Computationally Differentially Private Inner Product Protocols Imply Oblivious Transfer

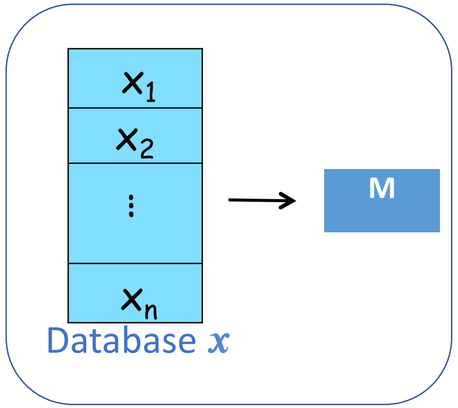
Chao Yan

Joint work with

Iftach Haitner, Noam Mazor, Jad Silbak and Eliad Tsfadia

Differential Privacy

Dwork, McSherry, Nissim, Smith 2006



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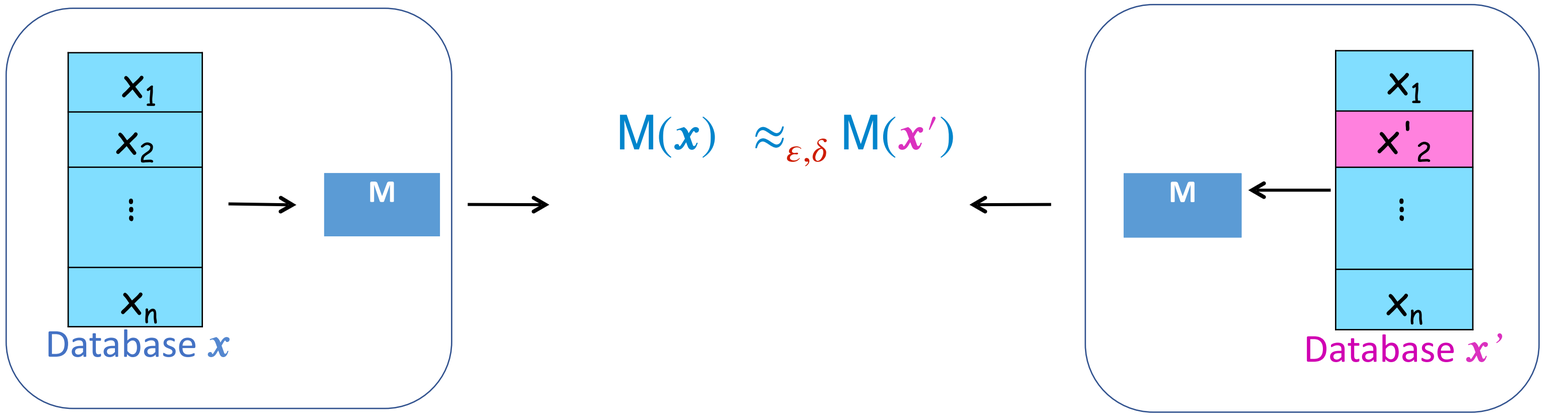

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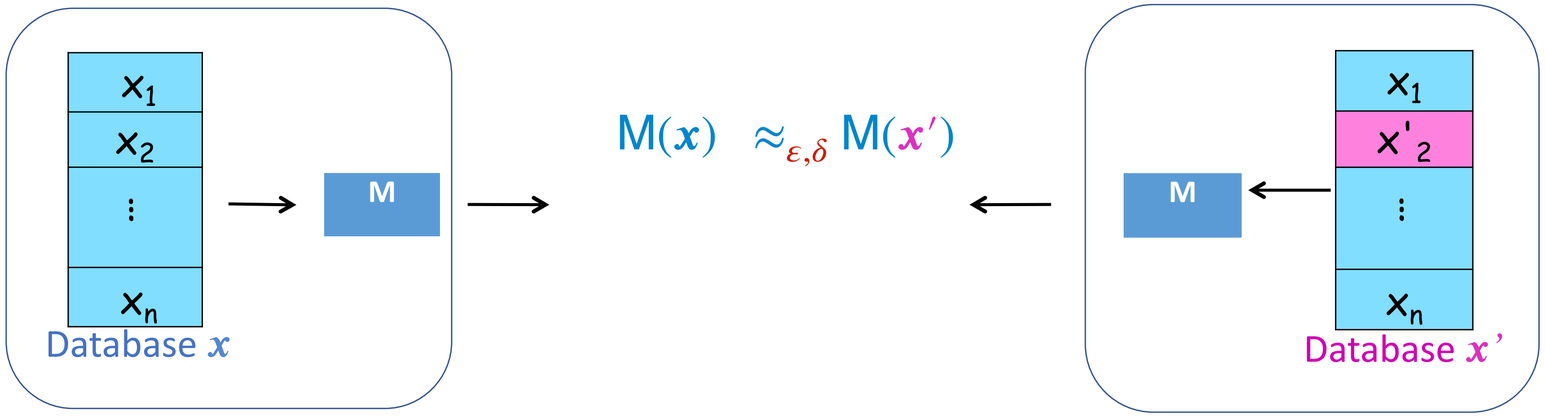

M is (ε, δ)-DP

$\forall$ neighboring databases x, x' and $\forall$ (unbounded) distinguisher D:

$$\Pr[\mathrm{D}(\mathrm{M}(x)) = 1] \leq e^{\varepsilon} \cdot \Pr[\mathrm{D}(\mathrm{M}(x')) = 1] + \delta$$

Centralized DP

Google

$\boldsymbol{x} = (x_1, \ldots, x_n)$

$$f(x) = \sum_{i=1}^{n} x_i$$

$$M(x) = \sum_{i=1}^{n} x_i + noise$$

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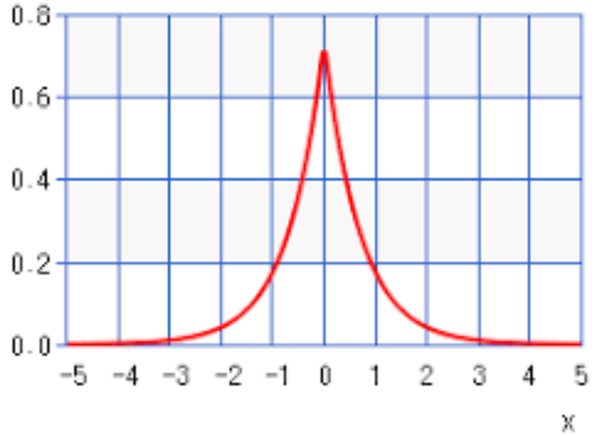
$\boldsymbol{x} = (x_1, \ldots, x_n)$

$$f(x) = \sum_{i=1}^{n} x_i$$

$$M(x) = \sum_{i=1}^{n} x_i + \textbf{Laplace}(1/\varepsilon)$$

w.h.p $noise \leq O(1)$



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$x = (x_1, \ldots, x_n)$

$$f(x) = \sum_{i=1}^{n} x_i$$

centralized-DP: M can access the entire database x

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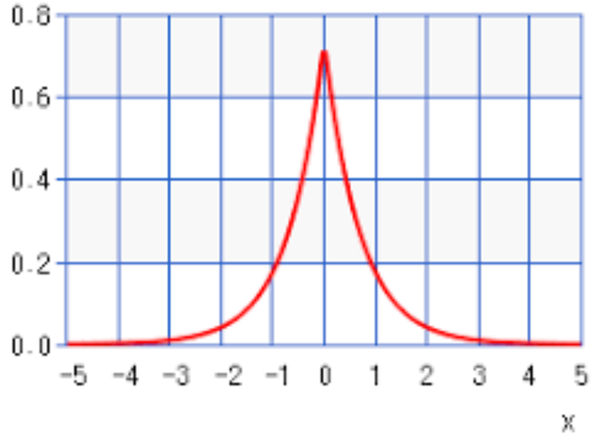
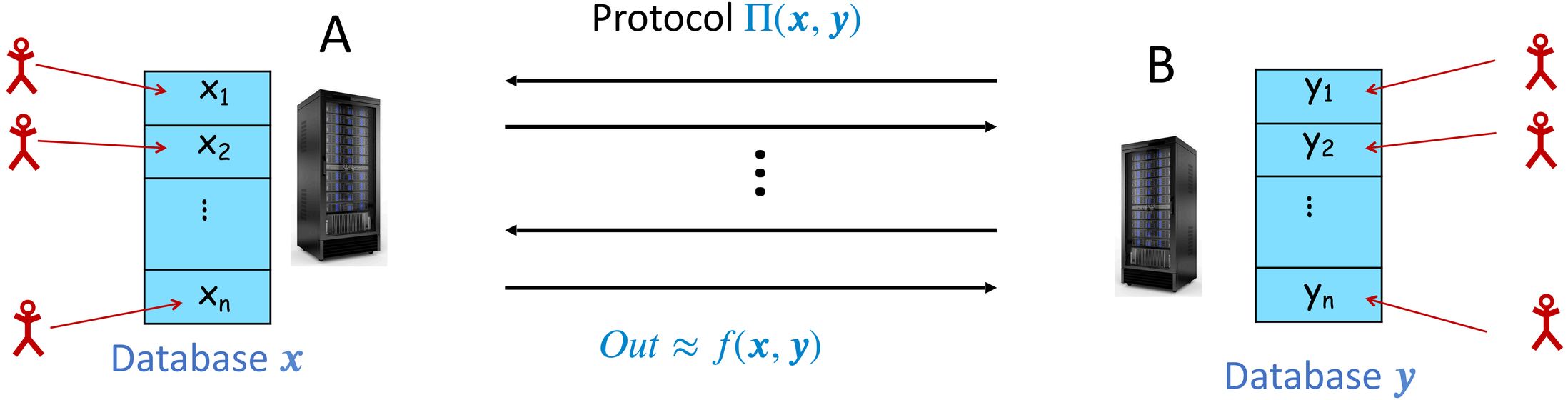


Two-Party DP


A
x_1
x_2
x_n
Database x
Protocol $\Pi(x, y)$
$Out \approx f(x, y)$
B
y_1
y_2
y_n
Database y

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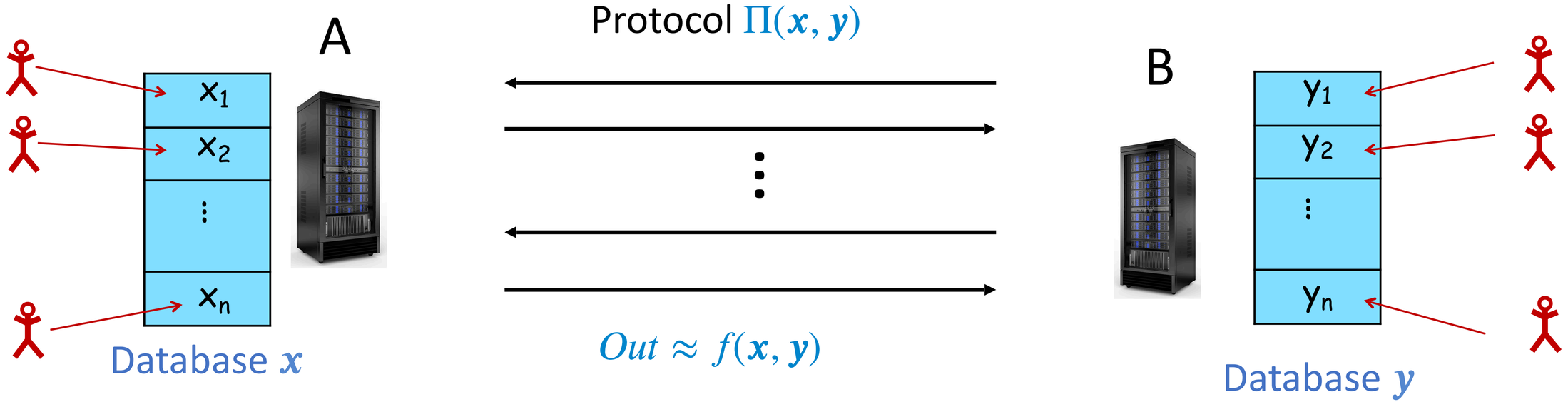

Goal: Estimate $f(x, y)$ while preserving (ε, δ)-DP:

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$view_A^{\Pi}(x, y)$ – A's view in $\Pi(x, y)$ (input, coins and transcript).

