Simple and General Counterexamples to Evasive LWE





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    - Our attack is an example of a “zeroizing” attack.
    - Questions the underlying philosophy of evasive LWE in the private-coin setting.
- **Concurrent work:** [Hsieh-Jain-Lin 25], [Agrawal-Modi-Yadav-Yamada 25] also show attacks on evasive LWE. More on this later.

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In this talk, we will treat $\mathbf{S}$ as a matrix rather than a vector.

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- LWE has proven to be **extremely fruitful**: e.g. Fully homomorphic encryption, attribute-based encryption, etc.
- However, some applications have still evaded us.
  - Some souped up "LWE++" seems sufficient. E.g. want to give out some "auxiliary" information **involving the trapdoor of $\mathbf{B}$…**

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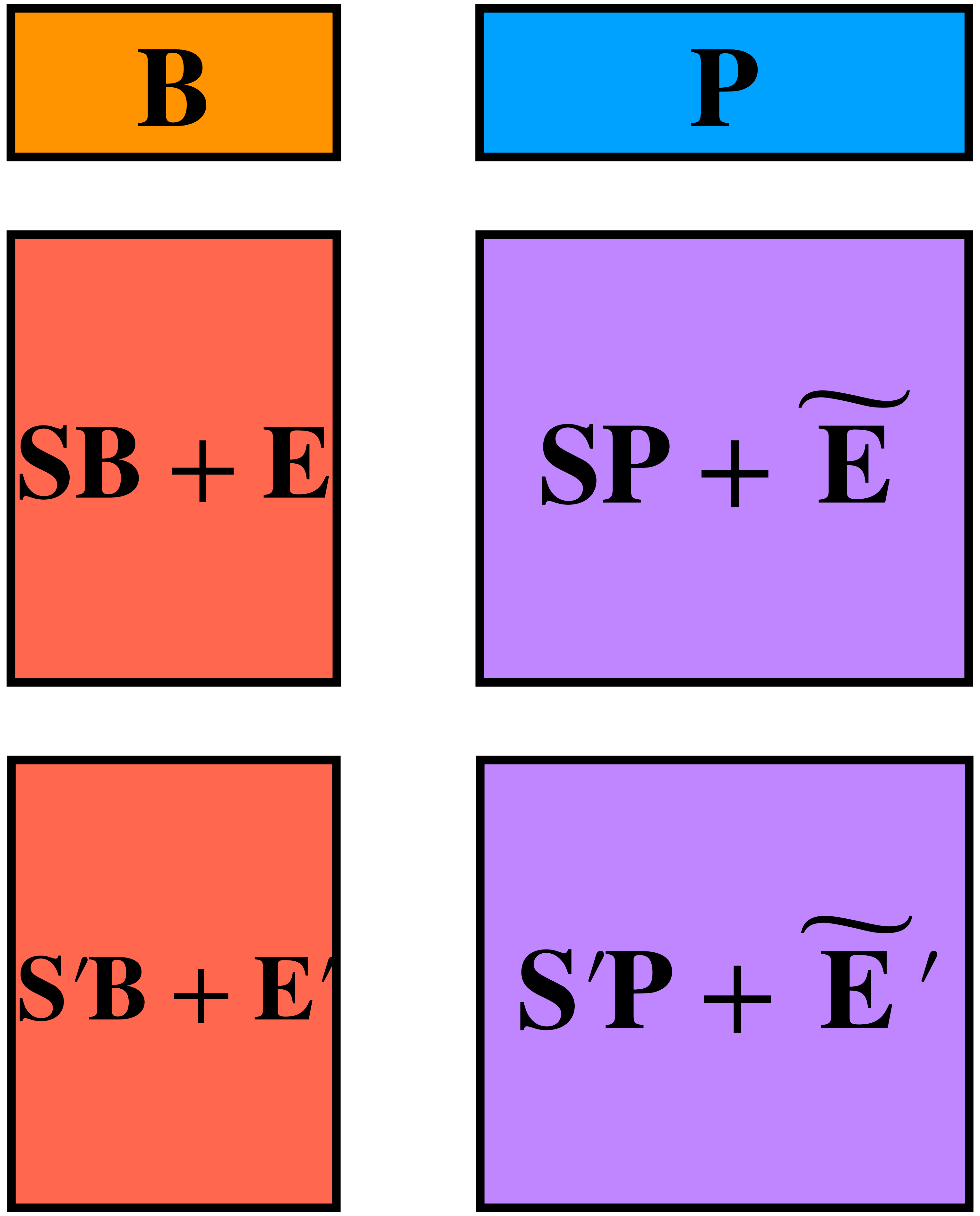

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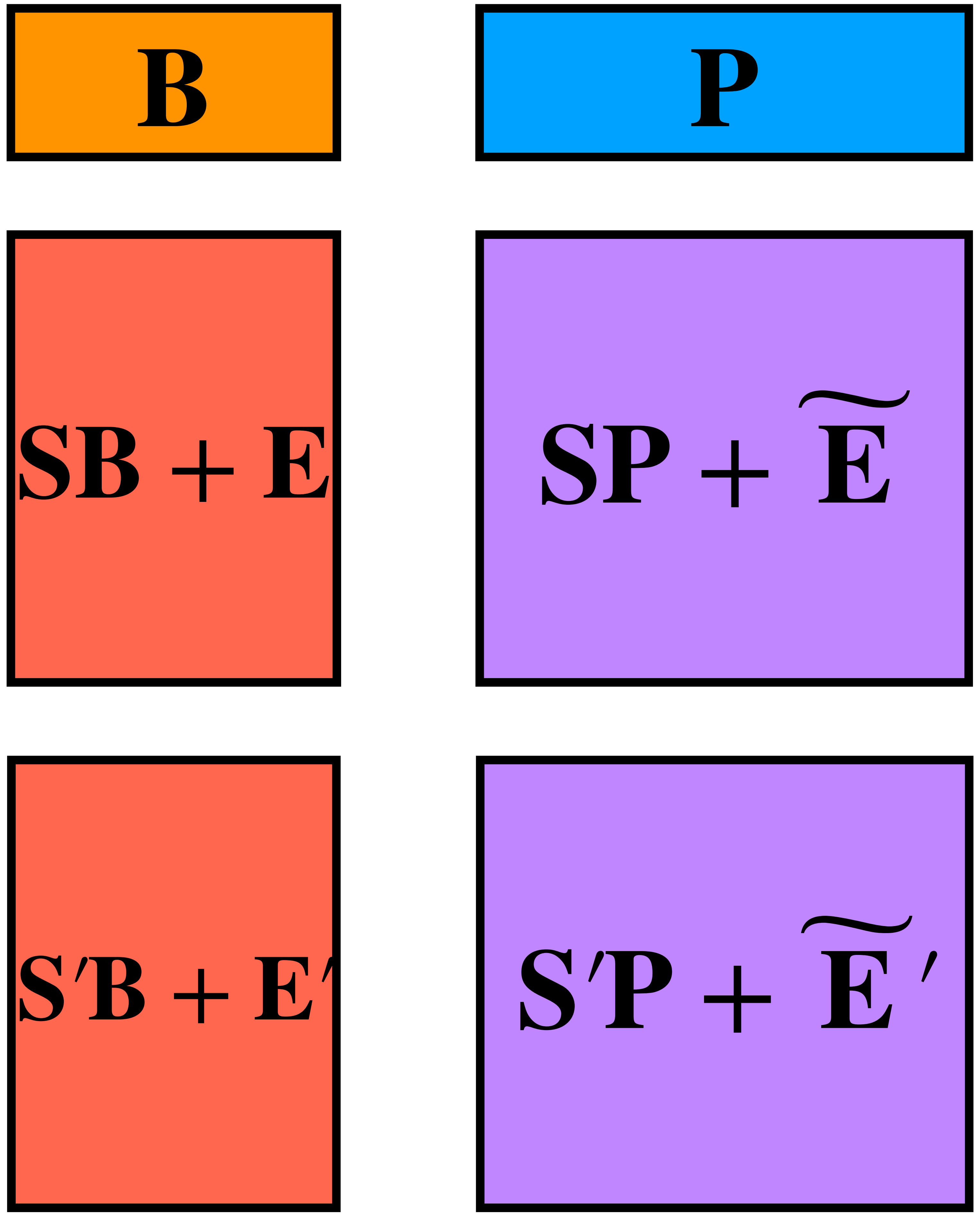

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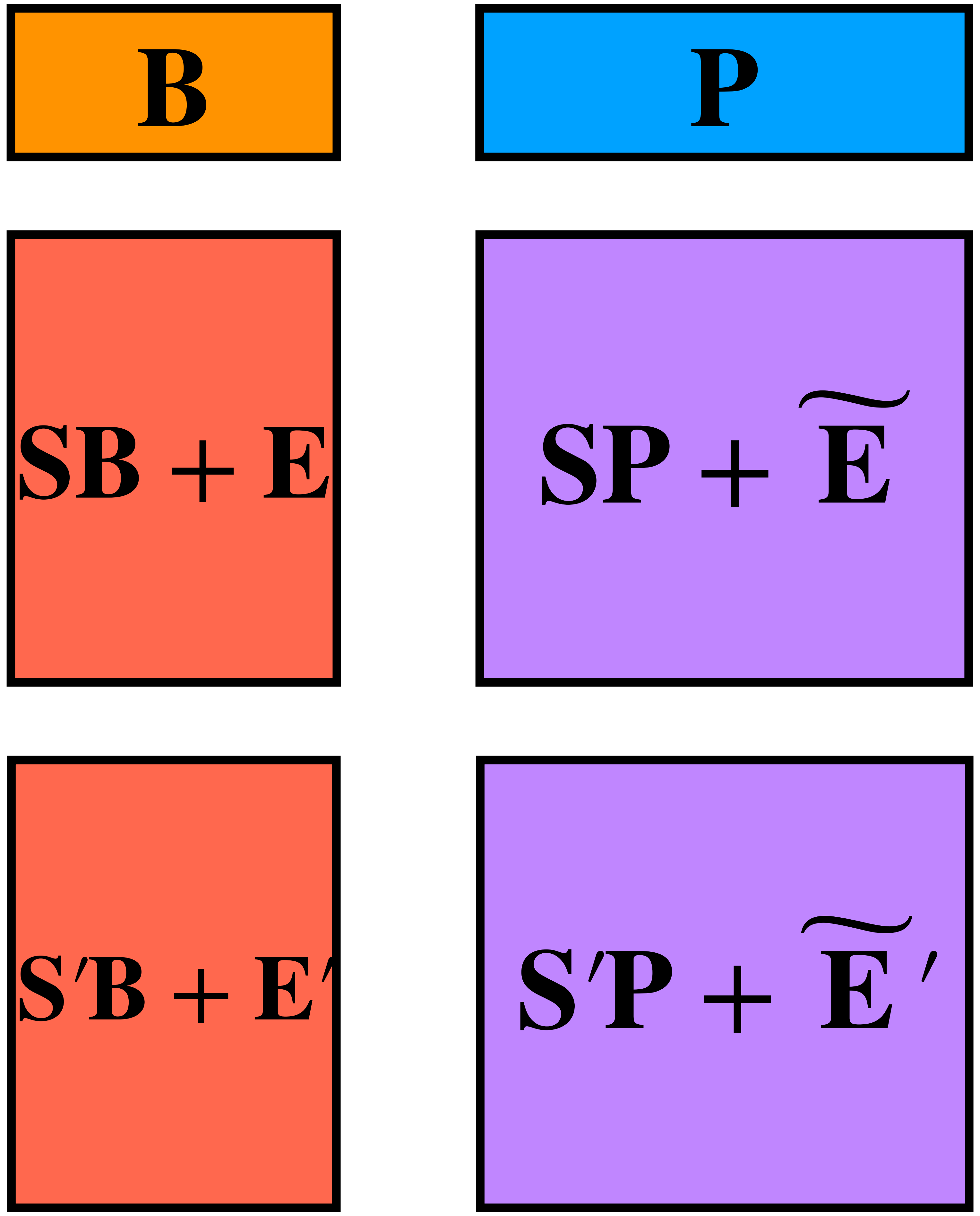

