Compiled Nonlocal Games from any Trapdoor Claw-Free Function

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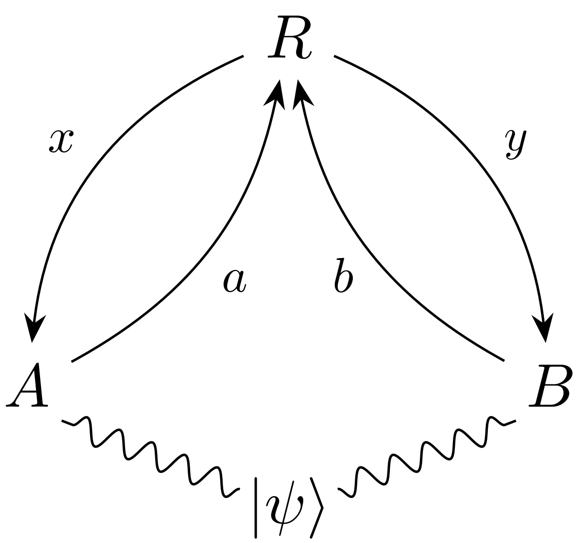
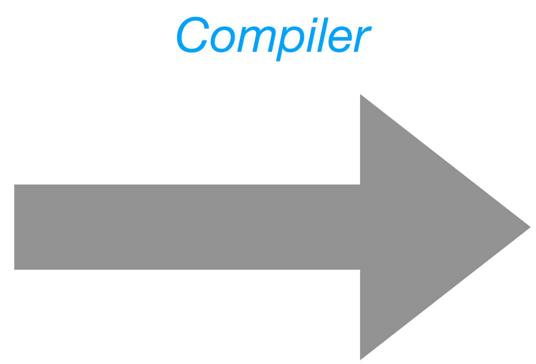
Nonlocal Game





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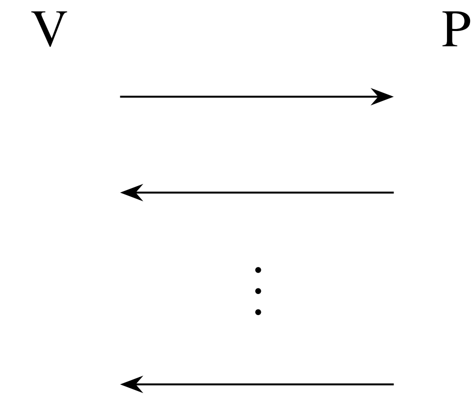
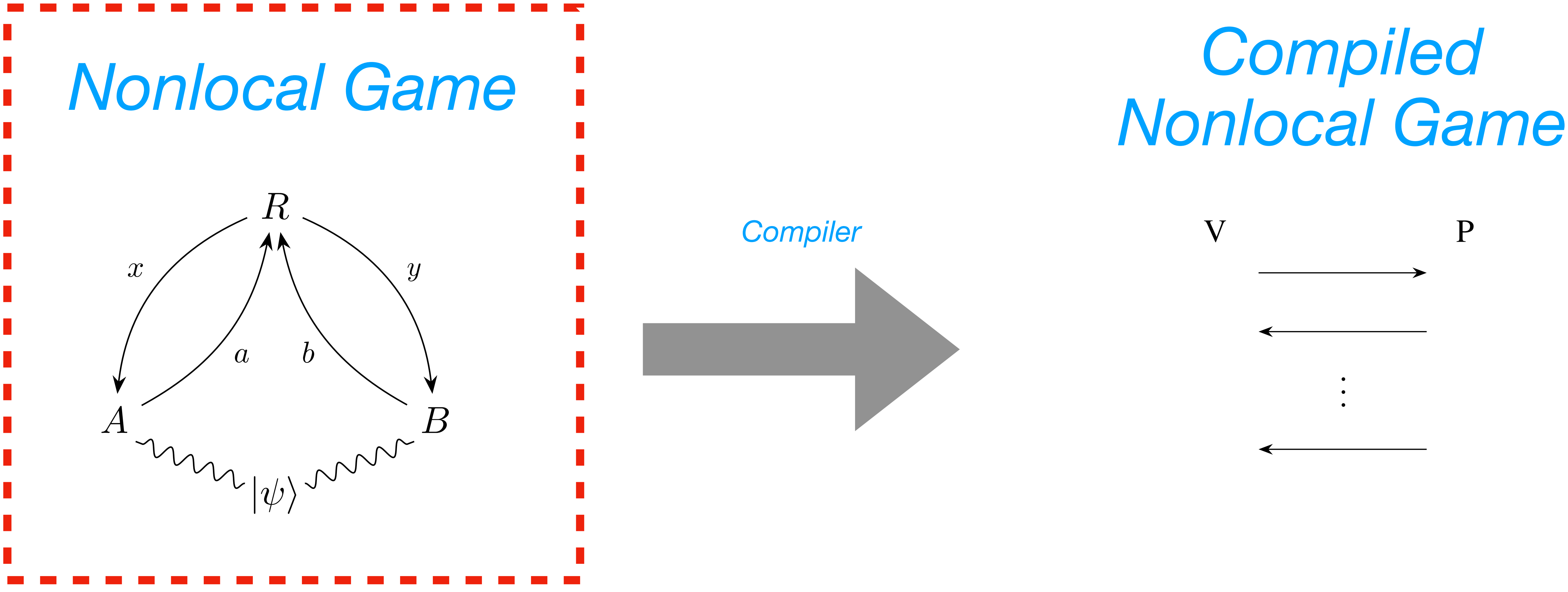


Compiler should preserve properties of the nonlocal game



Nonlocal Game
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x
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a
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A
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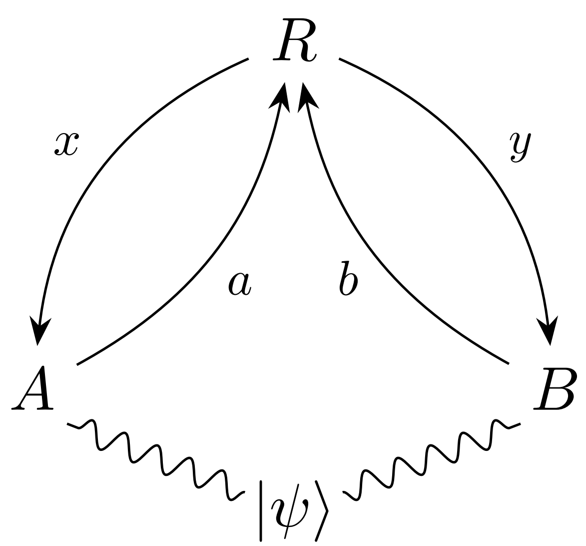
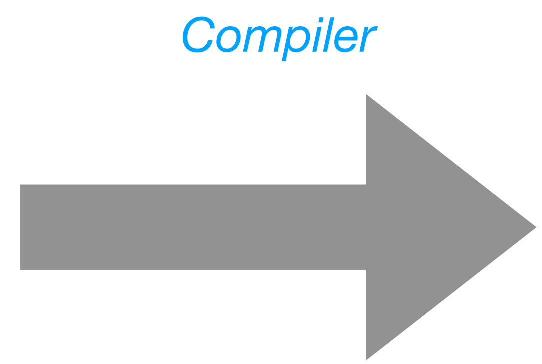
$|\psi\rangle$
Compiler
Compiled
Nonlocal Game
V
P