(and same for B)

Two-Party DP

$\boldsymbol{x} = (x_1, \ldots, x_n)$

$$out_1 = \sum_i x_i + Noise$$

$$out_2 = \sum_i y_i + Noise$$

$$out = out_1 + out_2$$

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Inner Product

A

$x \in \{-1,1\}^n$

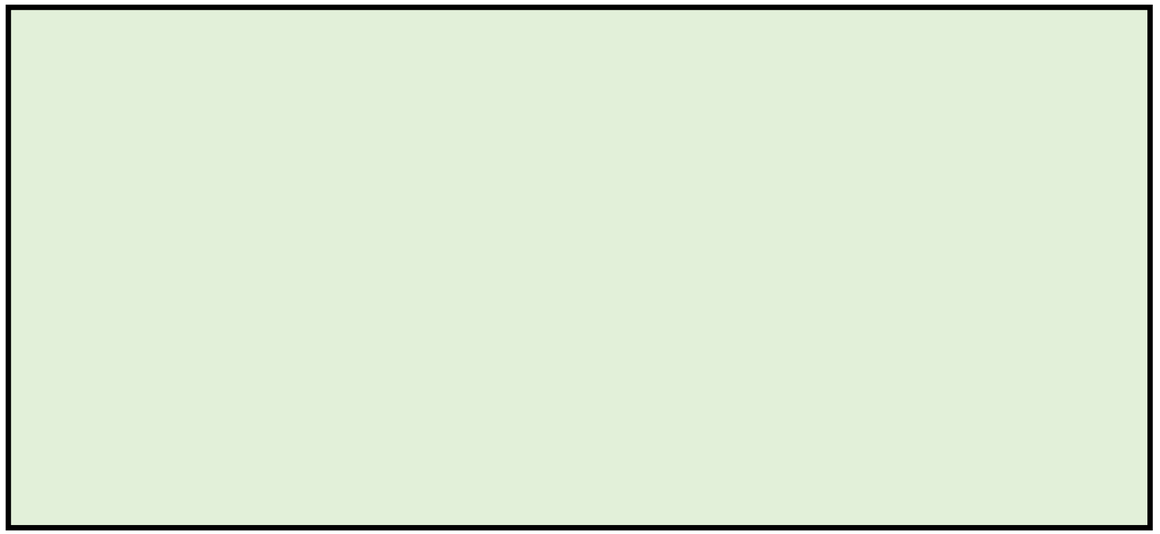
?

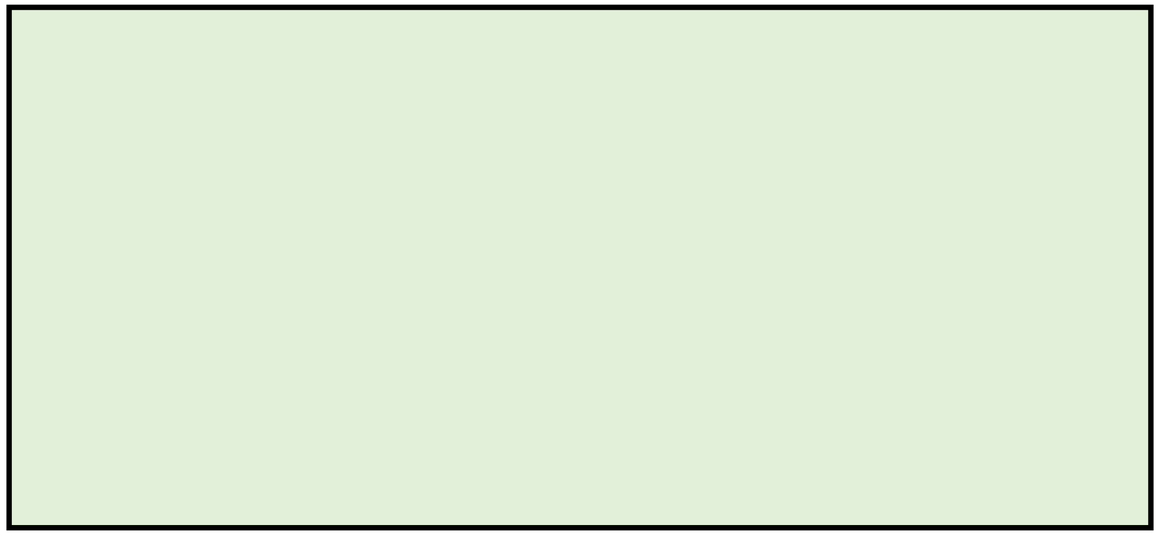
B

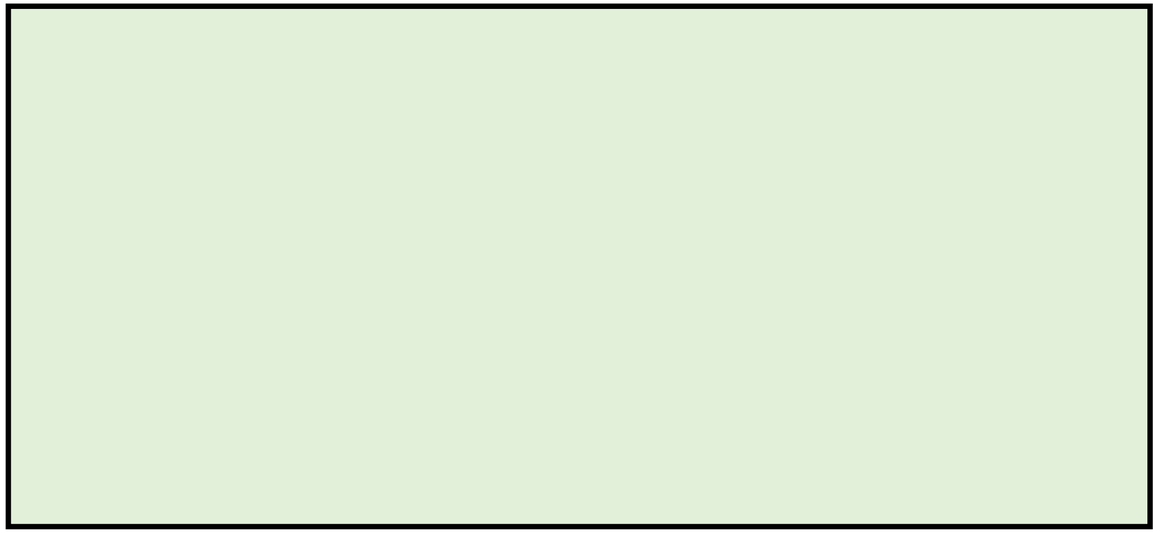
$y \in \{-1,1\}^n$

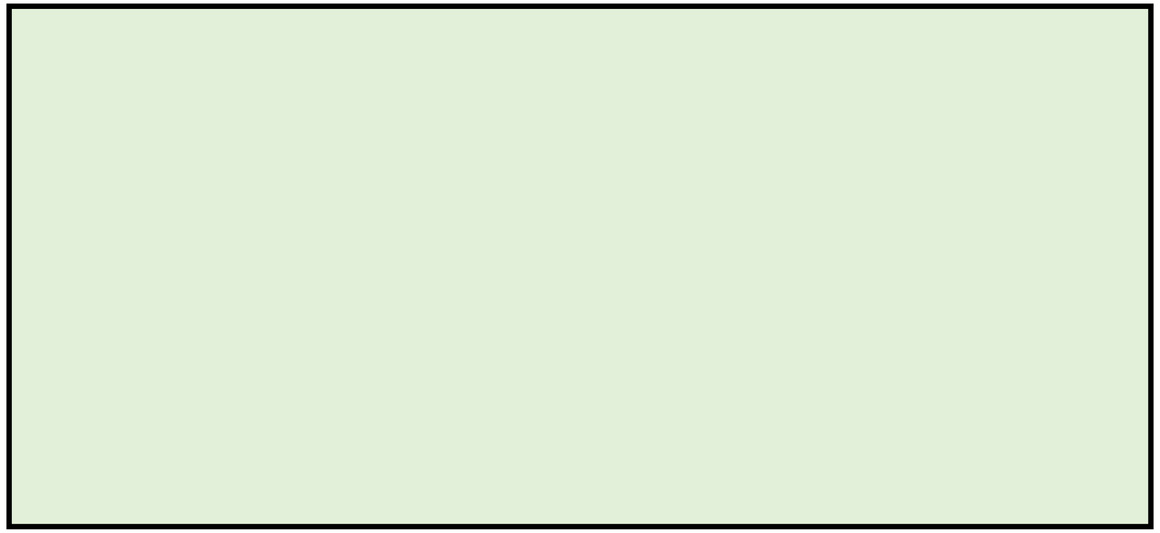
$\langle x, y \rangle = \sum_{i=1}^{n} x_i y_i$ – measures correlation between databases

DP Inner Product









DP Inner Product

Centralized Model

Can achieve constant error.

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Two-Party Protocol

For uniform inputs:

A: $x \in \{-1,1\}^n$ B: $y \in \{-1,1\}^n$

$$out = 0$$

- With prob. 0.99: $\left| out - \langle x, y \rangle \right| \approx \sqrt{n}$
- Can be generalized for every input distribution.

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McGregor, Mironov, Pitassi, Reingold, Talwar and Vadhan 2010

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Computational Two-Party DP

Can achieve constant error, using Cryptography

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Oblivious Transfer and Secure MPC

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M is (ε, δ)-**CDP**

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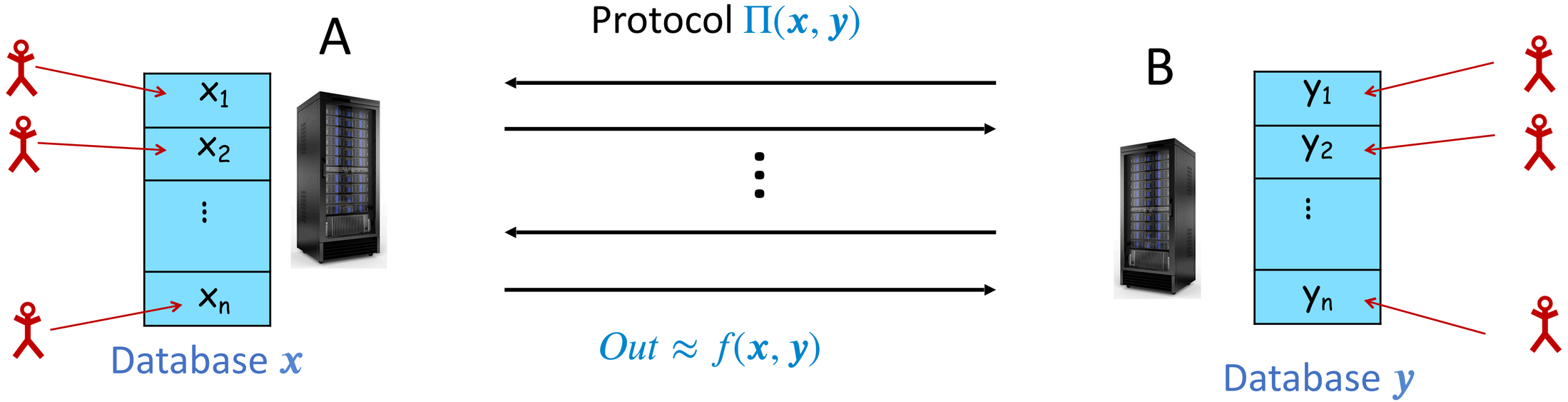
Two-Party CDP



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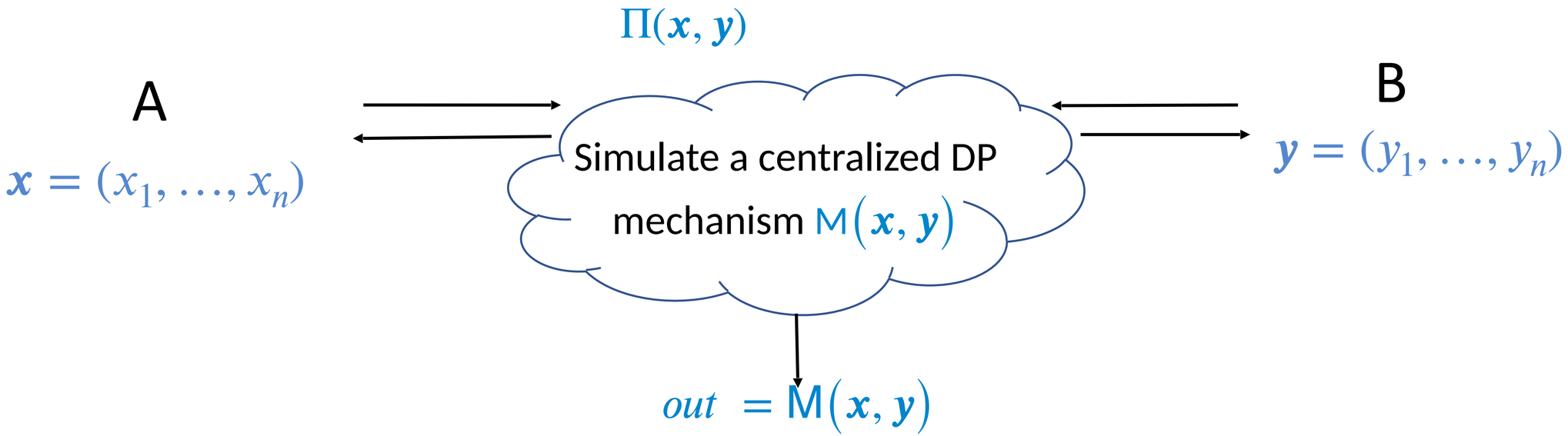
CDP via Secure Multiparty Computation