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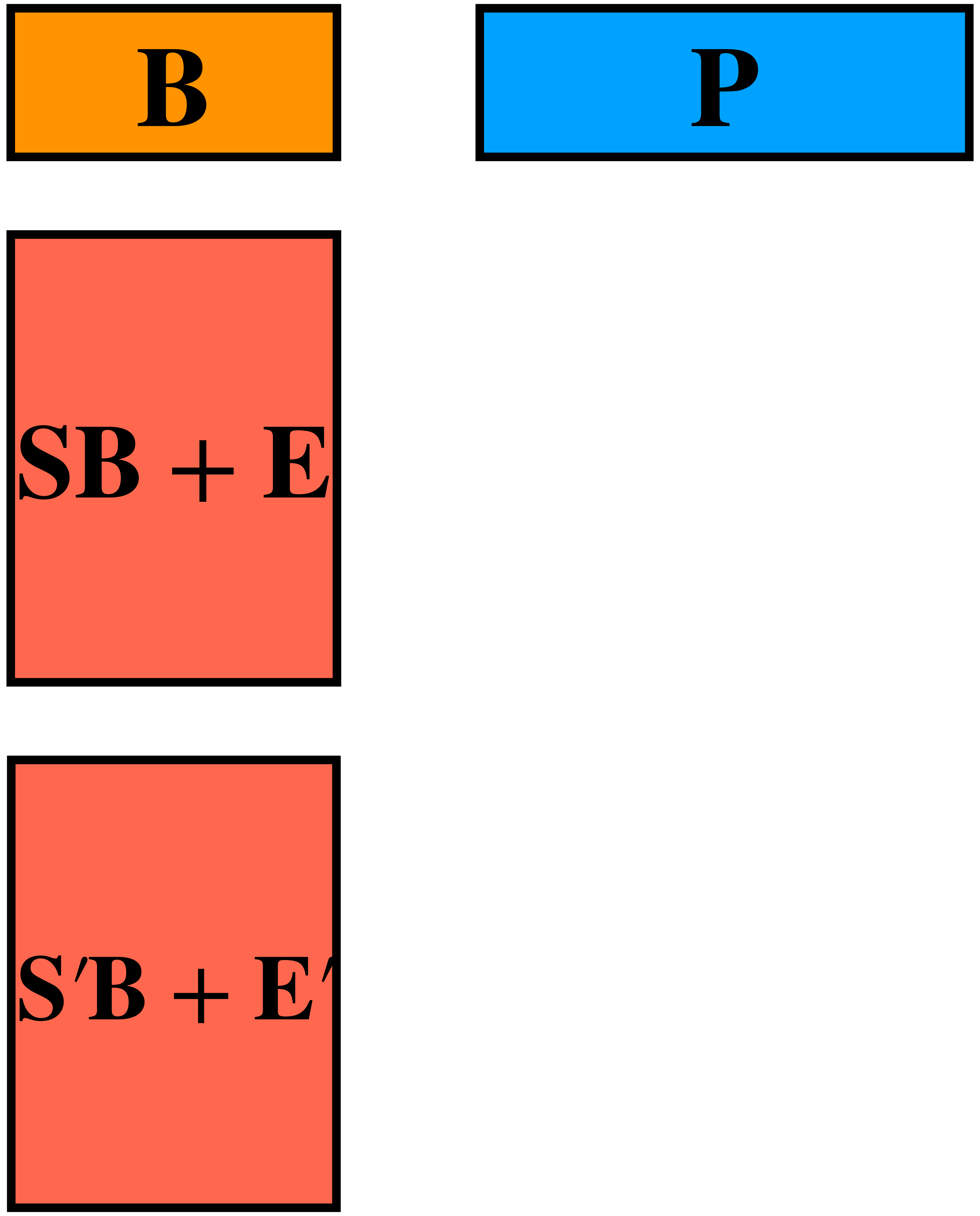

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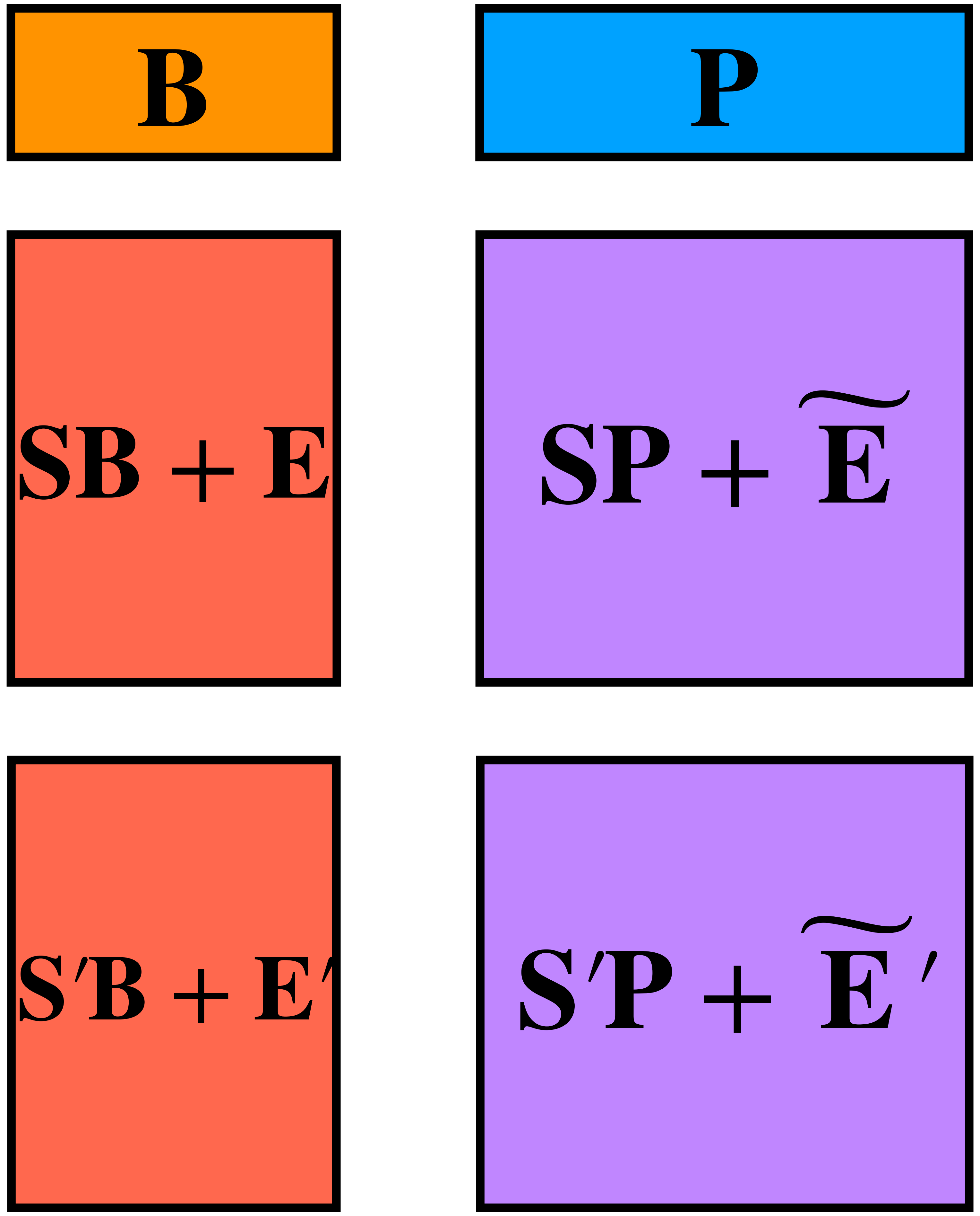

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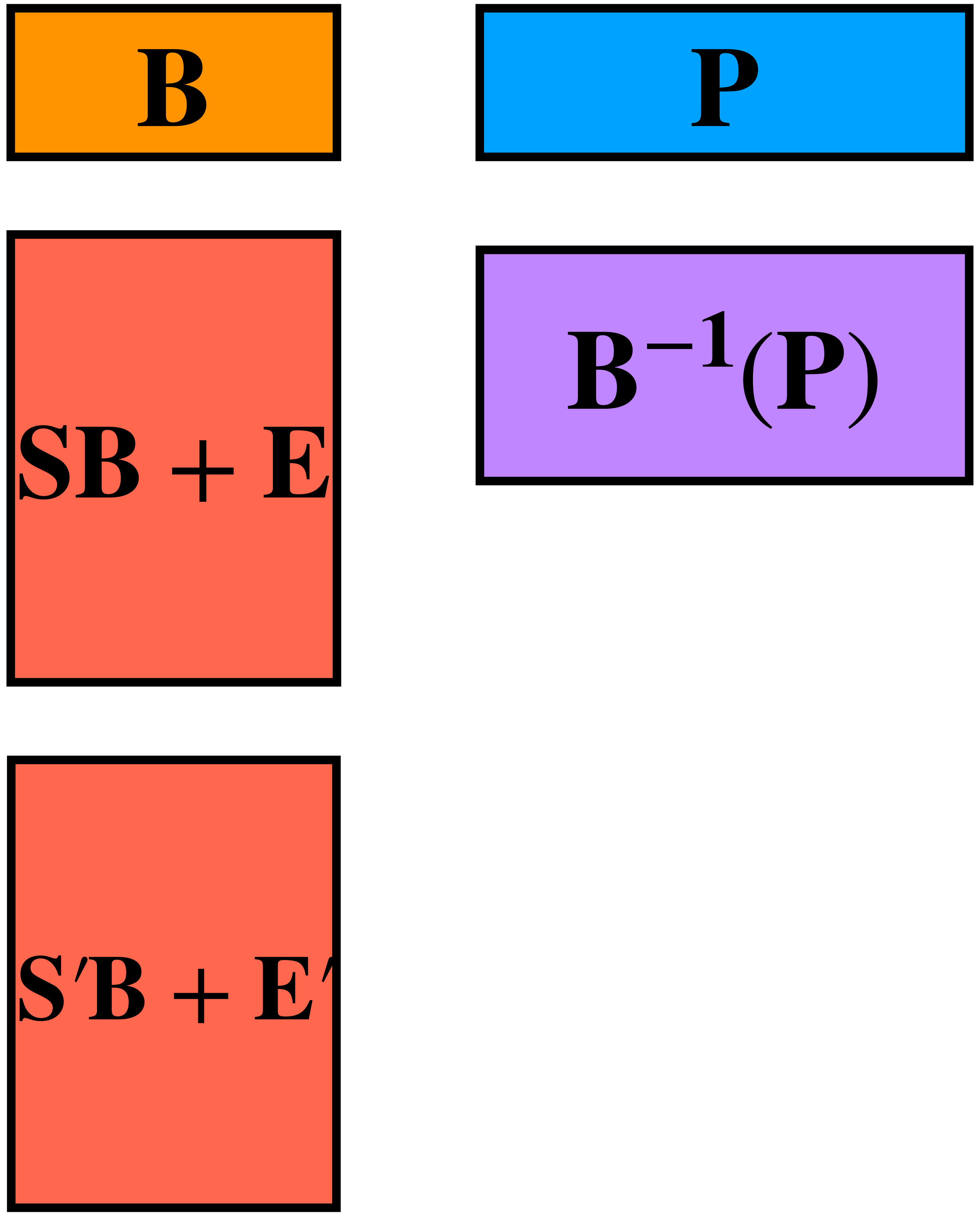

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Evasive LWE:
When can give out $\mathbf{B}^{-1}(\mathbf{P})$?

Evasive LWE [Wee '22, Tsabary '22]

- Let $\mathbf{S}, \mathbf{P}, \mathsf{aux} \leftarrow \mathsf{Samp}(\mathsf{rand})$.

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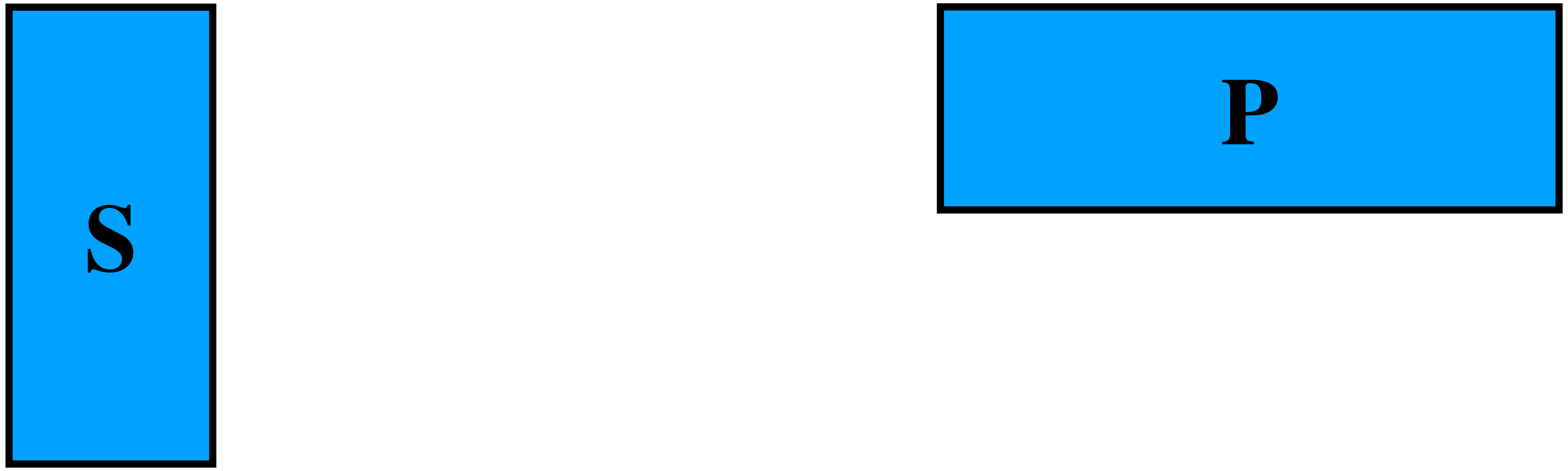

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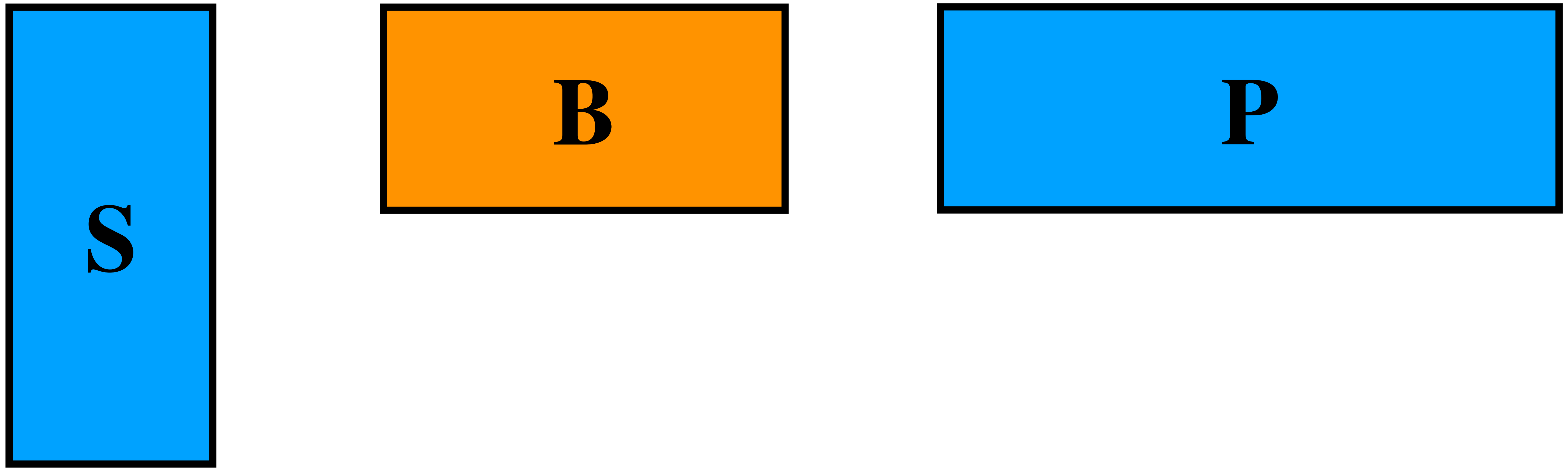

