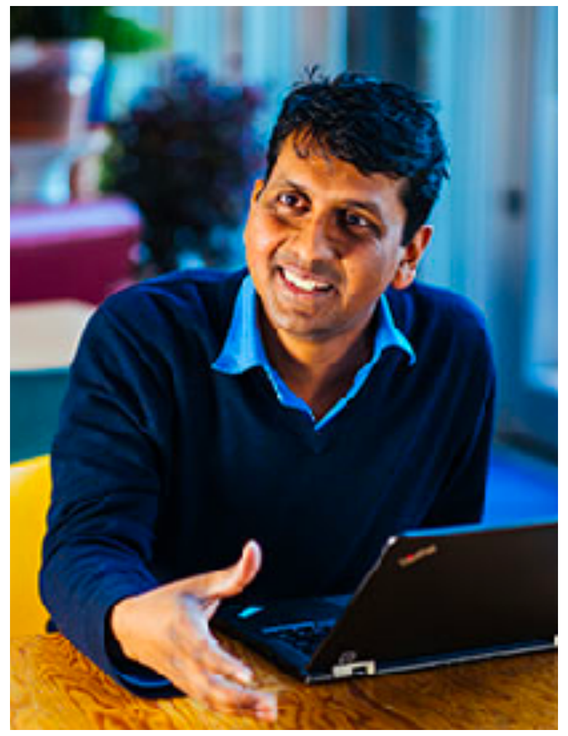
Pseudorandom Obfuscation

And applications









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Nico Döttling
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Abhishek Jain
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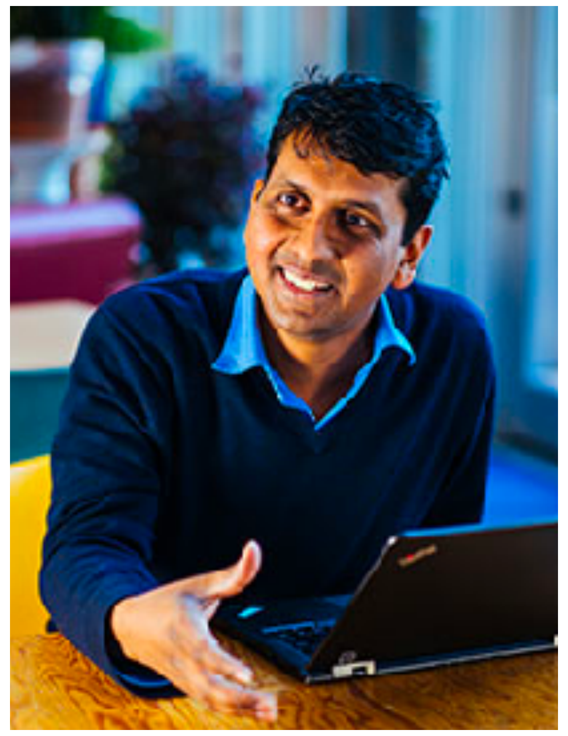

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Thank you Nico for many of these slides!

Indistinguishability Obfuscation

[BGI+01,GGH+13]

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C_0 C_1

Indistinguishability Obfuscation

[BGI+01,GGH+13]

$$C_0 \equiv C_1$$

Indistinguishability Obfuscation

[BGI+01,GGH+13]

$(x+y)(x-y)$ $\qquad$ $x^2 - y^2$

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Indistinguishability Obfuscation

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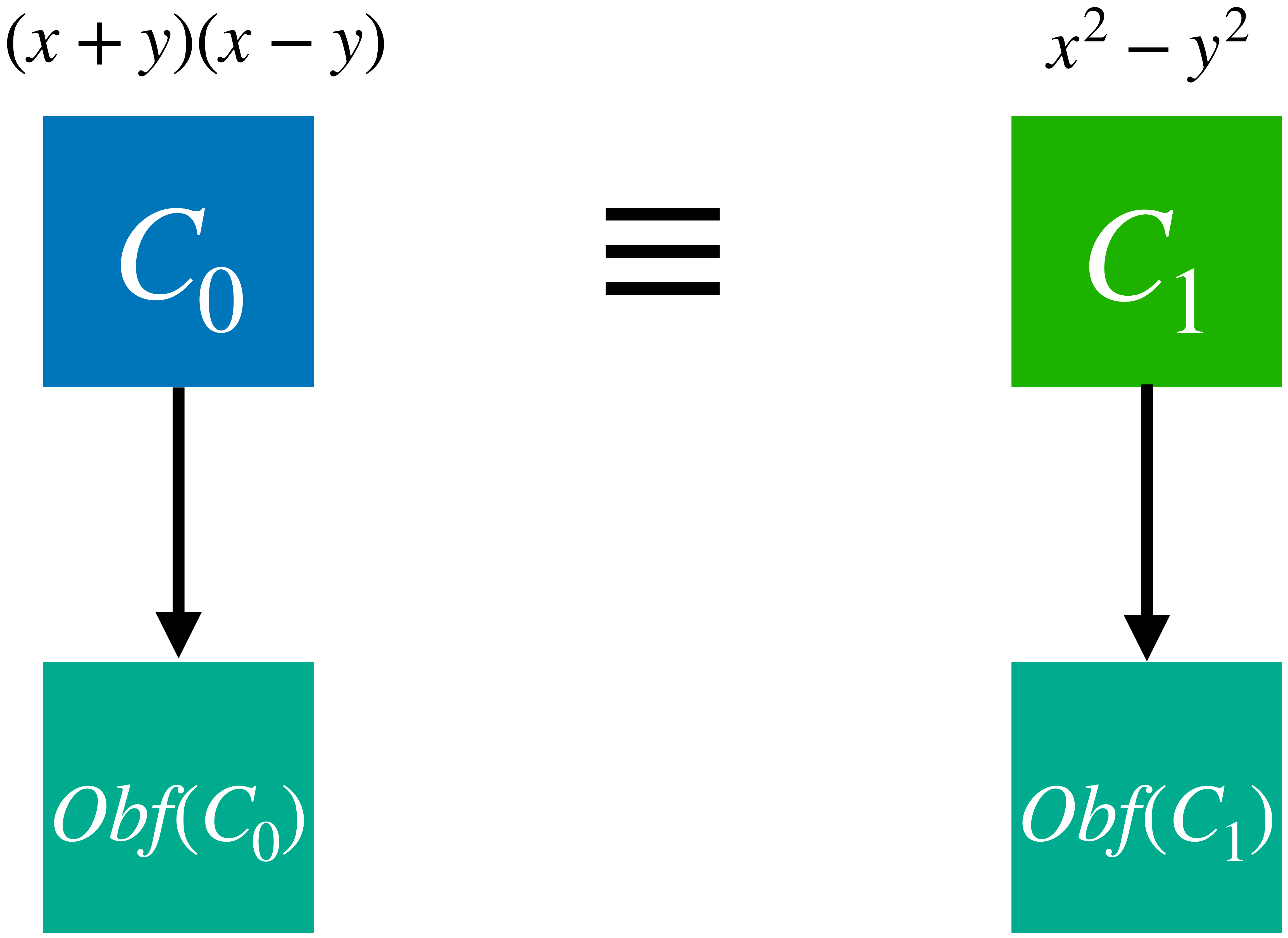

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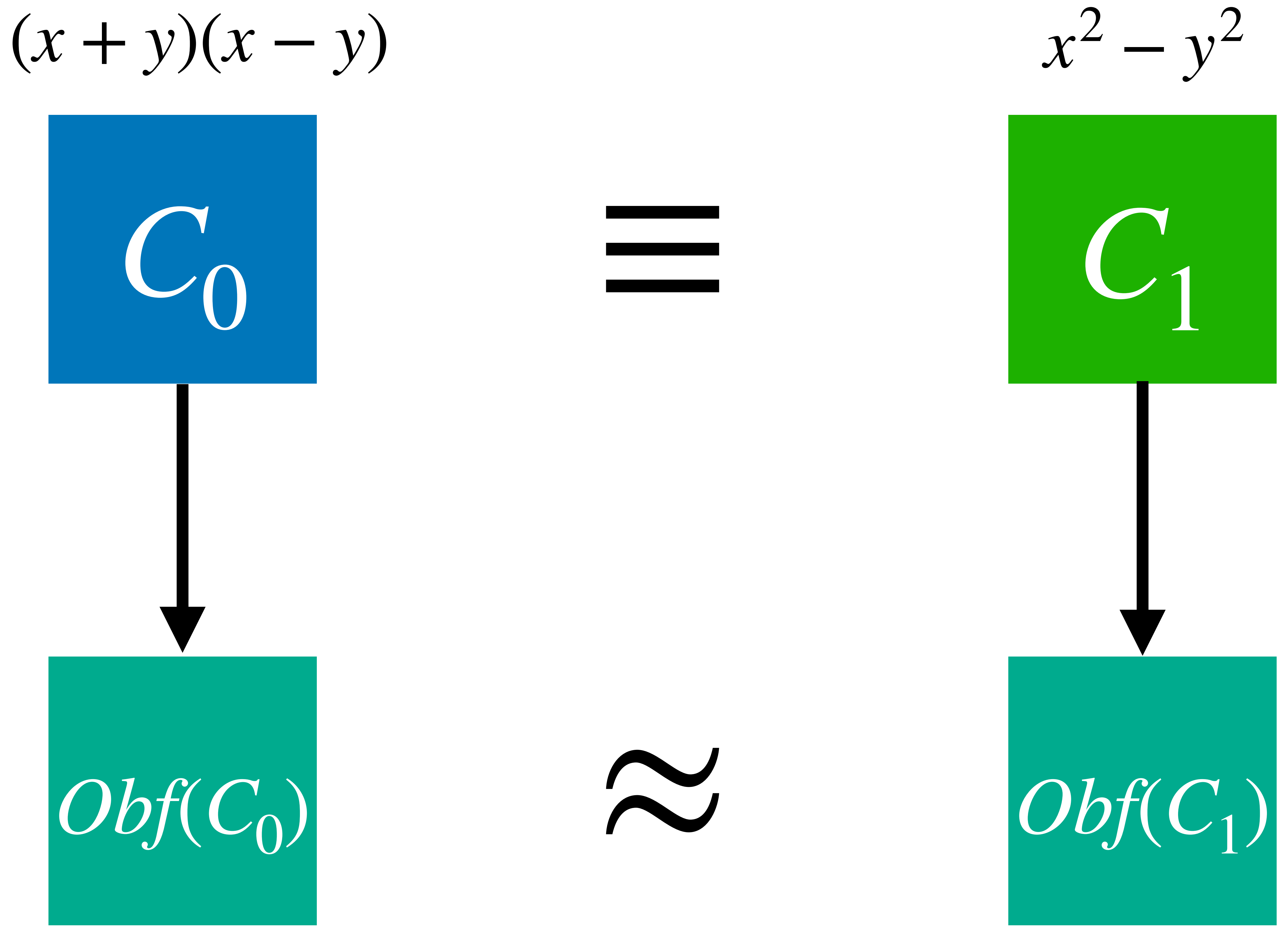

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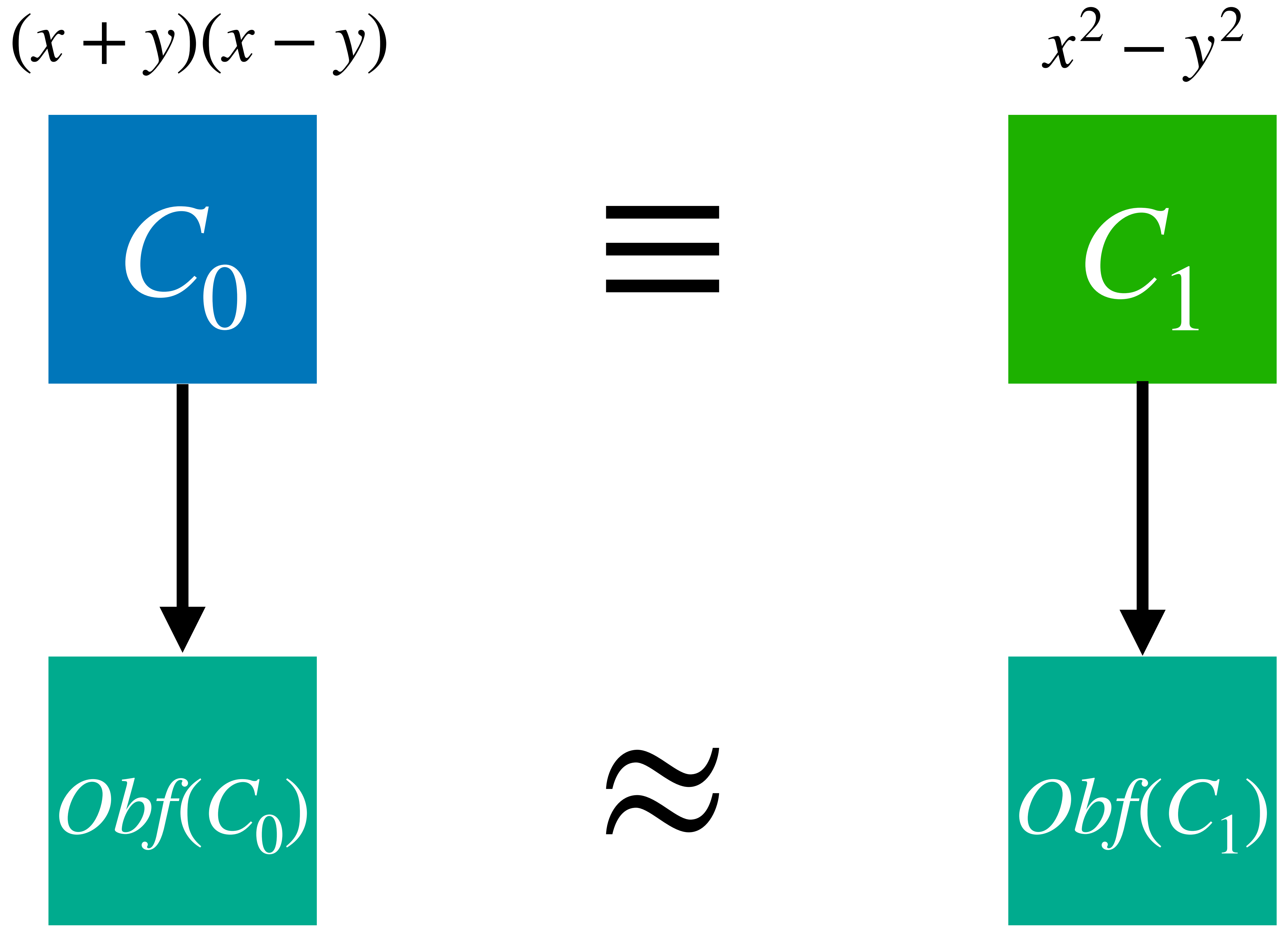

- Currently only candidates [JLS21, JLS22, RVV24] from standard assumptions follows the **[JLS20]** mould

iO: Swiss Army knife of Cryptography

(subexponential) iO (+ standard assumptions) is “crypto complete”

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Many of these applications involve obfuscating a cryptographic program. Can we leverage this?