Nonlocal Game





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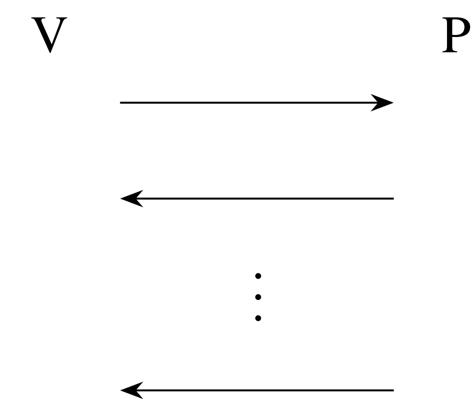
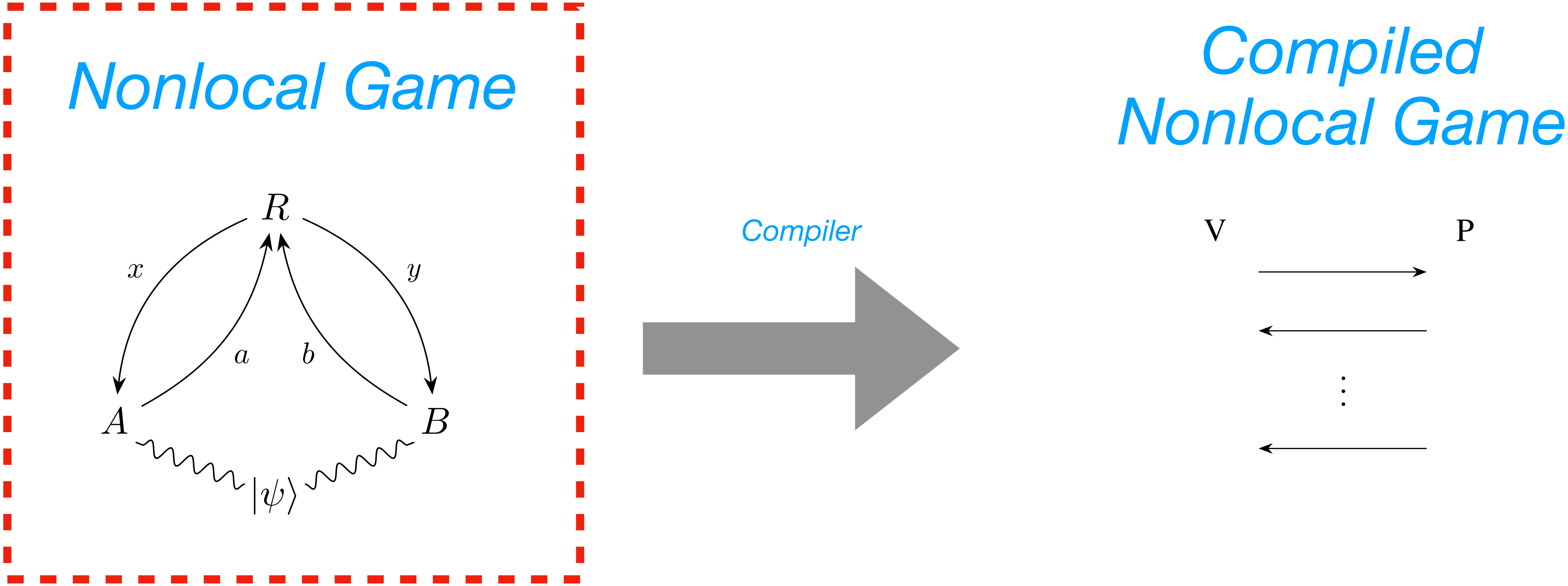


<u>CHSH Game</u> (Clauser–Horne–Shimony–Holt)

- Questions & answers: $x, y, a, b \in \{0,1\}$
- No communication between A and B
- Winning condition: $a \oplus b = x \cdot y$

x	y	winning condition
0	0	$a = b$
0	1	$a = b$
1	0	$a = b$
1	1	$a \neq b$



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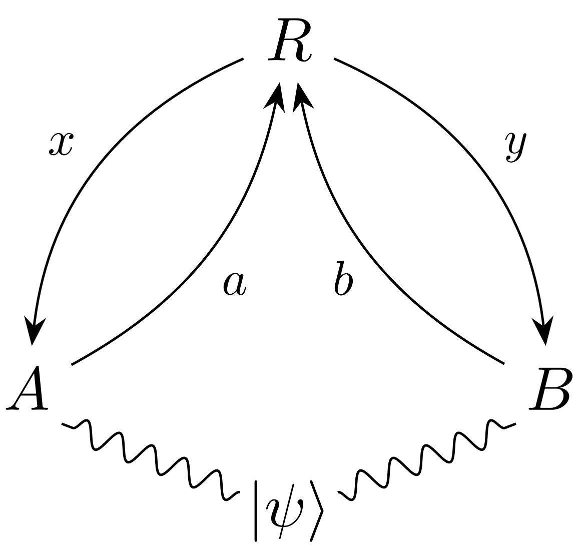
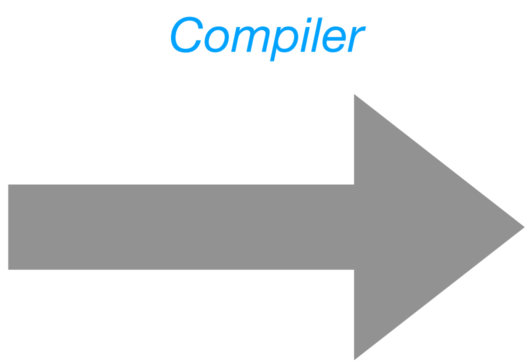
$\omega_c(\mathcal{G}) :=$ maximal winning probability for classical players

$\omega_q(\mathcal{G}) :=$ maximal winning probability for ⊗ quantum players

Nonlocal Game





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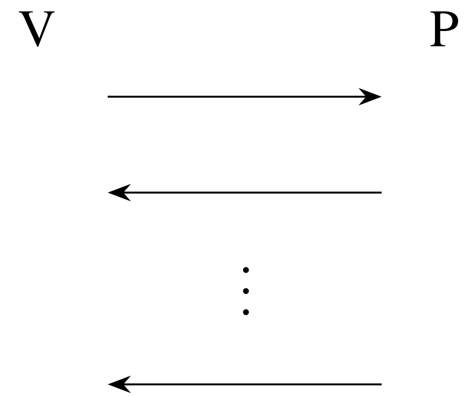

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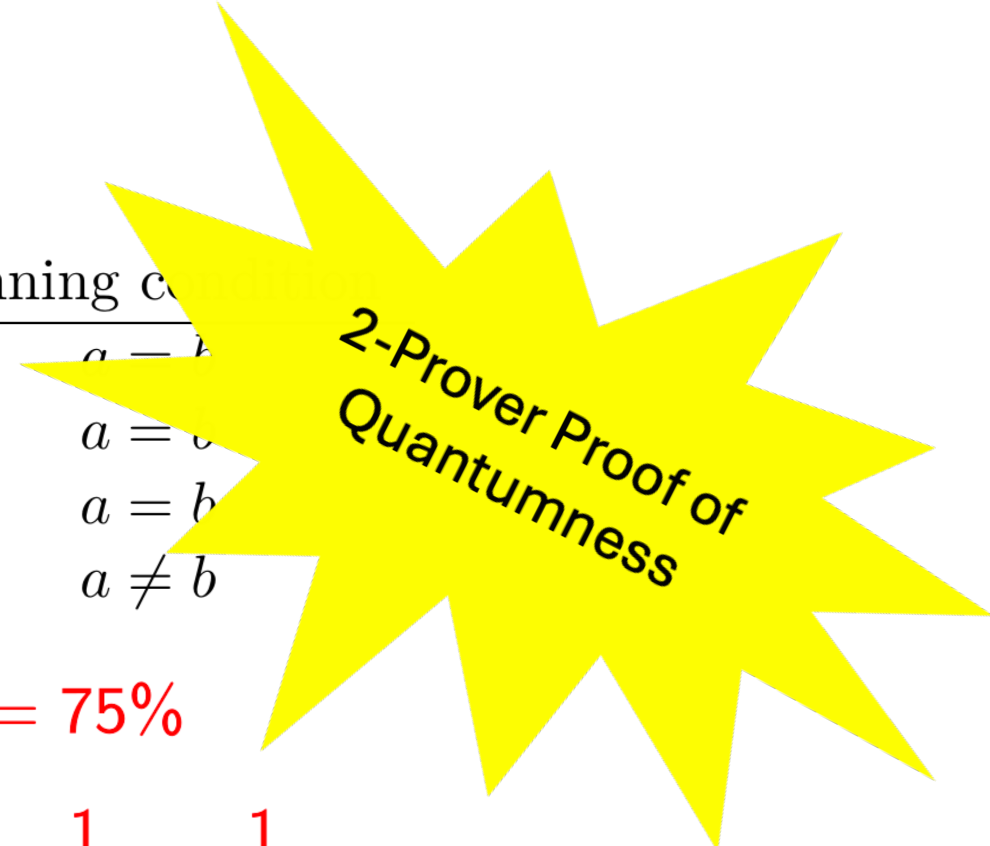

