$\omega(1/\lambda)$-Rate Boolean Garbling Scheme from Generic Groups

CRYPTO 2025

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JHU

Naman Kumar
Oregon State University

Garbled Circuits

[Yao’86]

Garbled Circuits

[Yao’86]

Garbler

Evaluator

Boolean Circuit C

Garbled Circuits

[Yao’86]

Garbler

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Boolean Circuit C $\xrightarrow{\text{Garble}}$ Garbled Circuit $\widehat{C}$

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Input x

Garbled Circuits

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Input x $\xrightarrow{\text{Encode}}$ Encoded input $\widehat{x}$

Garbled Circuits

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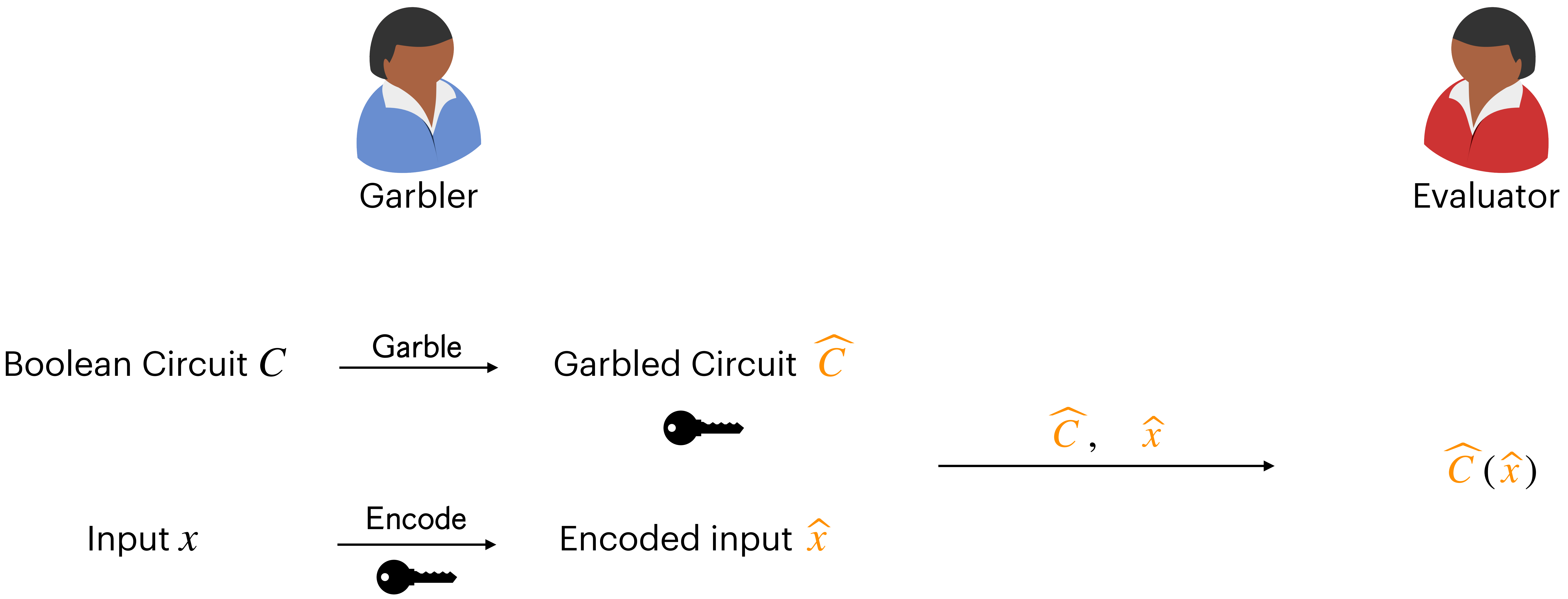

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$\xrightarrow{\widehat{C},\ \hat{x}}$

$\widehat{C}(\hat{x})$

Garbled Circuits

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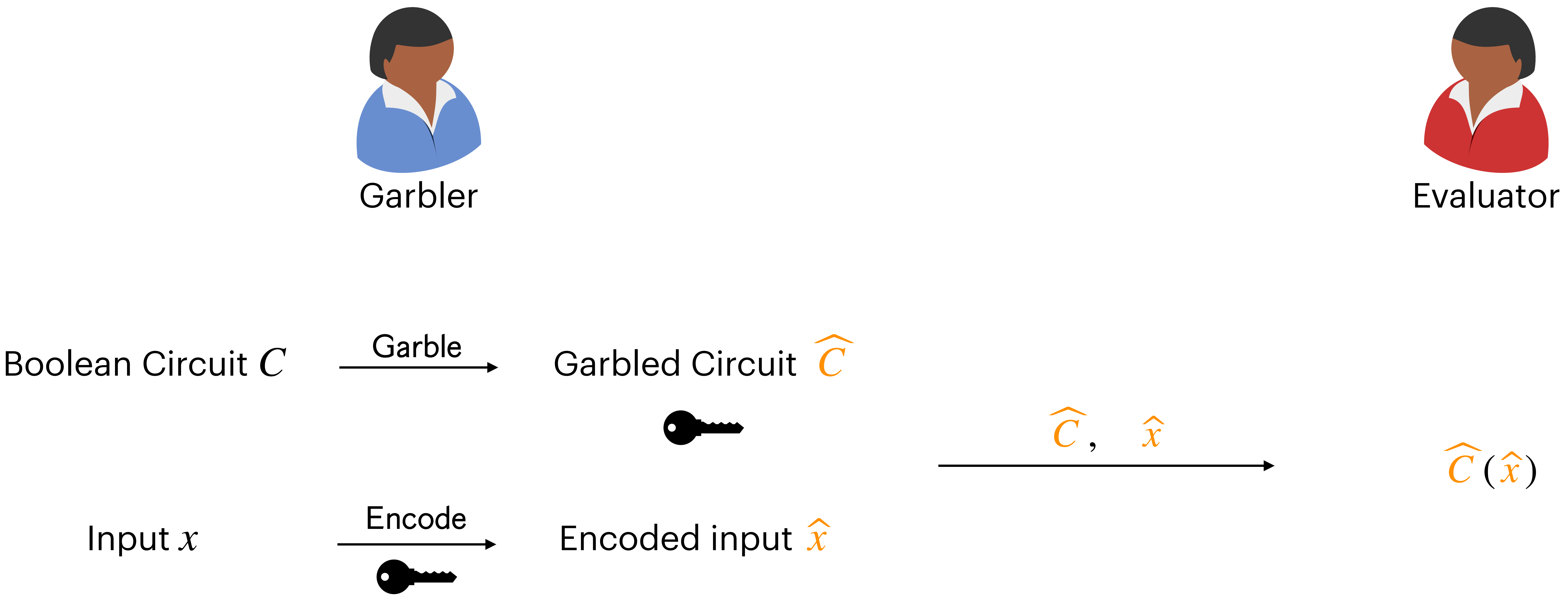

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🔑

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Correctness: $C(x) = \widehat{C}(\hat{x})$

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Privacy: $(\widehat{C}, \hat{x}) \leftarrow \text{Simulator}(C, C(x))$

Garbled Circuits

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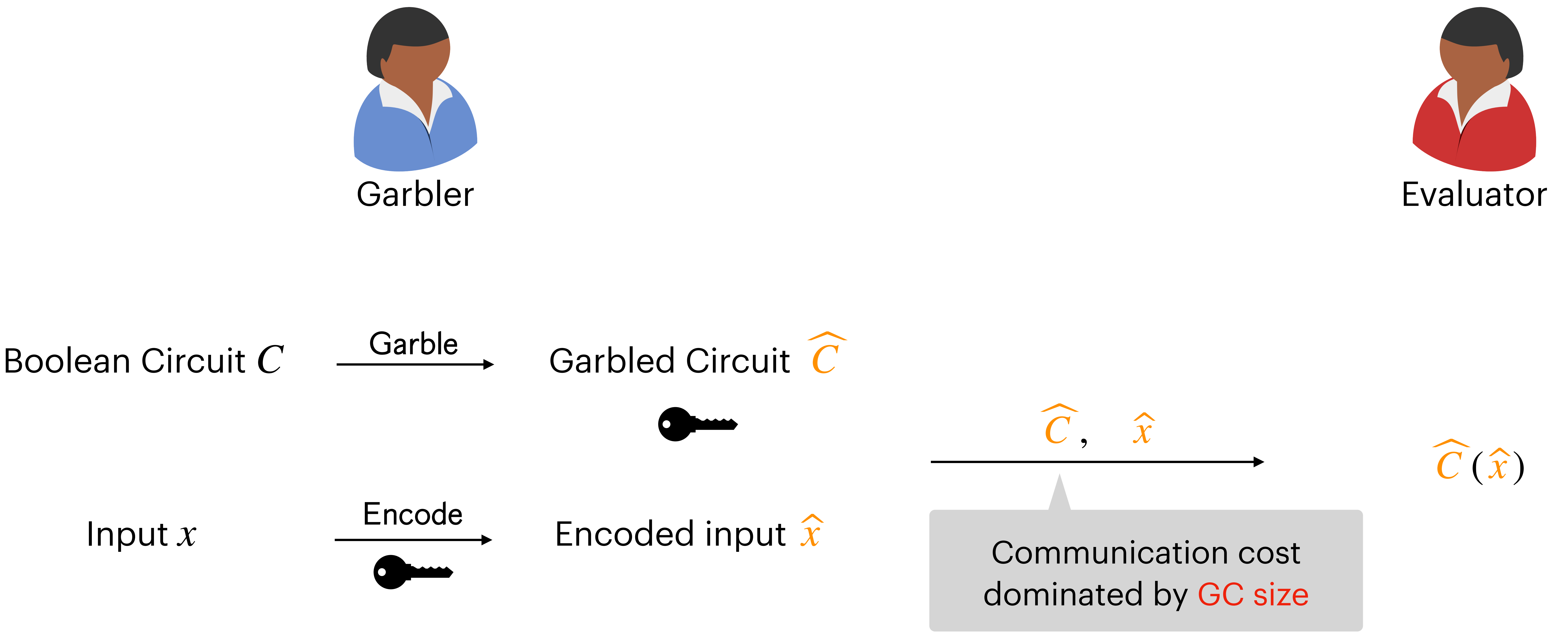

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Communication cost dominated by GC size

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Public circuit $\Longrightarrow$ $\widehat{C}$ can be smaller than C

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Rate: $\dfrac{\text{Number of gates in } C}{\text{Size of } \widehat{C} \text{ in bits}}$

Higher rate $\Longrightarrow$ smaller garbled circuits

Landscape of Garbling Schemes

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Rate-$\Omega(1/\lambda)$

Symmetric-key crypto   Black-box use of crypto

[Yao'86] [Lindell-Pinkas'07] [Koleshnikov-Schneider'08] [Zahur-Rosulek-Evans'15] [Rosulek-Roy'21]

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Rate-$\omega(1)$

ABE + FHE / iO + OWF Non-black-box use of crypto

[Boneh-Gentry-Grobunov-Halevi-Nikolaenko-Segev-Vaikuntanathan-Vinayagamurthy'14] [Lin-Pass'14]
[Goldwasser-Kalai-Popa-Vaikuntanathan-Zeldovich'13] [Koppula-Lewko-Waters'15] [Hsieh-Lin-Luo'23]

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Rate-1

Ring LWE / NTRU   Non-black-box use of crypto

[Liu-Wang-Yang-Yu'24]

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Can we get $\omega(1/\lambda)$-rate garbling while making black-box use of crypto?

Rate-1

Ring LWE / NTRU Non-black-box use of crypto

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Our Results

In the **Generic Group Model**, there exists a garbling scheme that garbles a boolean circuit C into $\widehat{C}$ such that

$$|\widehat{C}| = \frac{\lambda}{\sqrt{\log \lambda}} \cdot O(C) + \mathrm{poly}(\lambda)$$

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Assuming **Power-DDH** and a **Tweakable Correlation Robust Hash** function, there exists a garbling scheme that garbles a layered boolean circuit C into $\widehat{C}$ such that

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$\omega(1/\lambda)$-rate and black-box use of crypto

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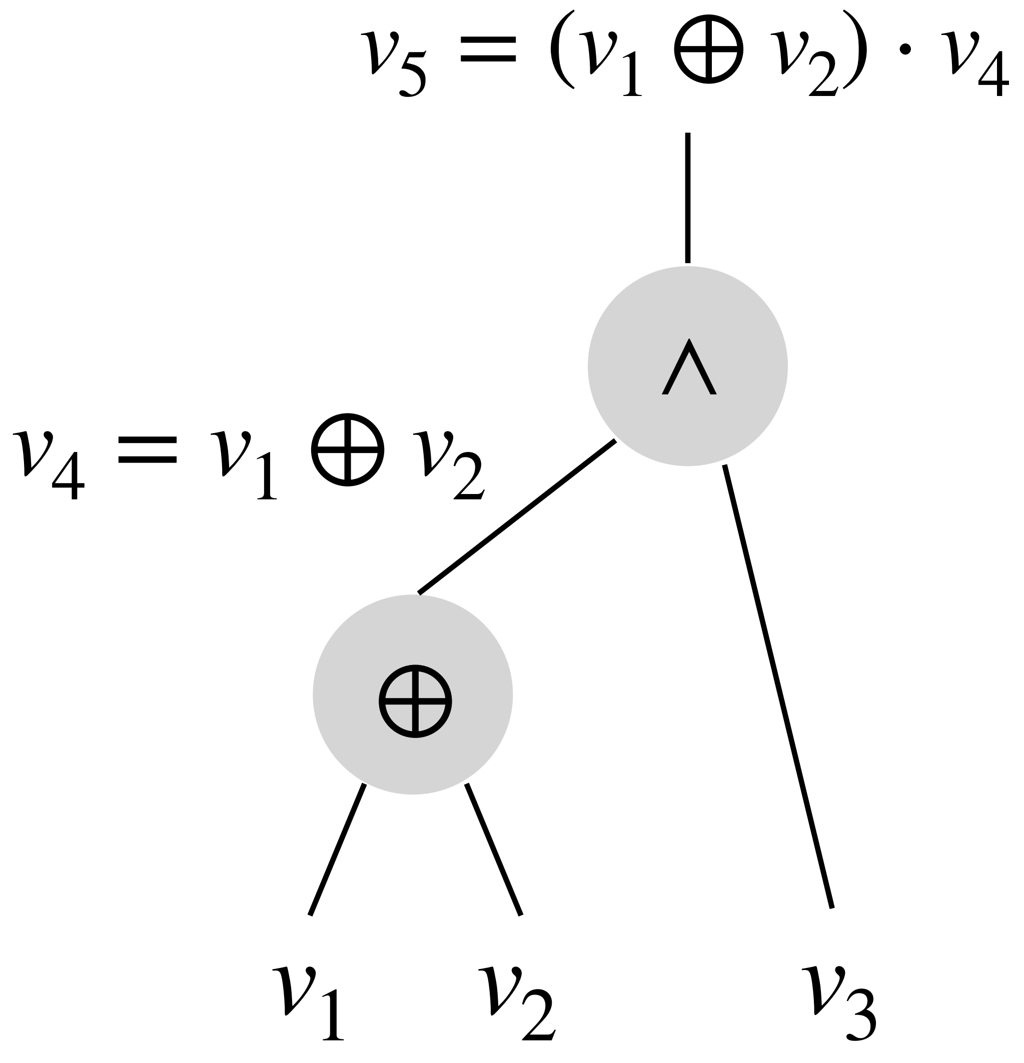
$\omega(1/\lambda)$-rate and black-box use of crypto

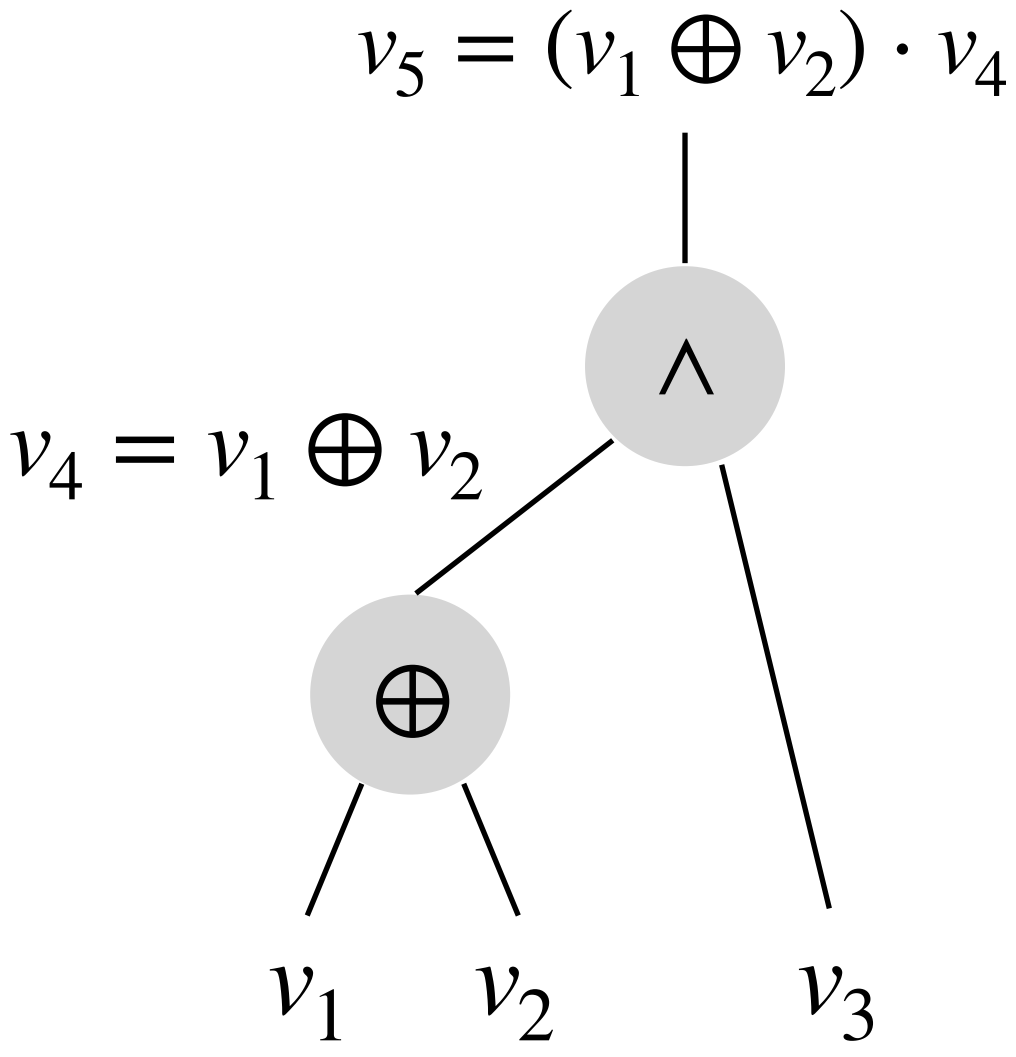
$o(\lambda)$-bits per gate scheme from assumptions not known to imply FHE

Template for Garbling Circuits



Boolean Circuit

Template for Garbling Circuits



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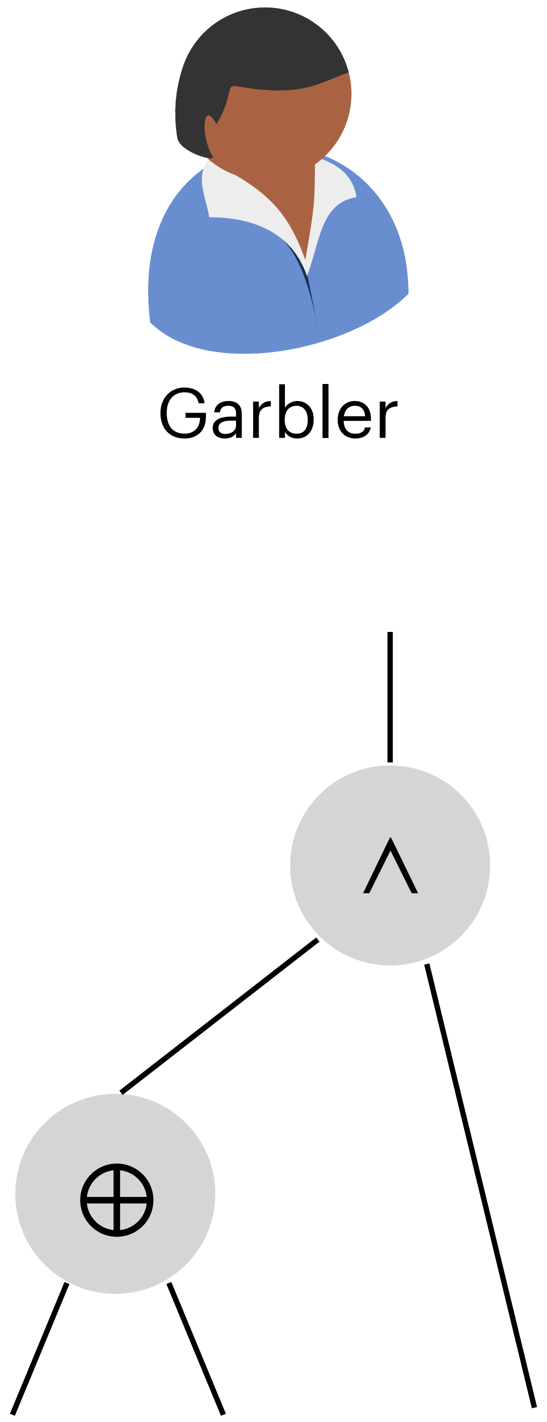
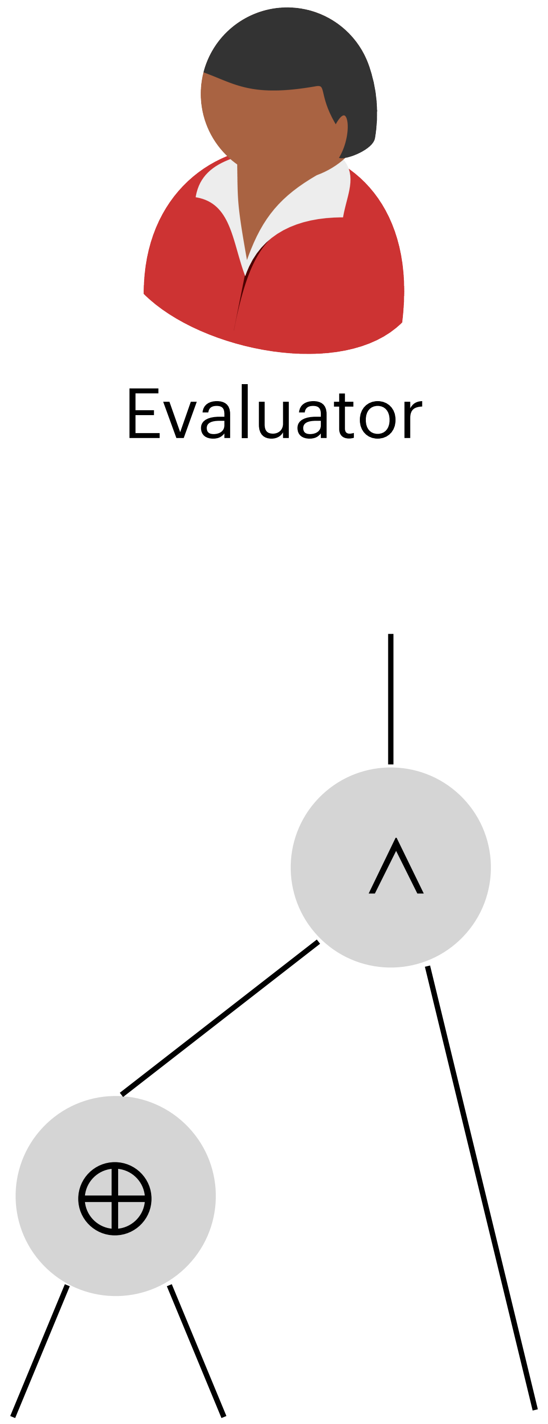
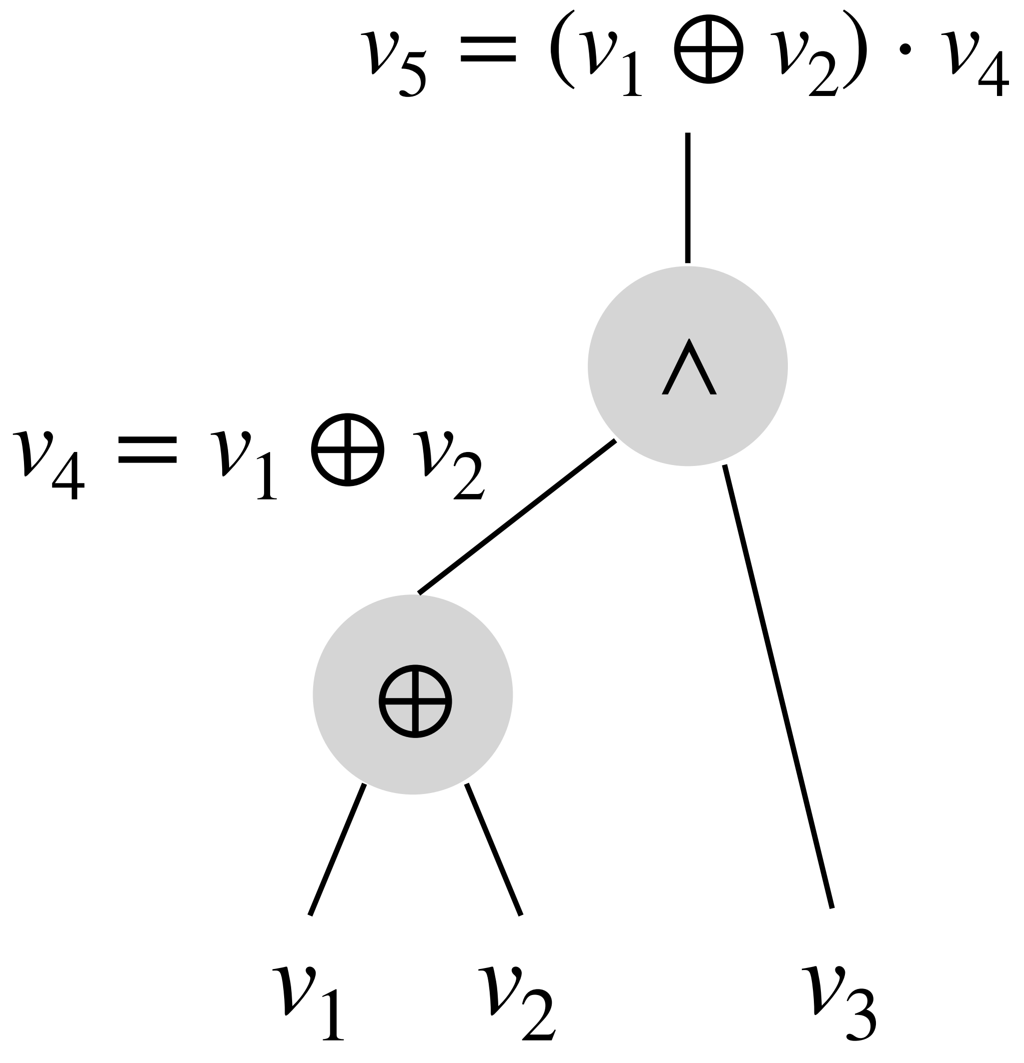


Garbler



Evaluator

Template for Garbling Circuits



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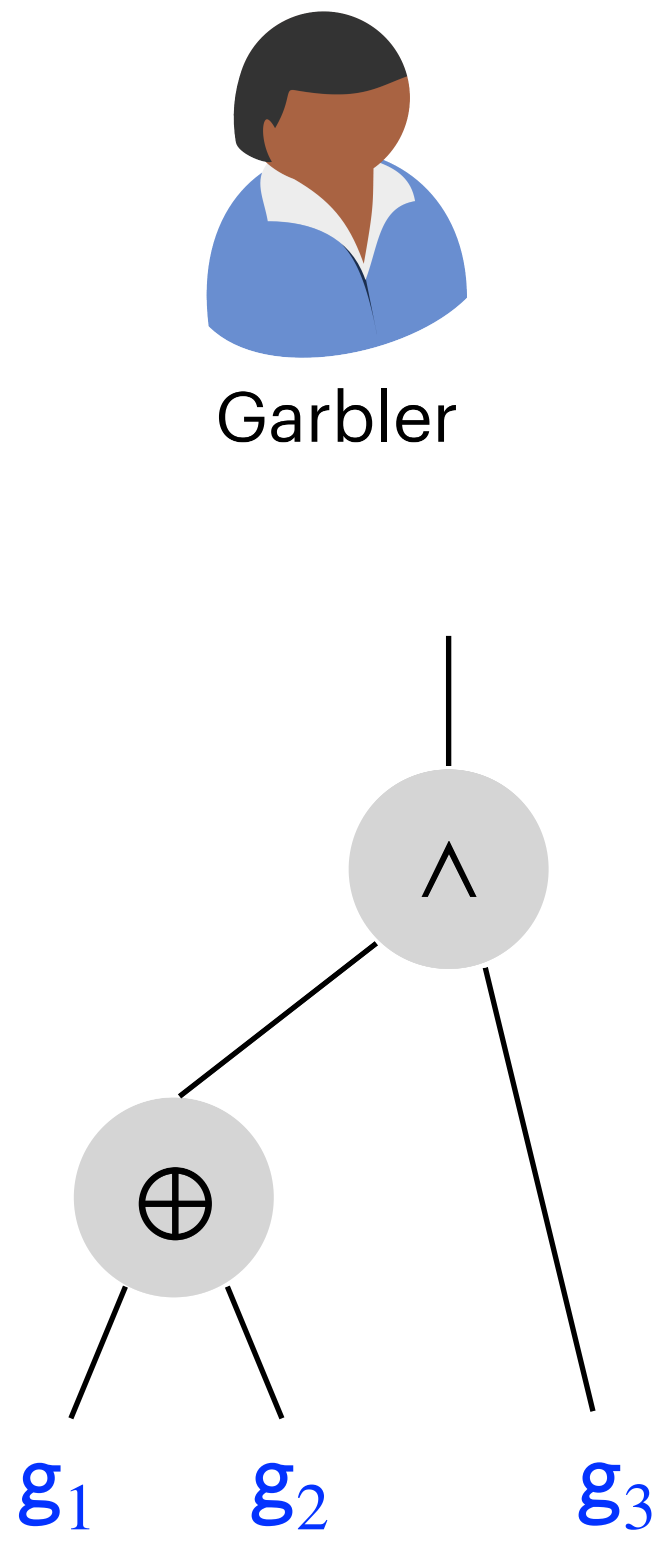
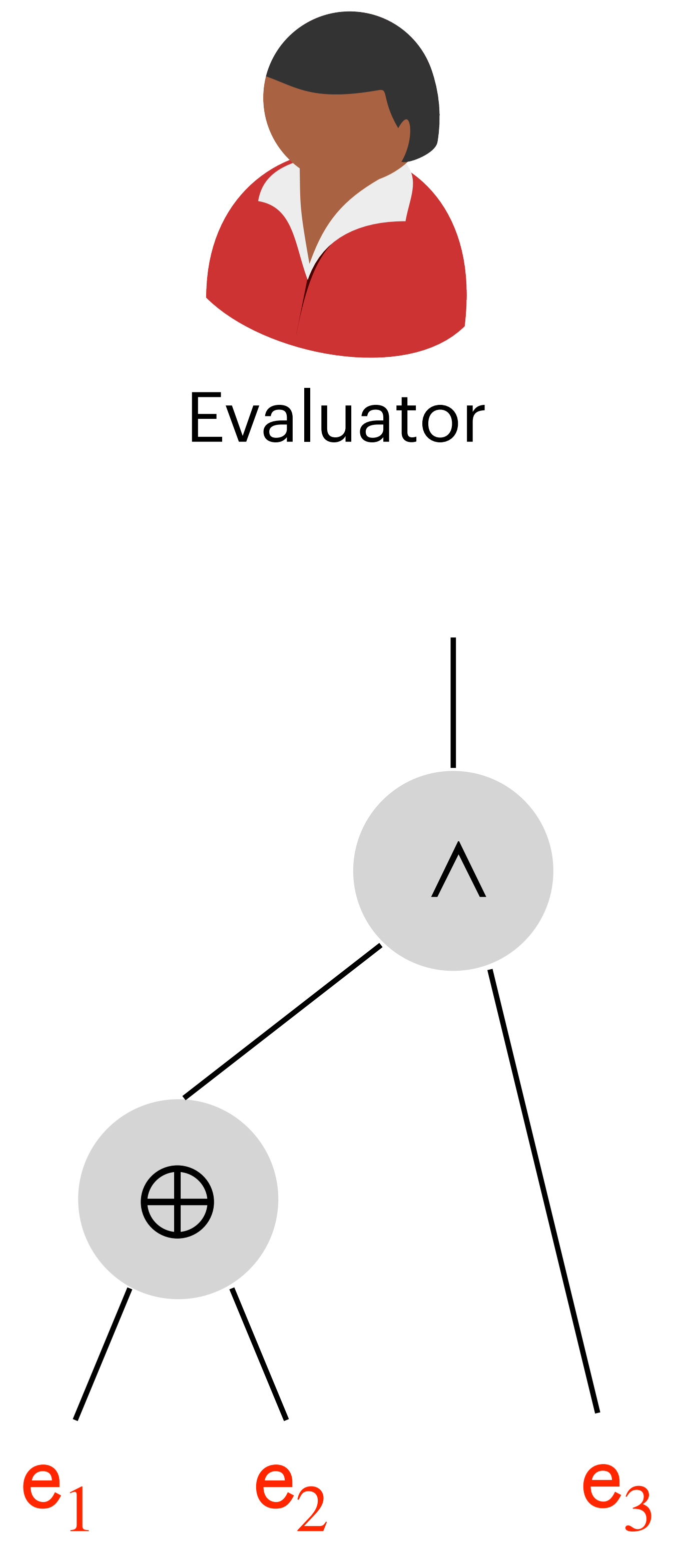