Is there a *different notion* of obfuscation that suffices for these applications?

Fully Homomorphic Encryption

a la [CLTV'15]

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- Leveled FHE: pk contains key chain of length N to support depth N computation.

$Enc_{pk_2}(sk_1)$ $Enc_{pk_3}(sk_2)$ $\cdots$ $Enc_{pk_{N-1}}(sk_{N-2})$ $Enc_{pk_N}(sk_{N-1})$

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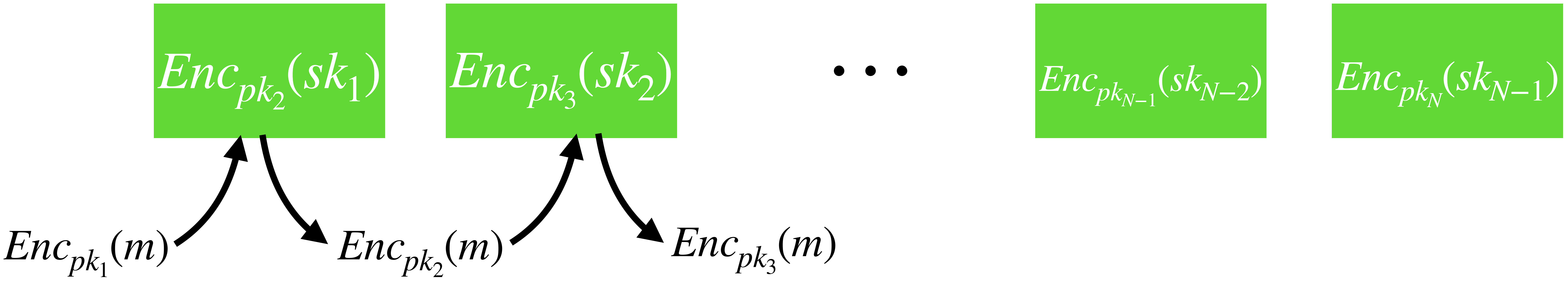

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F

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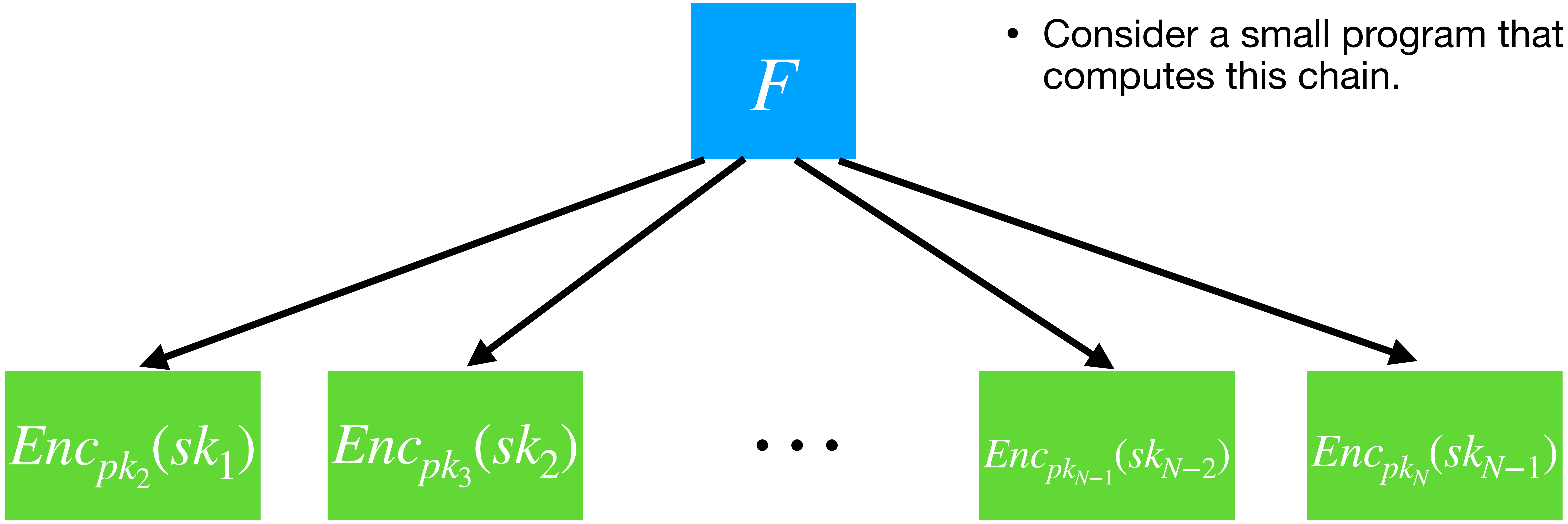

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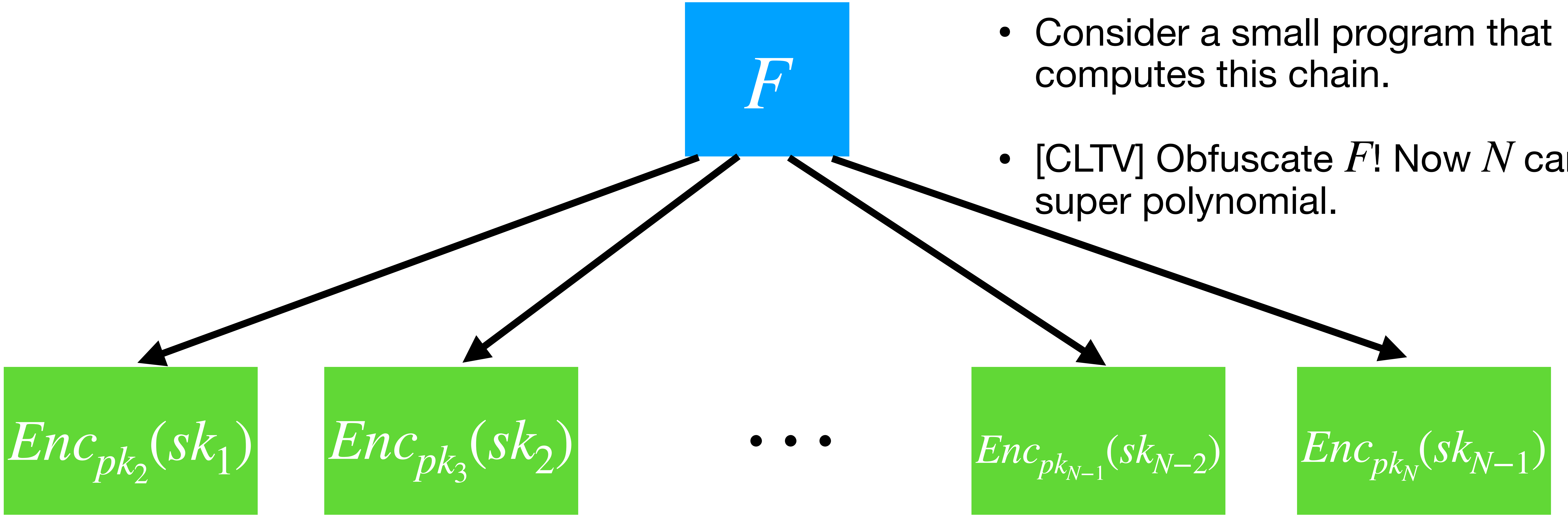

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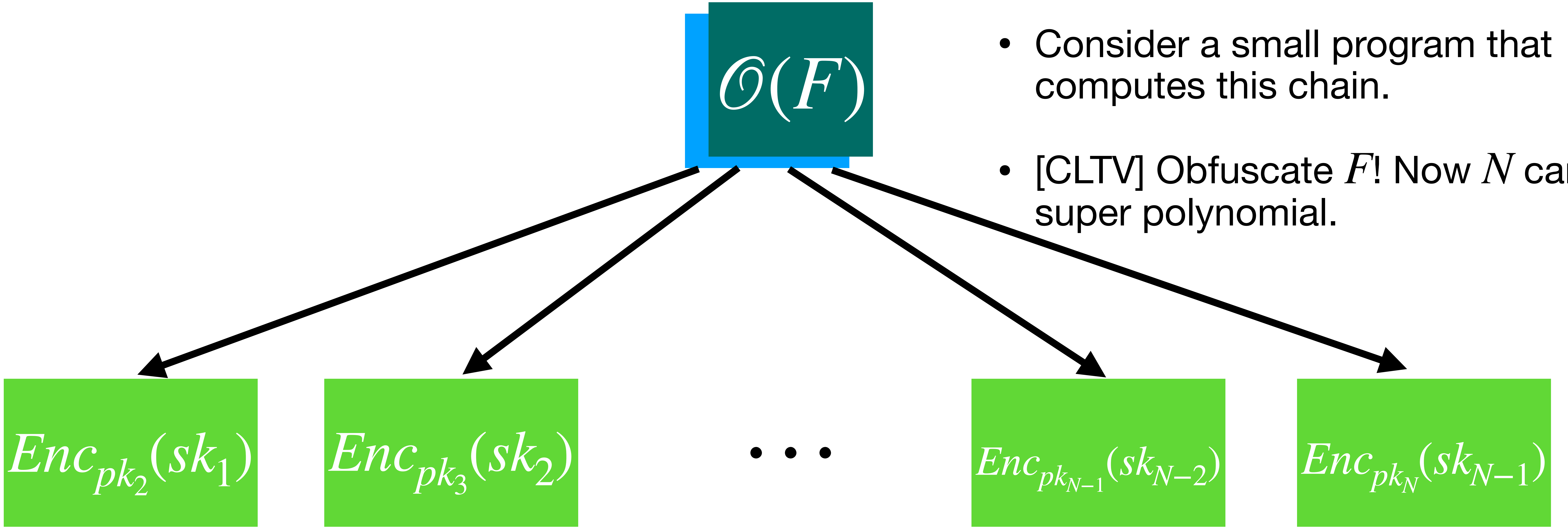

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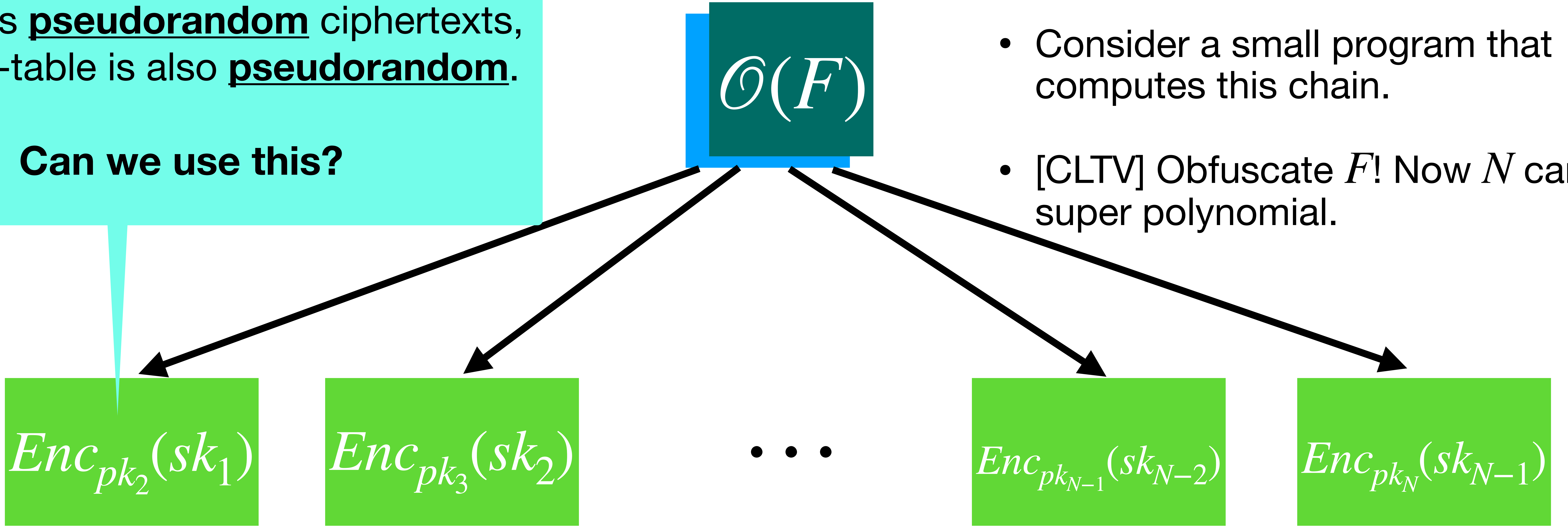

