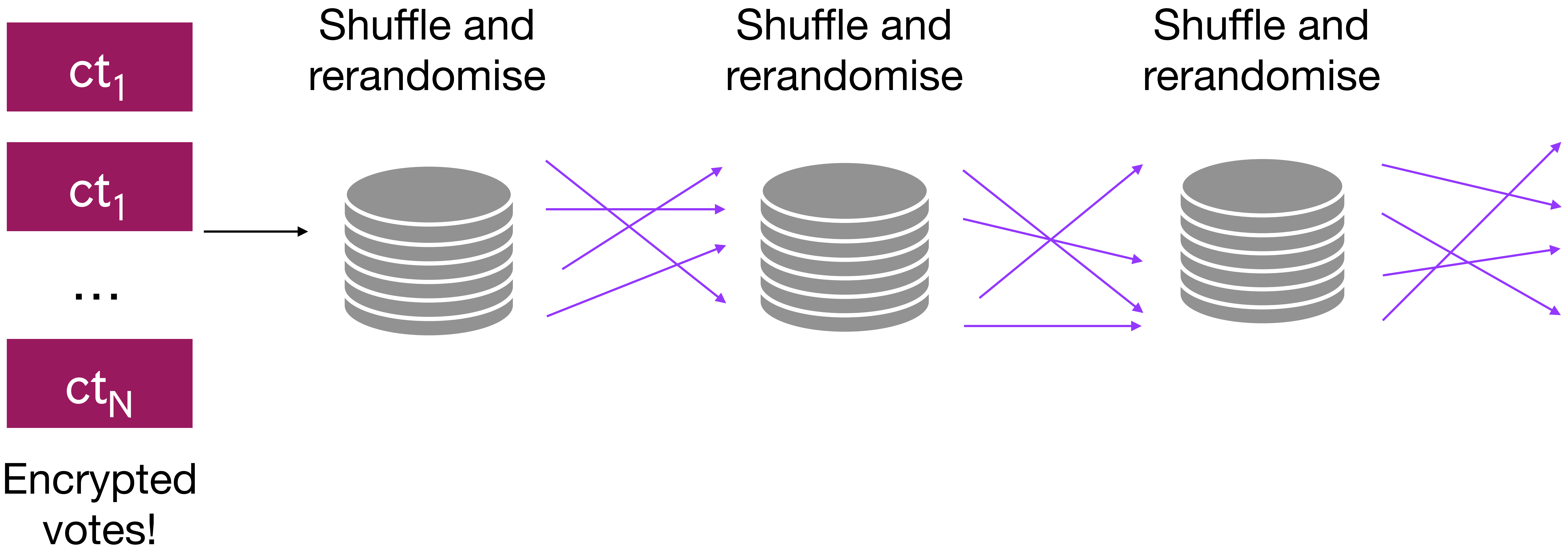

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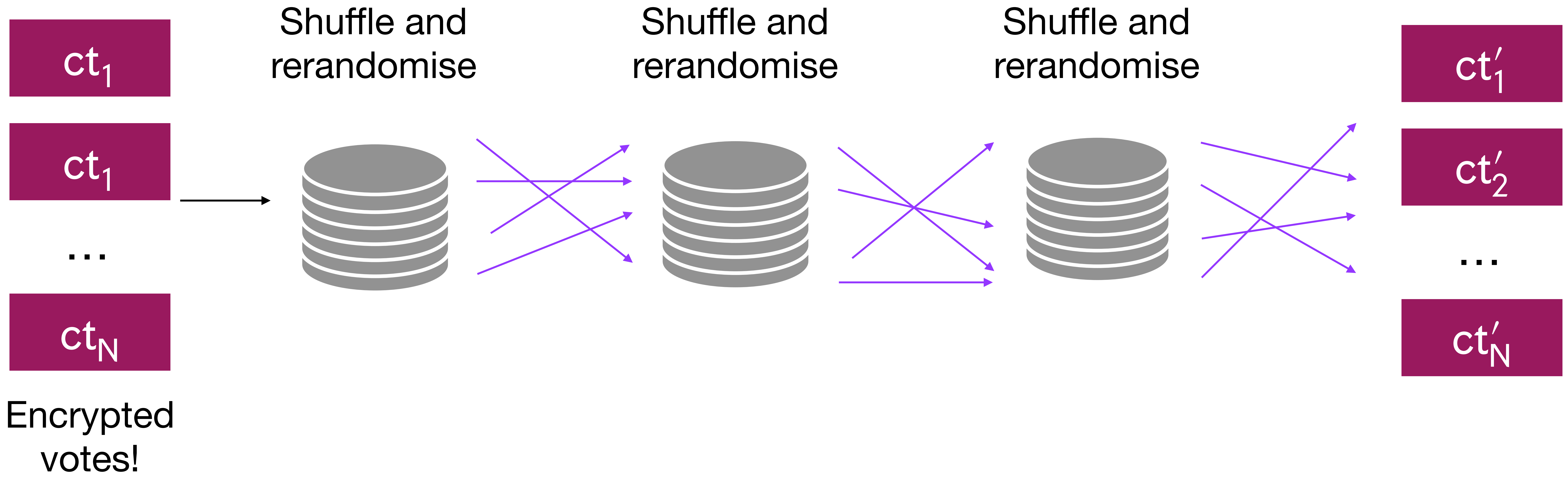

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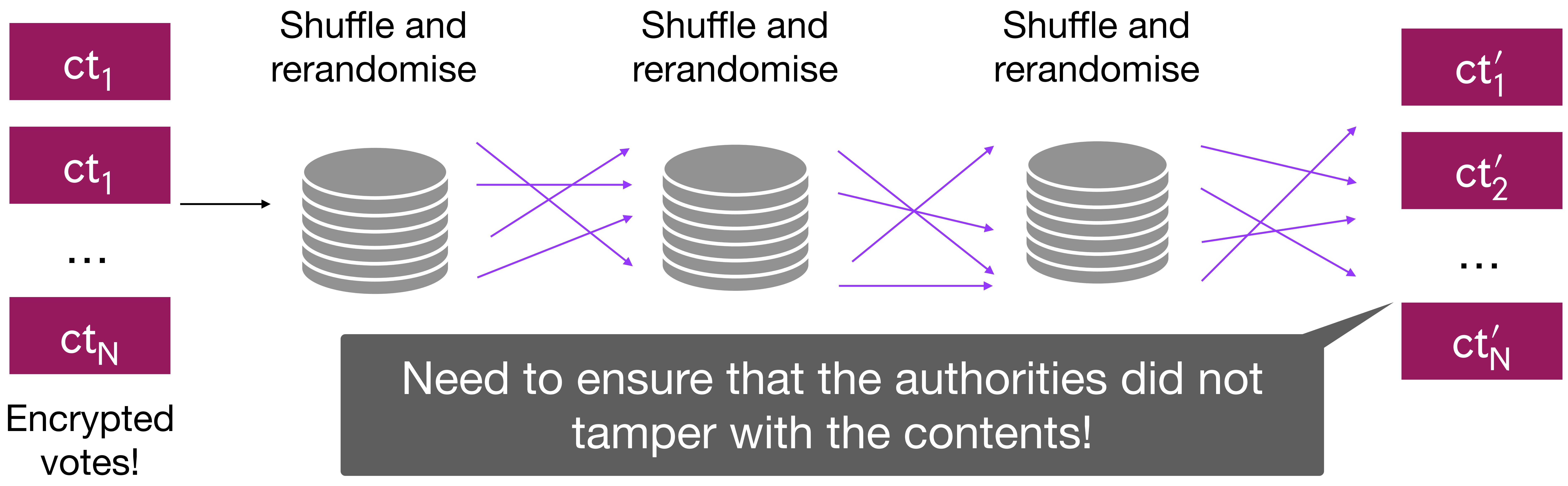

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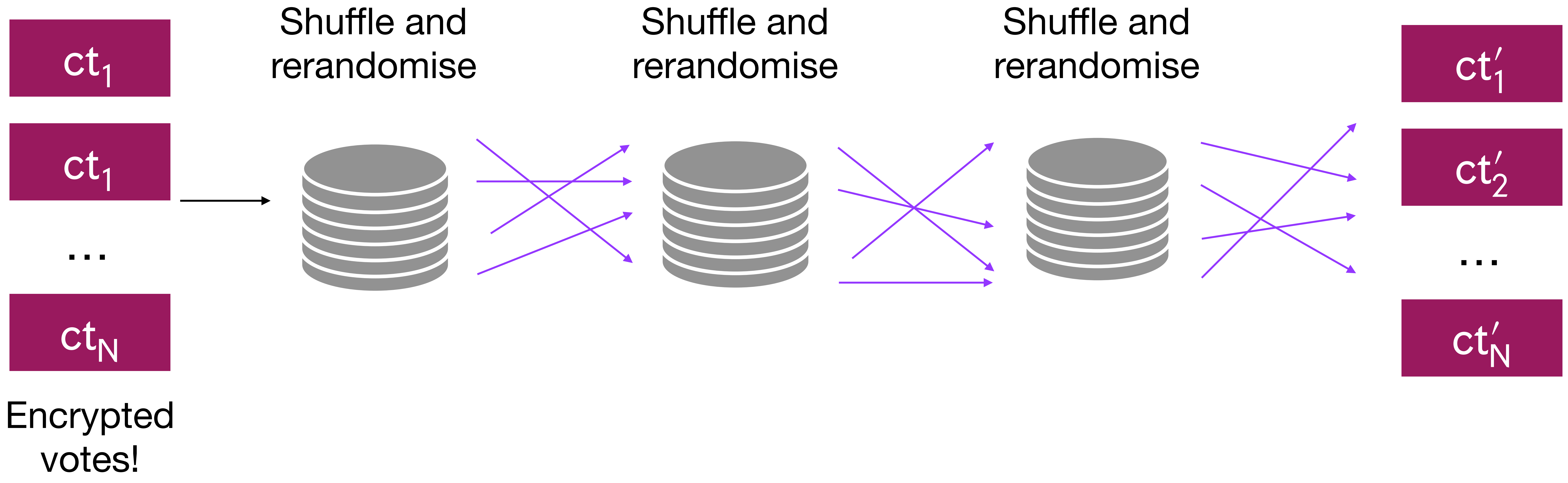

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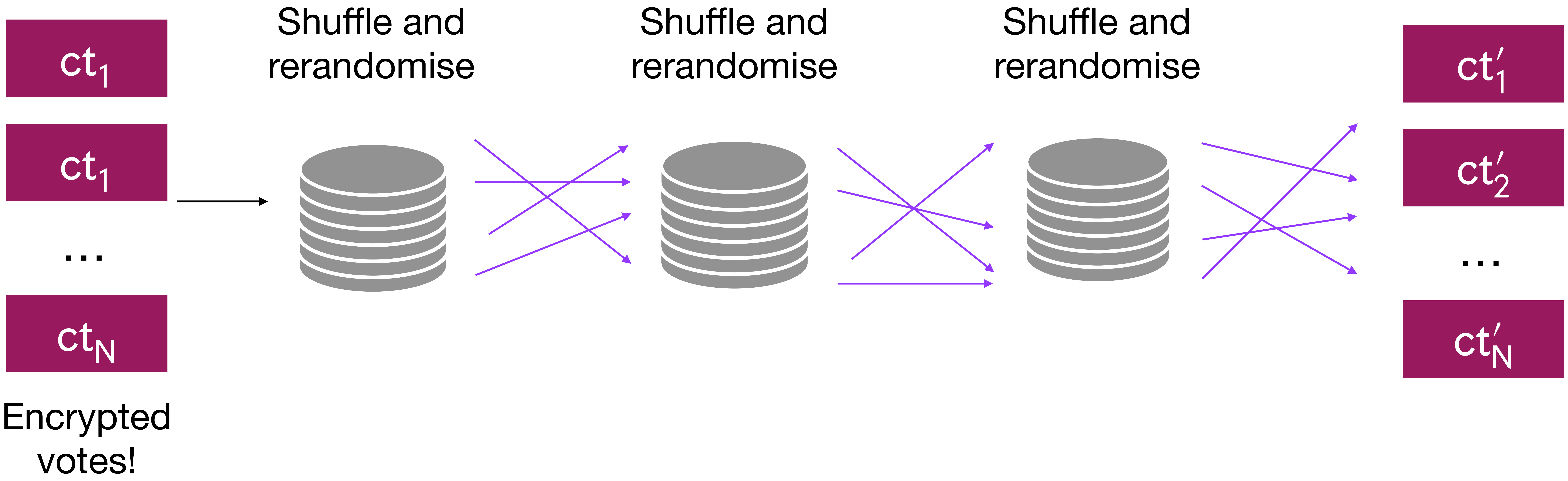

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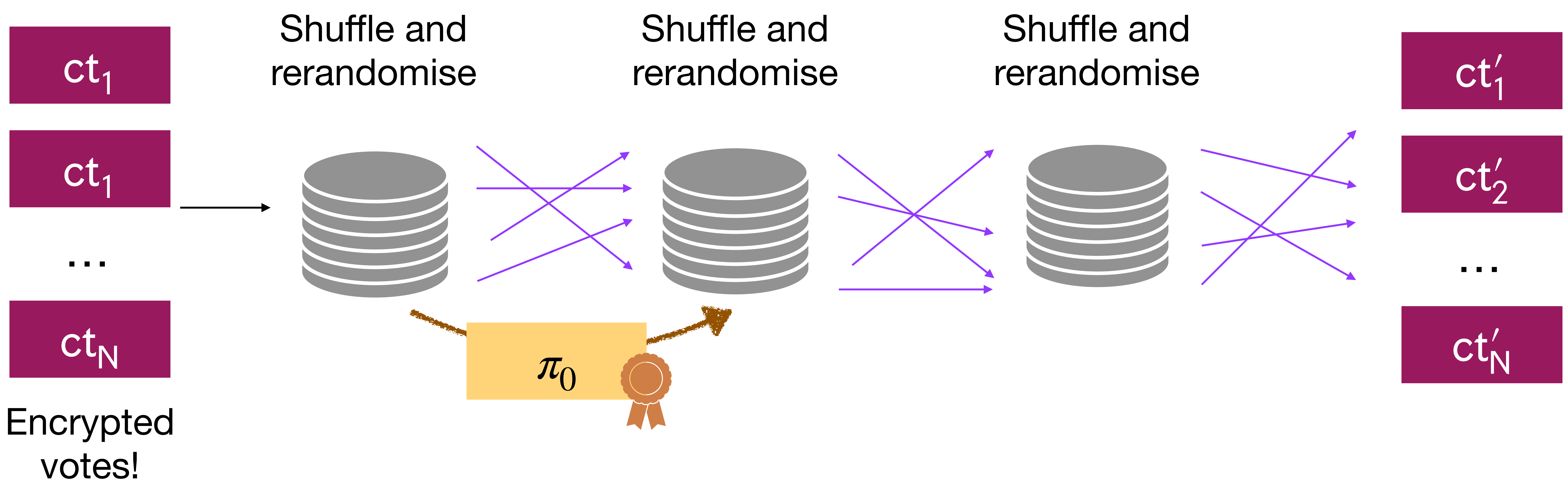

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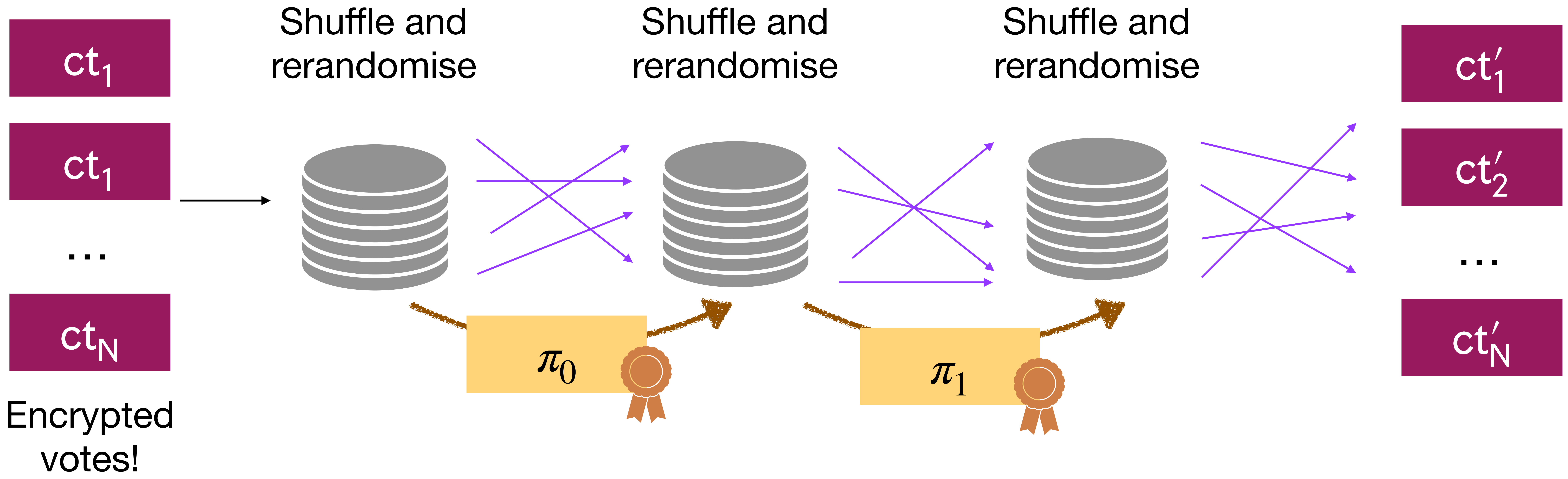

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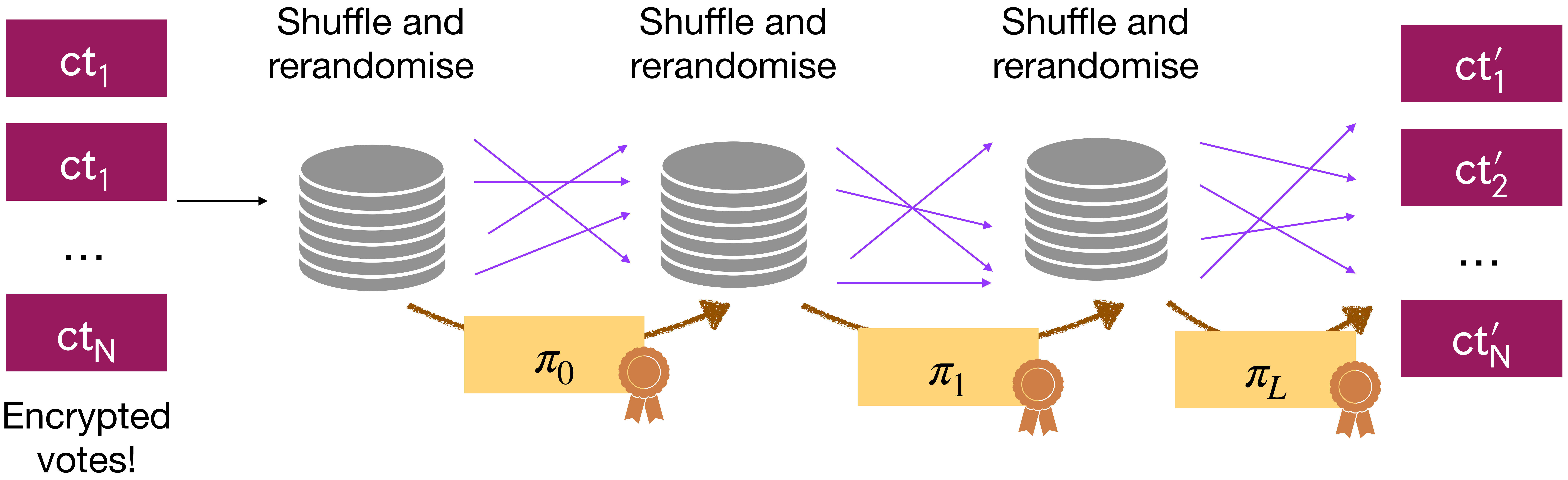

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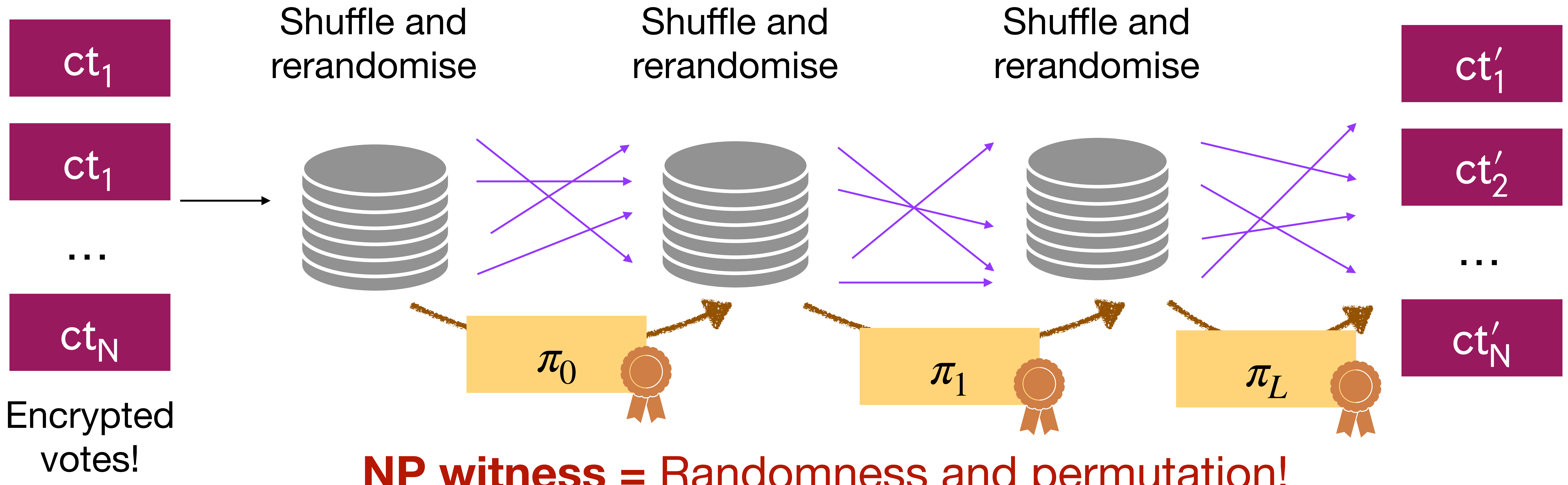

NP witness = Randomness and permutation!

How do we construct IVC?

Valiant's Recipe: Proof Merging

(Prior work)

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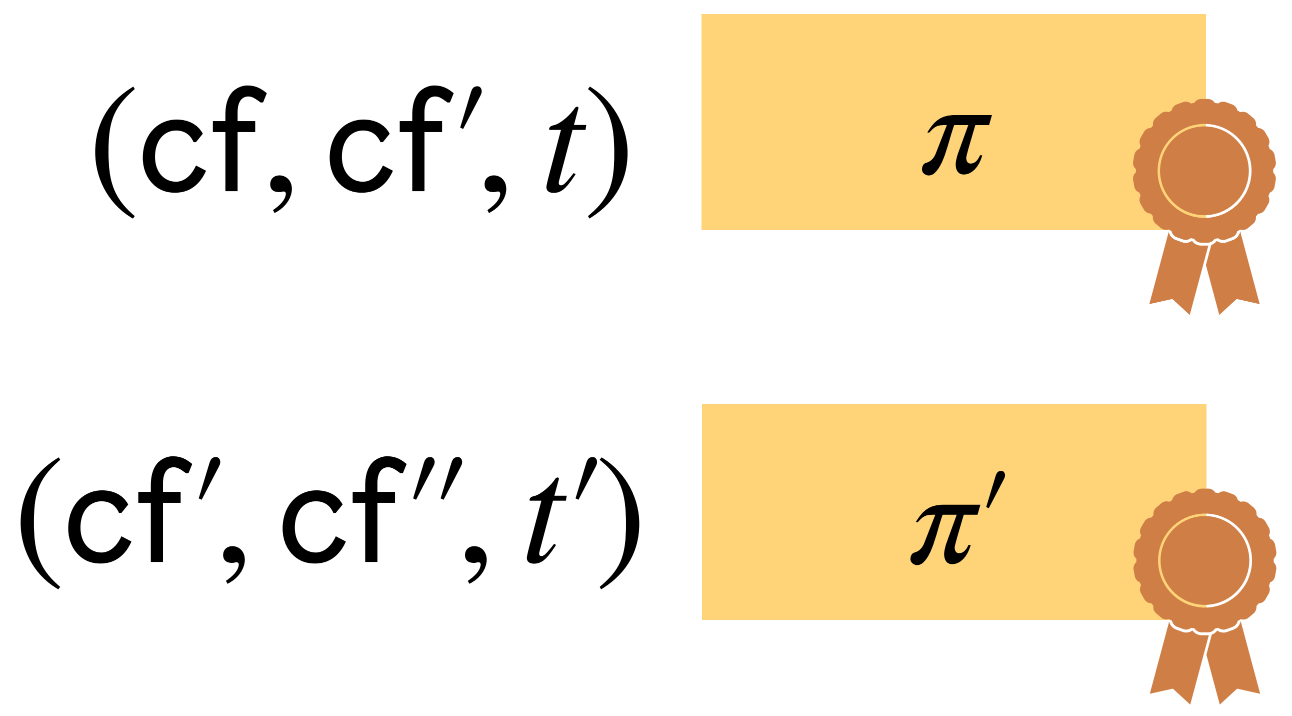

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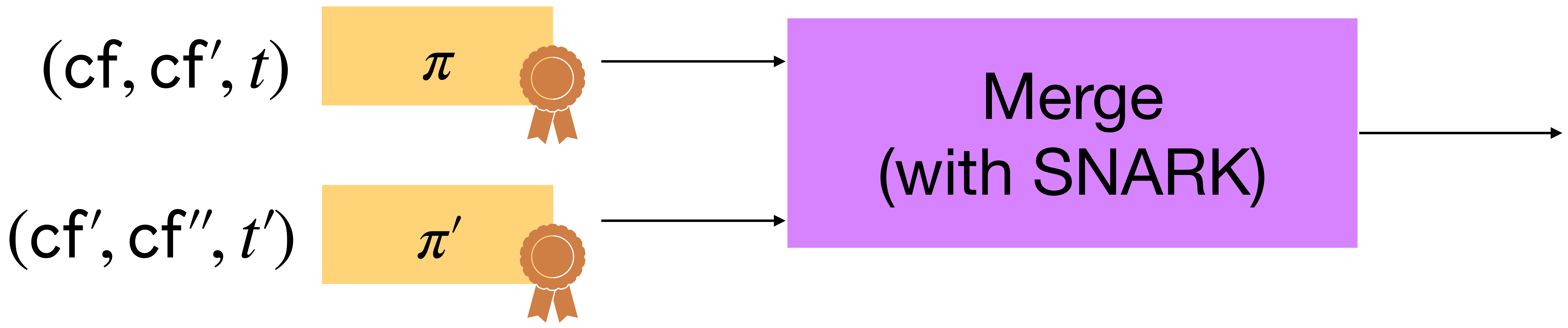

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- **Proof of knowledge:** If adversary gives **accepting** $(\mathsf{cf}, \mathsf{cf}'', t + t'), \pi''$, one can **extract** accepting tuples $(\mathsf{cf}, \mathsf{cf}', t), \pi$ and $(\mathsf{cf}', \mathsf{cf}'', t'), \pi'$.

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- **Succinctness:** $|\pi''| \approx |\pi|, |\pi'|$

Proof Merging → IVC! "Tree merging"

Level 3

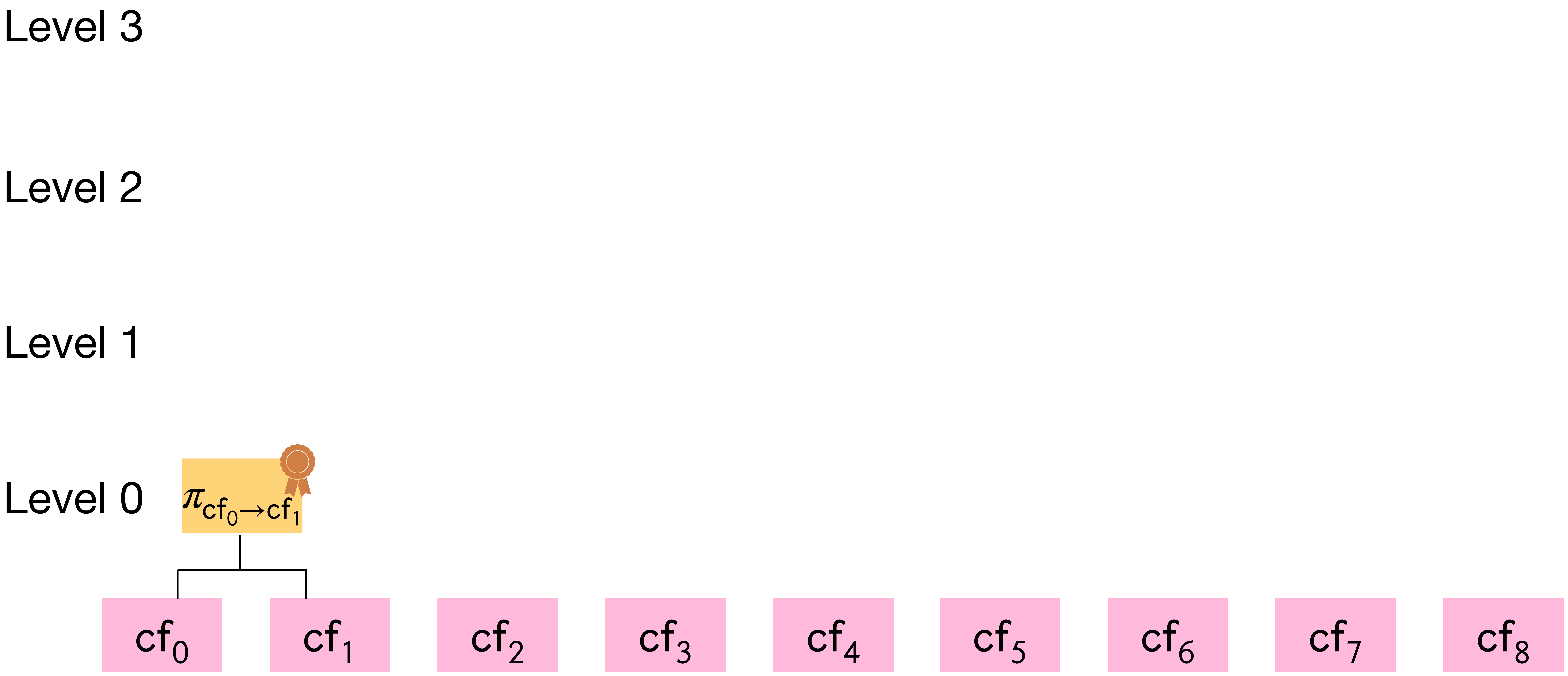
Level 2

Level 1

Level 0

cf_0 cf_1 cf_2 cf_3 cf_4 cf_5 cf_6 cf_7 cf_8

Proof Merging → IVC! “Tree merging”