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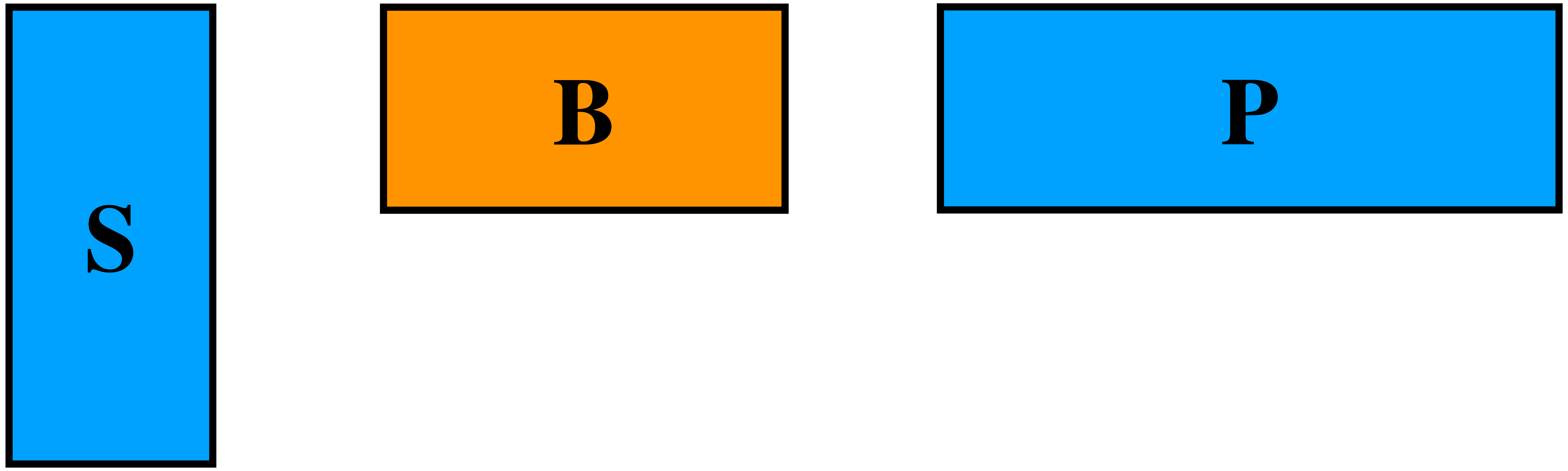

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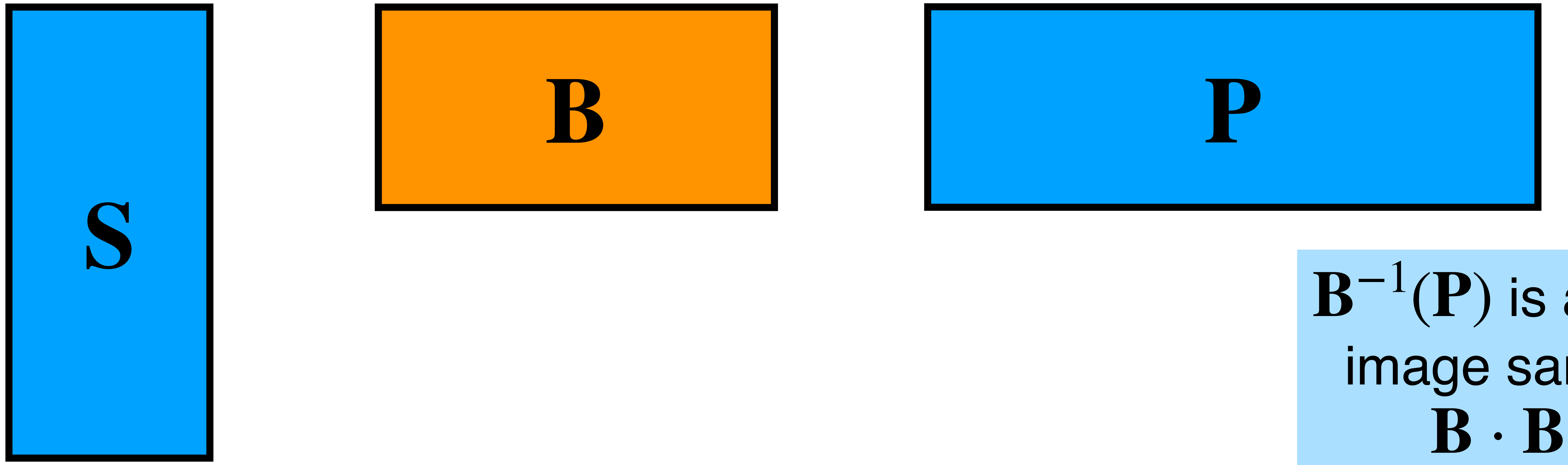

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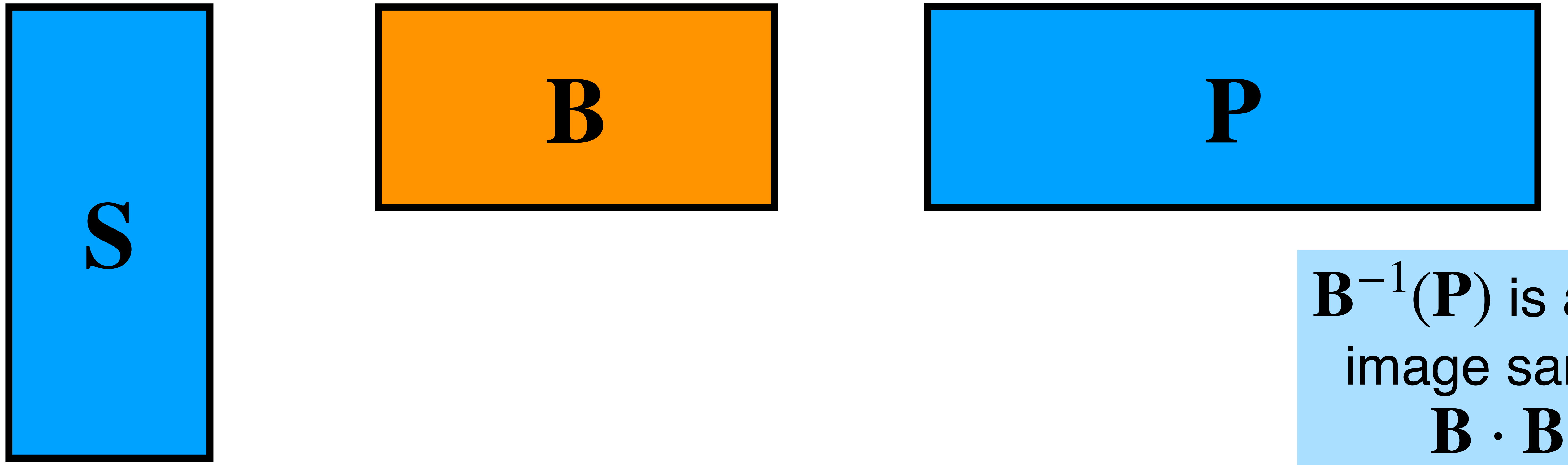

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A heuristic to justify the post-condition

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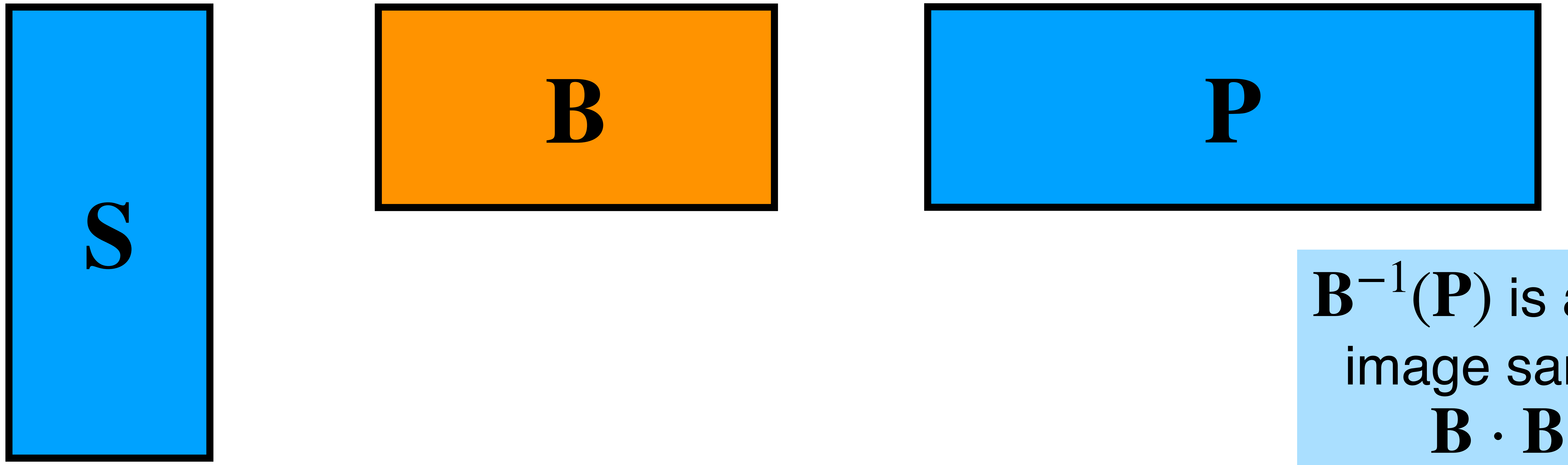

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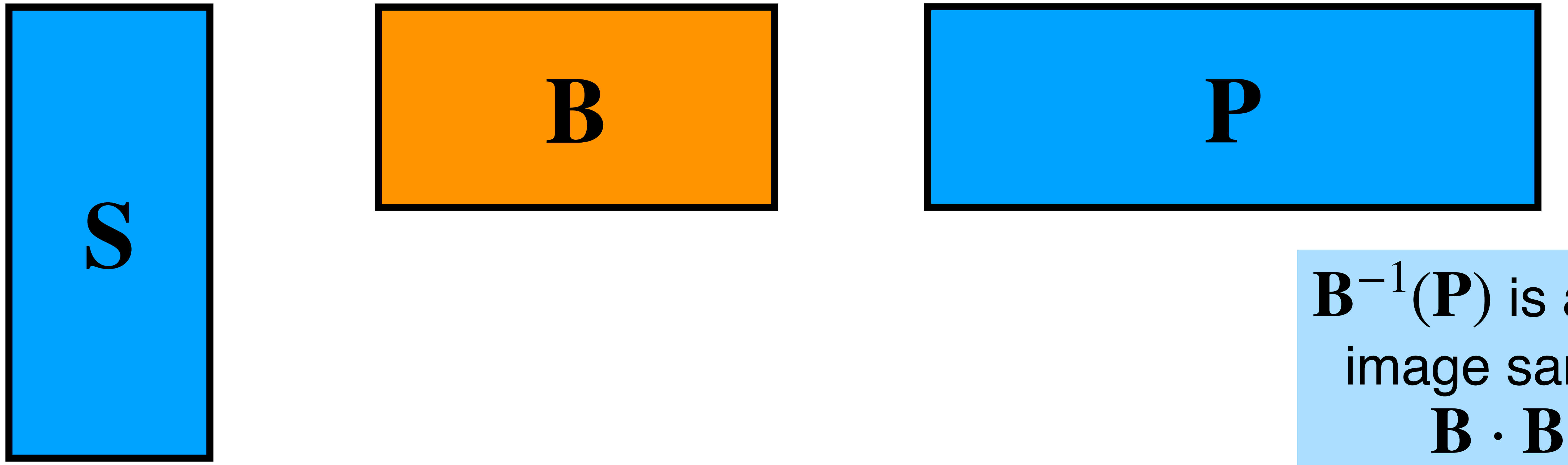

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Will omit aux for the next few slides.