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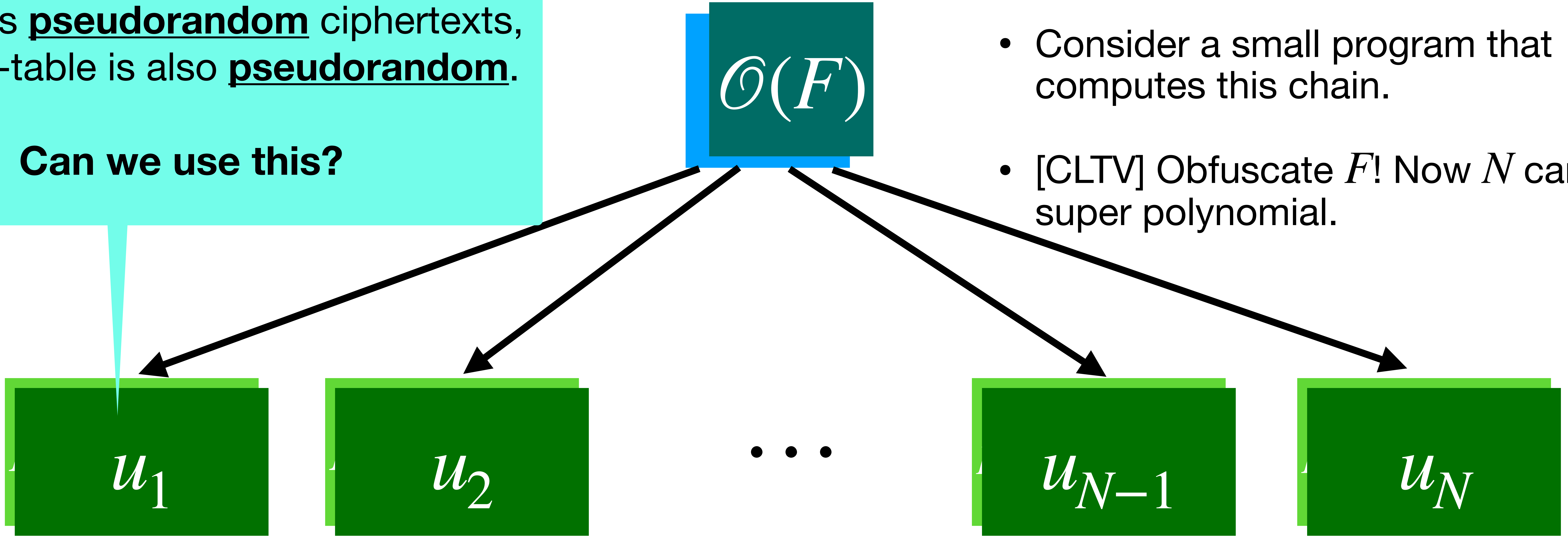

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    - 3 notions of PRO
    - Possibilities and impossibilities
    - PRO + Bilinear Maps = iO
- (Not in talk) The full version of this paper additionally includes a candidate construction of pseudorandom obfuscation from the evasive LWE heuristic.

Pseudorandom Obfuscation

Strongest Notion: Double Pseudorandomness (dPRO)

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$TT(C)$

Pseudorandom Obfuscation

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Strongest Notion: <u>Double</u> Pseudorandomness (dPRO)

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Pseudorandom Obfuscation

Strongest Notion: Double Pseudorandomness (dPRO)

$TT(C) \approx U$ given aux(C)

$\Downarrow$

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$TT(C)$ $\approx$ U given aux(C)

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xPRO: $|PRO(C)| = |TT(C)|^{1-\epsilon}$

Pseudorandom Obfuscation

Strongest Notion: Double Pseudorandomness (dPRO)

Precondition

$TT(C)$ $\approx$ U given aux(C)

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$PRO(C)$ $\approx$ u given aux(C)

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Pseudorandom Obfuscation

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Precondition

$TT(C) \approx U$ given aux(C)

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$PRO(C) \approx u$ given aux(C)

xPRO: $|PRO(C)| = |TT(C)|^{1-\epsilon}$

Pseudorandom Obfuscation

Precondition

$TT(C)$ $\approx$ U given $aux(C_b)$

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Pseudorandom Obfuscation

Medium notion

Precondition

$TT(C) \approx U$ given $aux(C_b)$

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Pseudorandom Obfuscation

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Pseudorandom Obfuscation

Medium notion

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$PRO(C_0) \approx PRO(C_1)$ given $aux(C_b)$

Pseudorandom Obfuscation

Medium notion

Precondition

$TT(C_b)$ $\approx$ U given $aux(C_b)$

Obfuscation itself doesn't have to be pseudorandom

Postcondition

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Over-simplified Sketch of Succinct Garbling

Via standard iO

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Size of program is independent of runtime of TM

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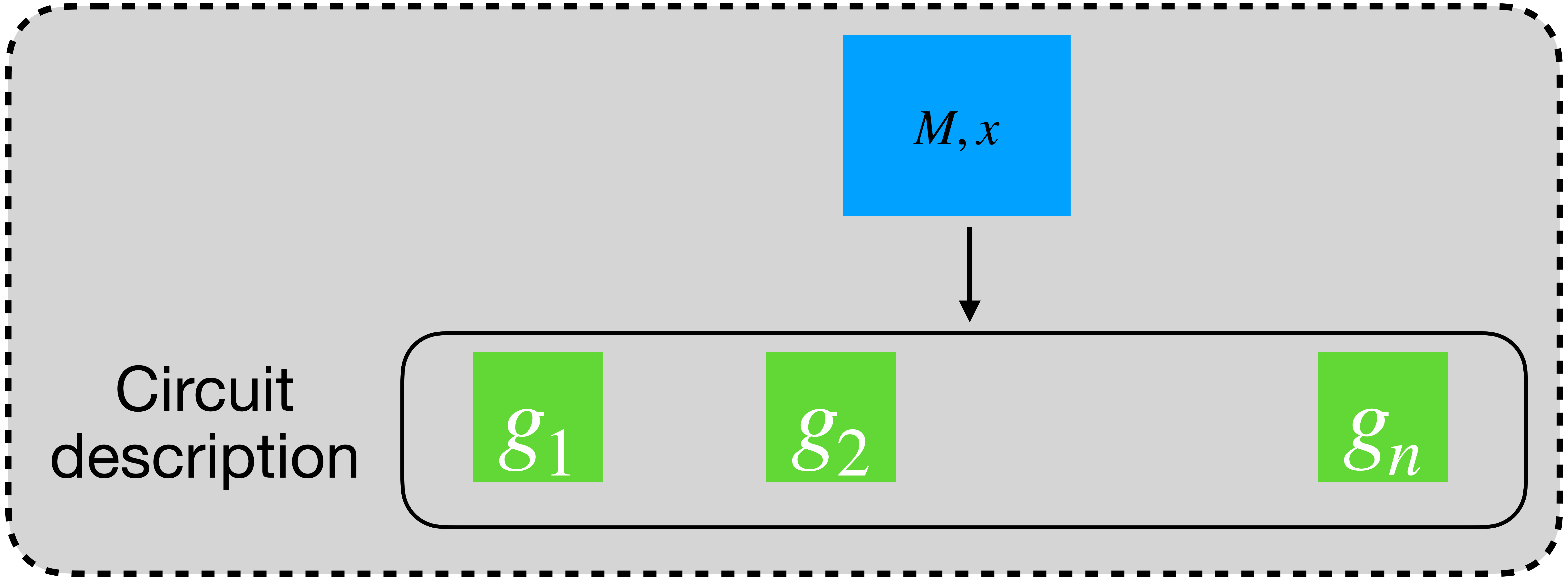

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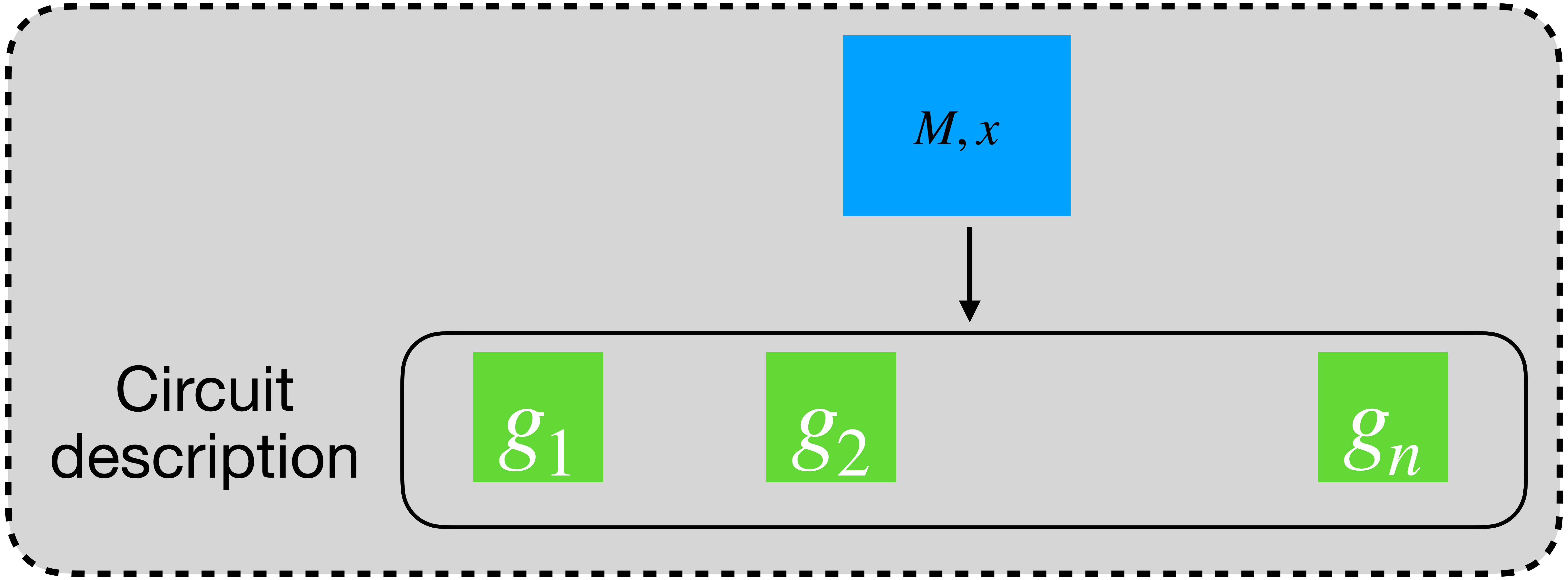

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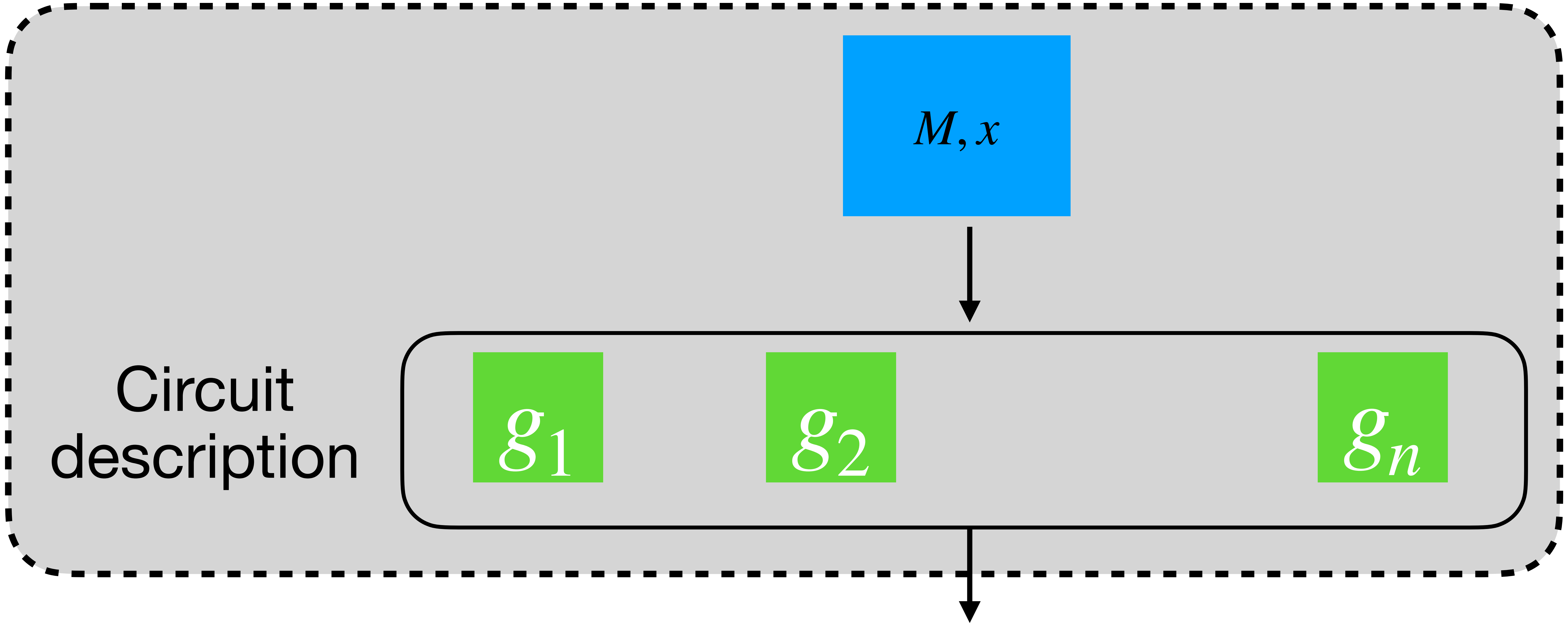