Invariant

$g_i \oplus e_i = v_i$



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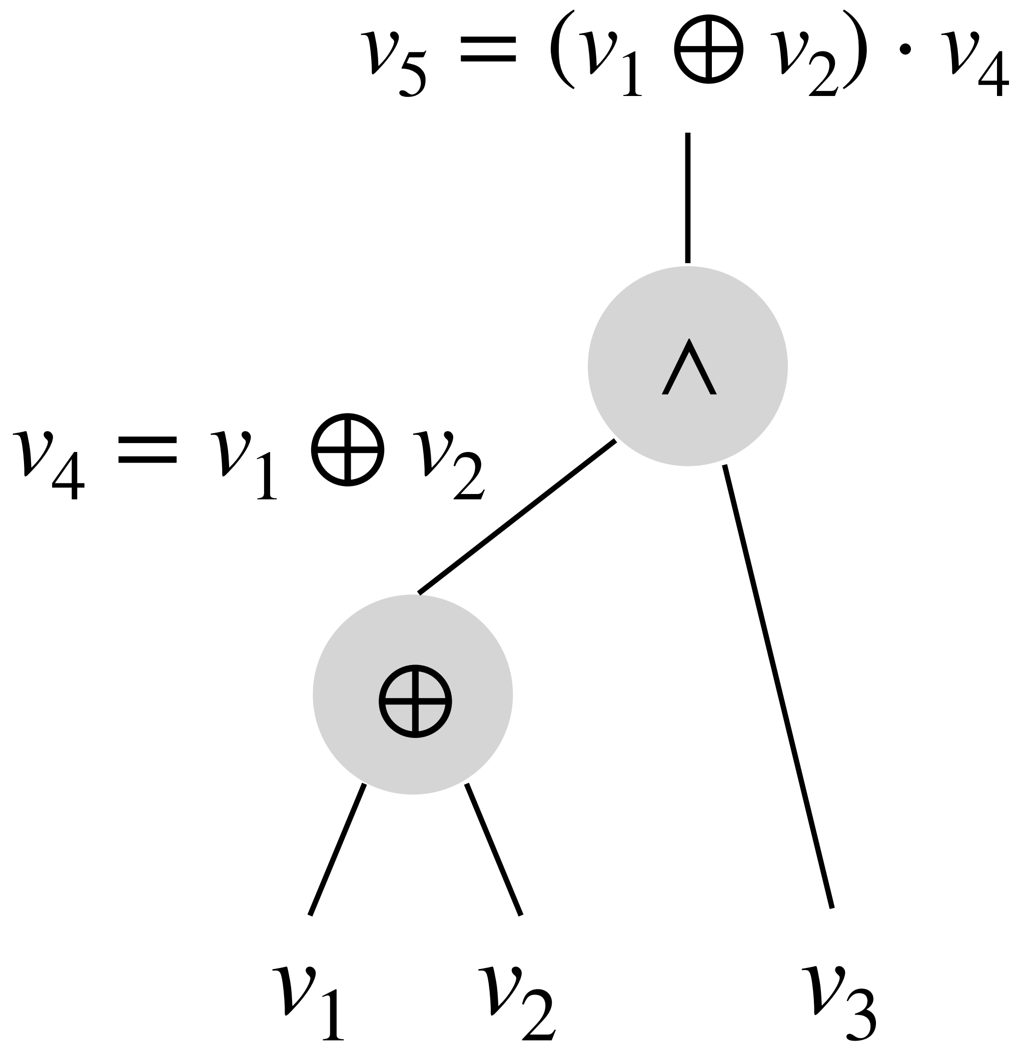

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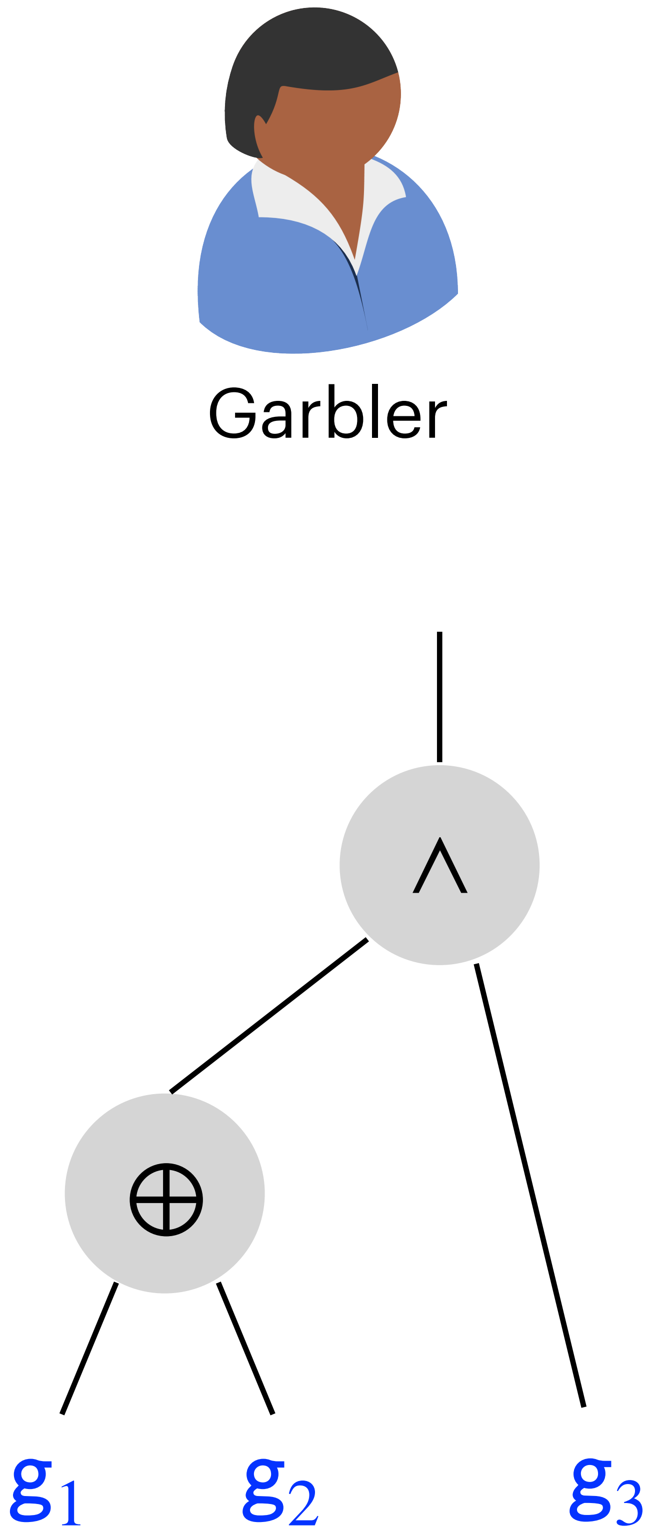

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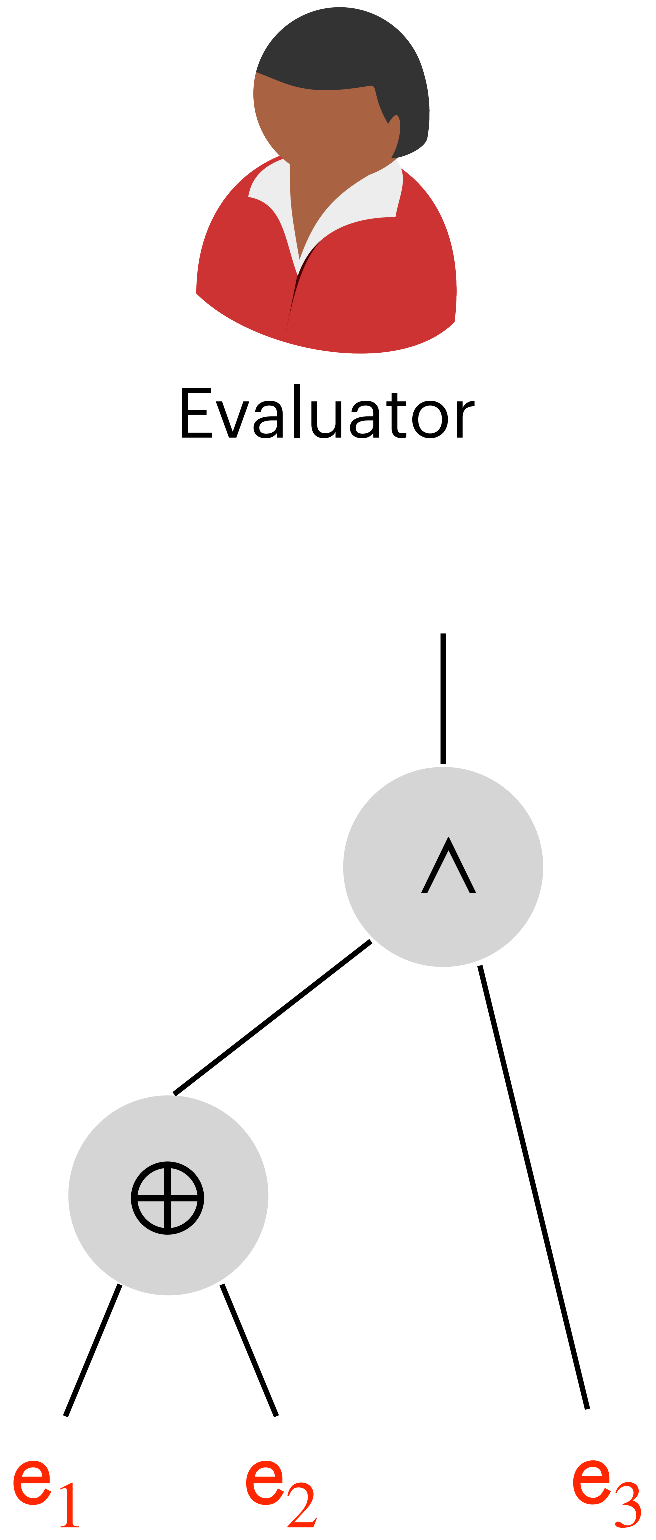
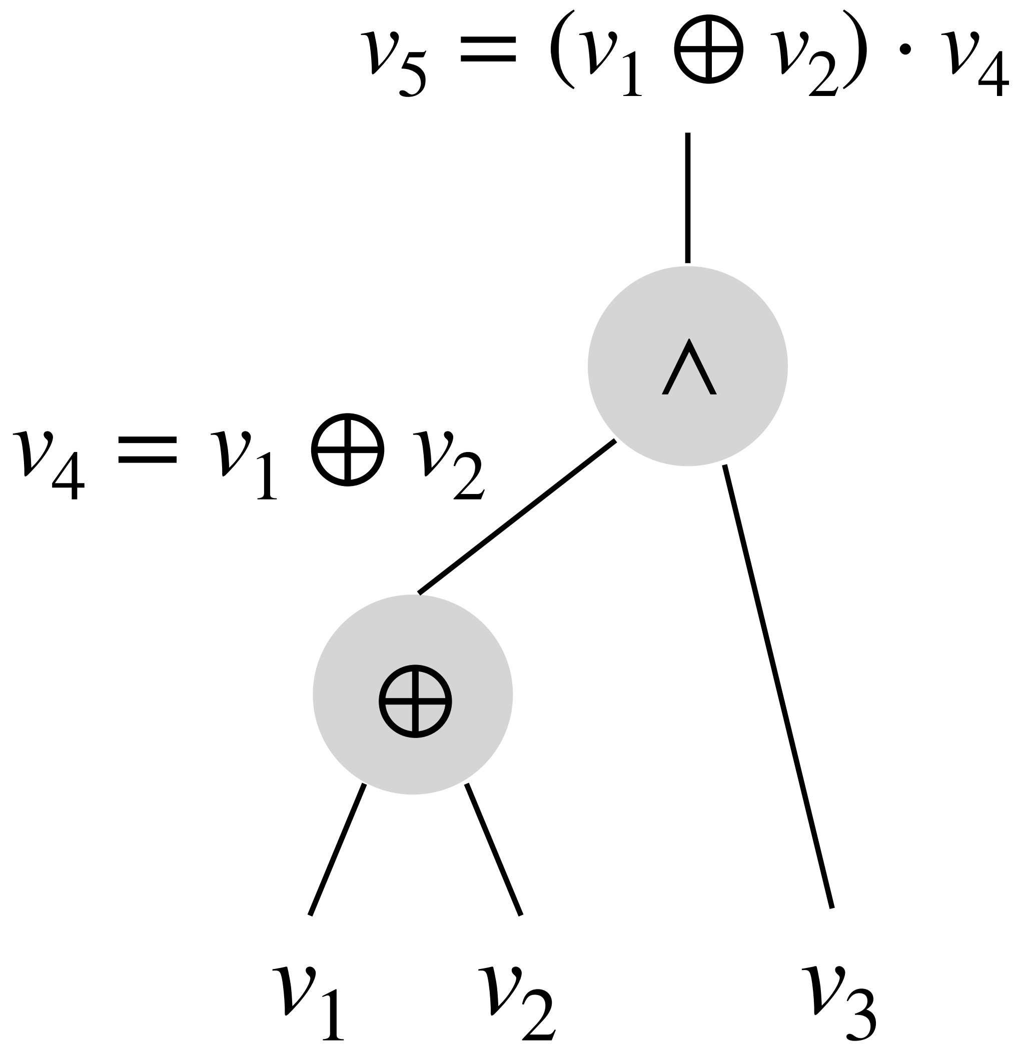


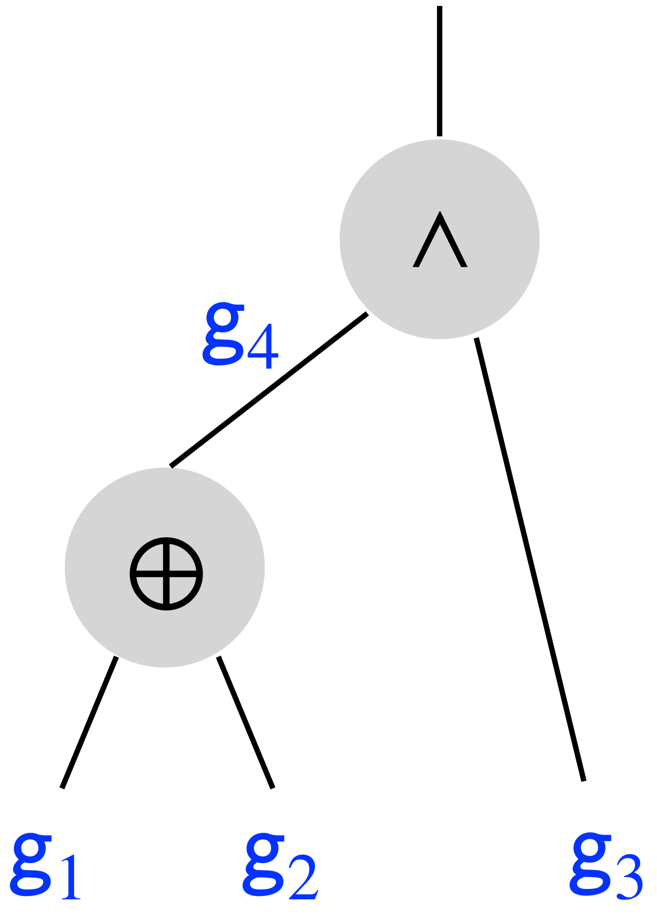
Garbled Circuit $\widehat{C}$

Template for Garbling Circuits



Boolean Circuit

Garbler



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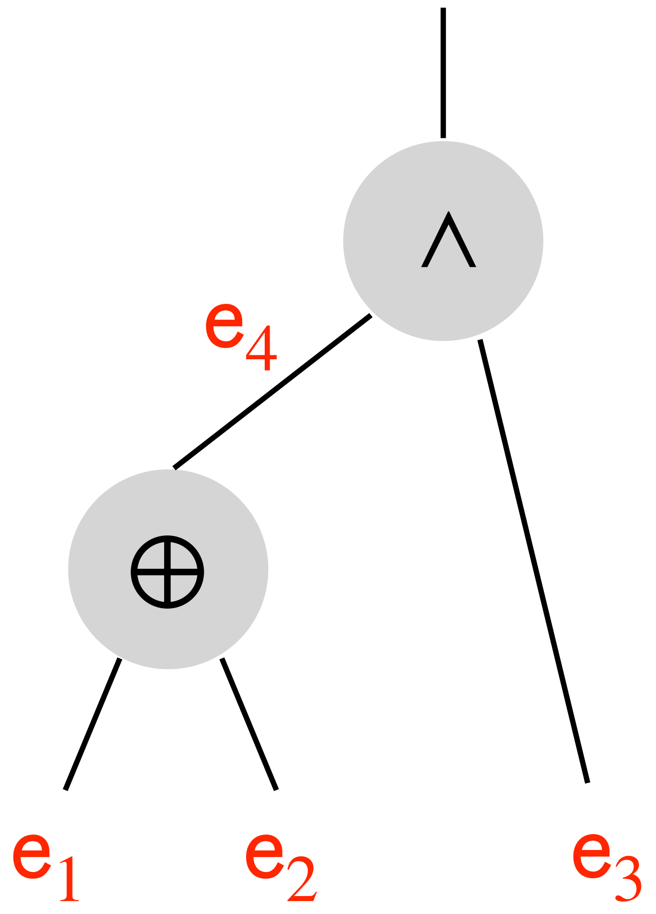
Evaluator



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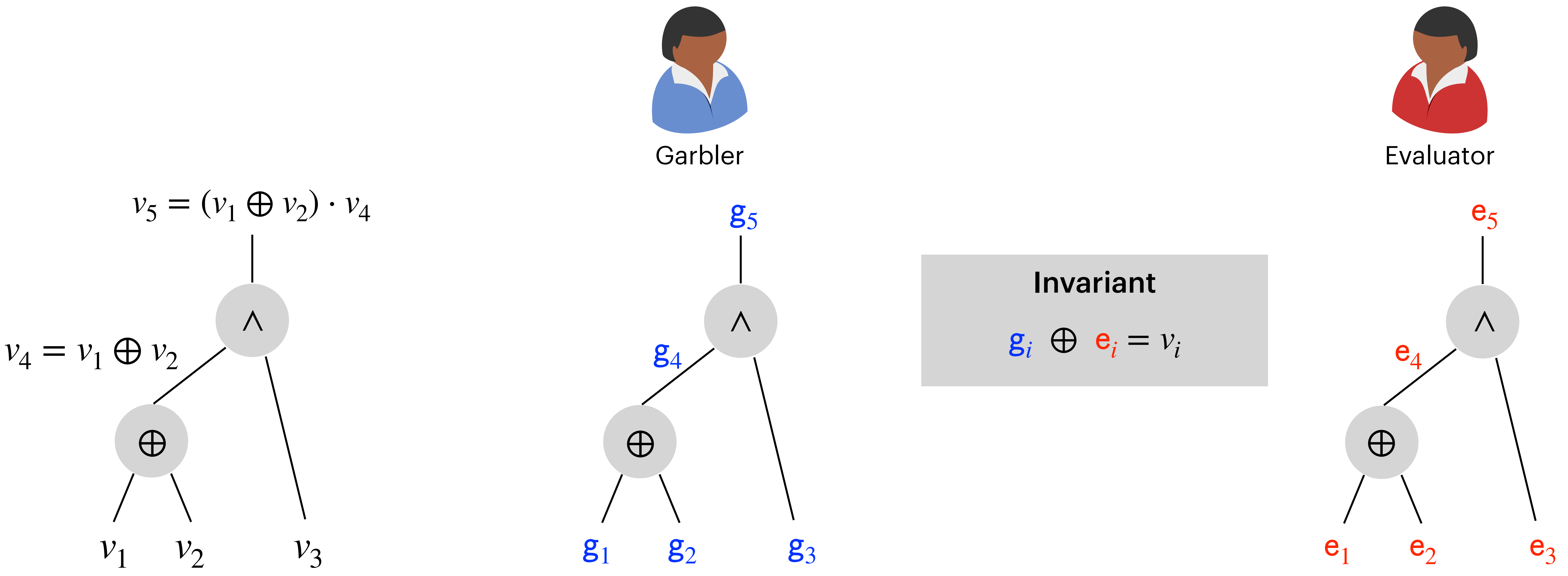

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Boolean Circuit

Garbled Circuit $\widehat{C}$

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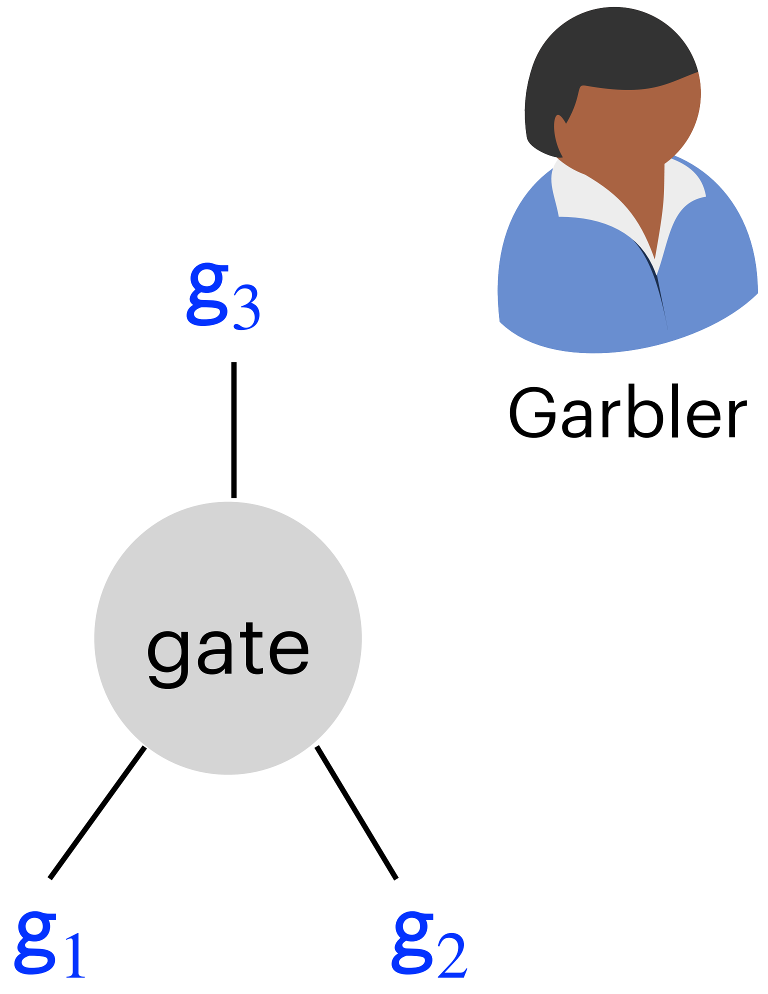

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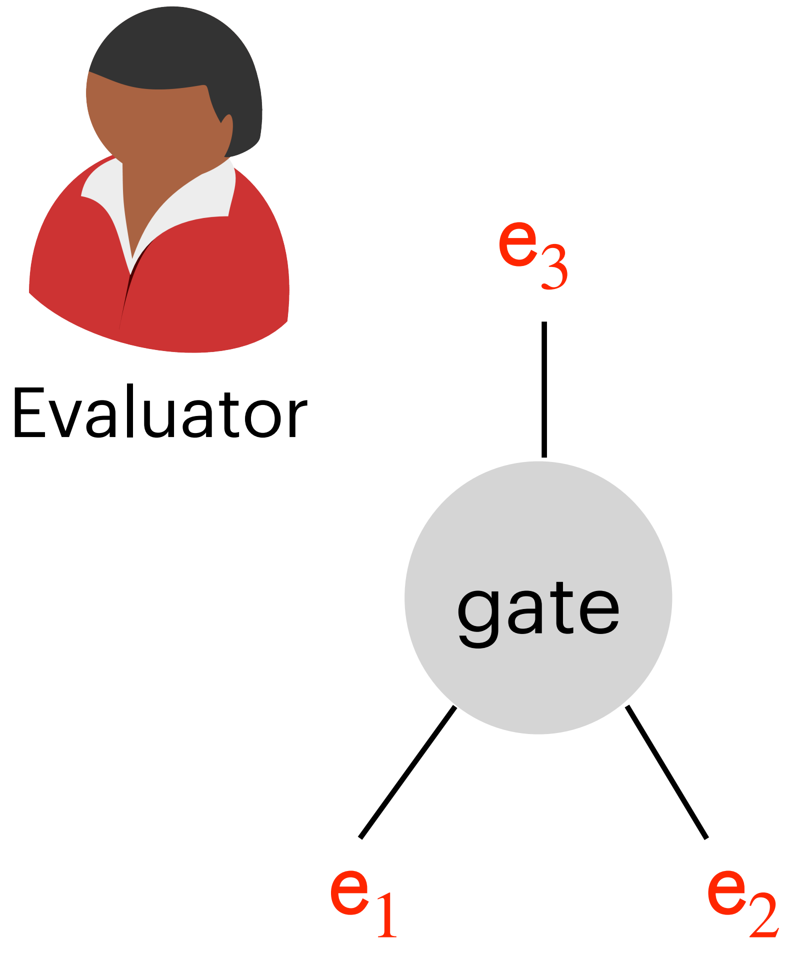

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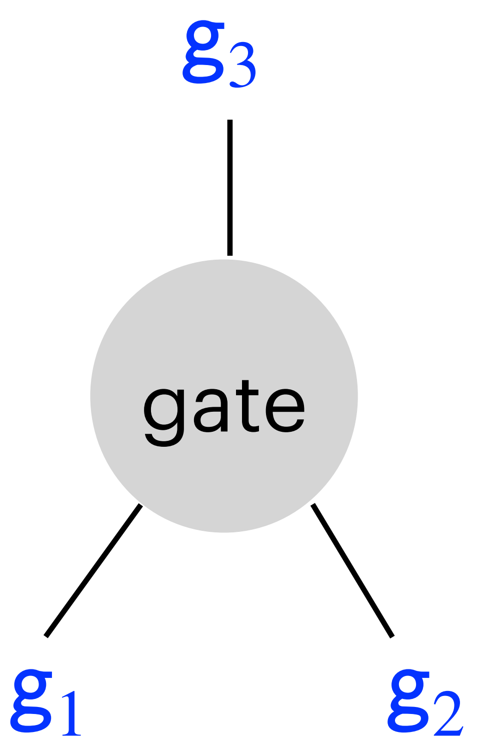

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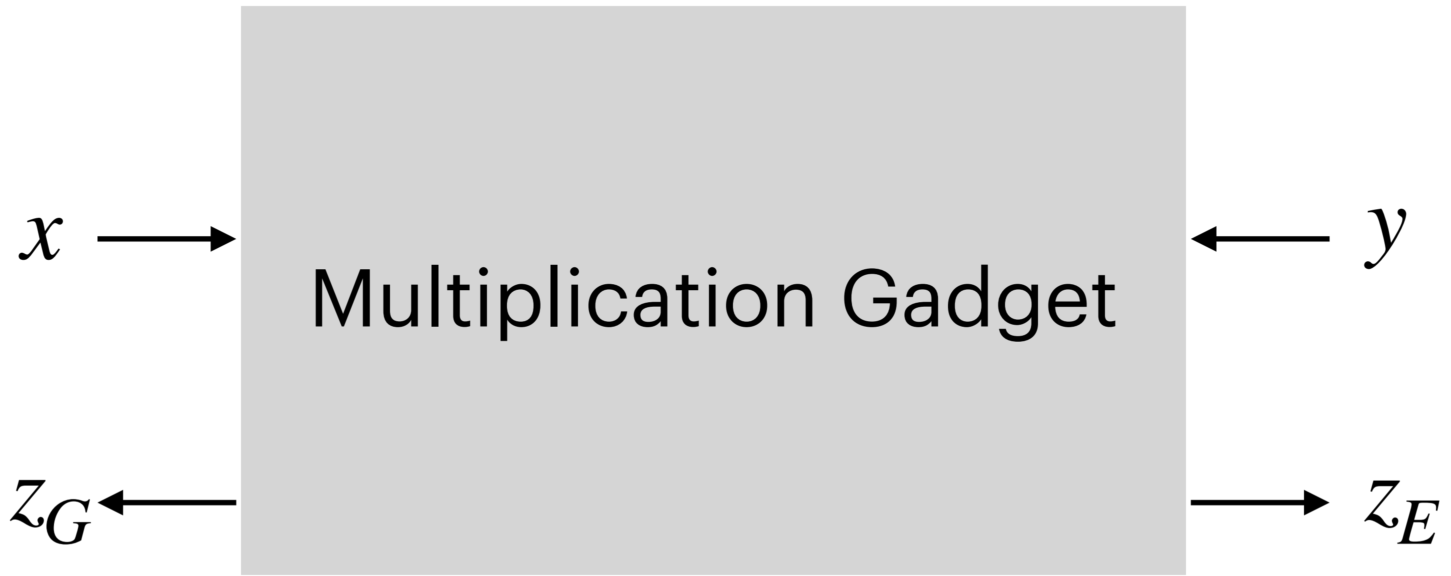
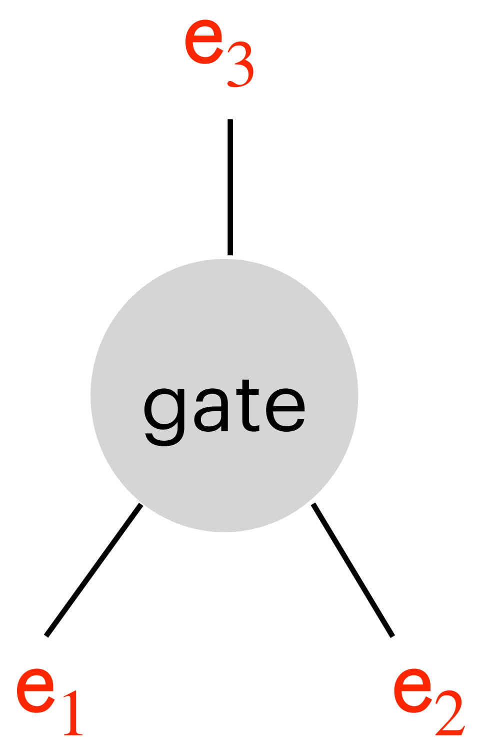


$$z_G \oplus z_E = xy$$

Evaluator



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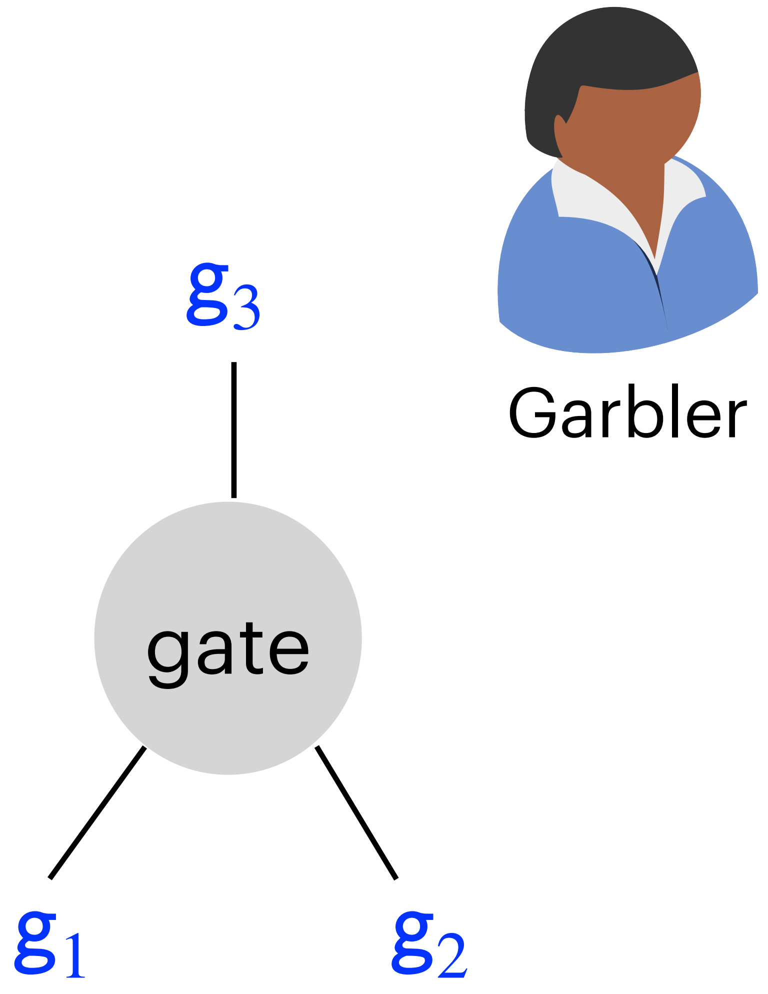