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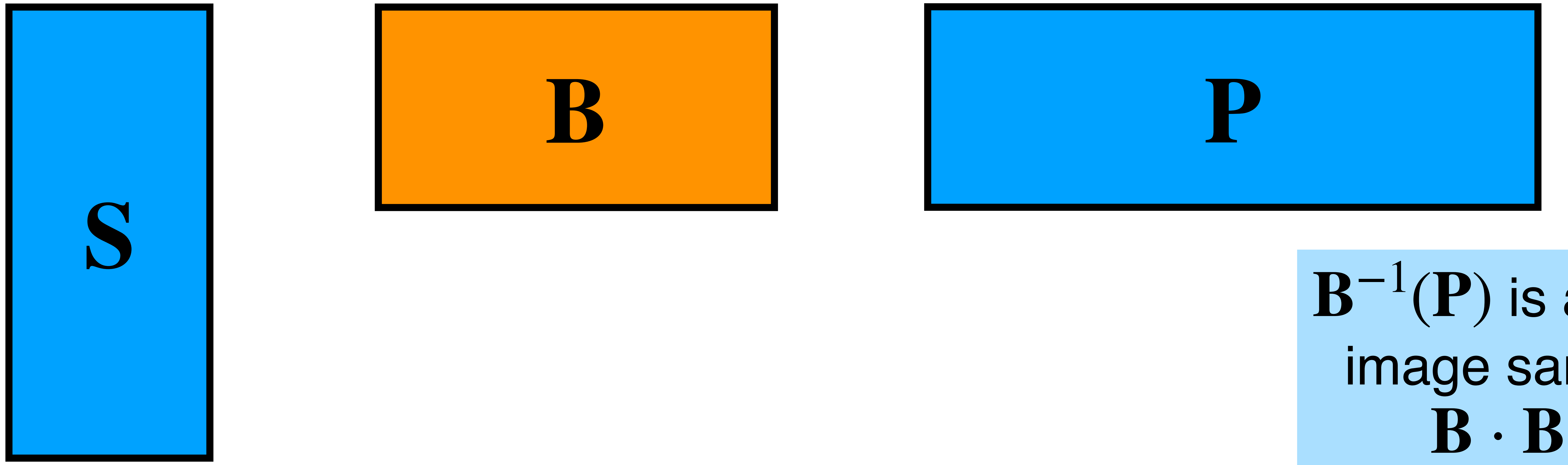

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$$(\mathbf{SB}+\mathbf{E}) \cdot \mathbf{B}^{-1}(\mathbf{P}) = \mathbf{EB}^{-1}(\mathbf{P})$$

Both $\mathbf{E}$ and $\mathbf{B}^{-1}(\mathbf{P})$ have low norm! We now have an equation over **integers,** AKA **"zeroizing"**

Evasive LWE: Design Philosophy

[Wee '22]

- Let $\mathbf{S}, \mathbf{P} \leftarrow \mathsf{Samp}(\mathsf{rand})$.

if $$(\mathbf{B}, \mathbf{P}, \mathbf{SB} + \mathbf{E}, \mathbf{SP} + \mathbf{E}') \approx_c (\mathbf{B}, \mathbf{P}, \mathscr{U}, \mathscr{U})$$

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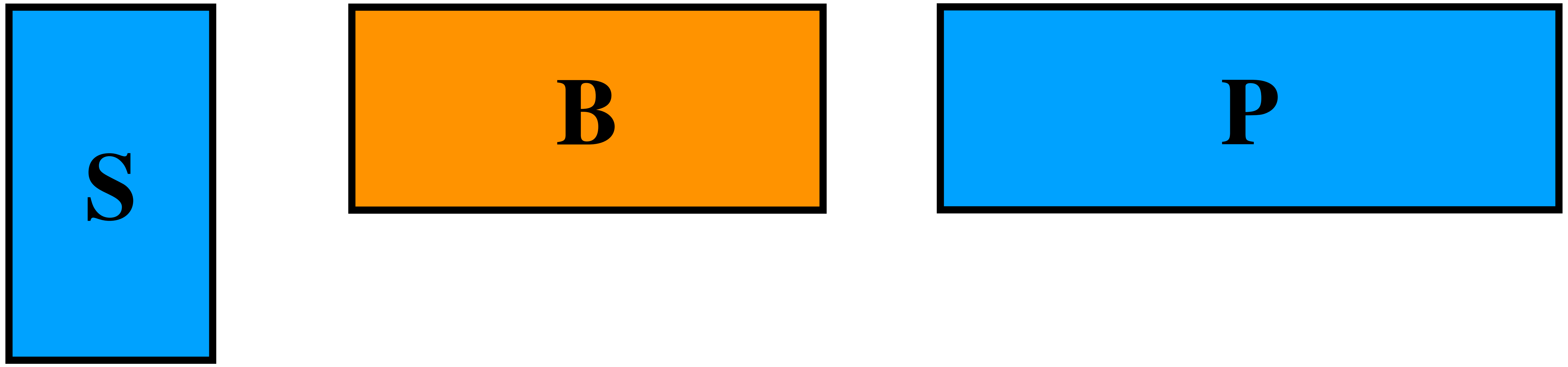

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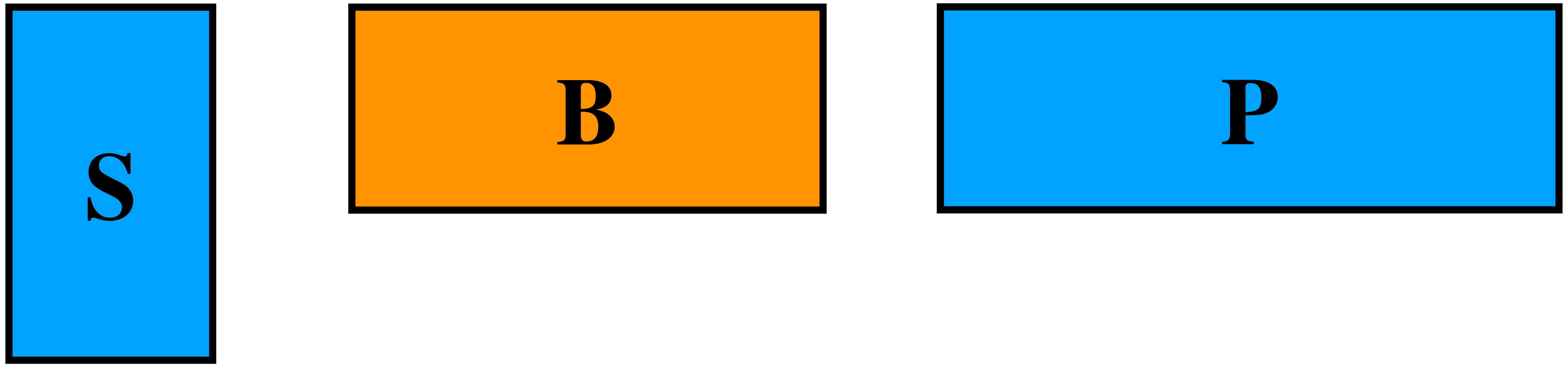
then $$(\mathbf{B}, \mathbf{P}, \mathbf{SB} + \mathbf{E}, \mathbf{B}^{-1}(\mathbf{P})) \approx_c (\mathbf{B}, \mathbf{P}, \mathscr{U}, \mathbf{B}^{-1}(\mathbf{P}))$$



Evasive LWE: Design Philosophy

[Wee '22]

- Let $\mathbf{S}, \mathbf{P} \leftarrow$ Samp(rand).

if $\quad (\mathbf{B}, \mathbf{P}, \mathbf{SB} + \mathbf{E}, \mathbf{SP} + \mathbf{E}') \approx_c (\mathbf{B}, \mathbf{P}, \mathscr{U}, \mathscr{U})$

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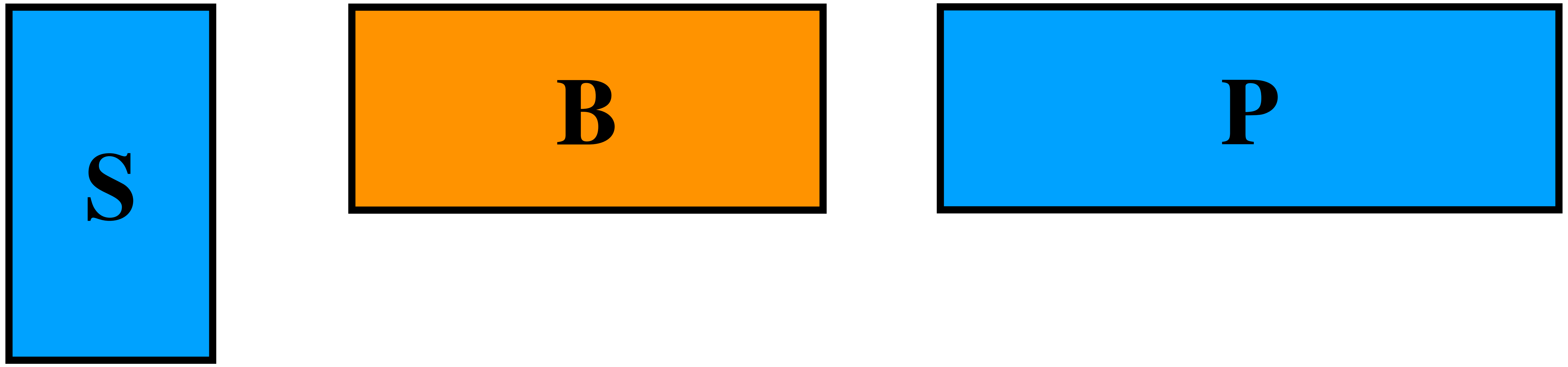

Evasive LWE: Design Philosophy

[Wee '22]

Hope: Hard to collect equations over integers if $\mathbf{SP}+\mathbf{E}' \approx_c \mathscr{U}$

- Let $\mathbf{S}, \mathbf{P} \leftarrow \mathsf{Samp}(\mathsf{rand})$.

if $\quad (\mathbf{B}, \mathbf{P}, \mathbf{SB}+\mathbf{E}, \mathbf{SP}+\mathbf{E}') \approx_c (\mathbf{B}, \mathbf{P}, \mathscr{U}, \mathscr{U})$

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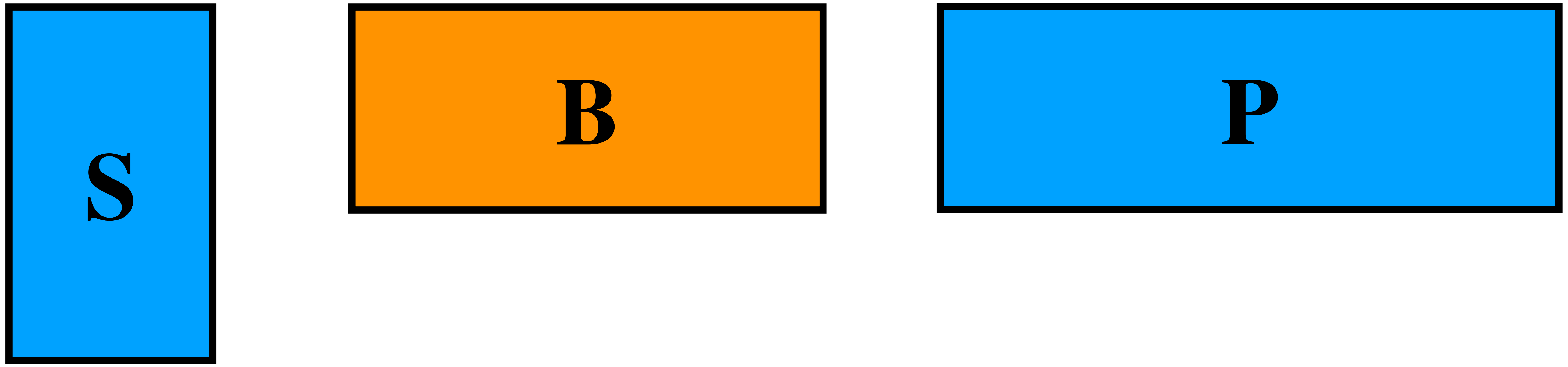
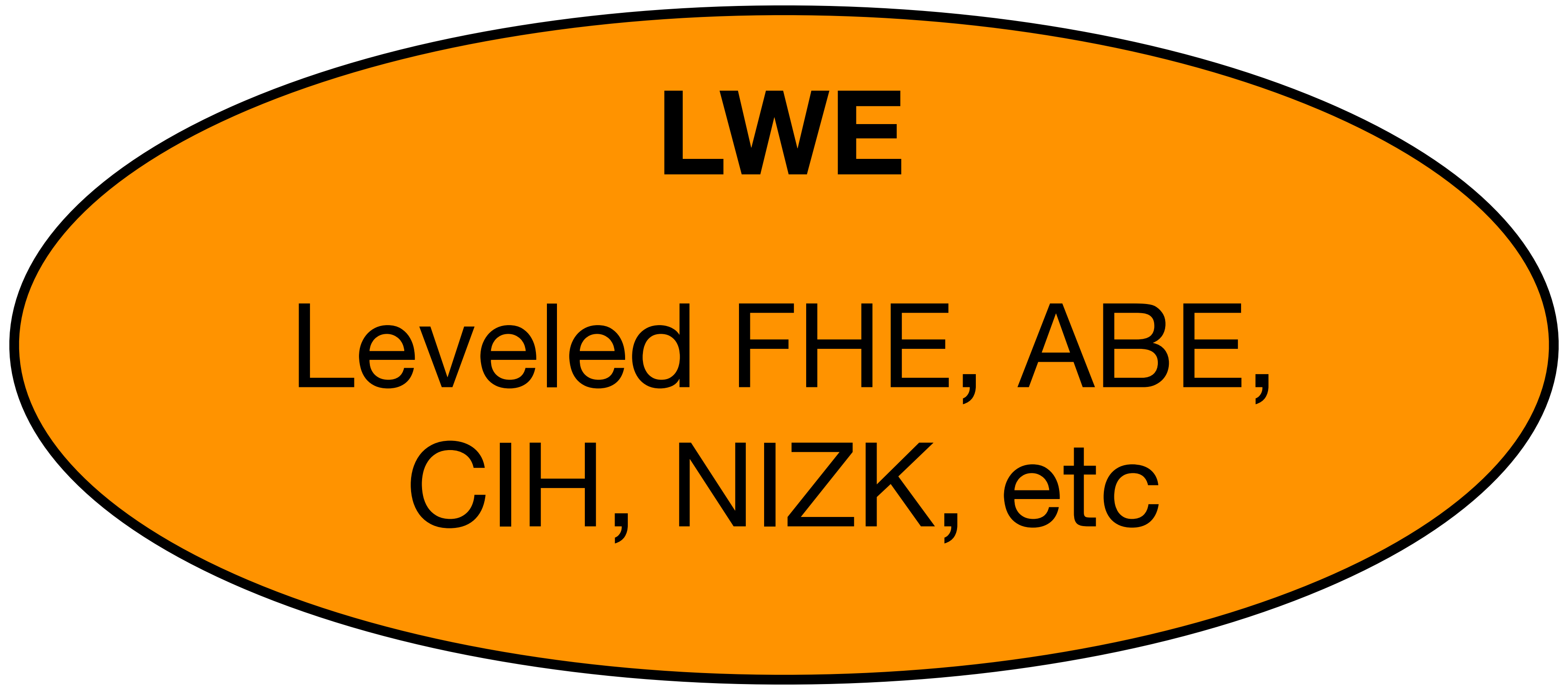