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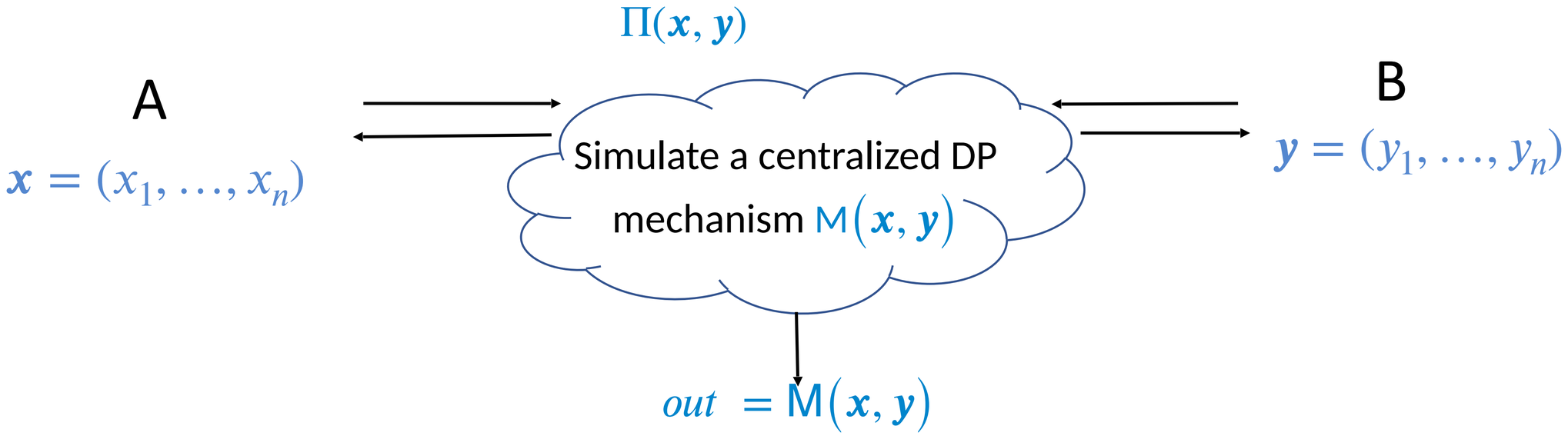

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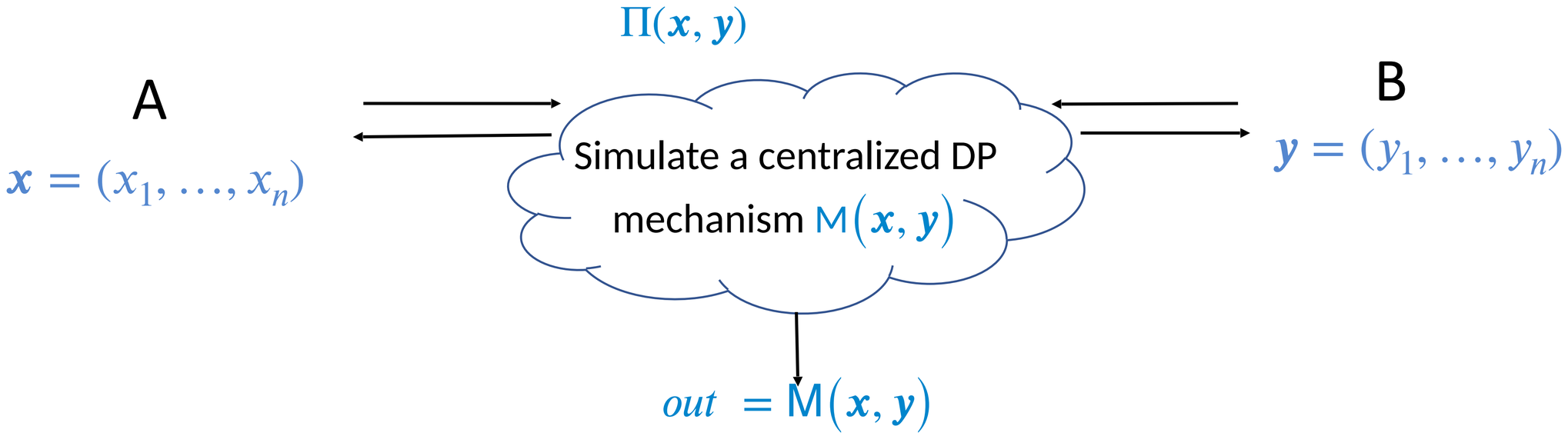

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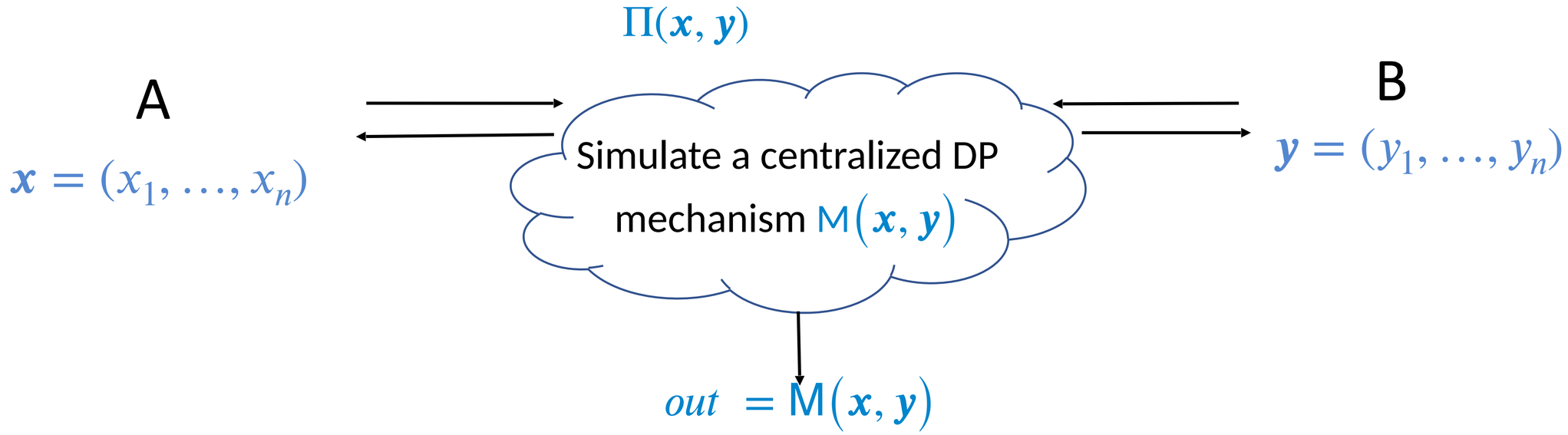

- M is (centralized) (ε, δ)-DP $\Longrightarrow$ Π is (ε, δ)-CDP.
- Secure MPC via *Oblivious Transfer* (OT).
- For computing IP, take $\mathsf{M}(\boldsymbol{x}, \boldsymbol{y}) = \langle \boldsymbol{x}, \boldsymbol{y} \rangle + Lap(2/\varepsilon)$.

The Complexity of Two-Party CDP

Using OT, we can construct very accurate CDP protocols!

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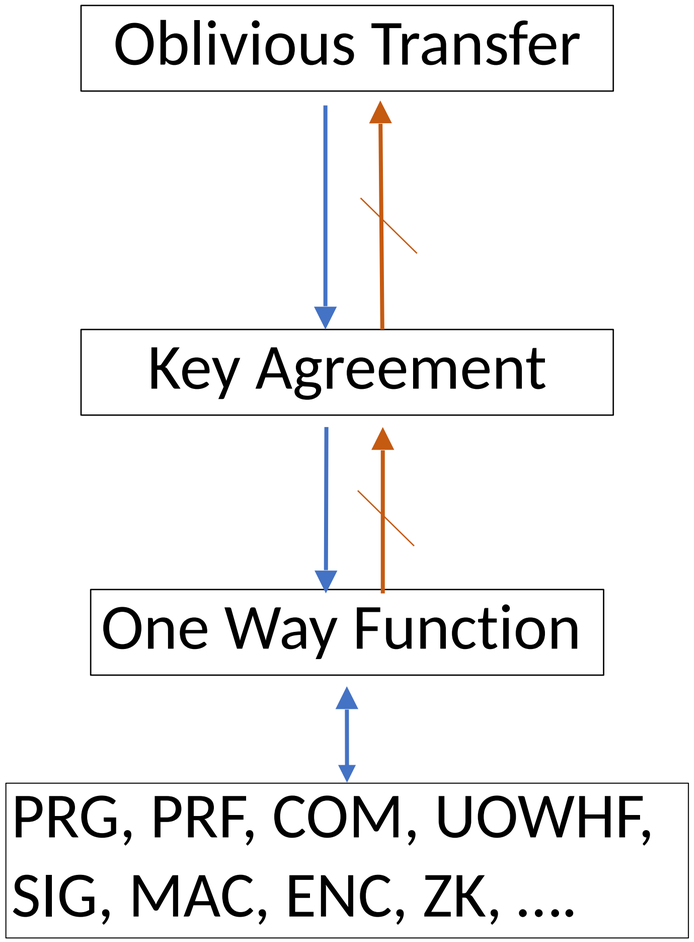
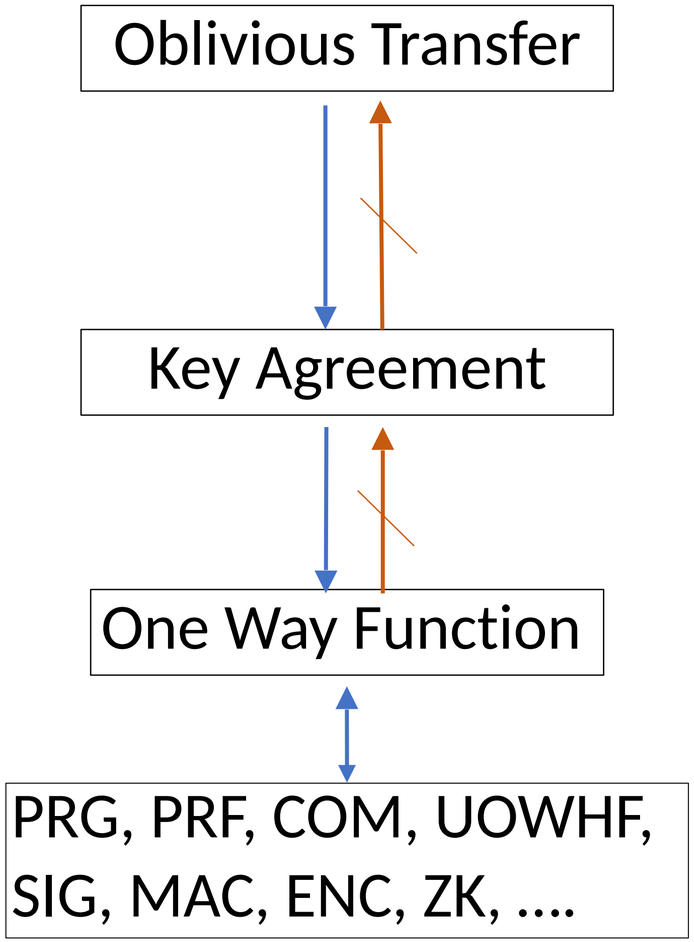
Do we have to use OT?

Complexity Hierarchy



Complexity Hierarchy

"Non-trivial":
Possible in two-party CDP
Impossible in two-party DP



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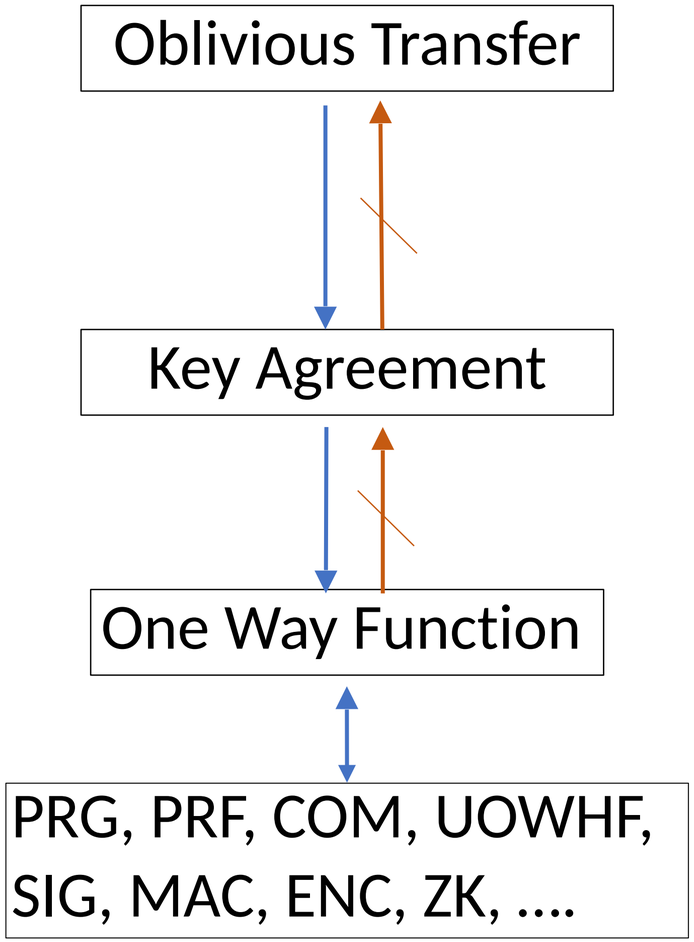