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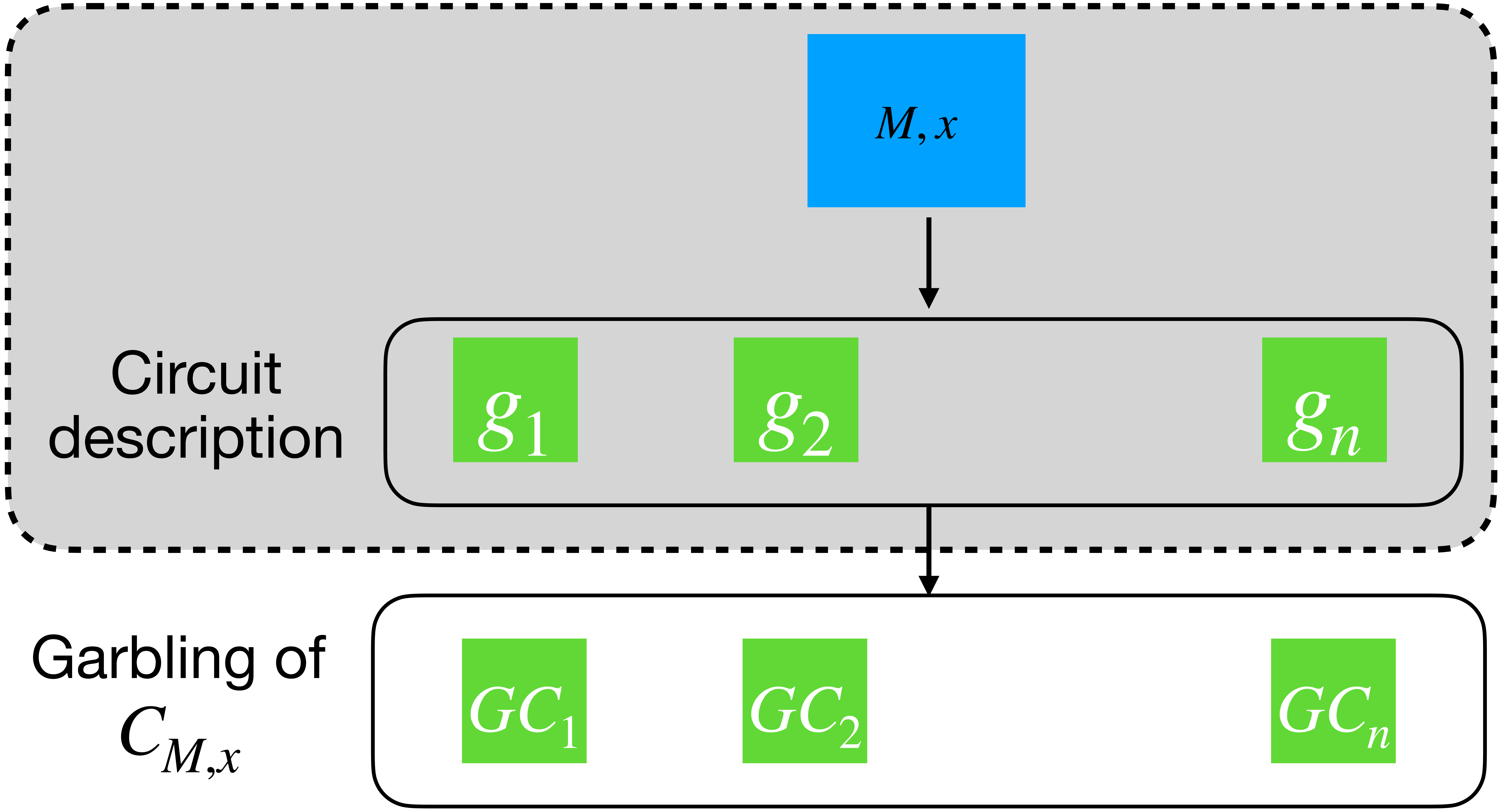

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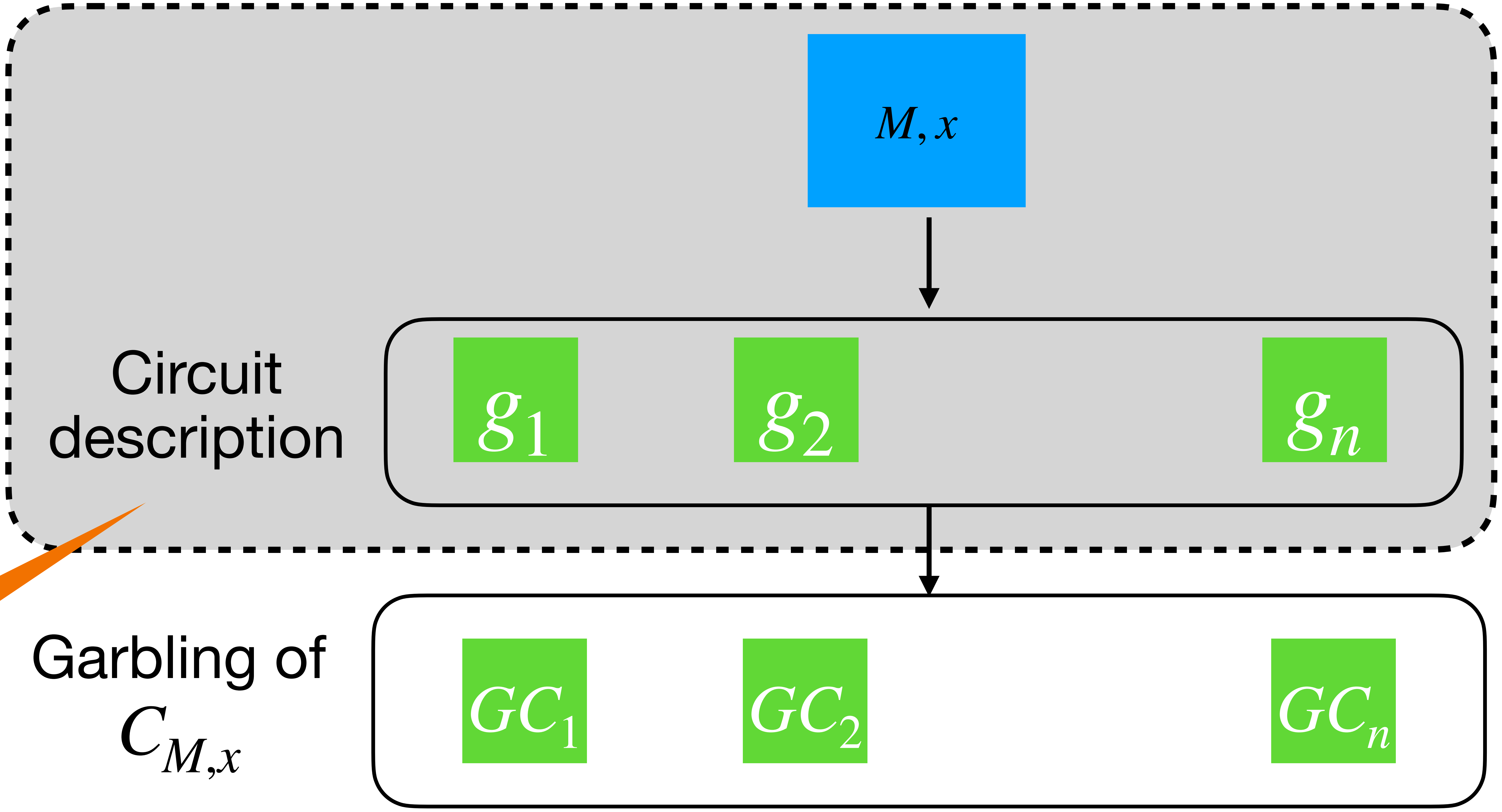
Garbling = Obfuscation of this program!



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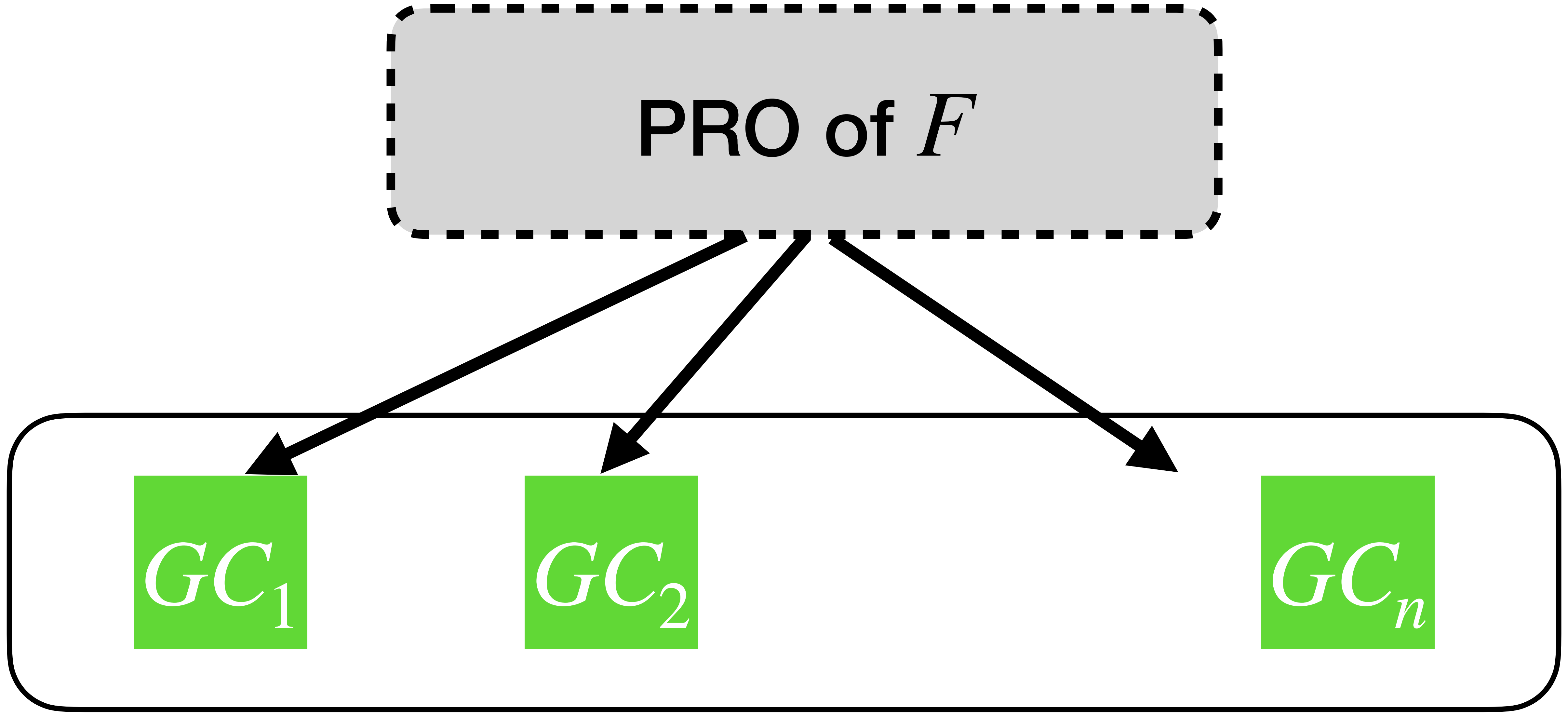

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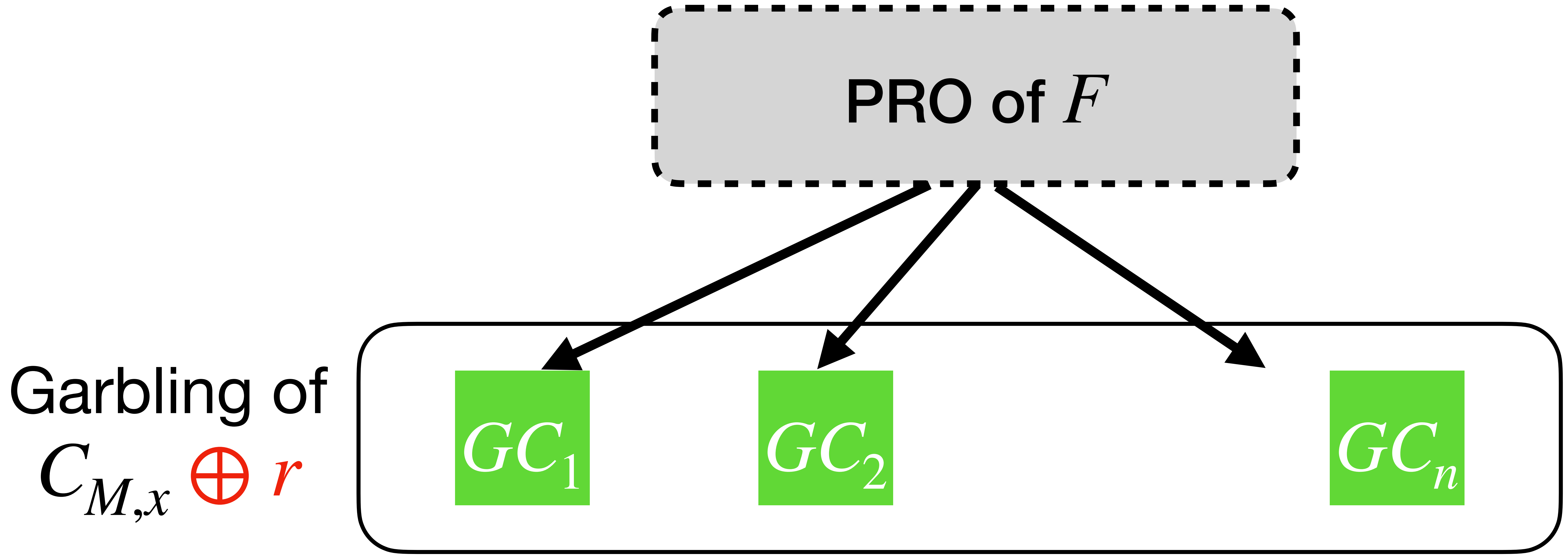