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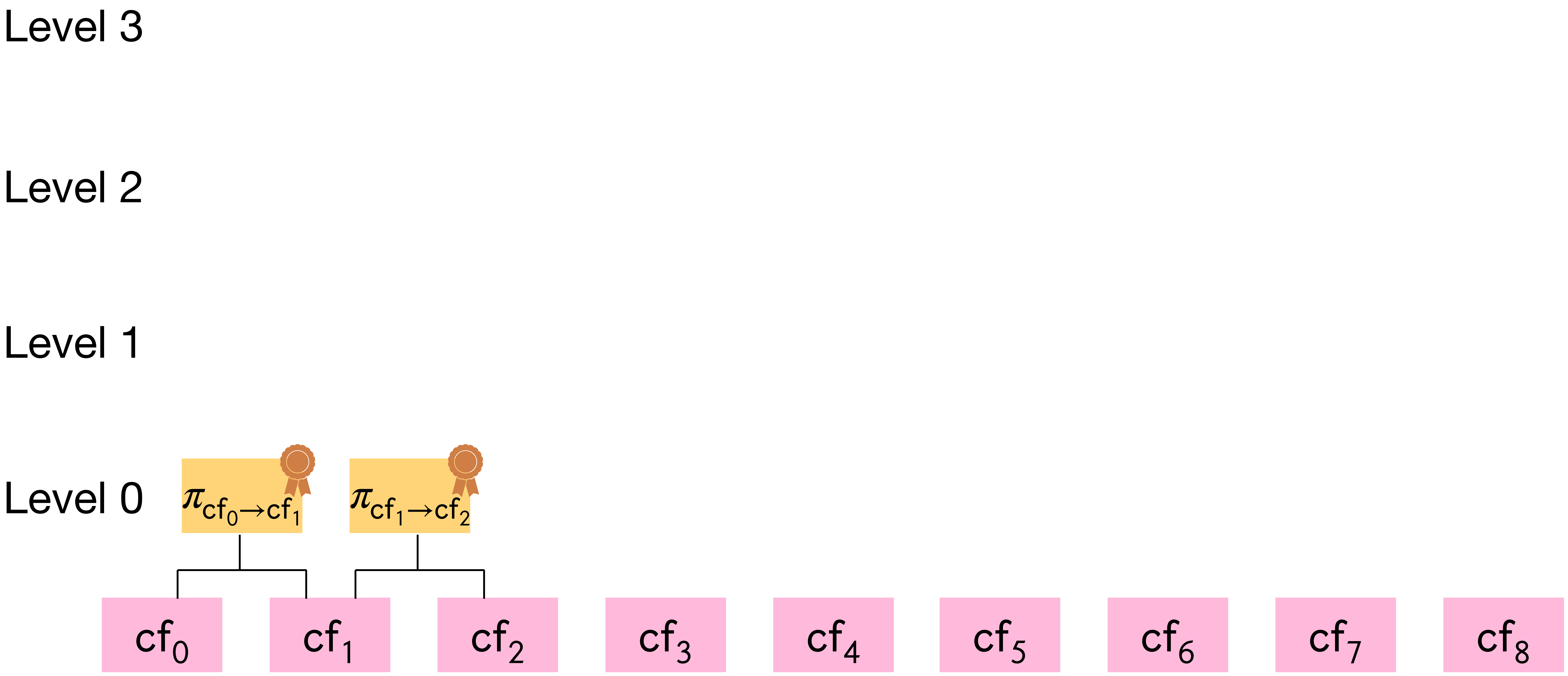

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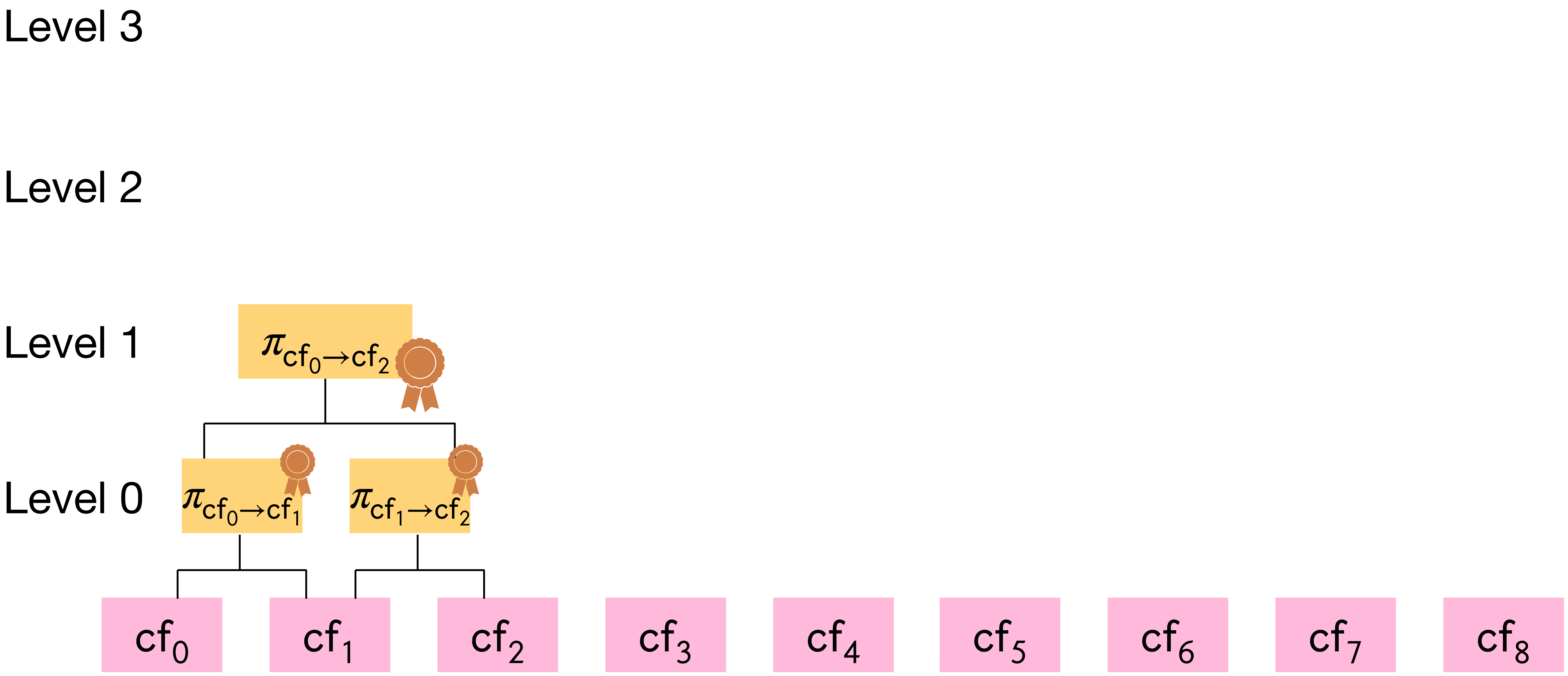

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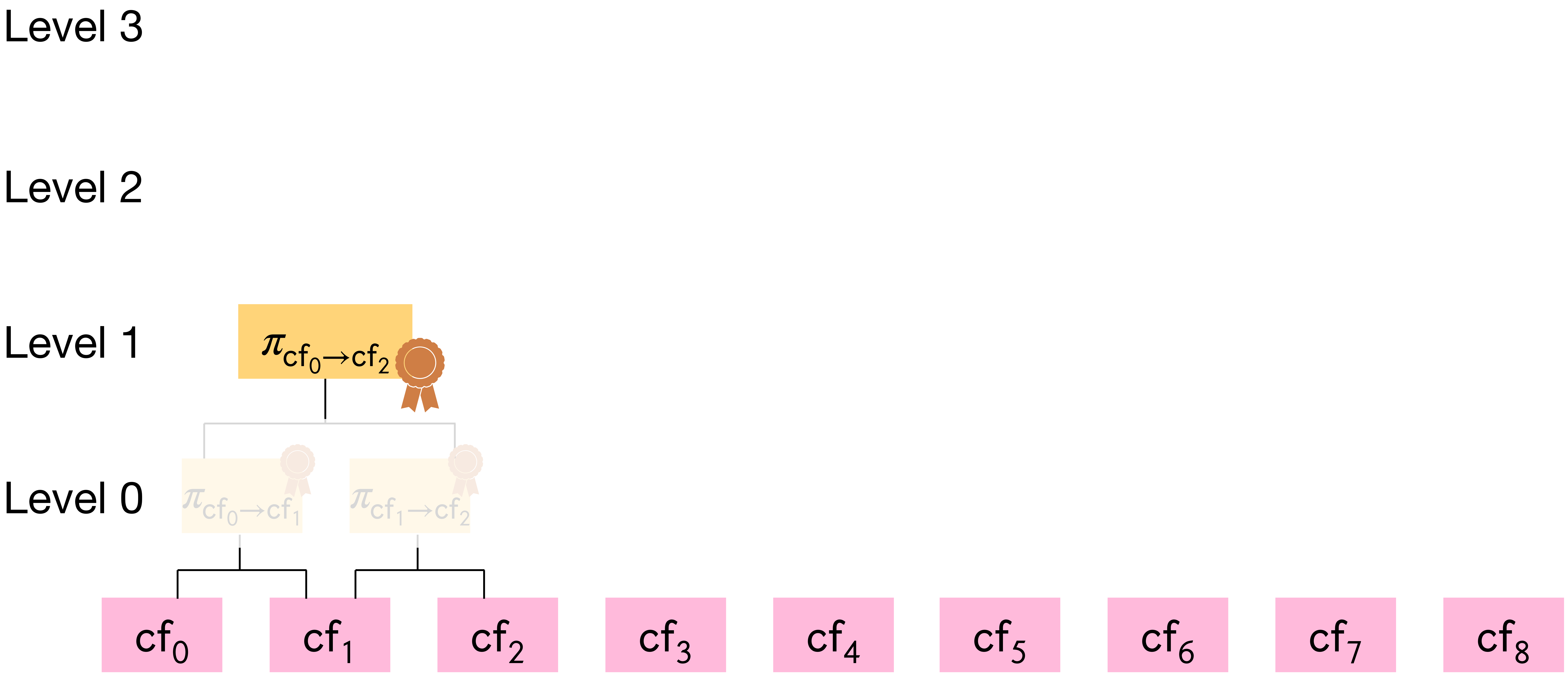

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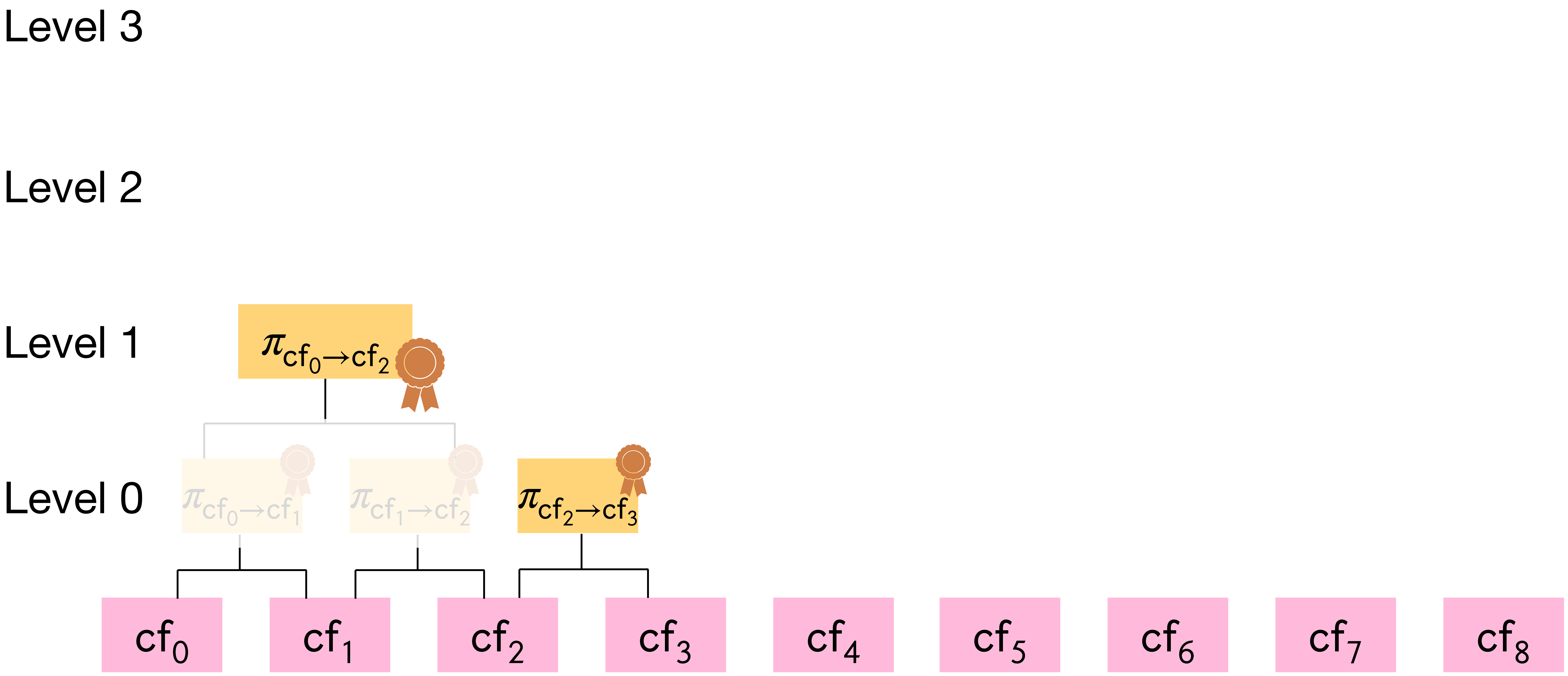

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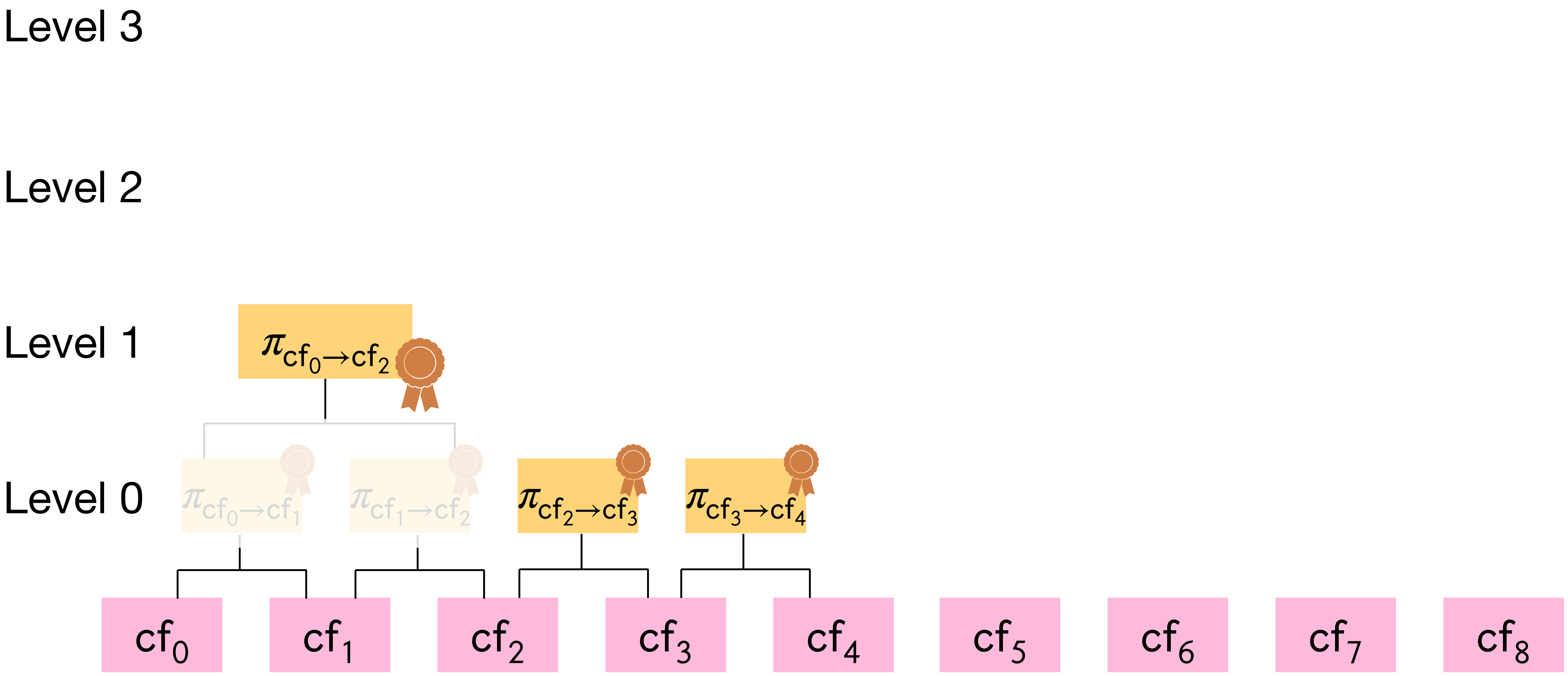

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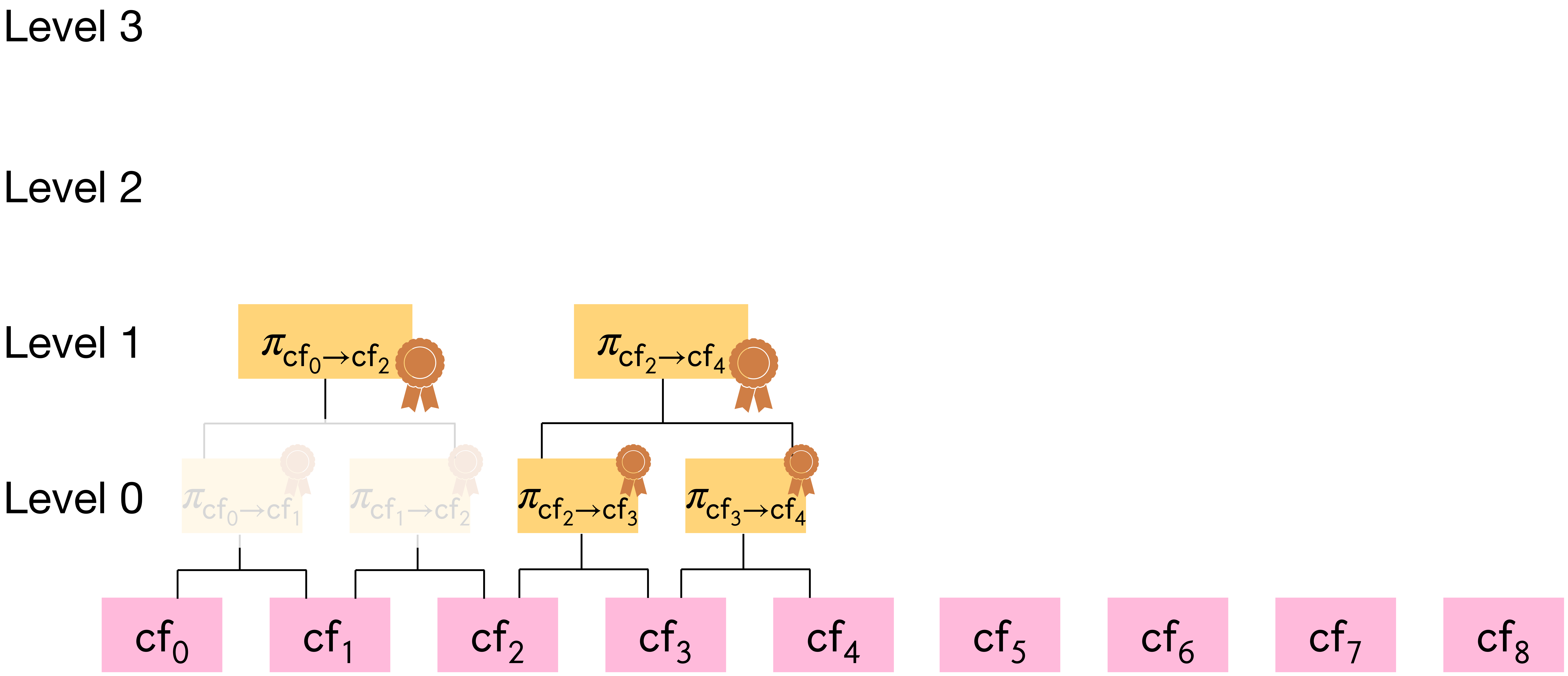

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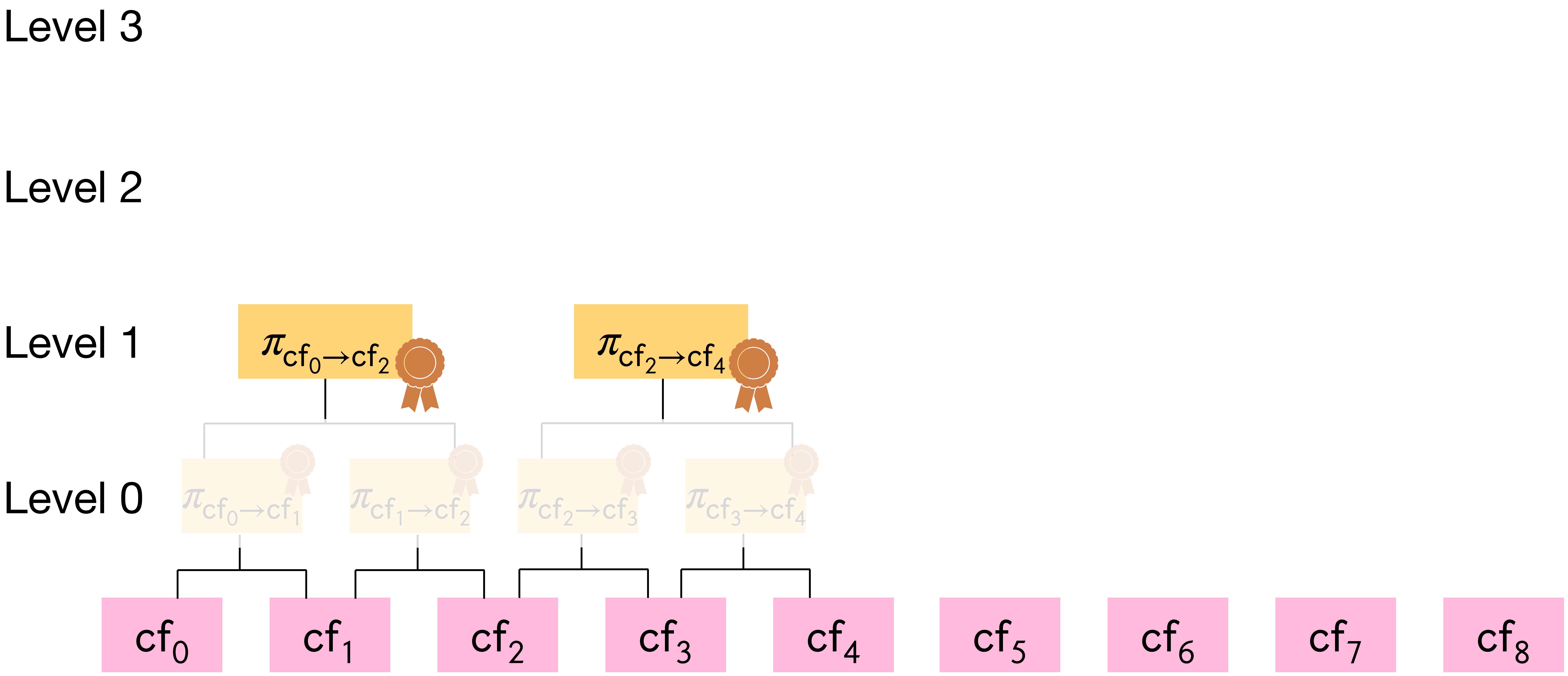

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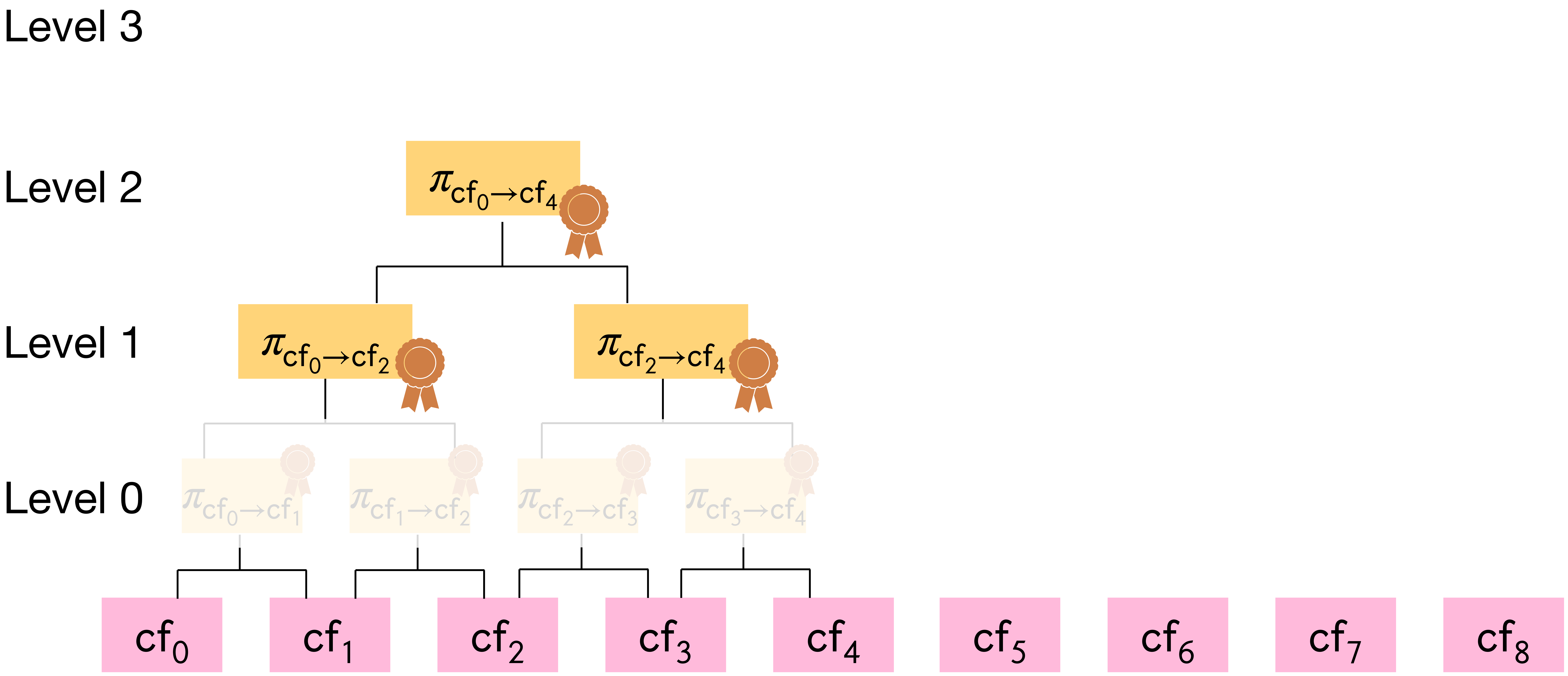

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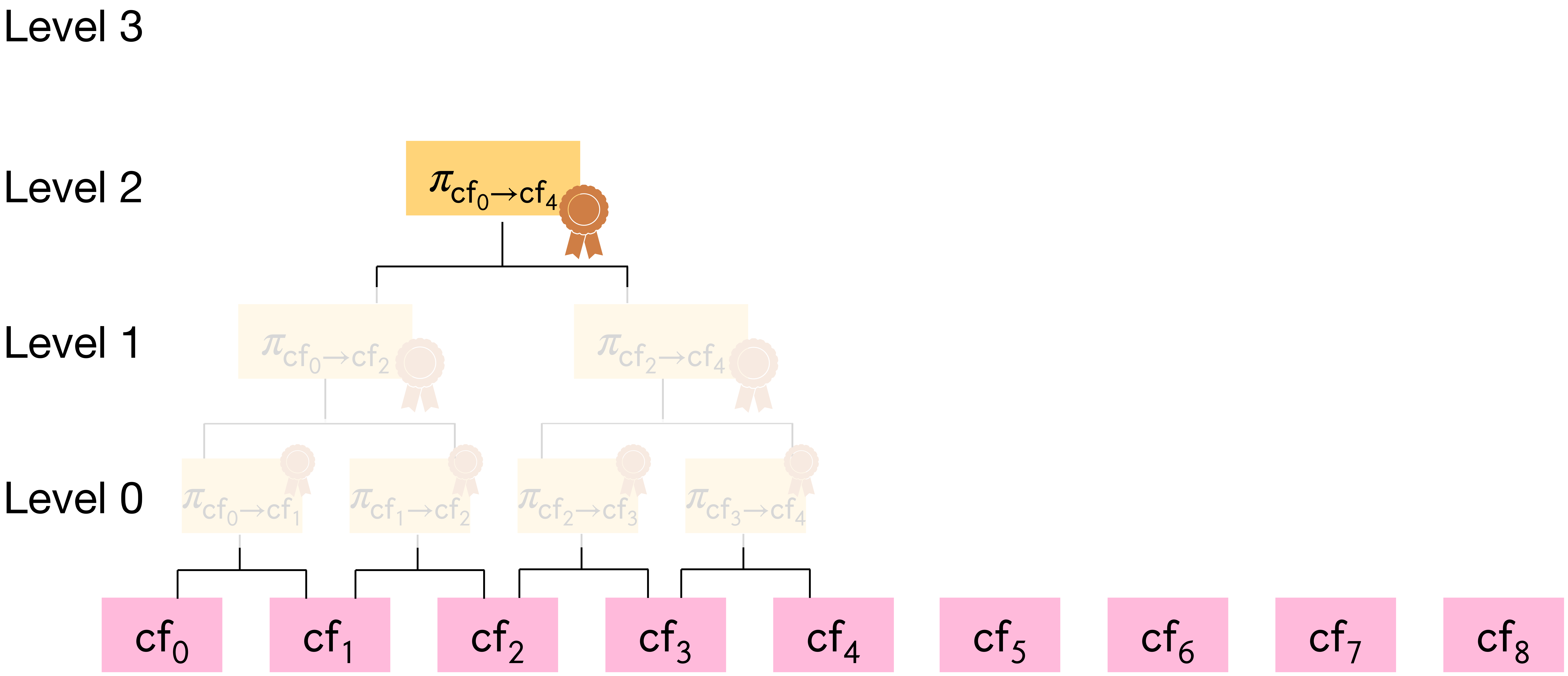

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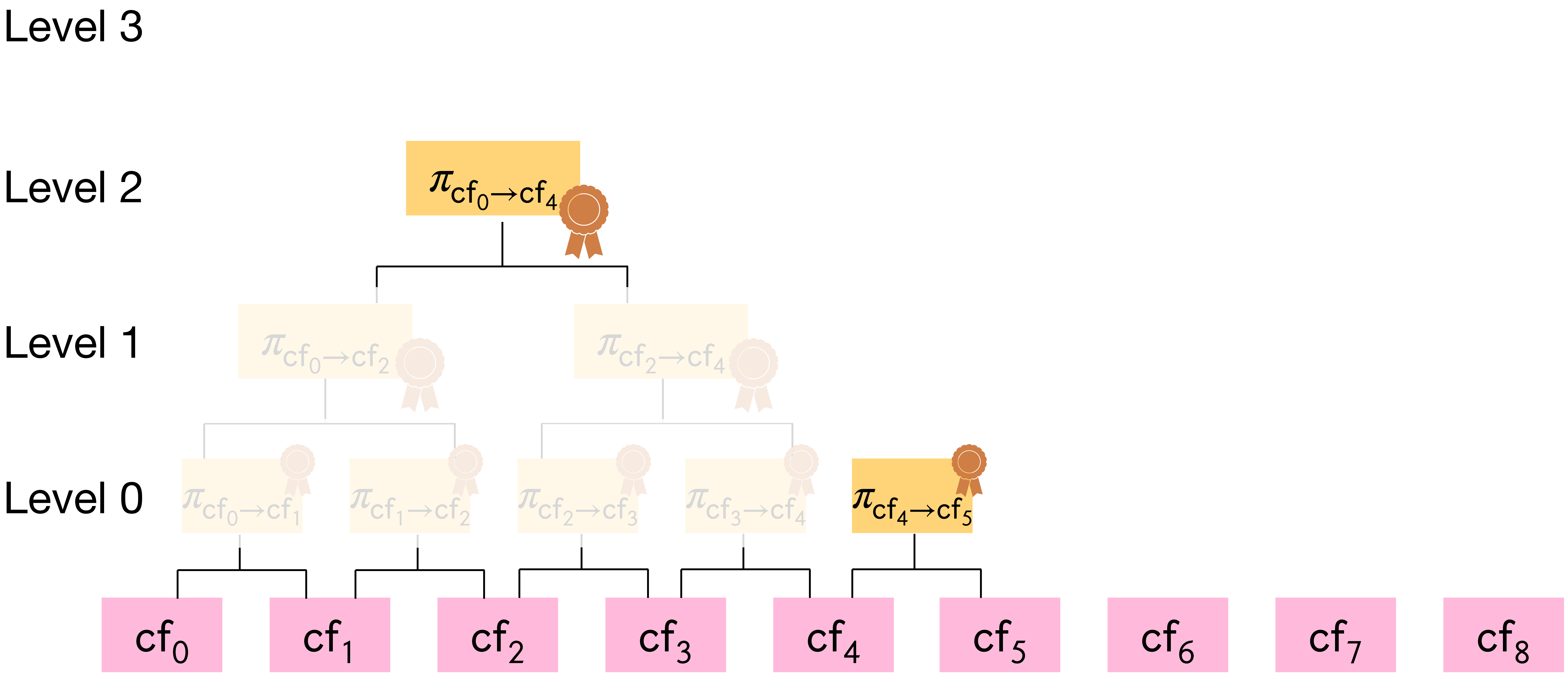