LWE Zoo



LWE Zoo

LWE

Leveled FHE, ABE, CIH, NIZK, etc

Optimal broadcast

Multi-Authority ABE

Succinct CP-ABE

Unleveled FHE

iO for circuits

Witness encryption

LWE Zoo

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*Succinct witness encryption**

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*not even known from iO

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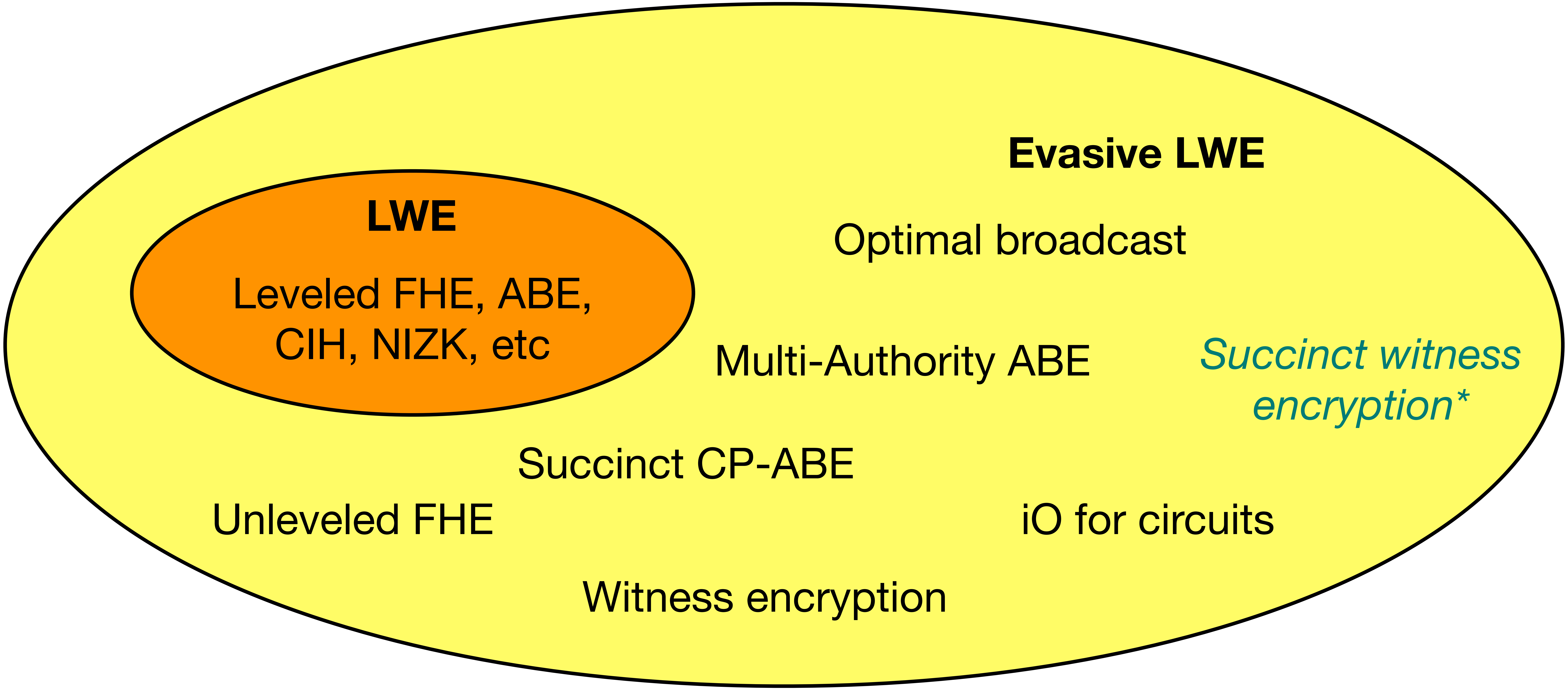
Evasive LWE Zoo



*not even known from iO

What can we do with Evasive LWE?

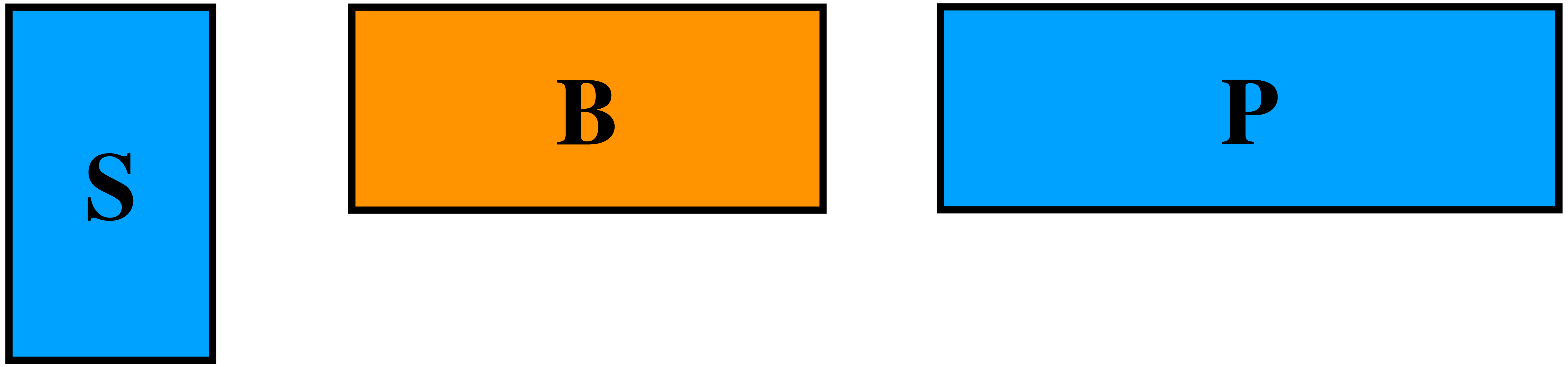
- Optimal Broadcast Encryption [Wee22]
- Multi-Authority ABE [WWW22]
- Unbounded depth ABE [HLL23]
- Witness Encryption [CVW18, VWW22]
- SNARKs for UP [**M**PV24]
- SNARGs for NP [JKL**M**24]
- ABE for TMs [AKY24]
- Pseudorandom Obfuscation (FHE, succinct WE) [DJM**M**PV25]
- Pseudorandom functional encryption [AKY24]
- Succinct iO for Turing Machines [JJ**M**P25]

<u>Public-Coin</u> Evasive LWE [Wee '22]

- Let $\mathbf{S}, \mathbf{P} \leftarrow \mathsf{Samp}(\mathsf{rand})$.

if $\quad (\mathbf{B}, \mathbf{P}, \mathbf{SB}+\mathbf{E}, \mathbf{SP}+\mathbf{E}', \mathsf{aux}=\mathsf{rand}) \approx_c (\mathbf{B}, \mathbf{P}, \mathcal{U}, \mathcal{U}, \mathsf{aux}=\mathsf{rand})$

then $\quad (\mathbf{B}, \mathbf{P}, \mathbf{SB}+\mathbf{E}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux}=\mathsf{rand}) \approx_c (\mathbf{B}, \mathbf{P}, \mathcal{U}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux}=\mathsf{rand})$



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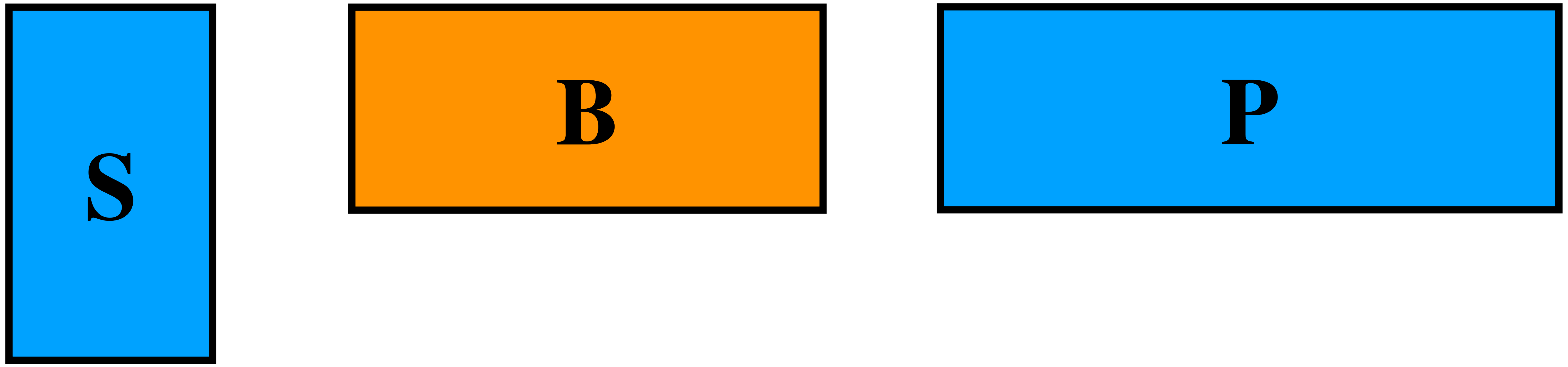
$$\text{then} \quad (\mathbf{B}, \mathbf{P}, \mathbf{SB} + \mathbf{E}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux} = \mathsf{rand}) \approx_c (\mathbf{B}, \mathbf{P}, \mathscr{U}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux} = \mathsf{rand})$$



Public-Coin Evasive LWE [Wee '22]

- Let $\mathbf{S}, \mathbf{P} \leftarrow \mathsf{Samp}(\mathsf{rand})$.

Randomness used to sample $\mathbf{S}, \mathbf{P}$ is public

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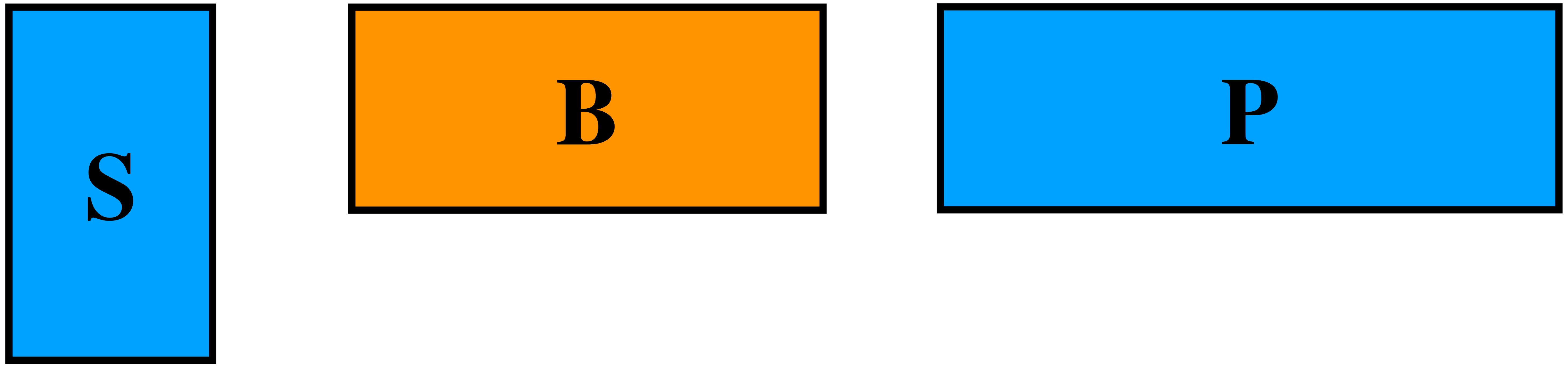

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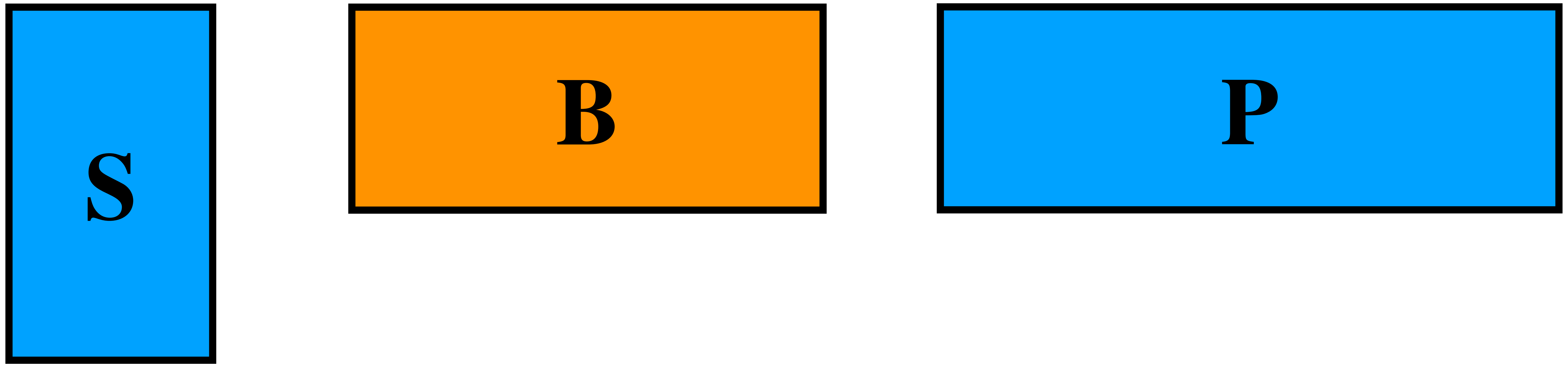