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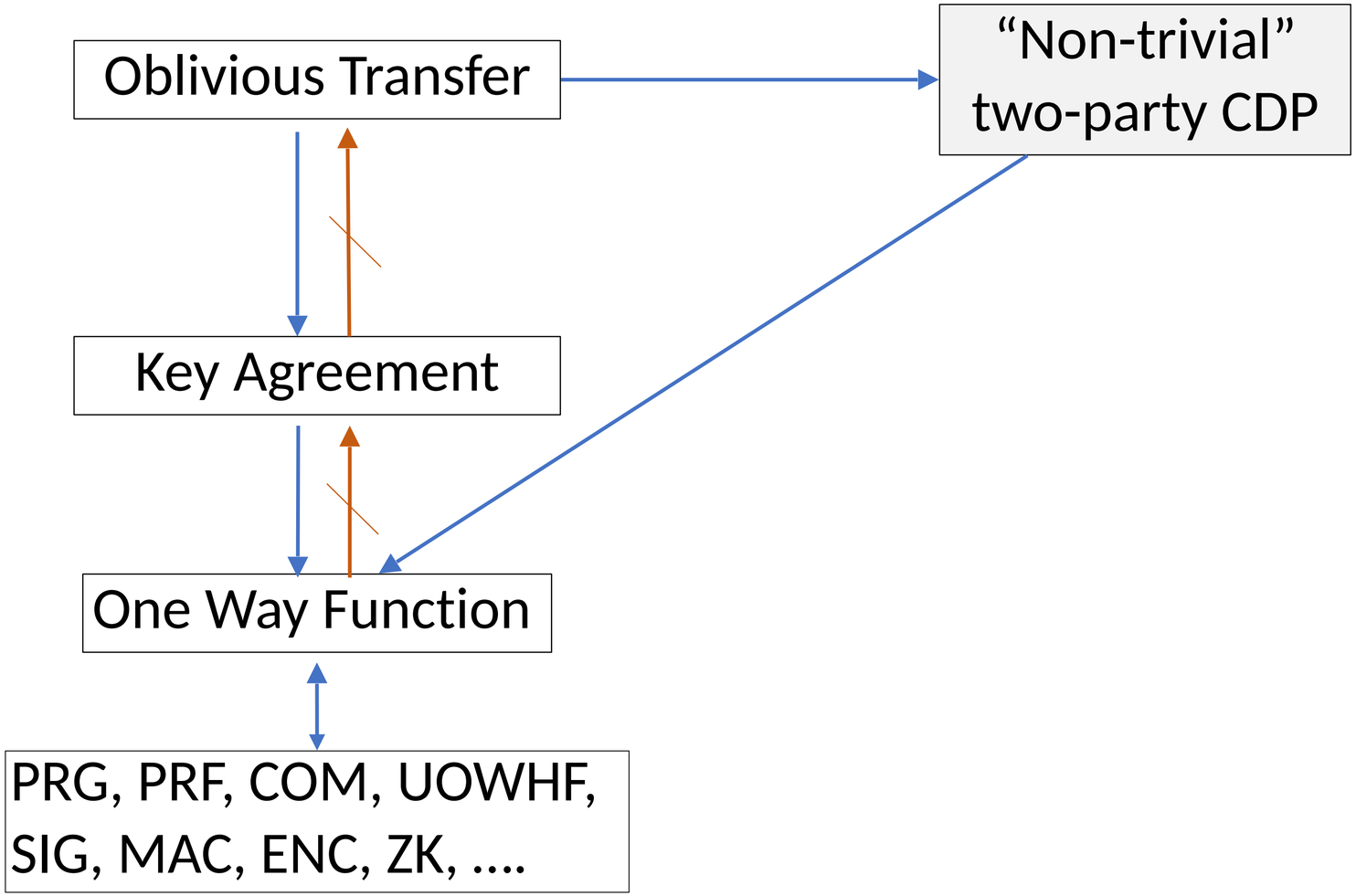

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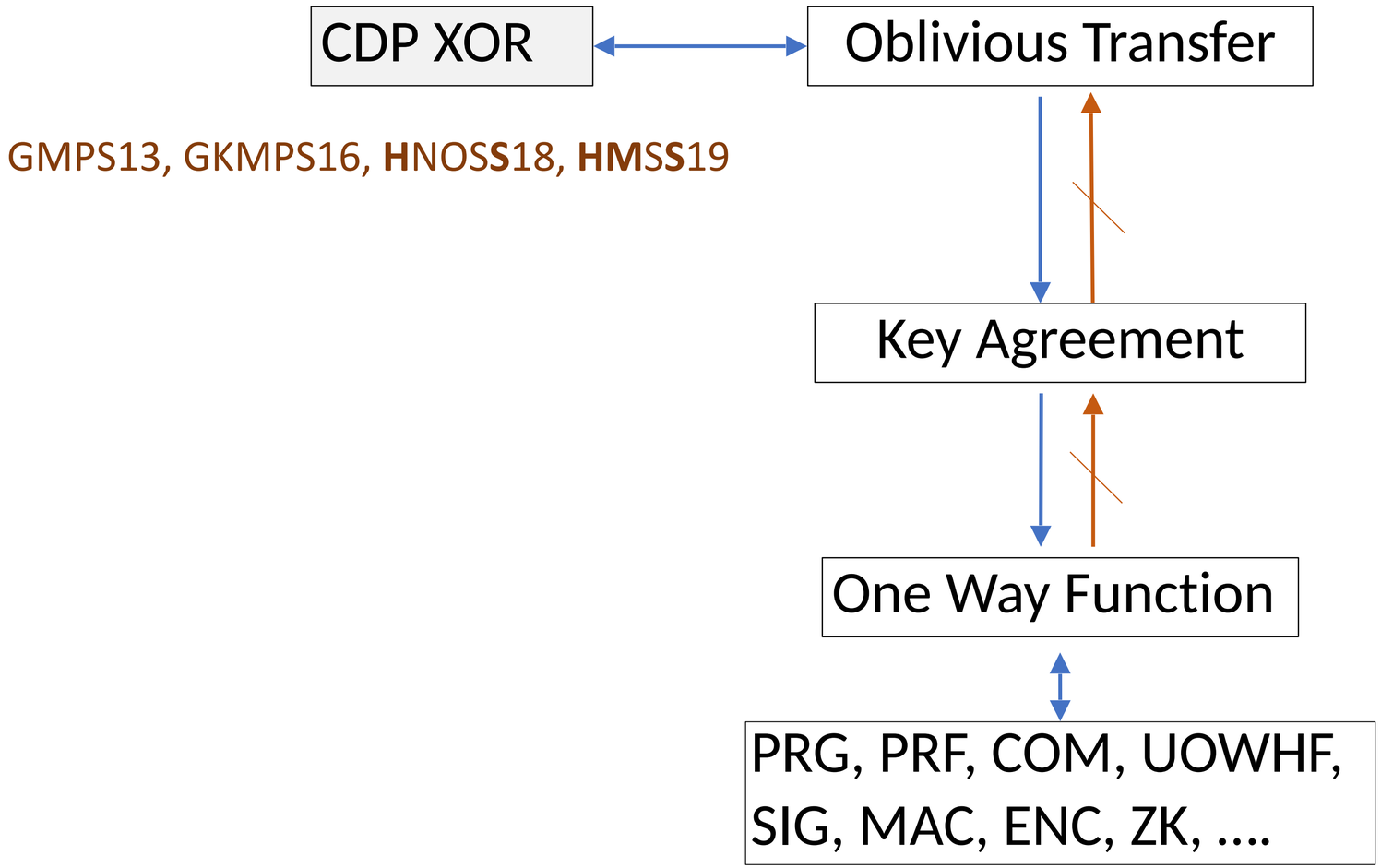

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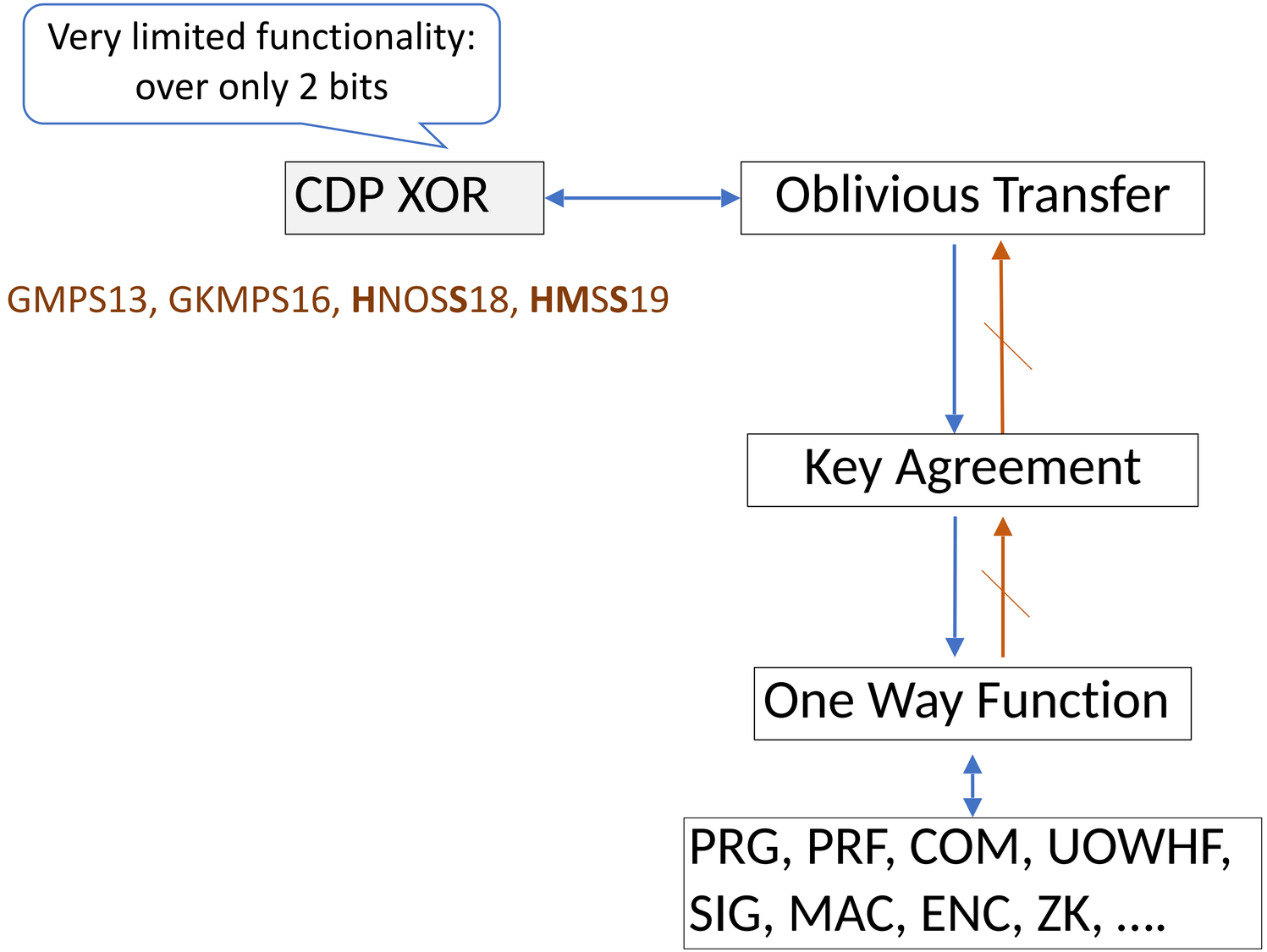

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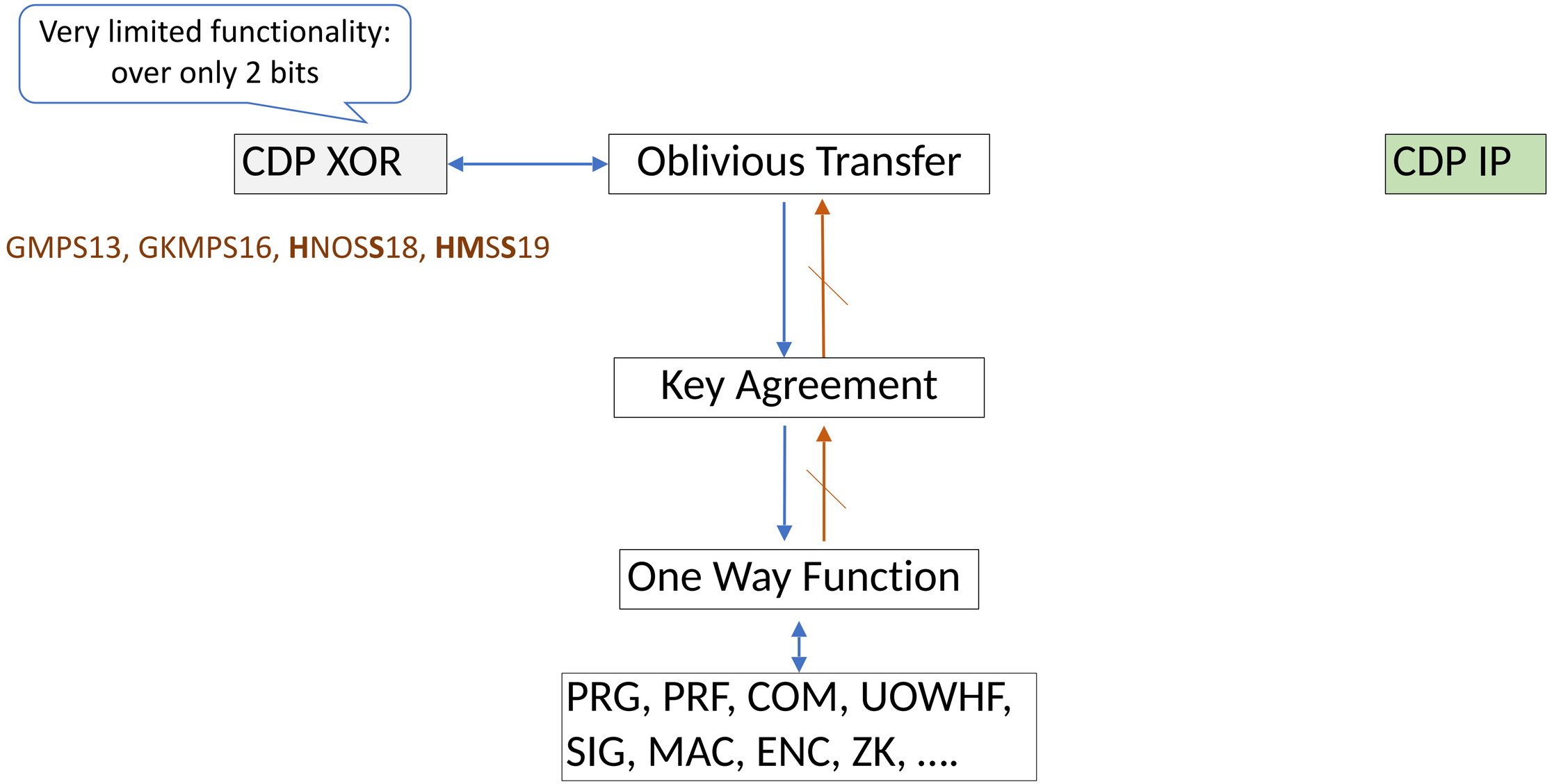