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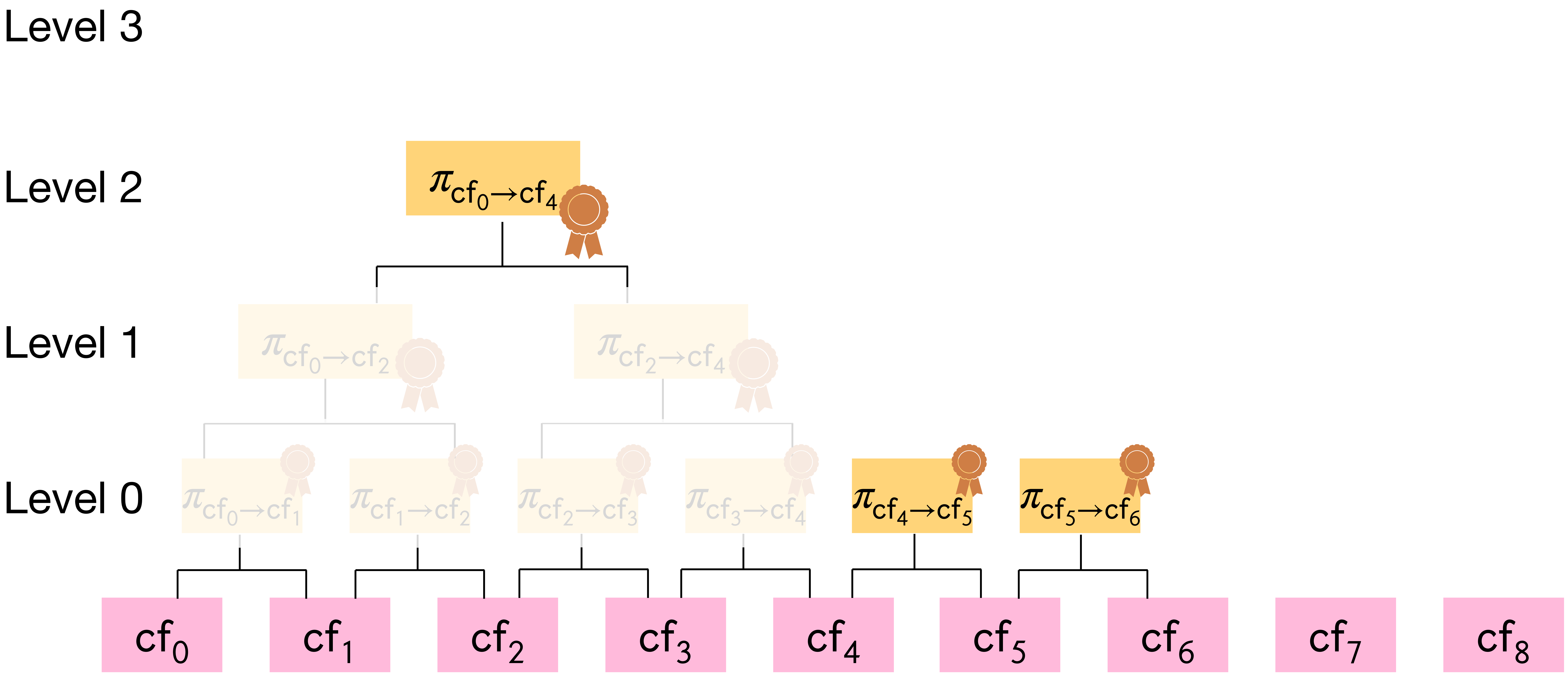

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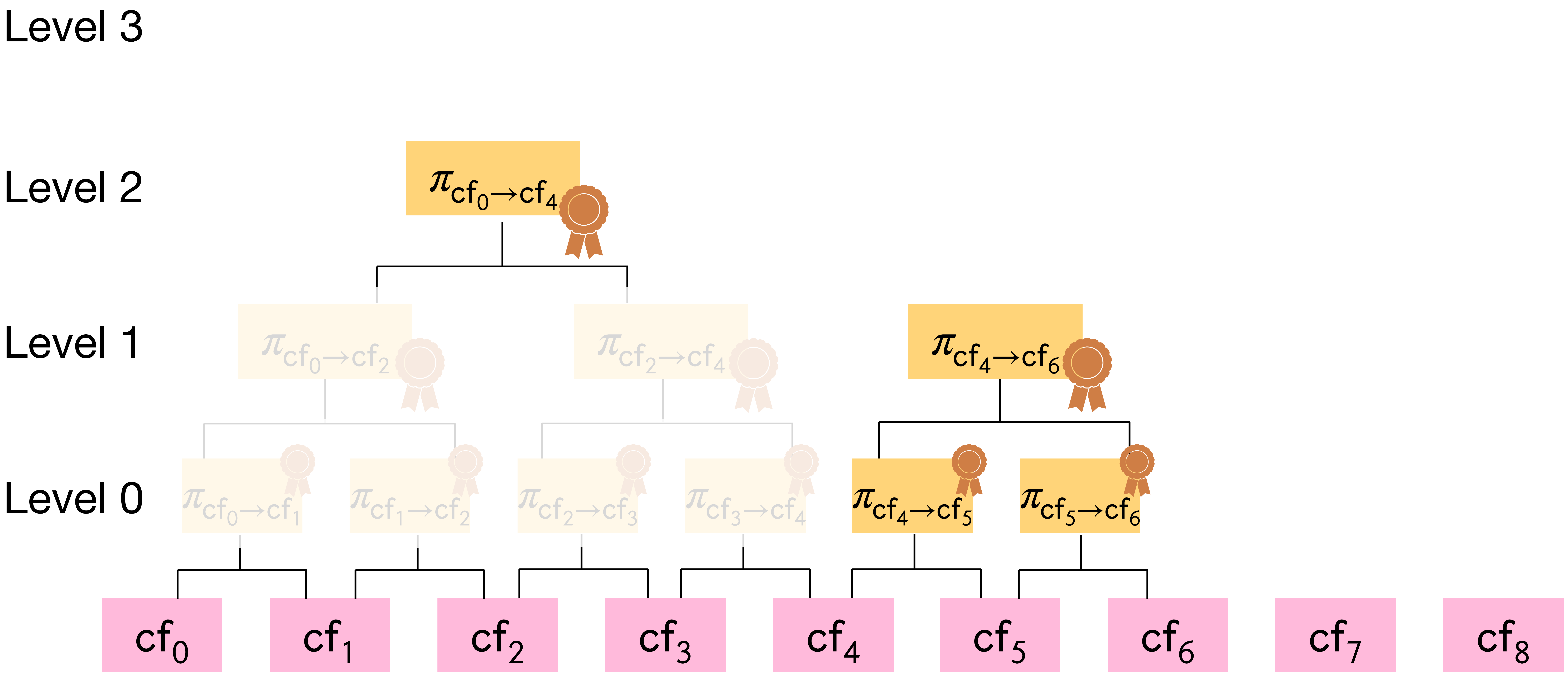

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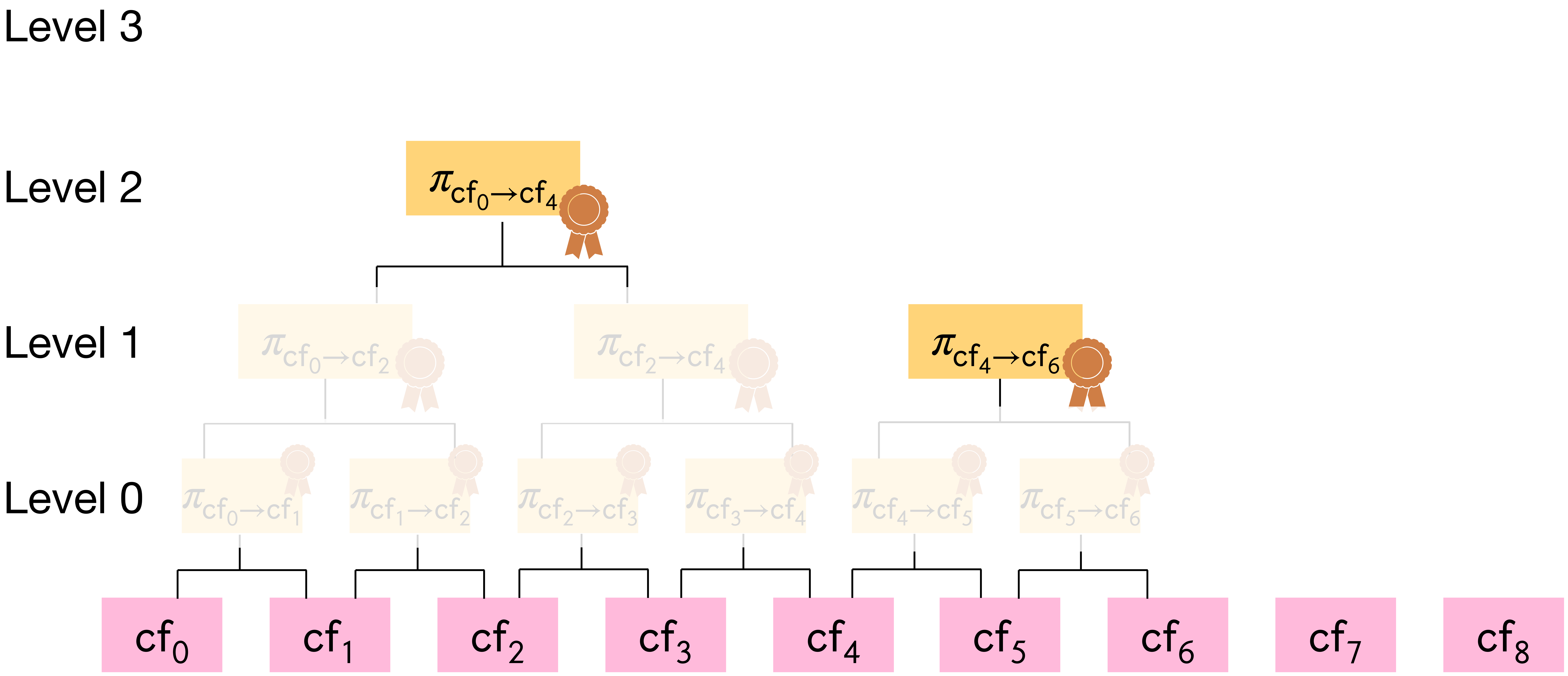

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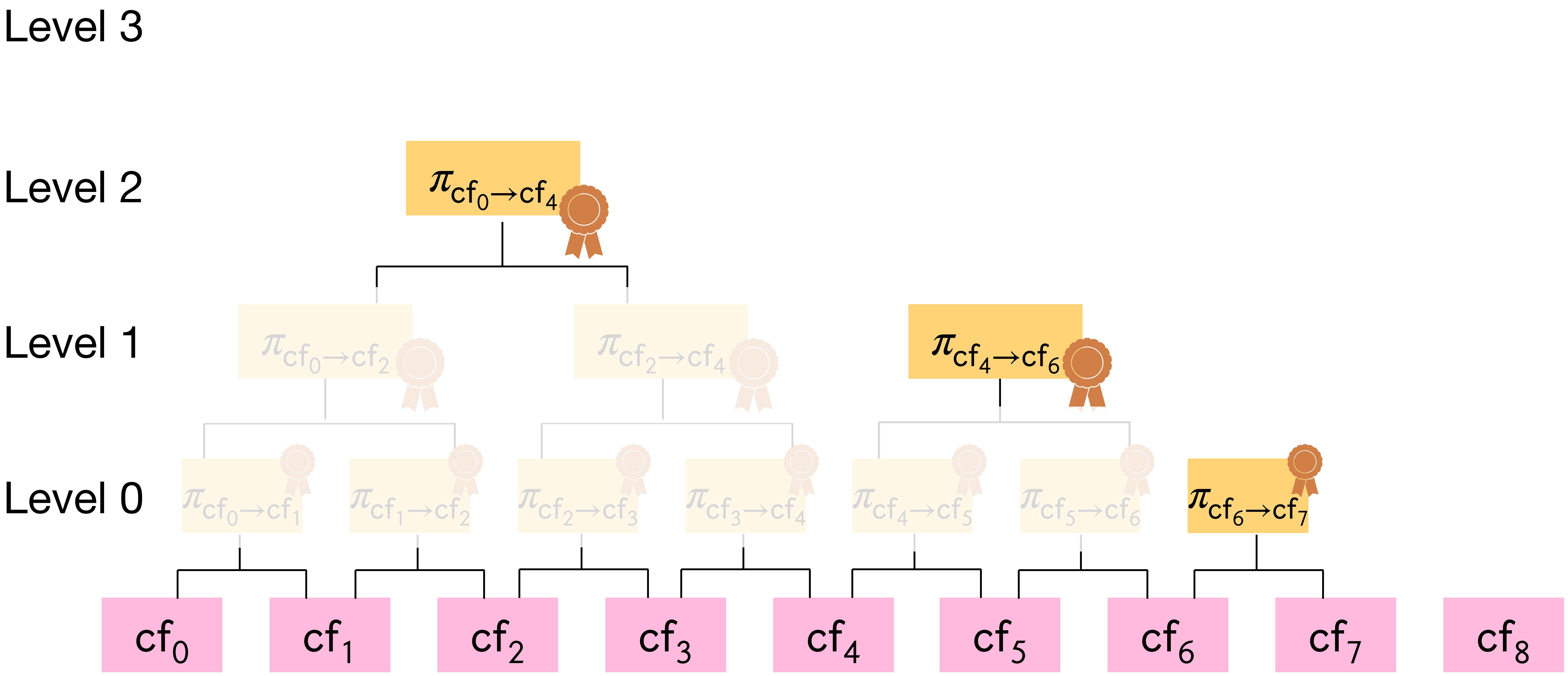

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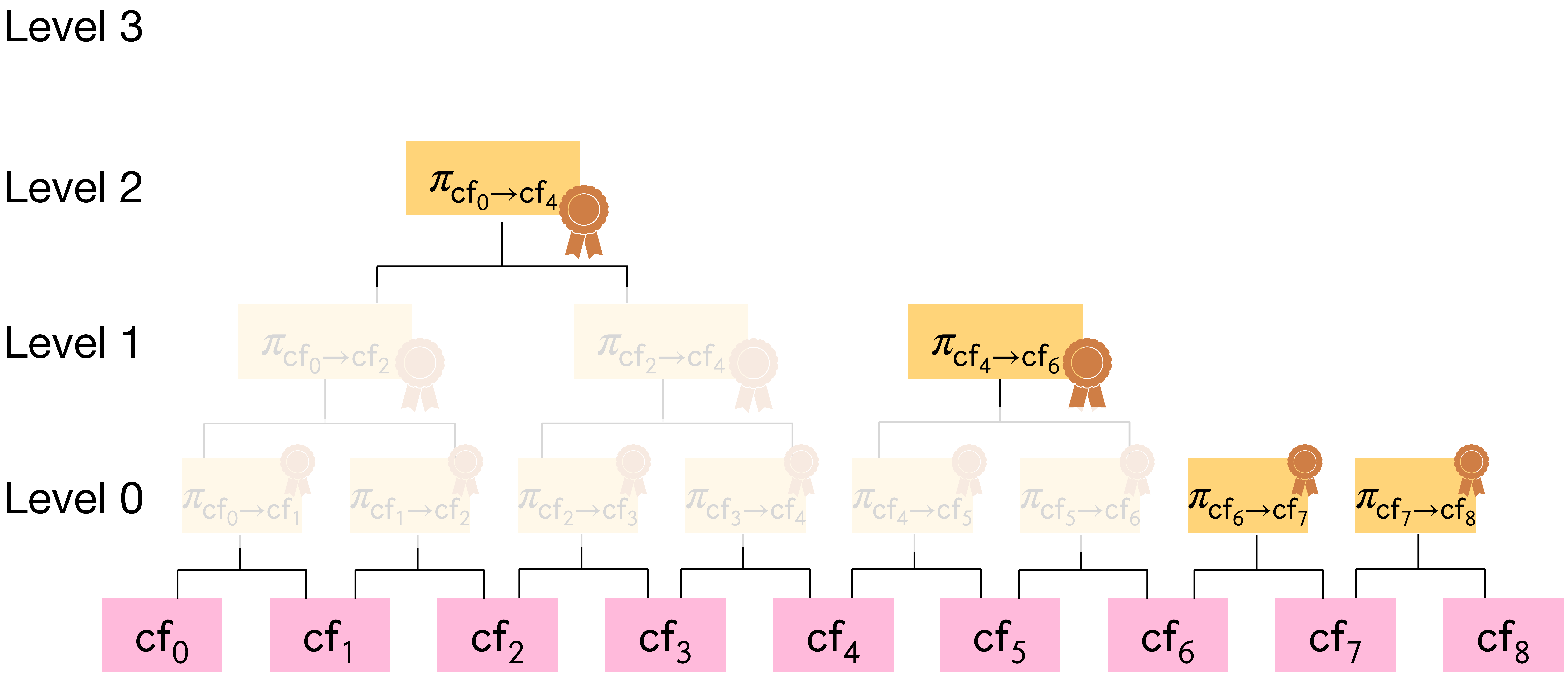

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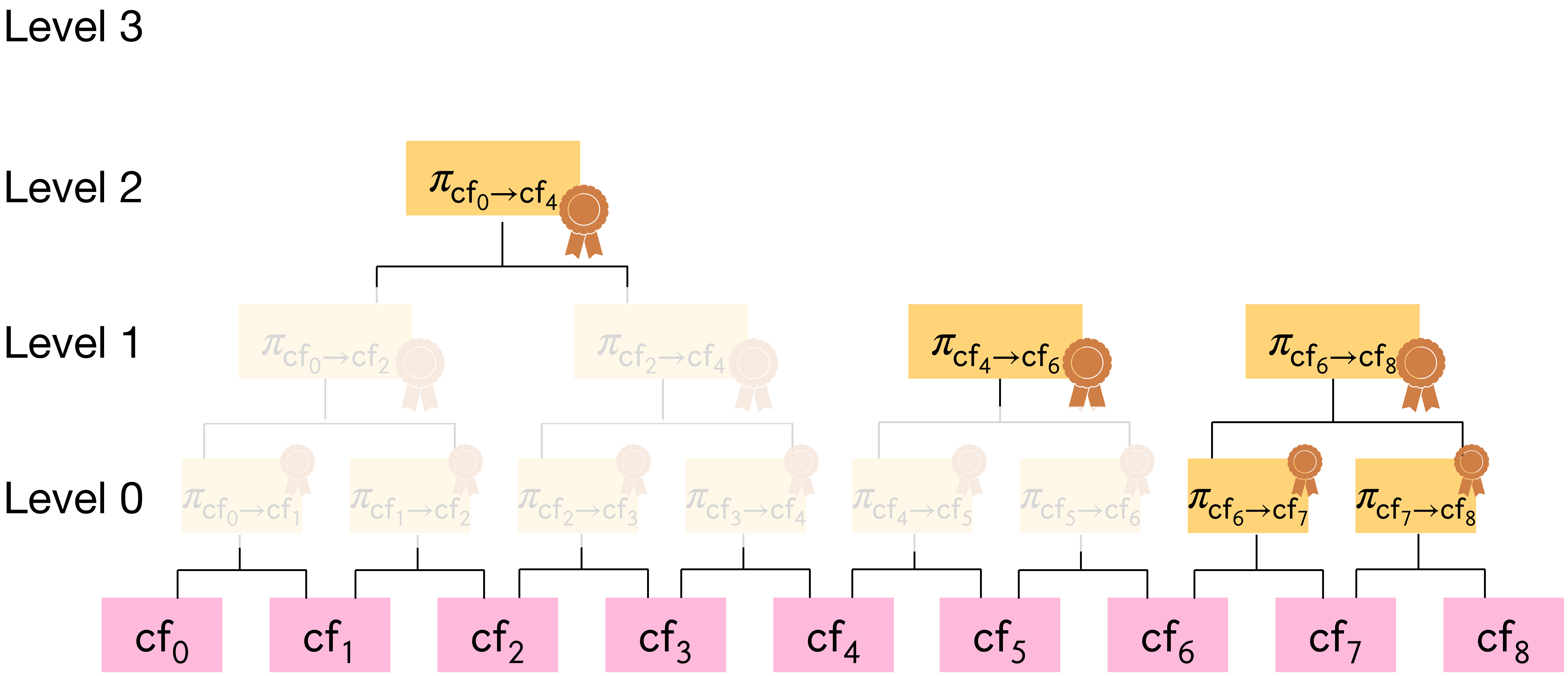

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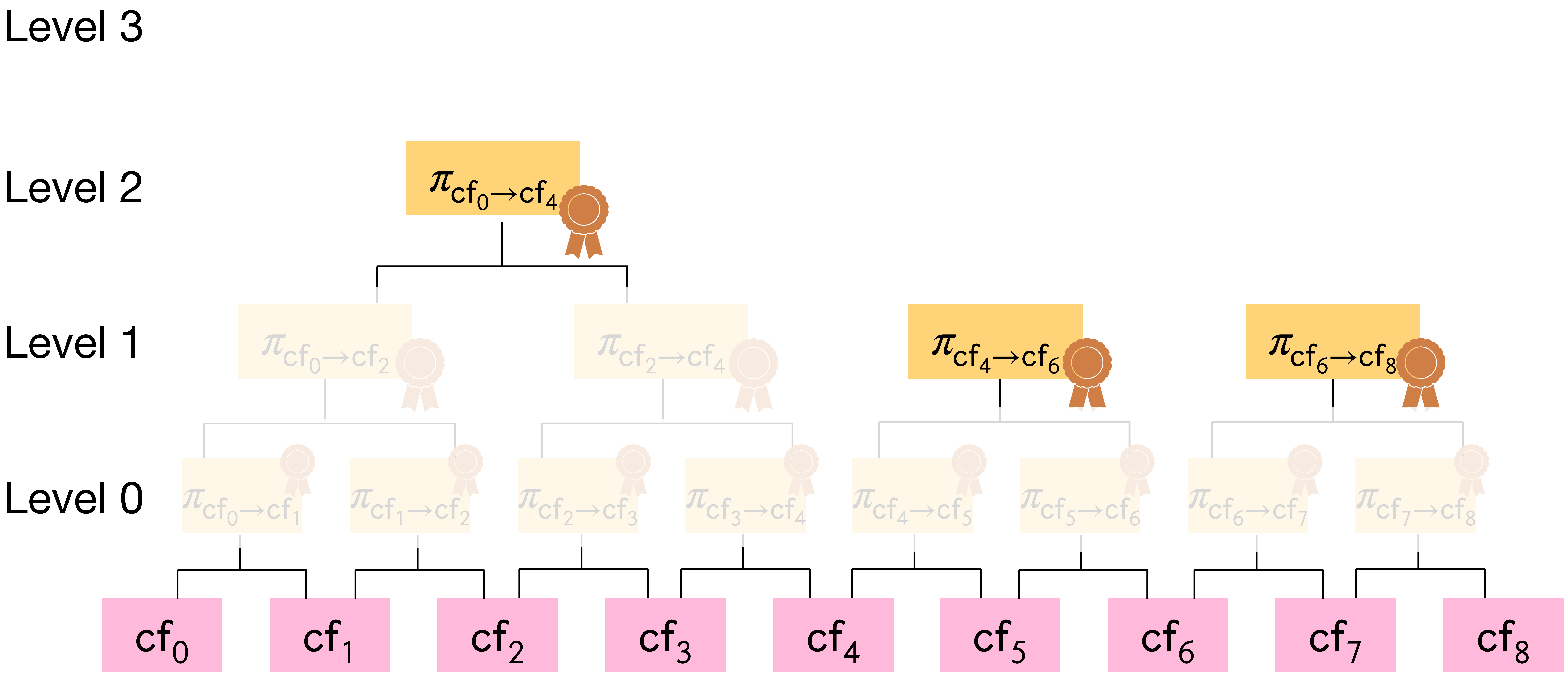

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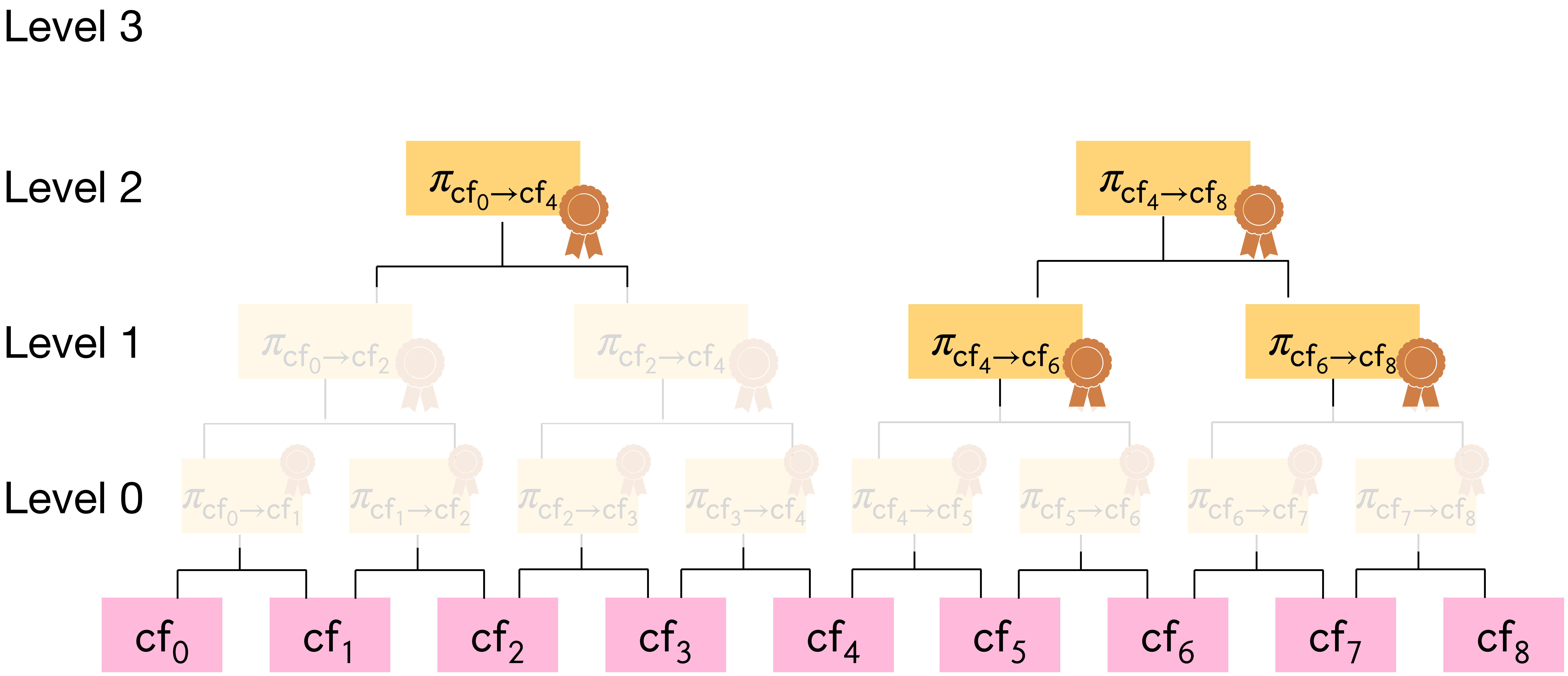

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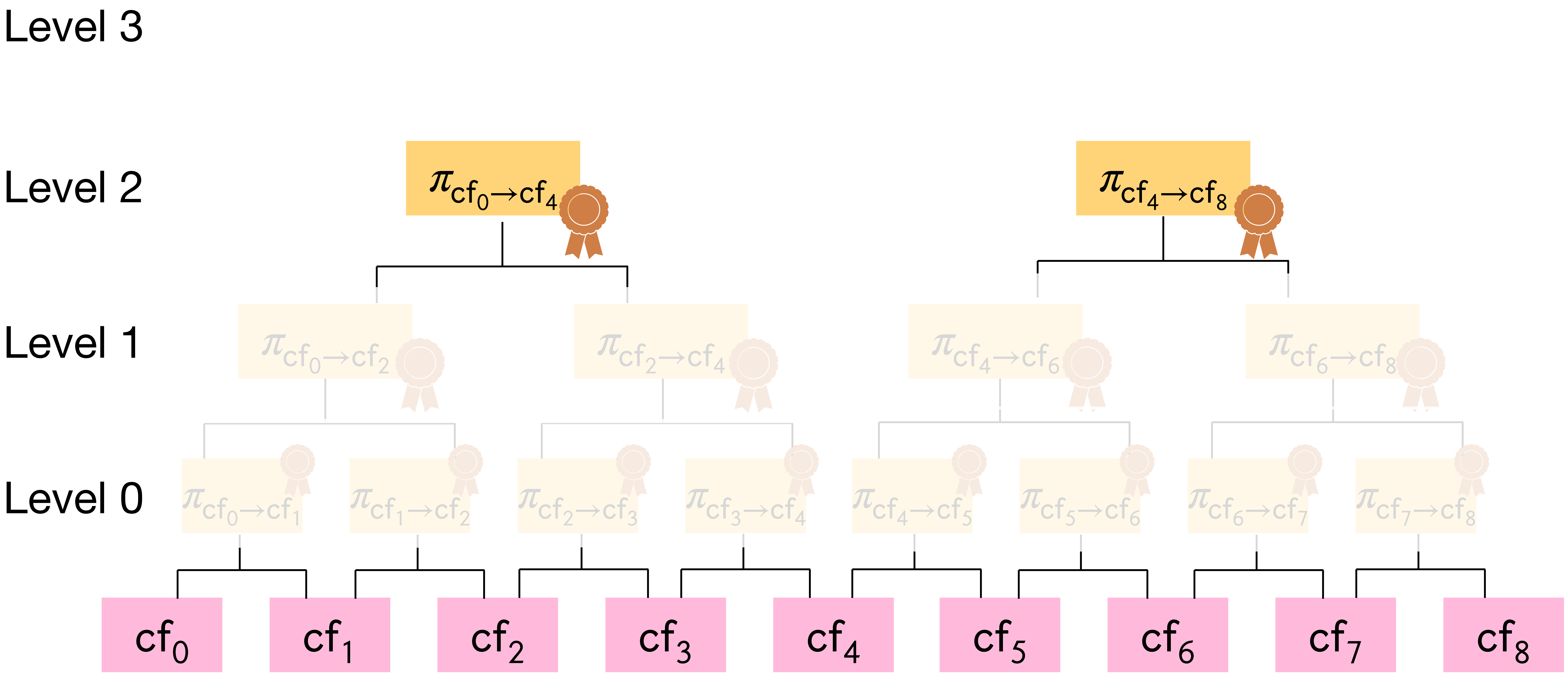

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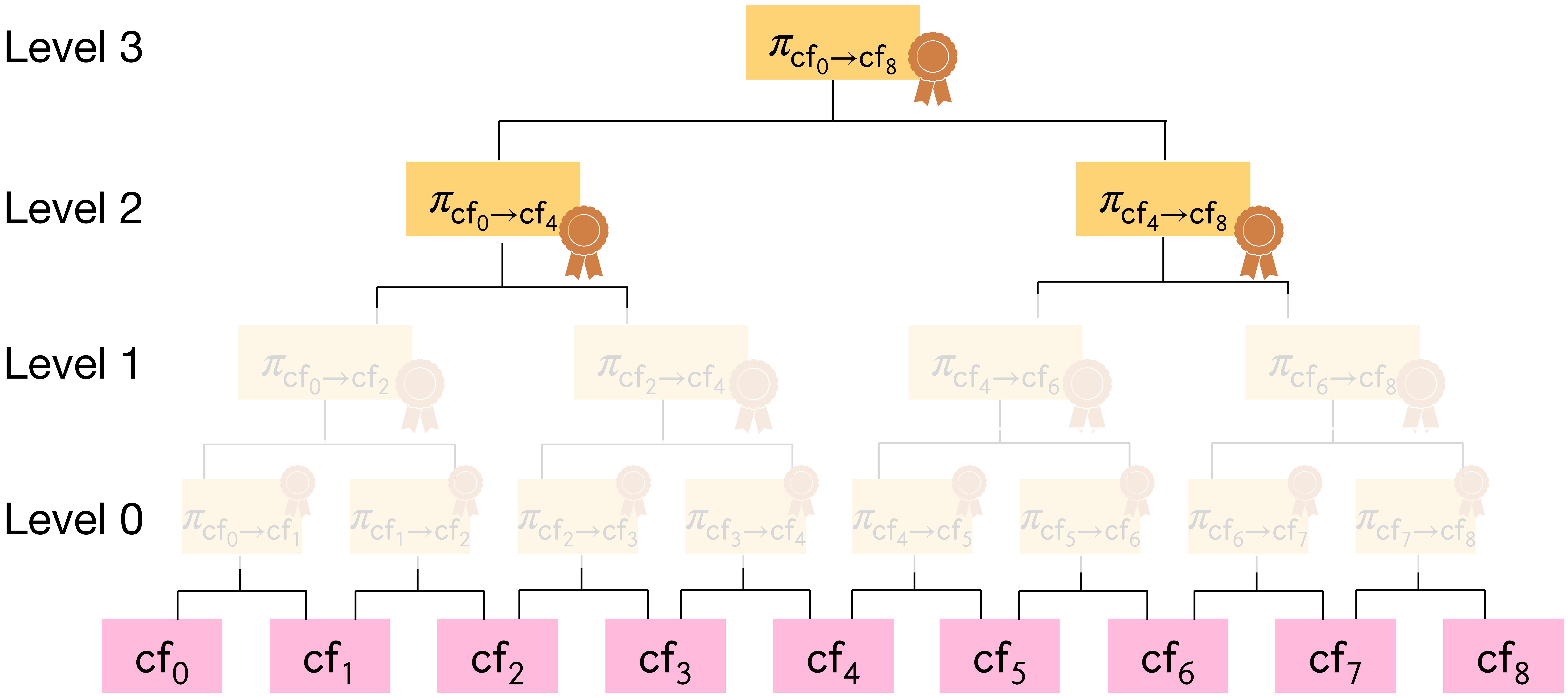