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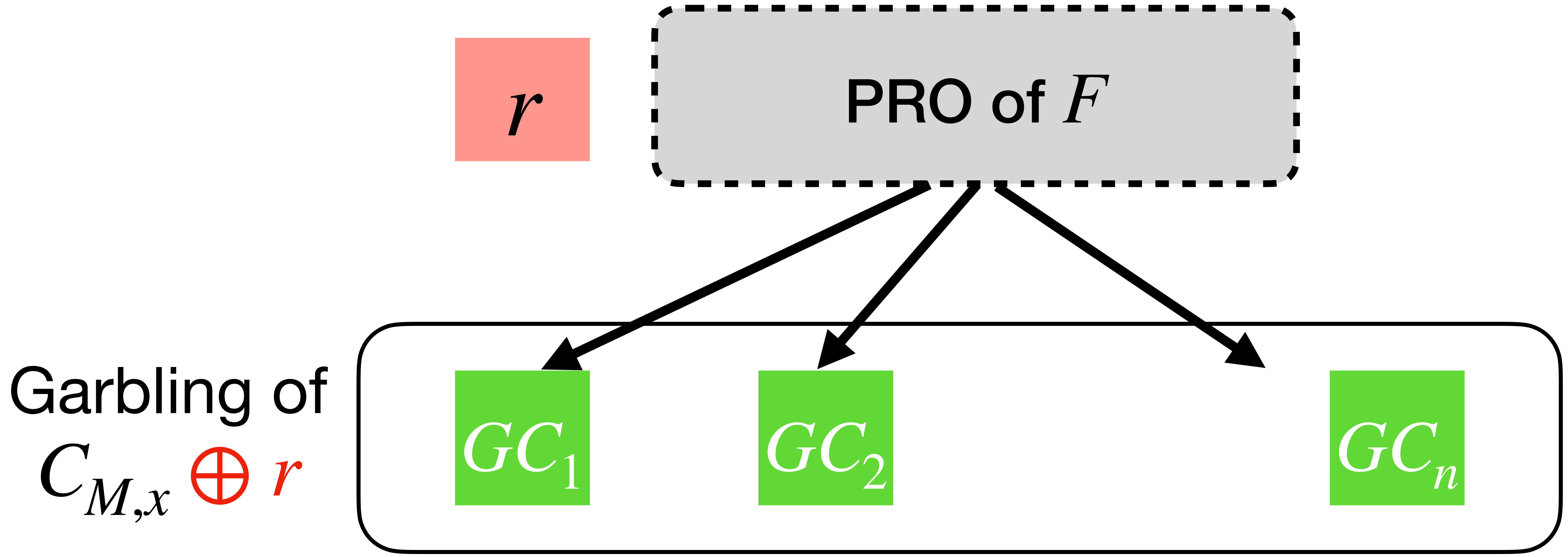

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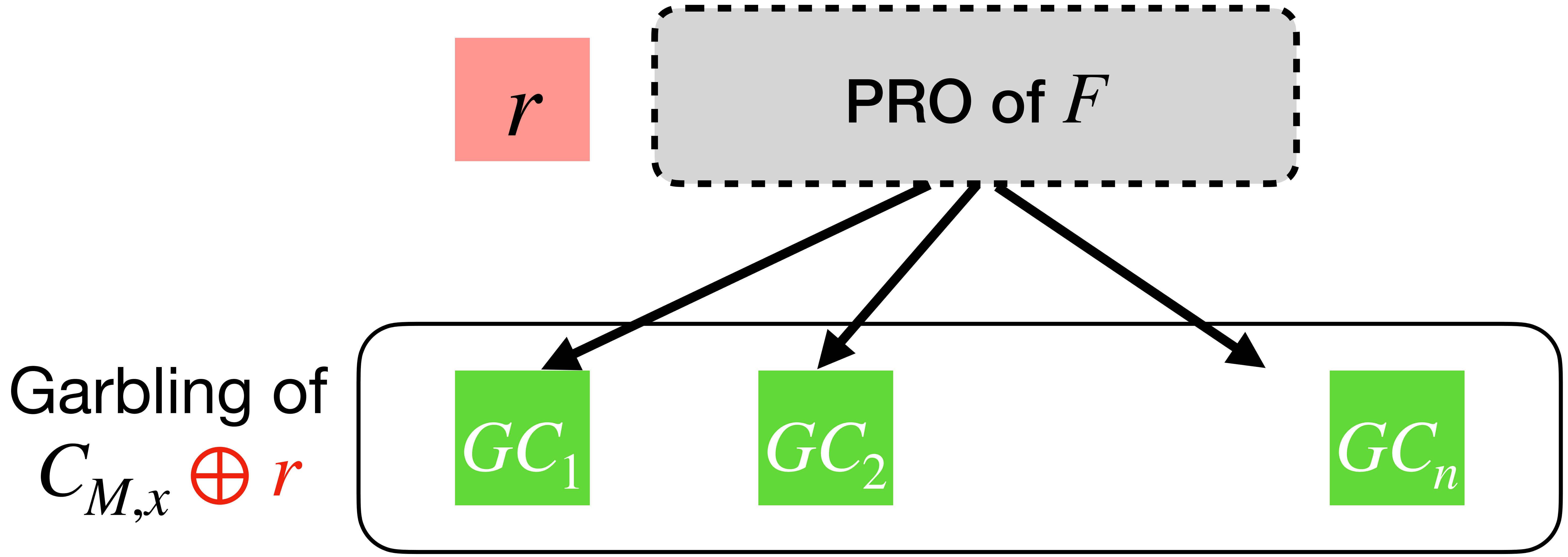

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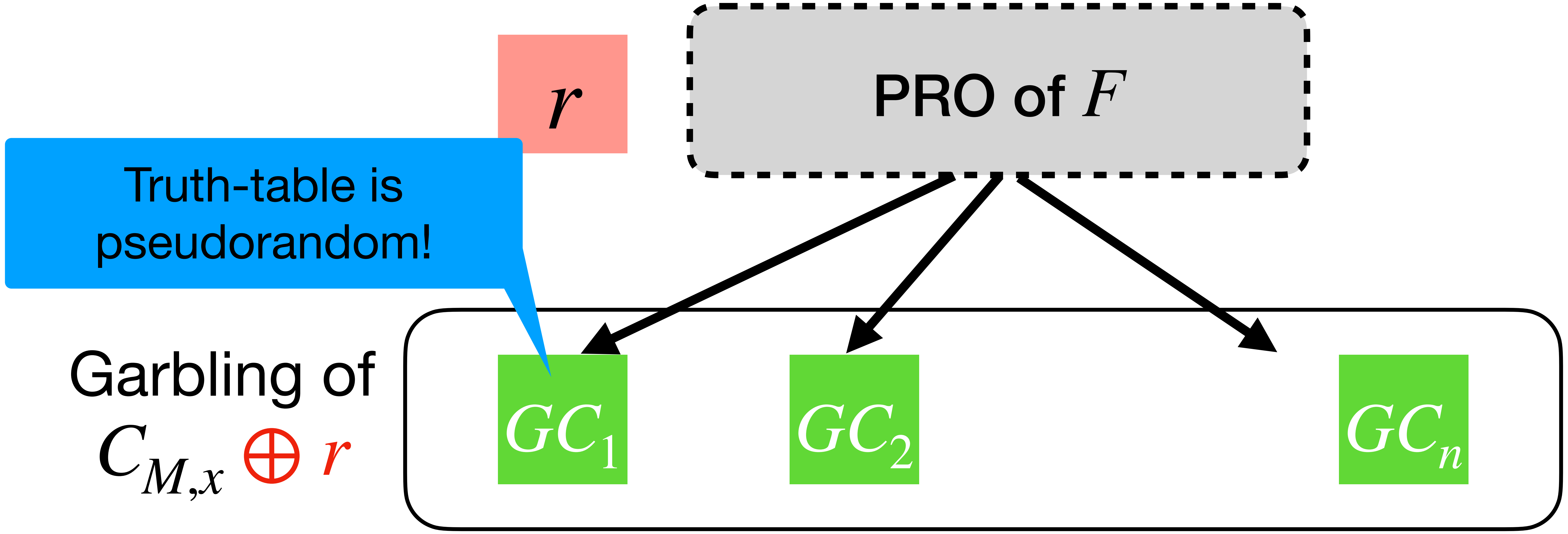

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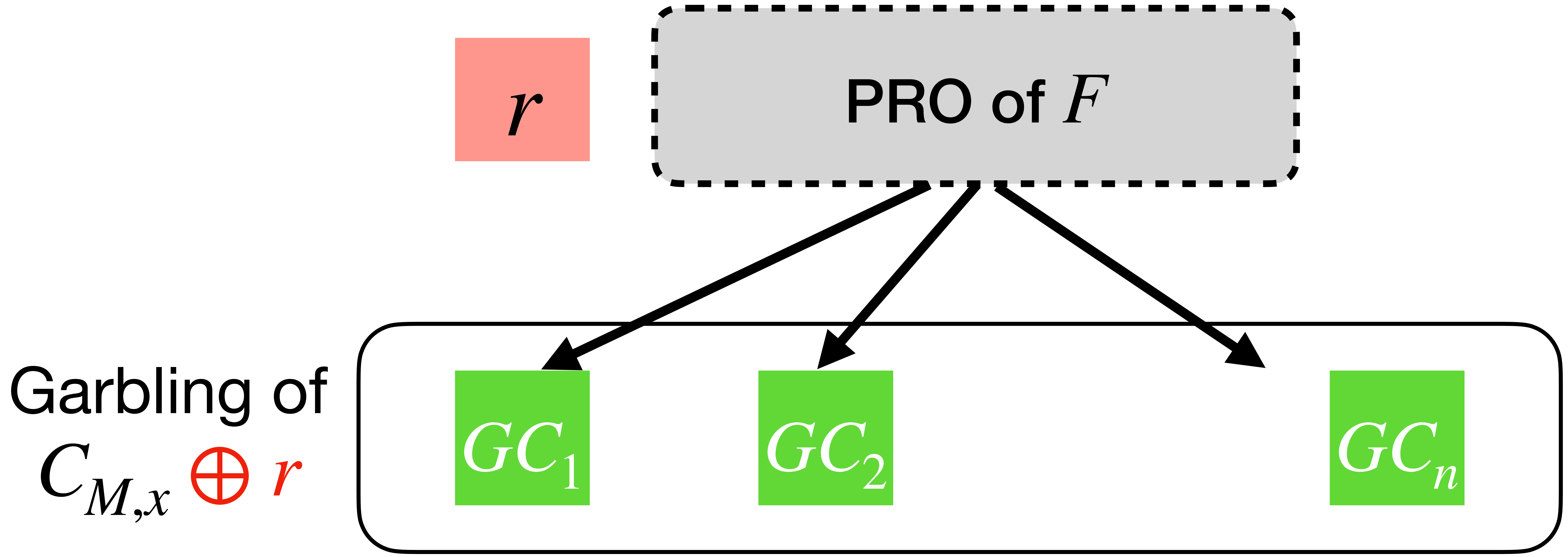

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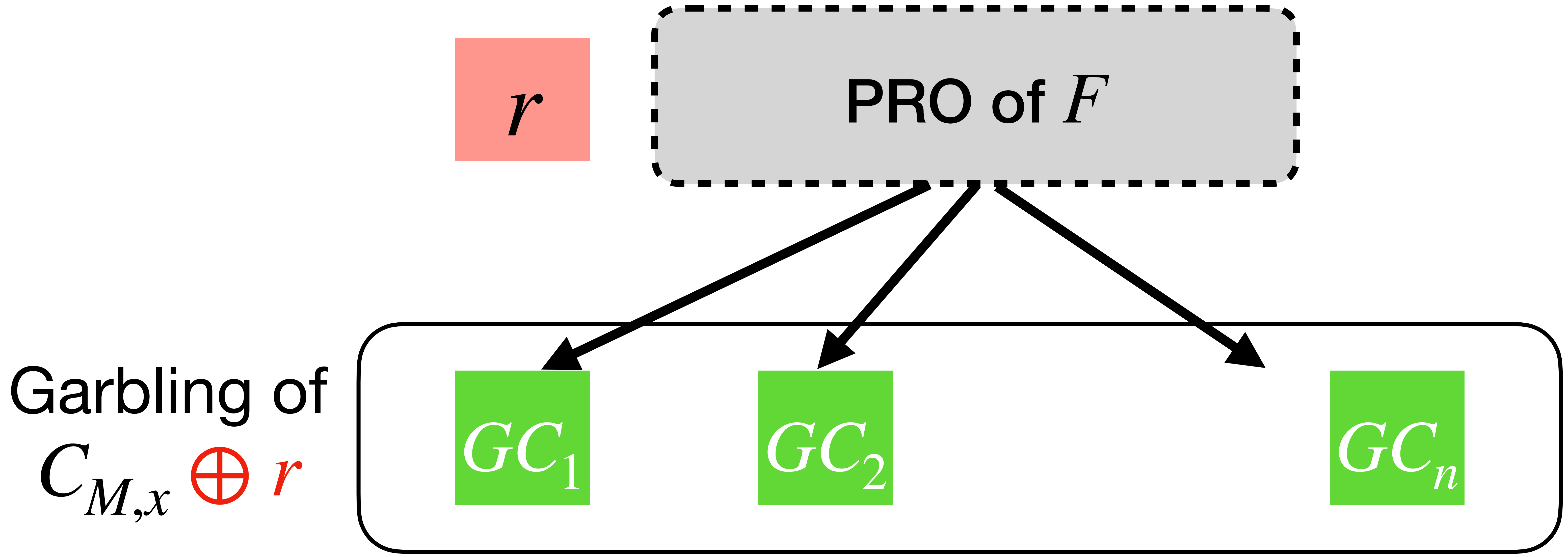

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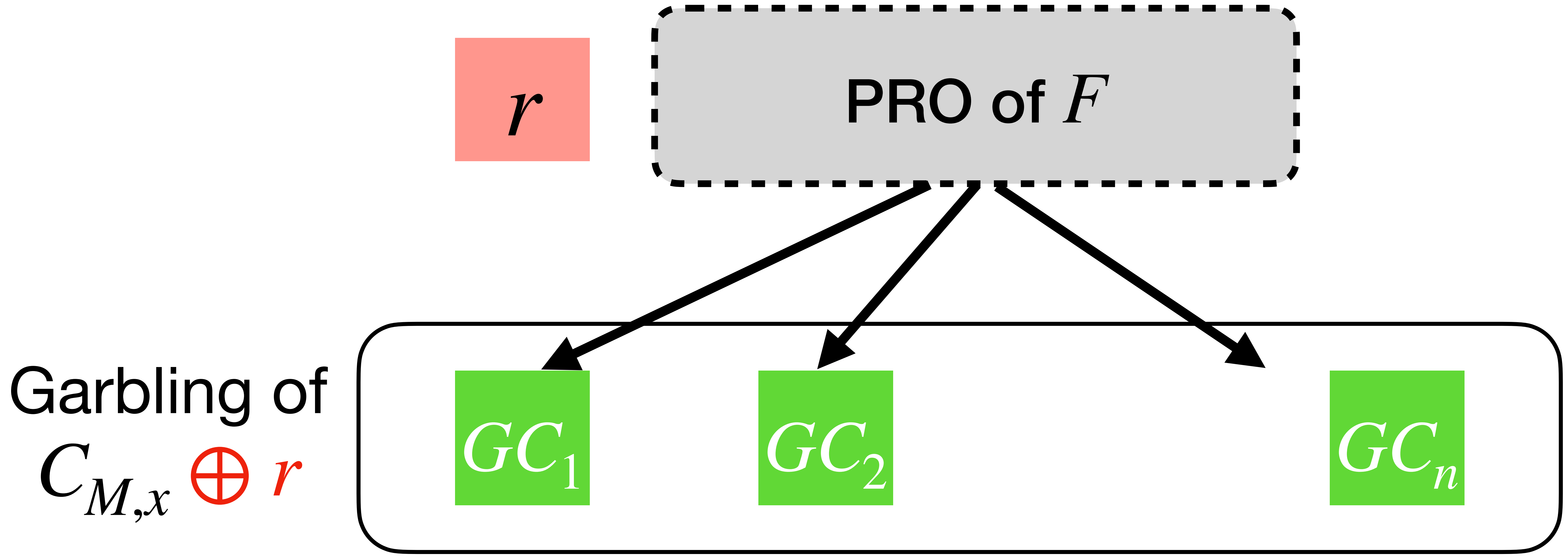