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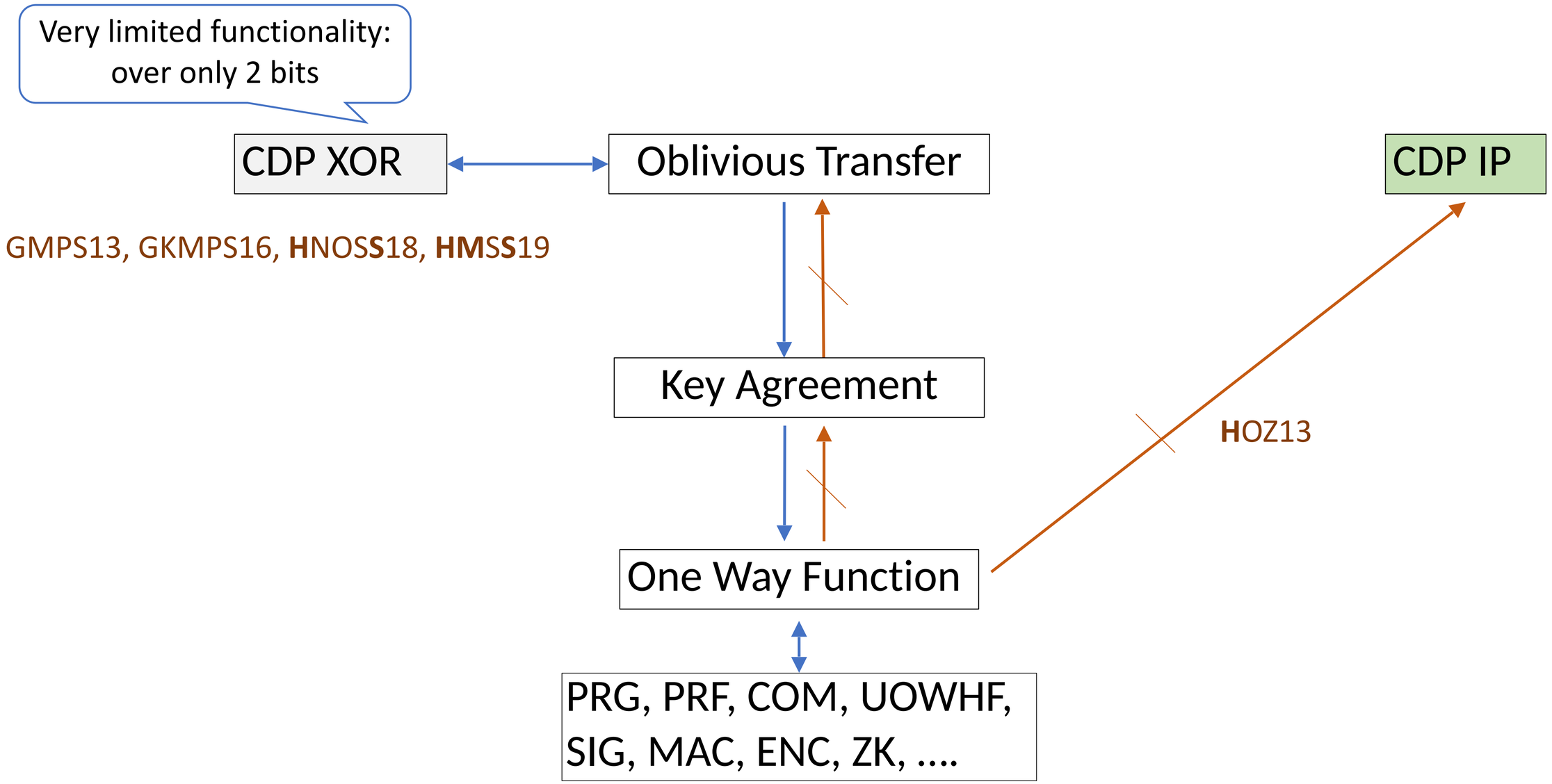

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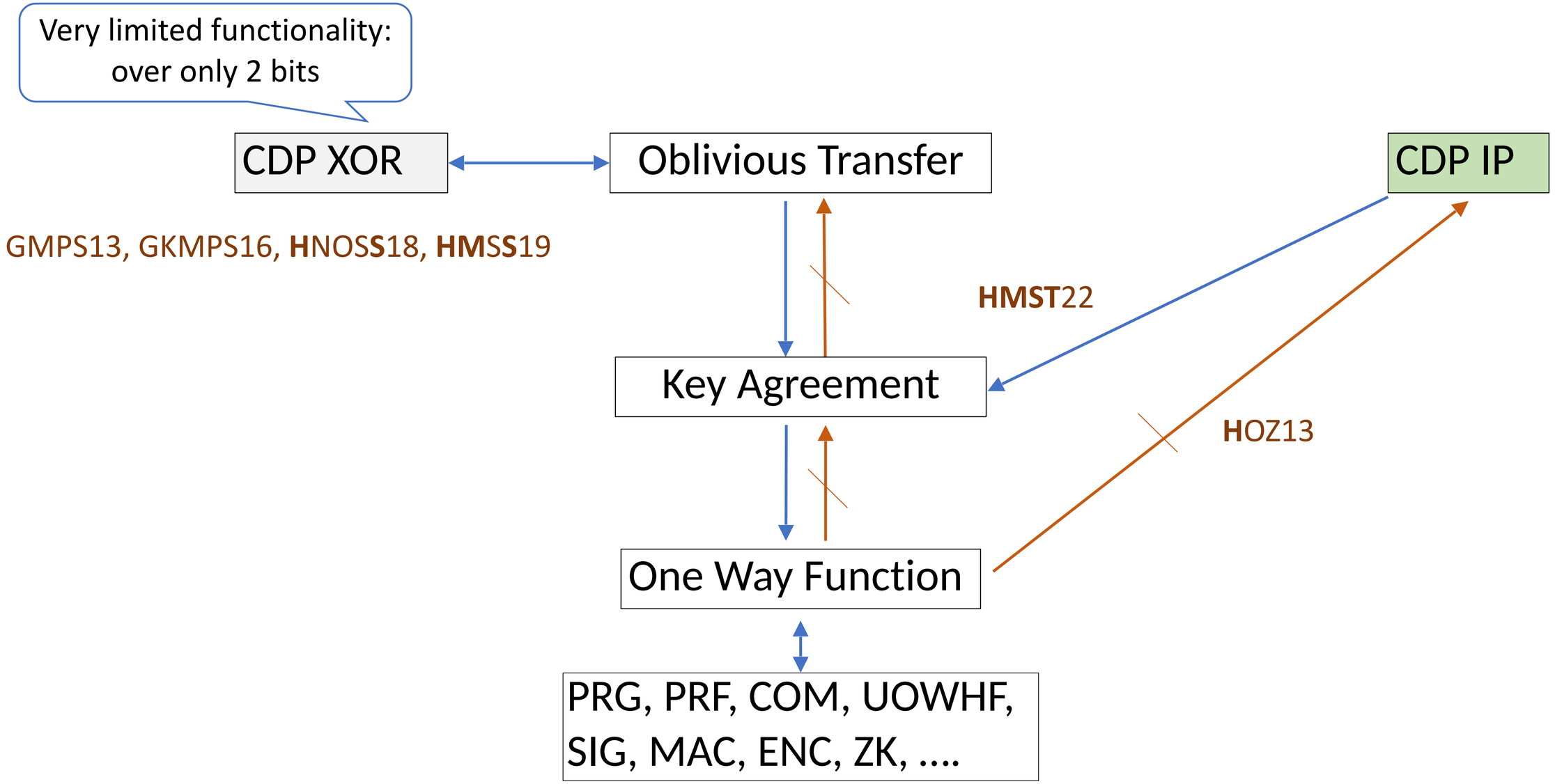

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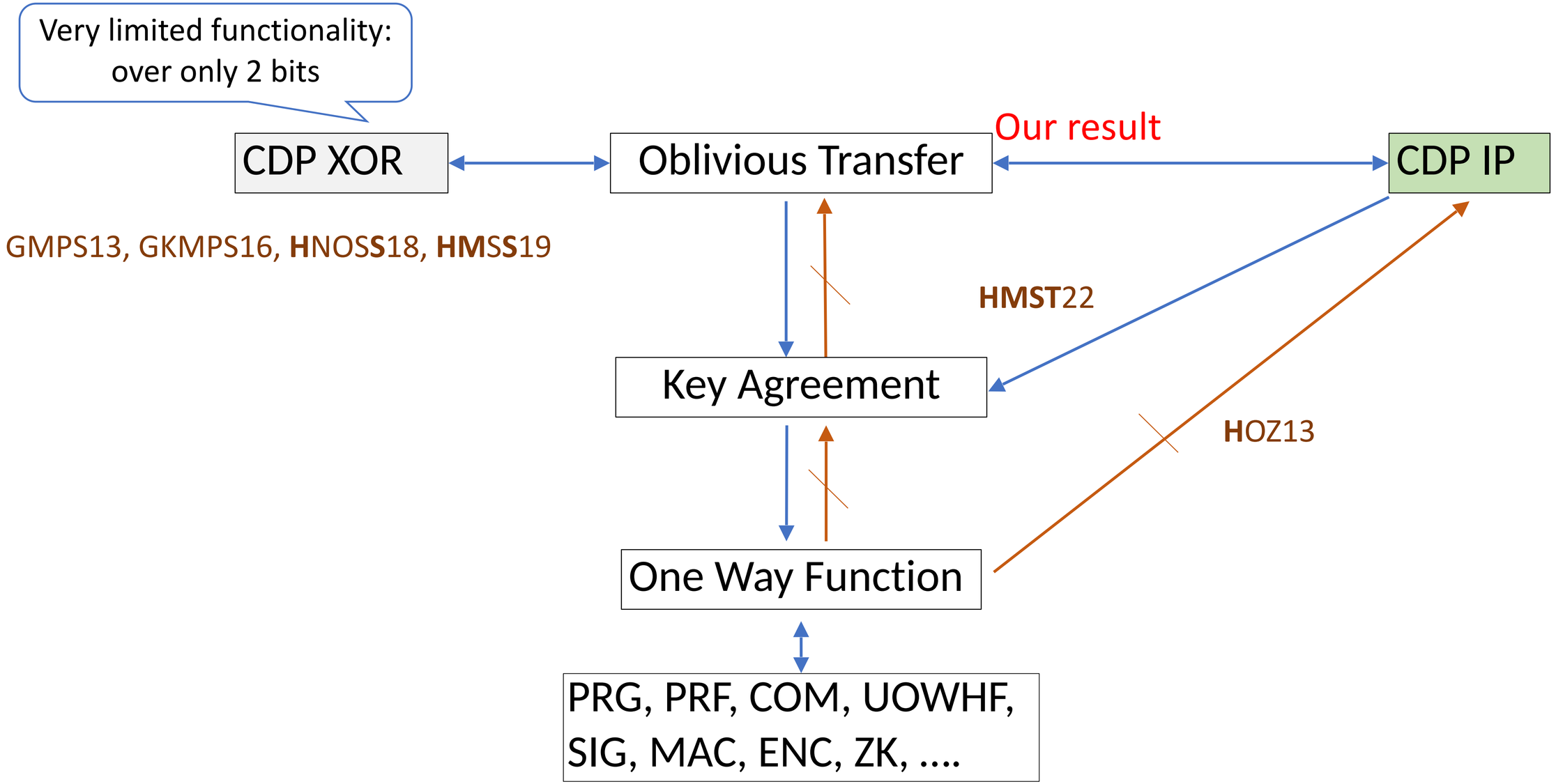

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- $\delta = (\frac{1}{n})$
- $$\Pr_{\substack{\boldsymbol{x}, \boldsymbol{y} \leftarrow \{-1,1\}^n \\ out \leftarrow \Pi(x, y)}} \left[\left| out - \langle \boldsymbol{x}, \boldsymbol{y} \rangle \right| < n^{1/6} \right] > 0.99$$

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Information theoretic barrier: $n^{1/2}$.