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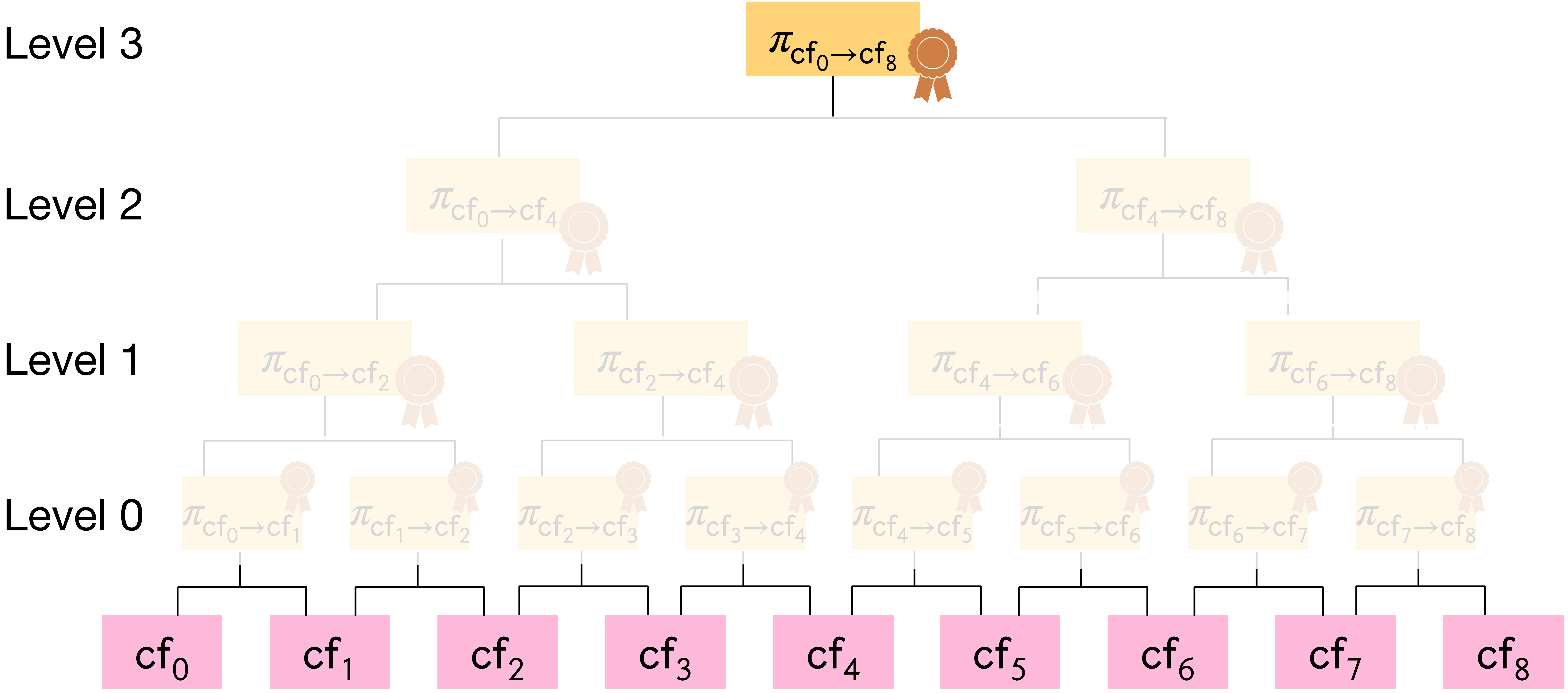

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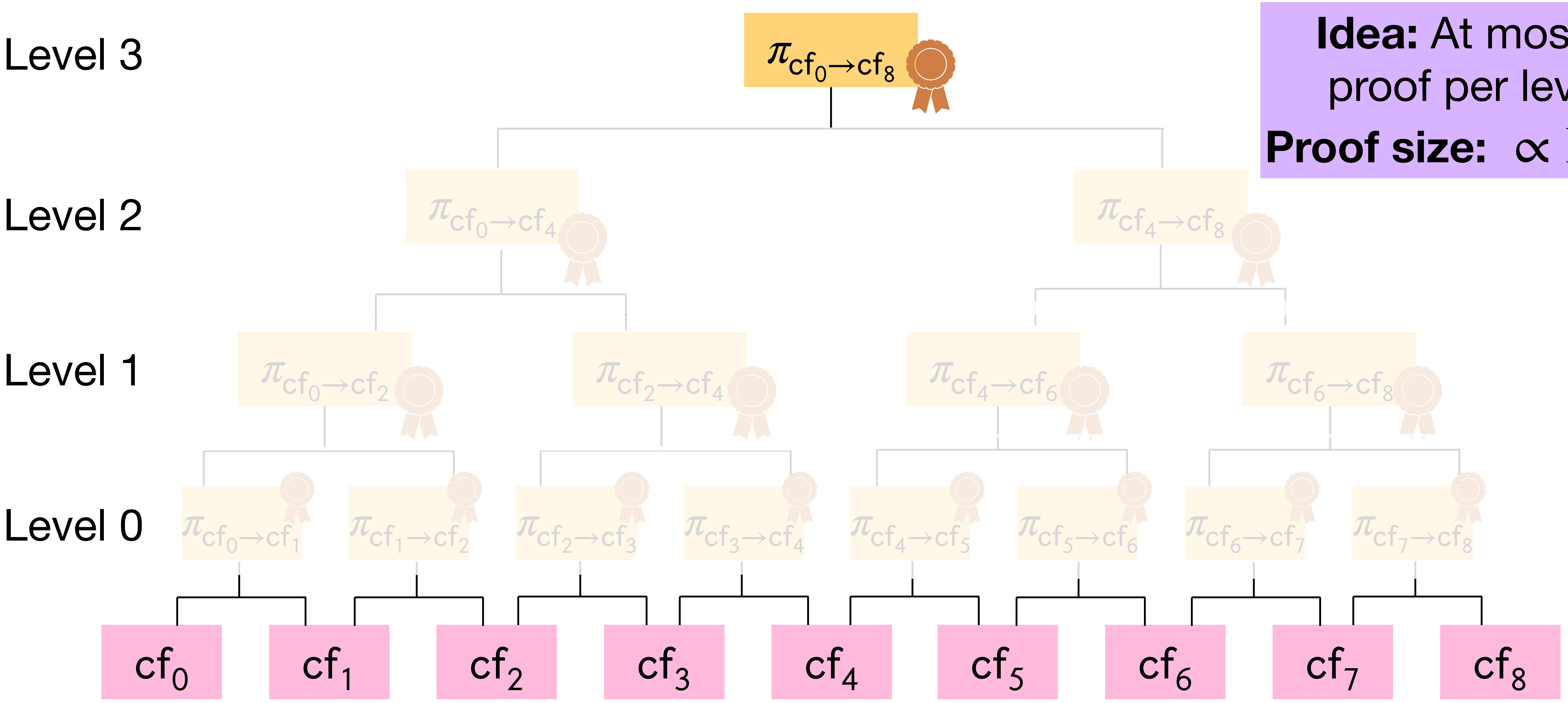

Idea: At most 1 proof per level

Proof size: $\propto \log T$

Valiant's Recipe: SNARK the SNARK!

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$(\mathsf{cf}, \mathsf{cf}', 2^k)$ π

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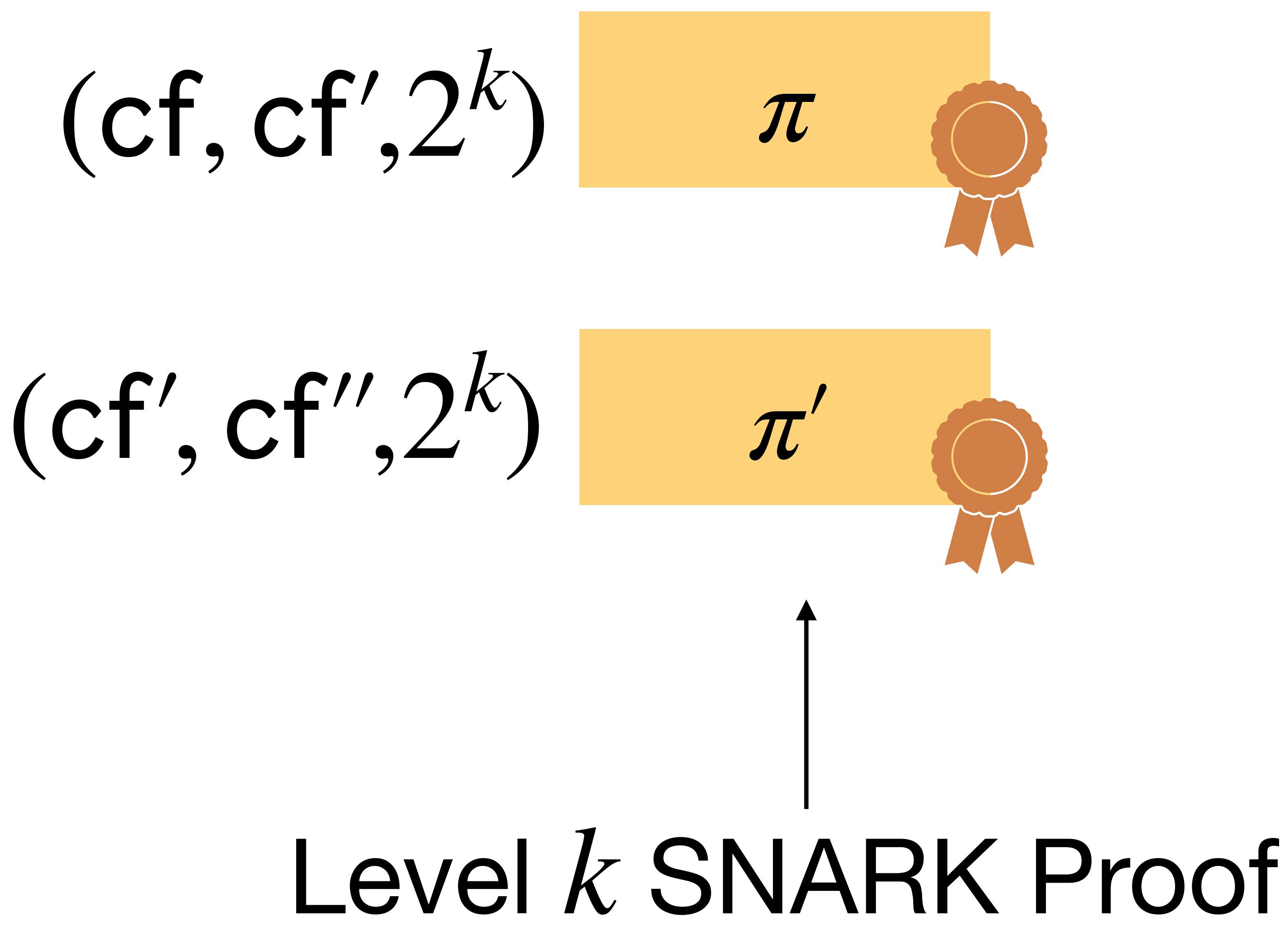

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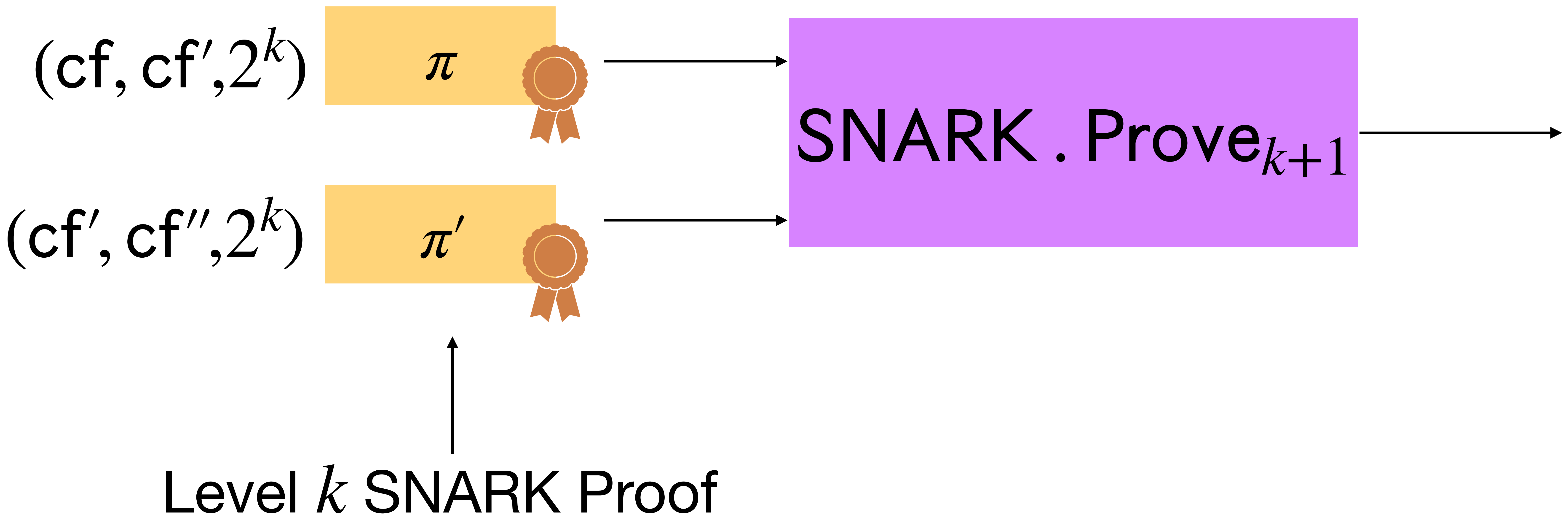

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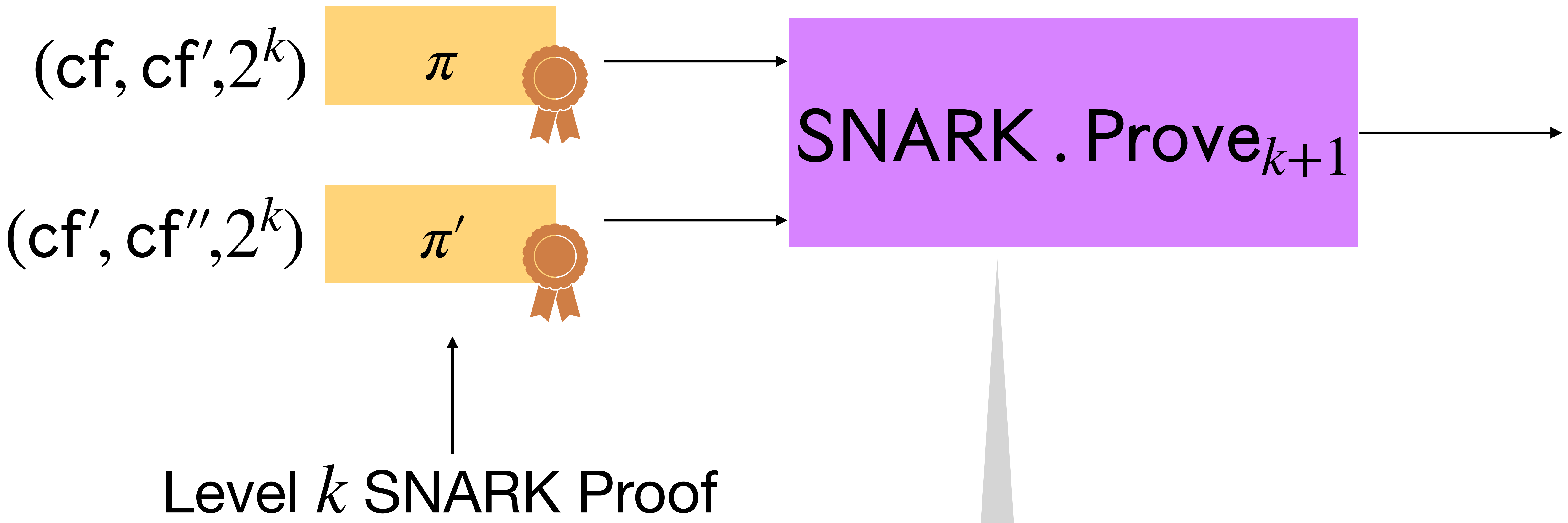

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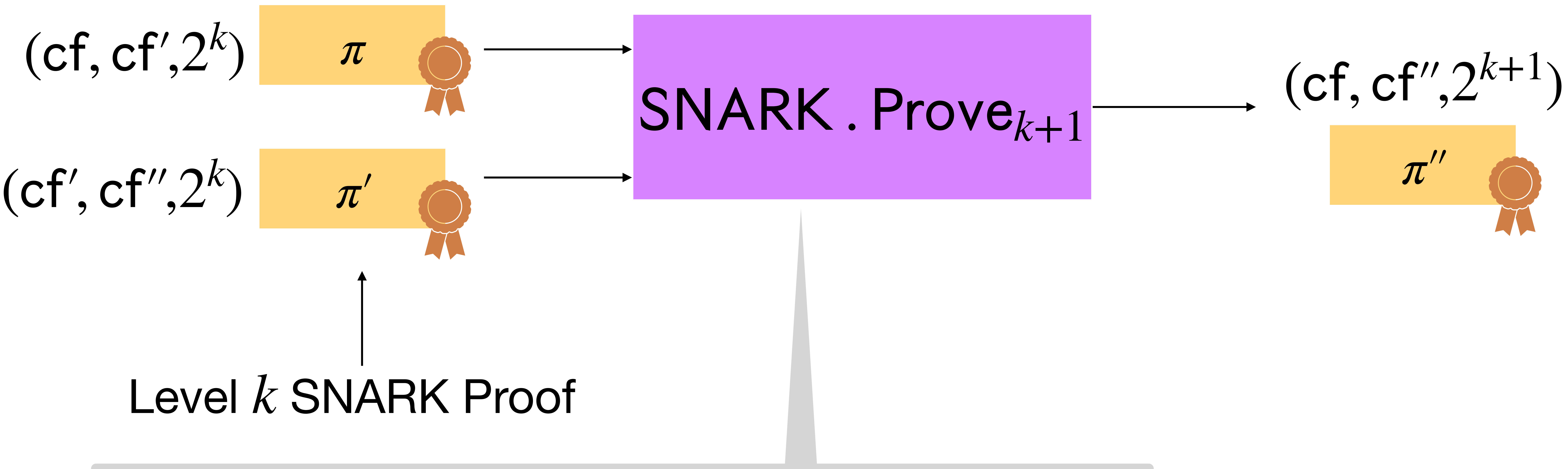

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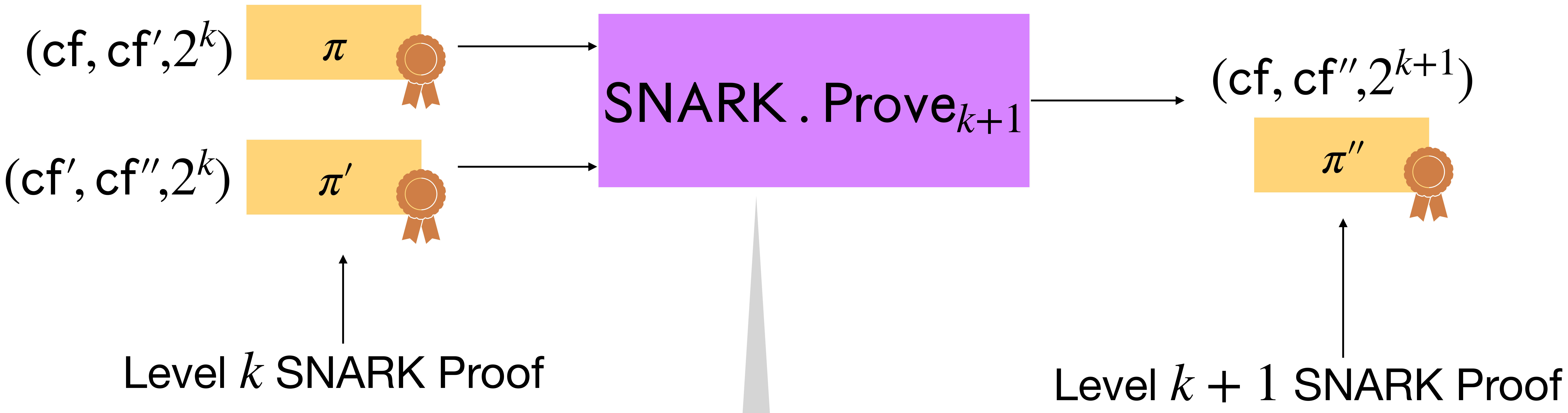

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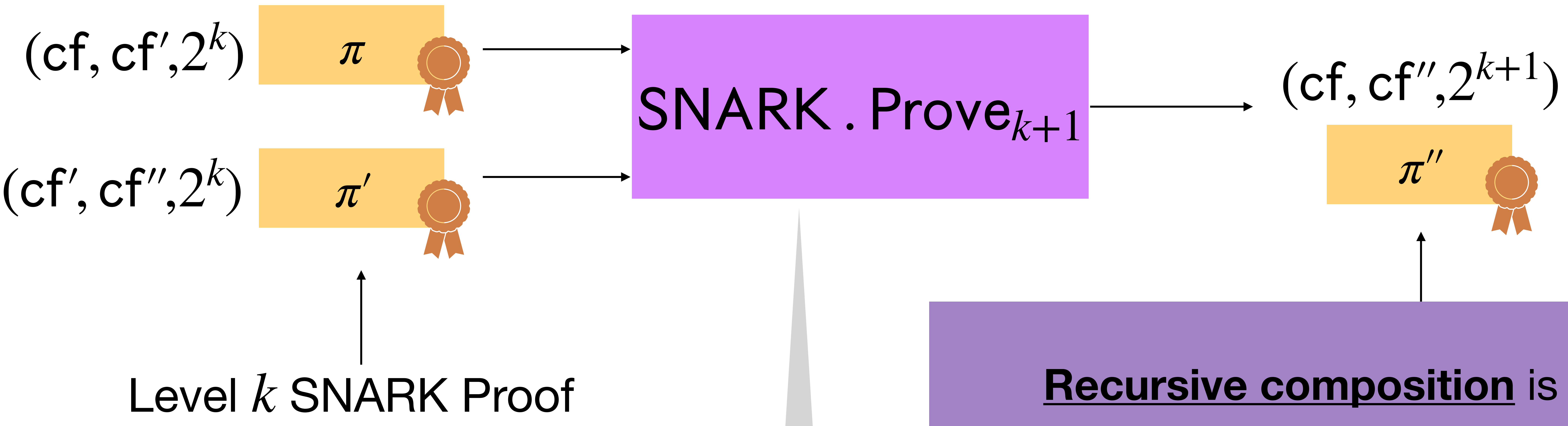

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Recursive composition is the backbone of many works! [CT10, BCCT13,BGH19, BCMS20, BDFG21, BCLMS21, KS22, CCS22, etc]

Proof strategy: EXTRACT!

Level 3

$\pi_{cf_0 \to cf_8}$

This is an intuitive sketch missing many details

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Level 3

$\pi_{\mathsf{cf}_0 \rightarrow \mathsf{cf}_8}$

Given cheating proof, recursively extract

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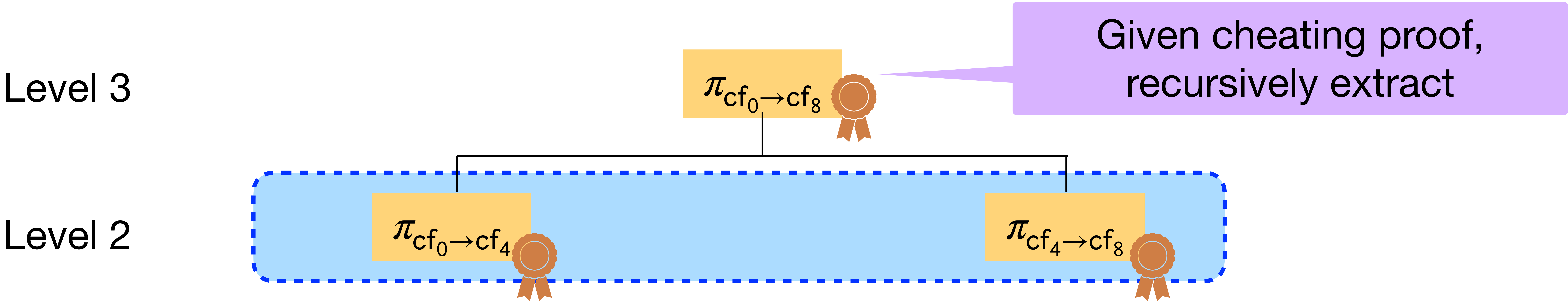

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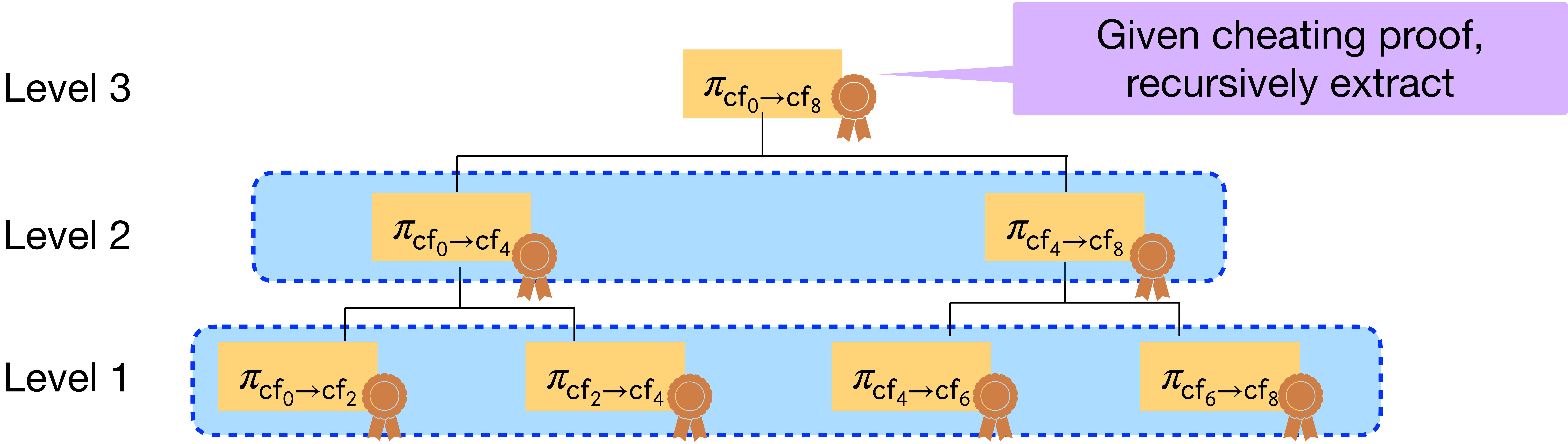

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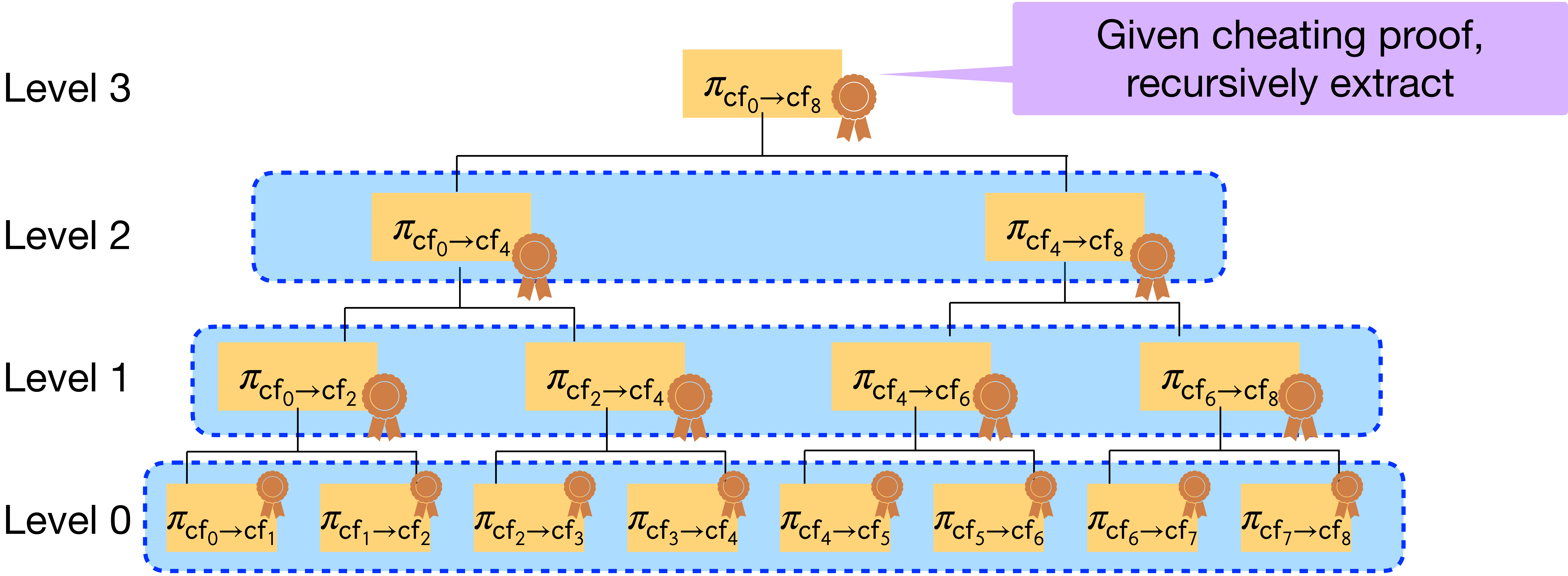