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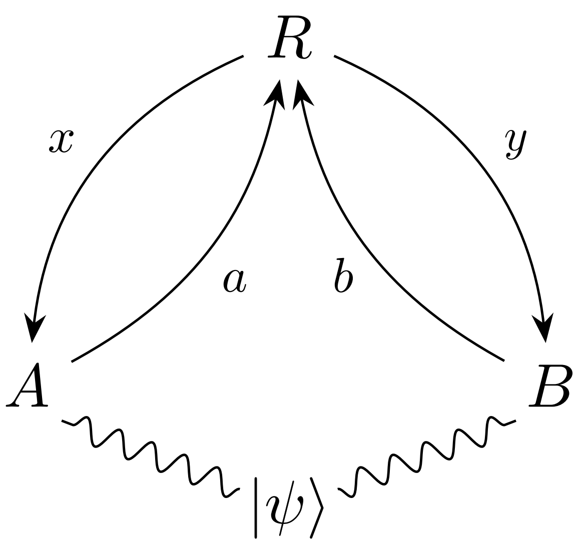
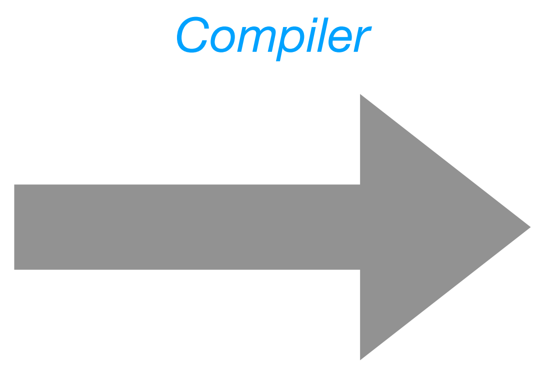
$\omega_c(\mathcal{G}_{\text{CHSH}}) = 75\%$

$\omega_q(\mathcal{G}_{\text{CHSH}}) = \frac{1}{2} + \frac{1}{2\sqrt{2}} \approx 85\%$

Nonlocal Game





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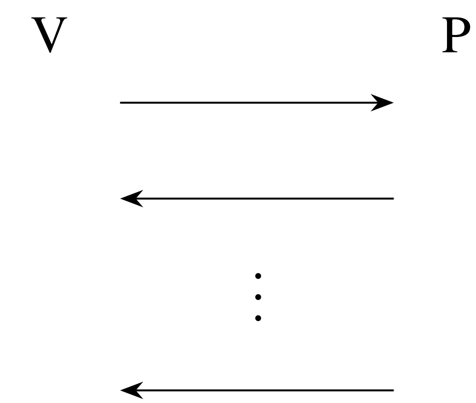
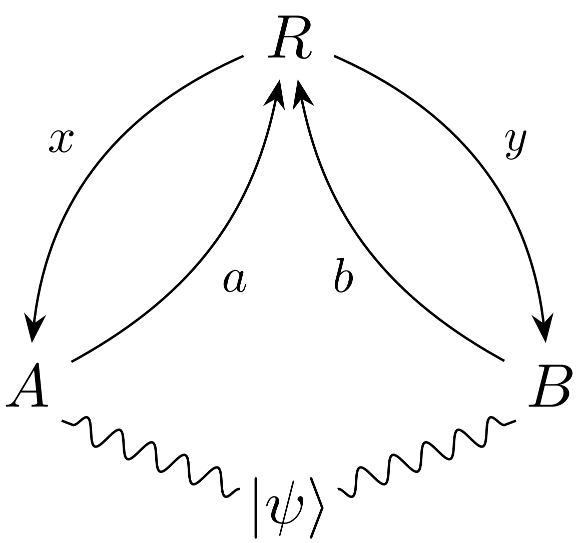
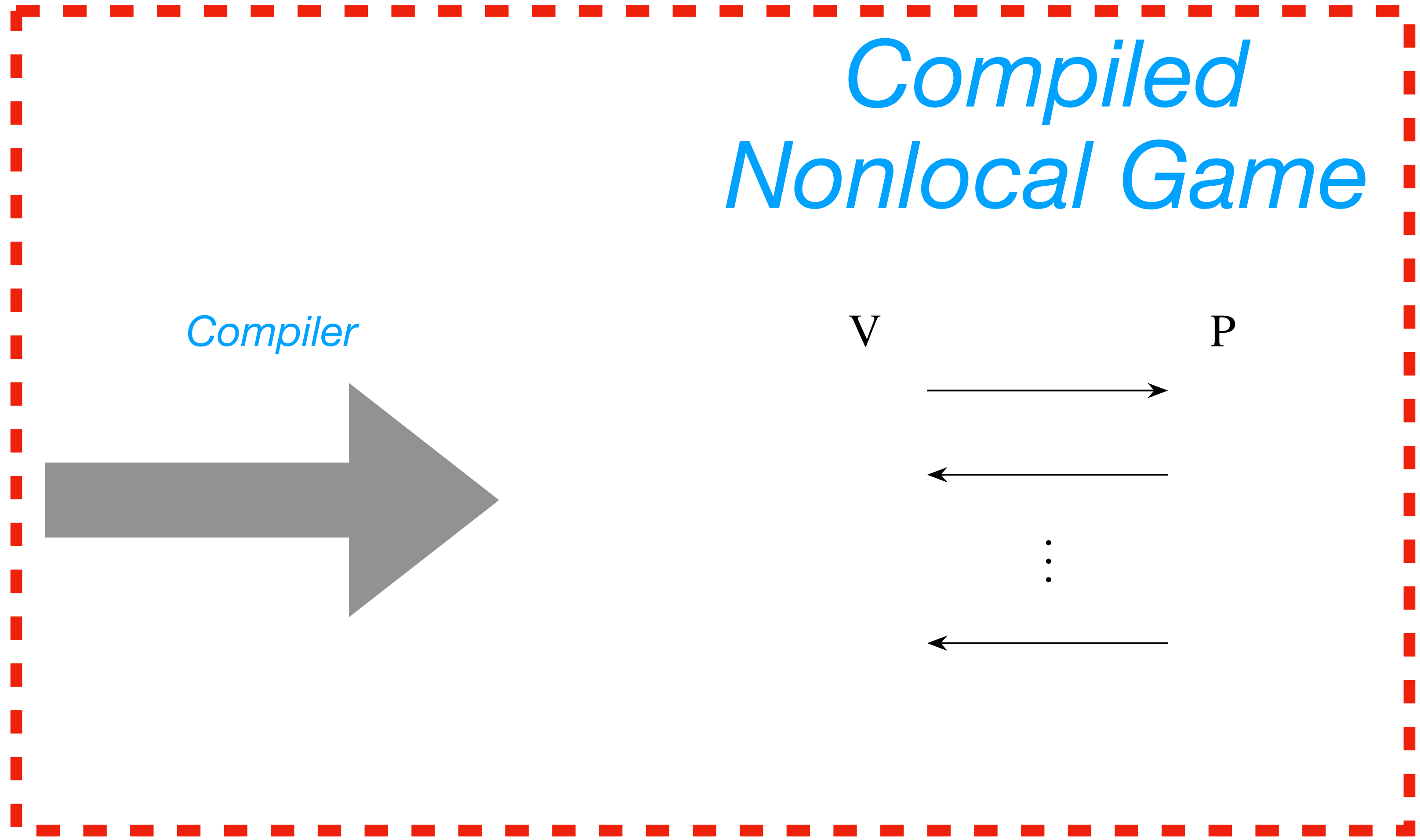