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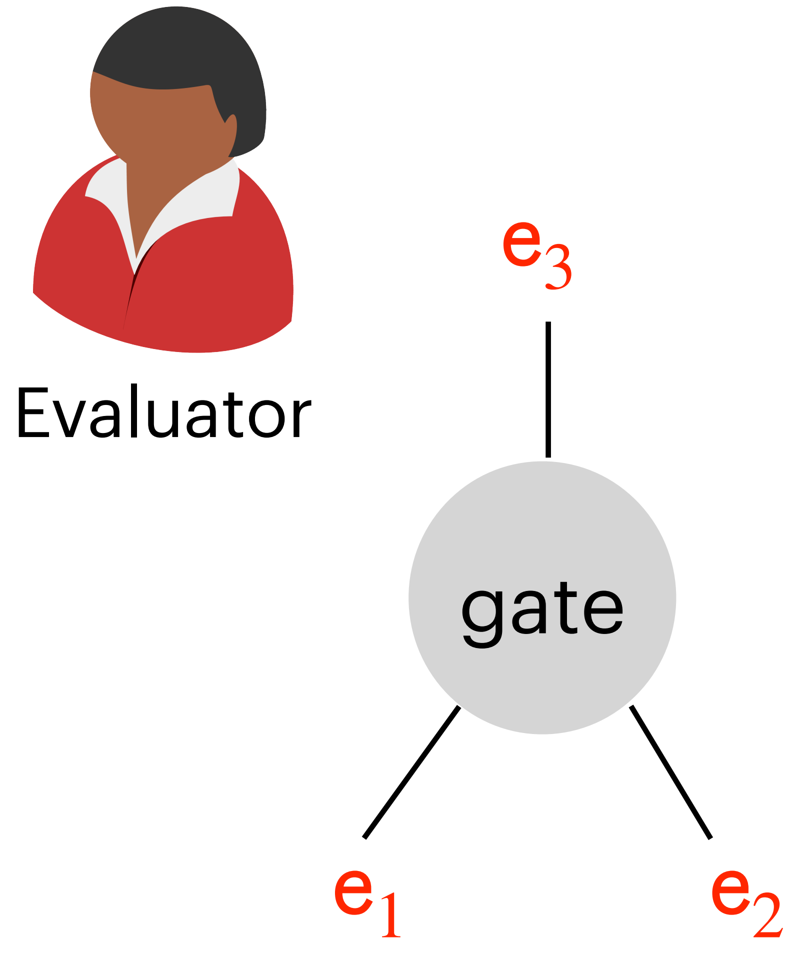

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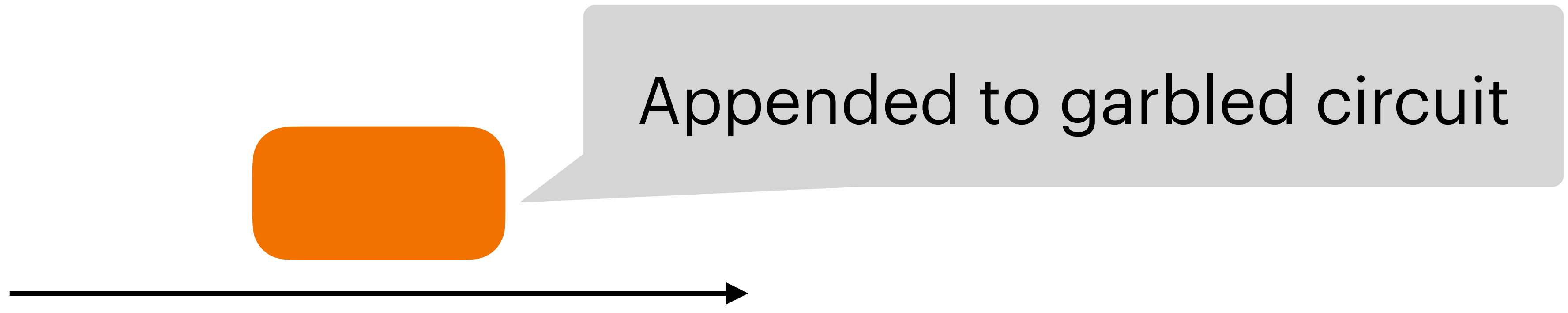
Instantiation:



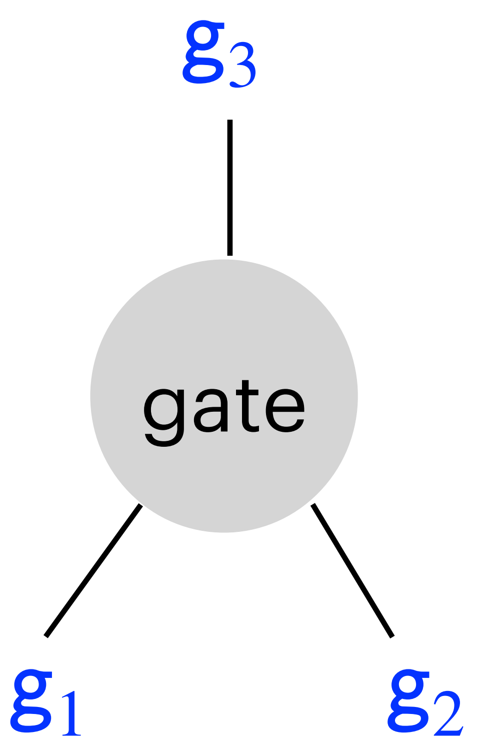
Appended to garbled circuit

Template for Garbling Circuits

Δ

Known to the Garbler

Garbler



g_3

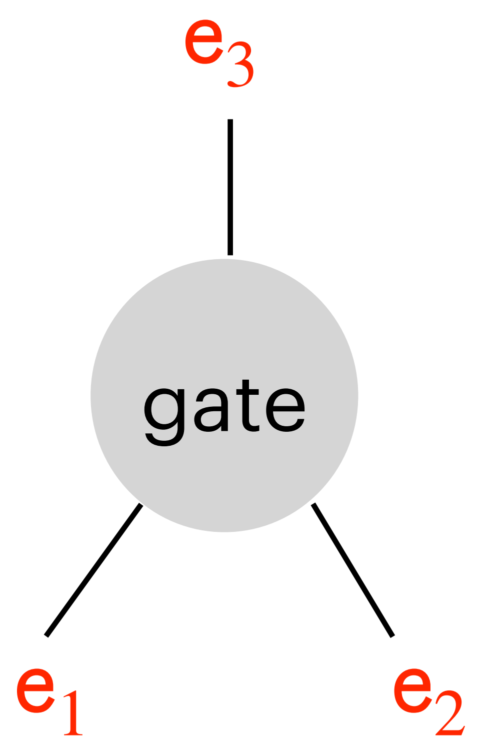
gate

g_1 g_2

Invariant

$g_i \oplus e_i = v_i$

Evaluator



e_3

gate

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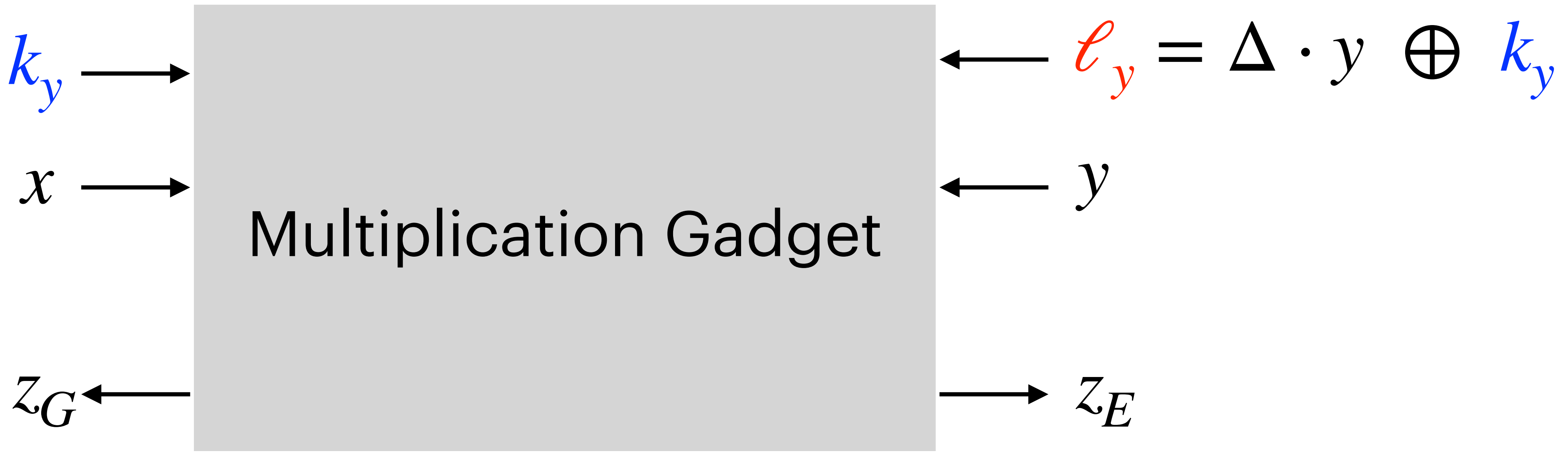


k_y

x

z_G

Multiplication Gadget

$\ell_y = \Delta \cdot y \oplus k_y$

y

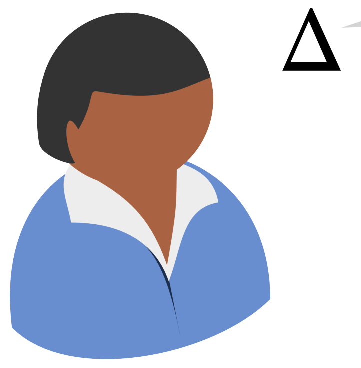
z_E

$z_G \oplus z_E = xy$

Instantiation:

Appended to garbled circuit

Template for Garbling Circuits



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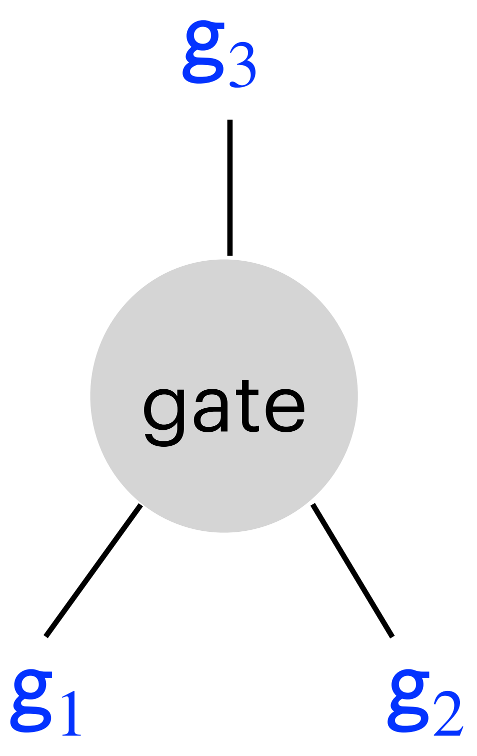

Invariant

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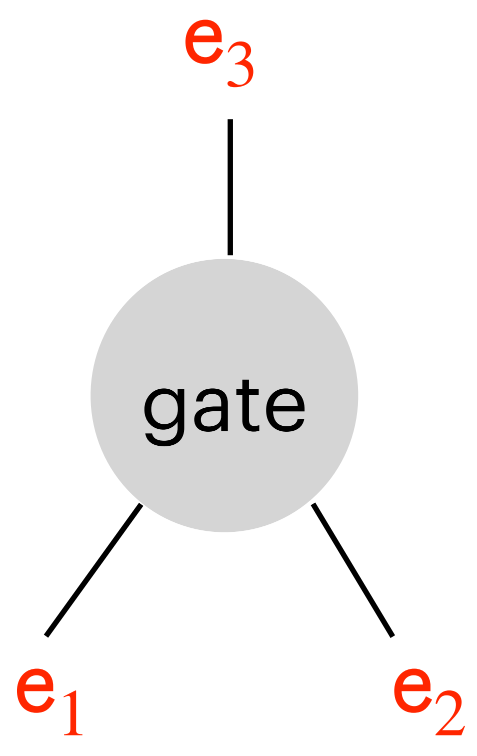
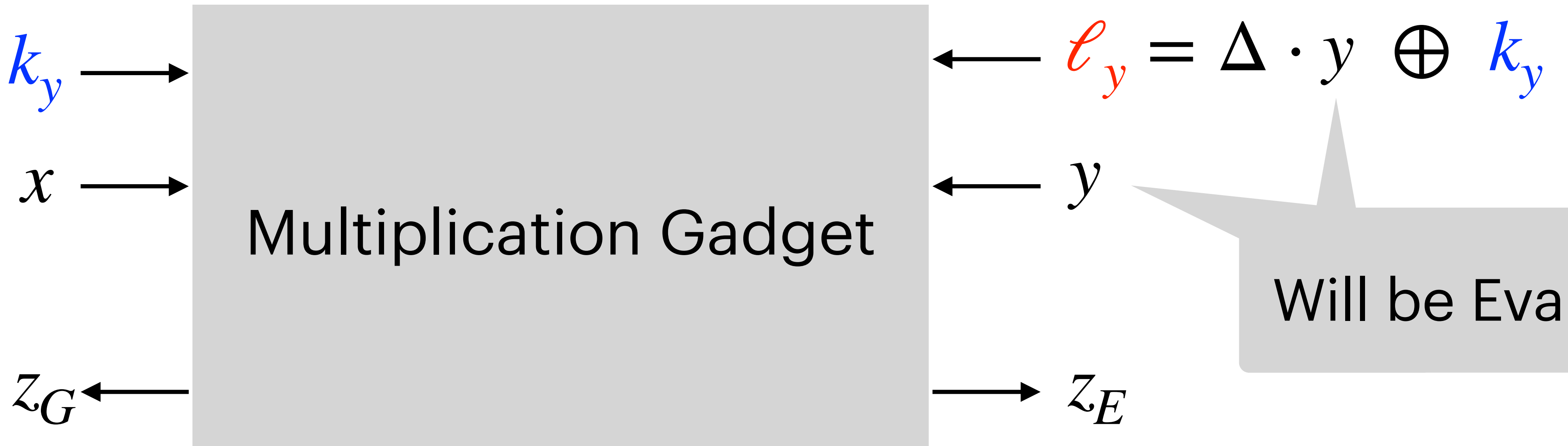
Evaluator





Will be Evaluator's share e

$z_G \oplus z_E = xy$

Instantiation:

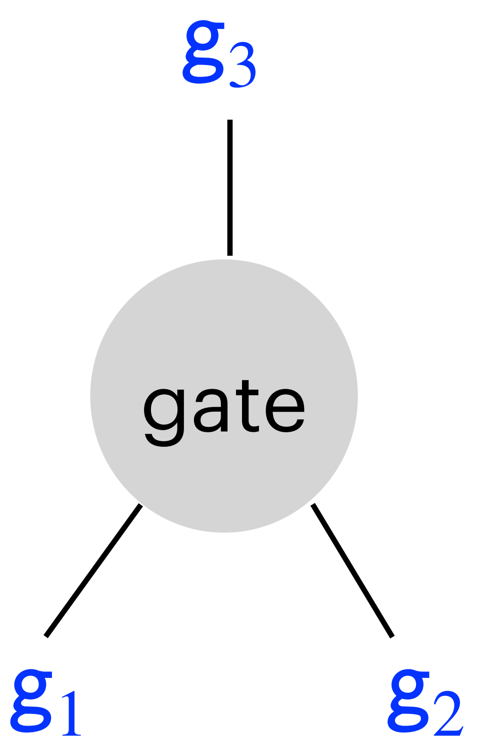
Appended to garbled circuit

Template for Garbling Circuits

Known to the Garbler

Δ

Garbler

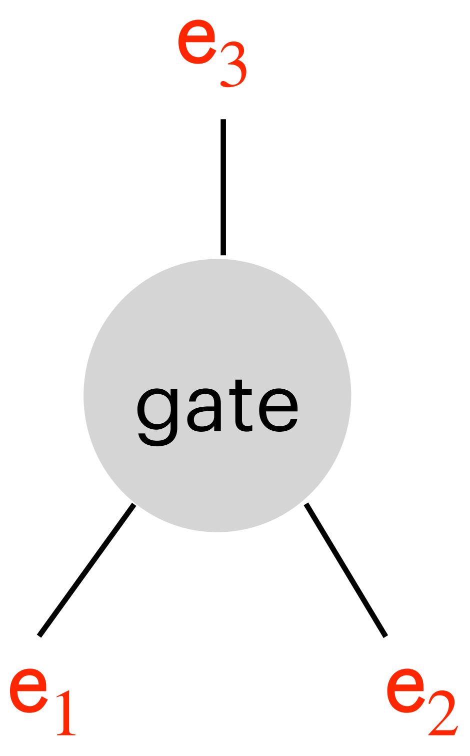


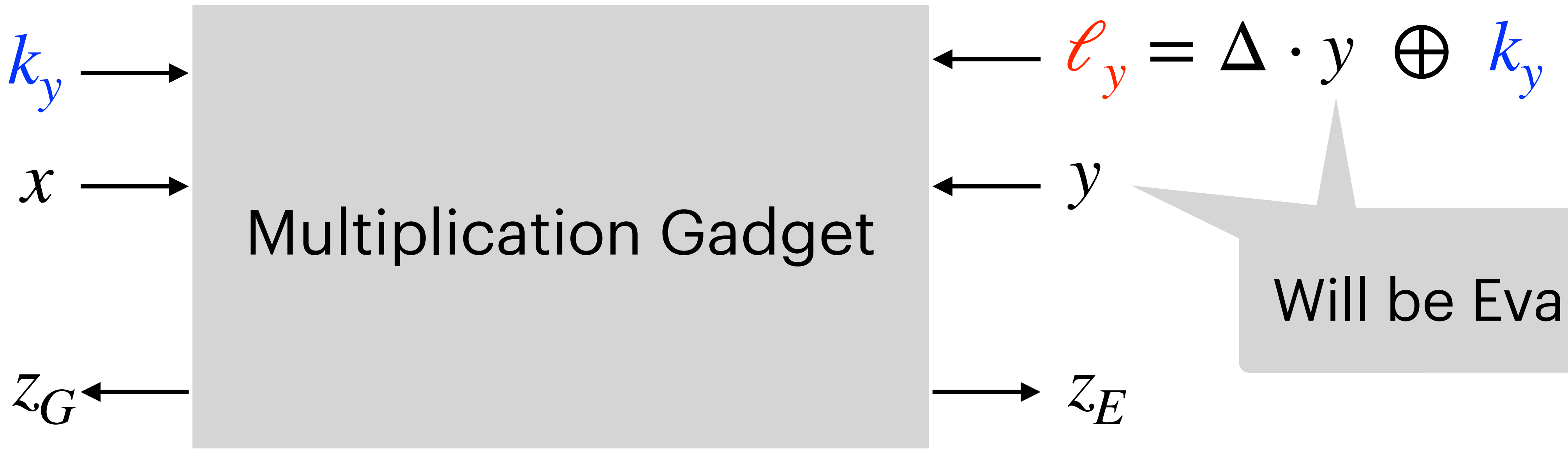
Invariant

$$g_i \oplus e_i = v_i$$

$$k_i \oplus \ell_i = \Delta \cdot e_i$$

Evaluator





Will be Evaluator’s share e

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Instantiation:

Appended to garbled circuit

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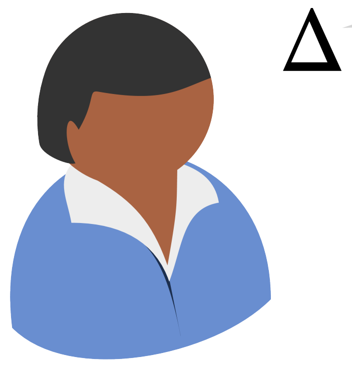

Known to the Garbler

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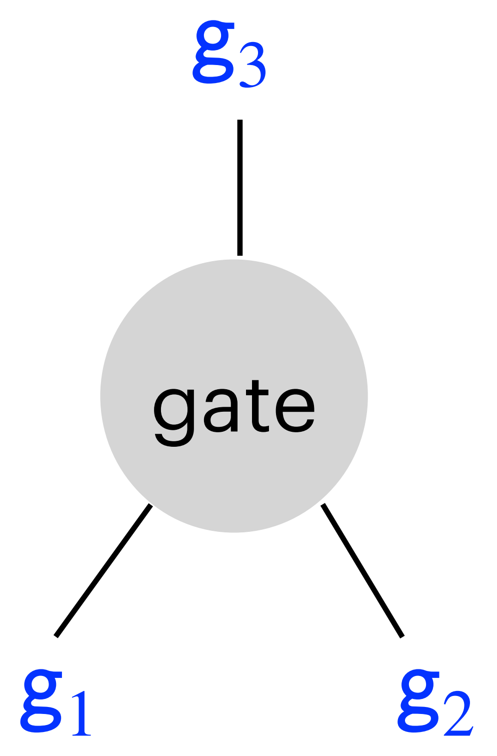

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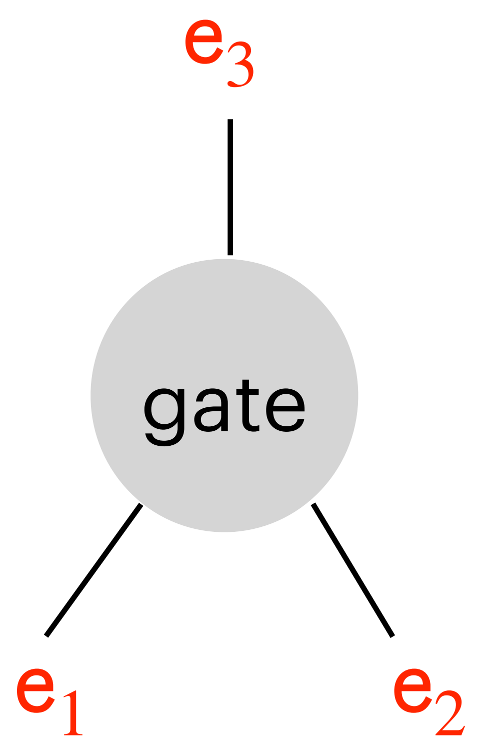

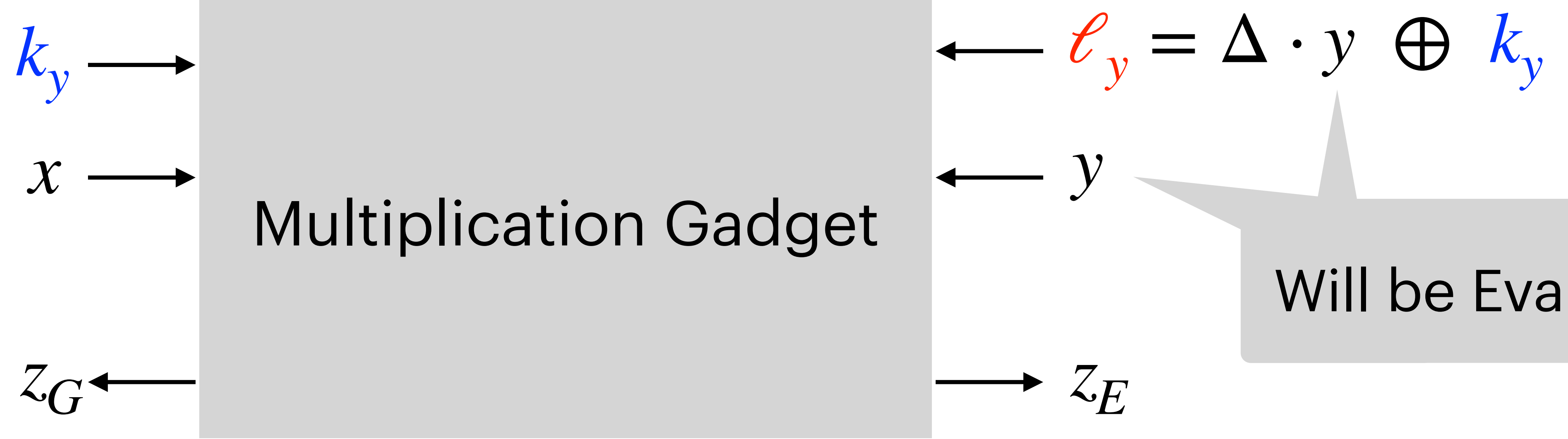

Will be Evaluator's share e

$$z_G \oplus z_E = xy$$

Suffices for garbling,

$O(1)$ invocations per gate

Appended to garbled circuit

Instantiation:

Towards $\omega(1/\lambda)$-Rate Garbling

Δ

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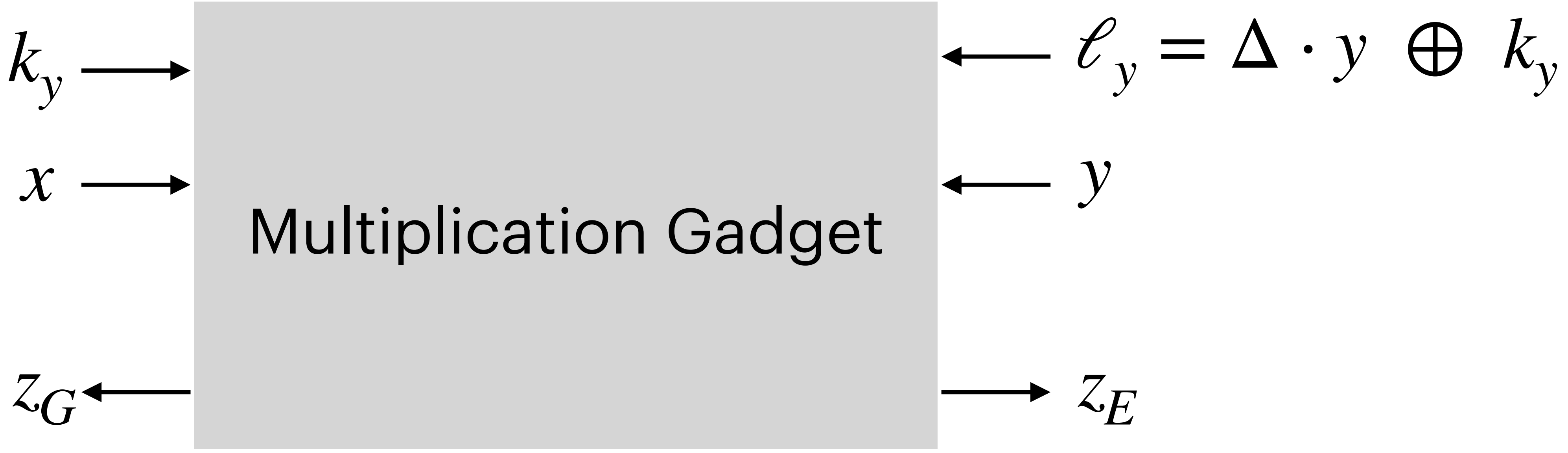


Multiplication Gadget

$k_y \rightarrow$

$x \rightarrow$

$z_G \leftarrow$

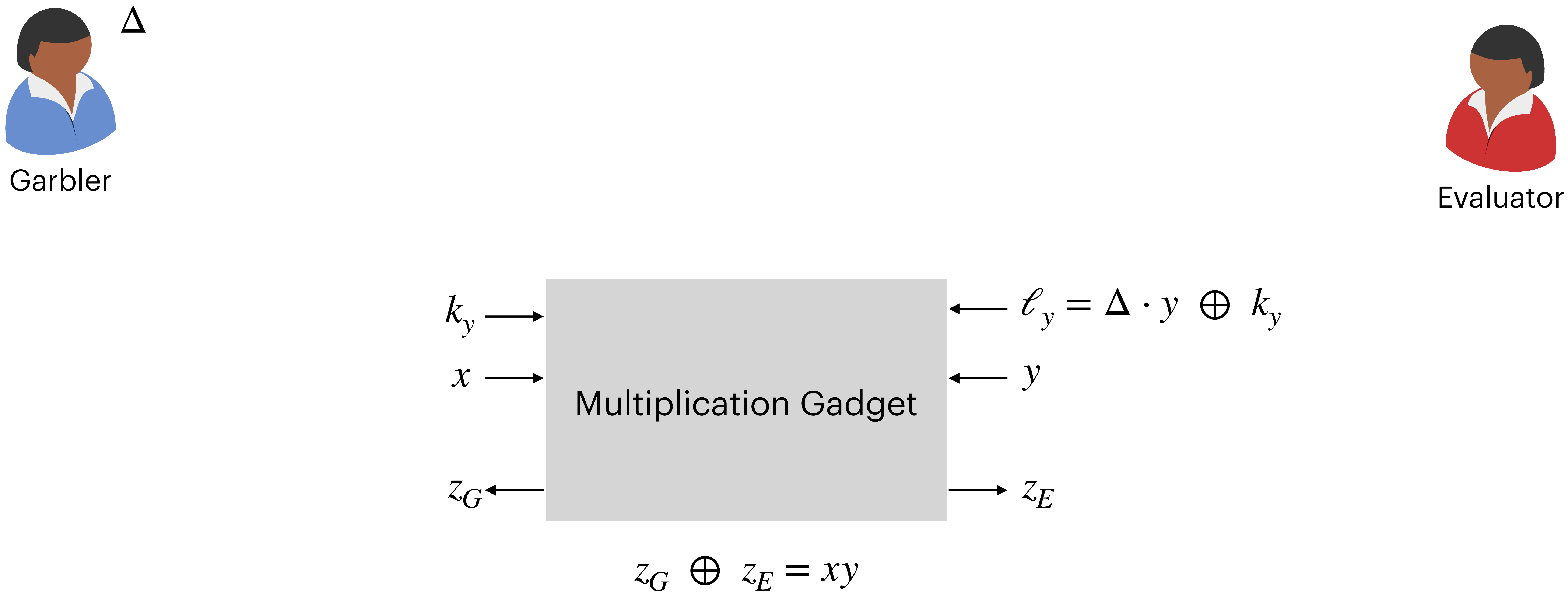
$\leftarrow \ell_y = \Delta \cdot y \oplus k_y$