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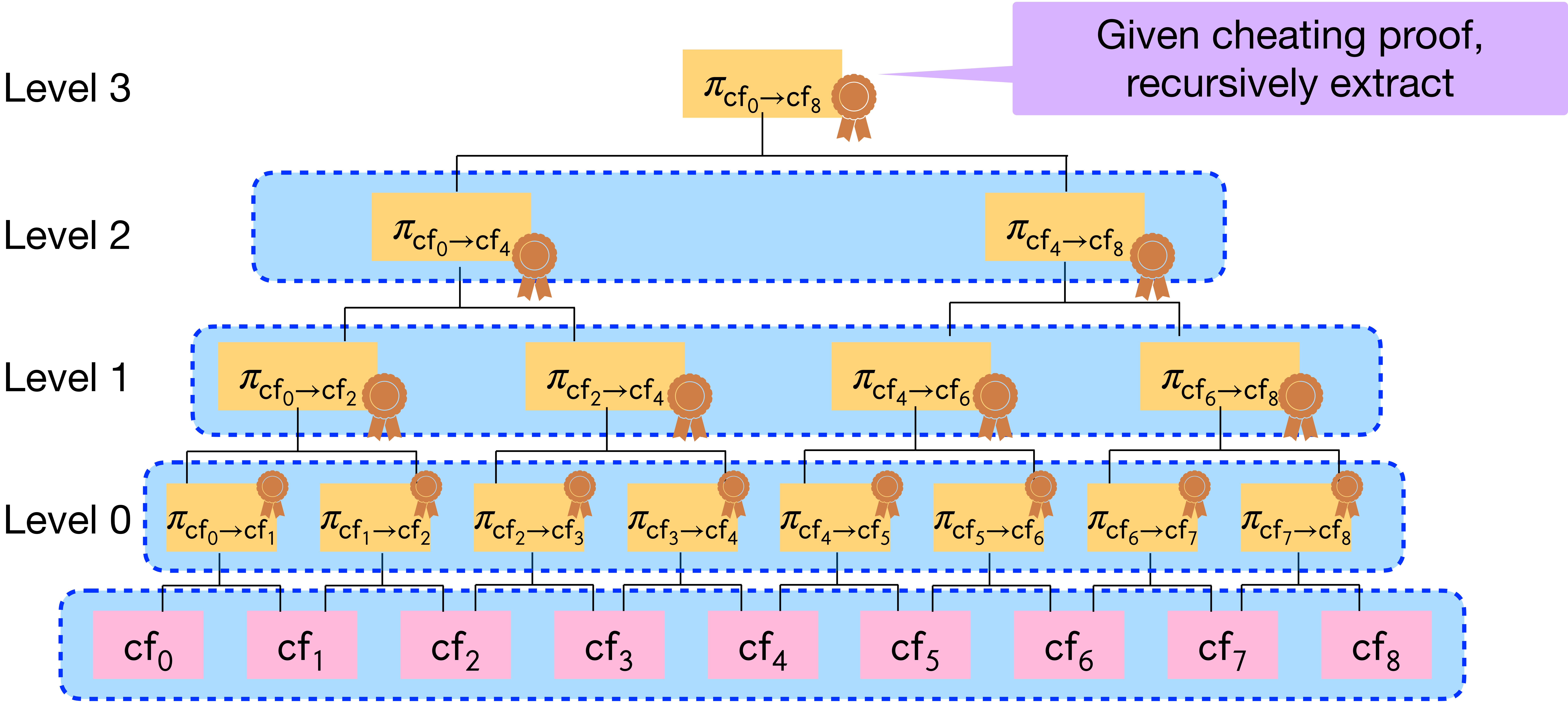

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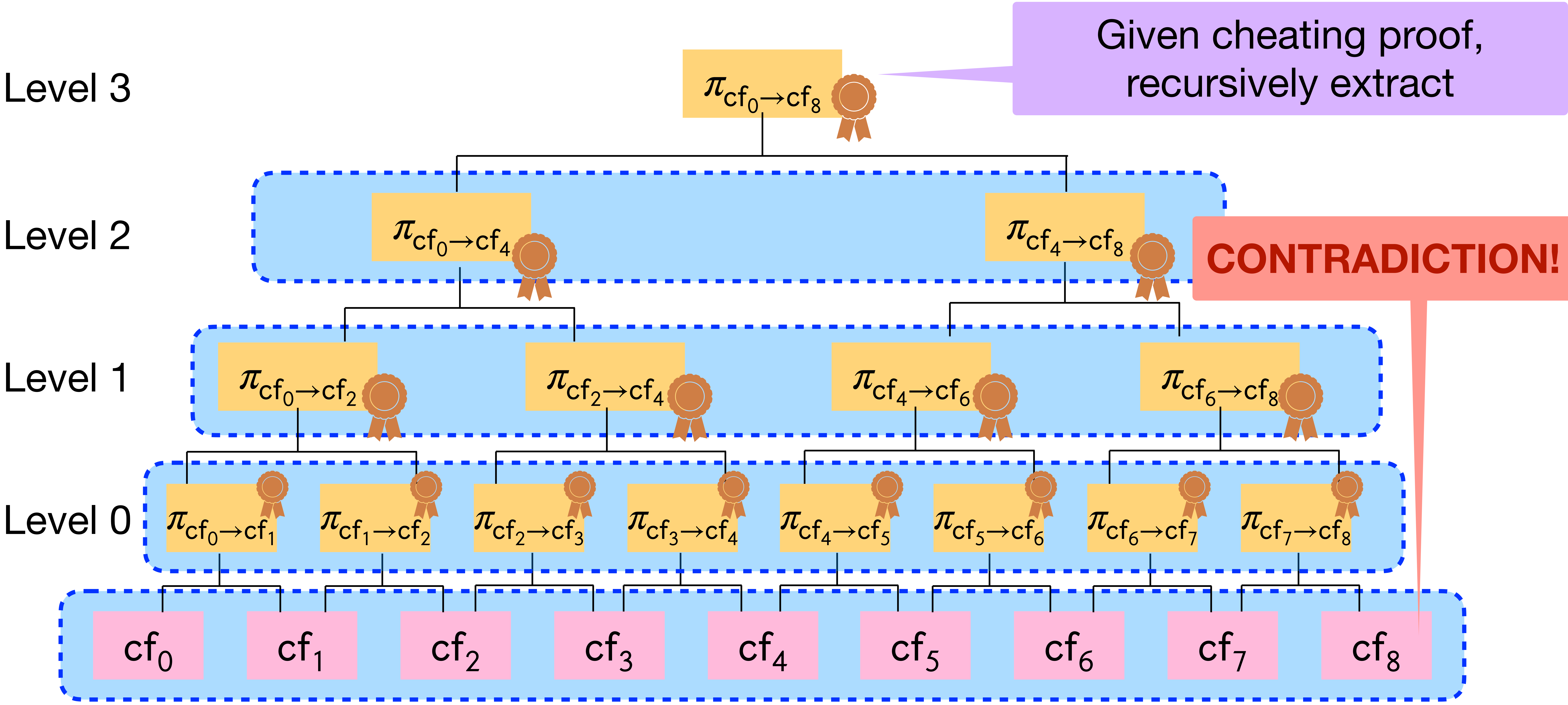

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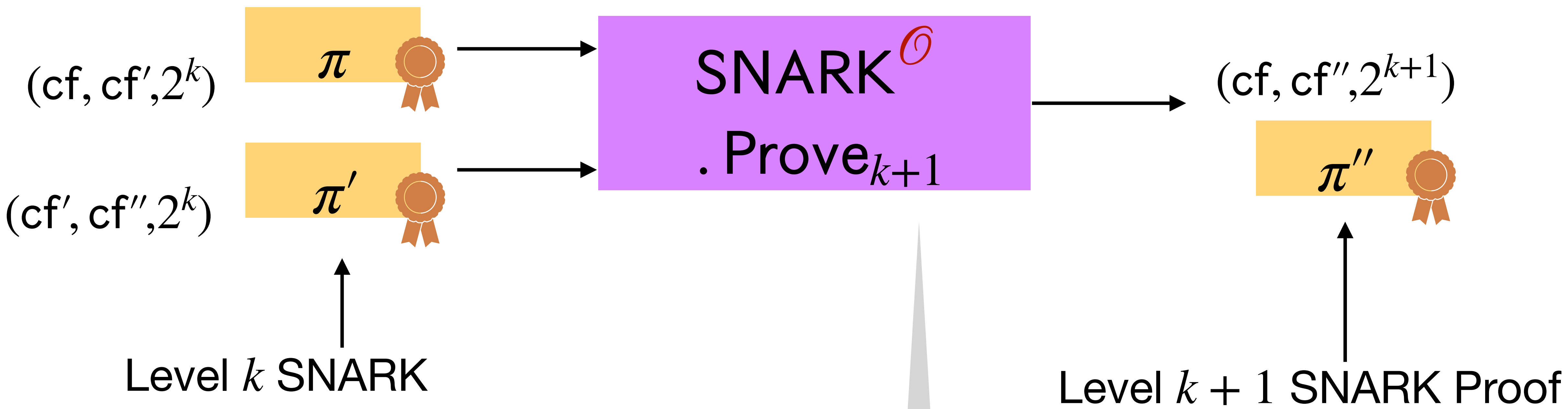

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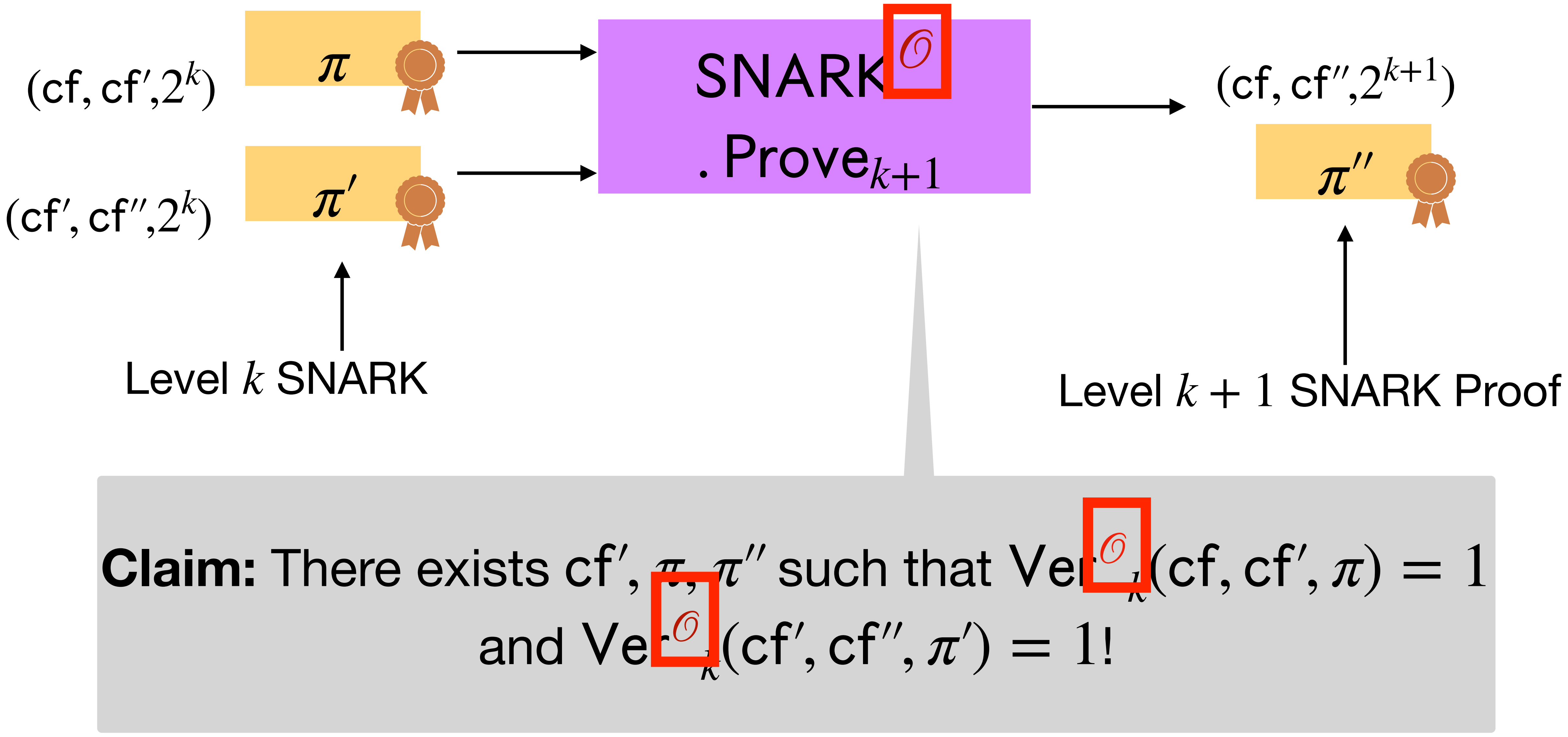

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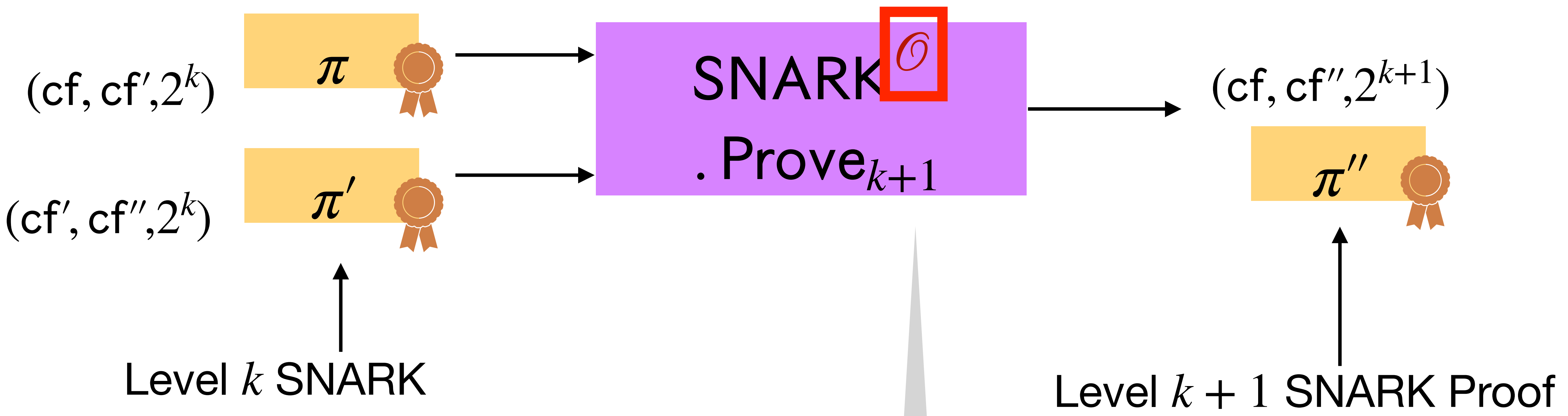

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  - Shows that SNARGs and IVC are fundamentally **different** problems

This Work:
How do we construct **IVC for NP**
from **standard assumptions**?

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- We show how to achieve ZK in both settings.

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Theorem [DJJMKS25]. There exists IVC for NP assuming subexponential hardness of:

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Independent of time-steps, but grows with intermediate configurations cf_i

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Theorem [PP22/DGKV22]. Assuming rate-1 somewhere extractable BARGs, there exists IVC for *deterministic* computations.

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- Addresses the common misconception that IVC for NP is **impossible** due to Gentry-Wichs.

Valiant's Recipe: Proof Merging

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- **Proof of knowledge:** If adversary gives **accepting** $(\mathsf{cf}, \mathsf{cf}'', t + t'), \pi''$, one can **extract** accepting tuples $(\mathsf{cf}, \mathsf{cf}', t), \pi$ and $(\mathsf{cf}', \mathsf{cf}'', t'), \pi'$.

- **Succinctness:** $|\pi''| \approx |\pi|, |\pi'|$

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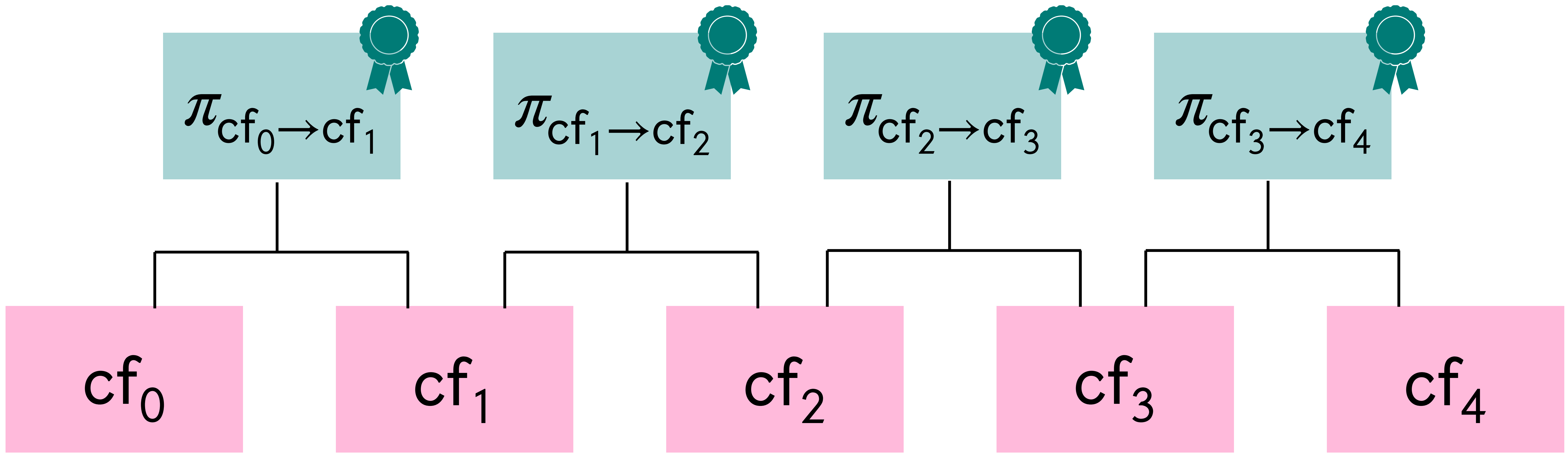
Achieved if BARG is rate-1

Construction

cf_0 cf_1 cf_2 cf_3 cf_4

Construction

Base case:
SNARG for NP!



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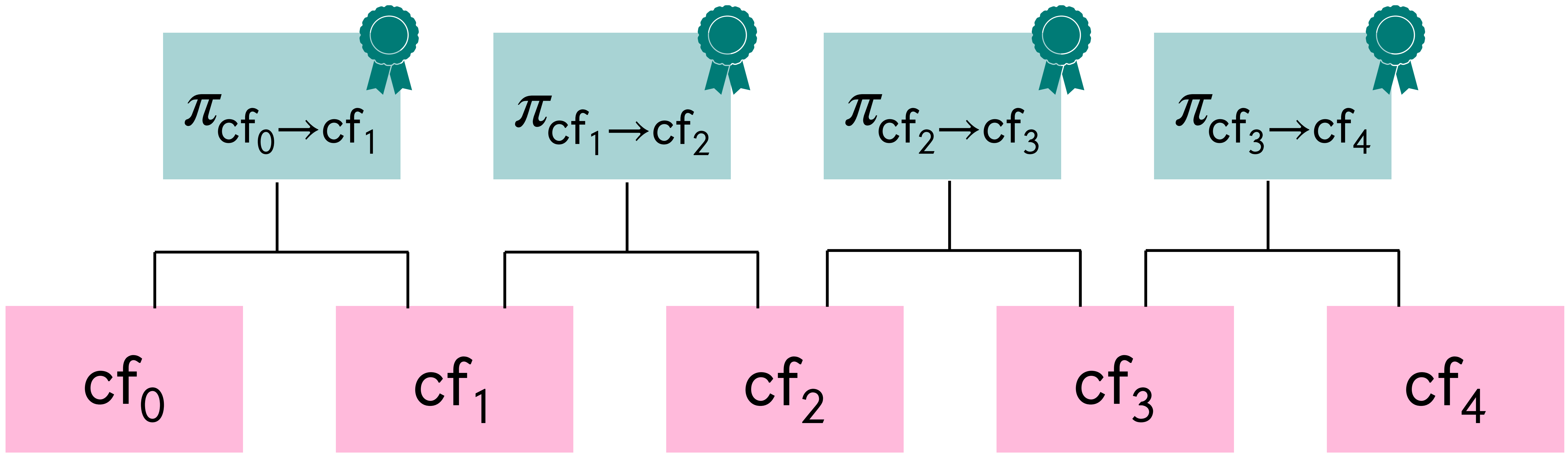

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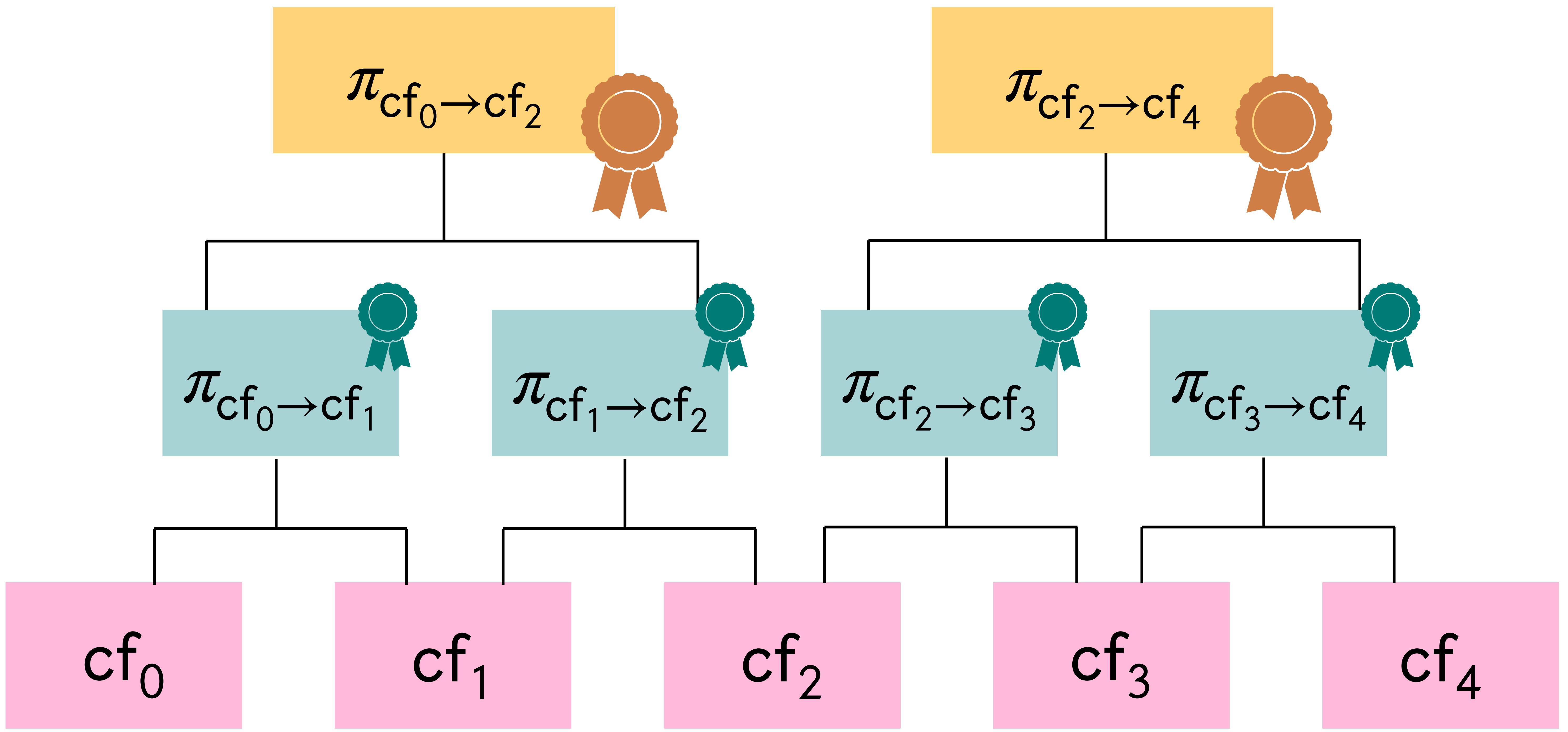

Construction

Level 1

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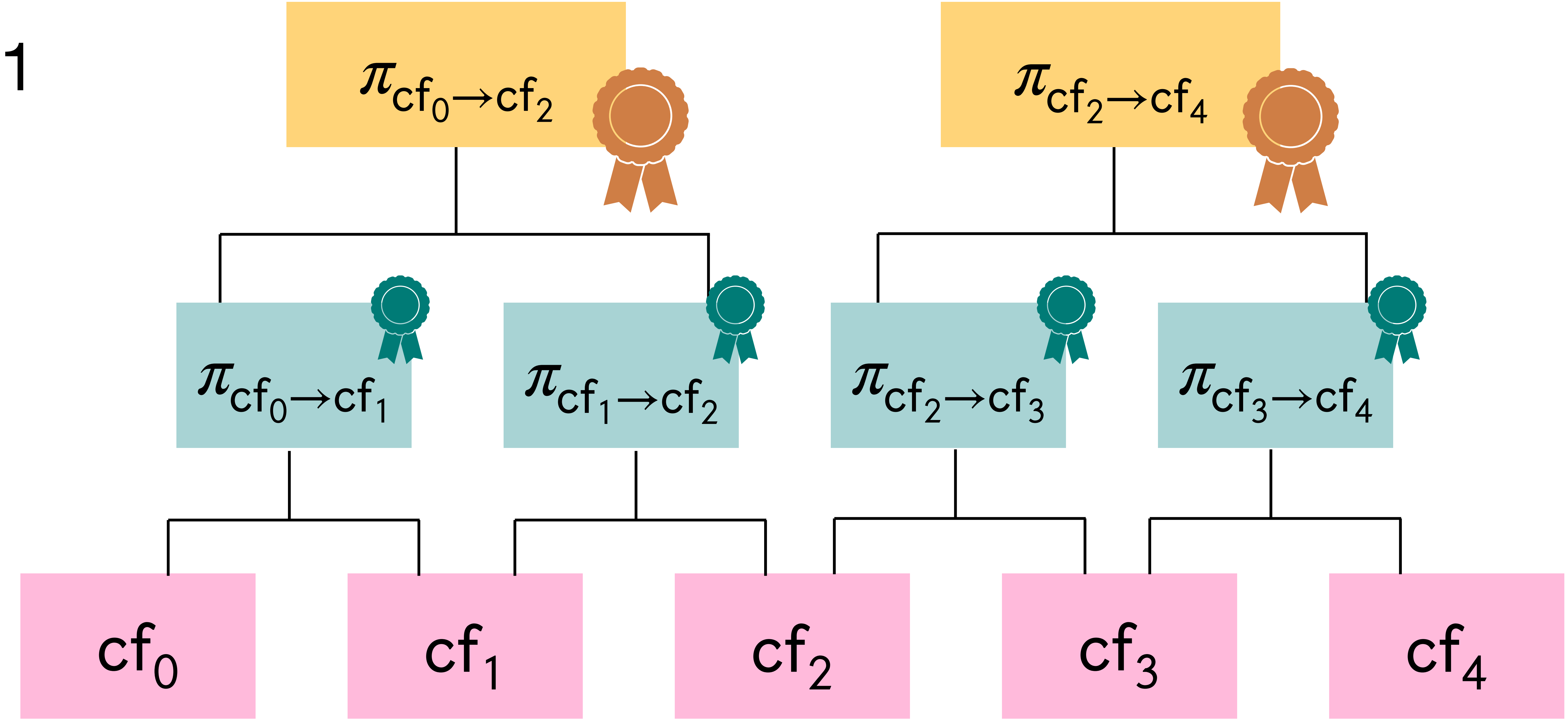

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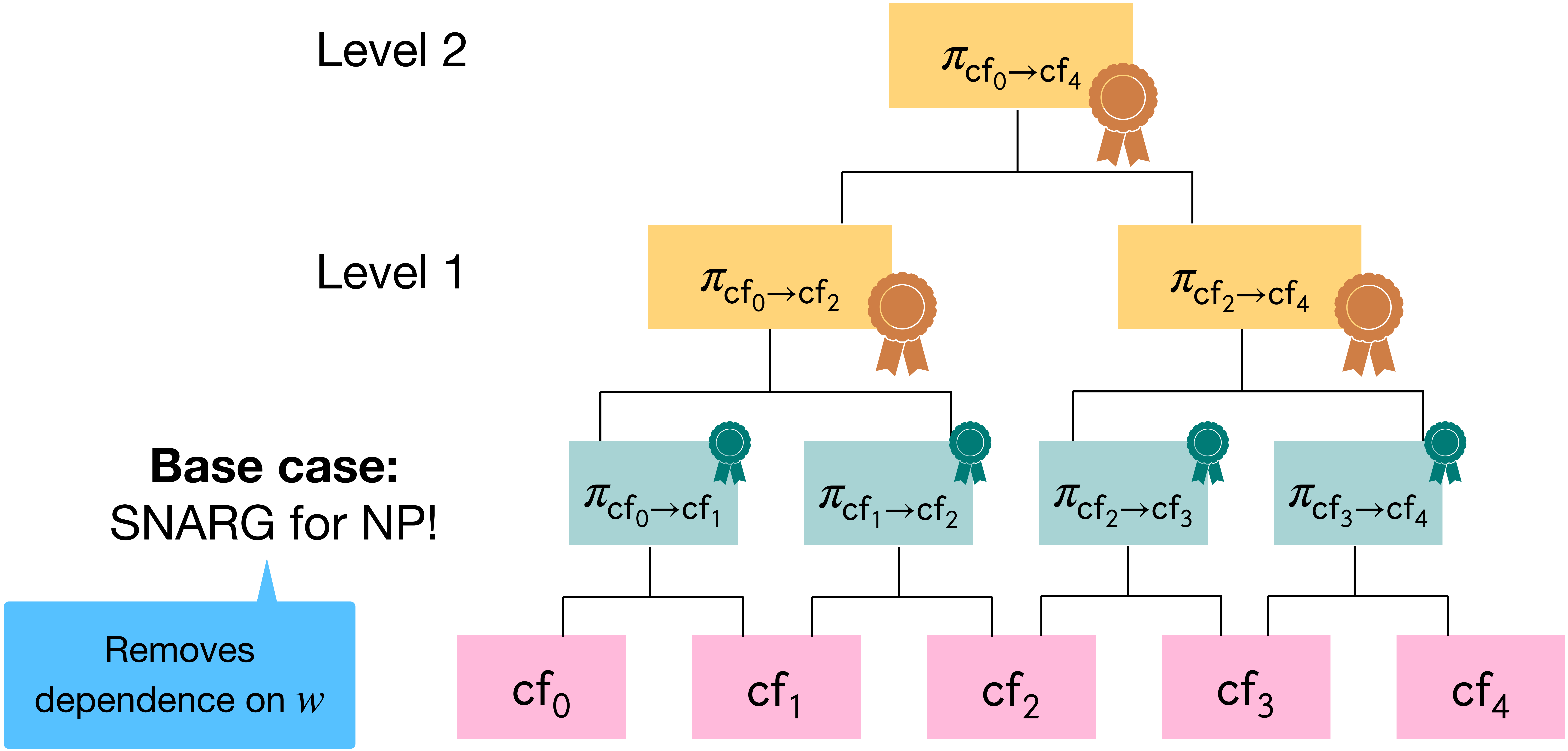