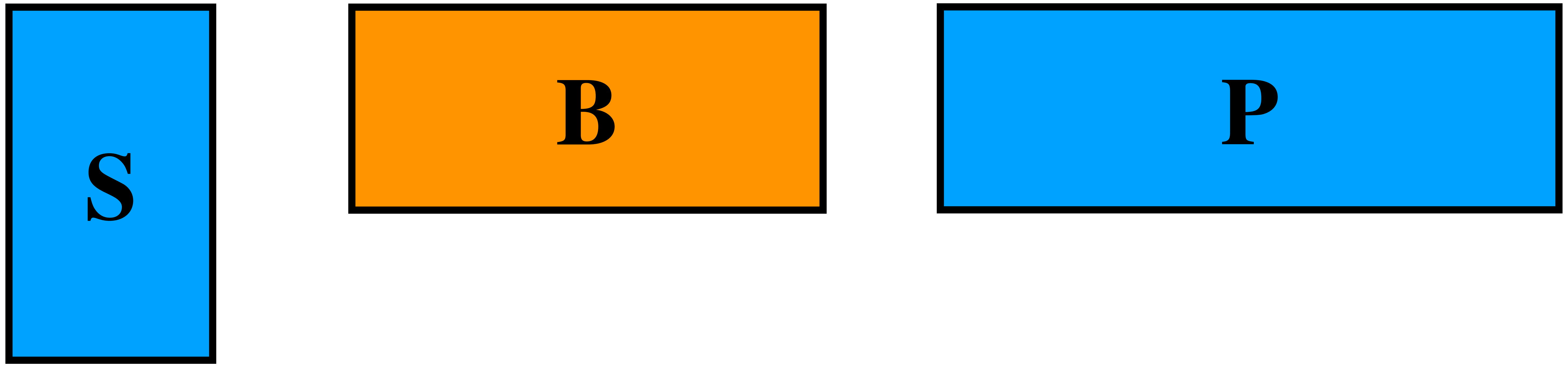
Private-Coin Evasive LWE

[VWW22, Tsabary 22]

- Let $\mathbf{S}, \mathbf{P}, \text{aux} \leftarrow \text{Samp(rand)}$.

$$\text{if} \quad (\mathbf{B}, \mathbf{P}, \mathbf{SB}+\mathbf{E}, \mathbf{SP}+\mathbf{E}', \text{aux}) \approx_c (\mathbf{B}, \mathbf{P}, \mathscr{U}, \mathscr{U}, \text{aux})$$

$$\text{then} \quad (\mathbf{B}, \mathbf{P}, \mathbf{SB}+\mathbf{E}, \mathbf{B}^{-1}(\mathbf{P}), \text{aux}) \approx_c (\mathbf{B}, \mathbf{P}, \mathscr{U}, \mathbf{B}^{-1}(\mathbf{P}), \text{aux})$$



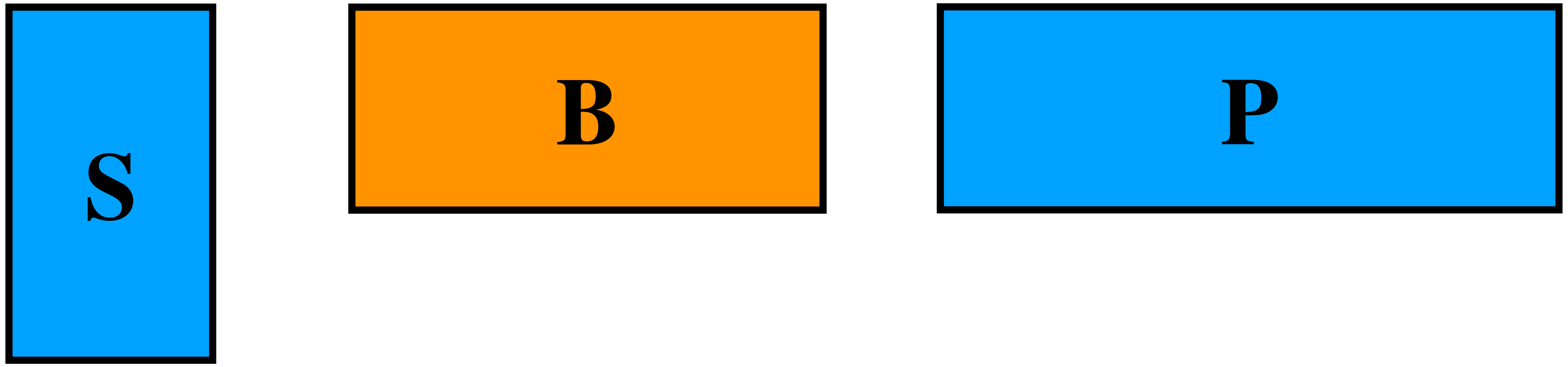
Private-Coin Evasive LWE

[VWW22, Tsabary 22]

- Let $\mathbf{S}, \mathbf{P}$, aux $\leftarrow$ Samp(rand).

if $(\mathbf{B}, \mathbf{P}, \mathbf{SB}+\mathbf{E}, \mathbf{SP}+\mathbf{E}', \text{aux}) \approx_c (\mathbf{B}, \mathbf{P}, \mathscr{U}, \mathscr{U}, \text{aux})$

then $(\mathbf{B}, \mathbf{P}, \mathbf{SB}+\mathbf{E}, \mathbf{B}^{-1}(\mathbf{P}), \text{aux}) \approx_c (\mathbf{B}, \mathbf{P}, \mathscr{U}, \mathbf{B}^{-1}(\mathbf{P}), \text{aux})$



<u>Private-Coin</u> Evasive LWE

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- Let $\mathbf{S}, \mathbf{P}, \mathsf{aux} \leftarrow \mathsf{Samp}(\mathsf{rand})$.

Randomness used to sample $\mathbf{S}, \mathbf{P}$, aux is **<u>private</u>**

if $\quad (\mathbf{B}, \mathbf{P}, \mathbf{SB}+\mathbf{E}, \mathbf{SP}+\mathbf{E}', \mathsf{aux}) \approx_c (\mathbf{B}, \mathbf{P}, \mathscr{U}, \mathscr{U}, \mathsf{aux})$

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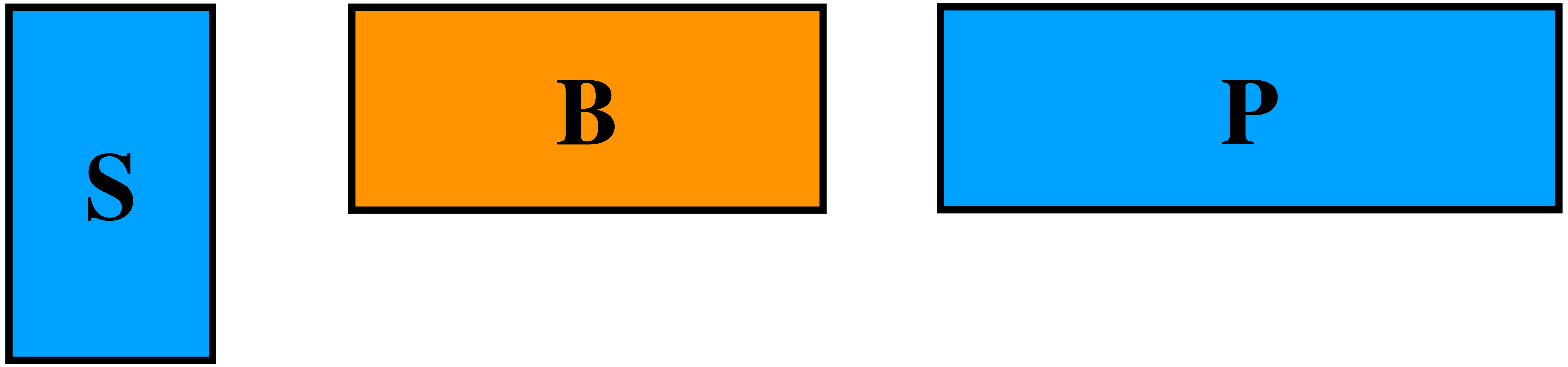

Private-Coin Evasive LWE

[VWW22, Tsabary 22]

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if $(\mathbf{B}, \mathbf{P}, \mathbf{SB} + \mathbf{E}, \mathbf{SP} + \mathbf{E}', \mathsf{aux}) \approx_c (\mathbf{B}, \mathbf{P}, \mathscr{U}, \mathscr{U}, \mathsf{aux})$

then $(\mathbf{B}, \mathbf{P}, \mathbf{SB} + \mathbf{E}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux}) \approx_c (\mathbf{B}, \mathbf{P}, \mathscr{U}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux})$

$\mathbf{S}$

Many variants! E.g. Fully available $\mathbf{B}, \mathbf{P}$, Hidden $\mathbf{B}, \mathbf{P}$, etc.

Evasive LWE Zoo

LWE

Leveled FHE, ABE, CIH, NIZK, etc

Optimal broadcast

Multi-Authority ABE

*Succinct witness encryption**

Succinct CP-ABE

iO for circuits

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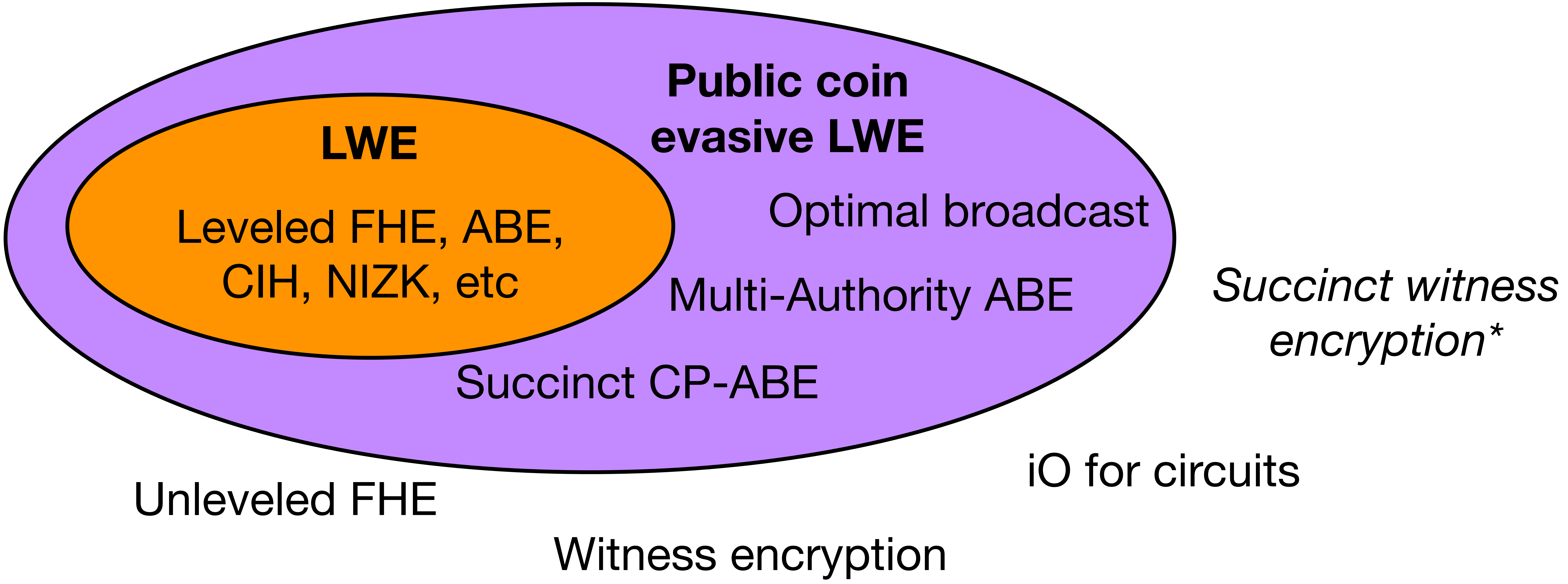
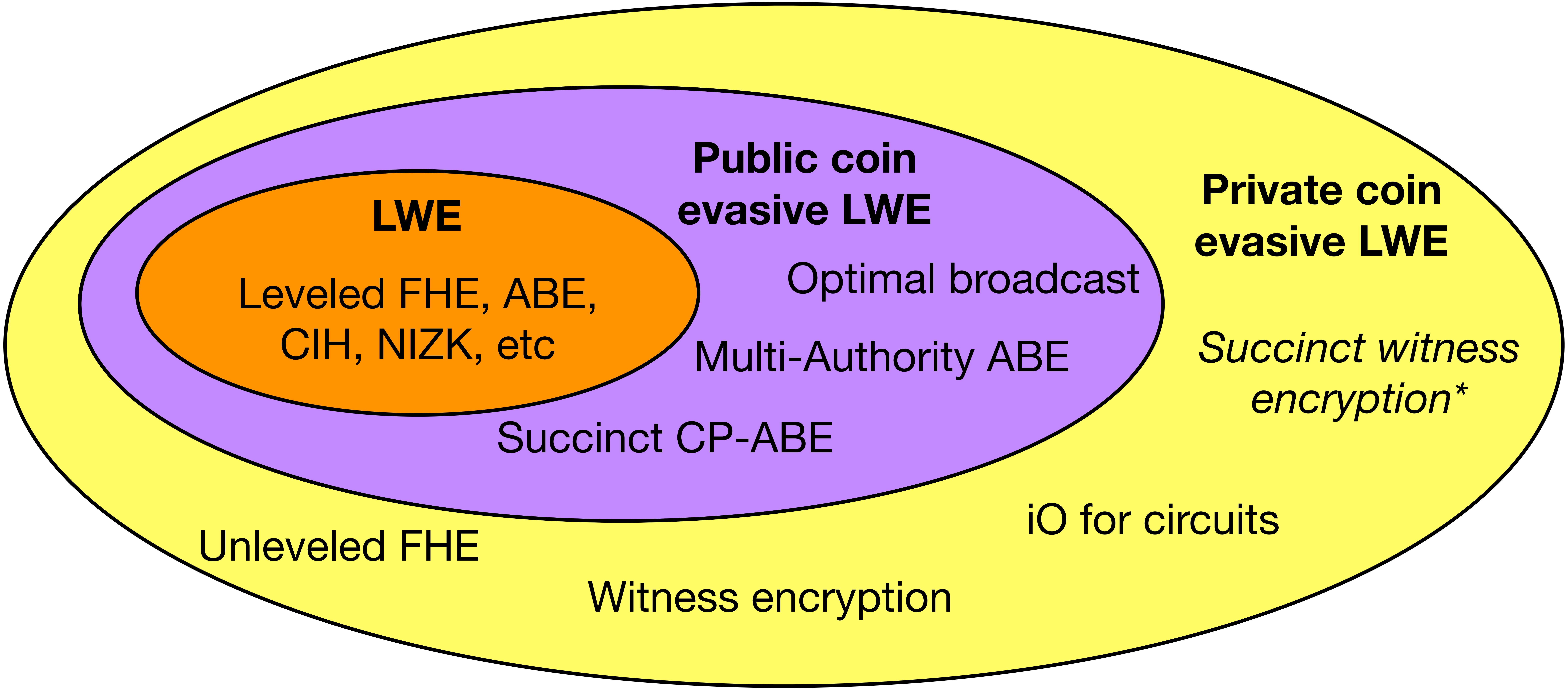