$\leftarrow y$

$\rightarrow z_E$

$$z_G \oplus z_E = xy$$

Towards $\omega(1/\lambda)$-Rate Garbling



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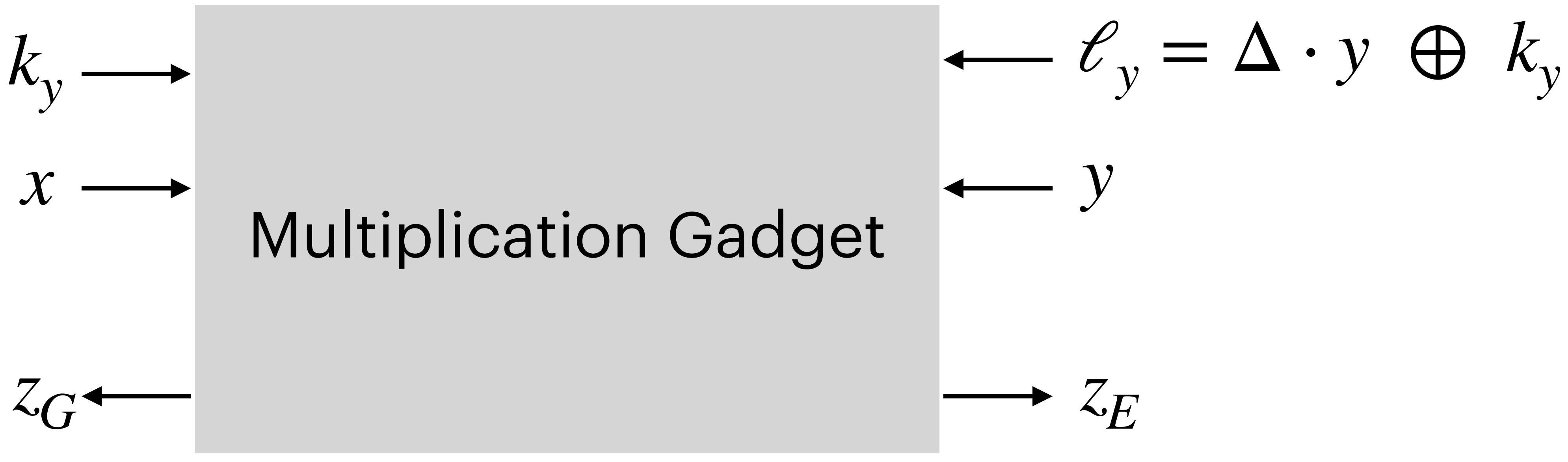
$\rightarrow z_E$

$z_G \oplus z_E = xy$

Yao's Garbling: Instantiation with $O(\lambda)$-bit message

Towards $\omega(1/\lambda)$-Rate Garbling



$$z_G \oplus z_E = xy$$

Yao's Garbling: Instantiation with $O(\lambda)$-bit message

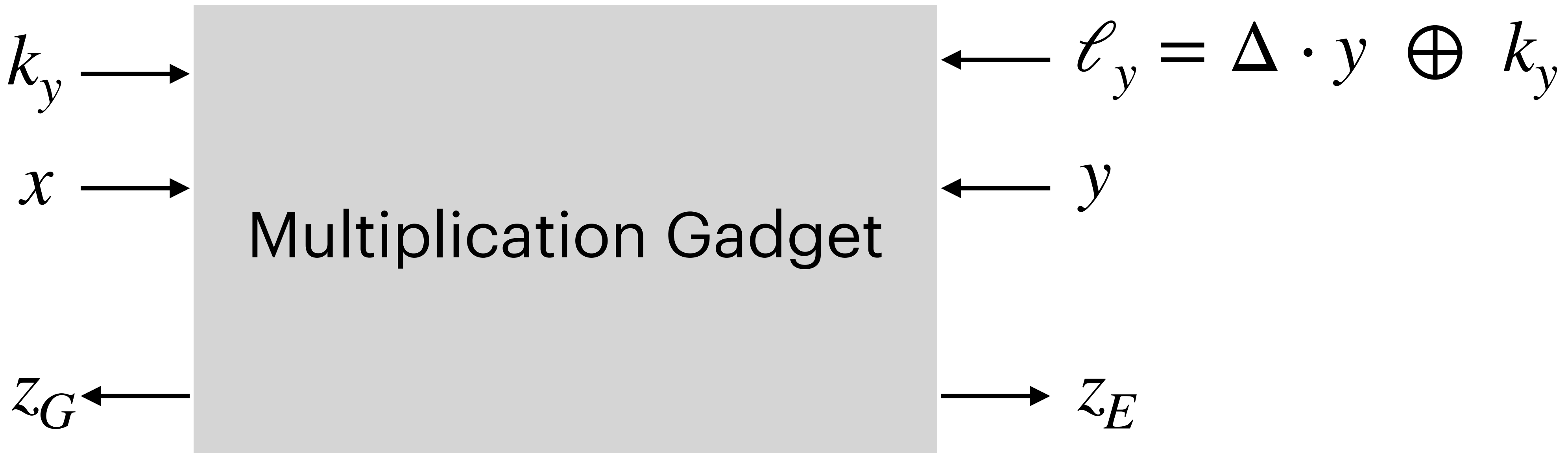
$\omega(1/\lambda)$**-Rate Garbling:** Requires instantiation with $o(\lambda)$-bit message

Towards $\omega(1/\lambda)$-Rate Garbling

Δ

Garbler

Evaluator



$$z_G \oplus z_E = xy$$

Yao's Garbling: Instantiation with $O(\lambda)$-bit message

$\omega(1/\lambda)$**-Rate Garbling:** Requires instantiation with $o(\lambda)$-bit message

Leverage recent advances in Arithmetic Garbling

Towards $\omega(1/\lambda)$-Rate Garbling

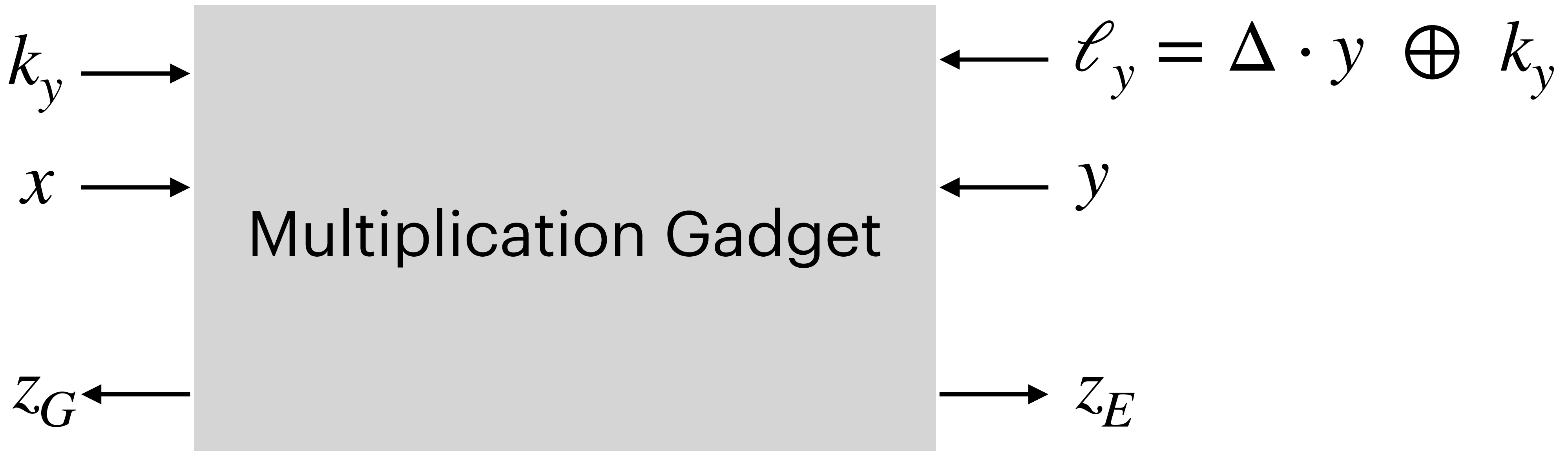
Multiplication Gadget from [Couteau-Hazay-H-Kumar'25]

Δ

Garbler

Evaluator

Multiplication Gadget for Integers



$$z_G \oplus z_E = xy$$

Towards $\omega(1/\lambda)$-Rate Garbling

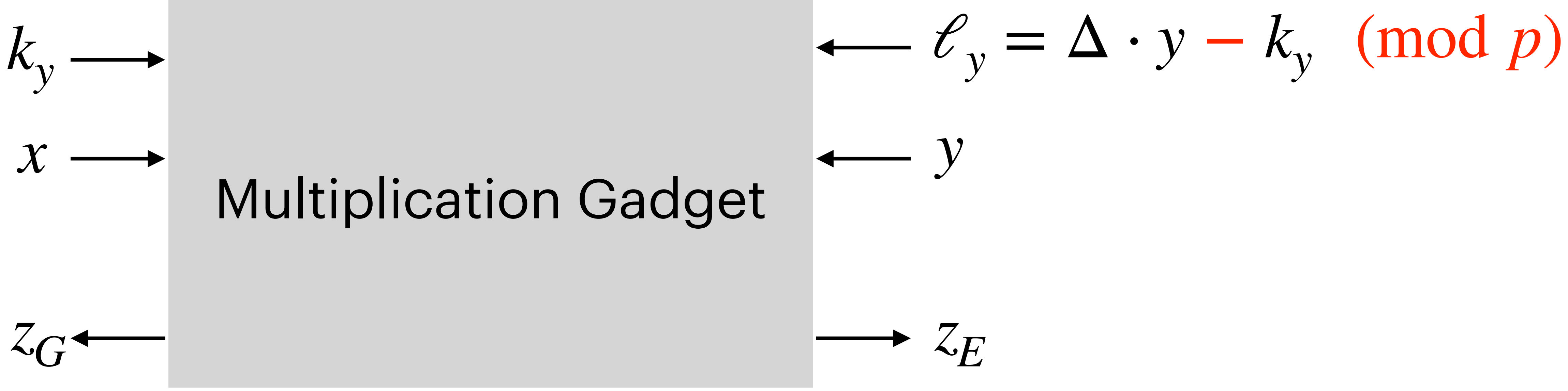
Multiplication Gadget from [Couteau-Hazay-H-Kumar'25]

Δ

Garbler

Evaluator

Multiplication Gadget for Integers



k_y

x

z_G

Multiplication Gadget

$\ell_y = \Delta \cdot y - k_y \pmod{p}$

y

z_E

$$z_G \oplus z_E = xy$$

Towards $\omega(1/\lambda)$-Rate Garbling

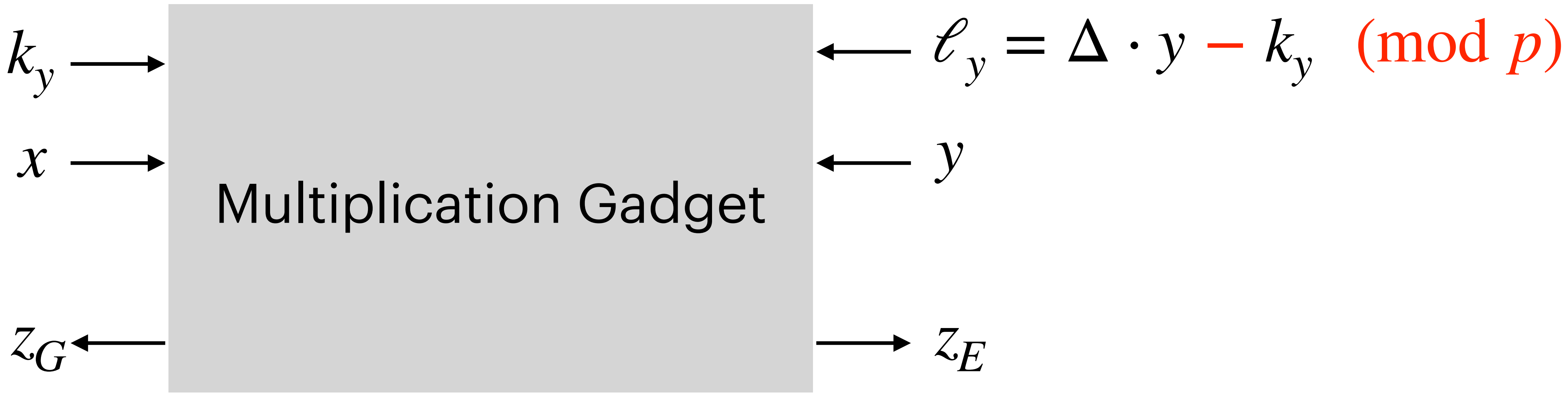
Multiplication Gadget from [Couteau-Hazay-H-Kumar’25]

Δ

Garbler

Evaluator

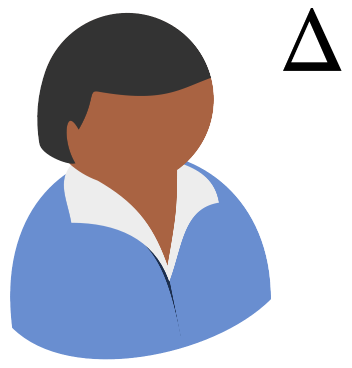
Multiplication Gadget for Integers



$$z_G + z_E = xy$$

Towards $\omega(1/\lambda)$-Rate Garbling

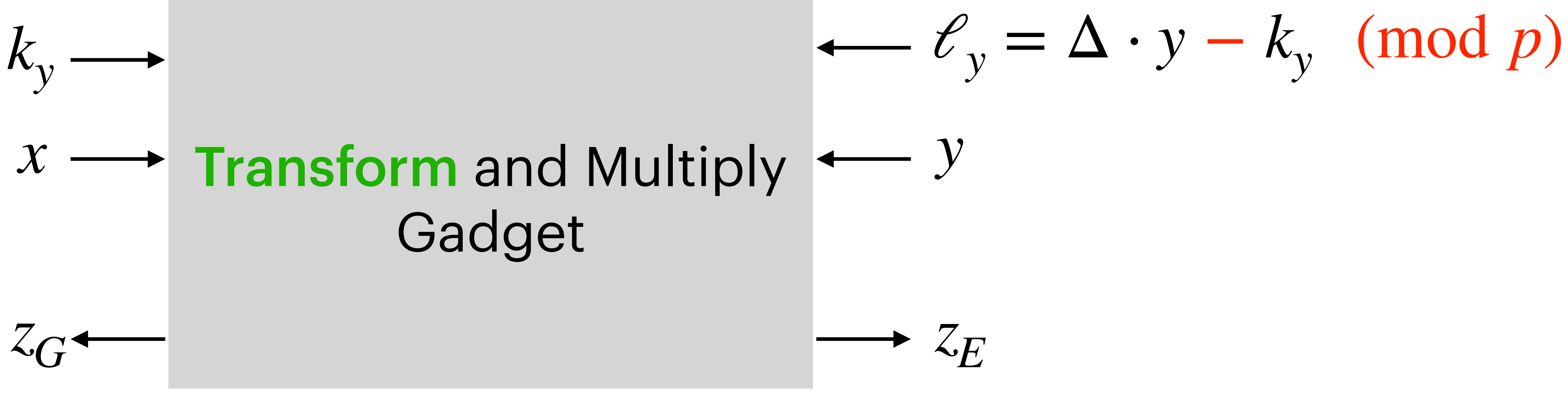
Multiplication Gadget from [Couteau-Hazay-H-Kumar’25]



Garbler

Evaluator

Multiplication Gadget for Integers



Transform and Multiply Gadget

k_y

x

z_G

$\ell_y = \Delta \cdot y - k_y \pmod{p}$

y

z_E

$$z_G + z_E = x \cdot f(y)$$

Towards $\omega(1/\lambda)$-Rate Garbling

Multiplication Gadget from [Couteau-Hazay-H-Kumar'25]

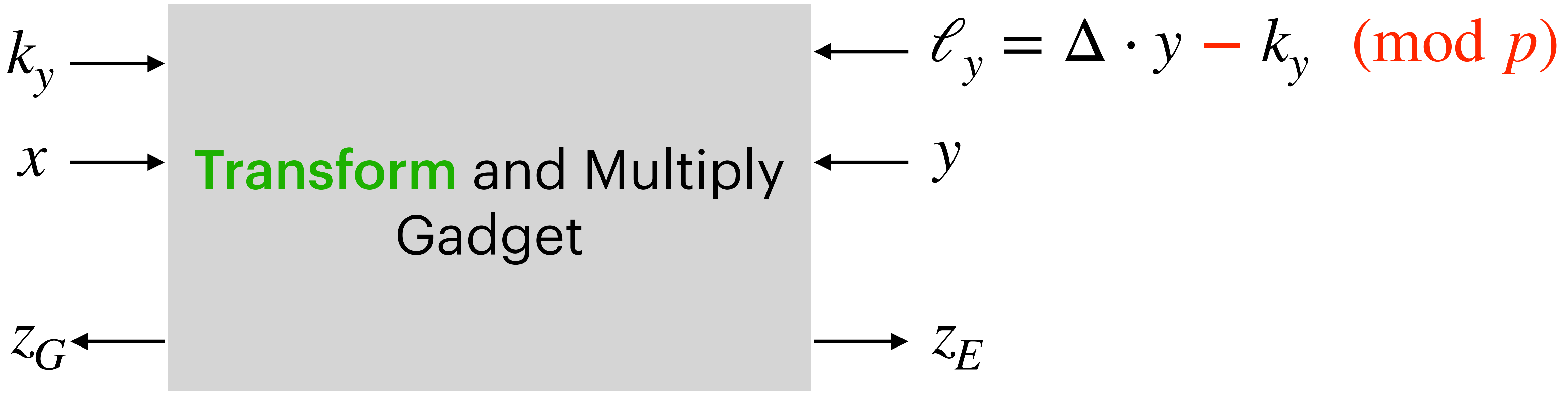
Δ

Garbler

Evaluator

Multiplication Gadget for Integers

Observation: Black-box use of crypto and secure in the GGM



$k_y \rightarrow$

$x \rightarrow$

$z_G \leftarrow$

Transform and Multiply Gadget

$\leftarrow \ell_y = \Delta \cdot y - k_y \pmod{p}$

$\leftarrow y$

$\rightarrow z_E$

$$z_G + z_E = x \cdot f(y)$$

Towards $\omega(1/\lambda)$-Rate Garbling

Multiplication Gadget from [Couteau-Hazay-H-Kumar'25]

Δ

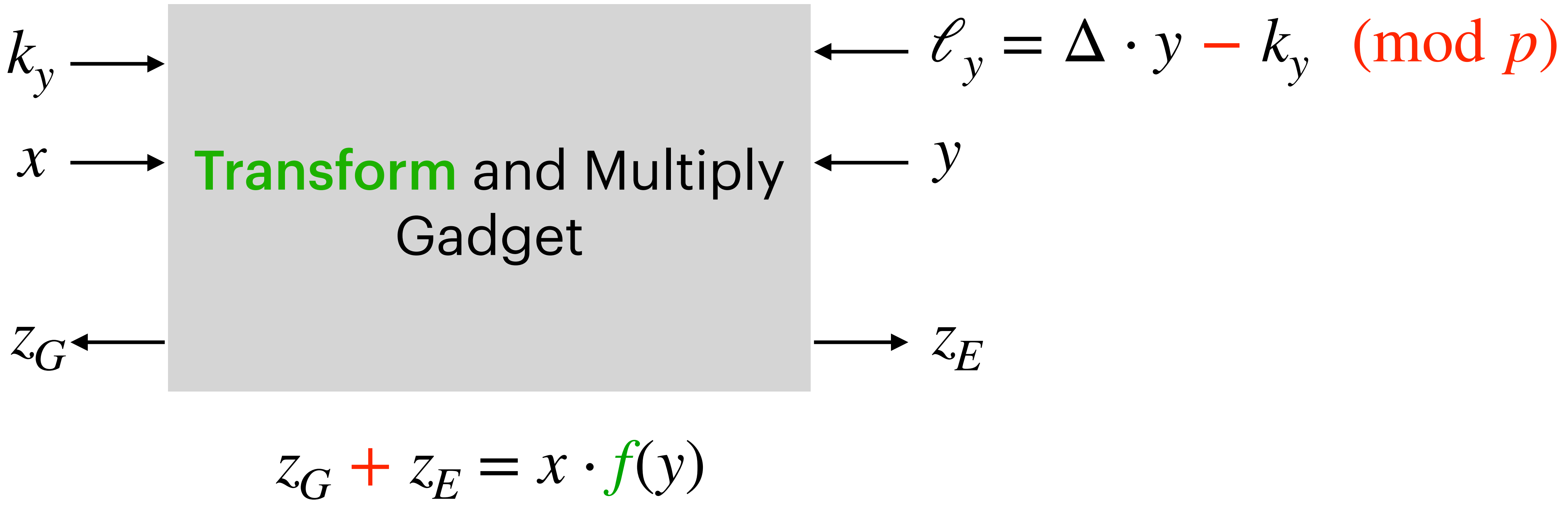
Garbler

Evaluator

Observation: Black-box use of crypto and secure in the GGM

Garbler sends $O(\lambda)$-bit message

Multiplication Gadget for Integers



$k_y \rightarrow$

$x \rightarrow$

Transform and Multiply Gadget

$\leftarrow \ell_y = \Delta \cdot y - k_y \pmod{p}$

$\leftarrow y$

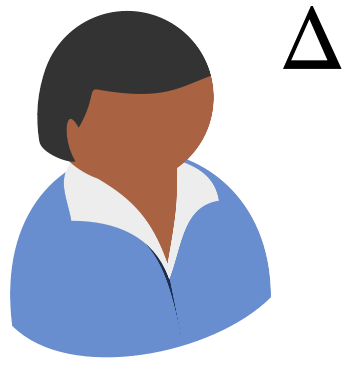
$z_G \leftarrow$

$\rightarrow z_E$

$z_G + z_E = x \cdot f(y)$

Towards $\omega(1/\lambda)$-Rate Garbling

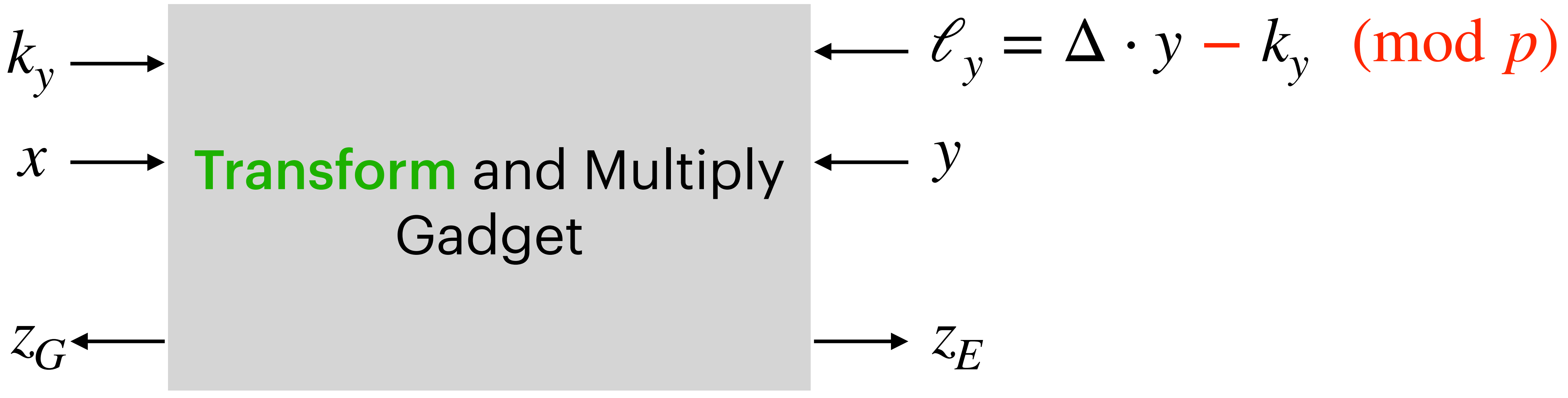
Multiplication Gadget from [Couteau-Hazay-H-Kumar'25]



Garbler

Evaluator

Multiplication Gadget for Integers



$$z_G + z_E = x \cdot f(y)$$

Observation: Black-box use of crypto and secure in the GGM

Garbler sends $O(\lambda)$-bit message

Rate of Arithmetic Garbling over $\mathbb{Z}_B$: $\dfrac{|C| \cdot \log B}{|\widehat{C}|}$

Towards $\omega(1/\lambda)$-Rate Garbling

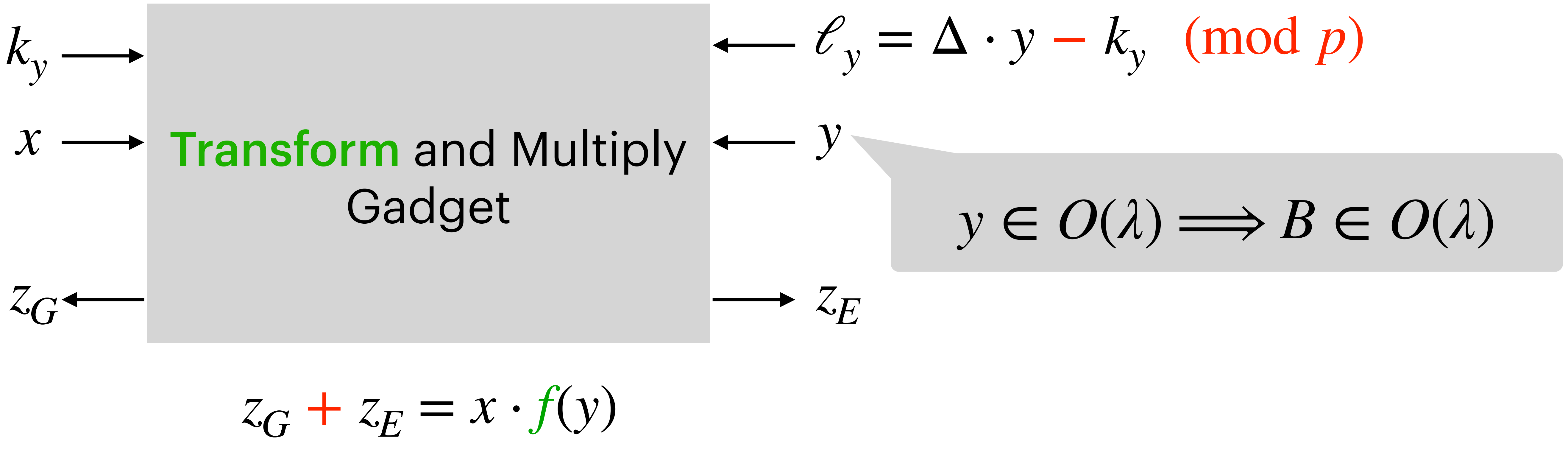
Multiplication Gadget from [Couteau-Hazay-H-Kumar’25]

Δ

Garbler

Evaluator

Multiplication Gadget for Integers



$k_y \rightarrow$

$x \rightarrow$

$z_G \leftarrow$

Transform and Multiply Gadget

$\leftarrow \ell_y = \Delta \cdot y - k_y \pmod{p}$

$\leftarrow y$

$\rightarrow z_E$

$y \in O(\lambda) \Longrightarrow B \in O(\lambda)$

$z_G + z_E = x \cdot f(y)$

Observation: Black-box use of crypto and secure in the GGM

Garbler sends $O(\lambda)$-bit message

Rate of Arithmetic Garbling over $\mathbb{Z}_B$: $\dfrac{|C| \cdot \log B}{|\widehat{C}|}$

Towards $\omega(1/\lambda)$-Rate Garbling

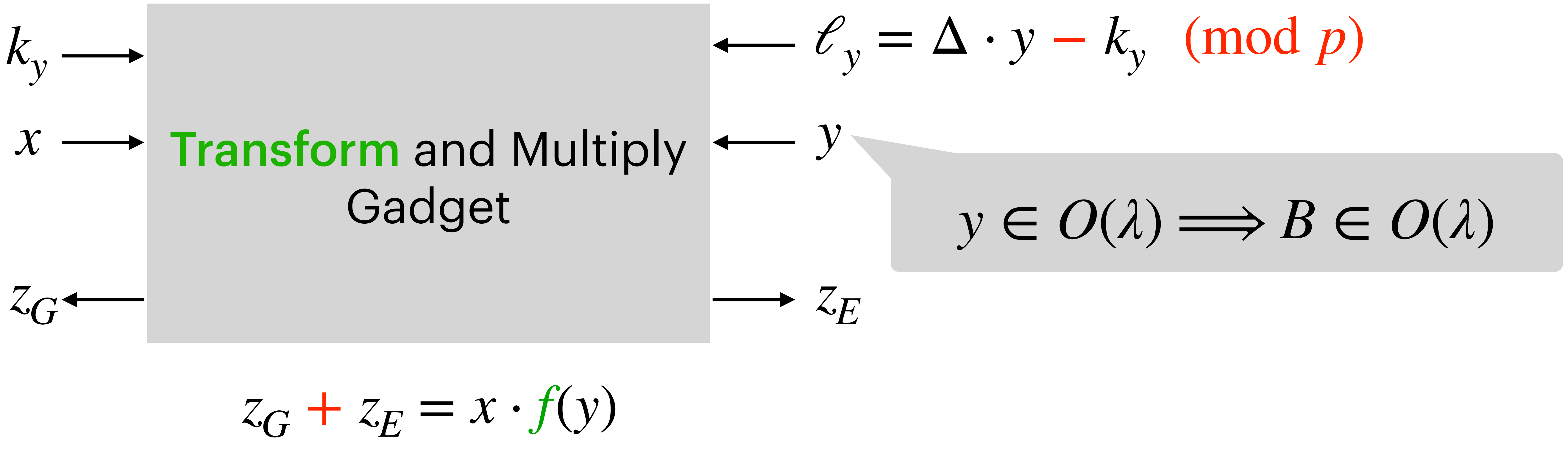
Multiplication Gadget from [Couteau-Hazay-H-Kumar’25]

Δ

Garbler

Evaluator

Multiplication Gadget for Integers



Transform and Multiply Gadget

$k_y \rightarrow$

$x \rightarrow$

$z_G \leftarrow$

$\leftarrow \ell_y = \Delta \cdot y - k_y \pmod{p}$

$\leftarrow y$

$\rightarrow z_E$

$y \in O(\lambda) \Longrightarrow B \in O(\lambda)$

$$z_G + z_E = x \cdot f(y)$$

Observation: Black-box use of crypto and secure in the GGM

Garbler sends $O(\lambda)$-bit message

Rate of Arithmetic Garbling over $\mathbb{Z}_B$: $\dfrac{|C| \cdot \log B}{|\widehat{C}|} = O\left(\dfrac{\log \lambda}{\lambda}\right)$

Towards $\omega(1/\lambda)$-Rate Garbling

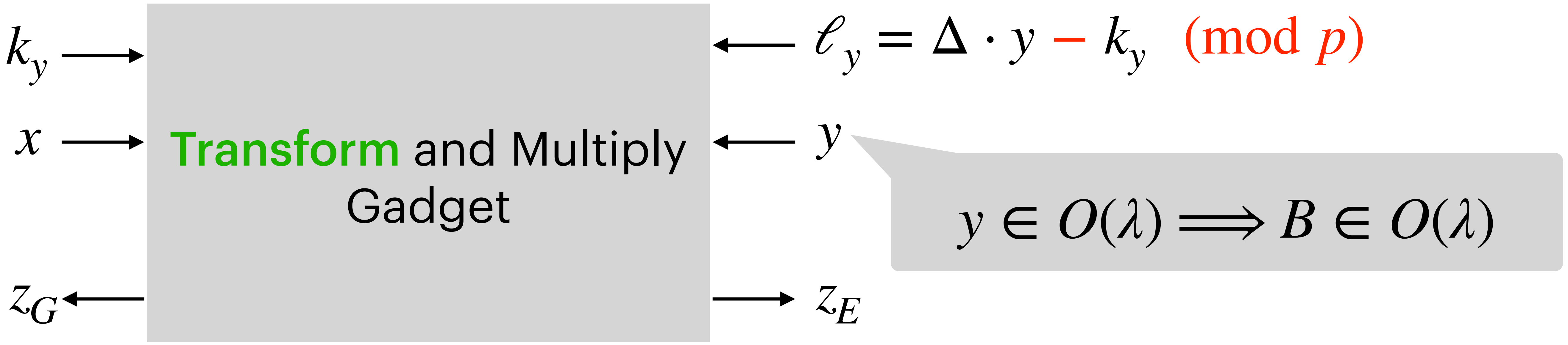
Multiplication Gadget from [Couteau-Hazay-H-Kumar'25]

Δ

Garbler

Evaluator

Multiplication Gadget for Integers



k_y

x

z_G

Transform and Multiply Gadget

$\ell_y = \Delta \cdot y - k_y \pmod{p}$

y

z_E

$y \in O(\lambda) \Longrightarrow B \in O(\lambda)$

$z_G + z_E = x \cdot f(y)$

Observation: Black-box use of crypto and secure in the GGM

Garbler sends $O(\lambda)$-bit message

Rate of Arithmetic Garbling over $\mathbb{Z}_B$: $\dfrac{|C| \cdot \log B}{|\widehat{C}|} = O\left(\dfrac{\log \lambda}{\lambda}\right)$