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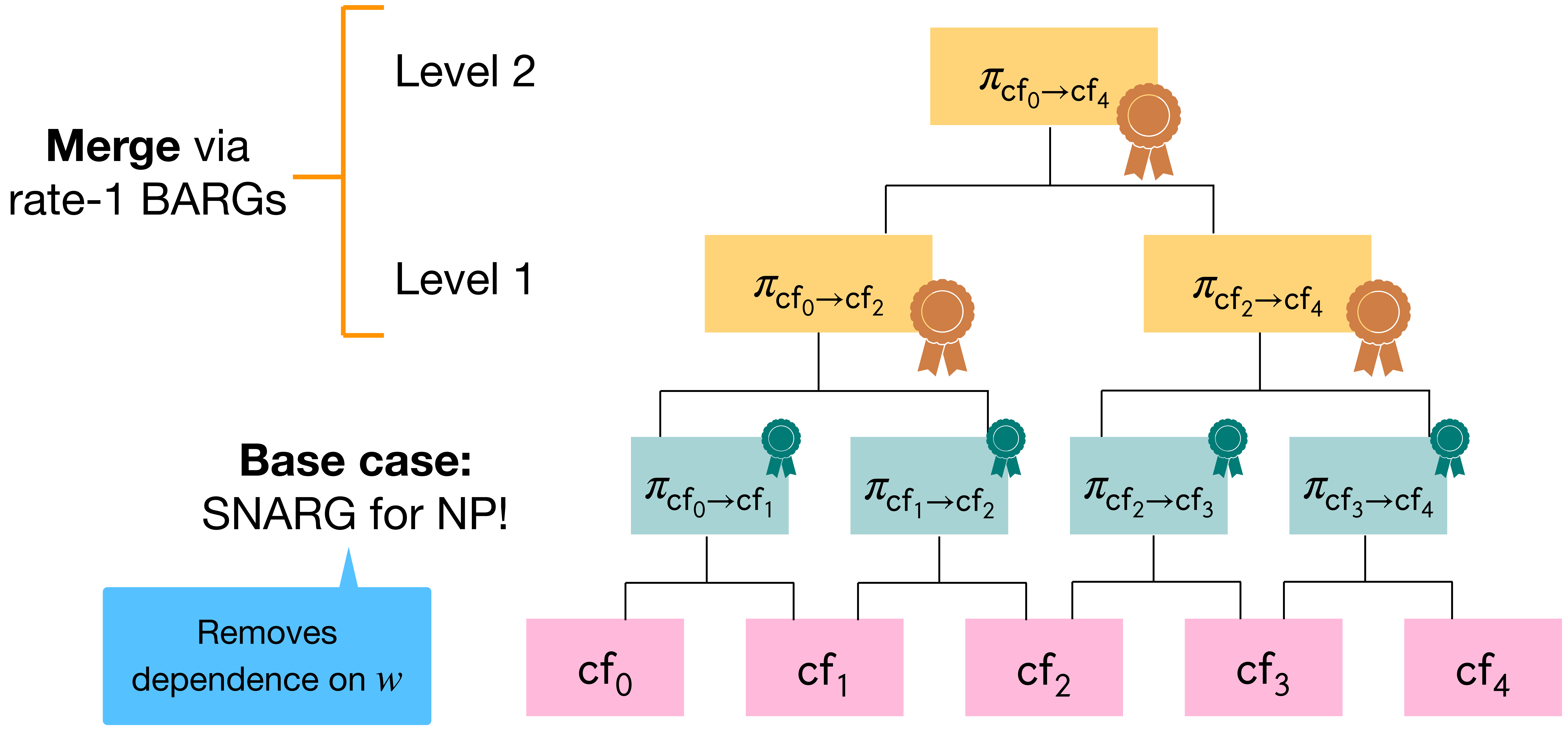
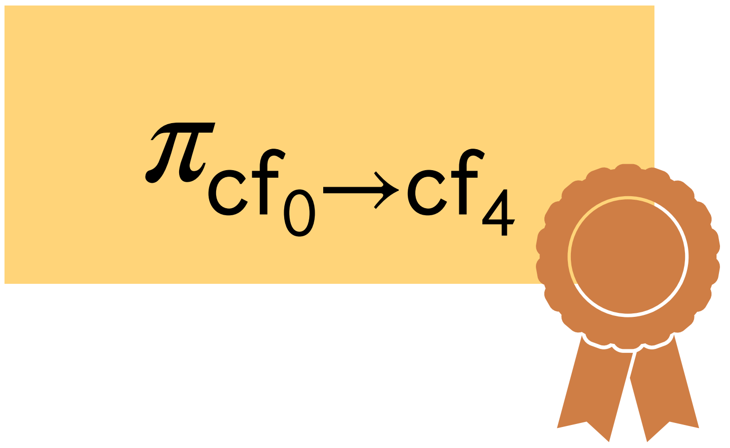


Security sketch with “Somewhere Soundness”

Level 2



Level 1

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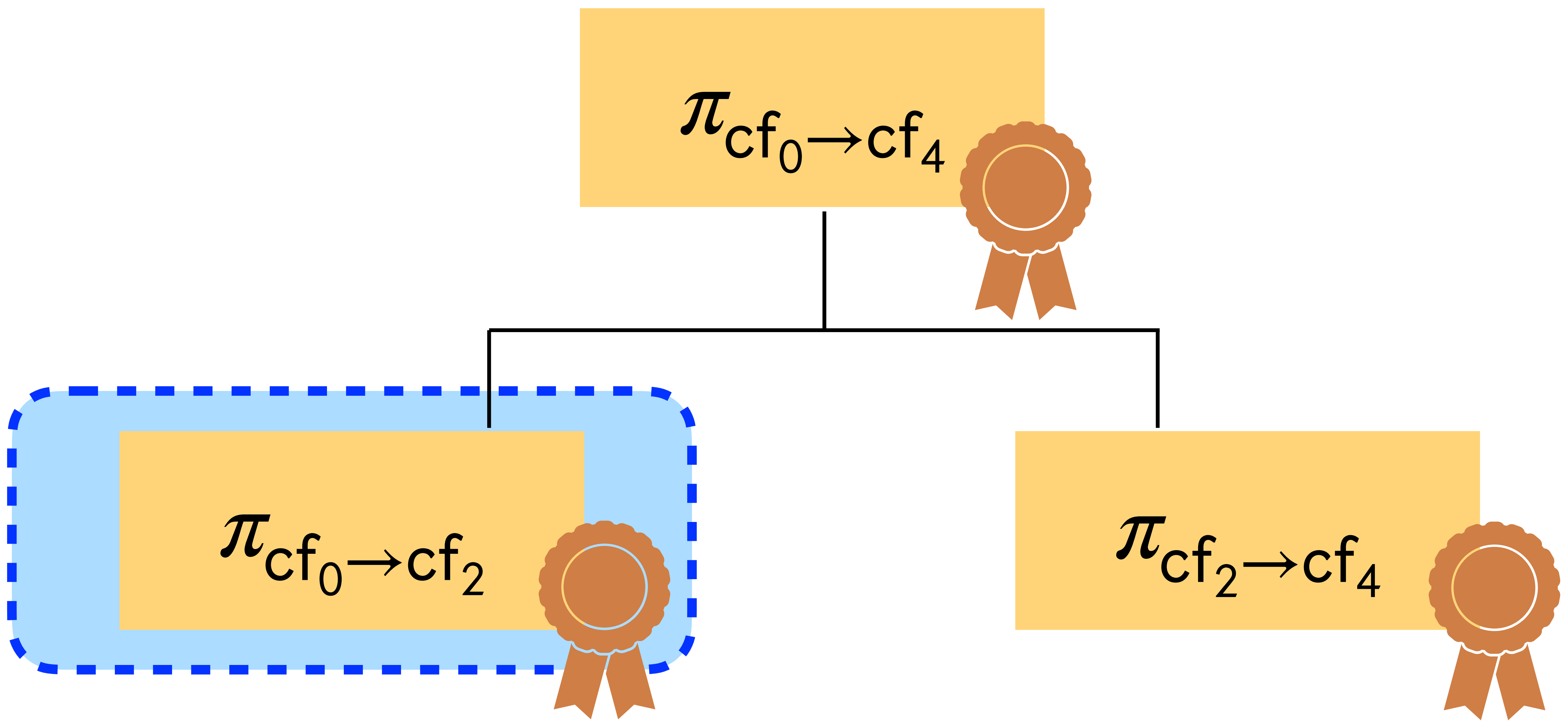

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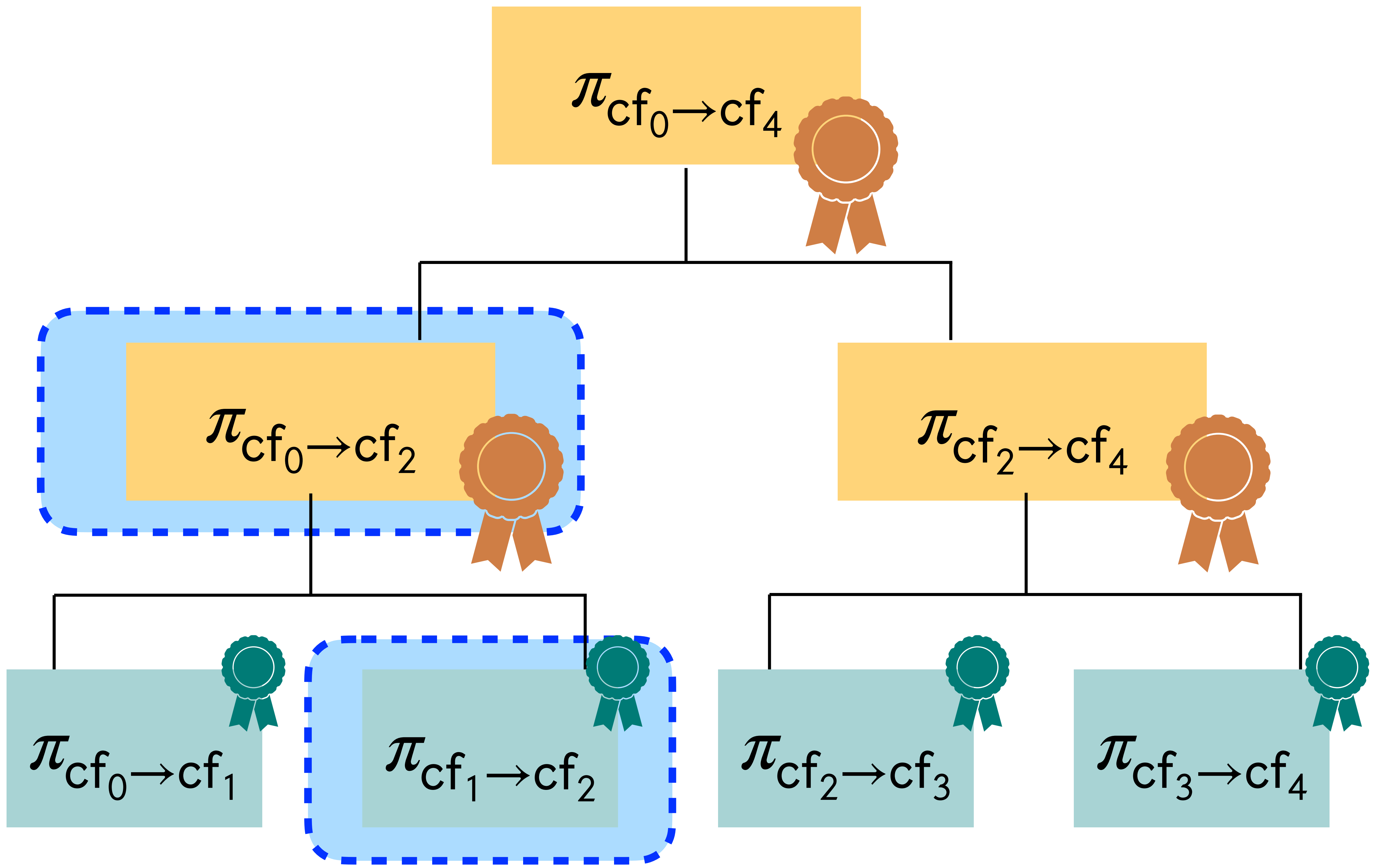

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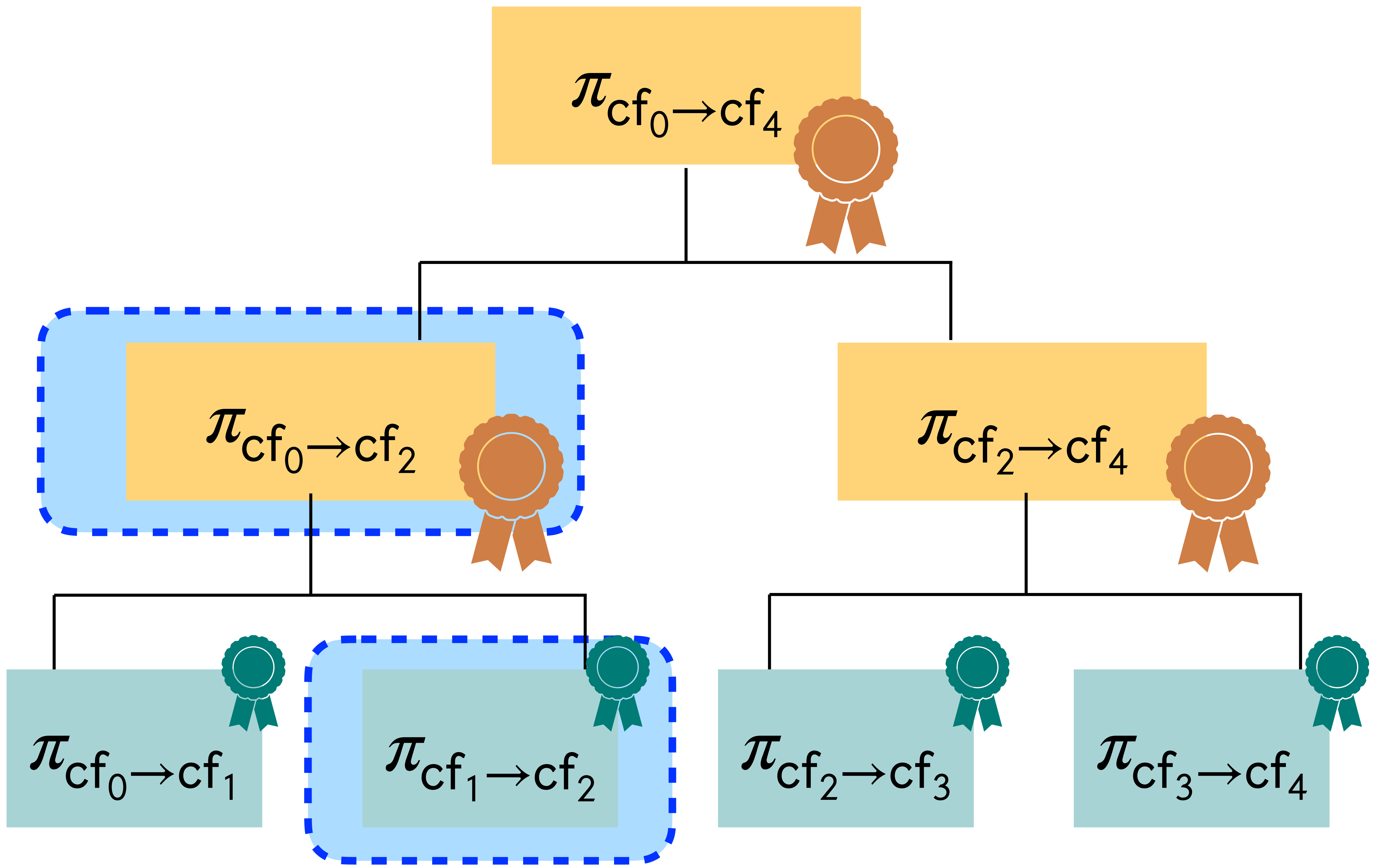
Base case:
SNARG for NP!



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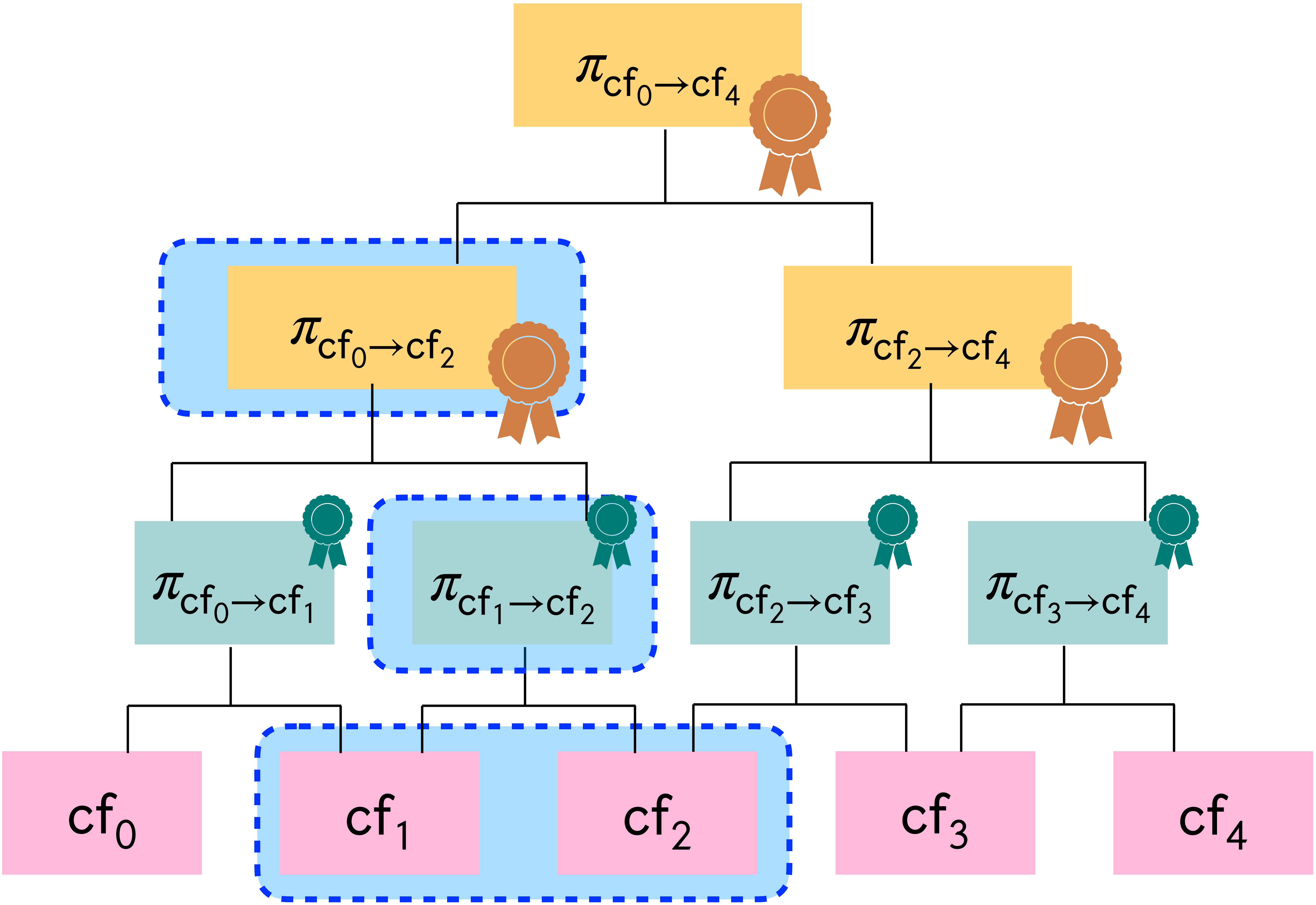

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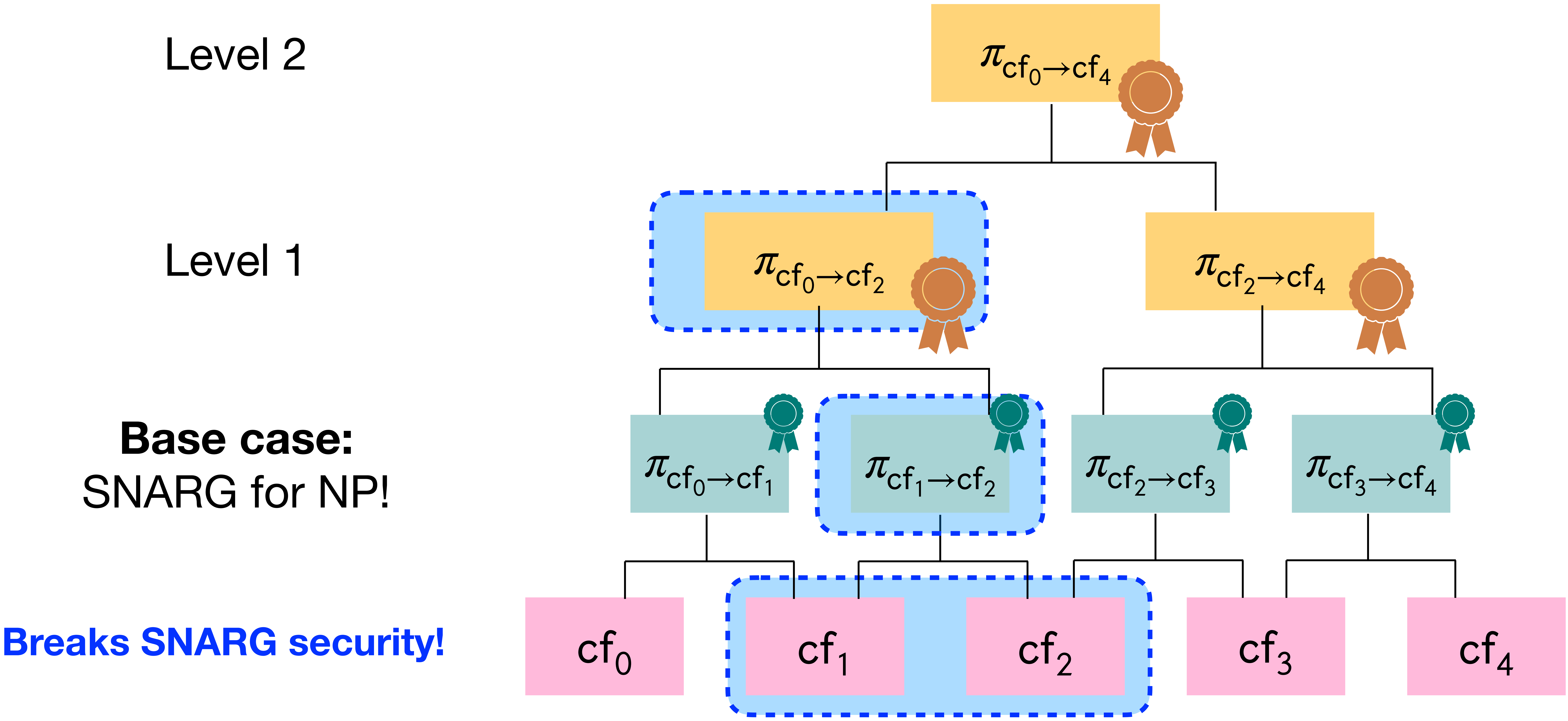

Security sketch with “Somewhere Soundness”

Our main technical contribution (over [DGKV/PP22]):
We show this works for *NP* with careful complexity leveraging

Level 1

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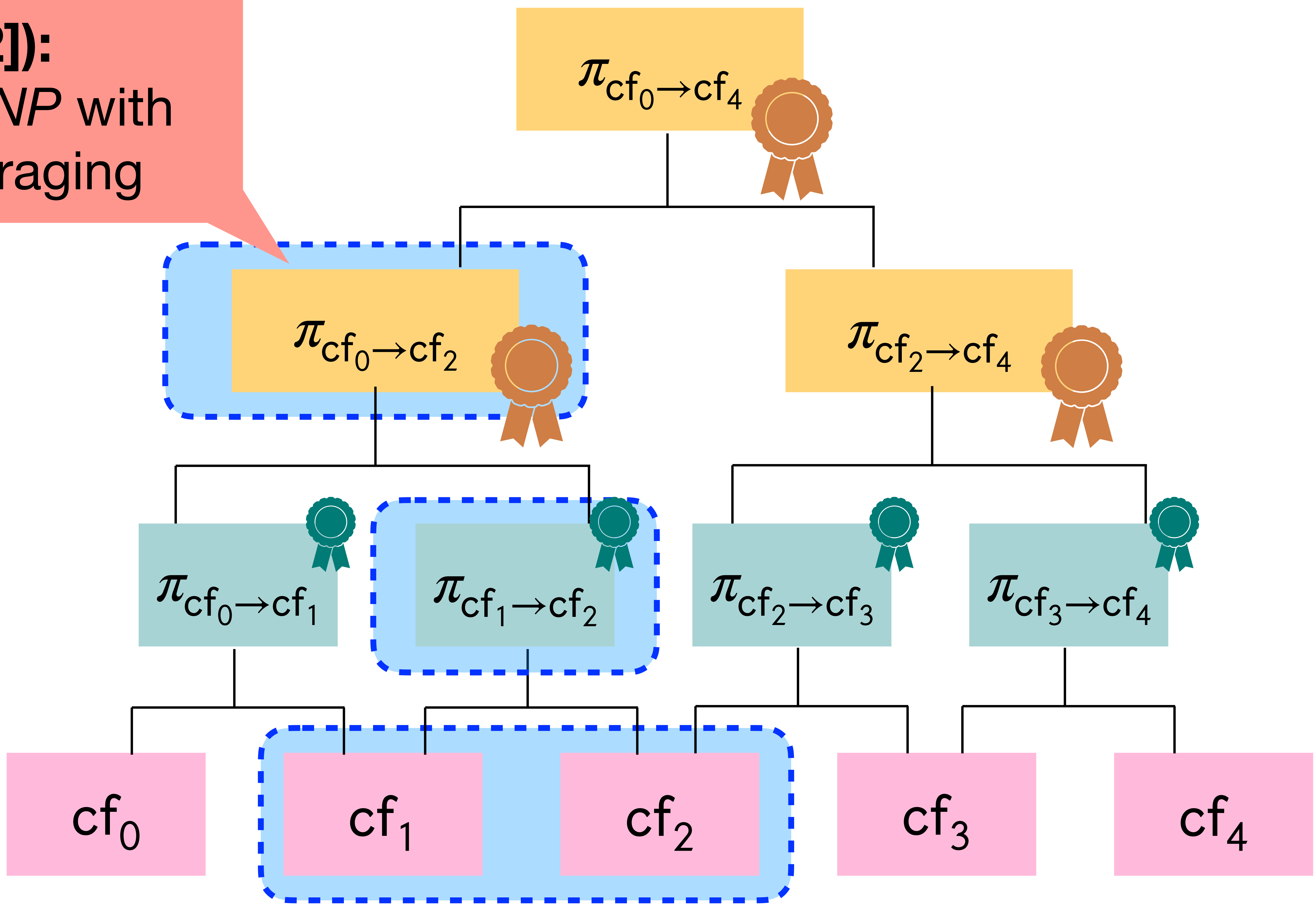
Breaks SNARG security!



IVC for NP!

Theorem [DJJMKS25]. There exists IVC for NP assuming subexponential hardness of:

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  - SNARGs without extraction? Smells like iO :)

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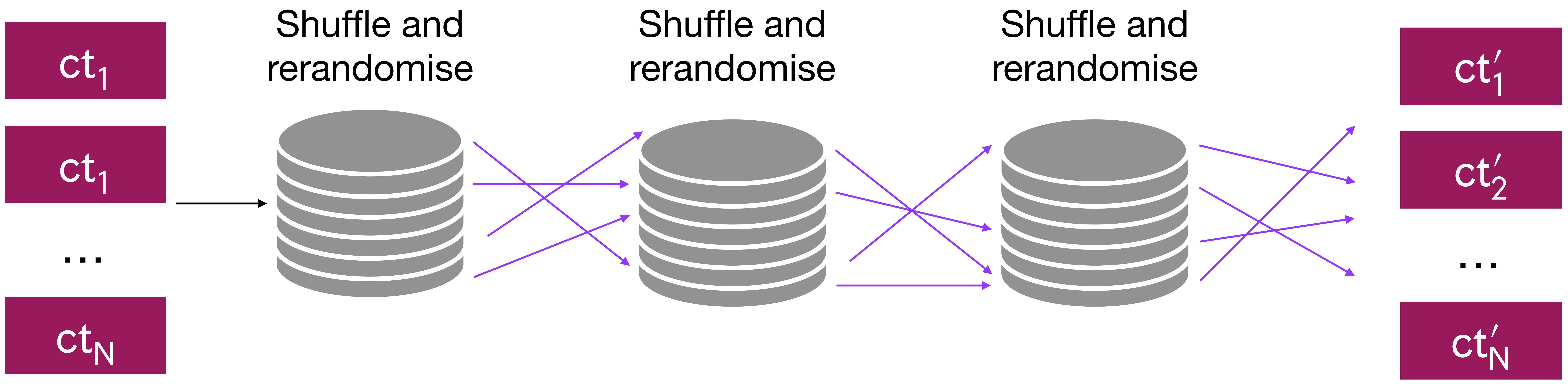
    - Can generate the CRS and guarantee soundness *only knowing* that a trapdoor exists

Verifiable Shuffling



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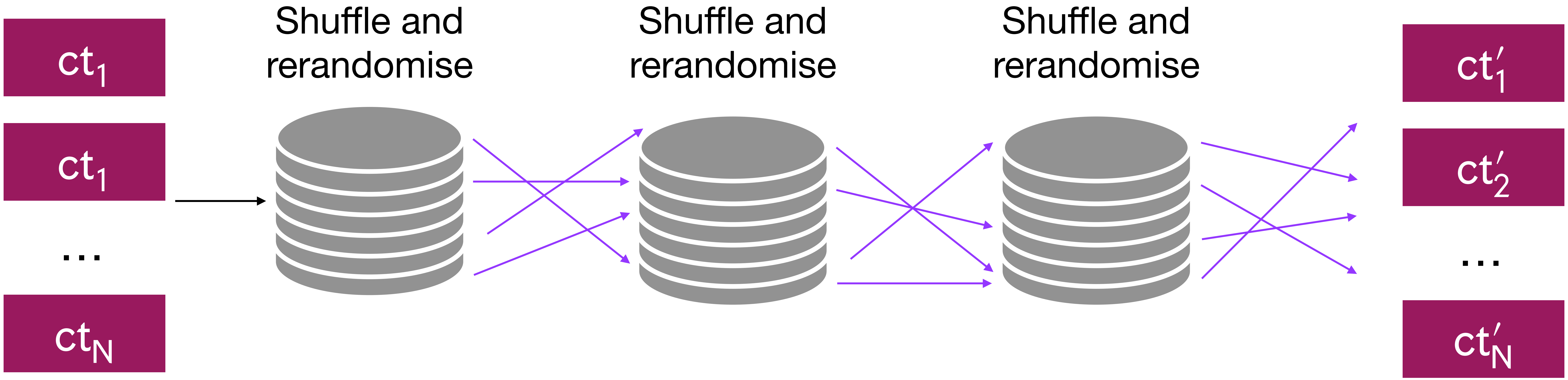

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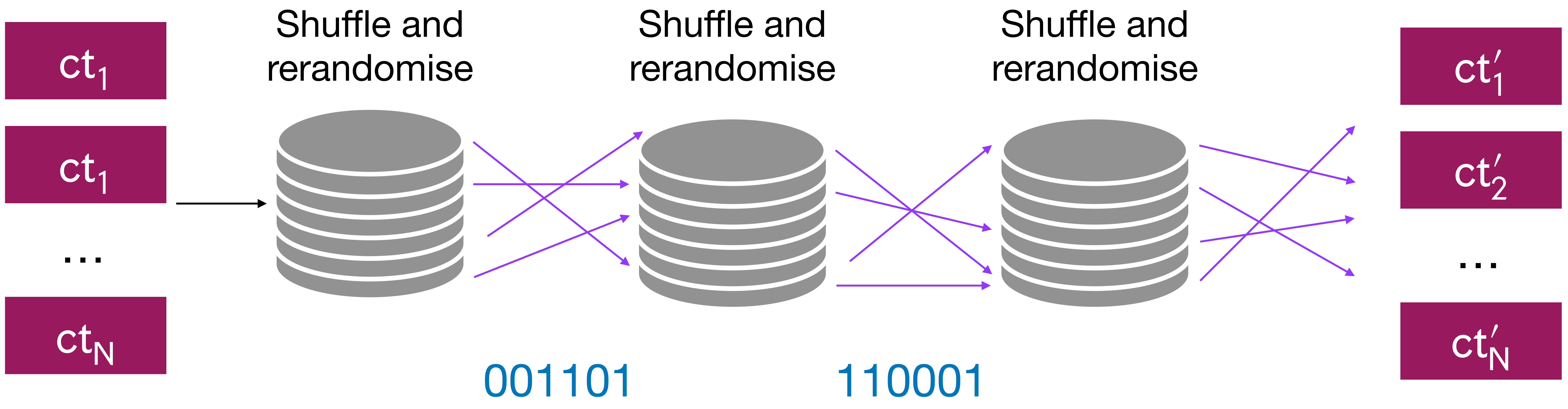

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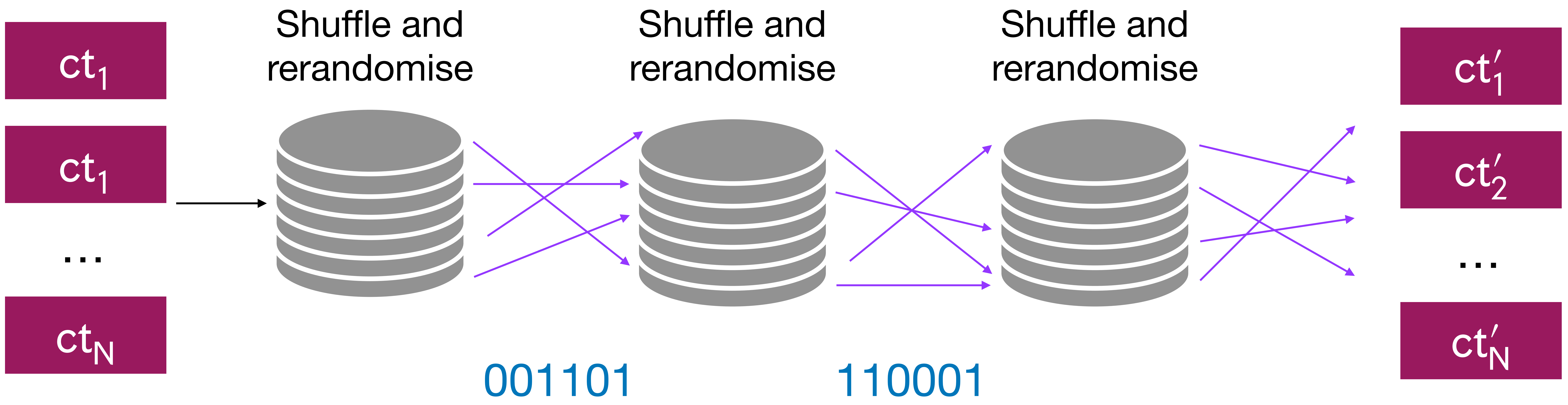

- Trapdoor = Secret key of rerandomisable scheme!
- Our work gives a **multi-hop** verifiable shuffling scheme with **short proofs** from **standard assumptions** in the plain model.