- Compiler should preserve properties of the nonlocal game

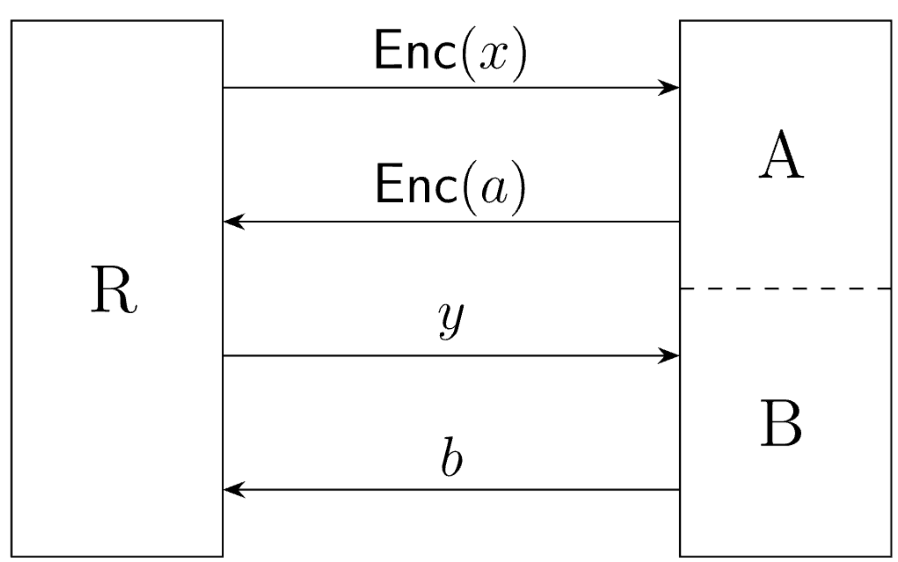
Nonlocal Game





KLVY Compiler

- Kalai, Lombardi, Vaikuntanathan, and Yang [KLVY21] compiler from Quantum Fully Homomorphic Encryption



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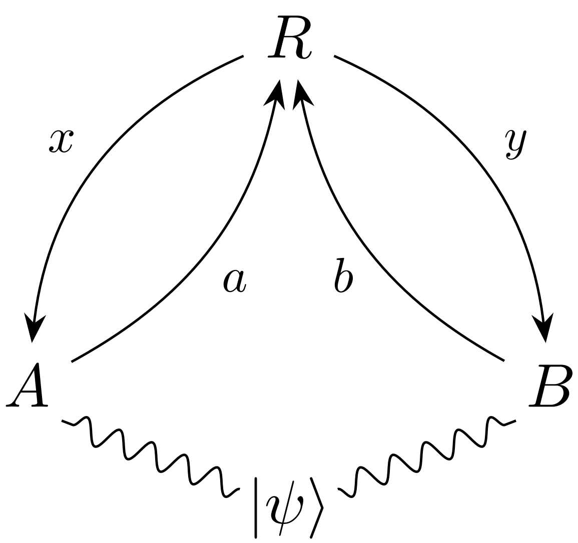


Compiled Nonlocal Game

KLVY Compiler

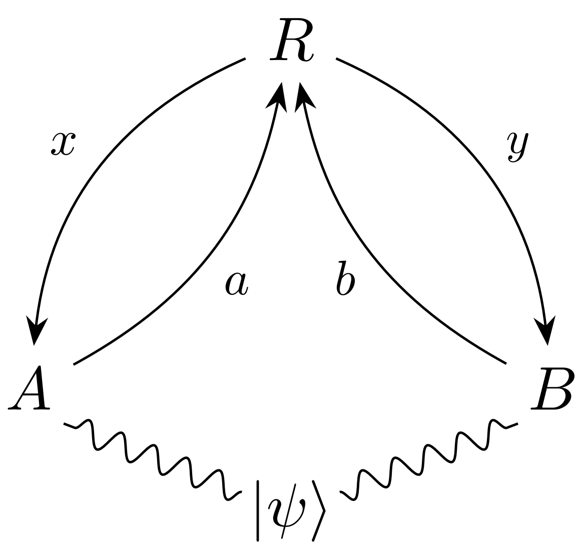
- Quantum Completeness ([KLVY23])

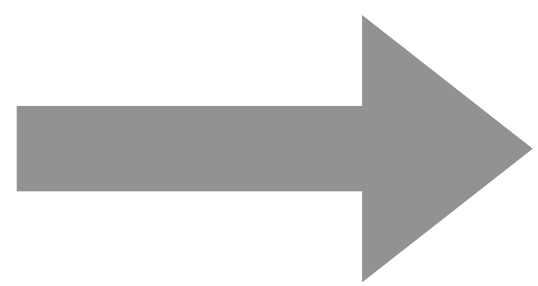
$$\omega_q(\mathcal{G}_{\mathsf{comp}}) \geq \omega_q(\mathcal{G})$$

- Classical Soundness ([KLVY23])

$$\omega_c(\mathcal{G}_{\mathsf{comp}}) \leq \omega_c(\mathcal{G})$$

Nonlocal Game





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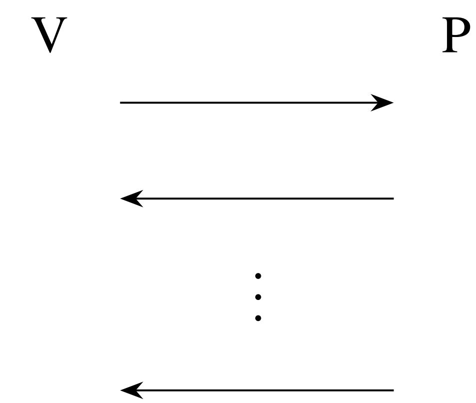

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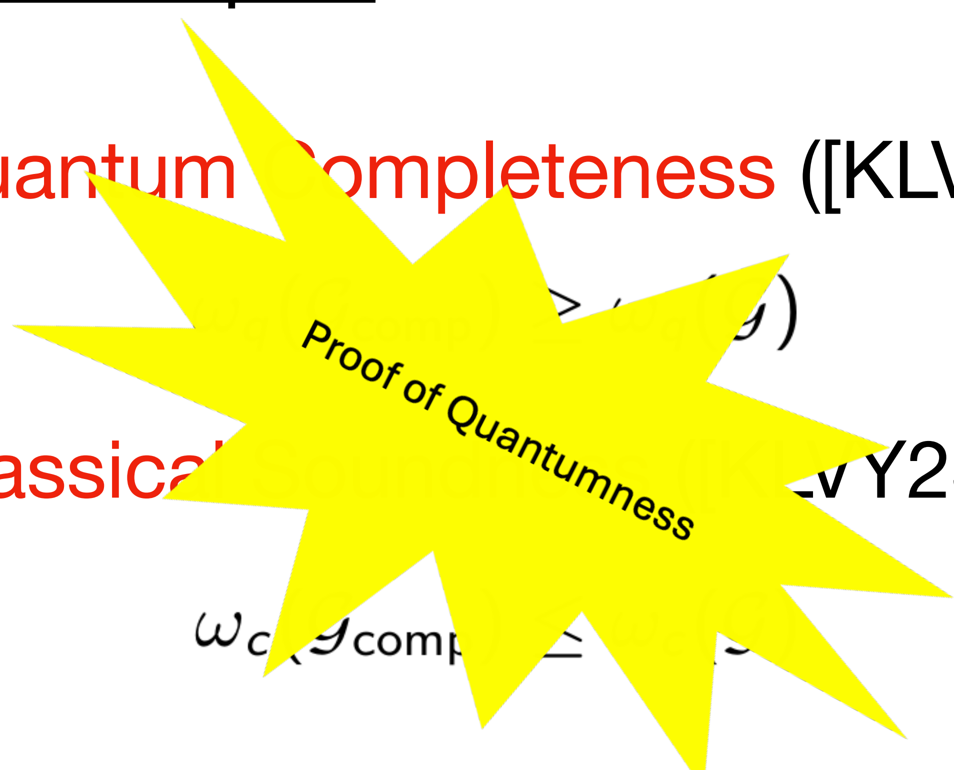

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$$\geq \qquad \mathcal{G})$$

- Classica ([K VY23])

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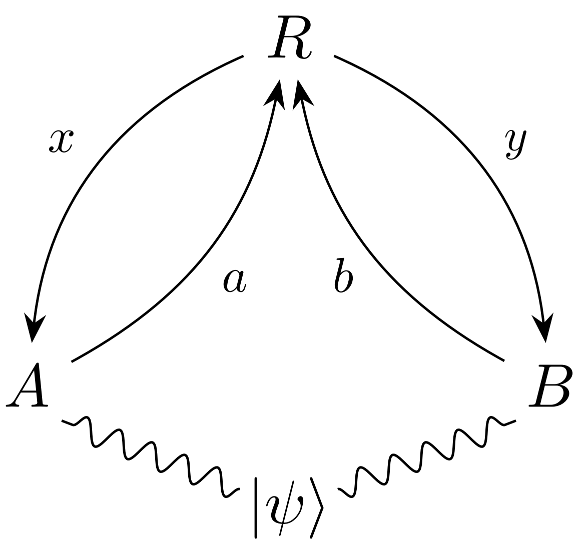