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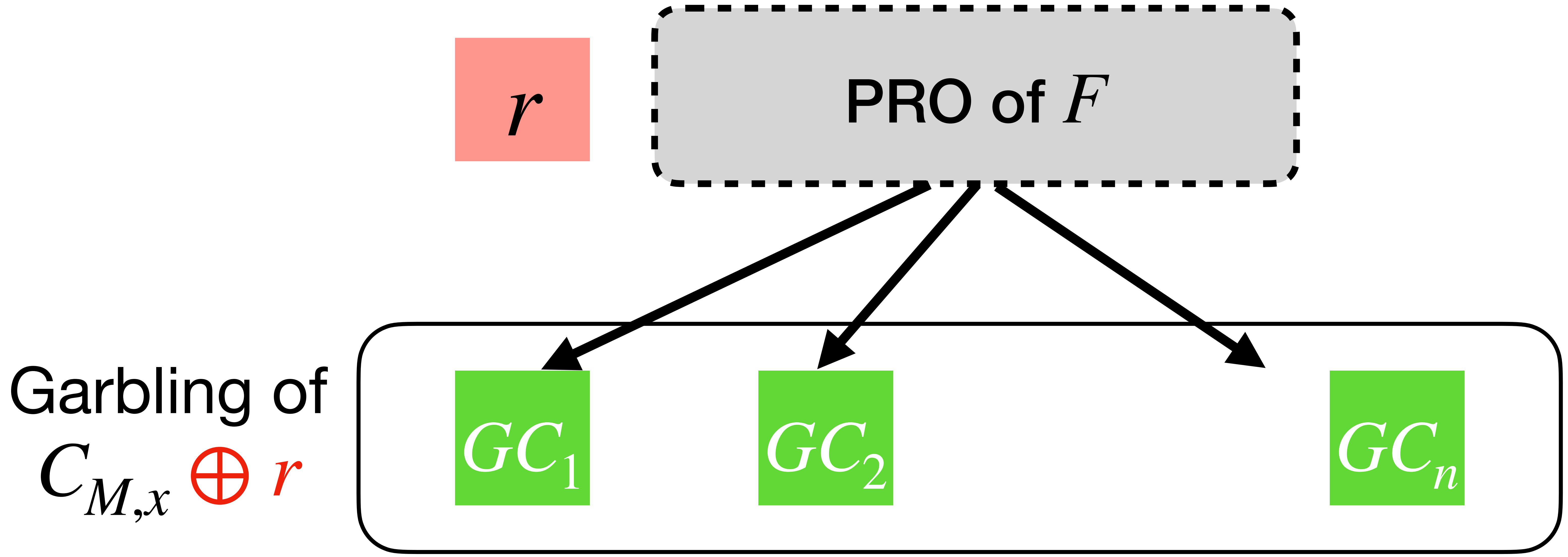

Simulator:

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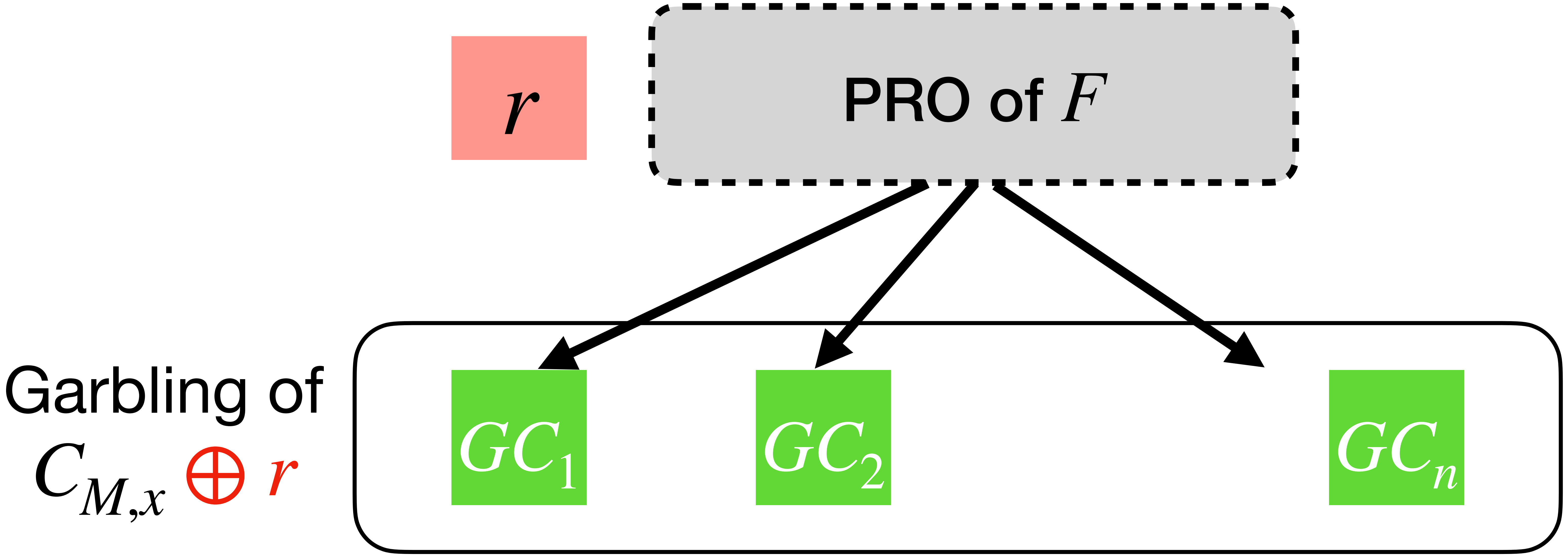

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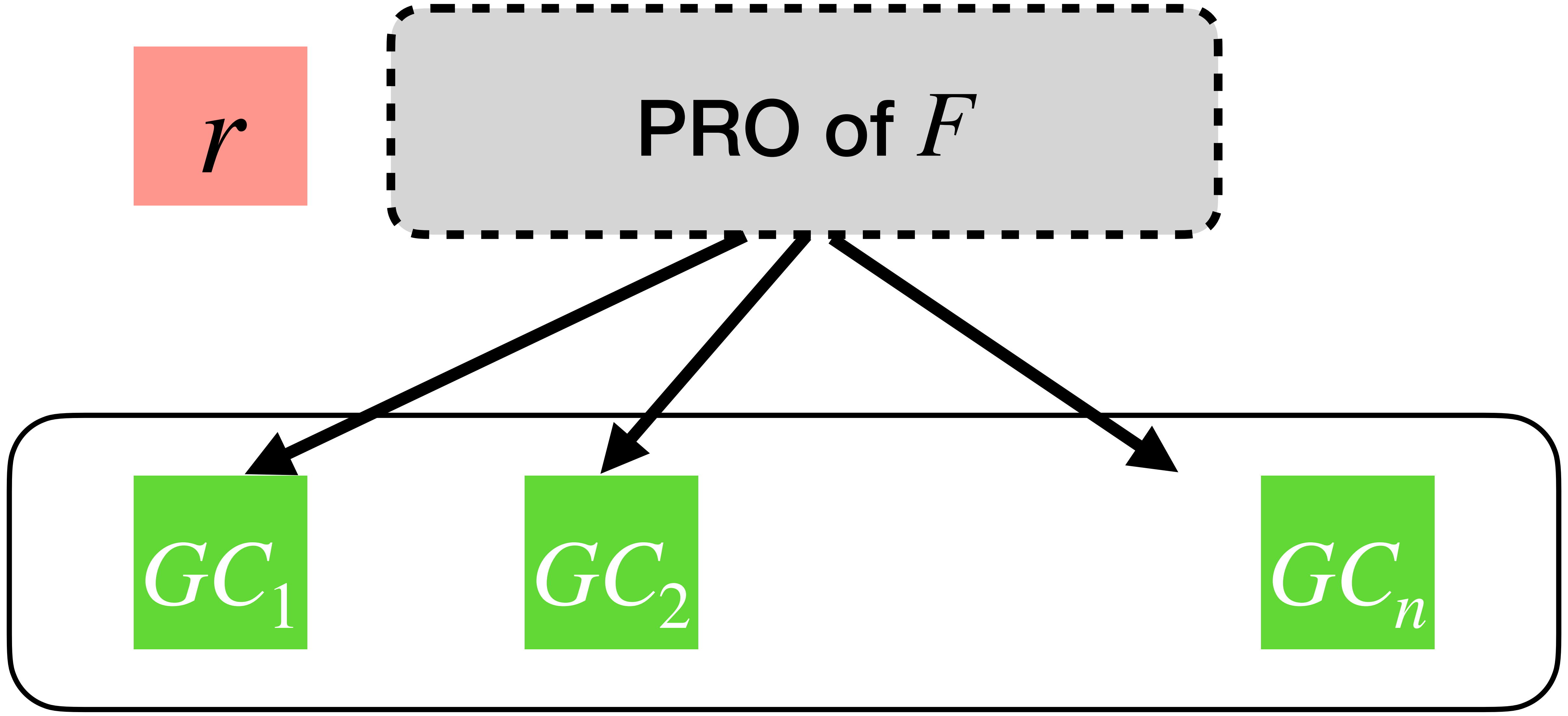
Garbling of $C_{M,x} \oplus r$



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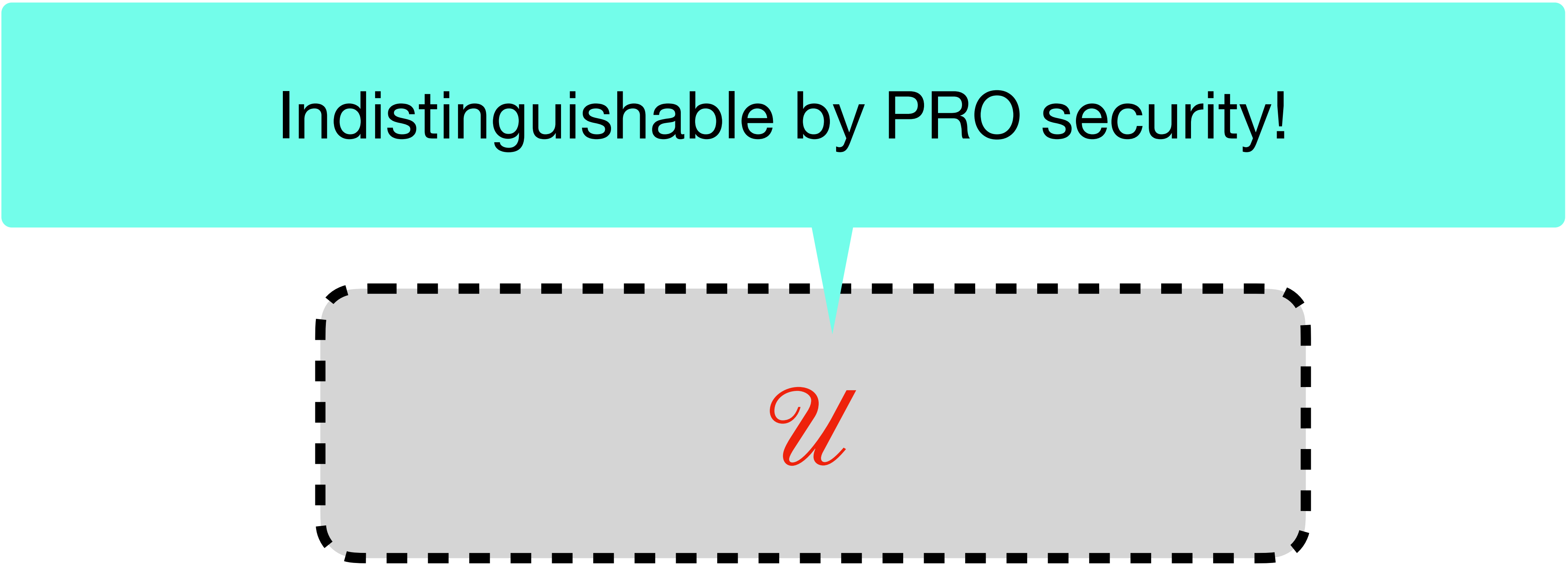
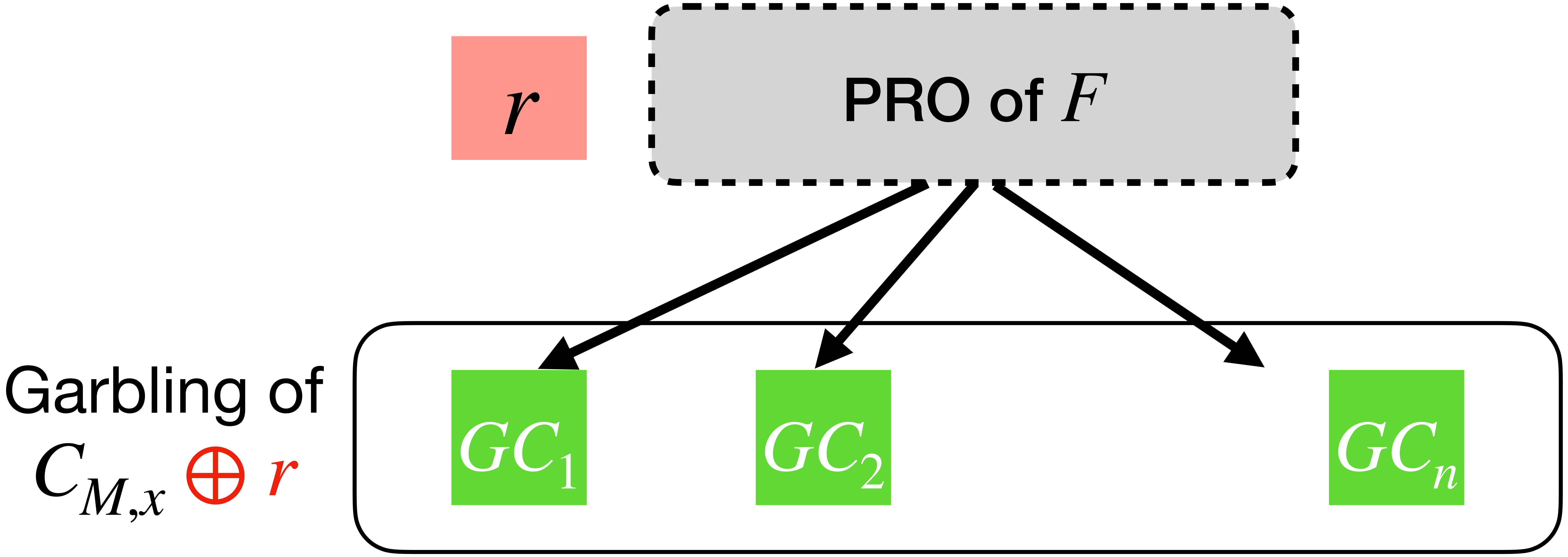