Rate of Boolean Garbling: $O\left(\dfrac{1}{\lambda}\right)$

Towards $\omega(1/\lambda)$-Rate Garbling

Multiplication Gadget from [Couteau-Hazay-H-Kumar'25]

Δ

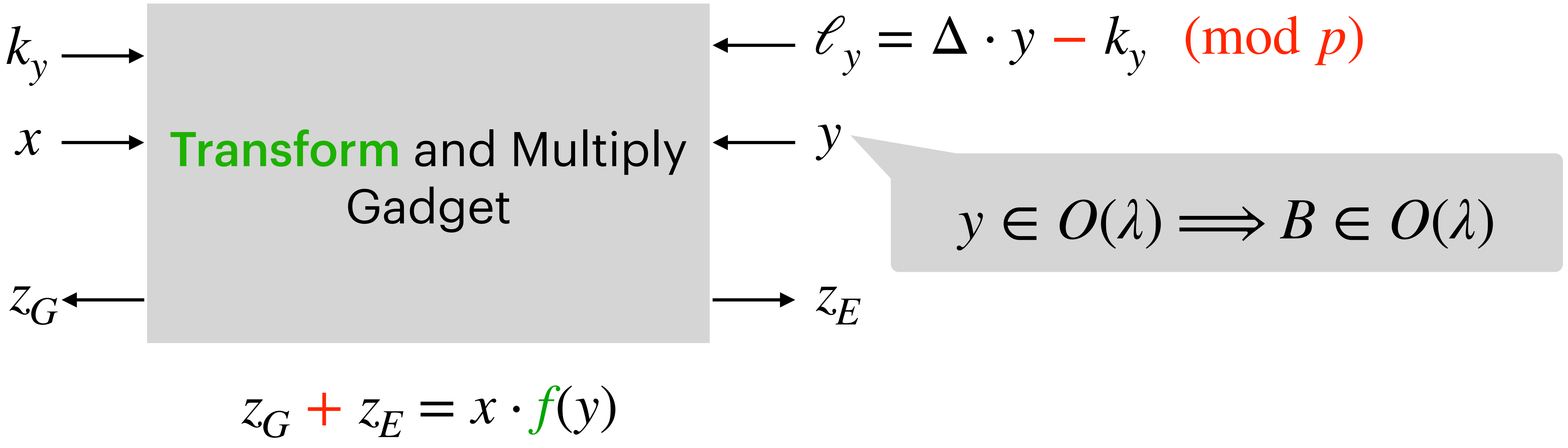
Garbler

Evaluator

Multiplication Gadget for Integers

Observation: Black-box use of crypto and secure in the GGM

Garbler sends $O(\lambda)$-bit message



$k_y \rightarrow$

$x \rightarrow$

$z_G \leftarrow$

Transform and Multiply Gadget

$\leftarrow \ell_y = \Delta \cdot y - k_y \pmod{p}$

$\leftarrow y$

$y \in O(\lambda) \implies B \in O(\lambda)$

$\rightarrow z_E$

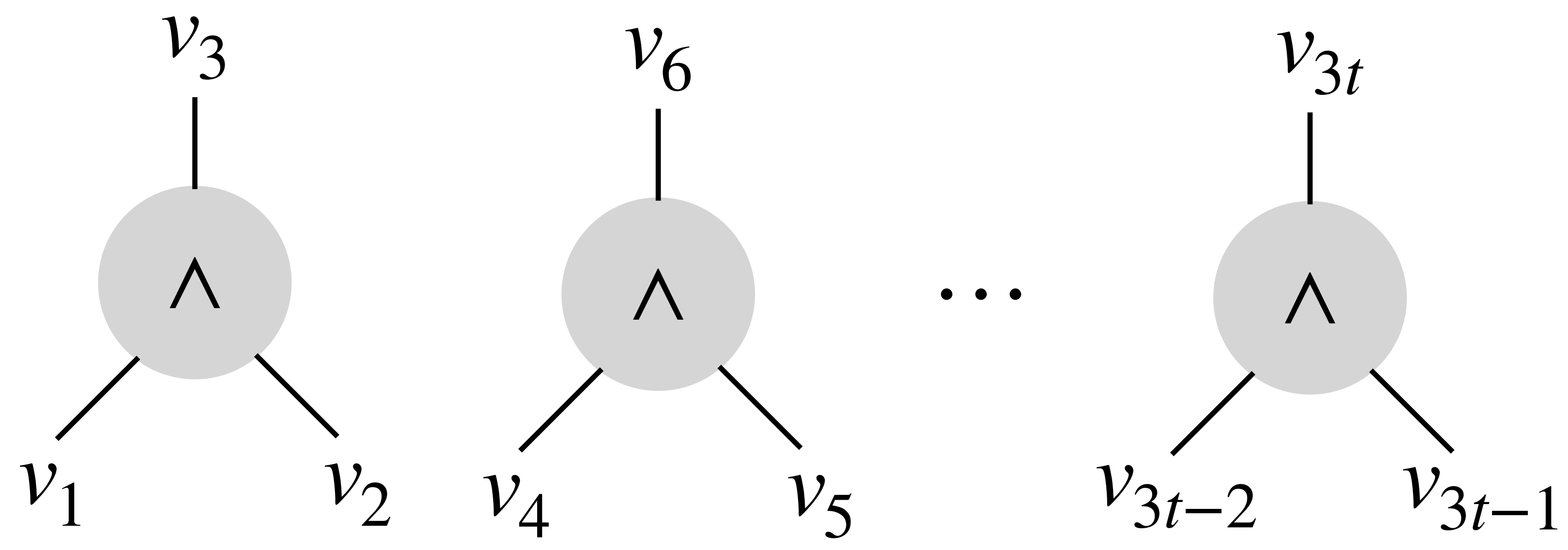
$z_G + z_E = x \cdot f(y)$

Rate of Arithmetic Garbling over $\mathbb{Z}_B$: $\dfrac{|C| \cdot \log B}{|\widehat{C}|} = O\left(\dfrac{\log \lambda}{\lambda}\right)$

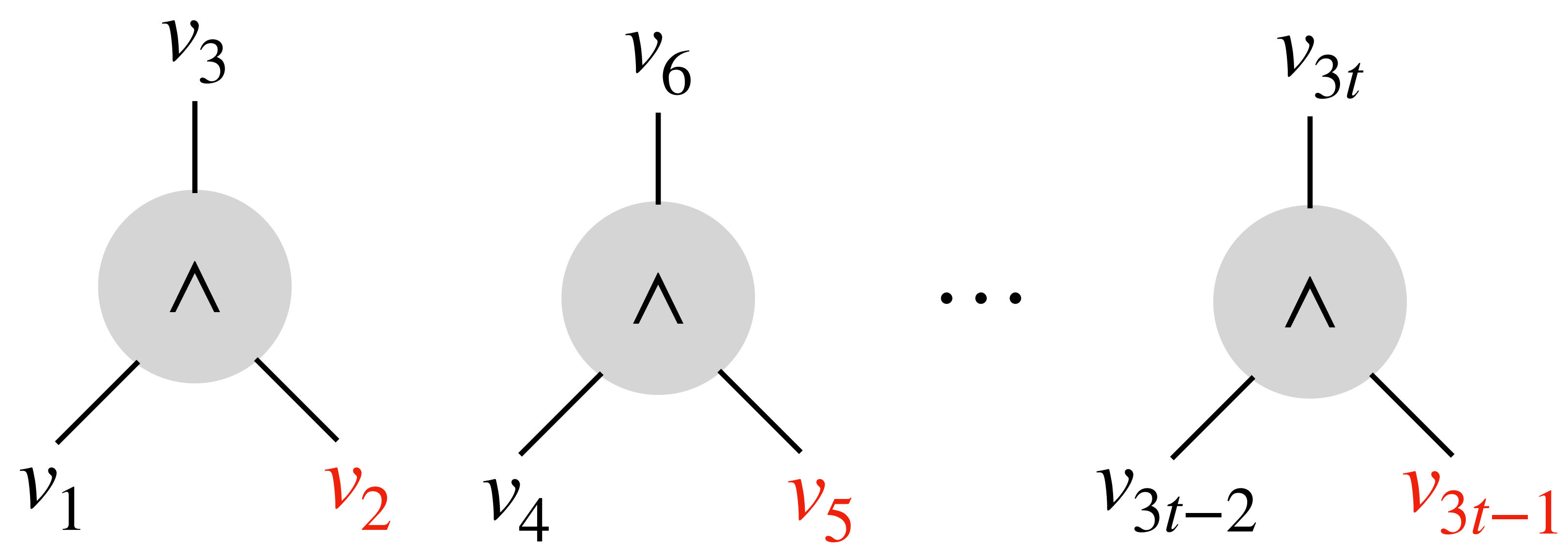
Rate of Boolean Garbling: $O\left(\dfrac{1}{\lambda}\right)$

Key Ingredient: A packing from $\mathbb{F}_2^t$ to $\log \lambda$-bit integers

Template for $\omega(1/\lambda)$-Rate Garbling



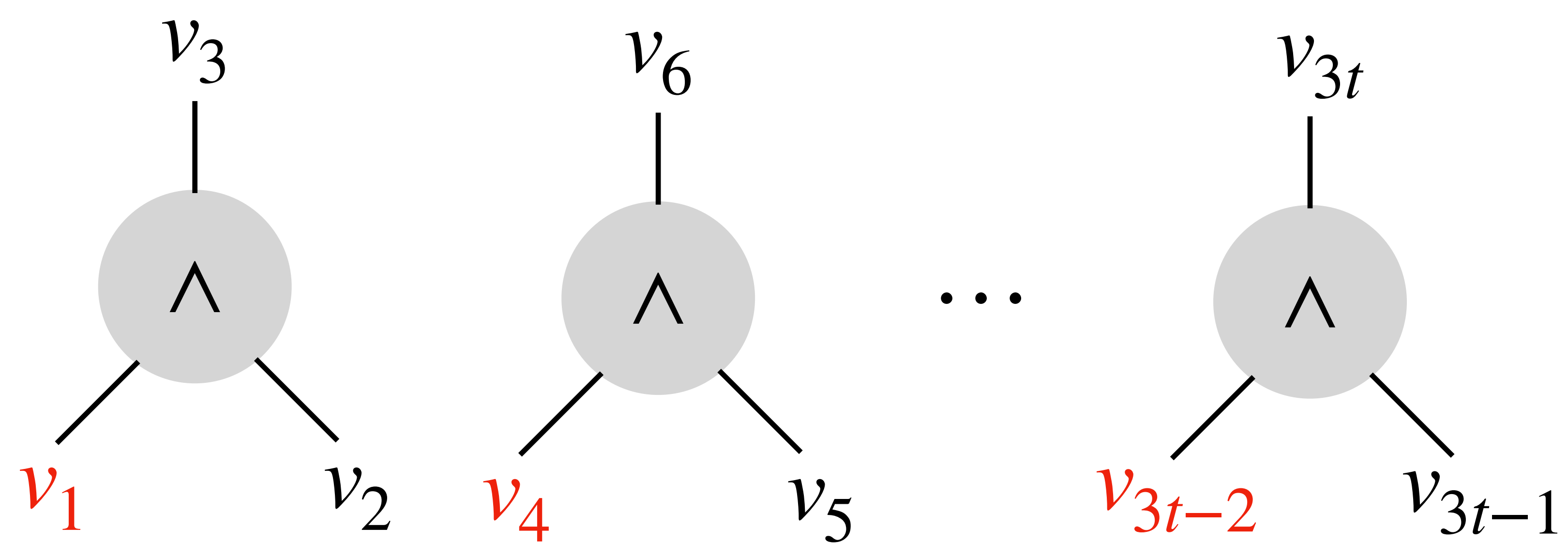
Template for $\omega(1/\lambda)$-Rate Garbling



1. Packing

$$V_{\mathsf{R}} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

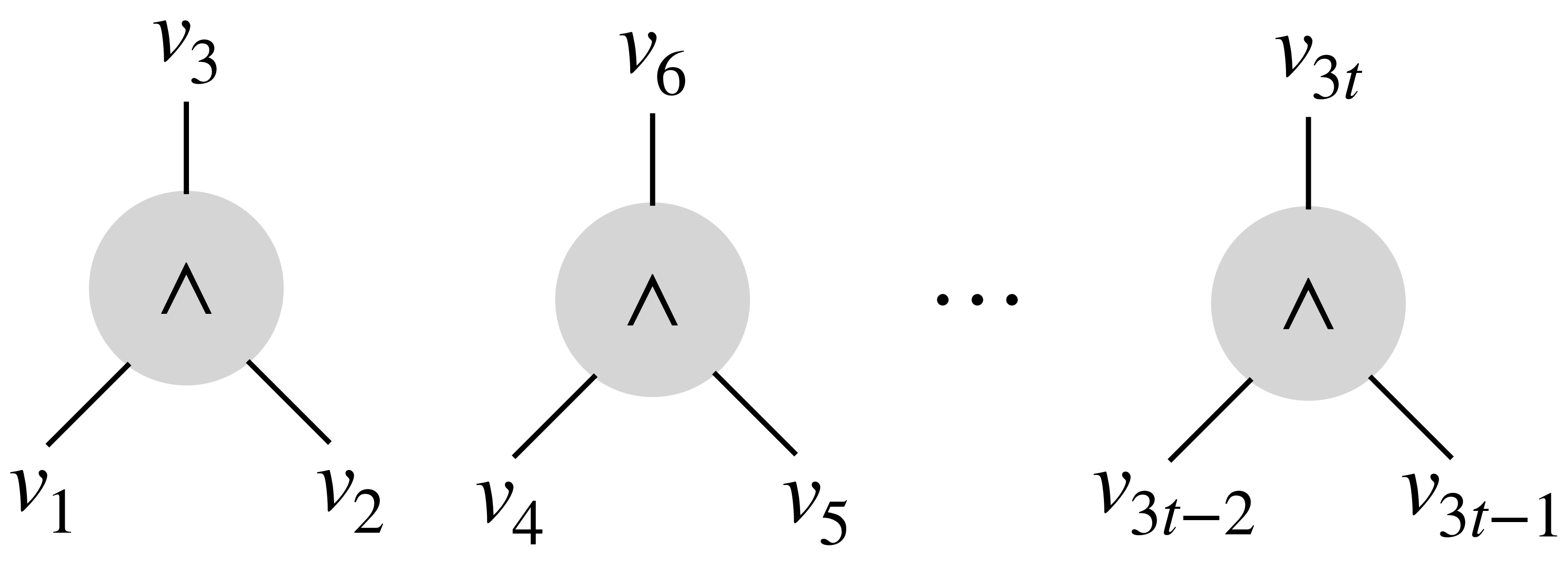
Template for $\omega(1/\lambda)$-Rate Garbling



1. Packing

$$V_\mathsf{R} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$
$$V_\mathsf{L} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

Template for $\omega(1/\lambda)$-Rate Garbling



1. Packing

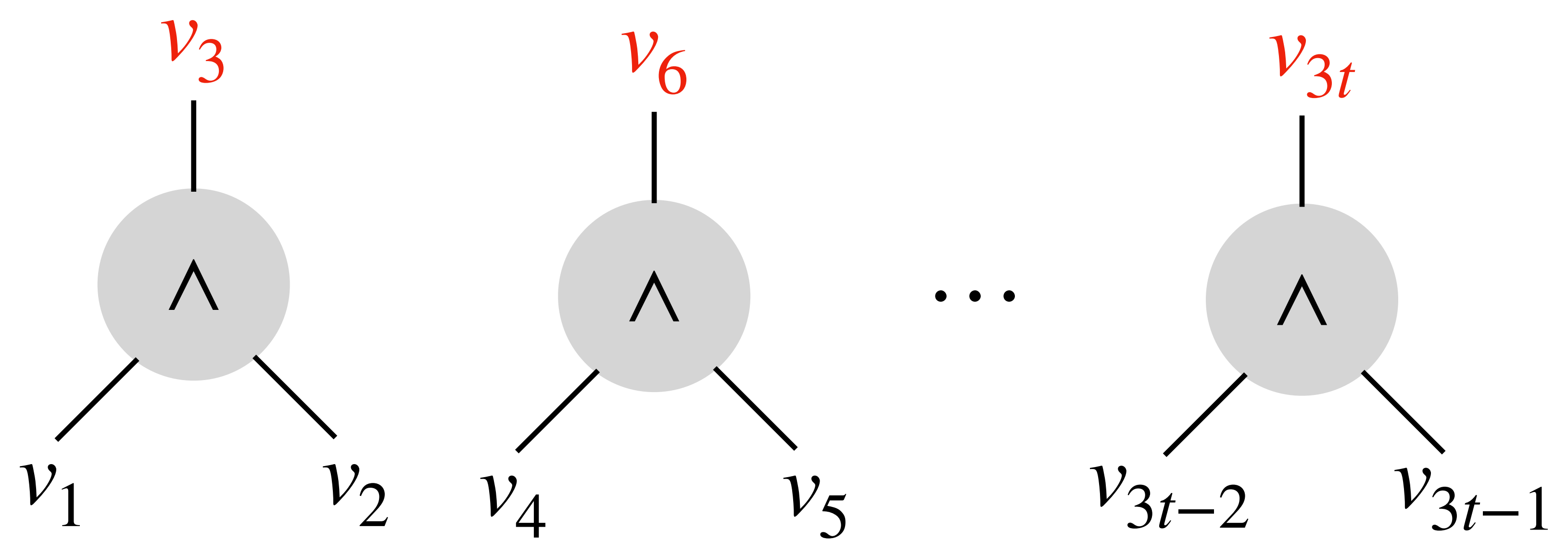
$$V_{\mathsf{R}} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_{\mathsf{L}} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

Template for $\omega(1/\lambda)$-Rate Garbling



1. Packing

$$V_{\mathsf{R}} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$
$$V_{\mathsf{L}} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

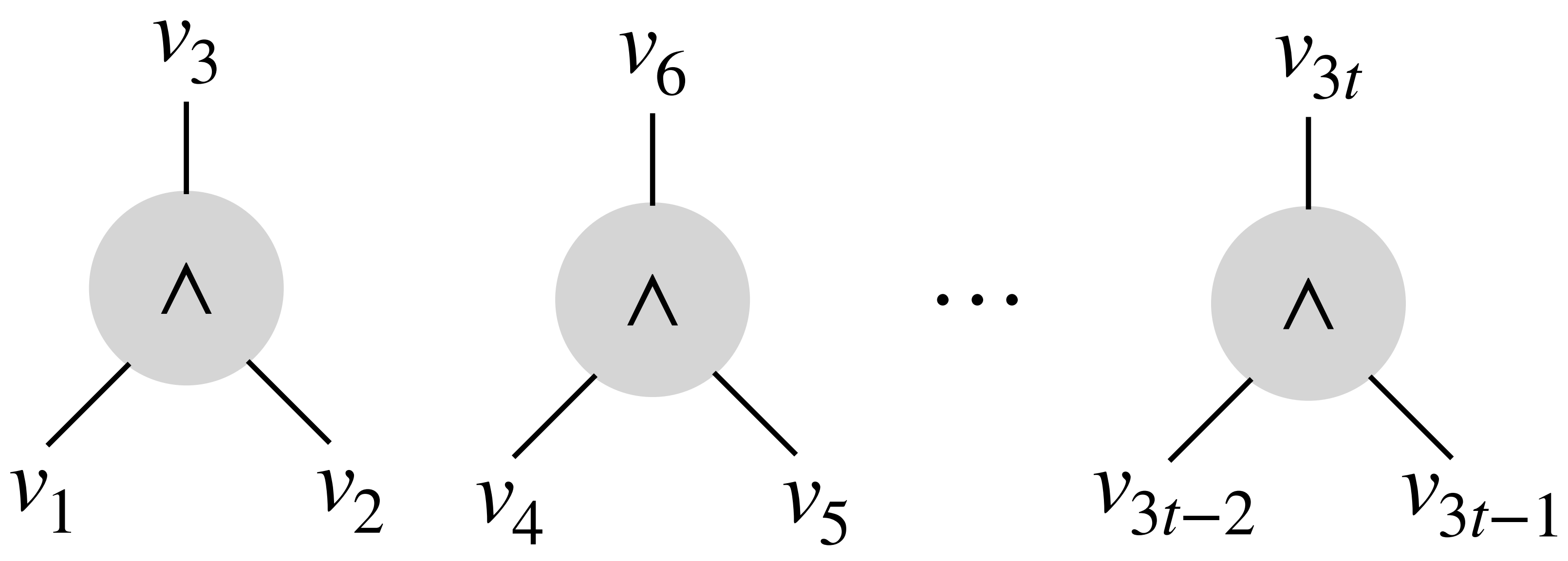
2. Gate Evaluation

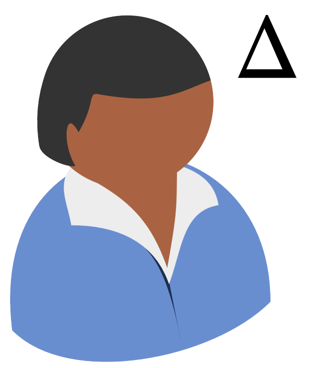
$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod{p}$$

Evaluator

1. Packing

$$V_\mathsf{R} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_\mathsf{L} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

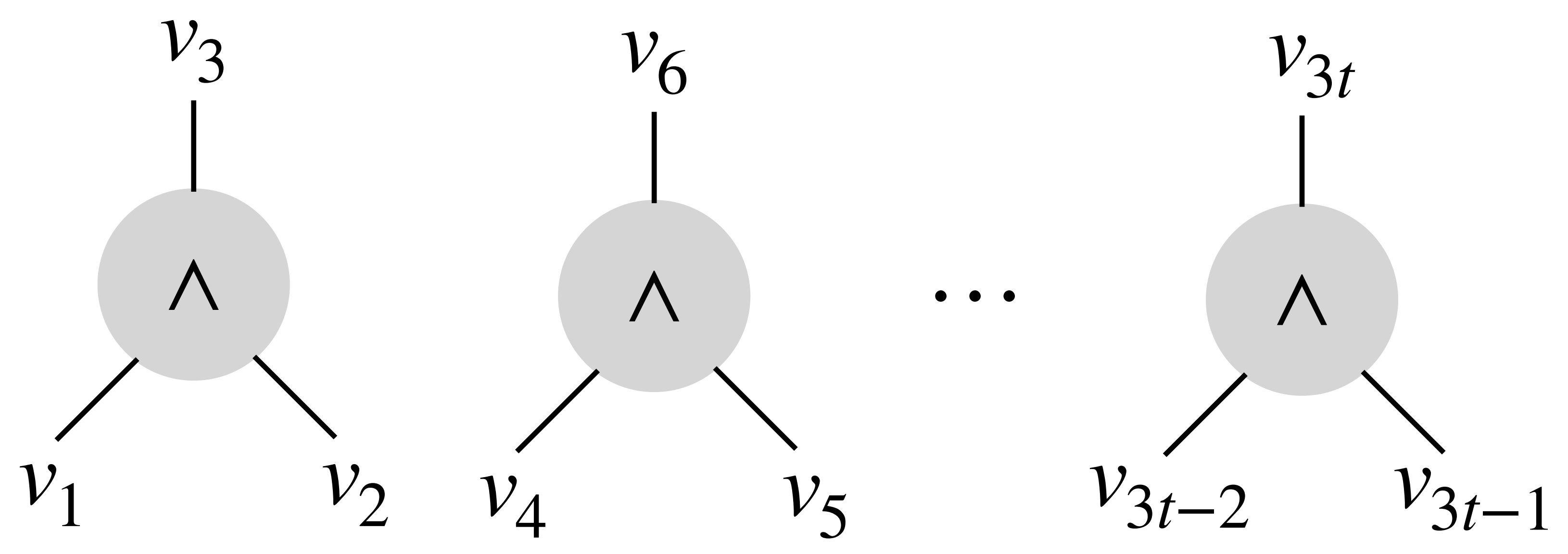
2. Gate Evaluation

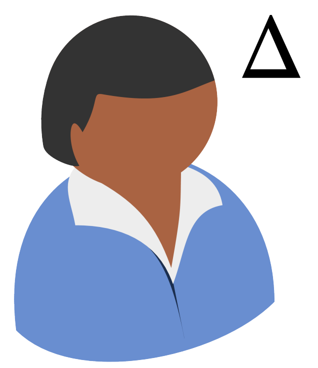
$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$g_i \oplus e_i = v_i$$

$$k_i + \ell_i = \Delta \cdot e_i \pmod p$$

Evaluator

1. Packing

$$V_R = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_L = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

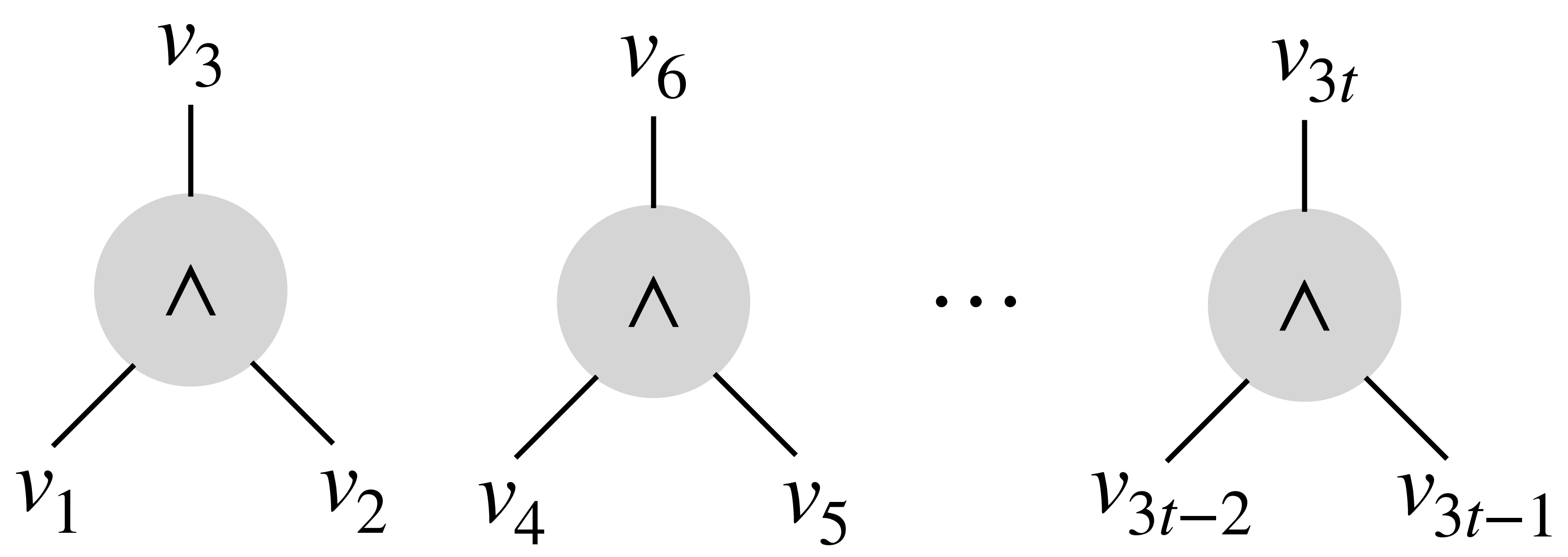
$$V = V_1 \cdot V_2$$

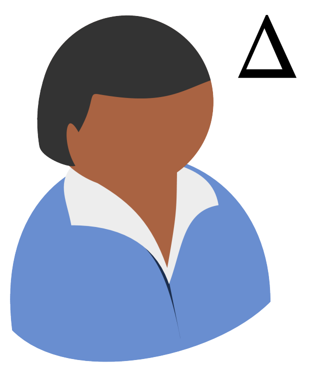
Evaluate over shares of packed encodings using multiplication gadget

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$g_i \oplus e_i = v_i$$

$$k_i + \ell_i = \Delta \cdot e_i \pmod{p}$$

Evaluator

1. Packing

$$V_R = \text{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_L = \text{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

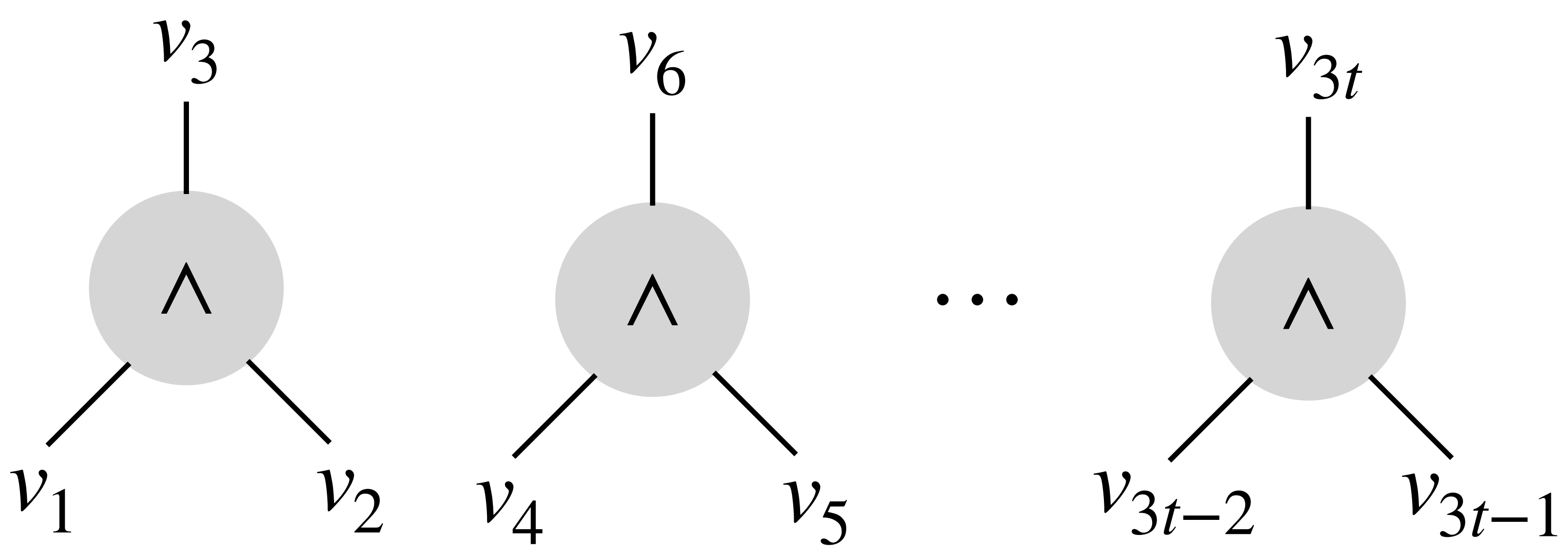
3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \text{UnPack}(V)$$

$$G_R = \text{Pack}(g_2, g_5, \ldots, g_{3t-1})$$

$$E_R = \text{Pack}(e_2, e_5, \ldots, e_{3t-1})$$

Template for $\omega(1/\lambda)$-Rate Garbling



1. Packing

$$V_R = \text{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

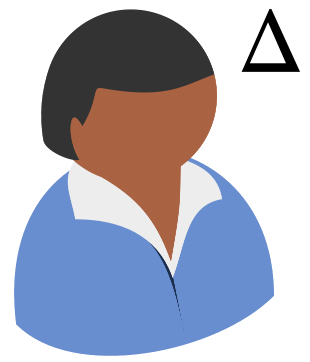
$$V_L = \text{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \text{UnPack}(V)$$