Evasive LWE Zoo

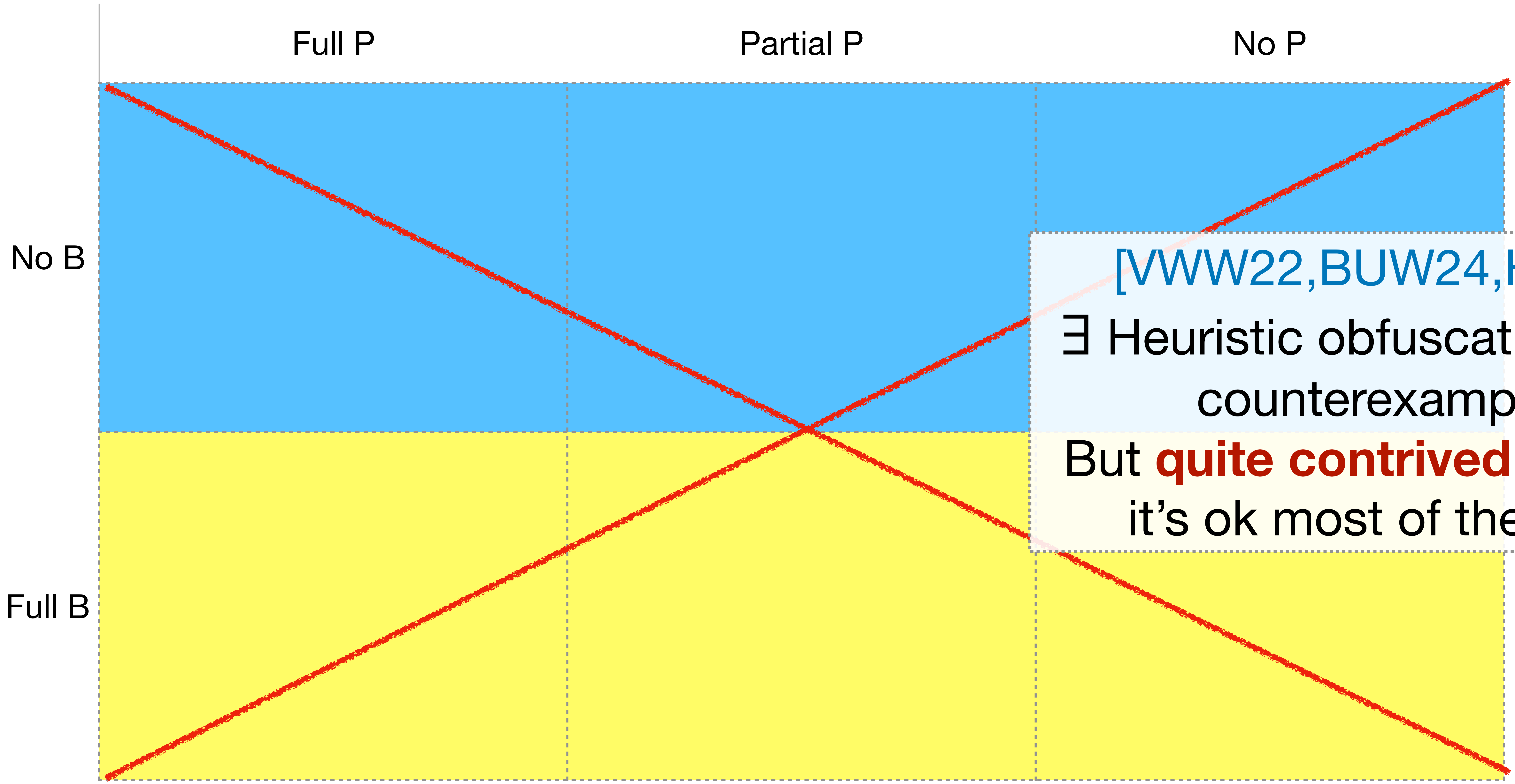


*not even known from iO

Private-Coin Evasive Attacks

	Full P	Partial P	No P
No B			
Full B			

Private-Coin Evasive Attacks

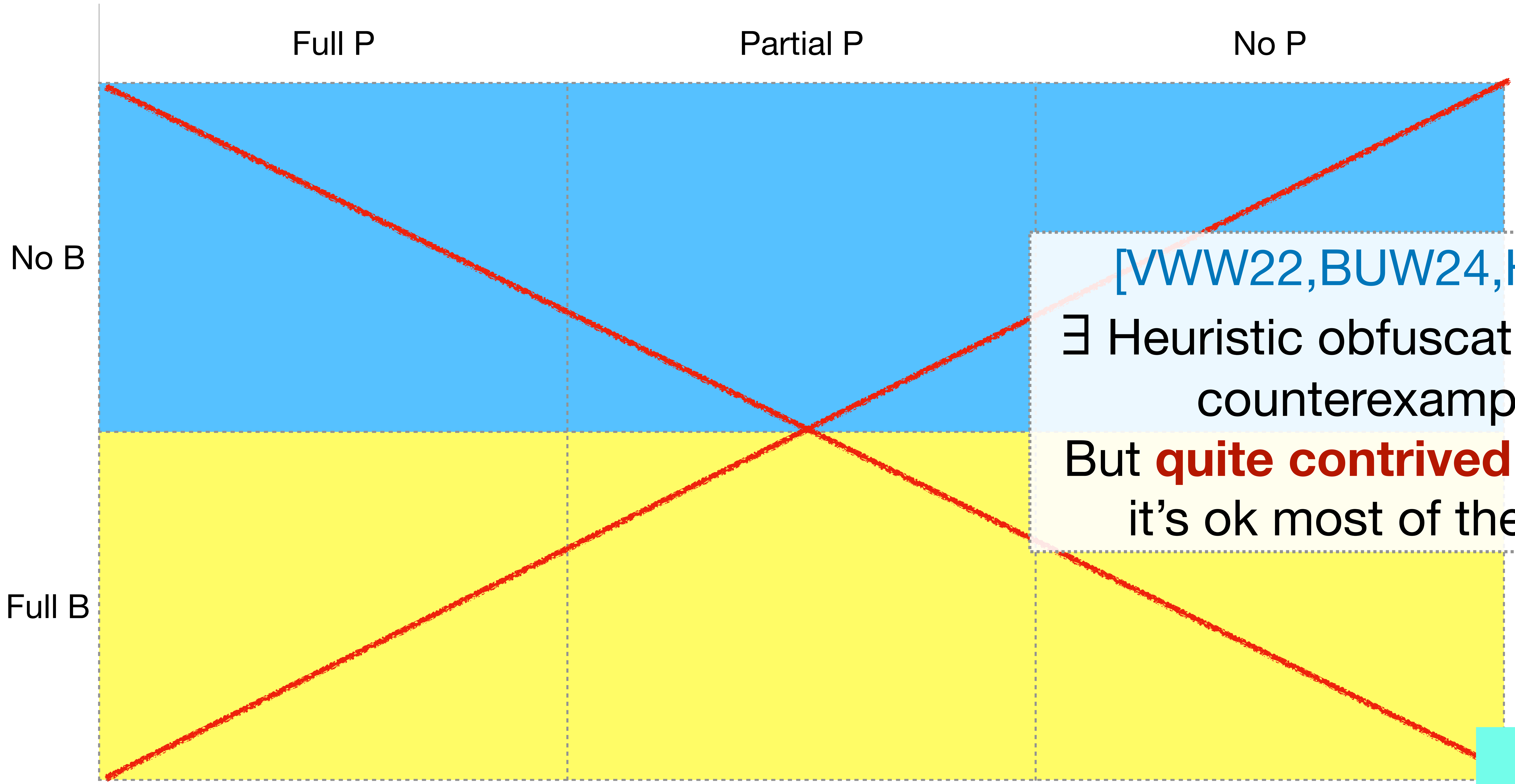


[VWW22,BUW24,HHY25]

∃ Heuristic obfuscation based counterexample!!!

But **quite contrived**… Maybe it’s ok most of the time?

Private-Coin Evasive Attacks



[VWW22,BUW24,HHY25]

∃ Heuristic obfuscation based counterexample!!!

But **quite contrived**... Maybe it's ok most of the time?

Bonus slide with attack. Ask later!

Private-Coin Evasive Attacks

[Brzuska-Unal-Woo '25]

	Full P	Partial P	No P
No B			
Full B			

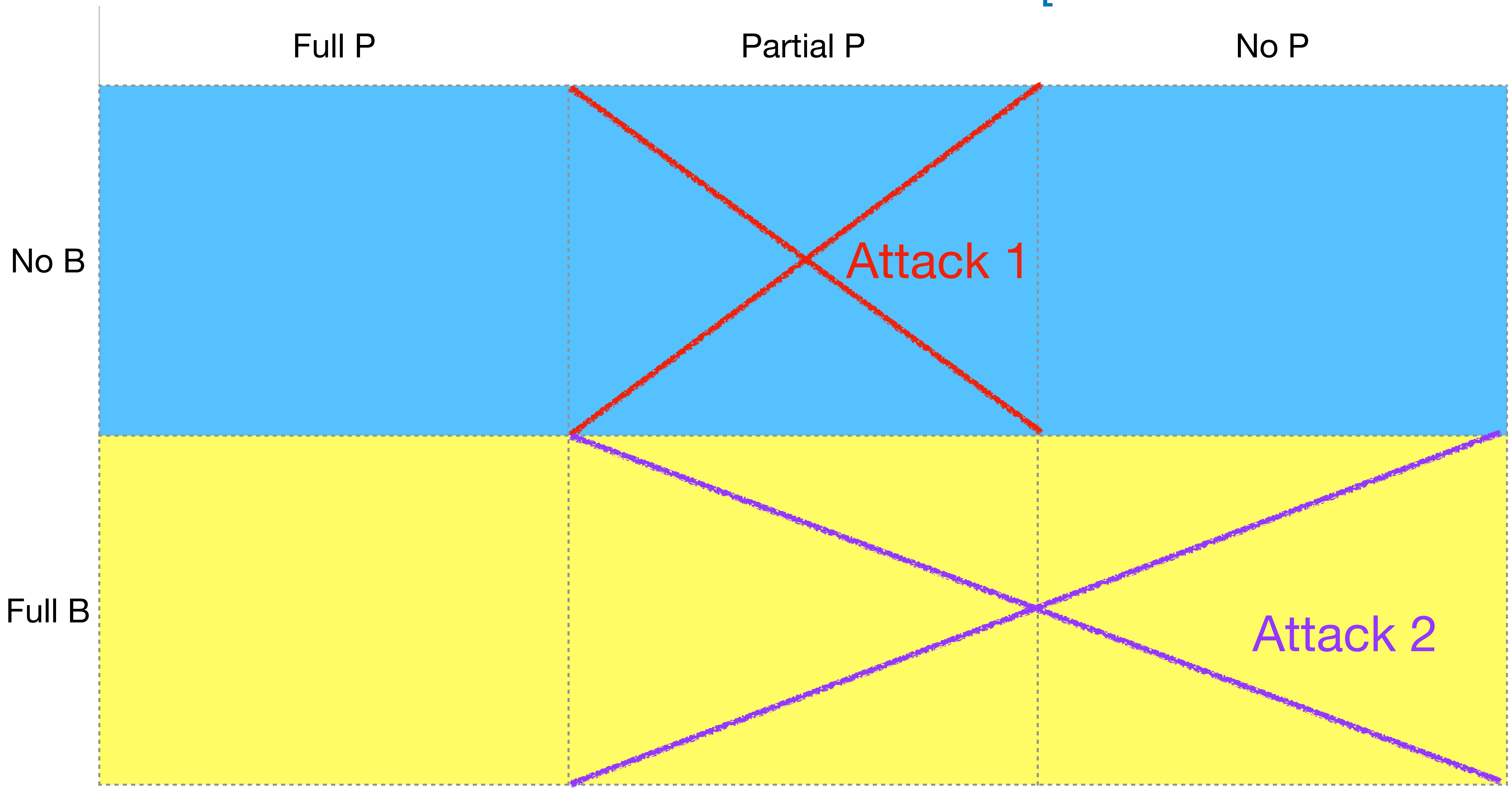
Private-Coin Evasive Attacks

[Brzuska-Unal-Woo ’25]

	Full P	Partial P	No P
No B		Attack 1	
Full B			

Private-Coin Evasive Attacks

[Brzuska-Unal-Woo ’25]



Private-Coin Evasive Attacks

[Brzuska-Unal-Woo ’25]

	Full P	Partial P	No P
No B			Private-coin hiding evasive LWE*
Full B	Private-coin binding evasive LWE		

*pre-condition needs to satisfy additional conditions, but we will gloss over this for this talk