Succinct Garbling from PRO

- Pick a garbling scheme with ***pseudorandom*** garbling [BLSV '18].
- $\mathsf{GC.Garb} = (r, PRO(F))$



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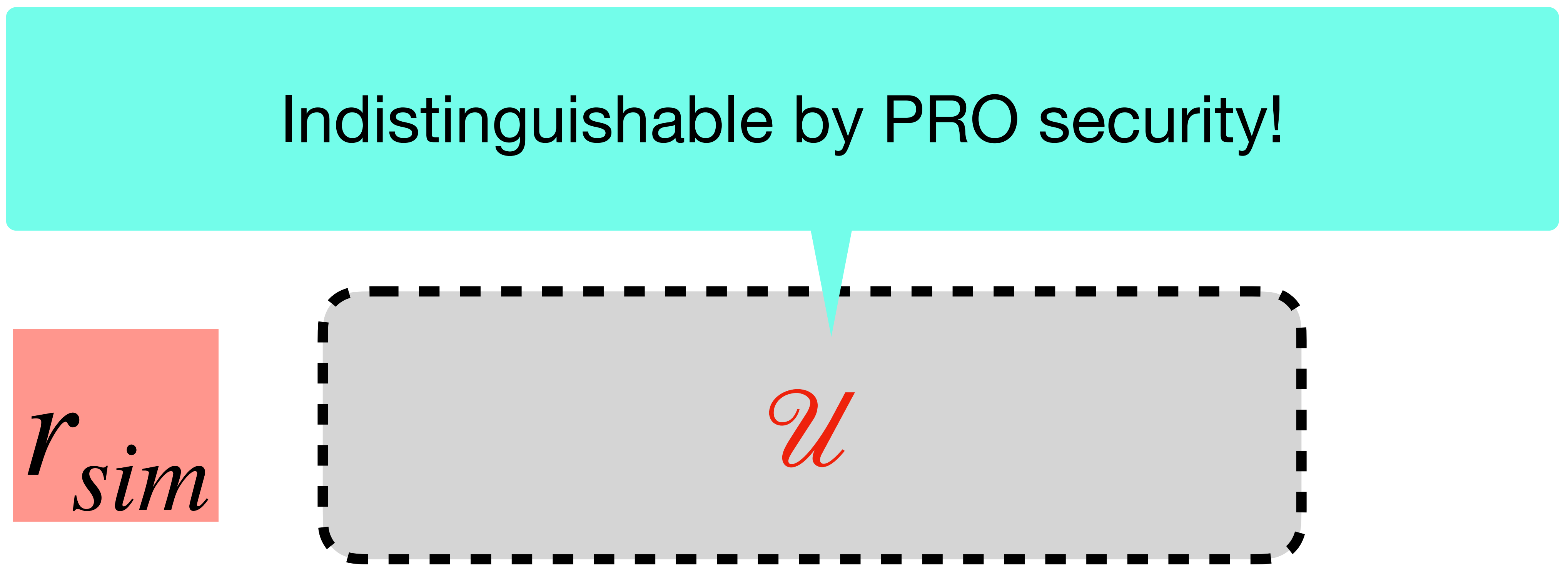

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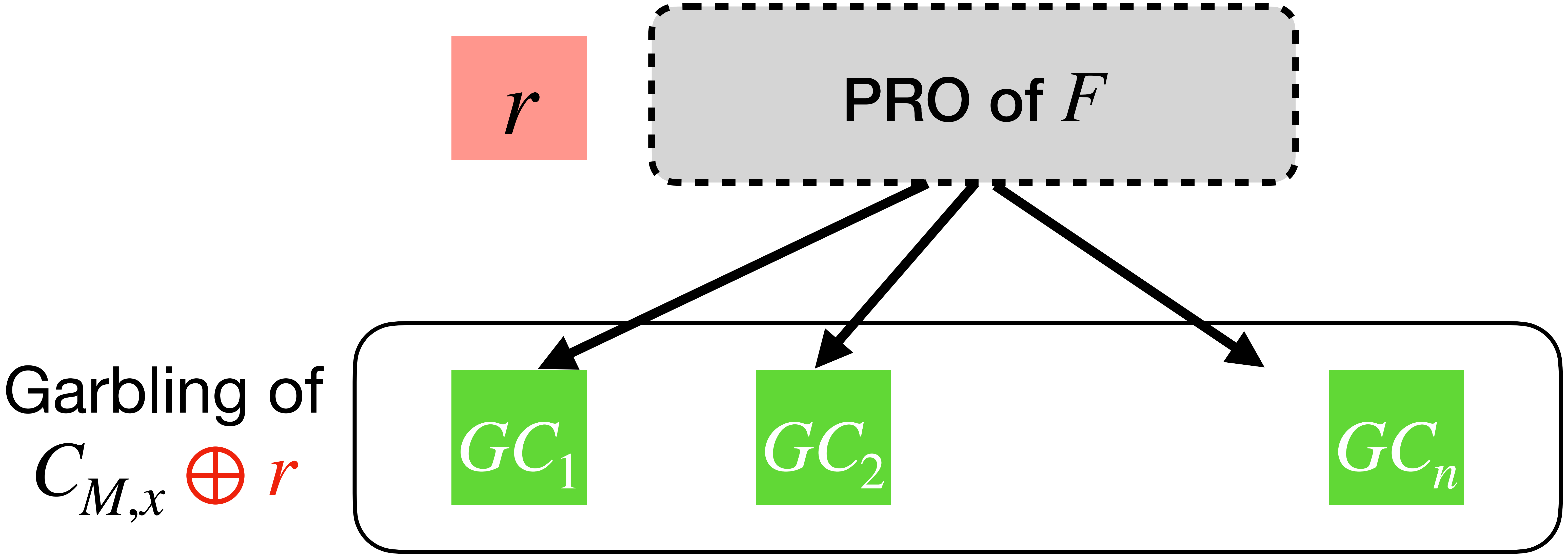
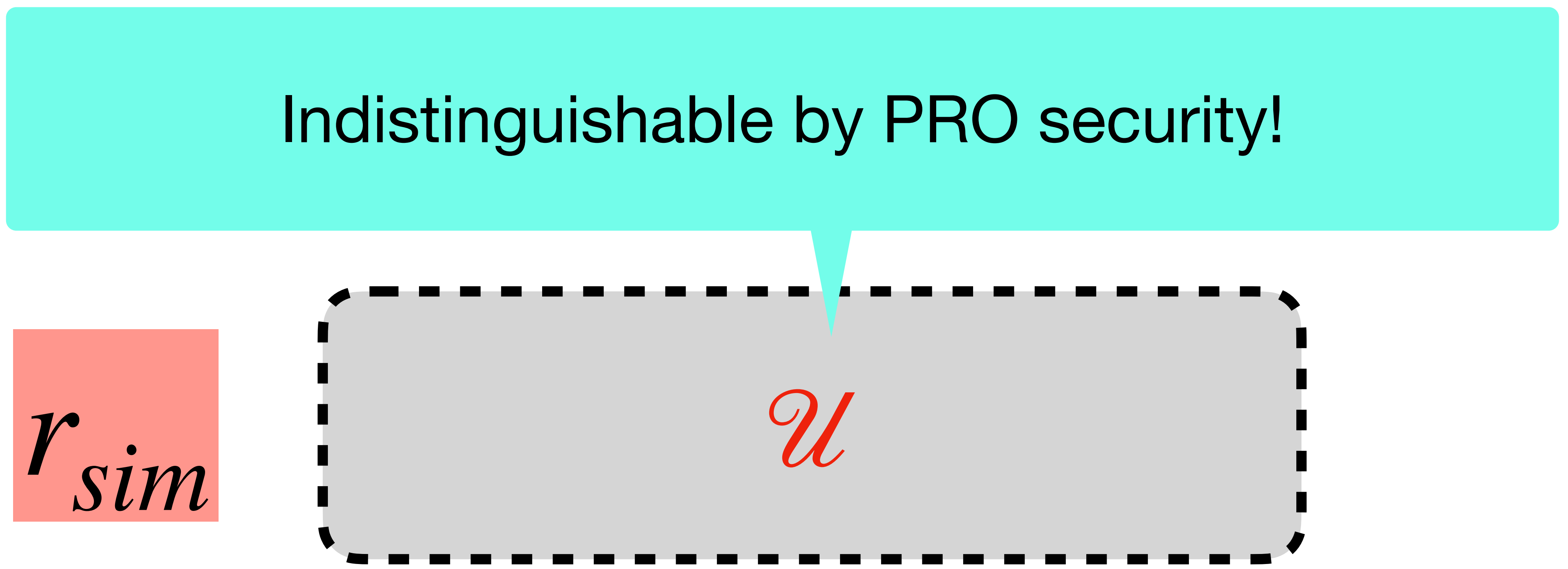


Simulator:

Set $r_{sim} = M(x) \oplus PRO.Eval(\mathscr{U})$



The notions and applications

	$f_K \equiv f_{K'}$	$TT(f_K)$ is pseudorandom
$\mathcal{O}(f_K) \approx_c \mathcal{O}(f_{K'})$	**iO**	**PRO** Null-iO [VWW22] SNARK for UP [**M**PV24] SNARG for NP [JKL**M**25] **This work:** (+LWE) FHE Succinct Garbling Succinct witness encryption
$\mathcal{O}(f_K) \approx_c \mathcal{U}$	**Impossible**	**dPRO**

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Go to bonus slide

A Weaker Notion

Indistinguishability PRO (iPRO)

$F_{K_0} \equiv F_{K_1},$

A Weaker Notion

Indistinguishability PRO (iPRO)

$F_{K_0} \equiv F_{K_1}$, $TT(F_K)$

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$\approx$

A Weaker Notion

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$\Downarrow$

$\approx$

A Weaker Notion

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$\Downarrow$

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A Weaker Notion

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- Implied by iO ⇒ no counterexamples!

A Weaker Notion

Indistinguishability PRO (iPRO)

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$$\Downarrow$$

$$iPRO(F_{K_0}) \approx iPRO(F_{K_1})$$

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- Implied by iO $\Rightarrow$ no counterexamples!

xiPRO: $|iPRO(C)| = |TT(C)|^{1-\epsilon}$

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$\mathcal{O}(f_K) \approx_c \mathcal{O}(f_{K'})$	**iO**		**PRO**
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$\mathcal{O}(f_K) \approx_c \mathcal{U}$	**Impossible**	**Same as dPRO**	**dPRO**

x-iPRO + Bilinear Maps = x-iO!