Garbler

Invariant

$$g_i \oplus e_i = v_i$$
$$k_i + \ell_i = \Delta \cdot e_i \pmod{p}$$

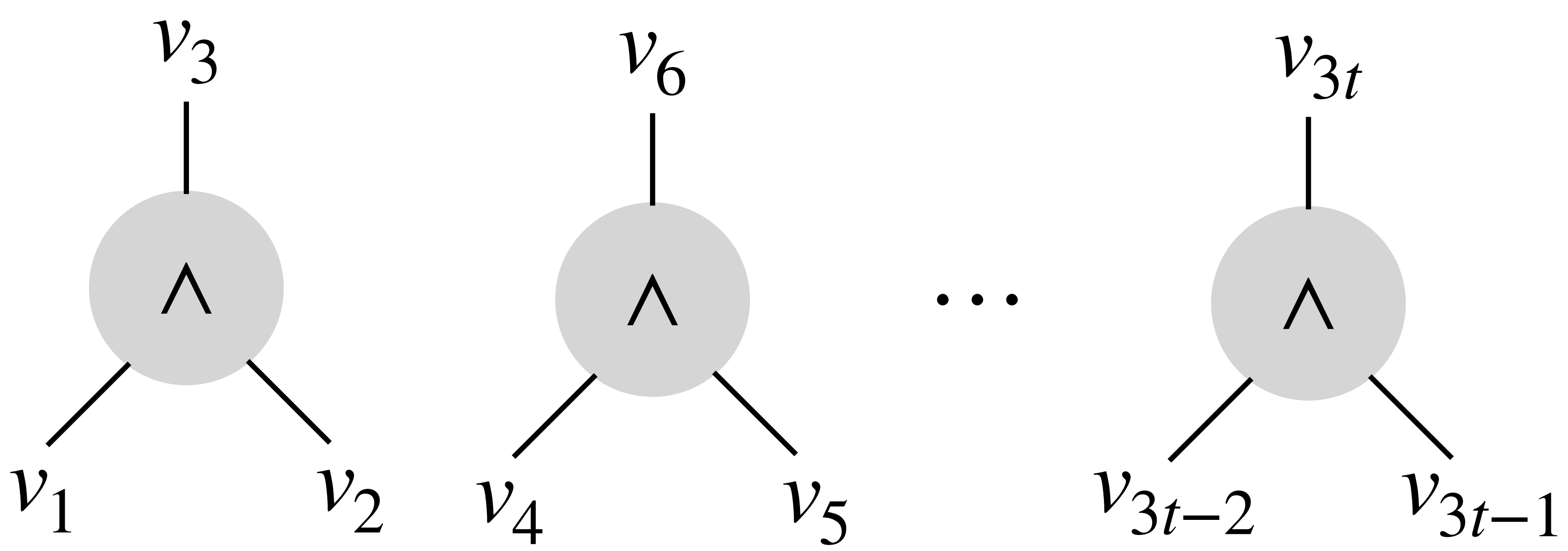
Evaluator

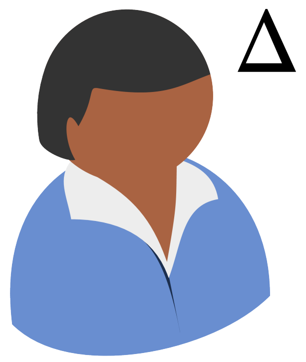
$G_R = \text{Pack}(g_2, g_5, \ldots, g_{3t-1})$

Pack is $\mathbb{F}_2$-linear

$E_R = \text{Pack}(e_2, e_5, \ldots, e_{3t-1})$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod p$$

Evaluator

1. Packing

$$V_\mathsf{R} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_\mathsf{L} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

$G_\mathsf{R} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$

Pack is $\mathbb{F}_2$-linear

$E_\mathsf{R} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$

We will need shares of $\Delta \cdot E_\mathsf{R}$ for evaluating the gate

K_R

$L_\mathsf{R} = \Delta \cdot E_\mathsf{R} - K_\mathsf{R} \pmod p$

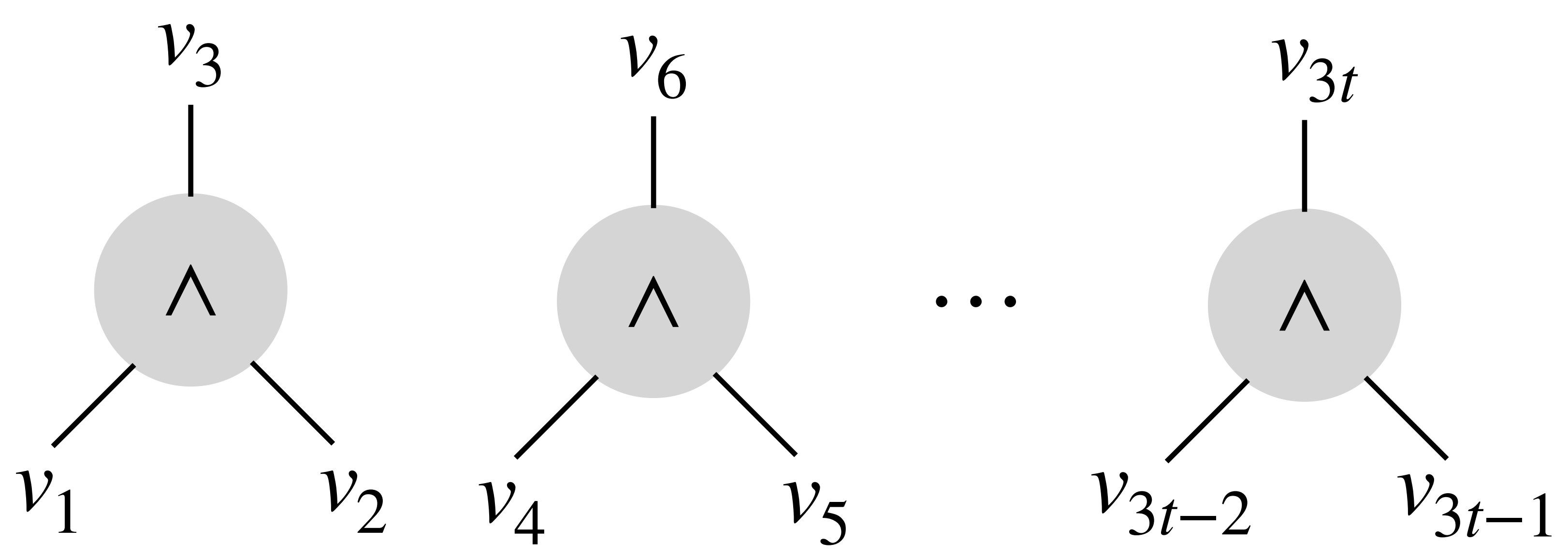
2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

Template for $\omega(1/\lambda)$-Rate Garbling



1. Packing

$$V_\mathsf{R} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

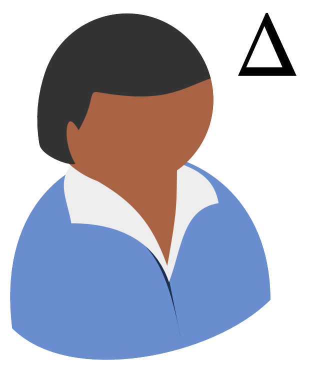
$$V_\mathsf{L} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$



Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod p$$

Evaluator

$$G_\mathsf{R} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$$

Pack is $\mathbb{F}_2$-linear

$$E_\mathsf{R} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$$

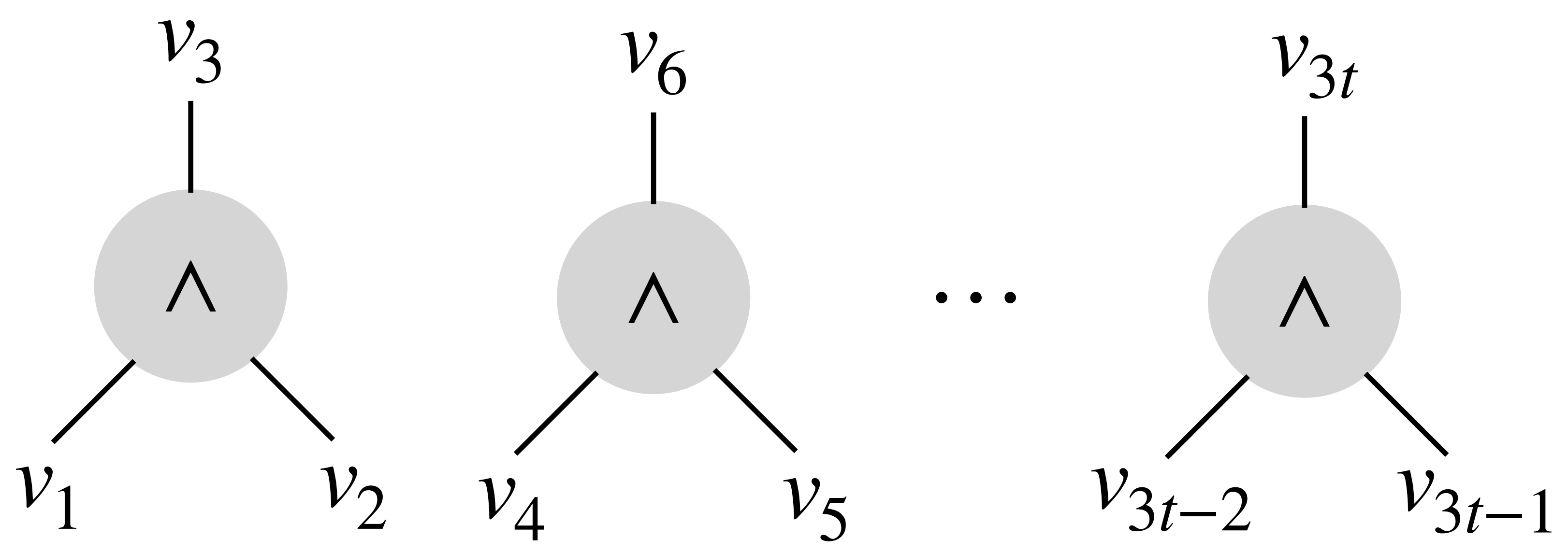
We will need shares of $\Delta \cdot E_\mathsf{R}$ for evaluating the gate

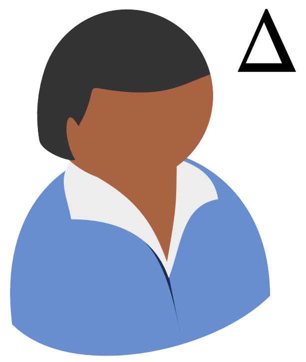
K_R

$$L_\mathsf{R} = \Delta \cdot E_\mathsf{R} - K_\mathsf{R} \pmod p$$

(k_i, ℓ_i) are shares over $\mathbb{Z}_p \Longrightarrow$ Can't use Pack directly

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod{p}$$

Evaluator

1. Packing

$$V_\mathsf{R} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_\mathsf{L} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

$$G_\mathsf{R} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$$

$$E_\mathsf{R} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$$

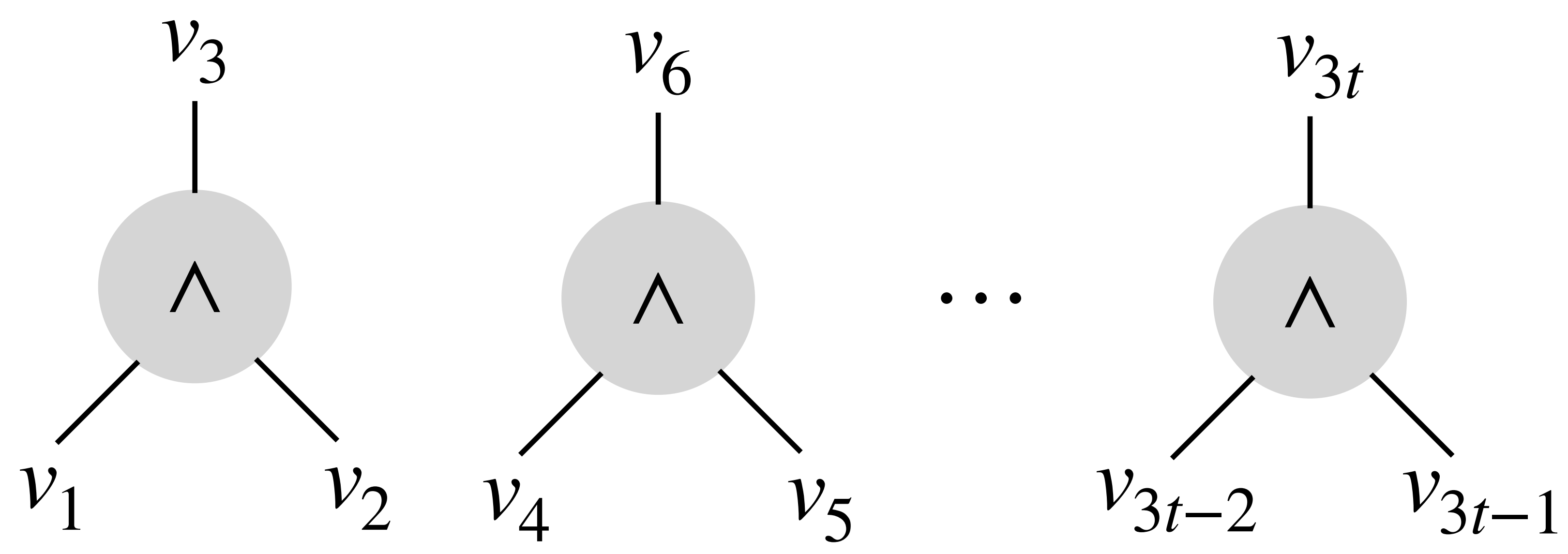
2. Gate Evaluation

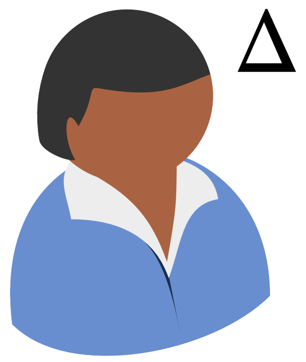
$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod{p}$$

Evaluator

1. Packing

$$V_{\mathsf{R}} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_{\mathsf{L}} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

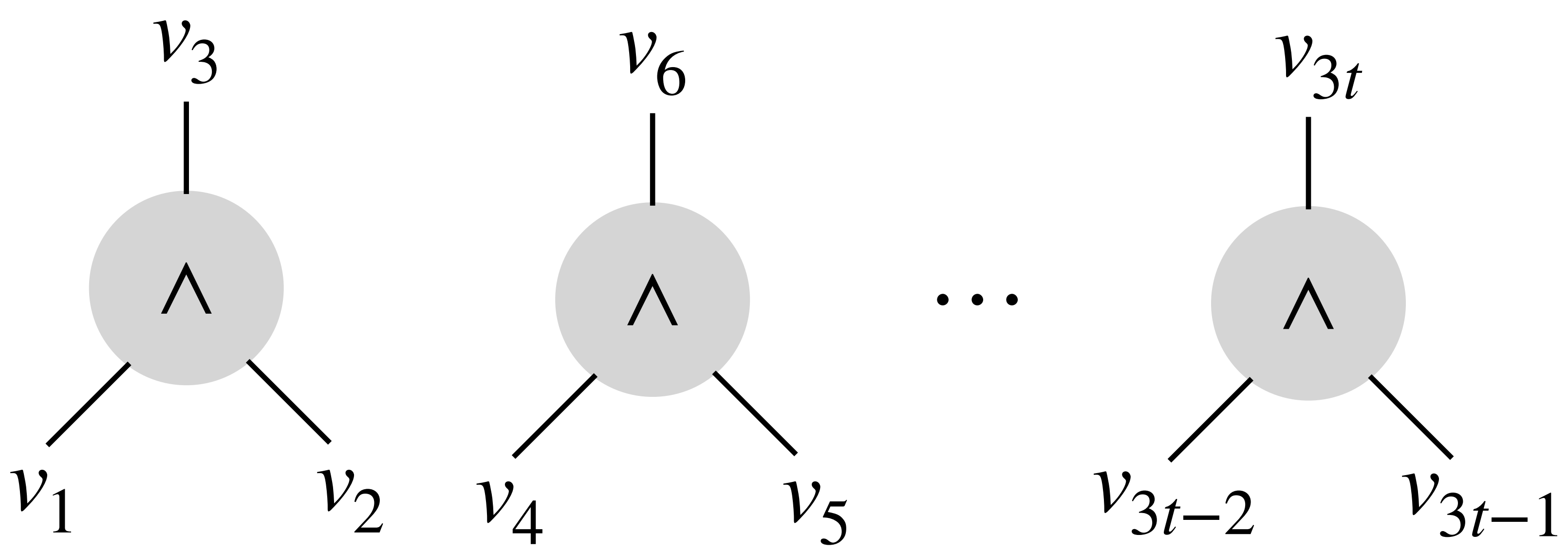
$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

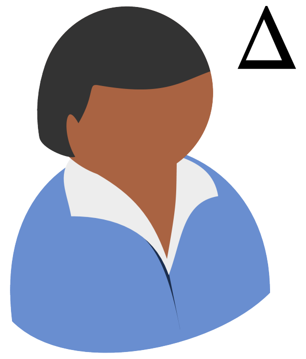
$$G_{\mathsf{R}} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$$

$$E_{\mathsf{R}} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$$

$$E^* = \sum_{i=1}^{t} \mathsf{e}_{3i-1} \cdot 2^i$$

Template for $\omega(1/\lambda)$-Rate Garbling





Δ

Garbler

Invariant

$$g_i \oplus e_i = v_i$$

$$k_i + \ell_i = \Delta \cdot e_i \pmod p$$

Evaluator

1. Packing

$$V_R = \text{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_L = \text{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \text{UnPack}(V)$$

$$G_R = \text{Pack}(g_2, g_5, \ldots, g_{3t-1})$$

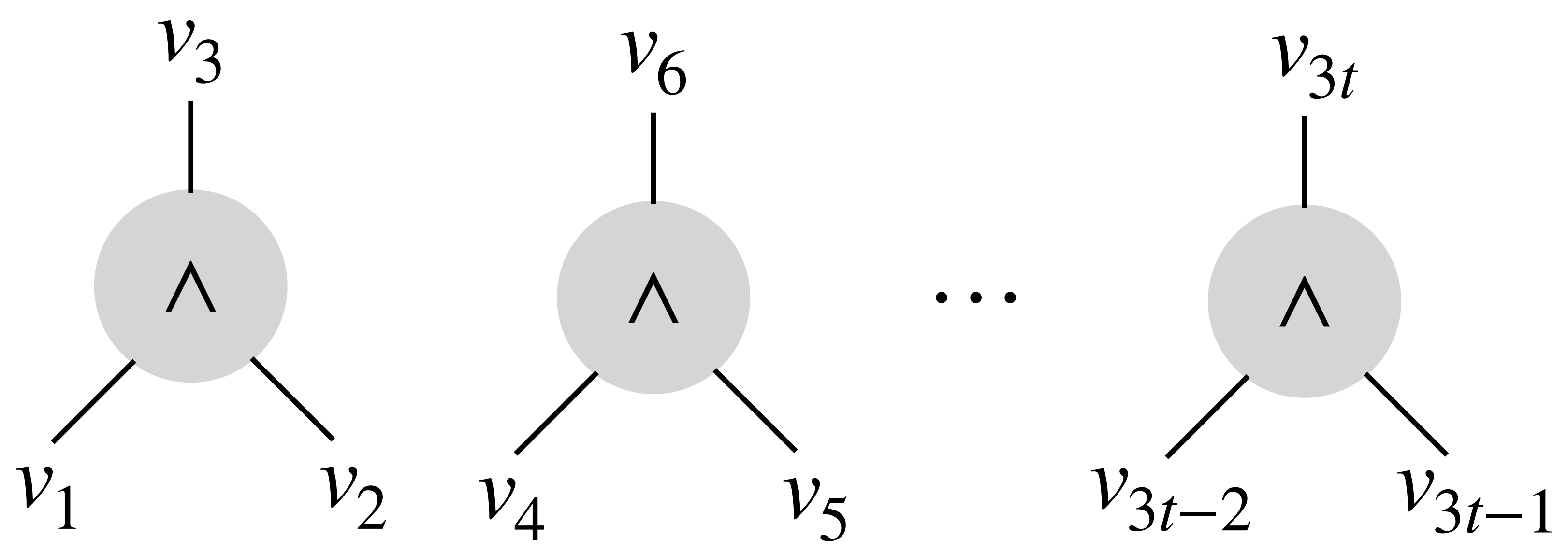
$$E_R = \text{Pack}(e_2, e_5, \ldots, e_{3t-1})$$

$$E^* = \sum_{i=1}^{t} e_{3i-1} \cdot 2^i$$

$$K^* = \sum_{i=1}^{t} k_{3i-1} \cdot 2^i \pmod p$$

$$L^* = \sum_{i=1}^{t} \ell_{3i-1} \cdot 2^i \pmod p$$

Template for $\omega(1/\lambda)$-Rate Garbling



1. Packing

$$V_{\mathrm{R}} = \mathrm{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

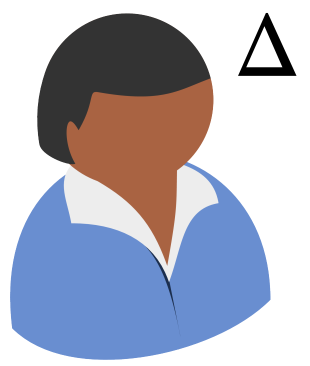
$$V_{\mathrm{L}} = \mathrm{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathrm{UnPack}(V)$$



Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod p$$

Evaluator

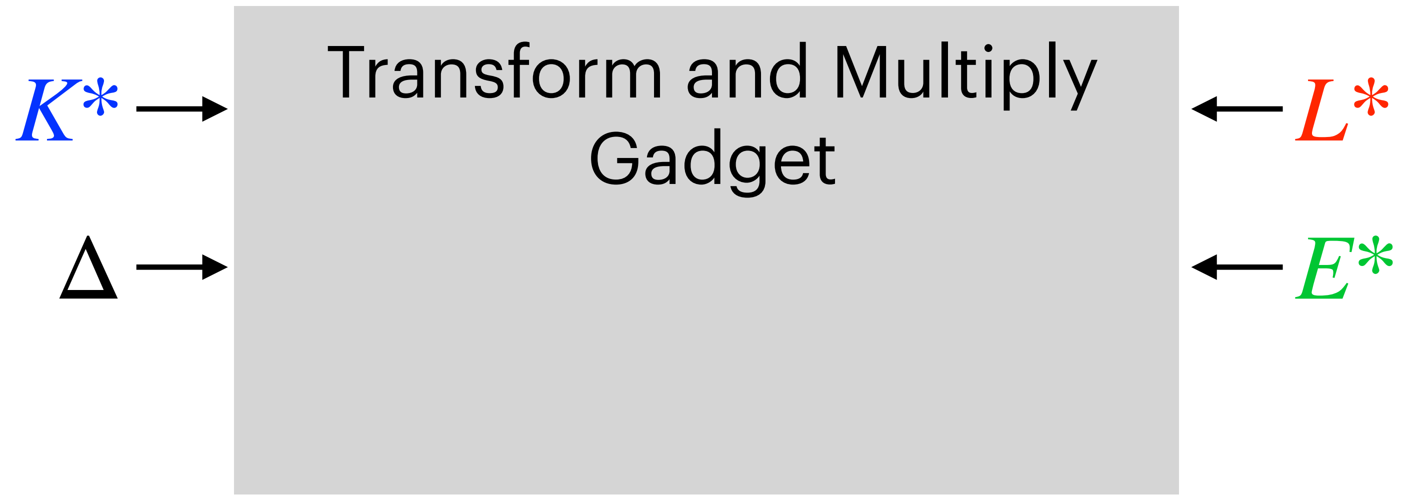
$$G_{\mathrm{R}} = \mathrm{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$$

$$E_{\mathrm{R}} = \mathrm{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$$

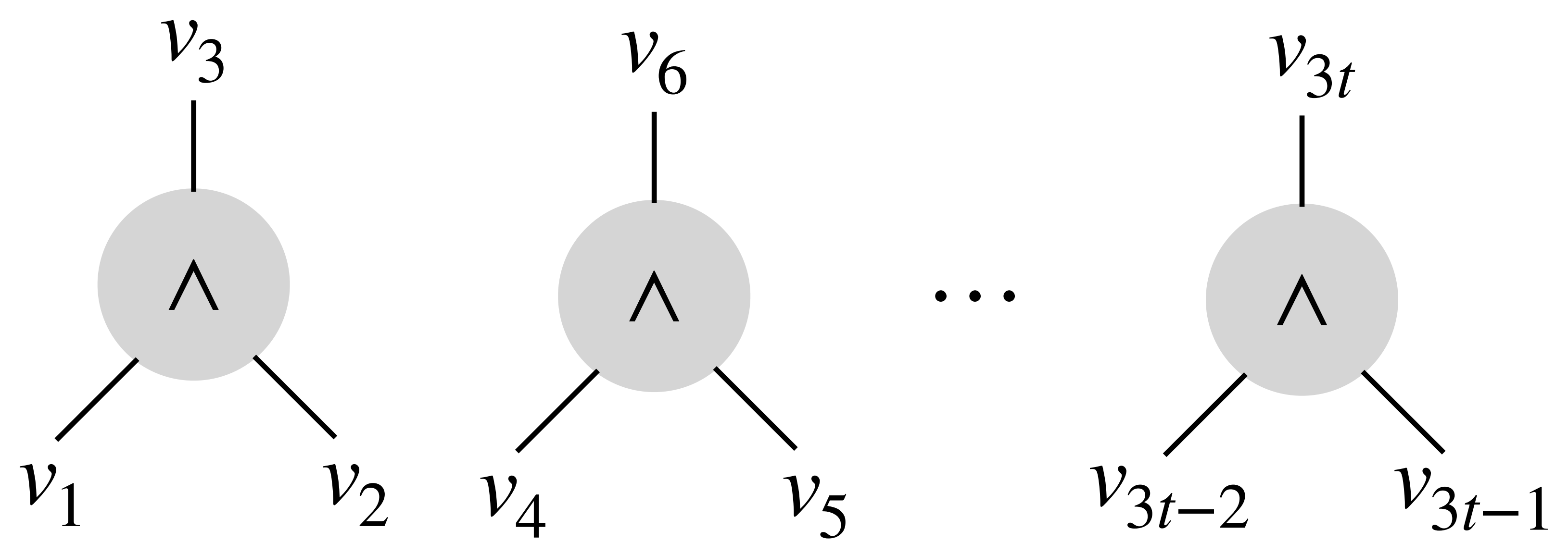
$$E^* = \sum_{i=1}^{t} \mathsf{e}_{3i-1} \cdot 2^i$$

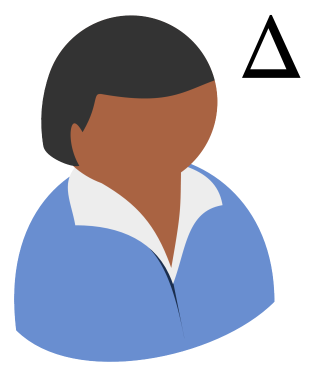
$$K^* = \sum_{i=1}^{t} k_{3i-1} \cdot 2^i \pmod p$$

$$L^* = \sum_{i=1}^{t} \ell_{3i-1} \cdot 2^i \pmod p$$



Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod{p}$$

Evaluator

1. Packing

$$V_{\mathsf{R}} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_{\mathsf{L}} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

$$G_{\mathsf{R}} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$$

$$E_{\mathsf{R}} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$$

$$E^* = \sum_{i=1}^{t} \mathsf{e}_{3i-1} \cdot 2^i$$

$$K^* = \sum_{i=1}^{t} k_{3i-1} \cdot 2^i \pmod{p}$$

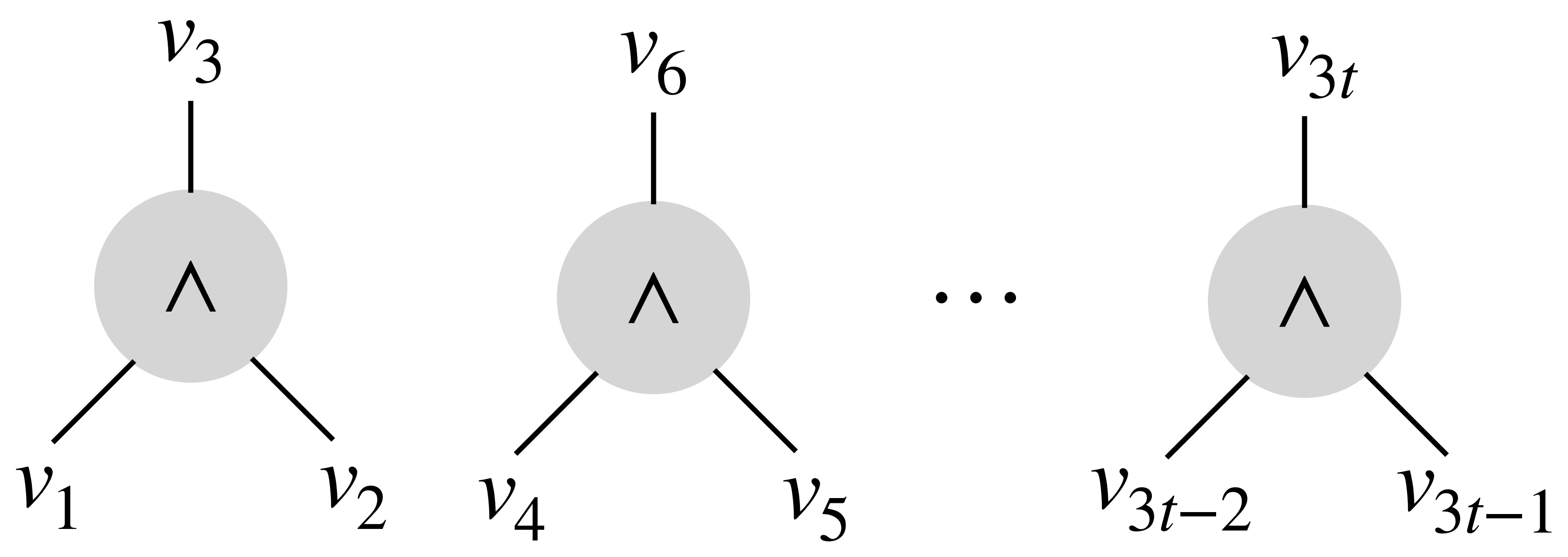
$$L^* = \sum_{i=1}^{t} \ell_{3i-1} \cdot 2^i \pmod{p}$$

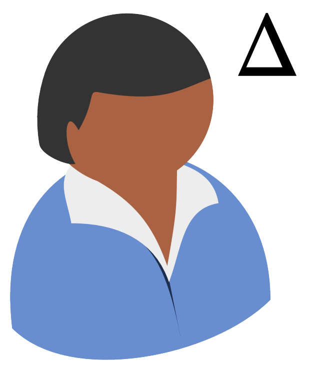
Transform and Multiply Gadget

$$K^* \rightarrow \qquad \leftarrow L^* = \Delta \cdot E^* - K^* \pmod{p}$$

$$\Delta \rightarrow \qquad \leftarrow E^*$$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$g_i \oplus e_i = v_i$