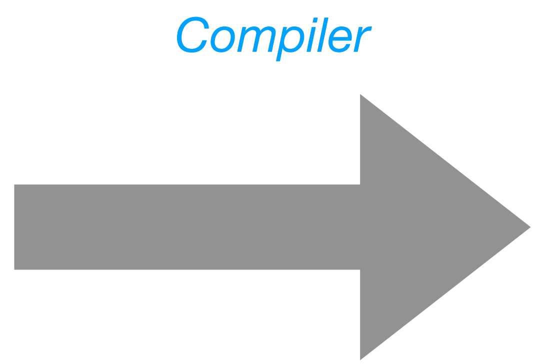

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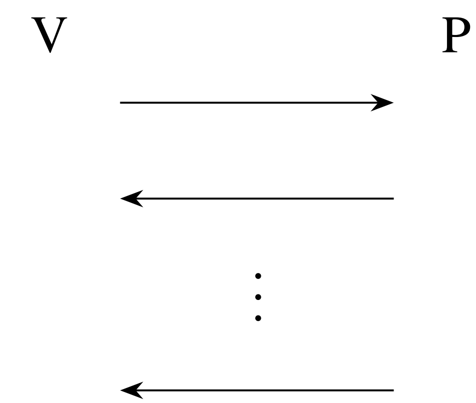

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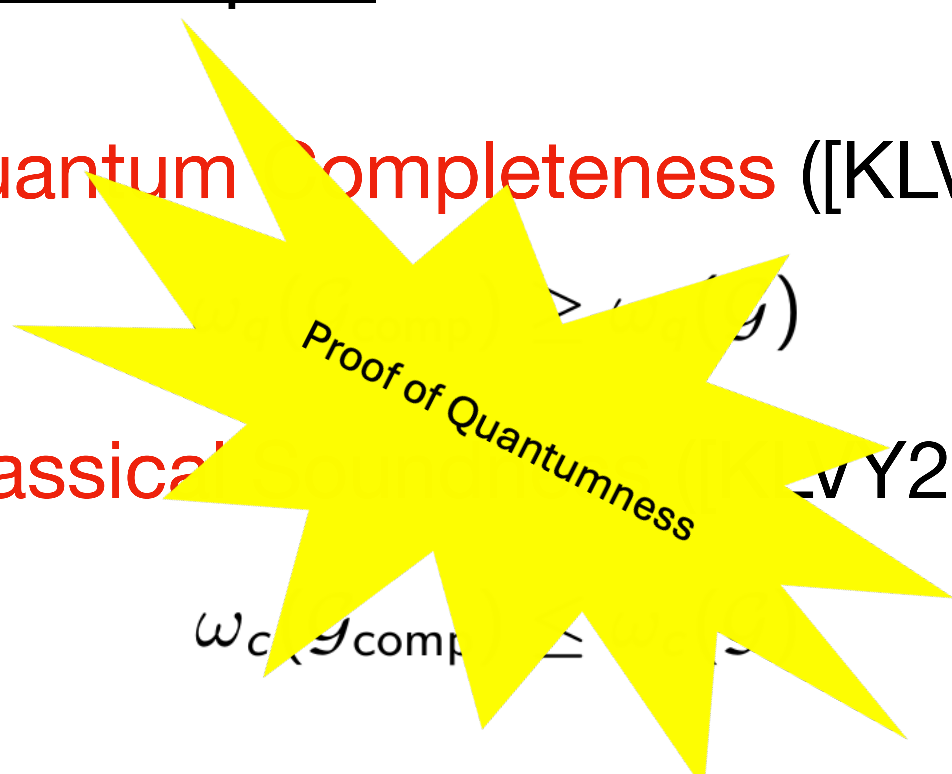


- Quantum [illegible]ompleteness ([KLVY23])

  ω_q[illegible]$) \geq$ [illegible]$)$

- Classica[illegible] [illegible]Y23])

  $\omega_c(\mathcal{G}_{\text{comp}}$[illegible]

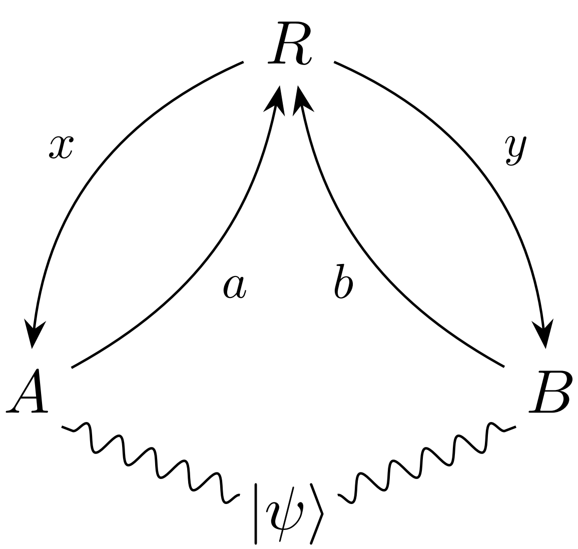
- Quantum Soundness ([NZ23, KMPSW24]):

$$\omega_q(\mathcal{G}_{\text{comp}}) \leq \omega_{qc}(\mathcal{G})$$

+

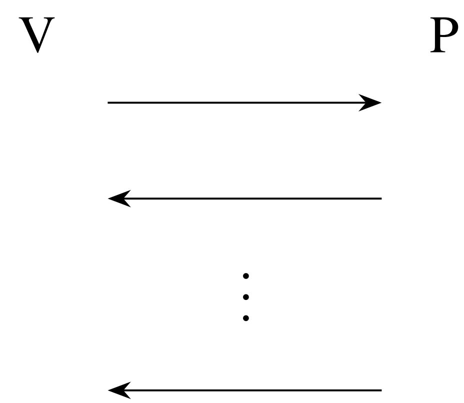
Rigidity

Nonlocal Game



Compiler

Compiled Nonlocal Game



<u>KLVY Compiler</u>

- Quantum Completeness ([KLVY23])