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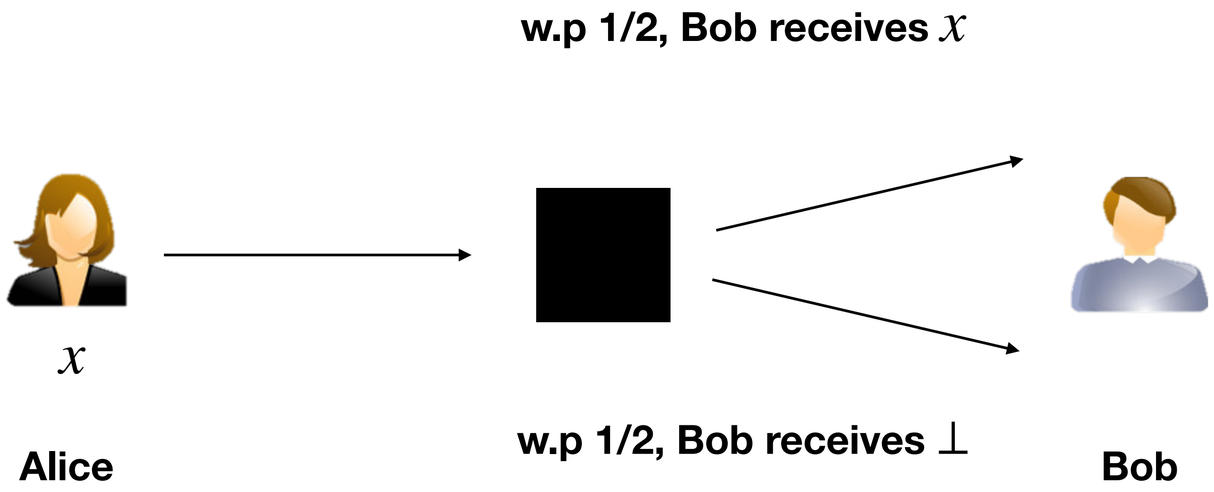
- Mildly accurate ⇒ Very accurate, $O(1)$

Information theoretic barrier: $n^{1/2}$.

Proof Overview

Rabin's OT

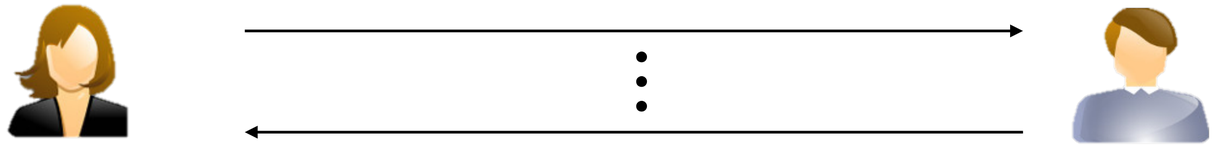
Rabin 1981



Alice doesn't know what Bob receives

Weak Erasure Channel

Wullschleger 2009



$out_A \in \{0,1\}$

$view_A$

$out_B \in \{0,1,\perp\}$

$view_B$

Weak Erasure Channel

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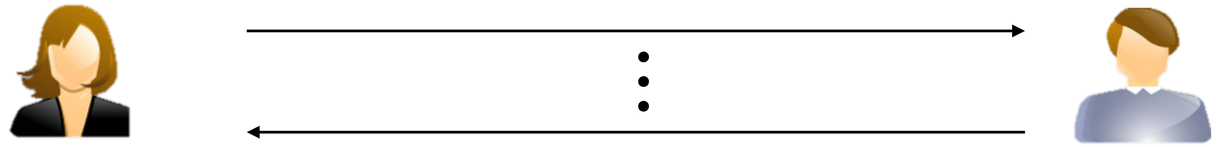

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w.p 1/2 (non-erasure) $out_B \neq \perp$

- **Agreement:** $\Pr[out_A = out_B] \geq 0.99$

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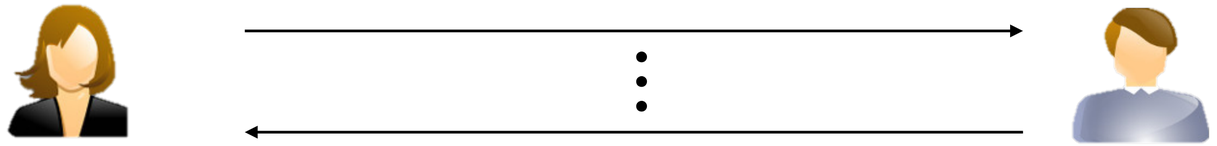

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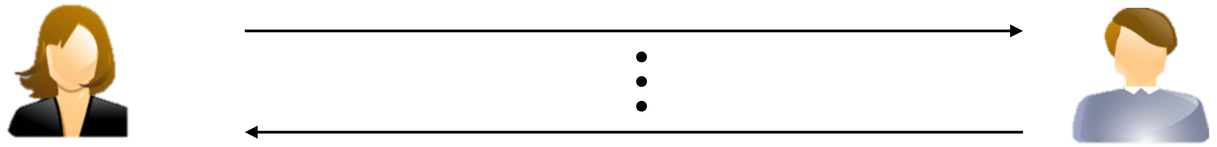
- **Secrecy: (If $out_B = \perp$, Bob doesn't know what the value of out_A)**

  $\forall$ **PPT** D, $\Pr[D(view_B) = out_A] \leq 0.01$

Weak Erasure Channel

Wullschleger 2009



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  $\forall$ **PPT** D, $\Pr[D(view_B) = out_A] \leq 0.01$

- **(Alice doesn't know if $out_B = \perp$)**

  $\forall$ **PPT** D, $|\Pr[D(view_A) = 1 \,|\, out_B \neq \perp] - \Pr[D(view_A) = 1 \,|\, out_B = \perp]| \leq 0.01$

Weak Erasure Channel

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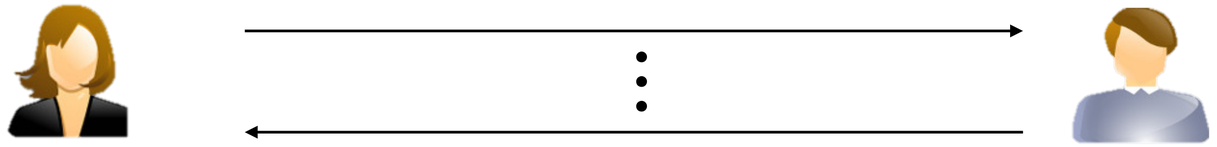

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WEC ➡ **OT**

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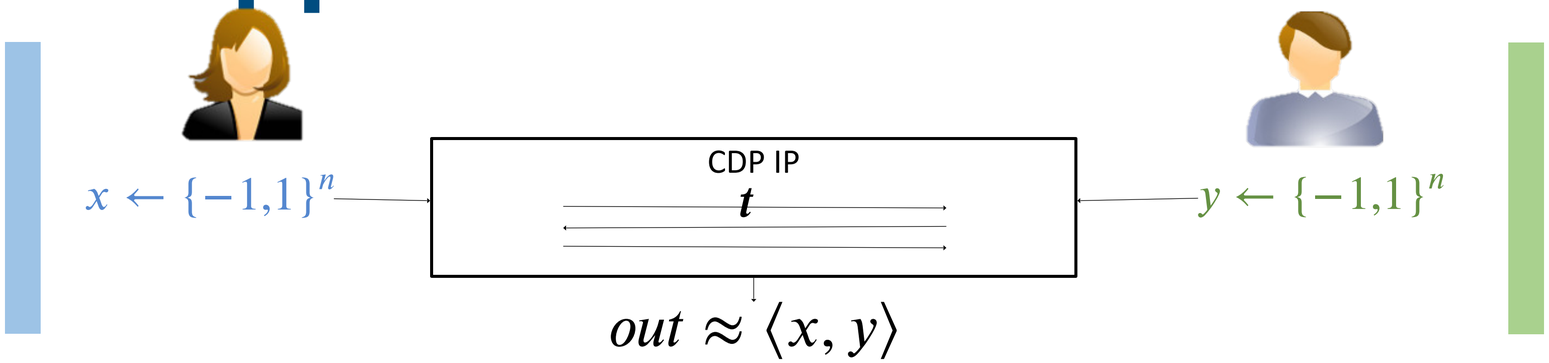
  $\forall$ **PPT** D, $|\Pr[D(view_A) = 1 \,|\, out_B \neq \perp] - \Pr[D(view_A) = 1 \,|\, out_B = \perp]| \leq 0.01$

CDP IP to Approximate Weak Erasure Channel



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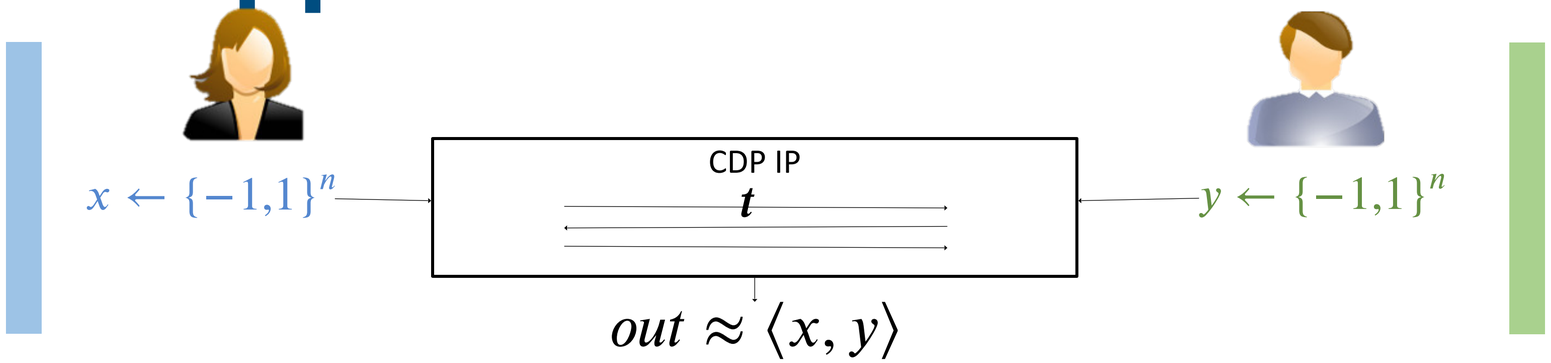
CDP IP to Approximate Weak Erasure Channel



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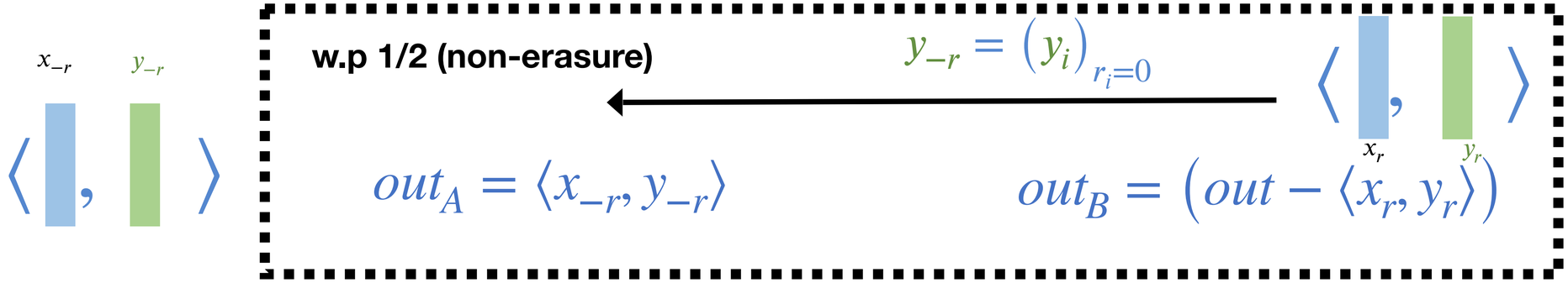

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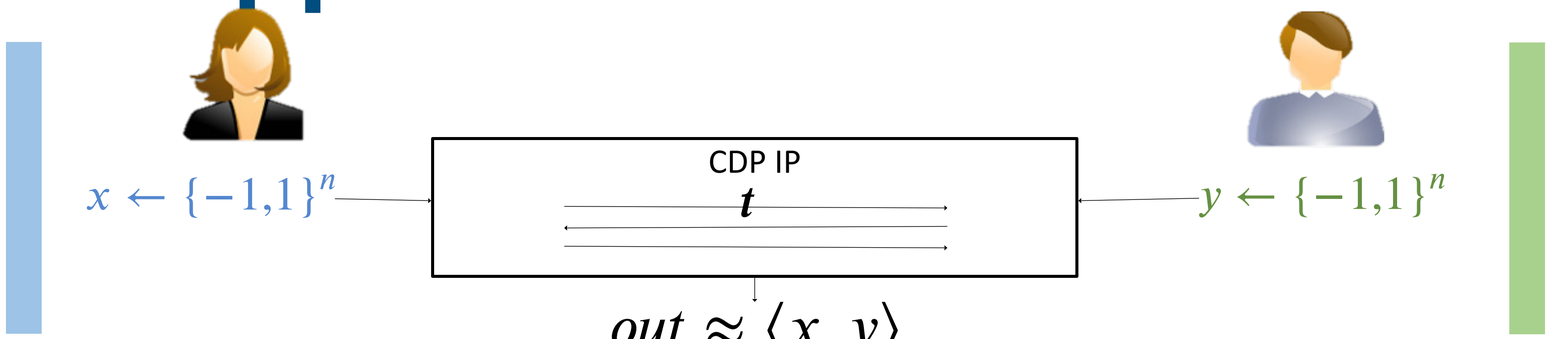