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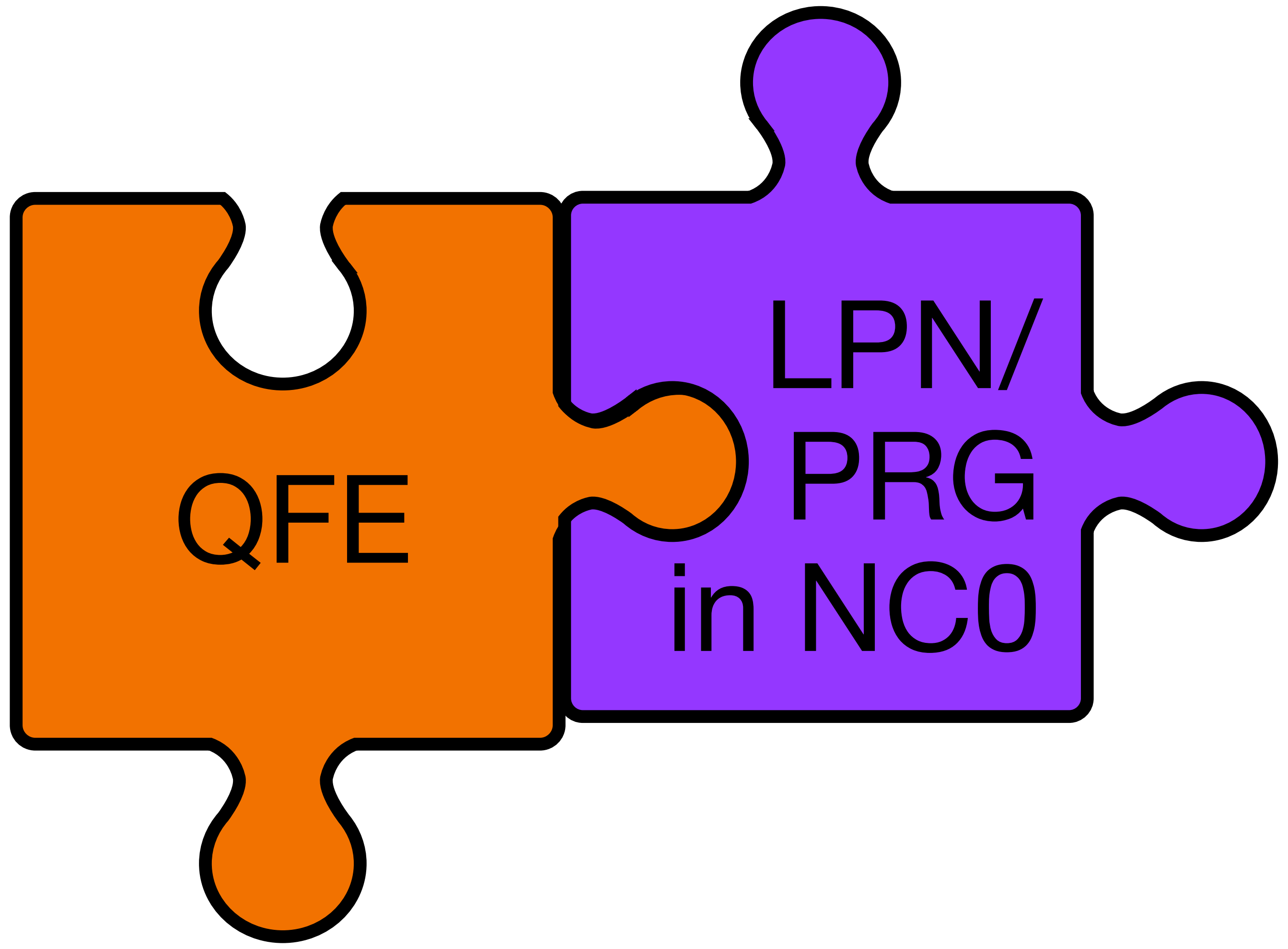

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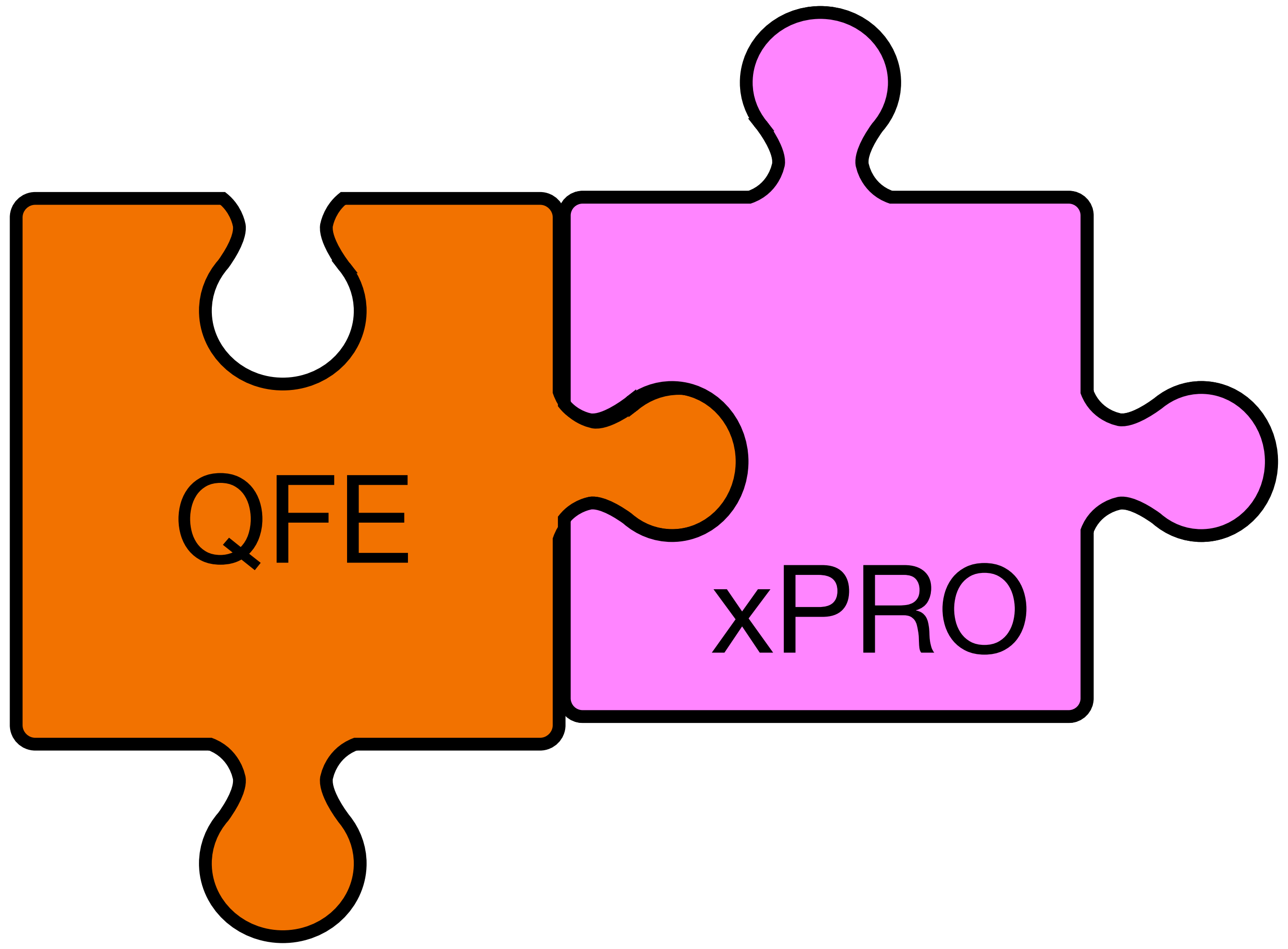

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    - **Open:** Can we construct iPRO from LPN variants/PRGs in NC0? Or even LWE?
- (Not in talk) We give a *candidate* construction via the evasive LWE heuristic (more on this in the next talk!)

Thank you for your attention!

Bonus slides

Counterexample to PRO

Precondition

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aux

$x = TT(PRF_K)$	WE("x is TT of small C",0^λ)

Counterexample to PRO

Precondition

Pick a witness encryption which is instance-hiding

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Counterexample to PRO

Postcondition

Back to main body

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$$x = TT(PRF_K) = TT(PRO(PRF_K))$$

Back to main body

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Back to main body

Counterexample to PRO

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$x = TT(PRF_K) = TT(PRO(PRF_K))$

aux

$PRO(PRF_K)$

WE(“x is TT of small C”,0^λ)

Back to main body

Counterexample to PRO

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$x = TT(PRF_K) = TT(PRO(PRF_K))$

aux

$PRO(PRF_K)$

WE(“x is TT of small C”,0^λ)

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Back to main body

Counterexample to PRO

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$x = TT(PRF_K) = TT(PRO(PRF_K))$

aux

$w =$ $PRO(PRF_K)$

WE("x is TT of small C",0^λ)

u'

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Back to main body

Counterexample to PRO

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aux

$w =$ $PRO(PRF_K)$

WE(“x is TT of small C”,0^λ)

$WE.Dec$

0^λ

u'

WE(“x is TT of small C”,0^λ)

Back to main body

Counterexample to PRO

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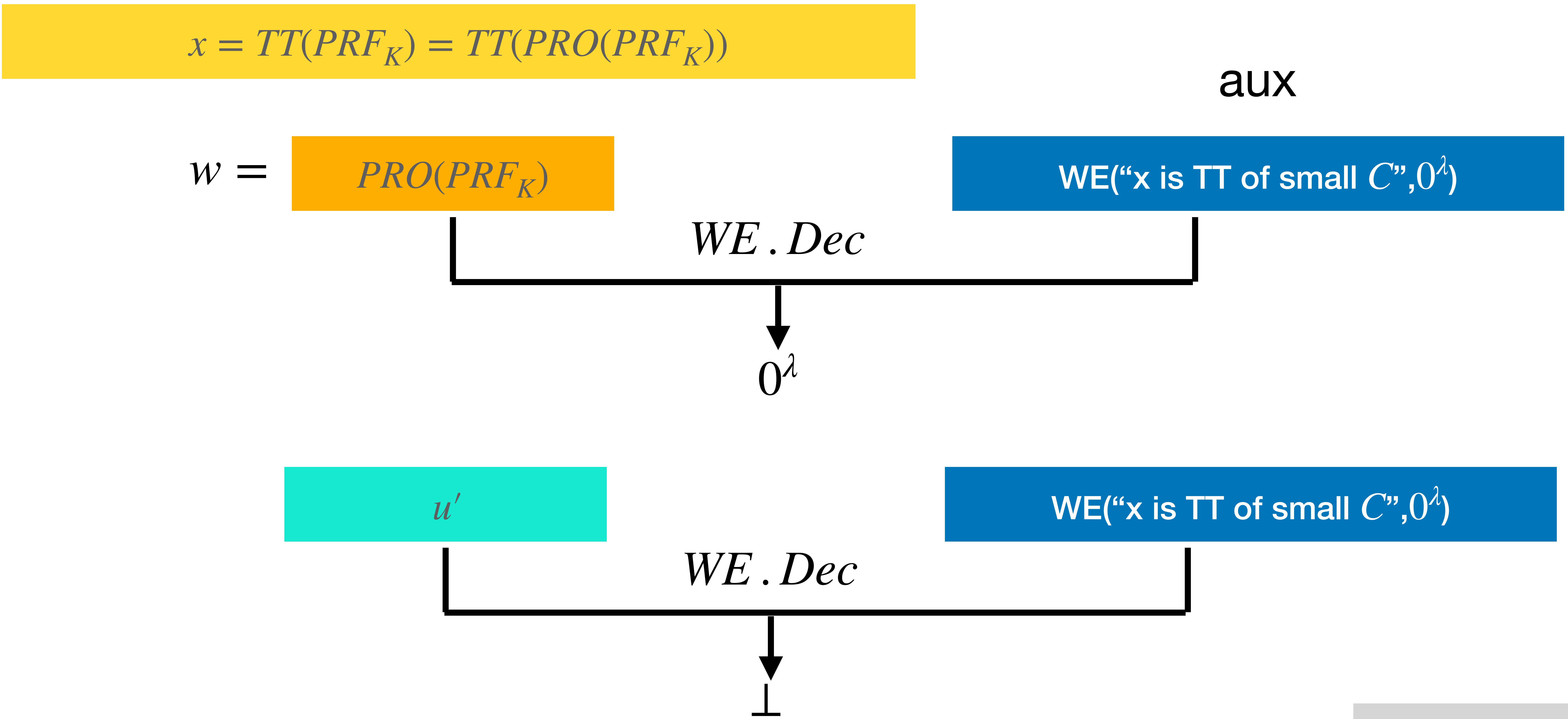

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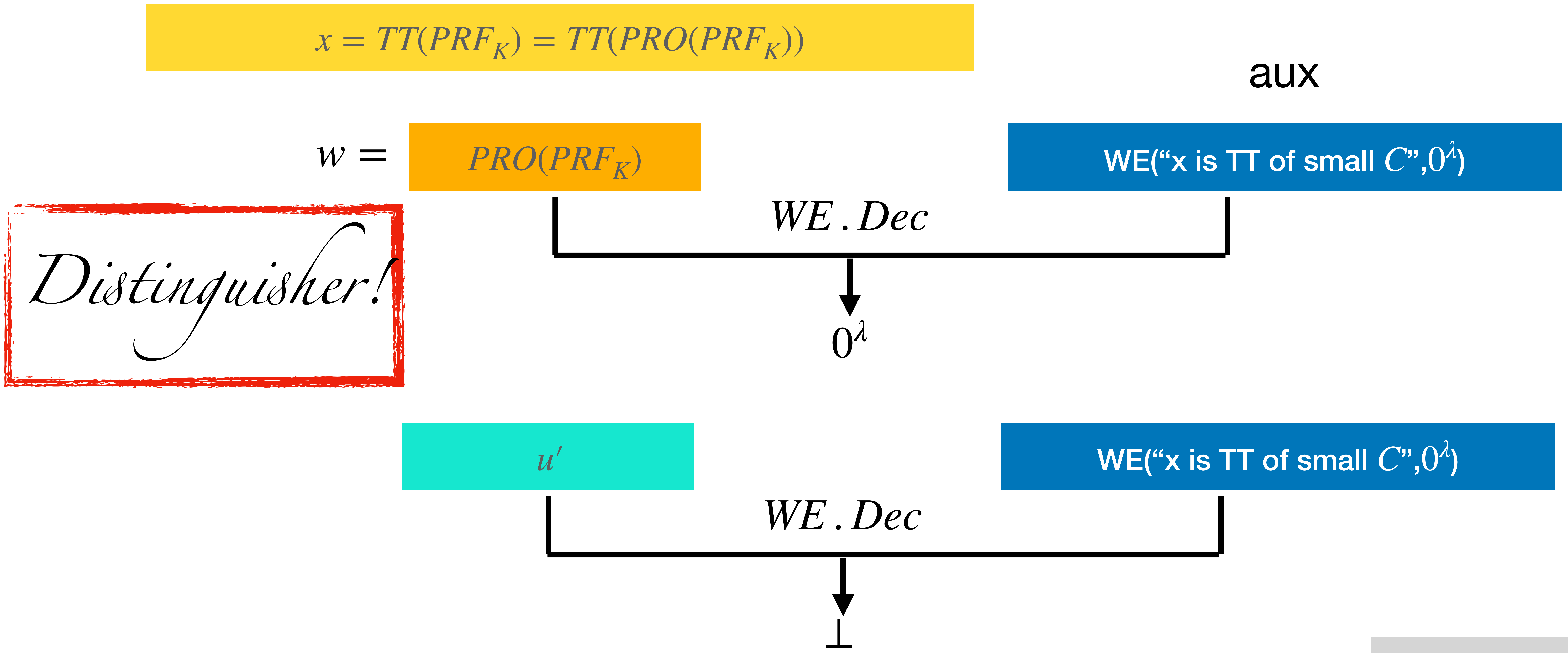
Counterexample to PRO

Postcondition



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