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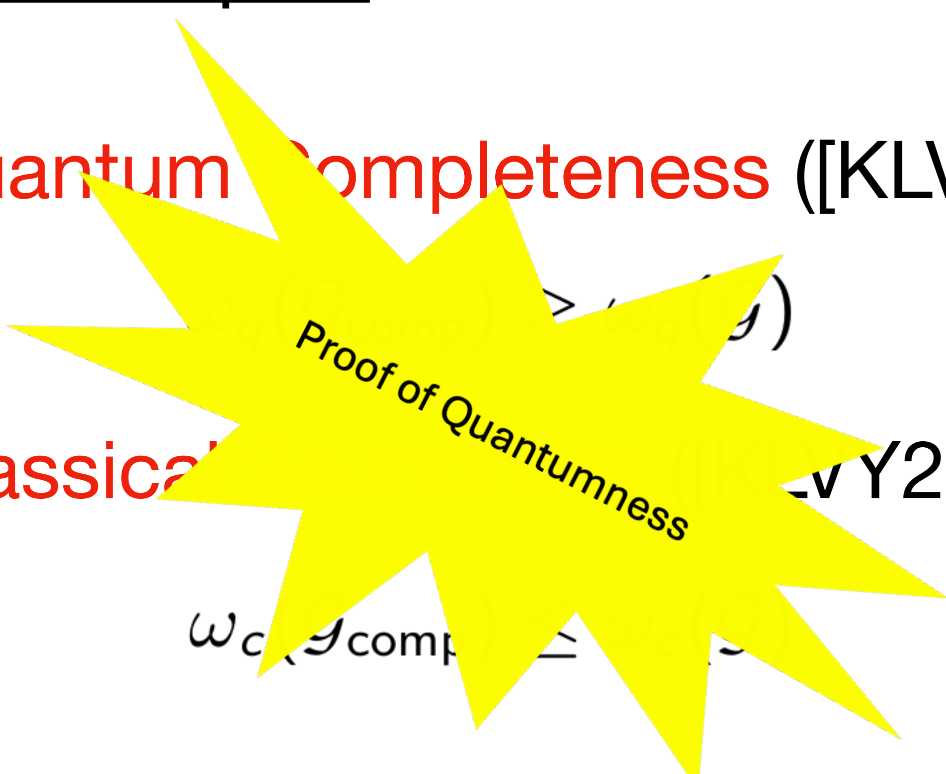
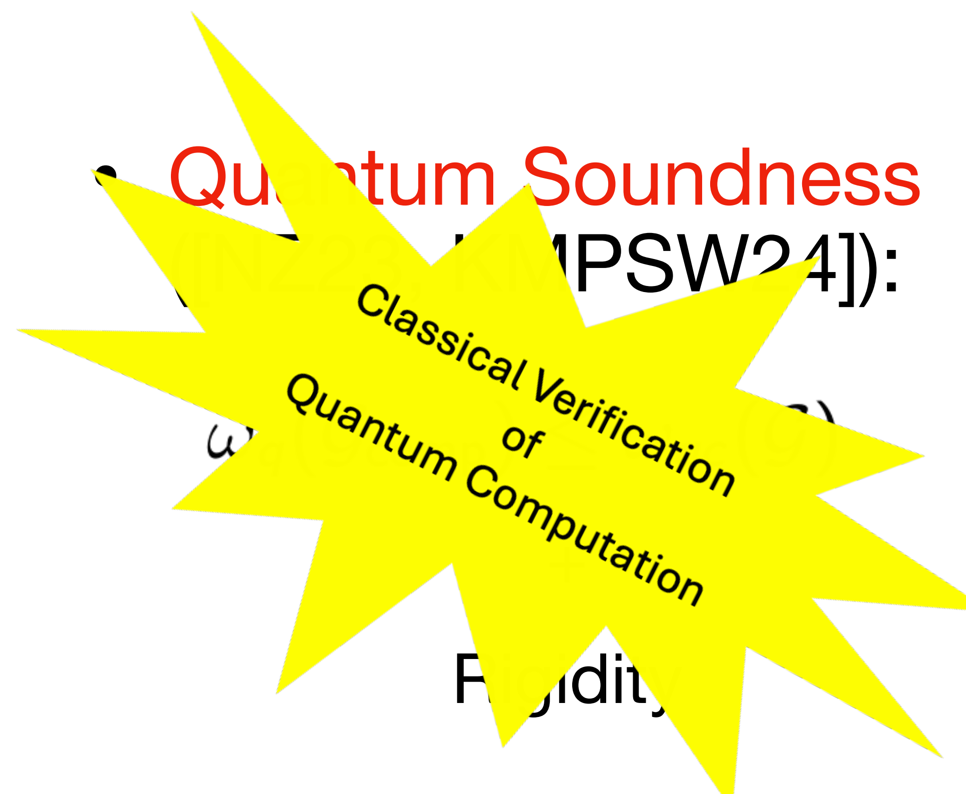
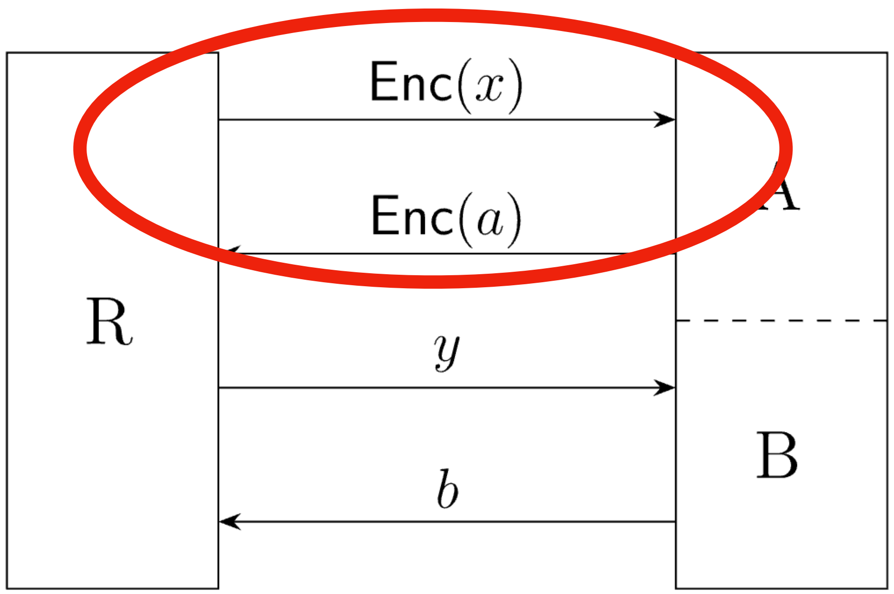
Rigidity





Our Compiler

- Idea 1: Replace QFHE with interactive blind classical delegation of any quantum computation



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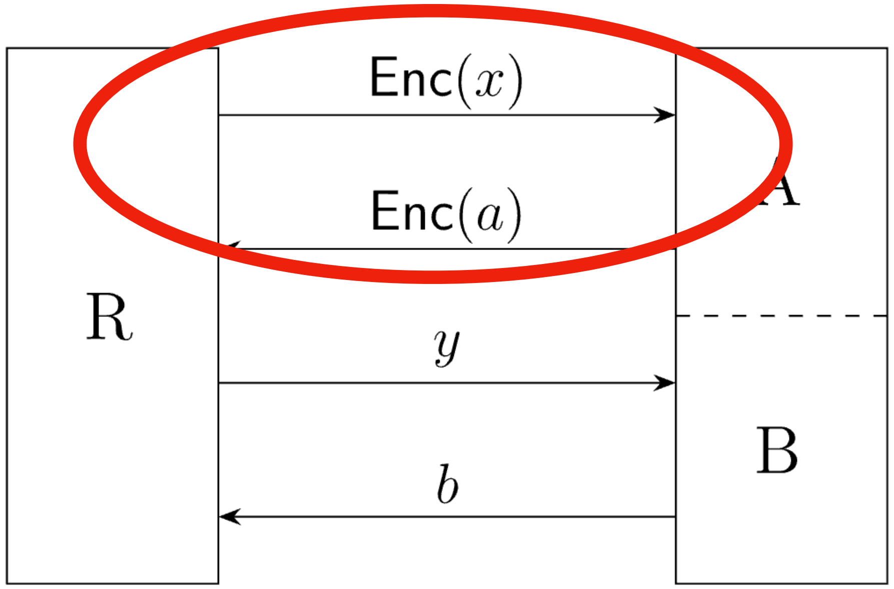
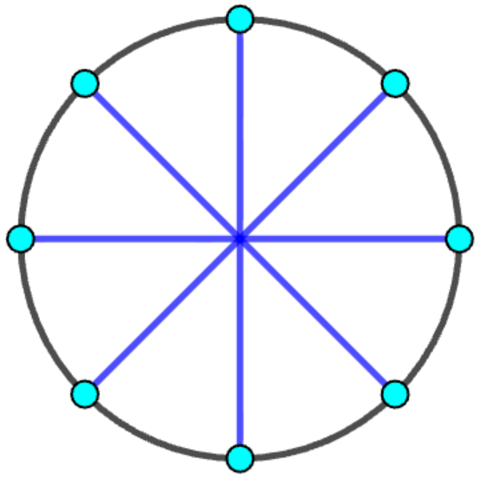


- Idea 2:

Blind classical delegation
=
Modification of universal blind quantum computation (UBQC) [BFK09] in the measurement-based quantum computation (MBQC) model
+
Blind remote state preparation (blind RSP)

Half-Blind Quantum Computation (HBQC)



(X, Y)-plane Bloch sphere

- Assumption: V can prepare single qubits in the state

$$|+_\theta\rangle := \frac{1}{\sqrt{2}}(|0\rangle + e^{i\theta}|1\rangle),$$

where $\theta \in \Theta := \{k \cdot \pi/4 \mid k = 0, \ldots, 7\}$ is uniformly random.