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x_{-r} y_{-r}

$\langle \quad , \quad \rangle$

w.p 1/2 (non-erasure)

$y_{-r} = (y_i)_{r_i=0}$

$\langle \quad , \quad \rangle$ (x_r, y_r)

$out_A = \langle x_{-r}, y_{-r} \rangle$

$out_B = \left(out - \langle x_r, y_r \rangle\right)$

w.p 1/2 (erasure)

$i_1, \ldots, i_k \leftarrow [n]$

$\tilde{y}_i \sim \{-1,1\}$ **for** $i \in \{i_1, \ldots, i_k\}$

$\tilde{y}_{-r} = (\tilde{y}_i)_{r_i=0}$

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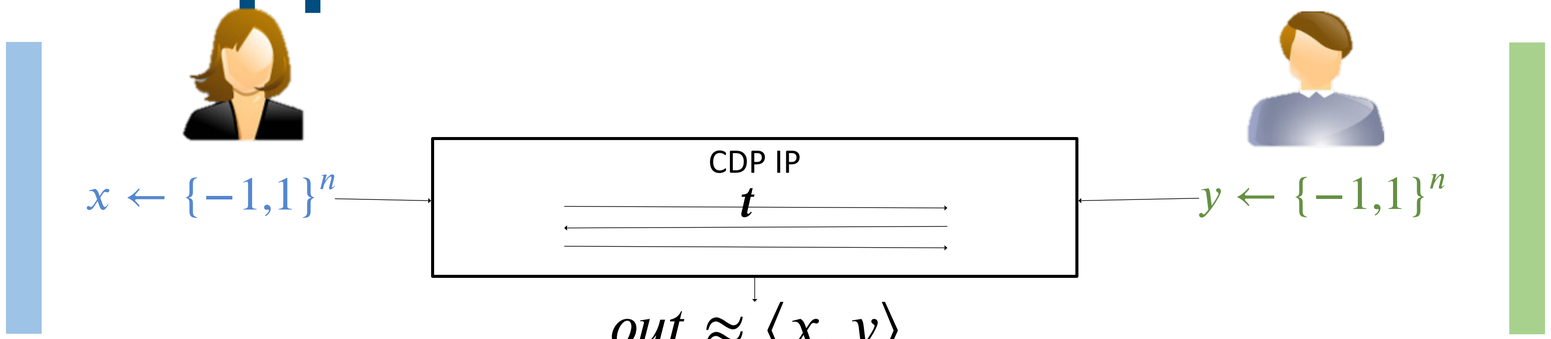
CDP IP to Approximate Weak Erasure Channel



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- **Alice doesn't know if** $out_B = \perp$
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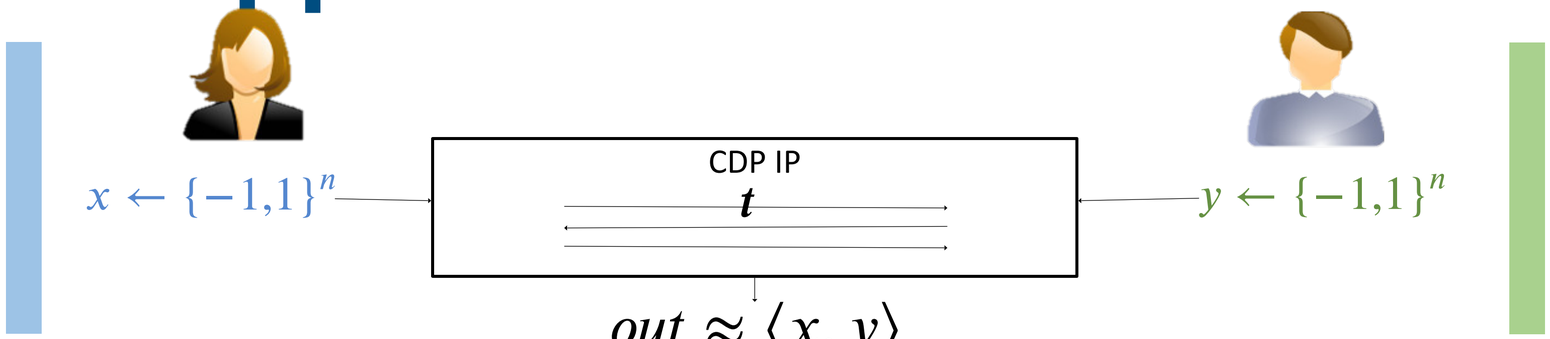

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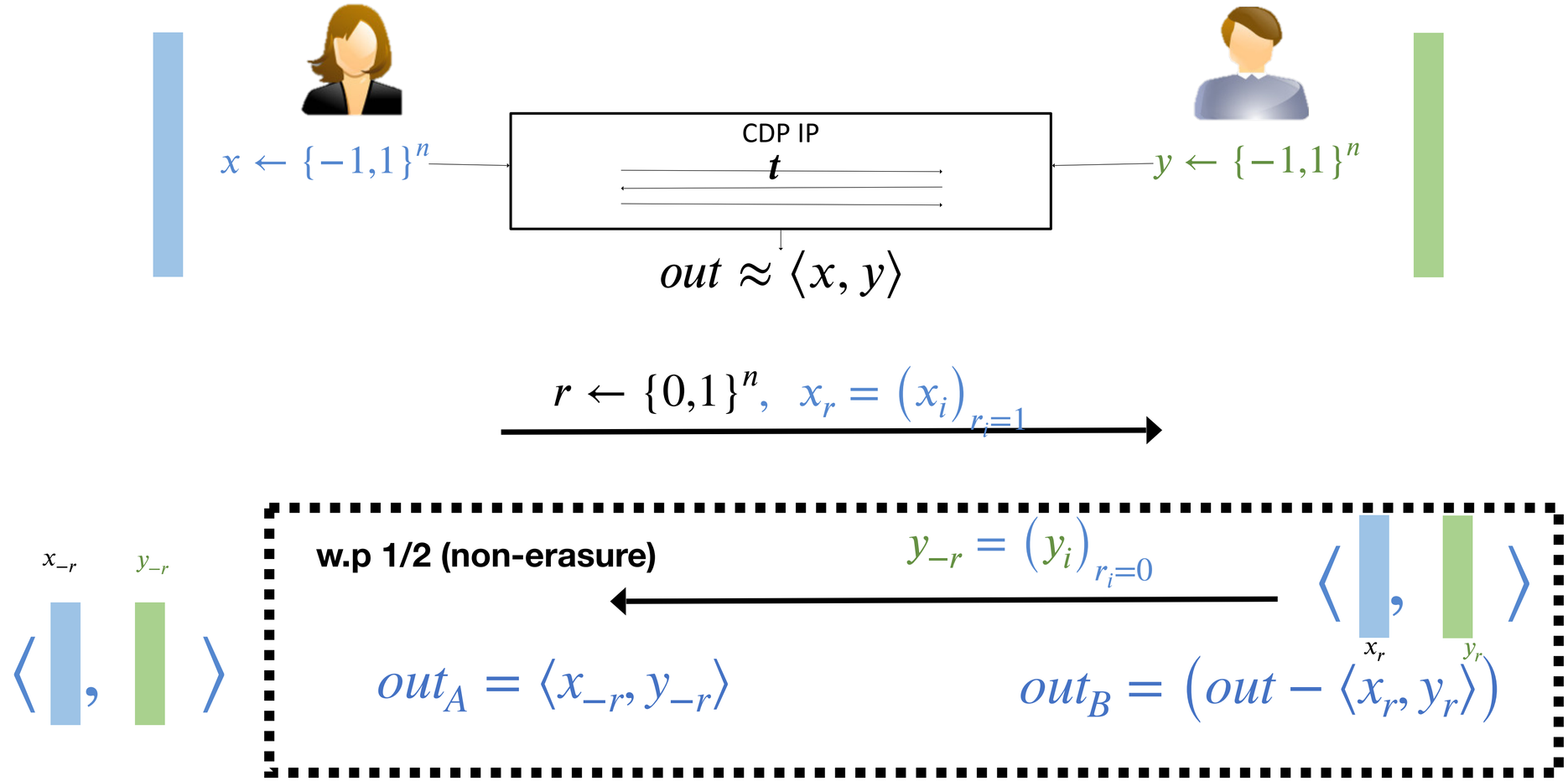
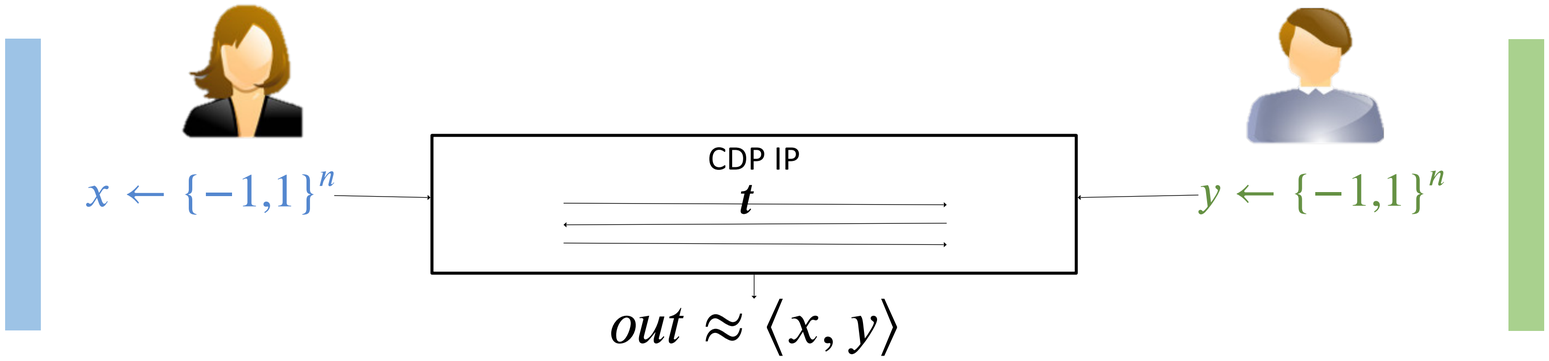
- **AWEC can be amplified to WEC**



- Agreement:

$Out \approx \langle X, Y \rangle \implies Out_A \approx Out_B$



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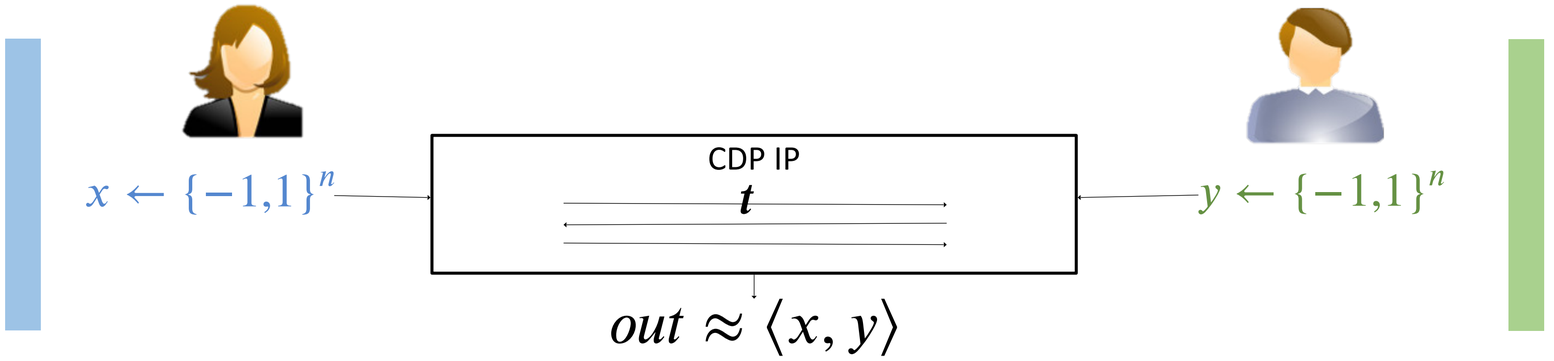
  HMST22: ∀ PPT Eve, Eve(T, R, X_R, Y_{-R}) is far from out_A.