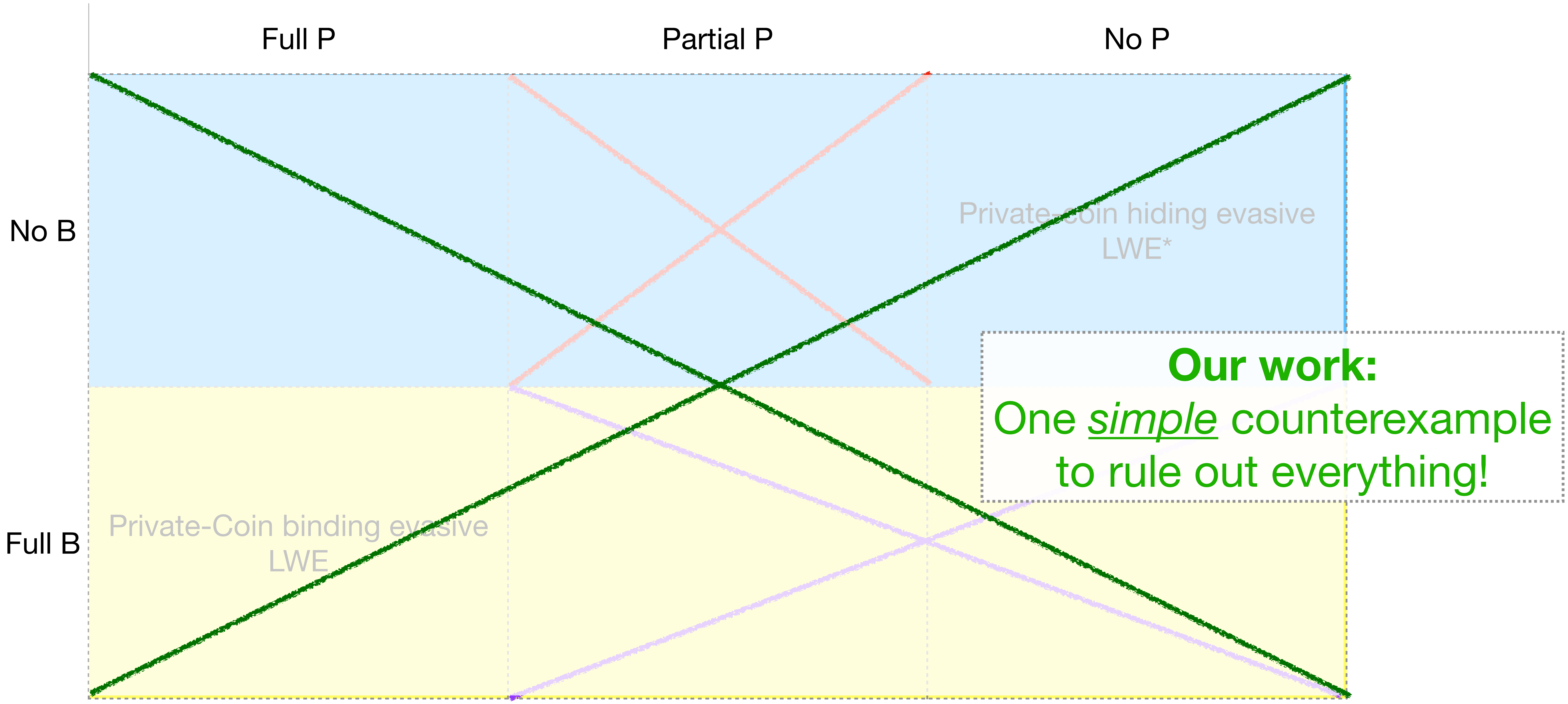
Private-Coin Evasive Attacks

	Full P	Partial P	No P
No B			Private-coin hiding evasive LWE*
Full B	Private-Coin binding evasive LWE		

Our work:
One *simple* counterexample to rule out everything!

*pre-condition needs to satisfy additional conditions, but we will gloss over this for this talk

Private-Coin Evasive Attacks



*pre-condition needs to satisfy additional conditions, but we will gloss over this for this talk

Our attack: Evasive LWE

- We give $\mathbf{S}, \mathbf{P}, \mathbf{aux} \leftarrow \mathsf{Samp}(\mathsf{rand})$ such that:

$$(\mathbf{B}, \mathbf{P}, \mathbf{SB} + \mathbf{E}, \mathbf{SP} + \mathbf{E}', \mathsf{aux}) \approx_c (\mathbf{B}, \mathbf{P}, \mathscr{U}, \mathscr{U}, \mathsf{aux})$$

$$(\mathbf{B}, \mathbf{P}, \mathbf{SB} + \mathbf{E}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux}) \not\approx_c (\mathbf{B}, \mathbf{P}, \mathscr{U}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux})$$

Our attack: Evasive LWE

- We give $\mathbf{S}, \mathbf{P}, \mathbf{aux} \leftarrow$ Samp(rand) such that:

Satisfies strongest pre-condition

$$(\mathbf{B}, \mathbf{P}, \mathbf{SB} + \mathbf{E}, \mathbf{SP} + \mathbf{E}', \mathsf{aux}) \approx_c (\mathbf{B}, \mathbf{P}, \mathscr{U}, \mathscr{U}, \mathsf{aux})$$

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- We give $\mathbf{S}, \mathbf{P}, \mathbf{aux} \leftarrow \mathsf{Samp}(\mathsf{rand})$ such that:

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Does not satisfy weakest post-condition

Our attack: Evasive LWE

- We give $\mathbf{S}, \mathbf{P}, \mathbf{aux} \leftarrow \mathsf{Samp}(\mathsf{rand})$ such that:

$$(\mathbf{B}, \mathbf{P}, \mathbf{SB} + \mathbf{E}, \mathbf{SP} + \mathbf{E}', \mathsf{aux}) \approx_c (\mathbf{B}, \mathbf{P}, \mathscr{U}, \mathscr{U}, \mathsf{aux})$$

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- $(\mathbf{S}, \mathbf{P}, \mathsf{aux} = \mathbf{SP} - 2\mathbf{T}) \leftarrow$ Samp, where:

Our attack: Evasive LWE

- We give $\mathbf{S}, \mathbf{P}, \mathbf{aux} \leftarrow$ Samp(rand) such that:

$$(\mathbf{B}, \mathbf{P}, \mathbf{SB} + \mathbf{E}, \mathbf{SP} + \mathbf{E}', \mathsf{aux}) \approx_c (\mathbf{B}, \mathbf{P}, \mathscr{U}, \mathscr{U}, \mathsf{aux})$$

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- $(\mathbf{S}, \mathbf{P}, \mathsf{aux} = \mathbf{SP} - 2\mathbf{T}) \leftarrow$ Samp, where:
  - $\mathbf{S}, \mathbf{P}$ have **uniform** $\mathbb{Z}_q$ entries, where q is **odd.**

Our attack: Evasive LWE

- We give $\mathbf{S}, \mathbf{P}, \mathbf{aux} \leftarrow$ Samp(rand) such that:

$$(\mathbf{B}, \mathbf{P}, \mathbf{SB} + \mathbf{E}, \mathbf{SP} + \mathbf{E}', \text{aux}) \approx_c (\mathbf{B}, \mathbf{P}, \mathcal{U}, \mathcal{U}, \text{aux})$$

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- $(\mathbf{S}, \mathbf{P}, \text{aux} = \mathbf{SP} - 2\mathbf{T}) \leftarrow$ Samp, where:
  - $\mathbf{S}, \mathbf{P}$ have **uniform $\mathbb{Z}_q$ entries**, where q is **odd.**
  - $\mathbf{T} \leftarrow [0,1,\ldots,\lfloor q/2 \rfloor]$, (i.e. $2\mathbf{T} \approx$ random matrix with even entries mod q).

Attack on post-condition

Attack on post-condition

Goal: $(\mathbf{SB}+\mathbf{E}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux}) \not\approx_c (\mathscr{U}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux})$ where $\mathsf{aux} = \mathbf{SP} - \mathbf{2T}$

Attack on post-condition

Goal: $(\mathbf{SB}+\mathbf{E}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux}) \not\approx_c (\mathscr{U}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux})$ where $\mathsf{aux} = \mathbf{SP} - 2\mathbf{T}$

Attack on post-condition

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LHS:

Attack on post-condition

Goal: $(\mathbf{SB}+\mathbf{E}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux}) \not\approx_c (\mathscr{U}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux})$ where $\mathsf{aux} = \mathbf{SP} - 2\mathbf{T}$

LHS:

$$(\mathbf{SB}+\mathbf{E}) \cdot \mathbf{B}^{-1}(\mathbf{P}) - \mathsf{aux} \pmod q$$

Attack on post-condition

Goal: $(\mathbf{SB}+\mathbf{E}, \mathbf{B}^{-1}(\mathbf{P}), \text{aux}) \not\approx_c (\mathscr{U}, \mathbf{B}^{-1}(\mathbf{P}), \text{aux})$ where $\text{aux} = \mathbf{SP} - 2\mathbf{T}$

LHS:

$$(\mathbf{SB}+\mathbf{E}) \cdot \mathbf{B}^{-1}(\mathbf{P}) - \text{aux} \pmod q$$

$$= (\mathbf{SP} + \mathbf{E} \cdot \mathbf{B}^{-1}(\mathbf{P})) - (\mathbf{SP} - 2\mathbf{T}) \pmod q$$

Attack on post-condition

Goal: $(\mathbf{SB}+\mathbf{E}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux}) \not\approx_c (\mathscr{U}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux})$ where $\mathsf{aux} = \mathbf{SP} - 2\mathbf{T}$

LHS:

$$(\mathbf{SB}+\mathbf{E}) \cdot \mathbf{B}^{-1}(\mathbf{P}) - \mathsf{aux} \pmod q$$
$$= (\mathbf{SP} + \mathbf{E}\cdot\mathbf{B}^{-1}(\mathbf{P})) - (\mathbf{SP} - 2\mathbf{T}) \pmod q$$
$$= \mathbf{E}\cdot\mathbf{B}^{-1}(\mathbf{P}) + 2\mathbf{T} \pmod q$$

Attack on post-condition

Goal: $(\mathbf{SB}+\mathbf{E}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux}) \not\approx_c (\mathcal{U}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux})$ where $\mathsf{aux} = \mathbf{SP} - 2\mathbf{T}$

LHS:

$$(\mathbf{SB}+\mathbf{E}) \cdot \mathbf{B}^{-1}(\mathbf{P}) - \mathsf{aux} \pmod q$$
$$= (\mathbf{SP} + \mathbf{E} \cdot \mathbf{B}^{-1}(\mathbf{P})) - (\mathbf{SP} - 2\mathbf{T}) \pmod q$$
$$= \mathbf{E} \cdot \mathbf{B}^{-1}(\mathbf{P}) + 2\mathbf{T} \pmod q$$

Because $\mathbf{E} \cdot \mathbf{B}^{-1}(\mathbf{P})$ is *small*, whp. does not wrap around $\bmod q$!

Attack on post-condition

Goal: $(\mathbf{SB}+\mathbf{E}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux}) \not\approx_c (\mathscr{U}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux})$ where $\mathsf{aux} = \mathbf{SP} - 2\mathbf{T}$

LHS:

$$(\mathbf{SB}+\mathbf{E}) \cdot \mathbf{B}^{-1}(\mathbf{P}) - \mathsf{aux} \pmod q$$
$$= (\mathbf{SP} + \mathbf{E} \cdot \mathbf{B}^{-1}(\mathbf{P})) - (\mathbf{SP} - 2\mathbf{T}) \pmod q$$
$$= \mathbf{E} \cdot \mathbf{B}^{-1}(\mathbf{P}) + 2\mathbf{T} \pmod q$$
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Because $\mathbf{E} \cdot \mathbf{B}^{-1}(\mathbf{P})$ is *small*, whp. does not wrap around $\bmod q$!

Attack on post-condition

Goal: $(\mathbf{SB}+\mathbf{E}, \mathbf{B}^{-1}(\mathbf{P}), \text{aux}) \not\approx_c (\mathscr{U}, \mathbf{B}^{-1}(\mathbf{P}), \text{aux})$ where $\text{aux} = \mathbf{SP} - \mathbf{2T}$

LHS:

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Leftover hash lemma!

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NOT in the row span of $\mathbf{B}^{-1}(\mathbf{P}) \pmod 2$ with high probability!
Recall $\mathbf{B}^{-1}(\mathbf{P})$ is wide.

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Zeroizing Attack!!

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By noise-flooding and picking $\mathbf{E}'' \ll \mathbf{E}', \mathbf{T}$.
(Pick q to be super polynomial.)

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LWE with ***even*** error (because q is odd)

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- **Open:** Can we achieve a similar story in the private-coin setting?

Thank you for your attention!

Heuristic Obfuscation-based Attack

[VWW22, BUW24, HHY25]

Return to
main body

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- Private-coin evasive LWE has a “contrived” obfuscation-based attack.
- $\mathbf{S}, \mathbf{P}, \mathsf{aux} = O_{\mathbf{W}} \leftarrow \mathsf{Samp}$,
  - $\mathbf{S}, \mathbf{P}$ are sampled uniformly,
  - $\mathbf{W} = \mathbf{SP} + \widetilde{\mathbf{E}}$ and O_W accepts low-rank $\mathbf{M_1}, \mathbf{M_2}$ such that $\mathbf{W} \approx \mathbf{M_1 M_2}$.

By LWE, $\mathbf{W} \approx_c \mathscr{U}$ in pre-condition, so $O_{\mathbf{W}} = \mathbf{Zero}$.

Pre ✓ $$(\mathbf{B}, \mathbf{P}, \mathbf{SB} + \mathbf{E}, \mathbf{SP} + \mathbf{E}', \mathsf{aux}) \approx_c (\mathbf{B}, \mathbf{P}, \mathscr{U}, \mathscr{U}, \mathsf{aux})$$

$$(\mathbf{B}, \mathbf{P}, \mathbf{SB} + \mathbf{E}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux}) \approx_c (\mathbf{B}, \mathbf{P}, \mathscr{U}, \mathbf{B}^{-1}(\mathbf{P}), \mathsf{aux})$$

Return to main body

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Clearly broken in post-condition!

Return to main body