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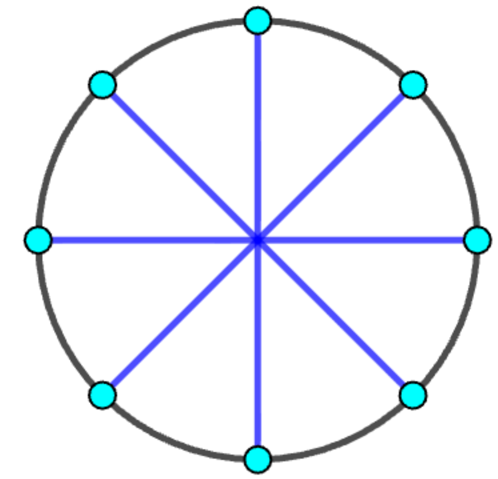

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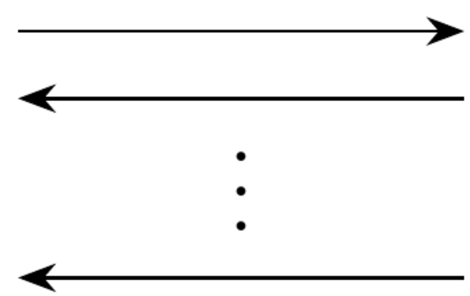
- HBQC protocol:

V(classical description of U)

P($|\psi\rangle$)

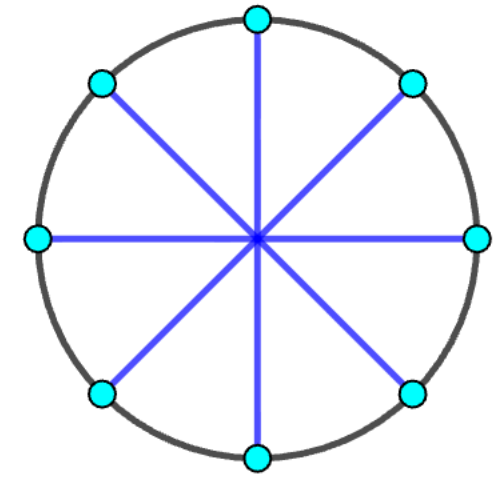


Outputs: a_i, b_i

Outputs: $(U' \otimes I)|\psi\rangle$

$$U' := \left(X^{a_1} Z^{b_1} \otimes \ldots \otimes X^{a_n} Z^{b_n}\right) U$$

Half-Blind Quantum Computation (HBQC)



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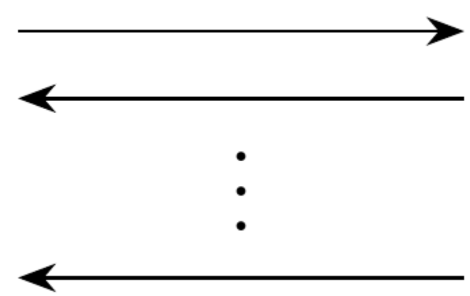

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- Security: Prover gains no information about U and a_i, b_i
- Security holds unconditionally (no computational assumptions!)

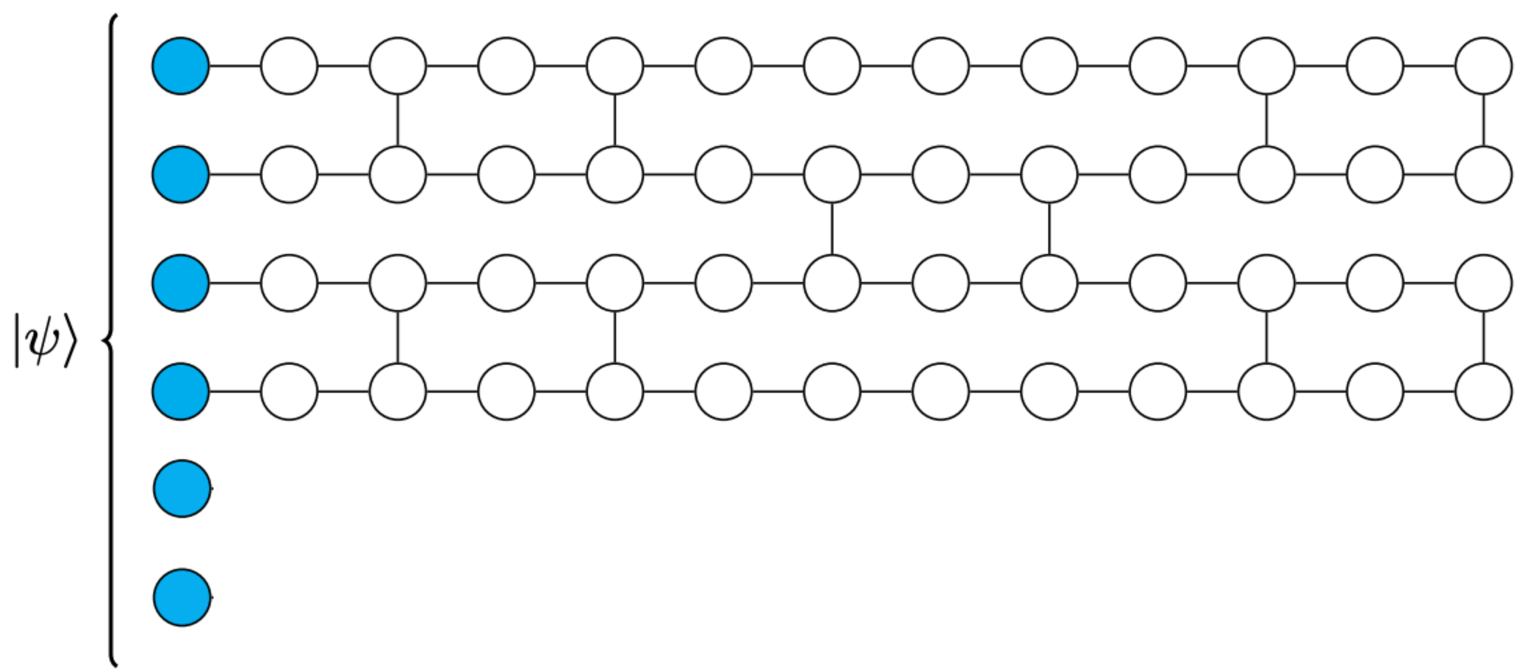
Half-Blind Quantum Computation (HBQC)

- HBQC protocol:

V(description of U)

P($|\psi\rangle$)

$|+_{\theta_{x,y}}\rangle$ $|+_{\theta_{x,y}}\rangle$ $|+_{\theta_{x,y}}\rangle$



V computes the updated measurement angle

⋮

P transmits the result

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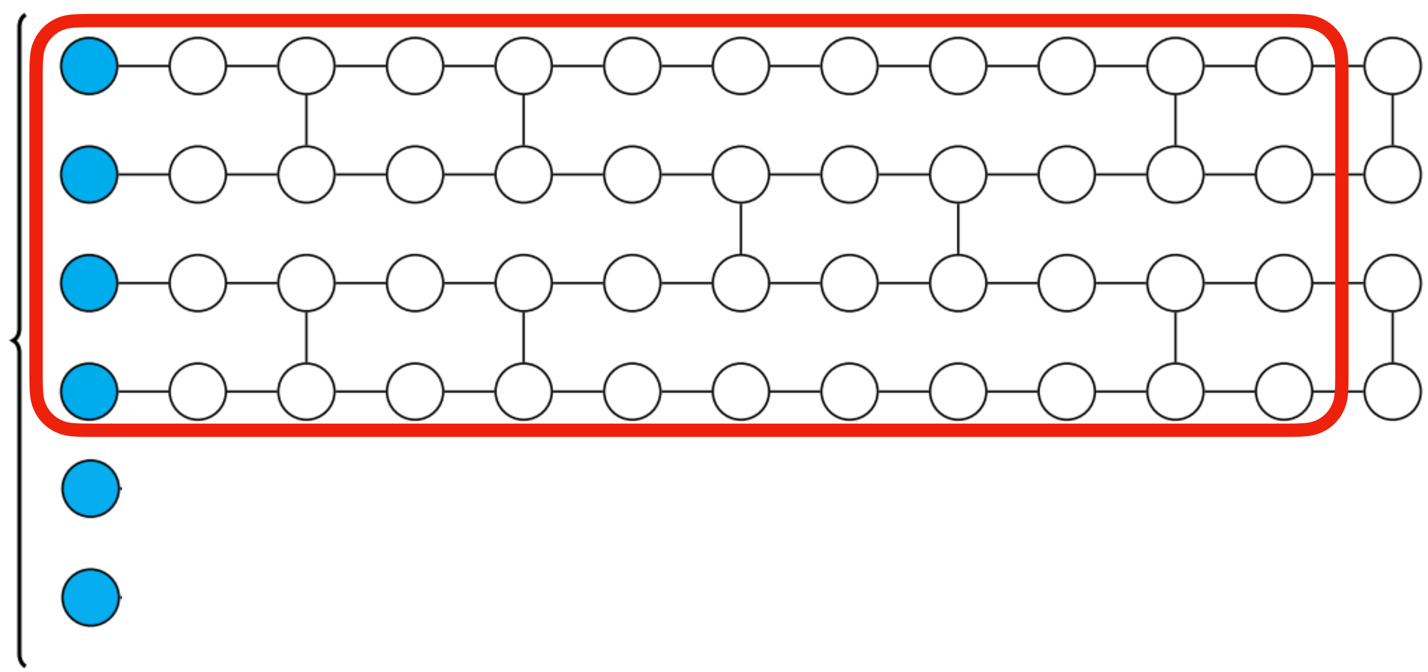
$|\psi\rangle$