$k_i + \ell_i = \Delta \cdot e_i \pmod{p}$

Evaluator

1. Packing

$V_R = \text{Pack}(v_2, v_5, \ldots, v_{3t-1})$

$V_L = \text{Pack}(v_1, v_4, \ldots, v_{3t-2})$

$G_R = \text{Pack}(g_2, g_5, \ldots, g_{3t-1})$

$E_R = \text{Pack}(e_2, e_5, \ldots, e_{3t-1})$

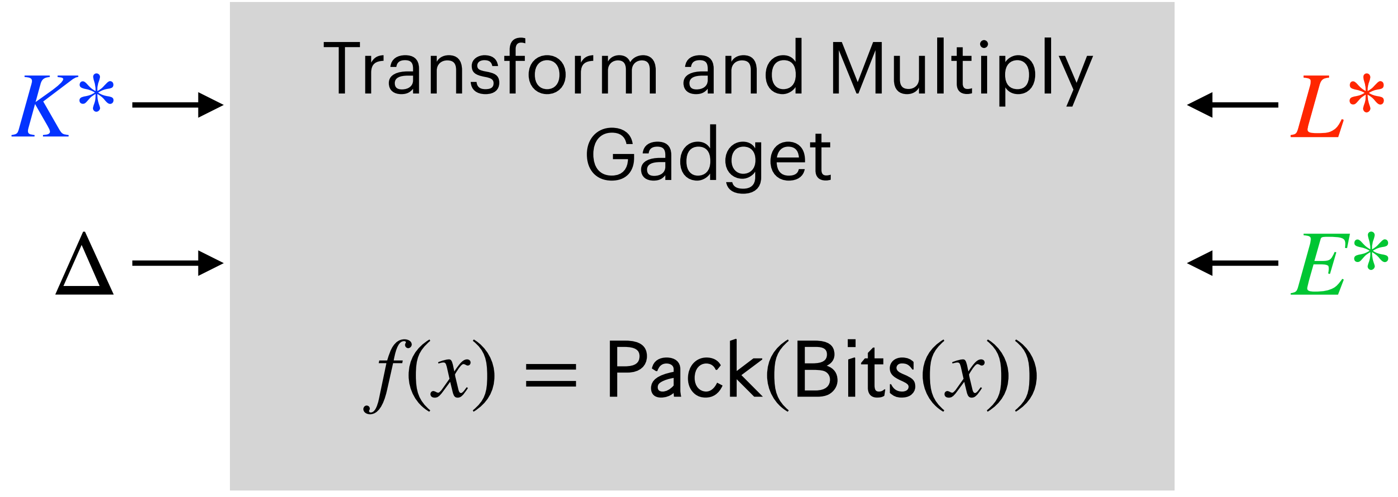
$$E^* = \sum_{i=1}^{t} e_{3i-1} \cdot 2^i$$

$$K^* = \sum_{i=1}^{t} k_{3i-1} \cdot 2^i \pmod{p}$$

$$L^* = \sum_{i=1}^{t} \ell_{3i-1} \cdot 2^i \pmod{p}$$

2. Gate Evaluation

$V = V_1 \cdot V_2$



Transform and Multiply Gadget

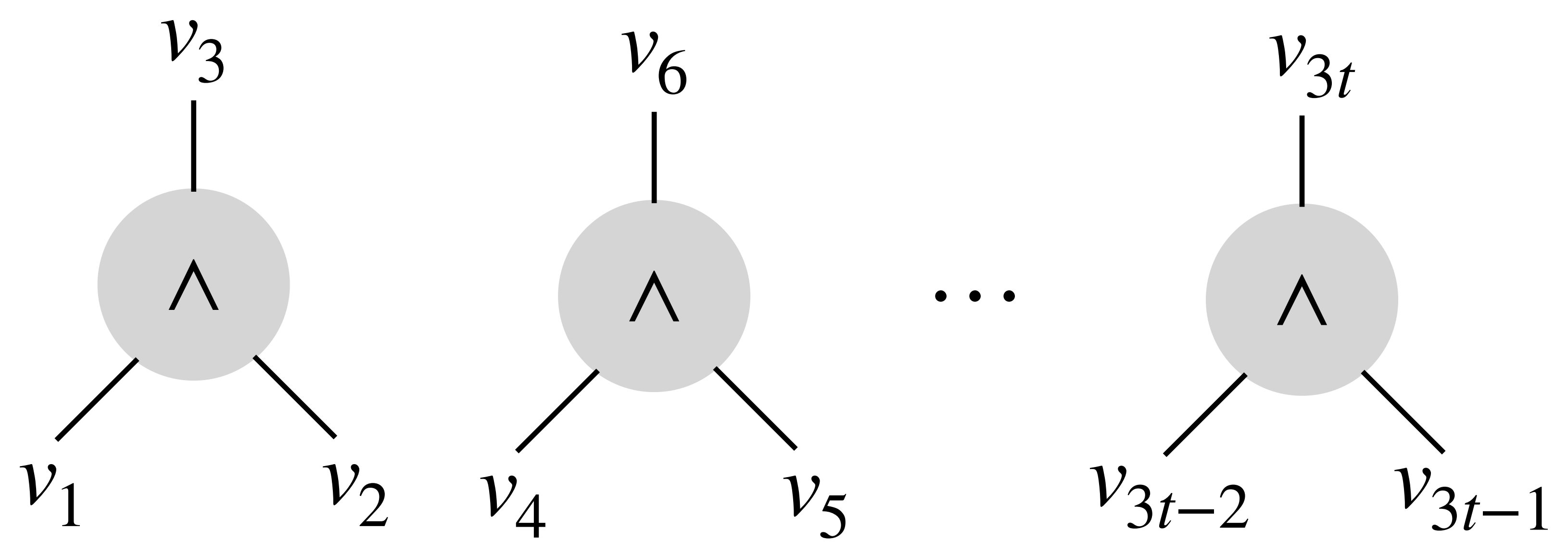
$f(x) = \text{Pack}(\text{Bits}(x))$

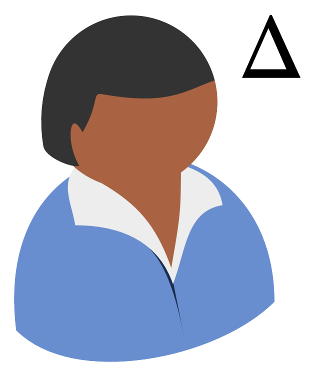
$L^* = \Delta \cdot E^* - K^* \pmod{p}$

3. Unpacking

$(v_3, v_6, \ldots, v_{3t}) = \text{UnPack}(V)$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$g_i \oplus e_i = v_i$$

$$k_i + \ell_i = \Delta \cdot e_i \pmod{p}$$

Evaluator

1. Packing

$$V_R = \text{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_L = \text{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \text{UnPack}(V)$$

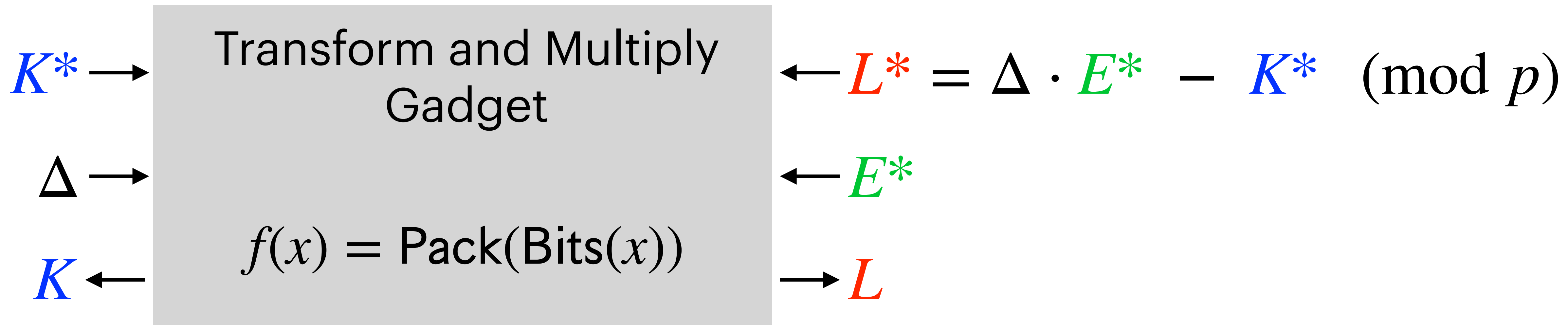
$$G_R = \text{Pack}(g_2, g_5, \ldots, g_{3t-1})$$

$$E_R = \text{Pack}(e_2, e_5, \ldots, e_{3t-1})$$

$$E^* = \sum_{i=1}^{t} e_{3i-1} \cdot 2^i$$

$$K^* = \sum_{i=1}^{t} k_{3i-1} \cdot 2^i \pmod{p}$$

$$L^* = \sum_{i=1}^{t} \ell_{3i-1} \cdot 2^i \pmod{p}$$



Transform and Multiply Gadget

$$f(x) = \text{Pack}(\text{Bits}(x))$$

$$K^* \rightarrow$$

$$\Delta \rightarrow$$

$$K \leftarrow$$

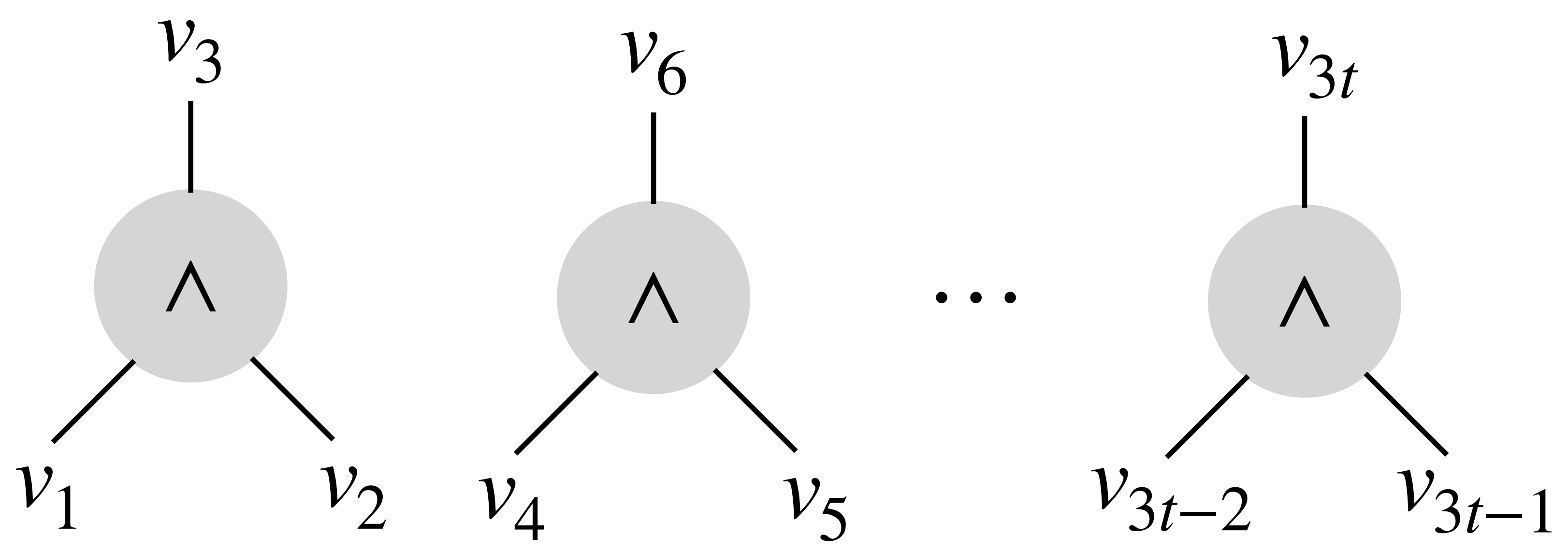
$$\leftarrow L^* = \Delta \cdot E^* - K^* \pmod{p}$$

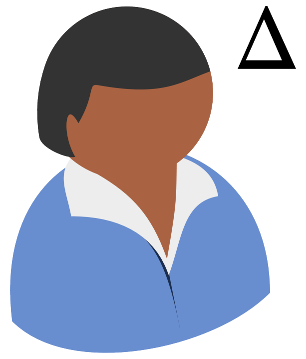
$$\leftarrow E^*$$

$$\rightarrow L$$

$$K + L = \Delta \cdot f(E^*)$$
$$= \Delta \cdot E_R$$

Template for $\omega(1/\lambda)$-Rate Garbling




Δ

Garbler

Invariant

$$g_i \oplus e_i = v_i$$

$$k_i + \ell_i = \Delta \cdot e_i \pmod p$$

Evaluator

1. Packing

$$G_R = \text{Pack}(g_2, g_5, \ldots, g_{3t-1})$$

$$E_R = \text{Pack}(e_2, e_5, \ldots, e_{3t-1})$$

$$V_R = \text{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_L = \text{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

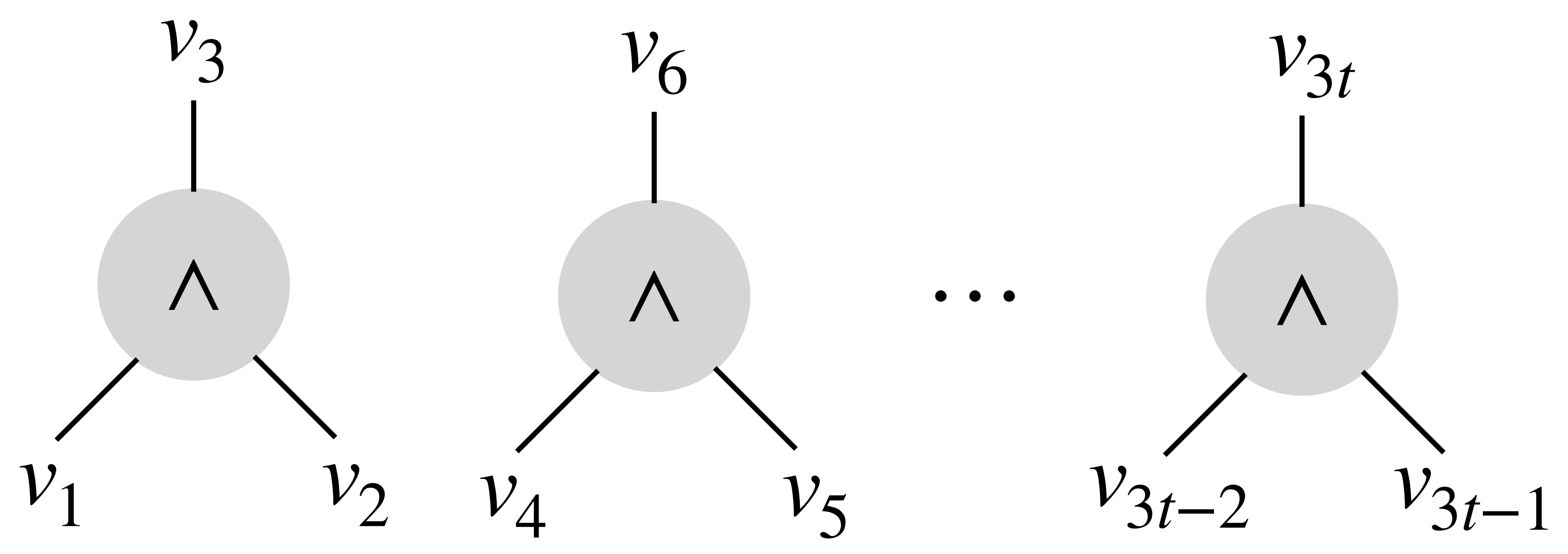
2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \text{UnPack}(V)$$

Template for $\omega(1/\lambda)$-Rate Garbling



1. Packing

$$V_\mathsf{R} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

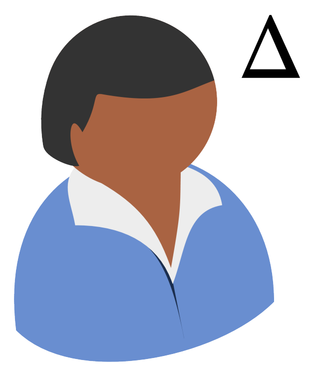
$$V_\mathsf{L} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$



Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod{p}$$

Evaluator

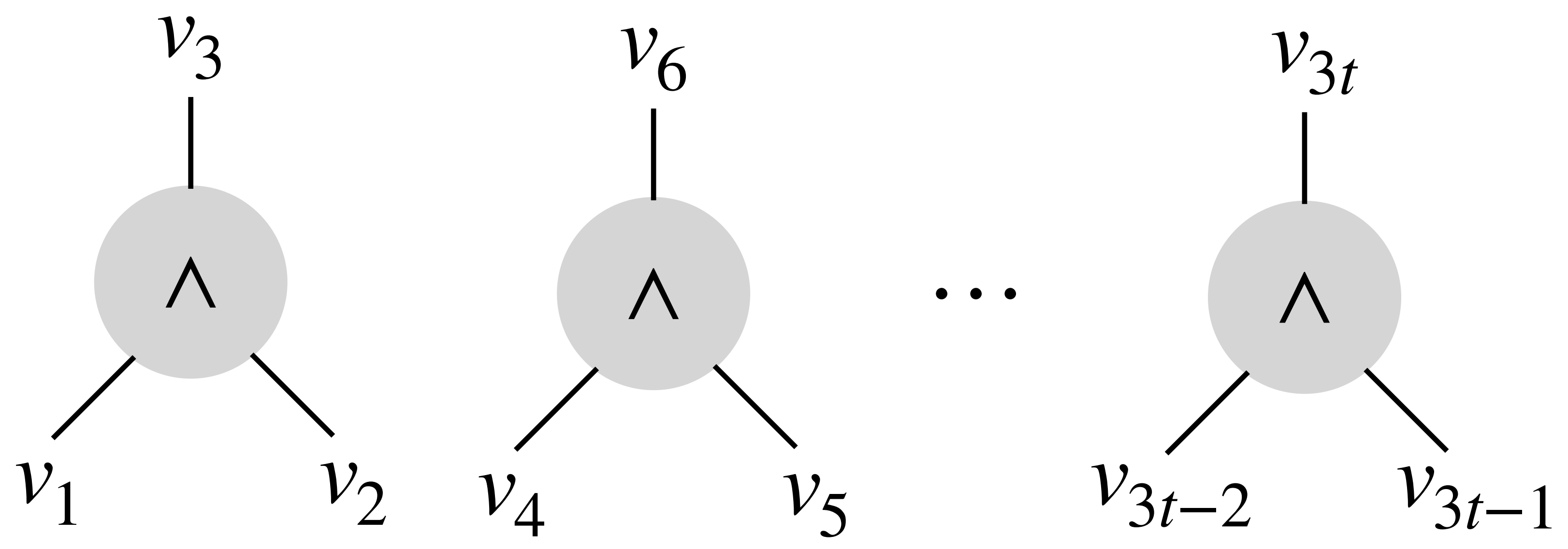
$$G_\mathsf{R} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$$

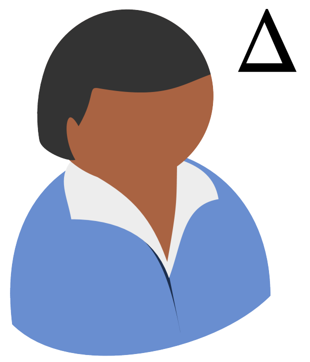
$$E_\mathsf{R} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$$

$$K_\mathsf{R}$$

$$L_\mathsf{R} = \Delta \cdot E_\mathsf{R} - K_\mathsf{R} \pmod{p}$$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$g_i \oplus e_i = v_i$$

$$k_i + \ell_i = \Delta \cdot e_i \pmod{p}$$

Evaluator

1. Packing

$$V_R = \text{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_L = \text{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

$$G_R = \text{Pack}(g_2, g_5, \ldots, g_{3t-1})$$

$$G_L = \text{Pack}(g_1, g_4, \ldots, g_{3t-2})$$

K_L K_R

$$E_R = \text{Pack}(e_2, e_5, \ldots, e_{3t-1})$$

$$E_L = \text{Pack}(e_1, e_4, \ldots, e_{3t-2})$$

$$L_R = \Delta \cdot E_R - K_R \pmod{p}$$

$$L_L = \Delta \cdot E_L - K_L \pmod{p}$$

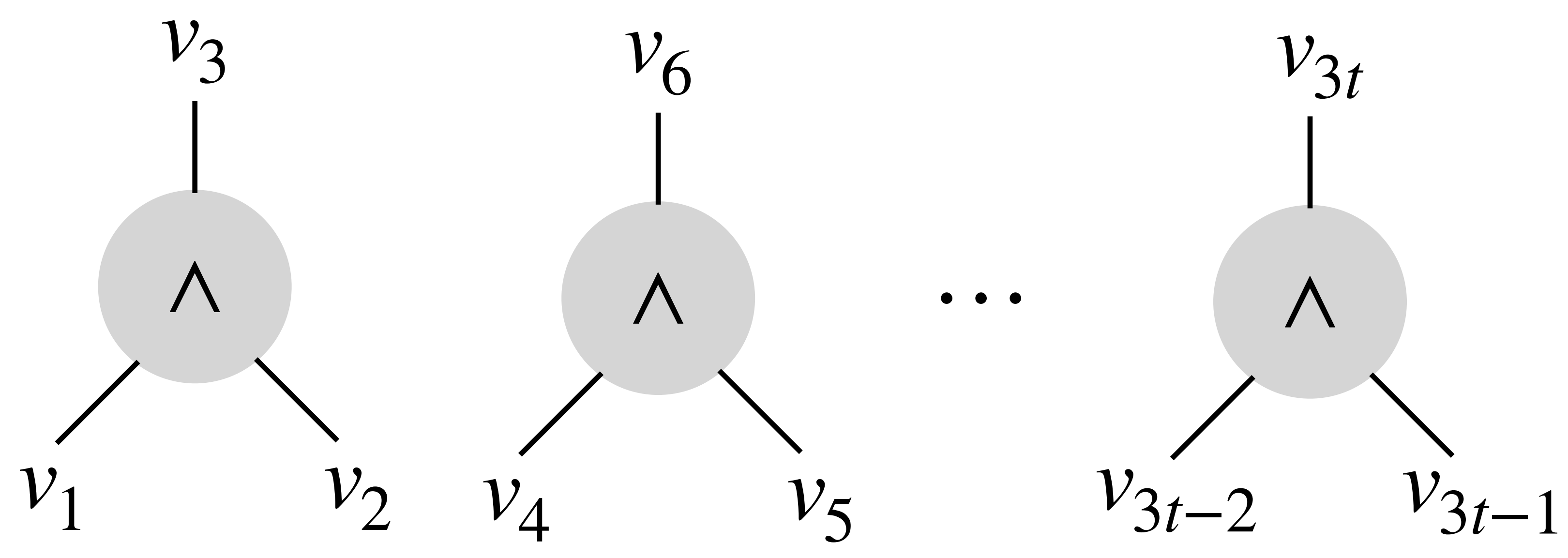
2. Gate Evaluation

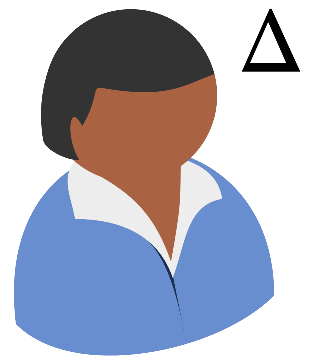
$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \text{UnPack}(V)$$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod{p}$$

Evaluator

1. Packing

$$V_\mathsf{R} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$
$$V_\mathsf{L} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

$$G_\mathsf{R} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$$
$$G_\mathsf{L} = \mathsf{Pack}(\mathsf{g}_1, \mathsf{g}_4, \ldots, \mathsf{g}_{3t-2})$$

K_L K_R

$$E_\mathsf{R} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$$
$$E_\mathsf{L} = \mathsf{Pack}(\mathsf{e}_1, \mathsf{e}_4, \ldots, \mathsf{e}_{3t-2})$$

$$L_\mathsf{R} = \Delta \cdot E_\mathsf{R} - K_\mathsf{R} \pmod{p}$$
$$L_\mathsf{L} = \Delta \cdot E_\mathsf{L} - K_\mathsf{L} \pmod{p}$$

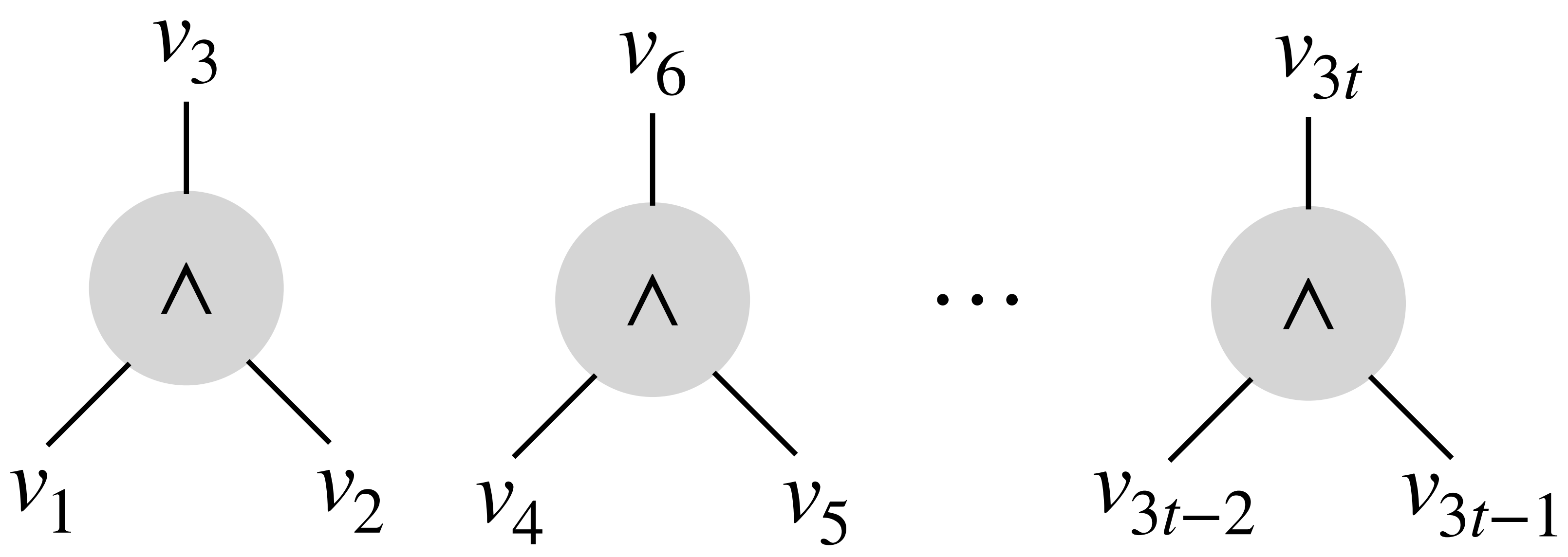
2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

Template for $\omega(1/\lambda)$-Rate Garbling



1. Packing

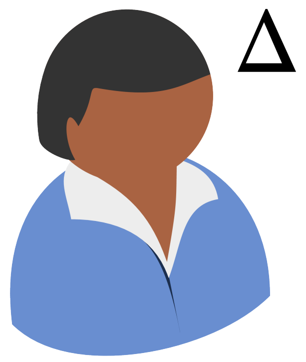
$$V_\mathsf{R} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$
$$V_\mathsf{L} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$



Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$
$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod{p}$$

Evaluator

$$G_\mathsf{R} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$$
$$G_\mathsf{L} = \mathsf{Pack}(\mathsf{g}_1, \mathsf{g}_4, \ldots, \mathsf{g}_{3t-2})$$

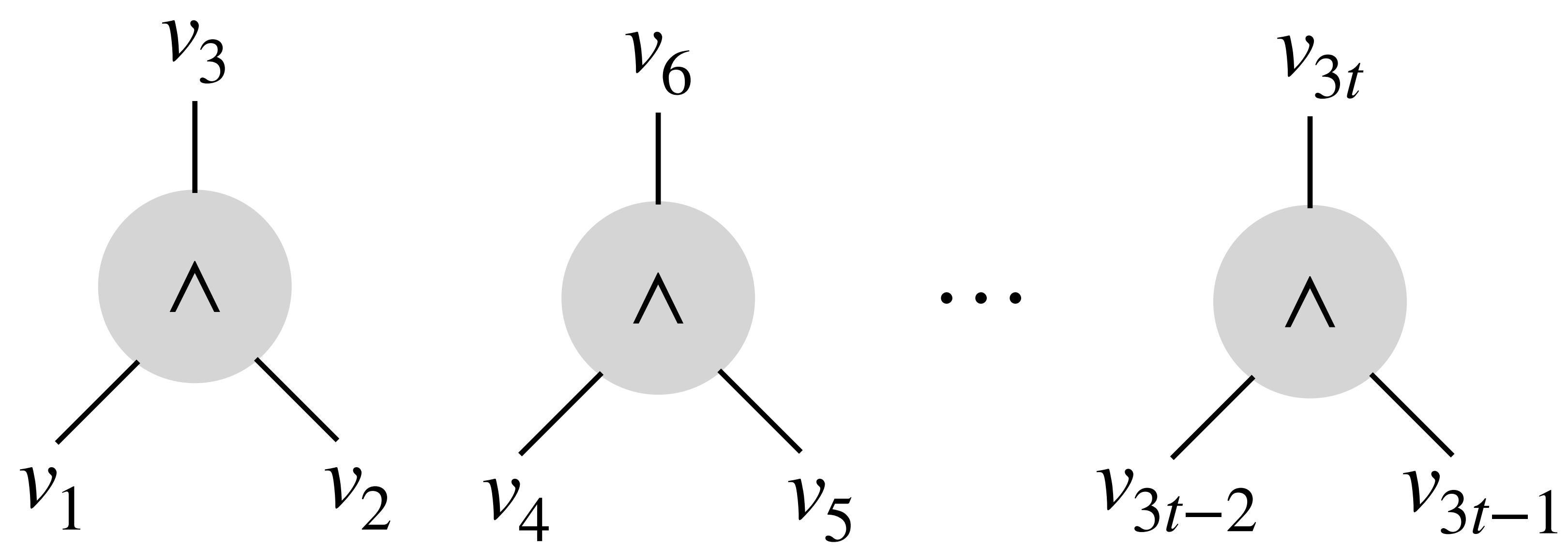
$$E_\mathsf{R} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$$
$$E_\mathsf{L} = \mathsf{Pack}(\mathsf{e}_1, \mathsf{e}_4, \ldots, \mathsf{e}_{3t-2})$$

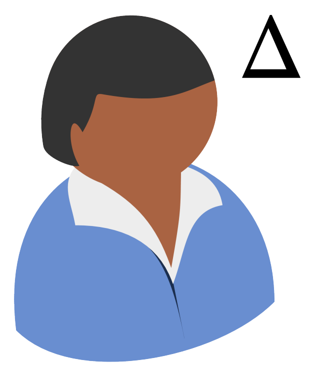
K_L K_R

$$L_\mathsf{R} = \Delta \cdot E_\mathsf{R} - K_\mathsf{R} \pmod{p}$$
$$L_\mathsf{L} = \Delta \cdot E_\mathsf{L} - K_\mathsf{L} \pmod{p}$$

$$V = V_1 \cdot V_2 = (G_\mathsf{L} + E_\mathsf{L}) \cdot (G_\mathsf{R} + E_\mathsf{R})$$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$
$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod{p}$$

Evaluator

1. Packing

$$V_\mathsf{R} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$
$$V_\mathsf{L} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

$$G_\mathsf{R} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$$
$$G_\mathsf{L} = \mathsf{Pack}(\mathsf{g}_1, \mathsf{g}_4, \ldots, \mathsf{g}_{3t-2})$$

$$E_\mathsf{R} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$$
$$E_\mathsf{L} = \mathsf{Pack}(\mathsf{e}_1, \mathsf{e}_4, \ldots, \mathsf{e}_{3t-2})$$

K_L K_R

$$L_\mathsf{R} = \Delta \cdot E_\mathsf{R} - K_\mathsf{R} \pmod{p}$$
$$L_\mathsf{L} = \Delta \cdot E_\mathsf{L} - K_\mathsf{L} \pmod{p}$$

2. Gate Evaluation

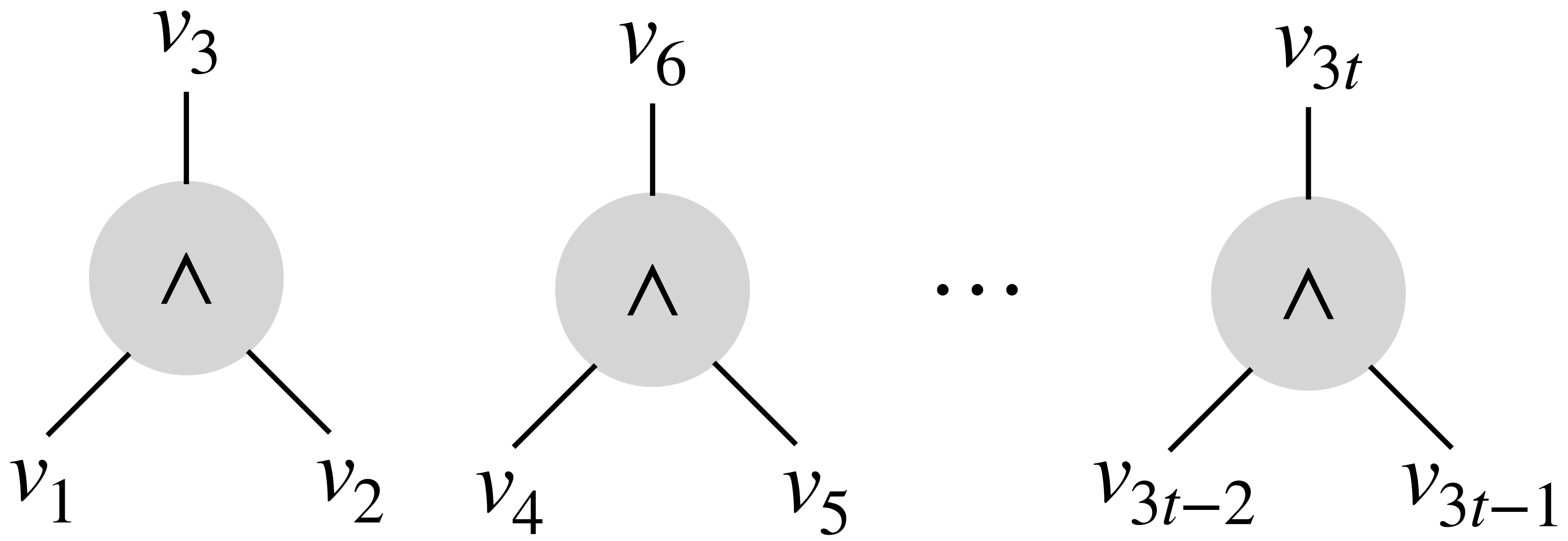
$$V = V_1 \cdot V_2$$

$$V = V_1 \cdot V_2 = (G_\mathsf{L} + E_\mathsf{L}) \cdot (G_\mathsf{R} + E_\mathsf{R})$$
$$= G_\mathsf{L} \cdot G_\mathsf{R} + G_\mathsf{L} \cdot E_\mathsf{R} + E_\mathsf{L} \cdot G_\mathsf{R} + E_\mathsf{L} \cdot E_\mathsf{R}$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