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Theorem [DJJMKS25]. There exists IVC for ***trapdoor computations*** $\mathscr{M}$ assuming subexponential hardness of iO and injective PRGs.

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  - **Idea:** Do proof *without* extraction!

Construction

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$\texttt{IVC}.V$

Construction

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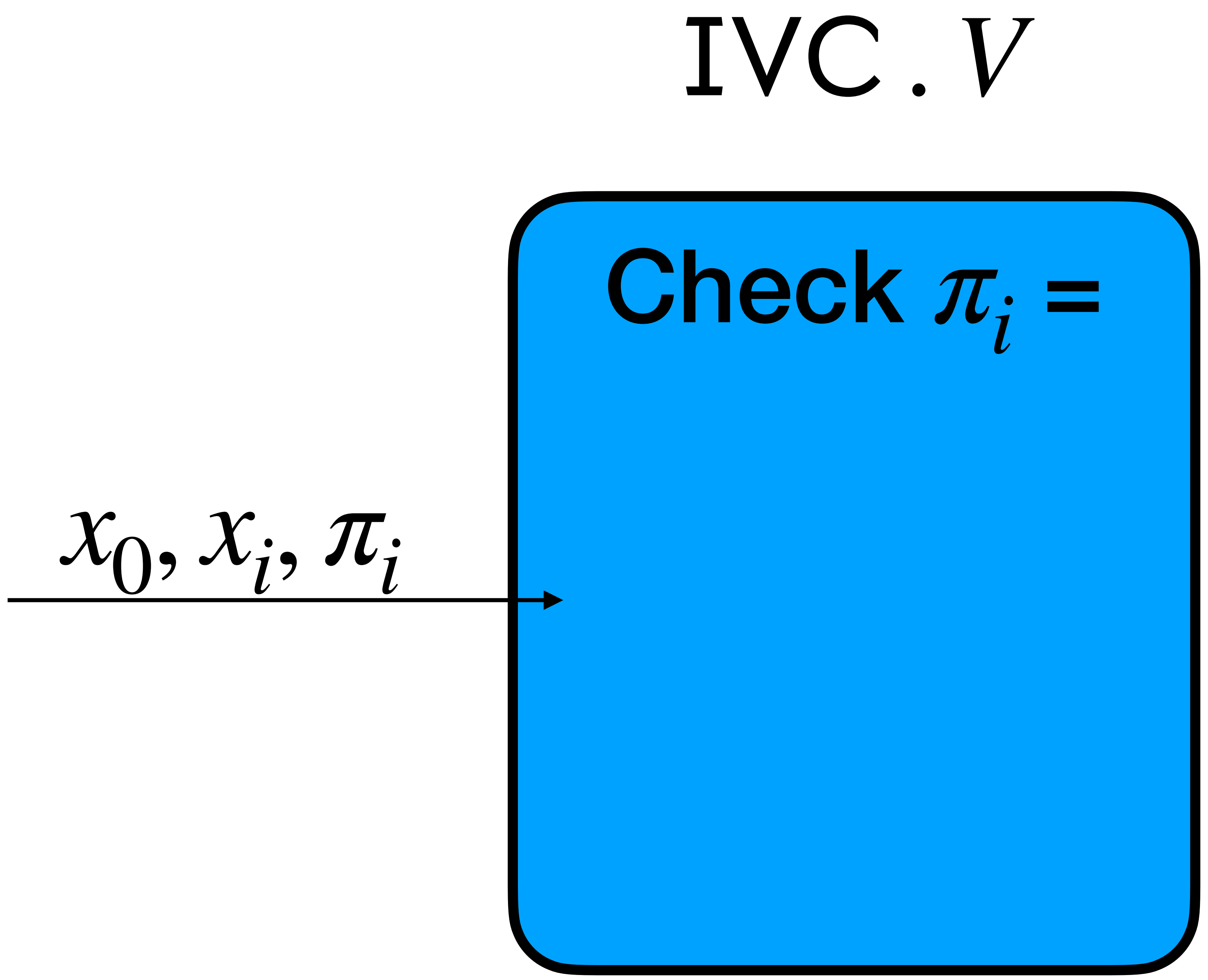
x_0, x_i, π_i →

Construction



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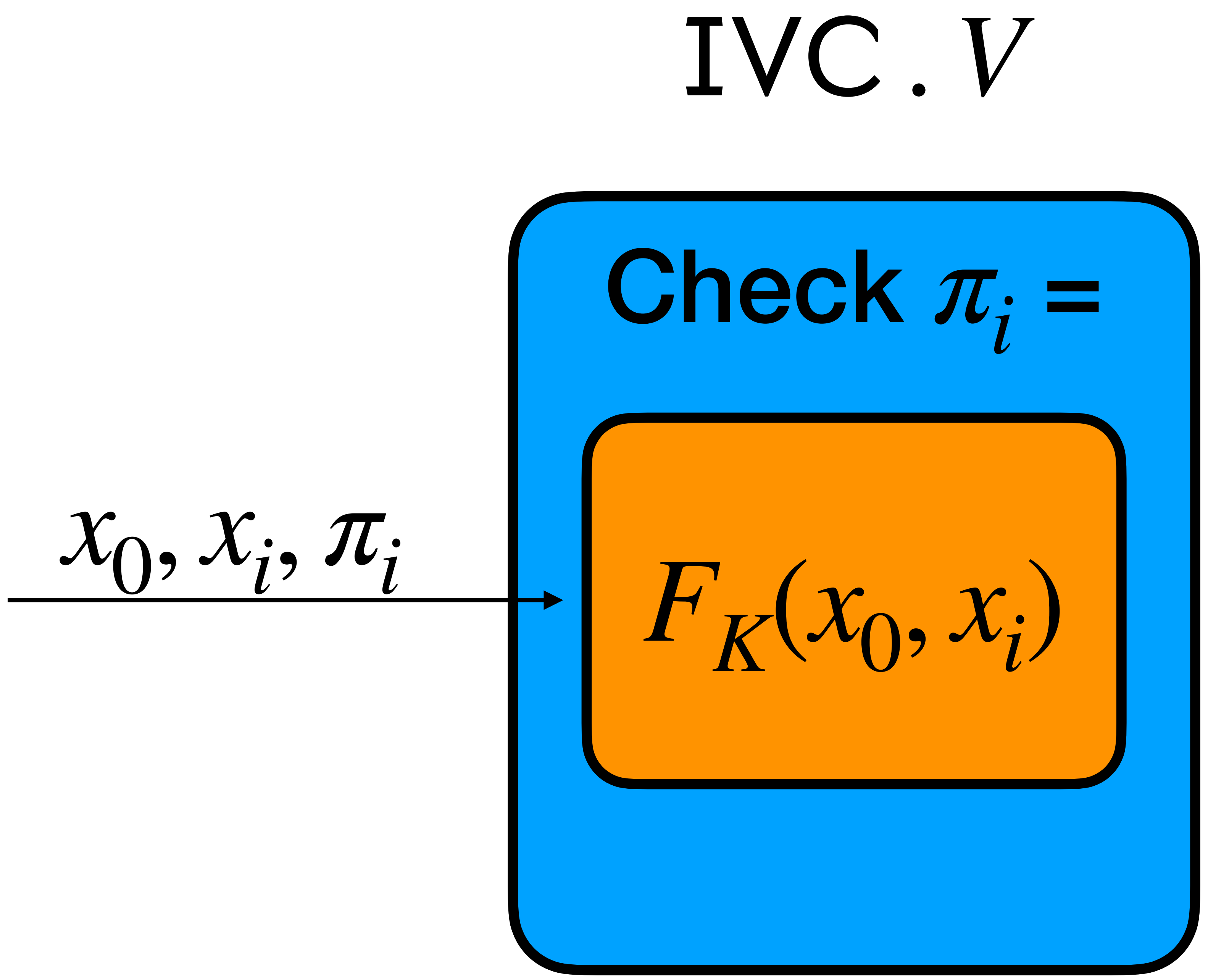

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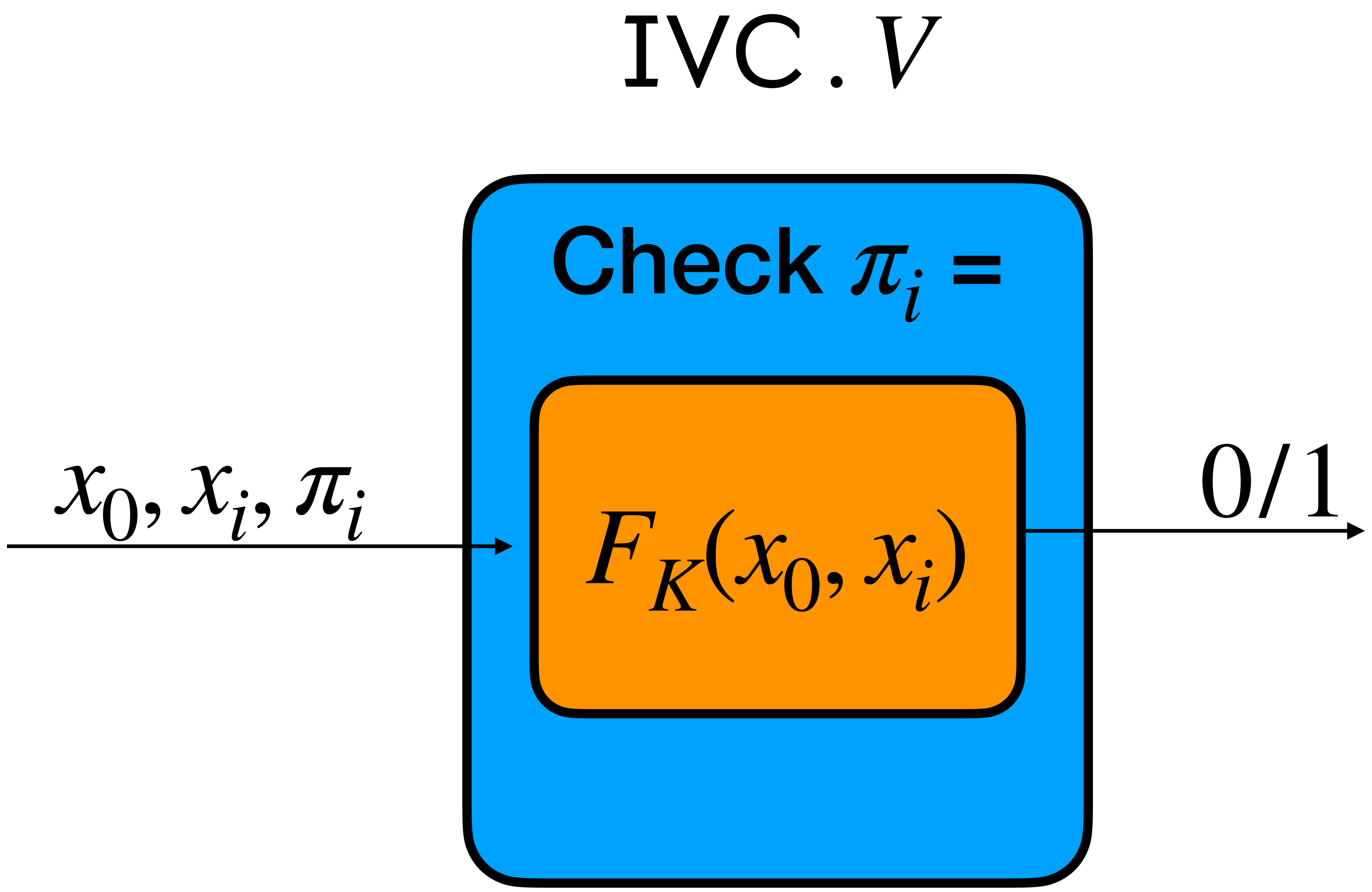

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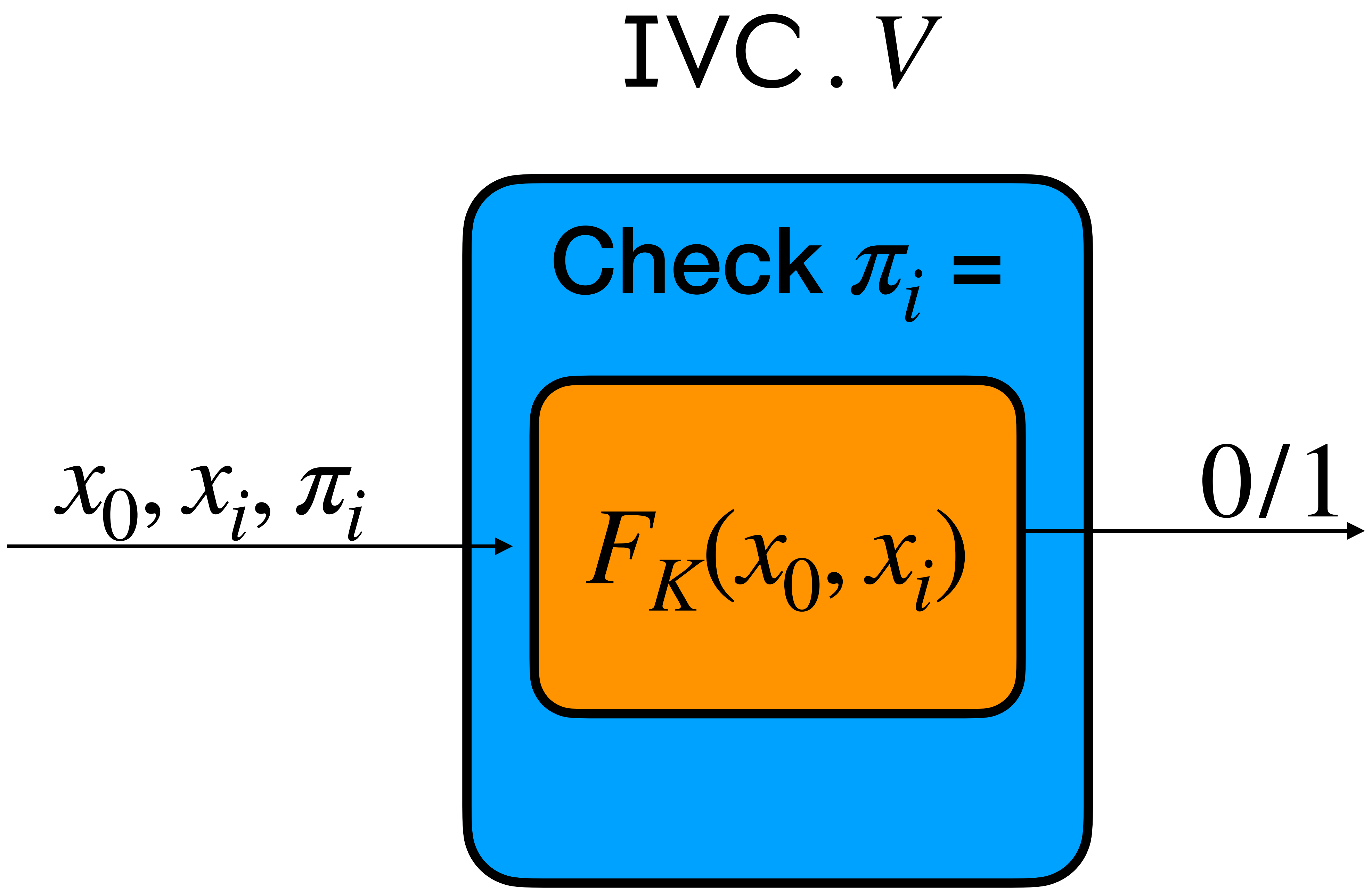

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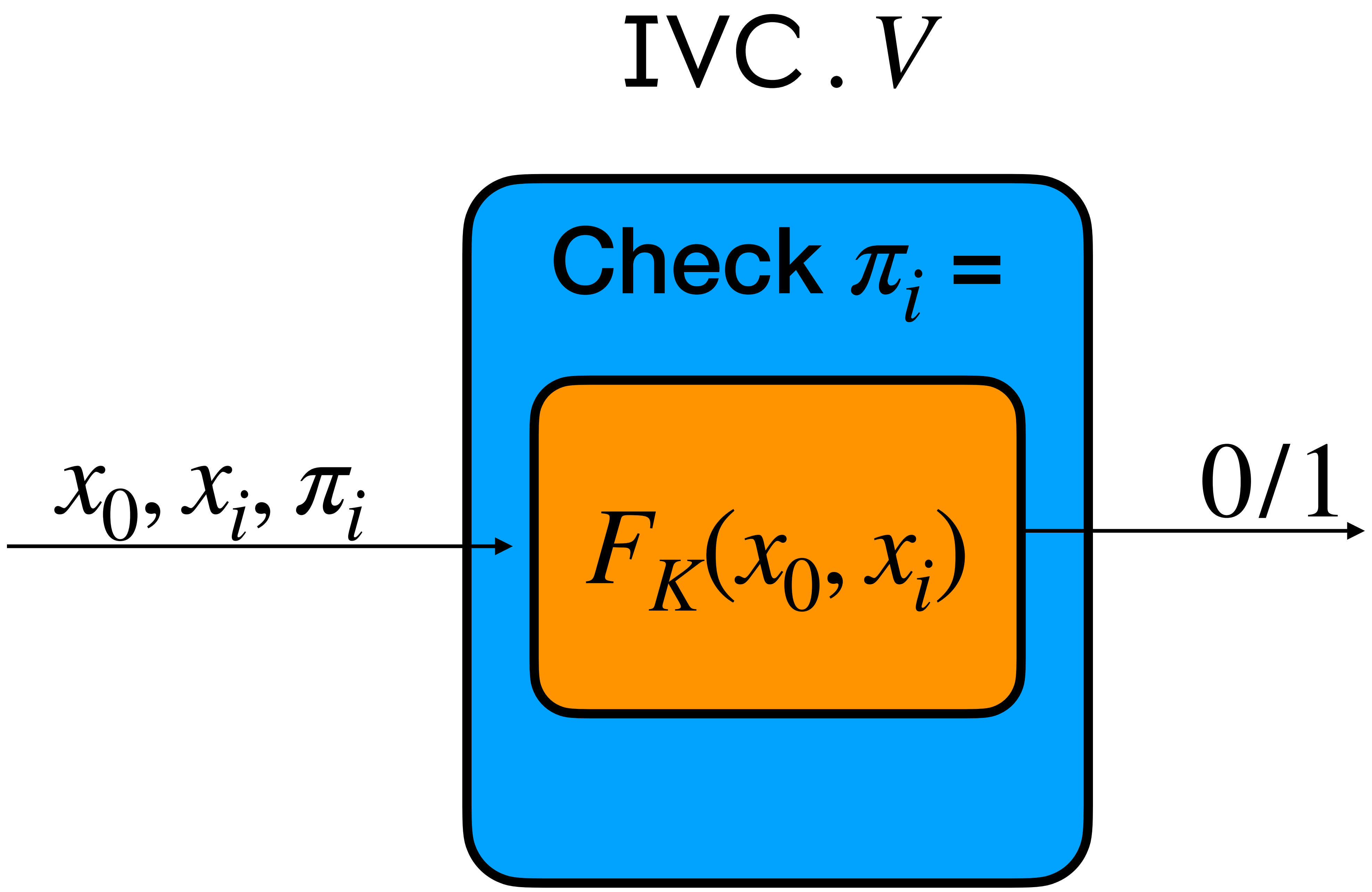

IVC.P

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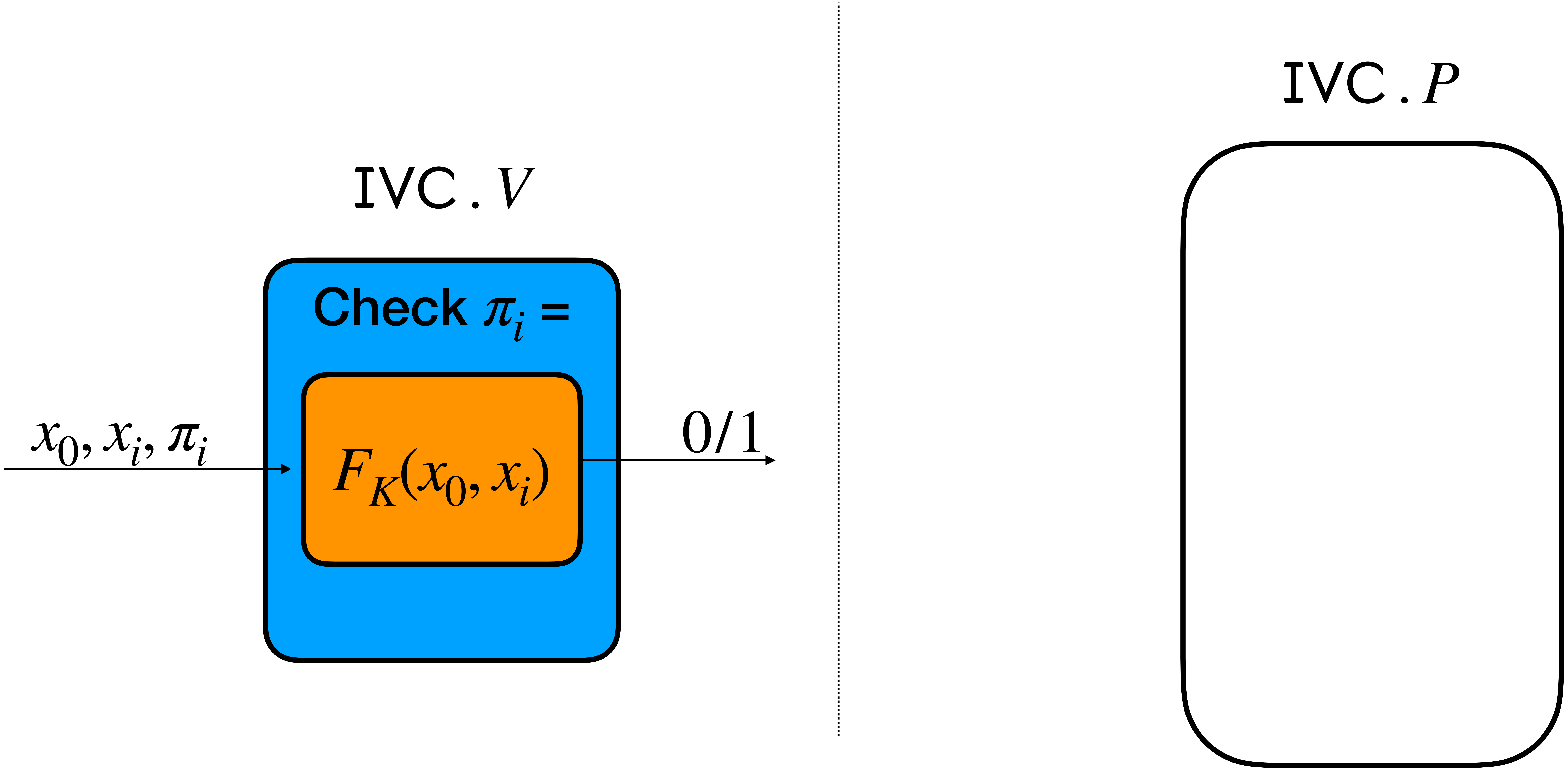

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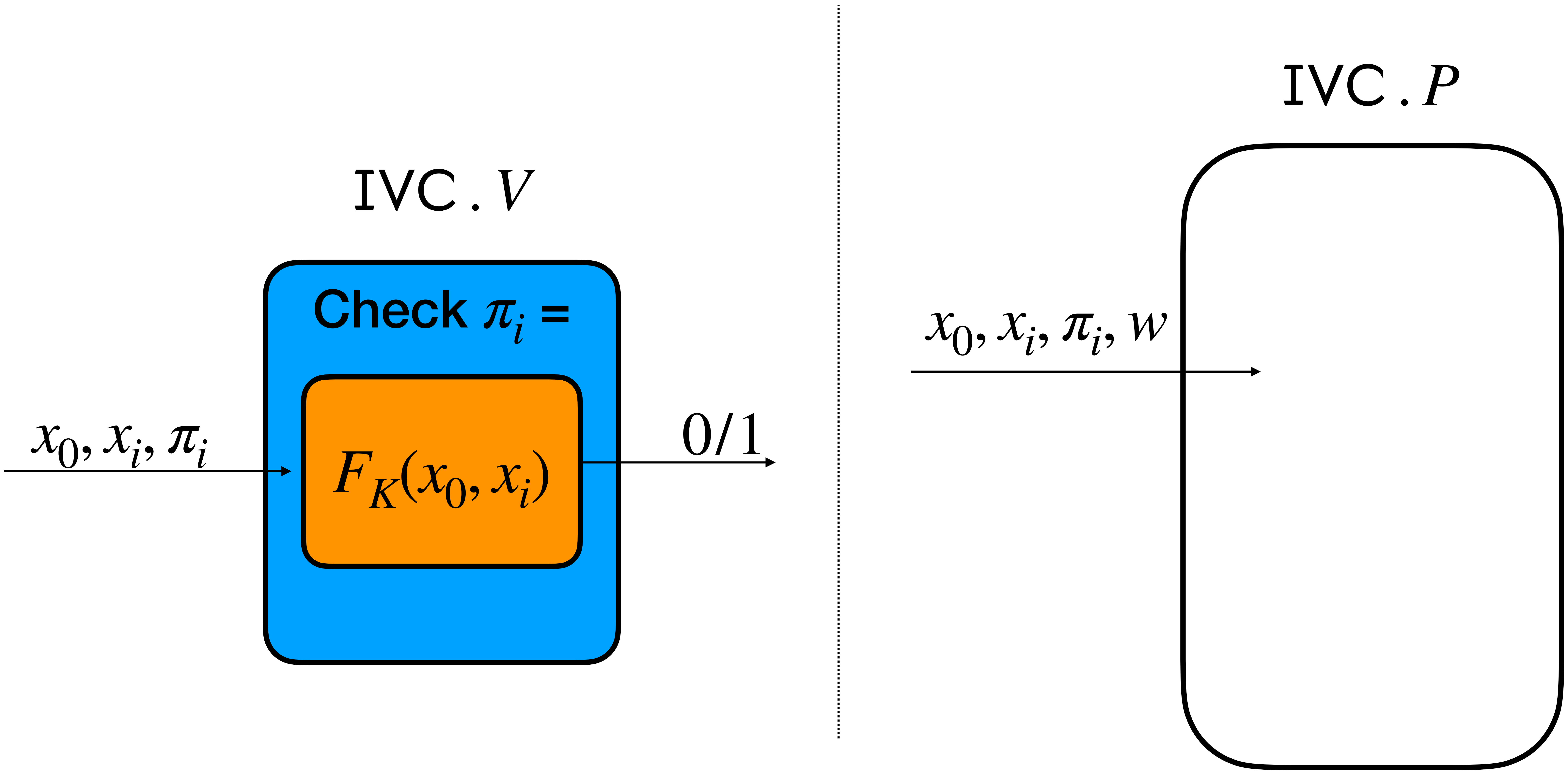

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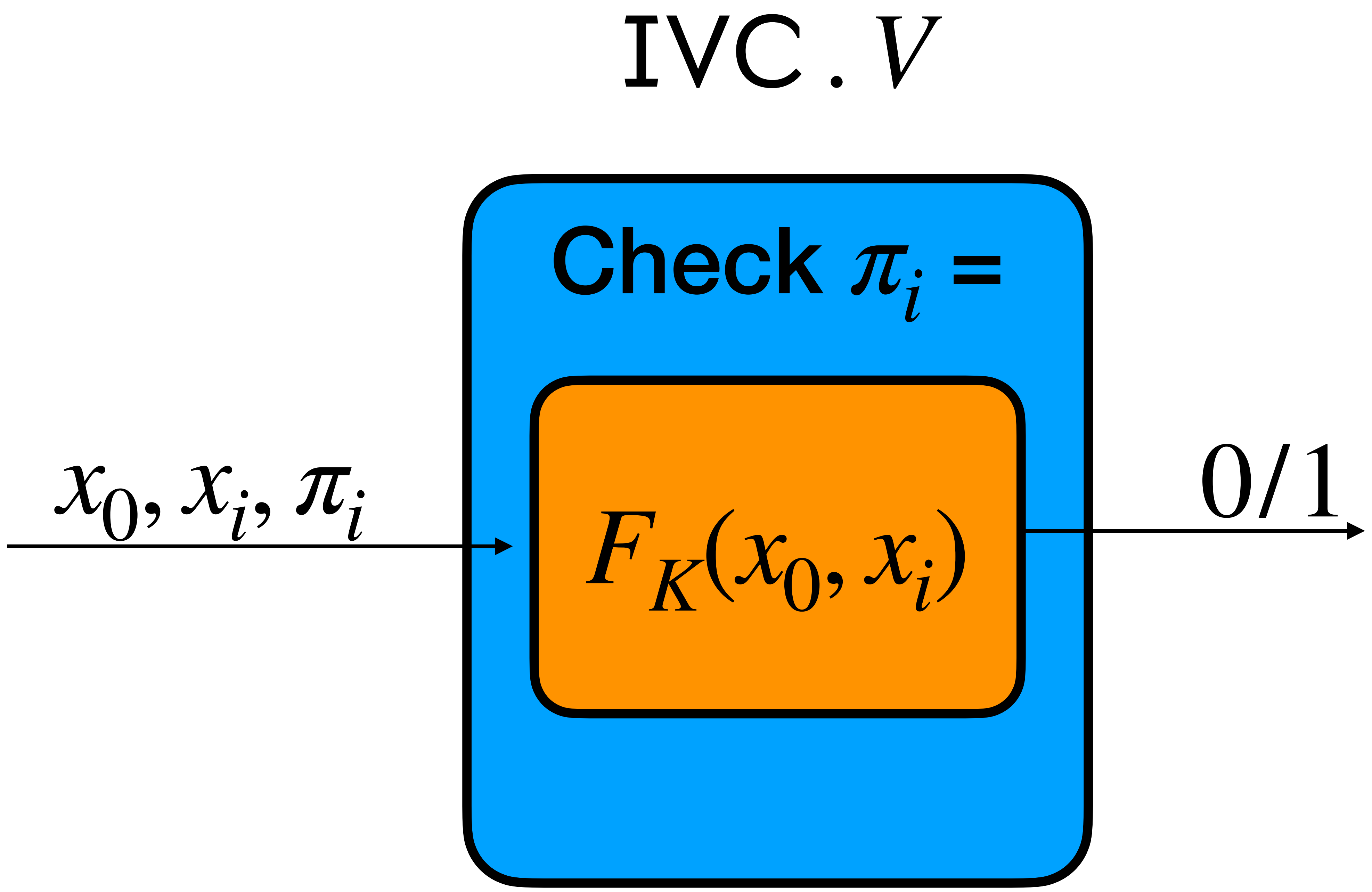