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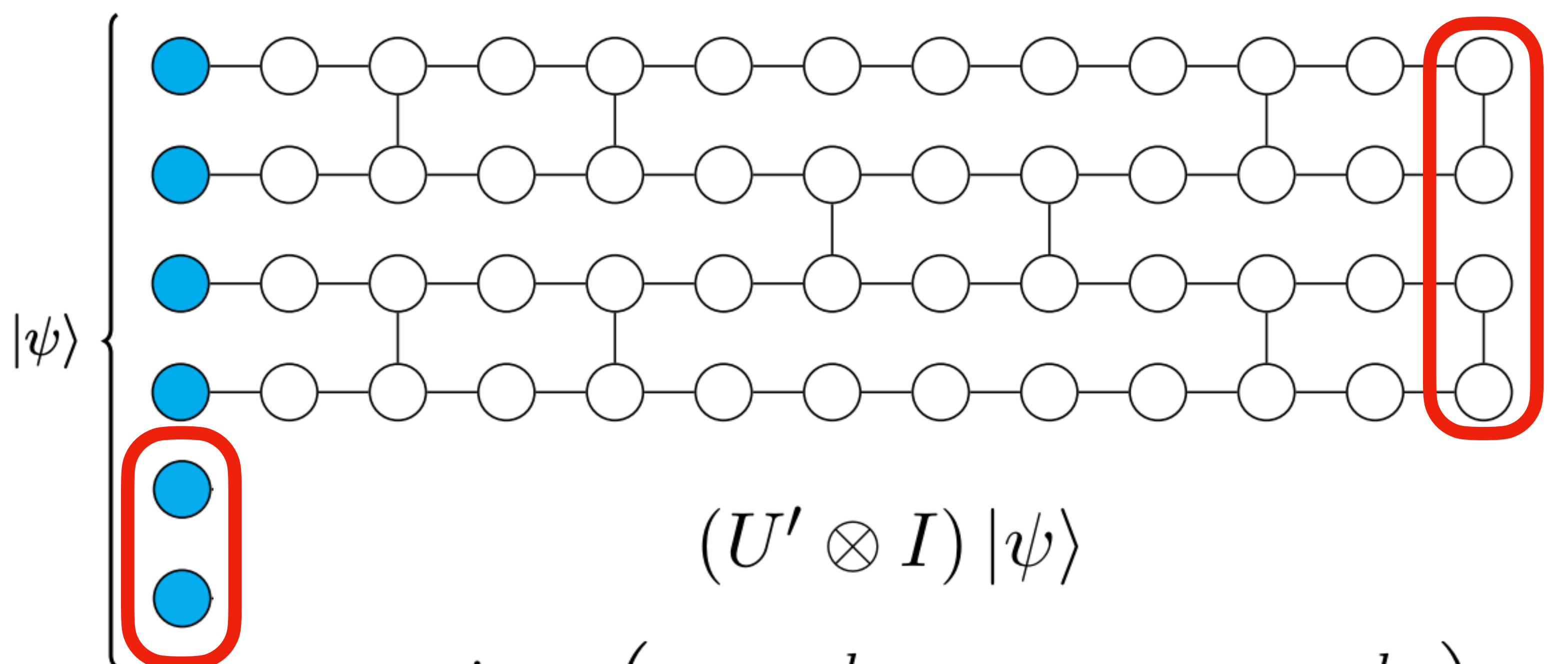

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Blind Remote State Preparation

- Blind RSP = Classical client delegates preparation of single-qubit states to quantum server without revealing information about the state

$$\left\{ |+_\theta\rangle := \frac{1}{\sqrt{2}} \left(|0\rangle + e^{i\theta \frac{\pi}{4}} |1\rangle \right), \theta \in \{0, \ldots, 7\} \right\}$$

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- Assumption: Existence of a post-quantum secure trapdoor claw-free function (TCF)

- TCF:

1. Family of injective function pairs $(f_0, f_1) : X \to Y$ with same image
2. Claw-freeness: Infeasible to find a claw (x_0, x_1) s.t. $f_0(x_0) = f_1(x_1)$
3. Given trapdoor and $y \in Y$ in the image, possible to efficiently invert to obtain a claw (x_0, x_1) s.t. $f_0(x_0) = f_1(x_1) = y$.

Blind Remote State Preparation

Subroutine parameterized by n and state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ held by the prover:

$V(n)$		$P(|\psi\rangle)$
$((f_0, f_1), \mathsf{td}) \leftarrow \mathsf{Gen}(1^\lambda)$	$n, (f_0, f_1)$ $\longrightarrow$	Compute claw state $\alpha|0, x_0\rangle + \beta|1, x_1\rangle$ with $f_0(x_0) = f_1(x_1) = y$
$(x_0, x_1) \leftarrow \mathsf{Invert}(\mathsf{td}, y)$	y $\longleftarrow$	
$r_0, r_1 \leftarrow_\$ \{0,1\}^{\mathsf{poly}(\lambda)}$	(r_0, r_1) $\longrightarrow$	1. Compute $\alpha|0, x_0, -x_0 r_0\rangle + \beta|1, x_1, x_1 r_1\rangle$ 2. Apply QFT_n to last register and measure; abort if outcome $\neq 1$. $\alpha|0, x_0\rangle + \omega_n^{x_0 r_0 + x_1 r_1}\beta|1, x_1\rangle$ 3. Measure last register in Hadamard basis with outcome $d \in \{0,1\}^{p(\lambda)}$. $\alpha|0\rangle + \beta(-1)^{d\cdot(x_0 \oplus x_1)}\omega_n^{x_0 r_0 + x_1 r_1}|1\rangle$
$b := d \cdot (x_0 \oplus x_1)$ $\theta := x_0 r_0 + x_1 r_1$	d $\longleftarrow$	

Blind Remote State Preparation

Blind RSP Protocol: Use the subroutine three times:

- Subroutine with $n = 2$ and $|+\rangle$.
  Let (b_1, θ_1) be the output of V, and $|\psi_1\rangle$ the output state of P.
- Subroutine with $n = 4$ and $|\psi_1\rangle$.
  Let (b_2, θ_2) be the output of V, and $|\psi_2\rangle$ the output state of P.
- Subroutine with $n = 8$ and $|\psi_2\rangle$.
  Let (b_3, θ_3) be the output of V, and $|\psi_3\rangle$ the output state of P.

The prover P holds the final state

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}(|0\rangle + (-1)^b \omega_8^\theta |1\rangle)$$

The verifier V holds

$$b := b_1 \oplus b_2 \oplus b_3 \text{ and } \theta := 4\theta_1 + 2\theta_2 + \theta_3 \mod 8.$$

- Security: From the prover's point of view, θ is uniformly random

Summary

- HBQC = generalized UBQC protocol [BFK09]
- TCF ⇒ blind RSP for $\left\{ |+_\theta\rangle := \frac{1}{\sqrt{2}} \left(|0\rangle + e^{i\theta\frac{\pi}{4}} |1\rangle \right), \theta \in \{0, \ldots, 7\} \right\}$
- HBQC + blind RSP ⇒ classical blind delegation of QC
- New compiler from classical blind delegation of QC, satisfying
  1. Quantum Completeness
  2. Quantum Soundness
- Extending Mahadev's result from
  '*specific TCF from LWE ⇒ CVQC*' to '*any TCF ⇒ CVQC*'

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Thank you for your attention!