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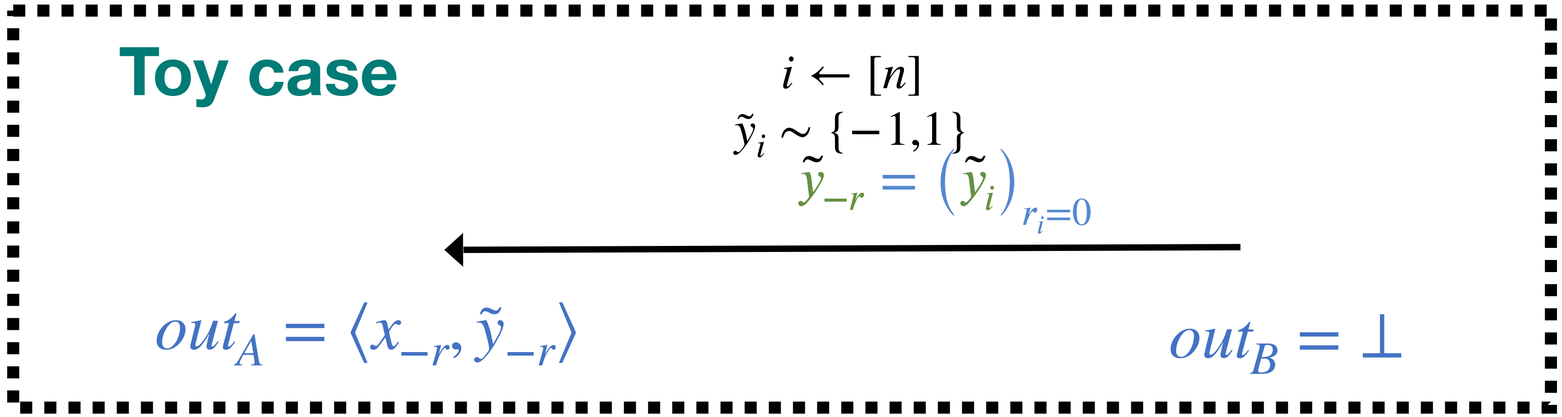
  ➢ If $out_B = \perp$, Bob doesn't know what the value of out_A

Secrecy of Bob



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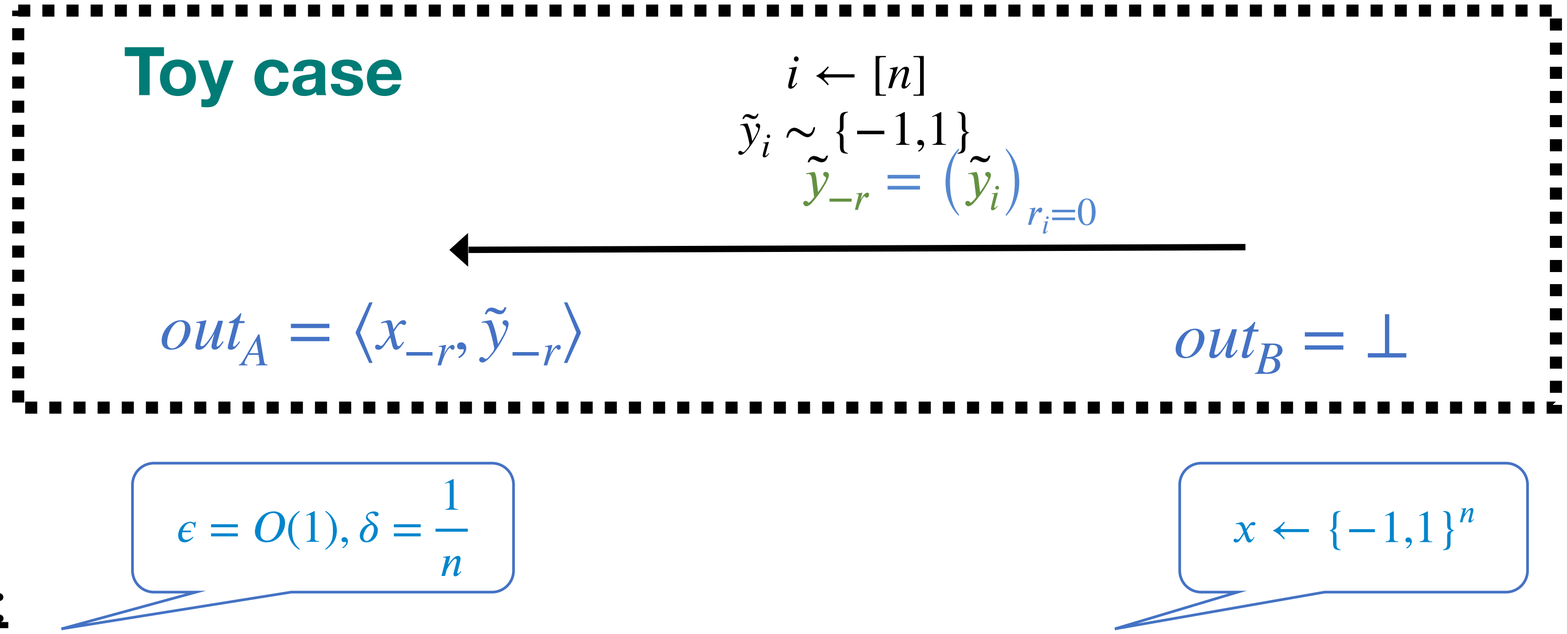

Key observation:

Given an $(\epsilon,\ \delta)$-DP mechanism M, and database $x = (x_1, \cdots,\ x_n)$ that has high min-entropy.

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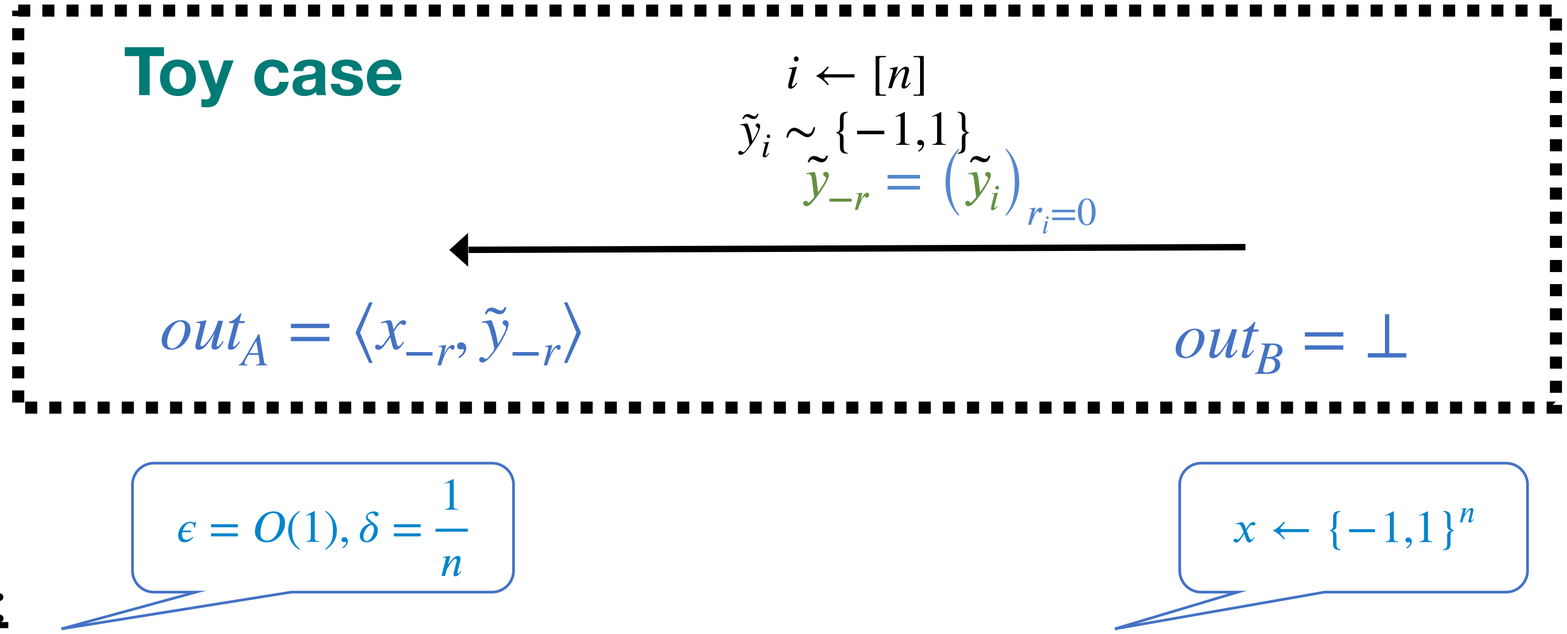

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- Let $i \leftarrow [n],$ and let $x^i = (x_1,\ \cdots, -x_i, \cdots,\ x_n)$.
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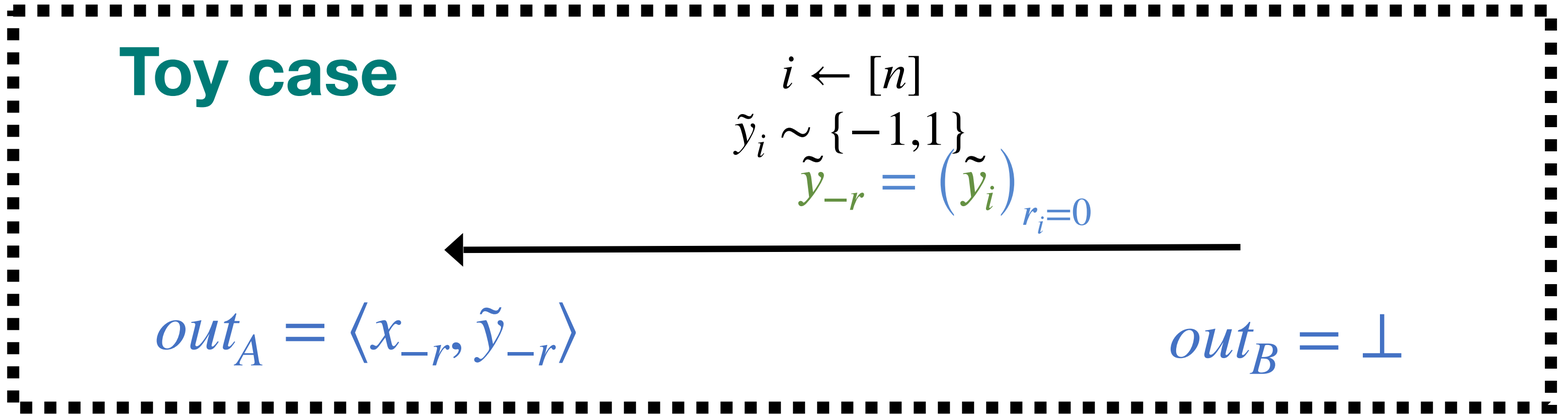

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$|\Pr[A(x_r, r, \mathrm{M}(x))=1] - \Pr[A(x_r^i, r, \mathrm{M}(x))]| > 1/n^{\frac{1}{3}}$

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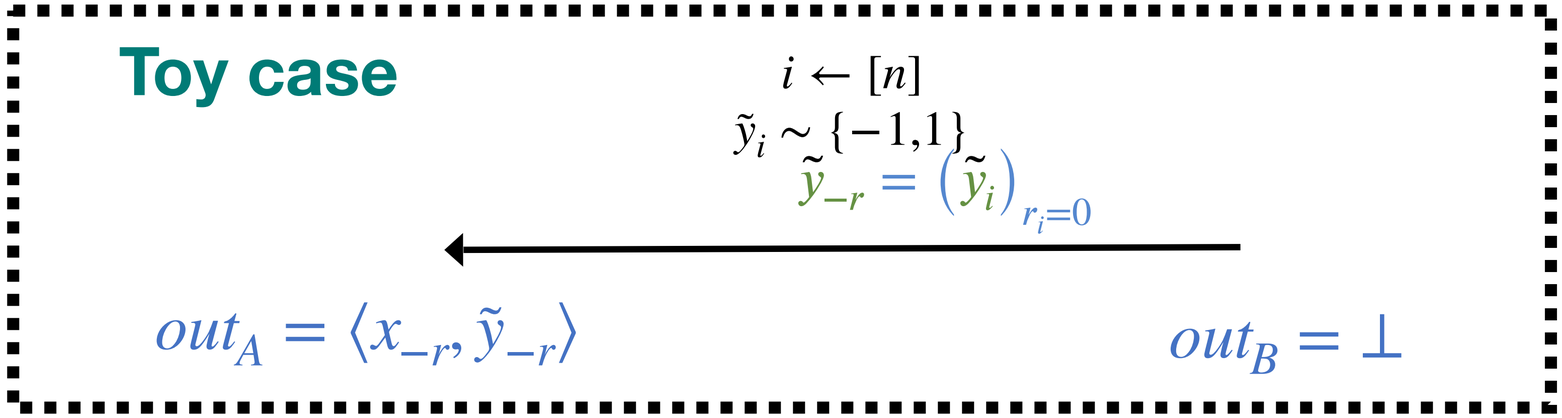

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Secrecy of Bob

Toy case

$i \leftarrow [n]$

$\tilde{y}_i \sim \{-1,1\}$

$\tilde{y}_{-r} = (\tilde{y}_i)_{r_i=0}$

$out_A = \langle x_{-r}, \tilde{y}_{-r} \rangle$

$out_B = \perp$

$\epsilon = O(1), \delta = \frac{1}{n}$

$x \leftarrow \{-1,1\}^n$

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Given an $(\epsilon,\ \delta)$-DP mechanism M, and database $x = (x_1, \cdots,\ x_n)$ that has high min-entropy.

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$\Rightarrow$ A can be used to contradict the $(\epsilon,\ \delta)$-DP of M.

A constructive proof:
works also for CDP

Secrecy of Bob

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can be extended to $k = O(n^{1/3})$ **random indexes**

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$\Rightarrow$**Alice doesn't know if** $out_B = \perp$

Conclusions & Open Problems

Mildly accurate CDP-IP $\Rightarrow$ OT

<u>Open Questions:</u>

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Thank You