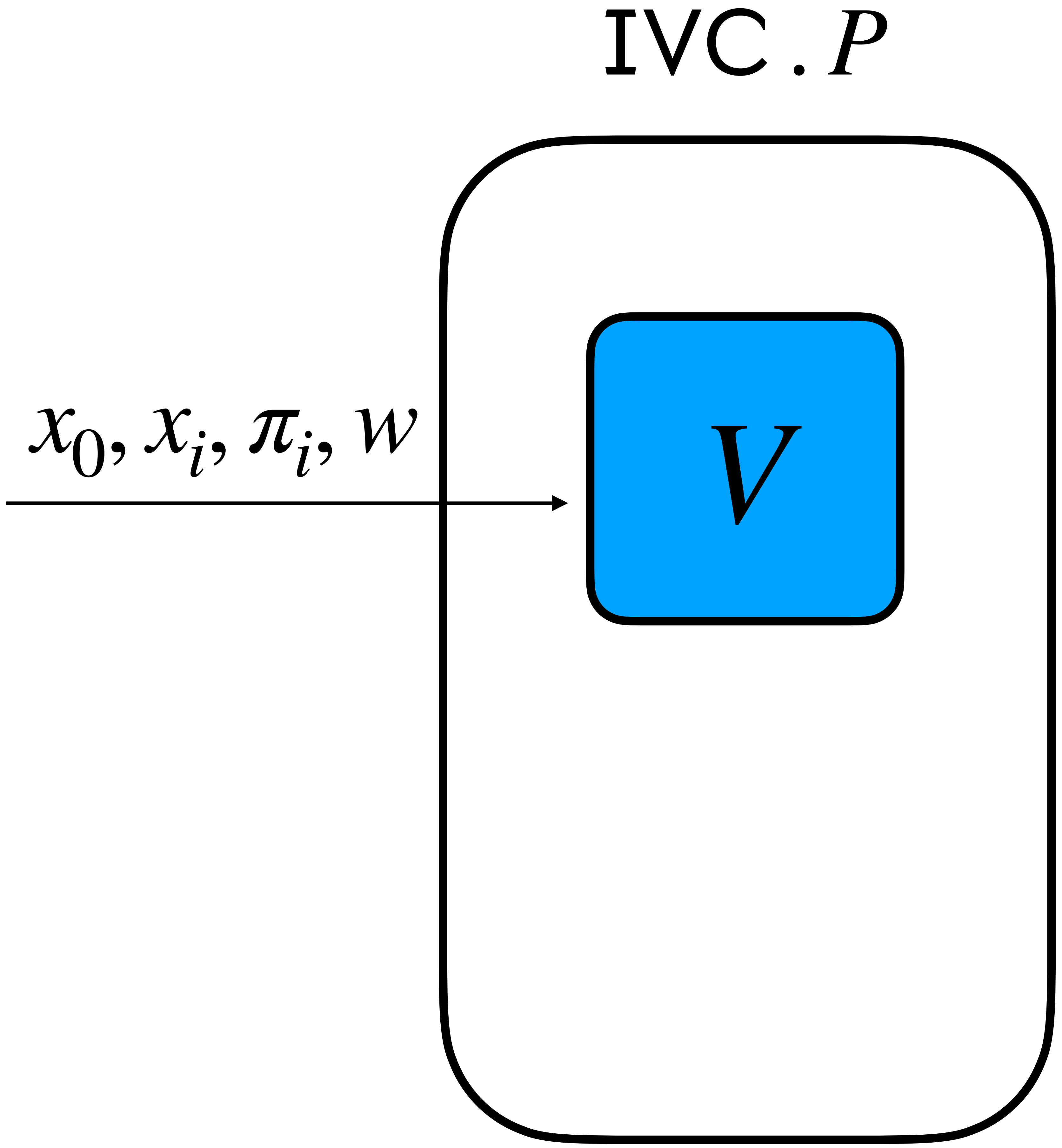

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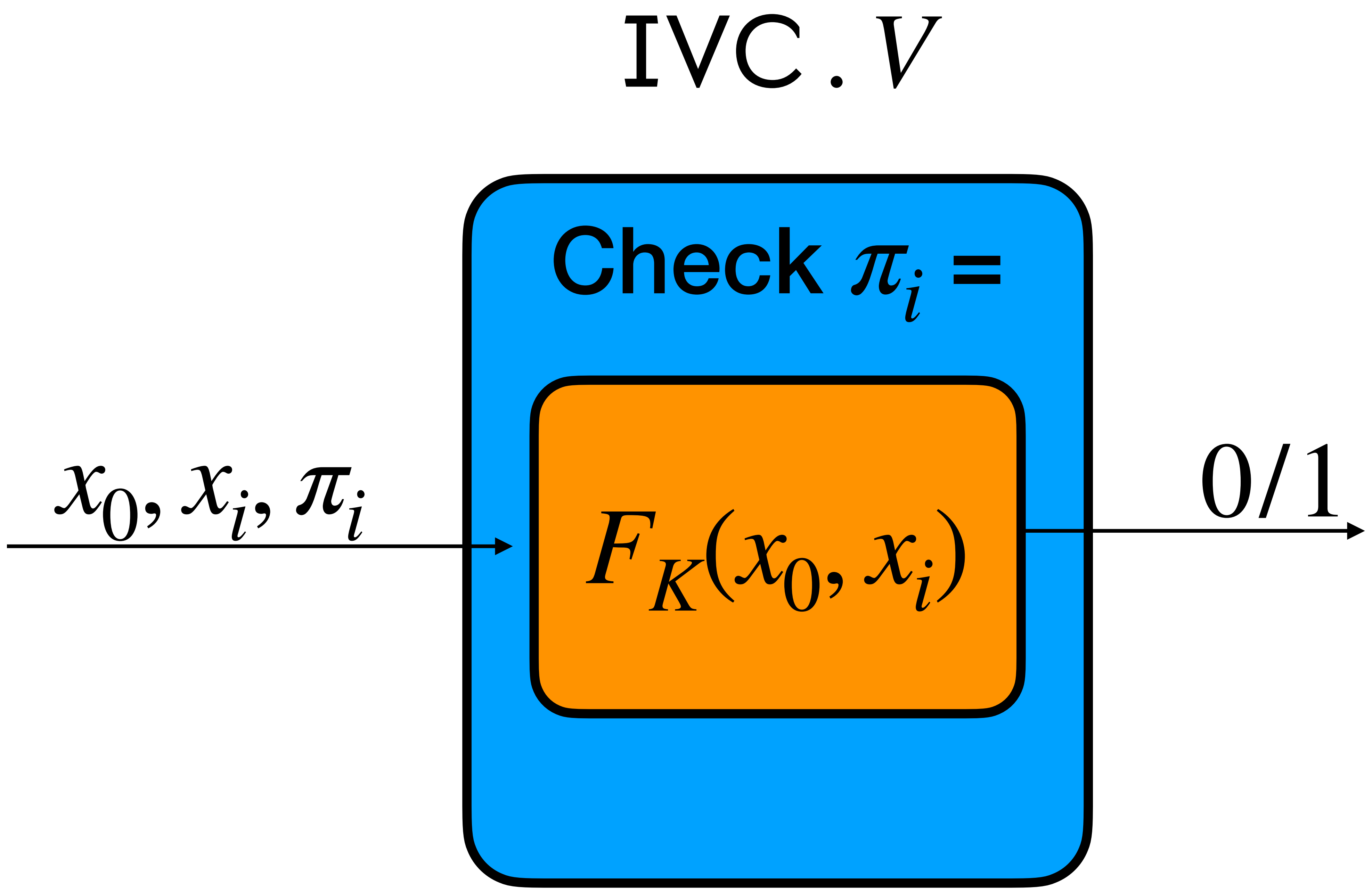

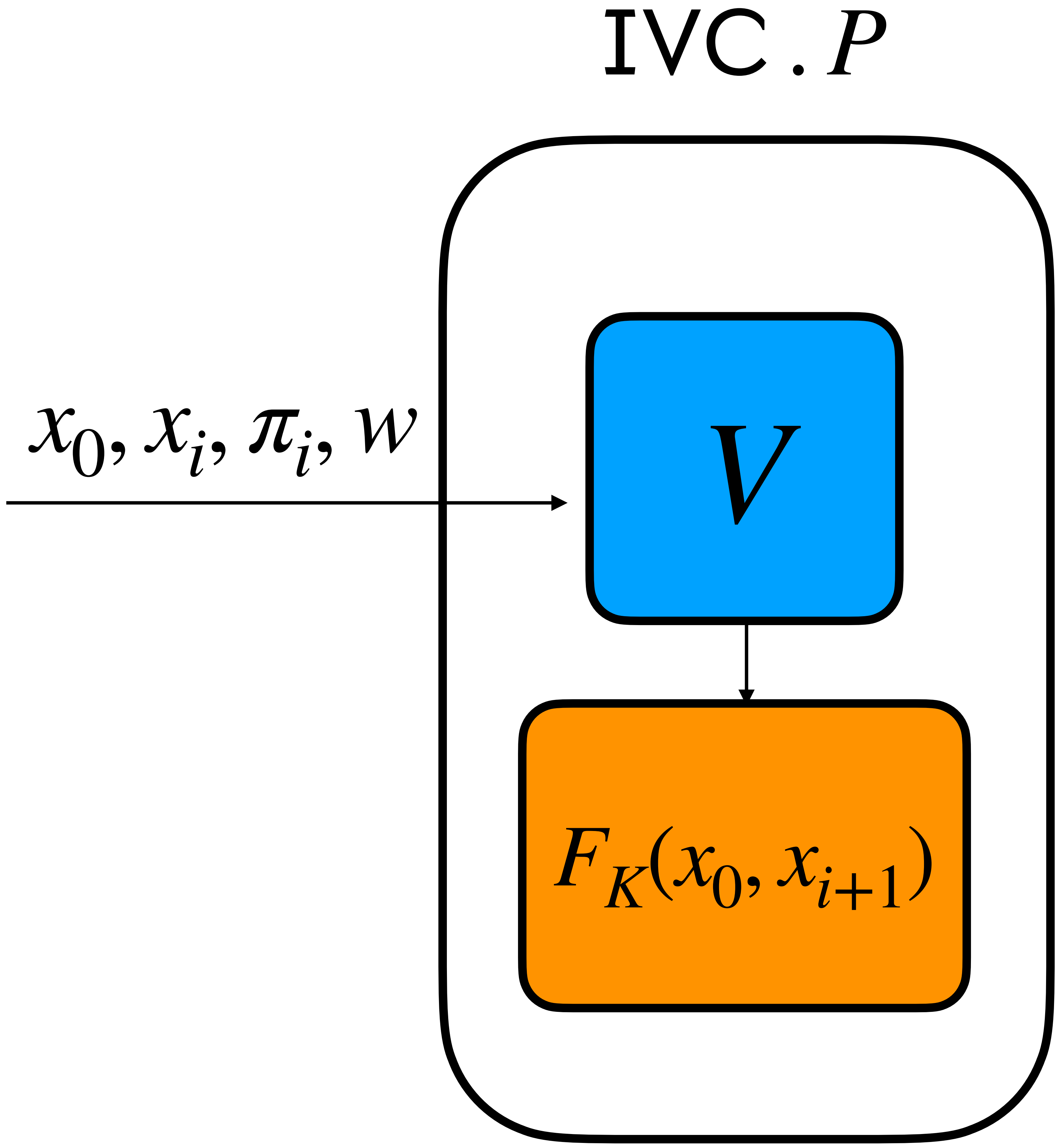

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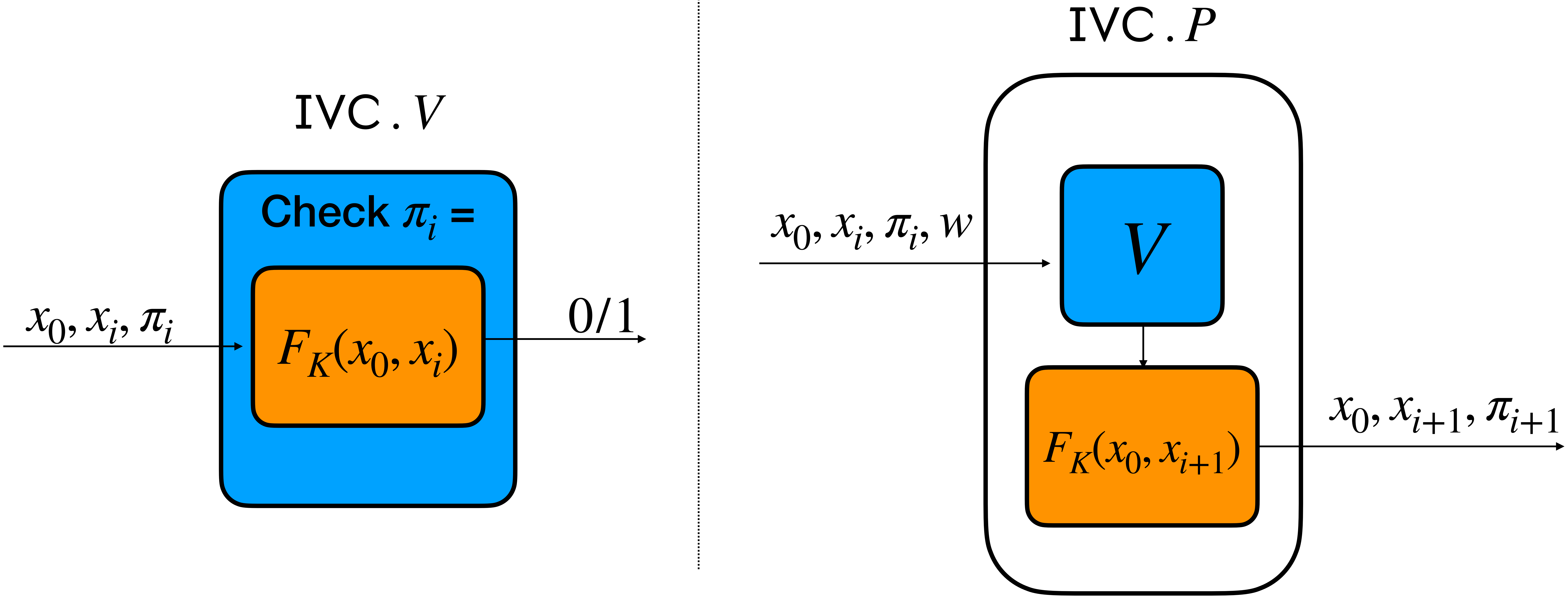

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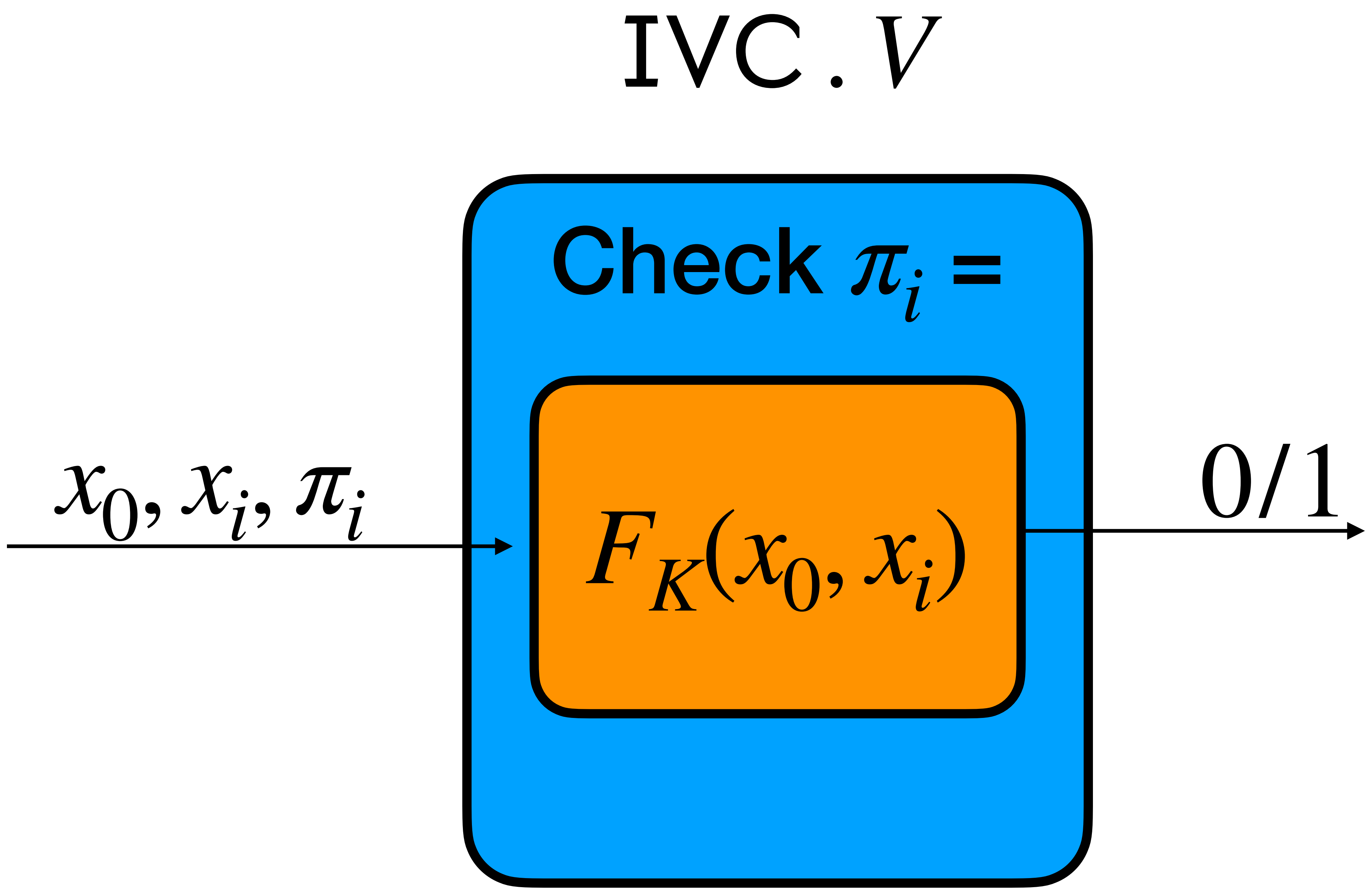

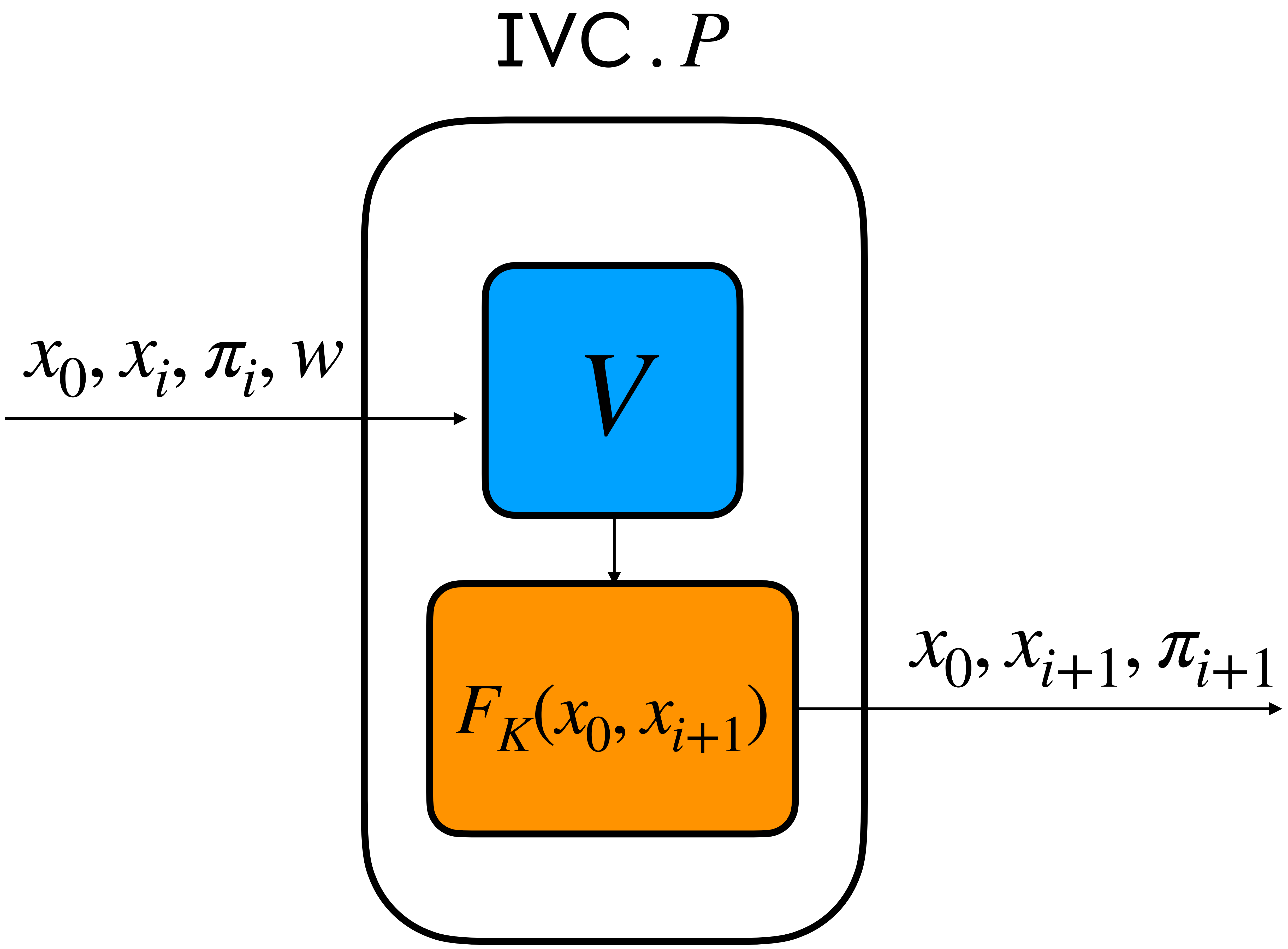

Obfuscate these programs and publish as CRS!!

High level idea!

High level idea!

H_{real}

High level idea!

The real scheme

High level idea!

H_{real}

iO Gymnastics! (See paper)

The real scheme

High level idea!

H_{real} → H_{ideal}

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H$_{real}$ — The real scheme

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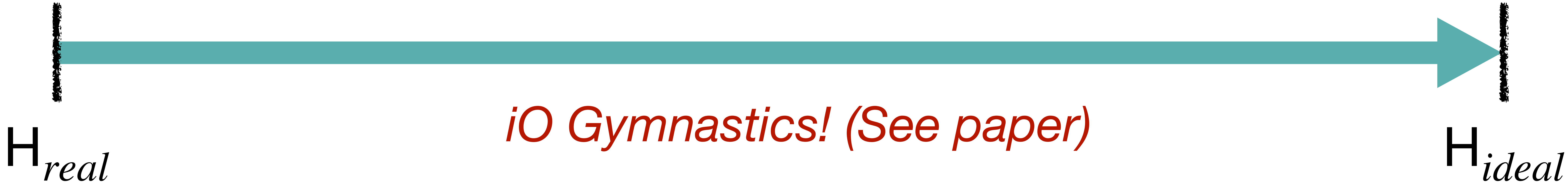

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- This approach **does not** use extraction and achieves poly(λ)-**sized proof**!

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    - Proof size: poly(λ).
    - **Open problem:** Can we extend this to all of NP?

Thank you for your attention!

Bonus Slides

Recall: Rate-1 Batch Arguments (BARG)

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w_1

w_2

…

w_k

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w_1

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$\ldots$

w_k

Usually only require
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Recall: Rate-1 Batch Arguments (BARG)

Common reference string crs(i^*)

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For construction, see [PP22, DGKV22, BDSZ24]

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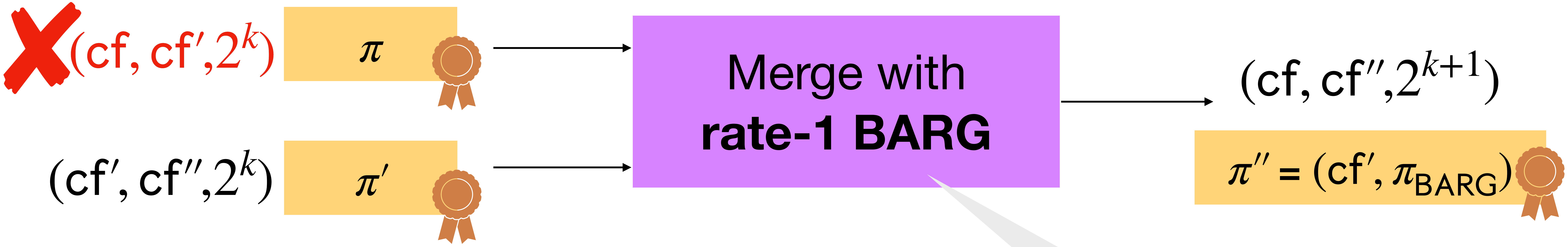
- Suppose $\mathscr{A}$ creates cheating proofs $(\mathsf{cf}', \pi_{\mathsf{BARG}})$ with **prob.** $\geq \epsilon$.
    - By **pigeonhole,** $\mathsf{cf} \nrightarrow \mathsf{cf}'$ with prob $\geq \epsilon/2$ or, $\mathsf{cf}' \nrightarrow \mathsf{cf}''$. WLOG first hop.

Proof Strategy: BARG the BARG



Construct a BARG proof π_{BARG} corresponding to:

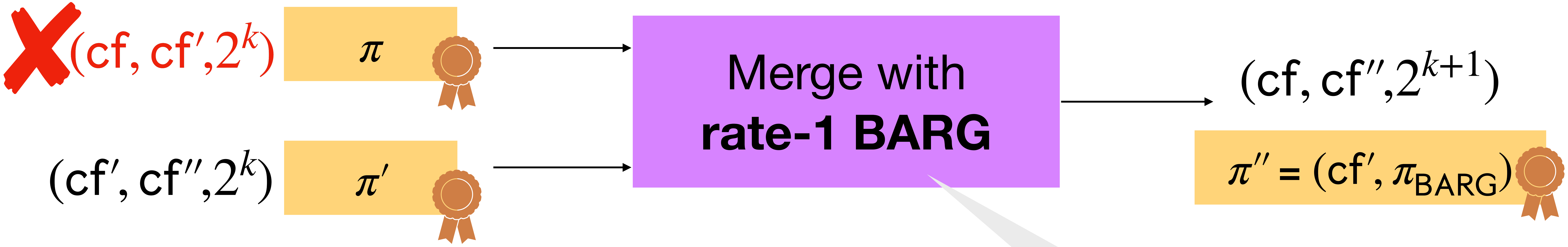
BARG Statement: $(\mathsf{cf}, \mathsf{cf}'), (\mathsf{cf}', \mathsf{cf}'')$.

BARG Witness: Level k proofs π, π''.

Level $k+1$ proof: $\pi'' = (\mathsf{cf}', \pi_{\mathsf{BARG}})$.

- Suppose $\mathscr{A}$ creates cheating proofs $(\mathsf{cf}', \pi_{\mathsf{BARG}})$ with **prob.** $\geq \epsilon$.
  - By **pigeonhole,** $\mathsf{cf} \not\rightarrow \mathsf{cf}'$ with prob $\geq \epsilon/2$ or, $\mathsf{cf}' \not\rightarrow \mathsf{cf}''$. WLOG first hop.

Proof Strategy: BARG the BARG



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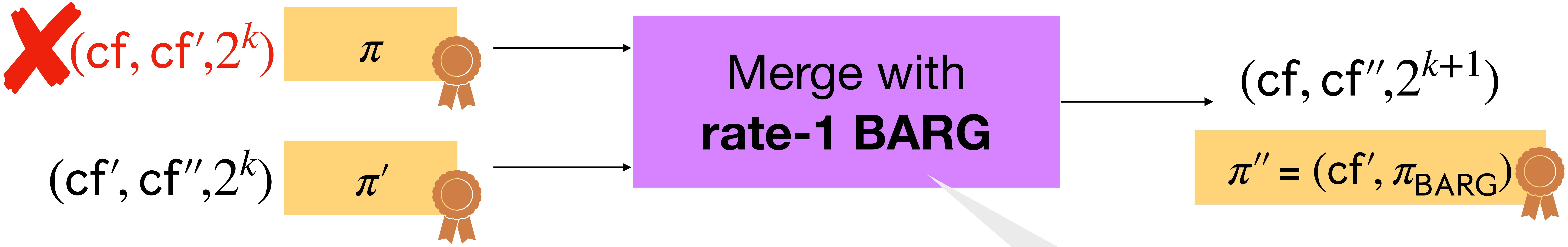
  - Switch BARG **CRS binding** accordingly!

Proof Strategy: BARG the BARG



Construct a BARG proof π_{BARG} corresponding to:

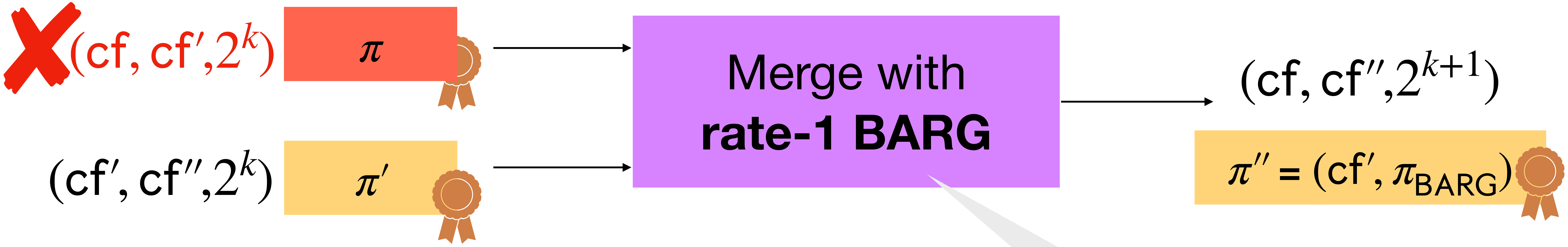
BARG Statement: $(\mathsf{cf}, \mathsf{cf}'), (\mathsf{cf}', \mathsf{cf}'')$.

BARG Witness: Level k proofs π, π''.

Level $k+1$ proof: $\pi'' = (\mathsf{cf}', \pi_{\mathsf{BARG}})$.

- Suppose $\mathscr{A}$ creates cheating proofs $(\mathsf{cf}', \pi_{\mathsf{BARG}})$ with **prob.** $\geq \epsilon$.
  - By **pigeonhole,** $\mathsf{cf} \nrightarrow \mathsf{cf}'$ with prob $\geq \epsilon/2$ or, $\mathsf{cf}' \nrightarrow \mathsf{cf}''$. WLOG first hop.
  - Switch BARG **CRS binding** accordingly!
    - By **index-hiding property** of the BARG, should still cheat on this hop!!

Proof Strategy: BARG the BARG



Construct a BARG proof π_{BARG} corresponding to:

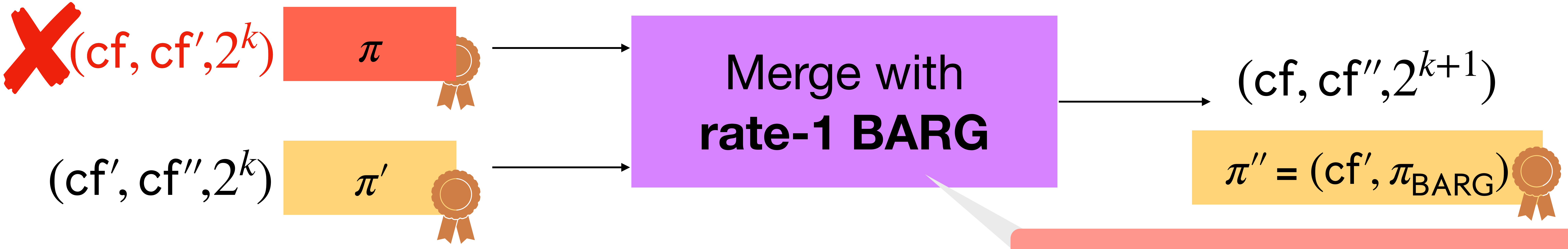
BARG Statement: (cf, cf′), (cf′, cf″).

BARG Witness: Level k proofs π, π''.

Level $k+1$ proof: $\pi'' = (\text{cf}', \pi_{\text{BARG}})$.

- Suppose $\mathscr{A}$ creates cheating proofs (cf′, π_{BARG}) with **prob.** $\geq \epsilon$.
  - By **pigeonhole,** cf $\nrightarrow$ cf′ with prob $\geq \epsilon/2$ or, cf′ $\nrightarrow$ cf″. WLOG first hop.
  - Switch BARG **CRS binding** accordingly!
    - By **index-hiding property** of the BARG, should still cheat on this hop!!
    - **Extract** π and recurse!!

Proof Strategy: BARG the BARG



Needs careful complexity leveraging for NP (this work, based on BKK+17)! Ask me later :)

- Suppose $\mathscr{A}$ creates cheating proofs $(\text{cf}', \pi_{\text{BARG}})$ with **prob.** $\geq \epsilon$.
  - By **pigeonhole,** $\text{cf} \nrightarrow \text{cf}'$ with prob $\geq \epsilon/2$ or $\text{cf}' \nrightarrow \text{cf}''$. WLOG first hop.
  - Switch BARG **CRS binding** accordingly!
    - By **index-hiding property** of the BARG, should still cheat on this hop!!
    - **Extract** π and recurse!!