Template for $\omega(1/\lambda)$-Rate Garbling



1. Packing

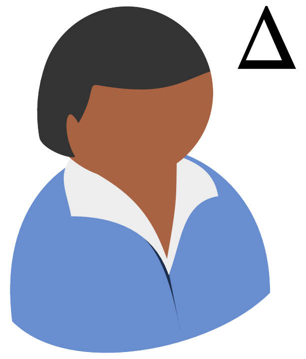
$$V_R = \text{Pack}(v_2, v_5, \ldots, v_{3t-1})$$
$$V_L = \text{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \text{UnPack}(V)$$



Δ

Garbler

Invariant

$$g_i \oplus e_i = v_i$$
$$k_i + \ell_i = \Delta \cdot e_i \pmod p$$

Evaluator

$$G_R = \text{Pack}(g_2, g_5, \ldots, g_{3t-1})$$
$$G_L = \text{Pack}(g_1, g_4, \ldots, g_{3t-2})$$

$K_L \quad K_R$

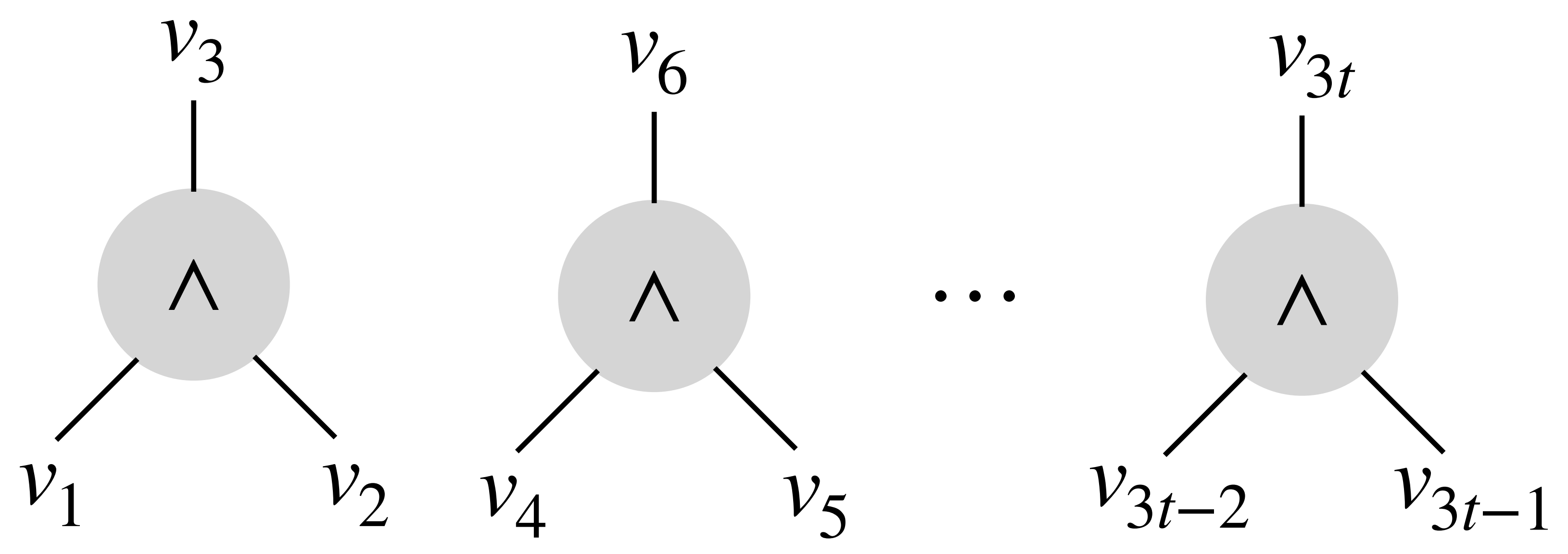
$$E_R = \text{Pack}(e_2, e_5, \ldots, e_{3t-1})$$
$$E_L = \text{Pack}(e_1, e_4, \ldots, e_{3t-2})$$

$$L_R = \Delta \cdot E_R - K_R \pmod p$$
$$L_L = \Delta \cdot E_L - K_L \pmod p$$

$$V = V_1 \cdot V_2 = (G_L + E_L) \cdot (G_R + E_R)$$
$$= G_L \cdot G_R + G_L \cdot E_R + E_L \cdot G_R + E_L \cdot E_R$$

Packed encoding is homomorphic

Template for $\omega(1/\lambda)$-Rate Garbling



Garbler Δ

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod p$$

Evaluator

1. Packing

$$V_\mathsf{R} = \text{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_\mathsf{L} = \text{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

$$G_\mathsf{R} = \text{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$$

$$G_\mathsf{L} = \text{Pack}(\mathsf{g}_1, \mathsf{g}_4, \ldots, \mathsf{g}_{3t-2})$$

$$E_\mathsf{R} = \text{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$$

$$E_\mathsf{L} = \text{Pack}(\mathsf{e}_1, \mathsf{e}_4, \ldots, \mathsf{e}_{3t-2})$$

$$\xrightarrow{\quad}$$

K_L K_R

$$L_\mathsf{R} = \Delta \cdot E_\mathsf{R} - K_\mathsf{R} \pmod p$$

$$L_\mathsf{L} = \Delta \cdot E_\mathsf{L} - K_\mathsf{L} \pmod p$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

$$V = V_1 \cdot V_2 = (G_\mathsf{L} + E_\mathsf{L}) \cdot (G_\mathsf{R} + E_\mathsf{R})$$

Packed encoding is homomorphic

$$= G_\mathsf{L} \cdot G_\mathsf{R} + G_\mathsf{L} \cdot E_\mathsf{R} + E_\mathsf{L} \cdot G_\mathsf{R} + E_\mathsf{L} \cdot E_\mathsf{R}$$

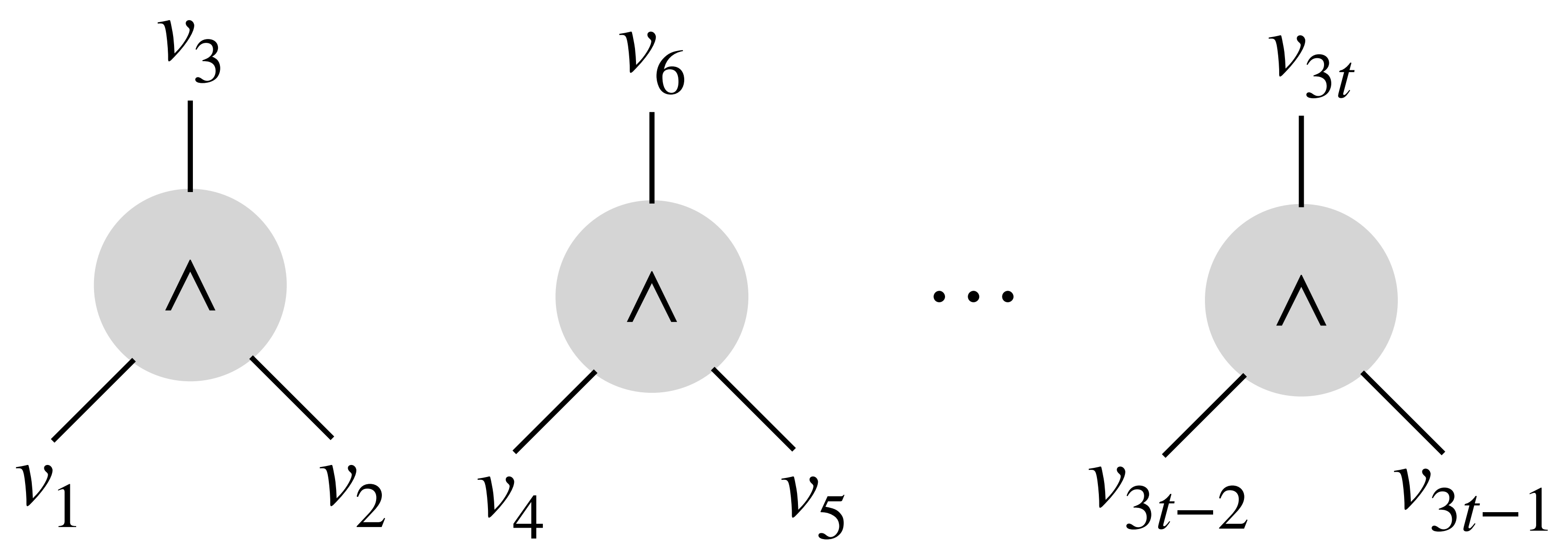
Locally computed by Garbler

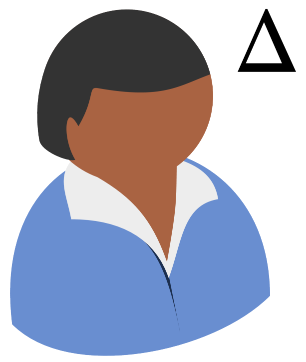
Locally computed by Evaluator

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \text{UnPack}(V)$$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$
$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod{p}$$

Evaluator

1. Packing

$$V_\mathsf{R} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$
$$V_\mathsf{L} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

$$G_\mathsf{R} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$$
$$G_\mathsf{L} = \mathsf{Pack}(\mathsf{g}_1, \mathsf{g}_4, \ldots, \mathsf{g}_{3t-2})$$

$$E_\mathsf{R} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$$
$$E_\mathsf{L} = \mathsf{Pack}(\mathsf{e}_1, \mathsf{e}_4, \ldots, \mathsf{e}_{3t-2})$$

K_L K_R

$$L_\mathsf{R} = \Delta \cdot E_\mathsf{R} - K_\mathsf{R} \pmod{p}$$
$$L_\mathsf{L} = \Delta \cdot E_\mathsf{L} - K_\mathsf{L} \pmod{p}$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

$$V = V_1 \cdot V_2 = (G_\mathsf{L} + E_\mathsf{L}) \cdot (G_\mathsf{R} + E_\mathsf{R})$$

Packed encoding is homomorphic

$$= G_\mathsf{L} \cdot G_\mathsf{R} + G_\mathsf{L} \cdot E_\mathsf{R} + E_\mathsf{L} \cdot G_\mathsf{R} + E_\mathsf{L} \cdot E_\mathsf{R}$$

Locally computed by Garbler

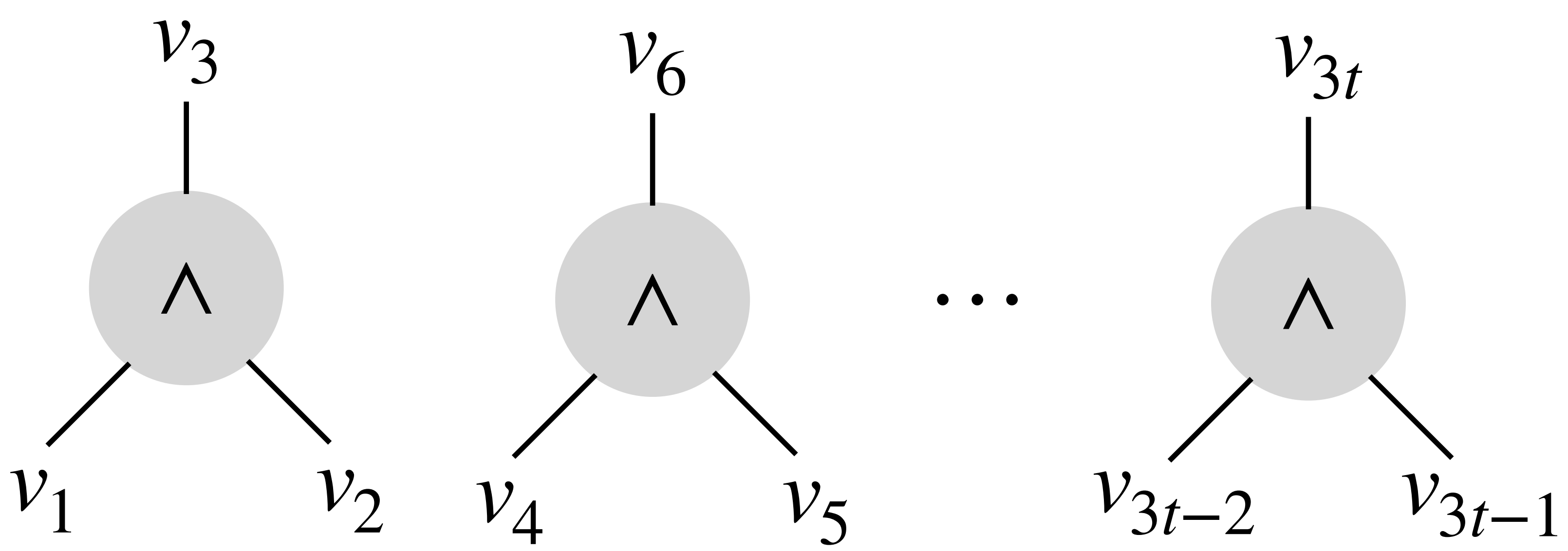
Use multiplication gadget

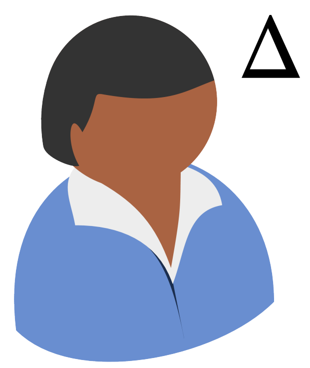
Locally computed by Evaluator

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod{p}$$

Evaluator

1. Packing

$$V_{\mathsf{R}} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_{\mathsf{L}} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

$$G_{\mathsf{R}} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$$

$$G_{\mathsf{L}} = \mathsf{Pack}(\mathsf{g}_1, \mathsf{g}_4, \ldots, \mathsf{g}_{3t-2})$$

$$E_{\mathsf{R}} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$$

$$E_{\mathsf{L}} = \mathsf{Pack}(\mathsf{e}_1, \mathsf{e}_4, \ldots, \mathsf{e}_{3t-2})$$

K_{L} K_{R}

$$L_{\mathsf{R}} = \Delta \cdot E_{\mathsf{R}} - K_{\mathsf{R}} \pmod{p}$$

$$L_{\mathsf{L}} = \Delta \cdot E_{\mathsf{L}} - K_{\mathsf{L}} \pmod{p}$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

$$V = V_1 \cdot V_2 = (G_{\mathsf{L}} + E_{\mathsf{L}}) \cdot (G_{\mathsf{R}} + E_{\mathsf{R}})$$

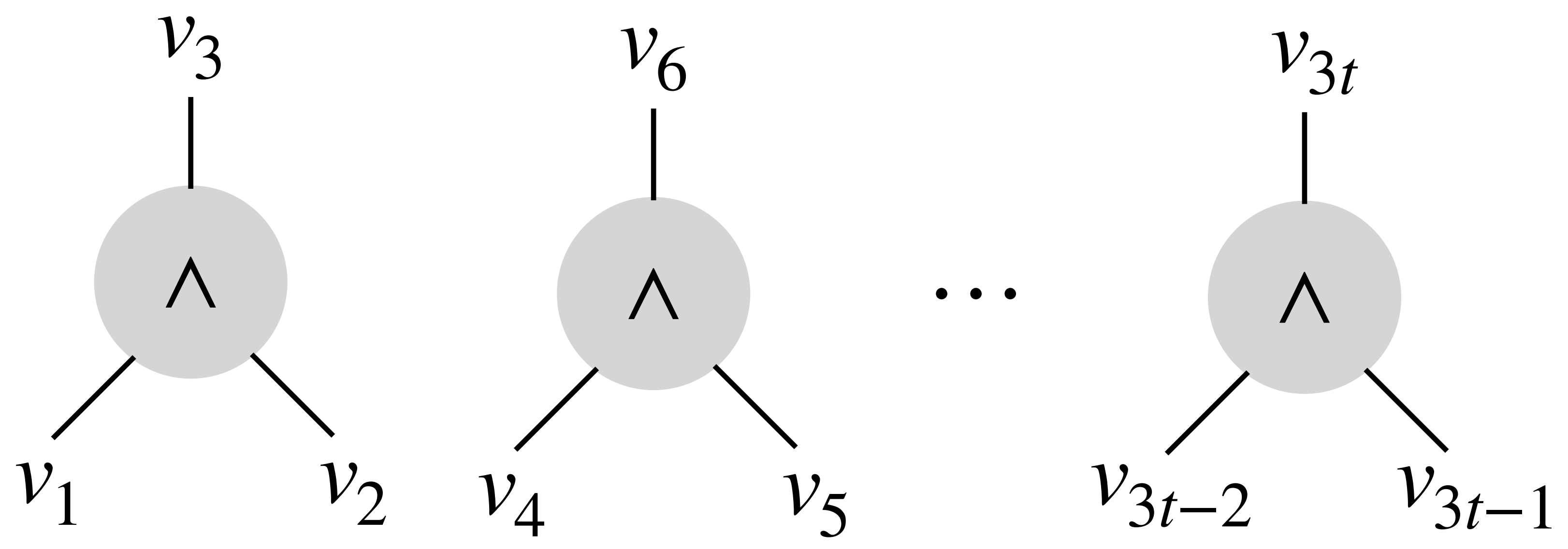
$$= G_{\mathsf{L}} \cdot G_{\mathsf{R}} + G_{\mathsf{L}} \cdot E_{\mathsf{R}} + E_{\mathsf{L}} \cdot G_{\mathsf{R}} + E_{\mathsf{L}} \cdot E_{\mathsf{R}}$$

Use multiplication gadget

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

Template for $\omega(1/\lambda)$-Rate Garbling



1. Packing

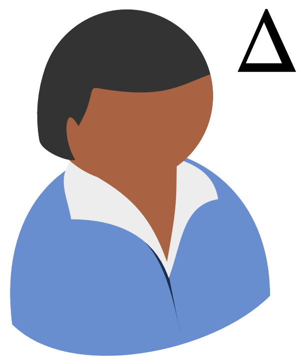
$$V_R = \text{Pack}(v_2, v_5, \ldots, v_{3t-1})$$
$$V_L = \text{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \text{UnPack}(V)$$



Δ

Garbler

Invariant

$$g_i \oplus e_i = v_i$$
$$k_i + \ell_i = \Delta \cdot e_i \pmod{p}$$

Evaluator

$$G_R = \text{Pack}(g_2, g_5, \ldots, g_{3t-1})$$
$$G_L = \text{Pack}(g_1, g_4, \ldots, g_{3t-2})$$

$K_L \quad K_R$

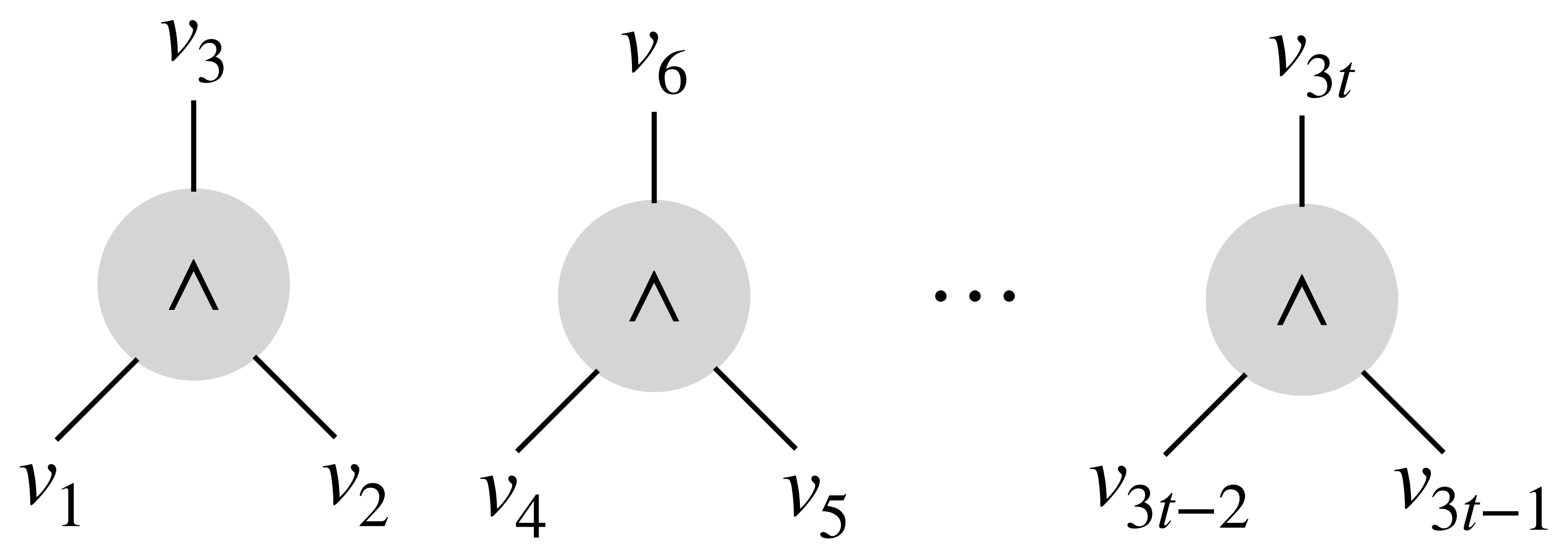
G

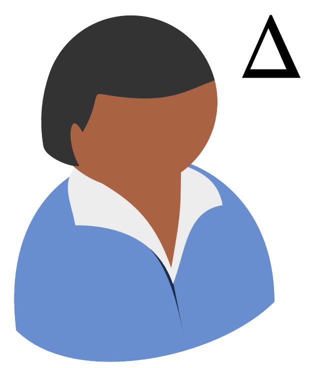
$$E_R = \text{Pack}(e_2, e_5, \ldots, e_{3t-1})$$
$$E_L = \text{Pack}(e_1, e_4, \ldots, e_{3t-2})$$

$$L_R = \Delta \cdot E_R - K_R \pmod{p}$$
$$L_L = \Delta \cdot E_L - K_L \pmod{p}$$

$$E = \text{Pack}(v_3, \ldots, v_{3t}) - G$$

Template for $\omega(1/\lambda)$-Rate Garbling





Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$
$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod p$$

1. Packing

$$V_{\mathsf{R}} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$
$$V_{\mathsf{L}} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

$$G_{\mathsf{R}} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$$
$$G_{\mathsf{L}} = \mathsf{Pack}(\mathsf{g}_1, \mathsf{g}_4, \ldots, \mathsf{g}_{3t-2})$$

$$E_{\mathsf{R}} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$$
$$E_{\mathsf{L}} = \mathsf{Pack}(\mathsf{e}_1, \mathsf{e}_4, \ldots, \mathsf{e}_{3t-2})$$

K_{L} K_{R}

$$L_{\mathsf{R}} = \Delta \cdot E_{\mathsf{R}} - K_{\mathsf{R}} \pmod p$$
$$L_{\mathsf{L}} = \Delta \cdot E_{\mathsf{L}} - K_{\mathsf{L}} \pmod p$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

G

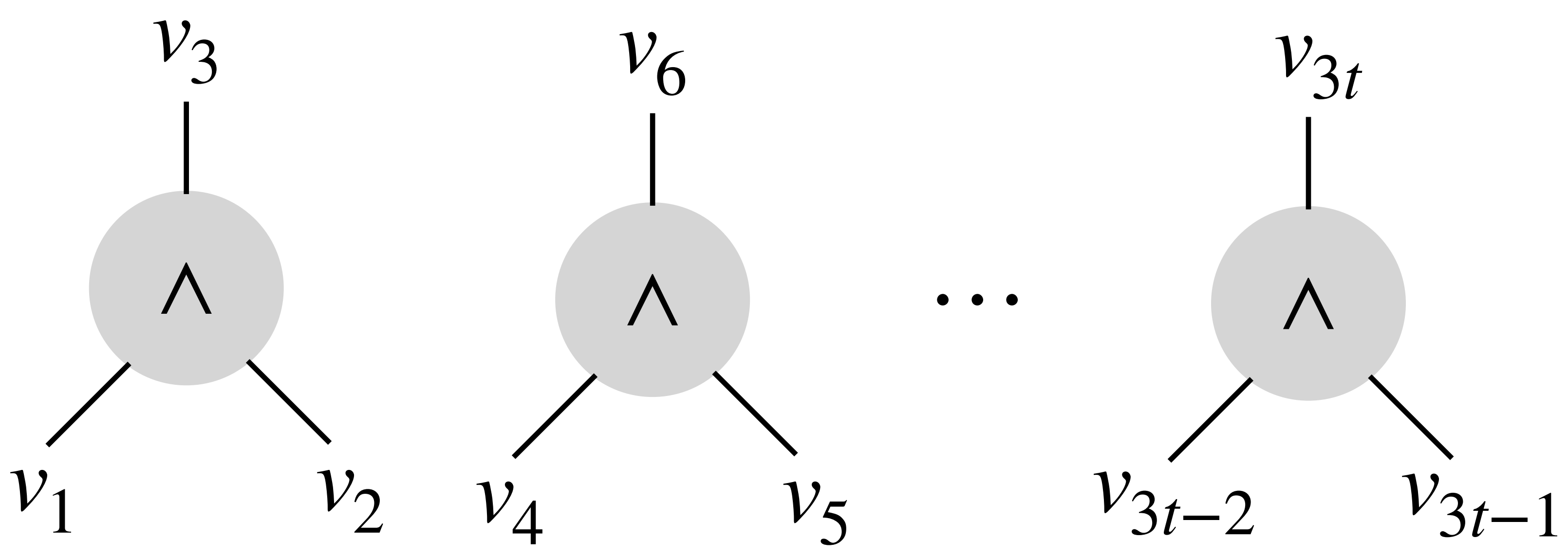
$$E = \mathsf{Pack}(v_3, \ldots, v_{3t}) - G$$

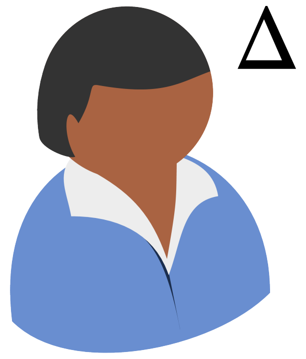
3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

Technically involved but transform and multiply gadget suffices for computing shares of $\Delta \cdot E$

Template for $\omega(1/\lambda)$-Rate Garbling





Δ

Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod p$$

Evaluator

1. Packing

$$V_\mathsf{R} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_\mathsf{L} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

$$G_\mathsf{R} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$$

$$G_\mathsf{L} = \mathsf{Pack}(\mathsf{g}_1, \mathsf{g}_4, \ldots, \mathsf{g}_{3t-2})$$

$$E_\mathsf{R} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$$

$$E_\mathsf{L} = \mathsf{Pack}(\mathsf{e}_1, \mathsf{e}_4, \ldots, \mathsf{e}_{3t-2})$$

K_L K_R

$$L_\mathsf{R} = \Delta \cdot E_\mathsf{R} - K_\mathsf{R} \pmod p$$

$$L_\mathsf{L} = \Delta \cdot E_\mathsf{L} - K_\mathsf{L} \pmod p$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

G

K

$$E = \mathsf{Pack}(v_3, \ldots, v_{3t}) - G$$

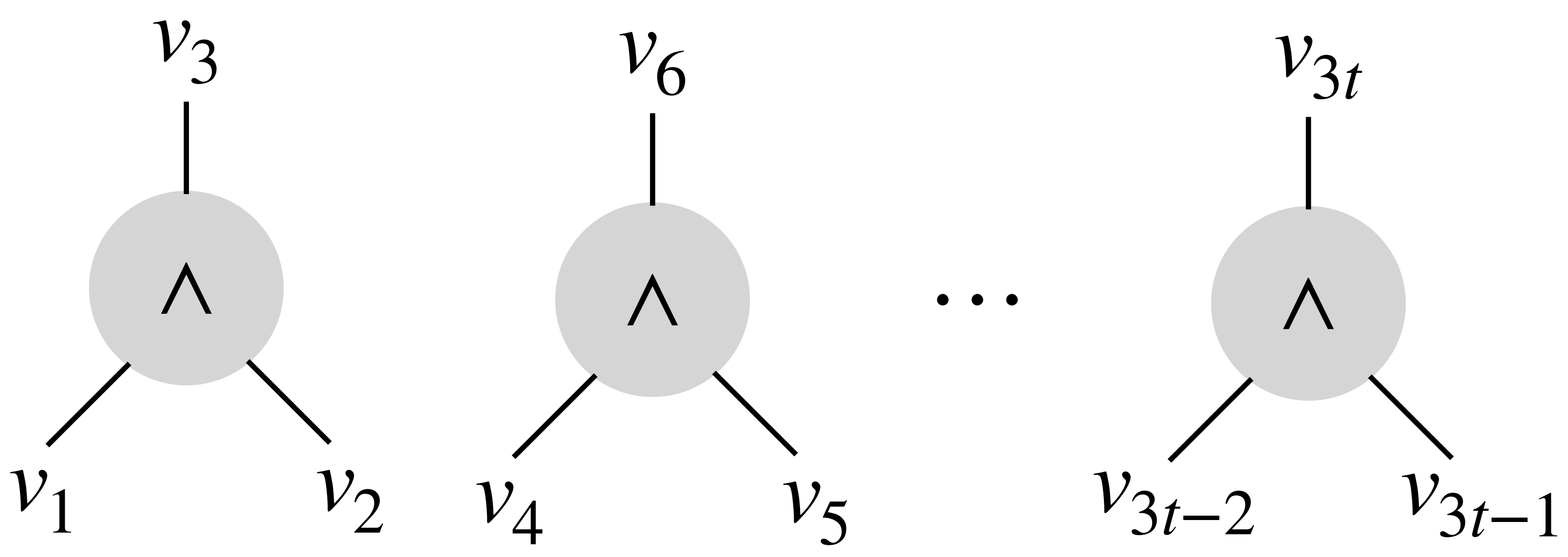
$$L = \Delta \cdot E - K \pmod p$$

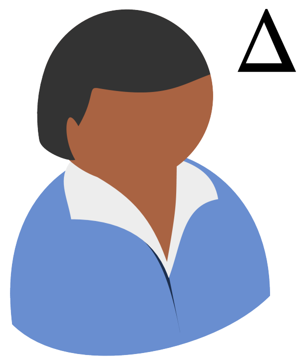
3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

Technically involved but transform and multiply gadget suffices for computing shares of $\Delta \cdot E$

Template for $\omega(1/\lambda)$-Rate Garbling




Δ

Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod{p}$$


Evaluator

1. Packing

$$V_\mathsf{R} = \mathsf{Pack}(v_2, v_5, \dots, v_{3t-1})$$

$$V_\mathsf{L} = \mathsf{Pack}(v_1, v_4, \dots, v_{3t-2})$$

$$G_\mathsf{R} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \dots, \mathsf{g}_{3t-1})$$

$$G_\mathsf{L} = \mathsf{Pack}(\mathsf{g}_1, \mathsf{g}_4, \dots, \mathsf{g}_{3t-2})$$

$$E_\mathsf{R} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \dots, \mathsf{e}_{3t-1})$$

$$E_\mathsf{L} = \mathsf{Pack}(\mathsf{e}_1, \mathsf{e}_4, \dots, \mathsf{e}_{3t-2})$$

$$K_\mathsf{L} \quad K_\mathsf{R}$$

$$L_\mathsf{R} = \Delta \cdot E_\mathsf{R} - K_\mathsf{R} \pmod{p}$$

$$L_\mathsf{L} = \Delta \cdot E_\mathsf{L} - K_\mathsf{L} \pmod{p}$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

$$G$$

$$K$$

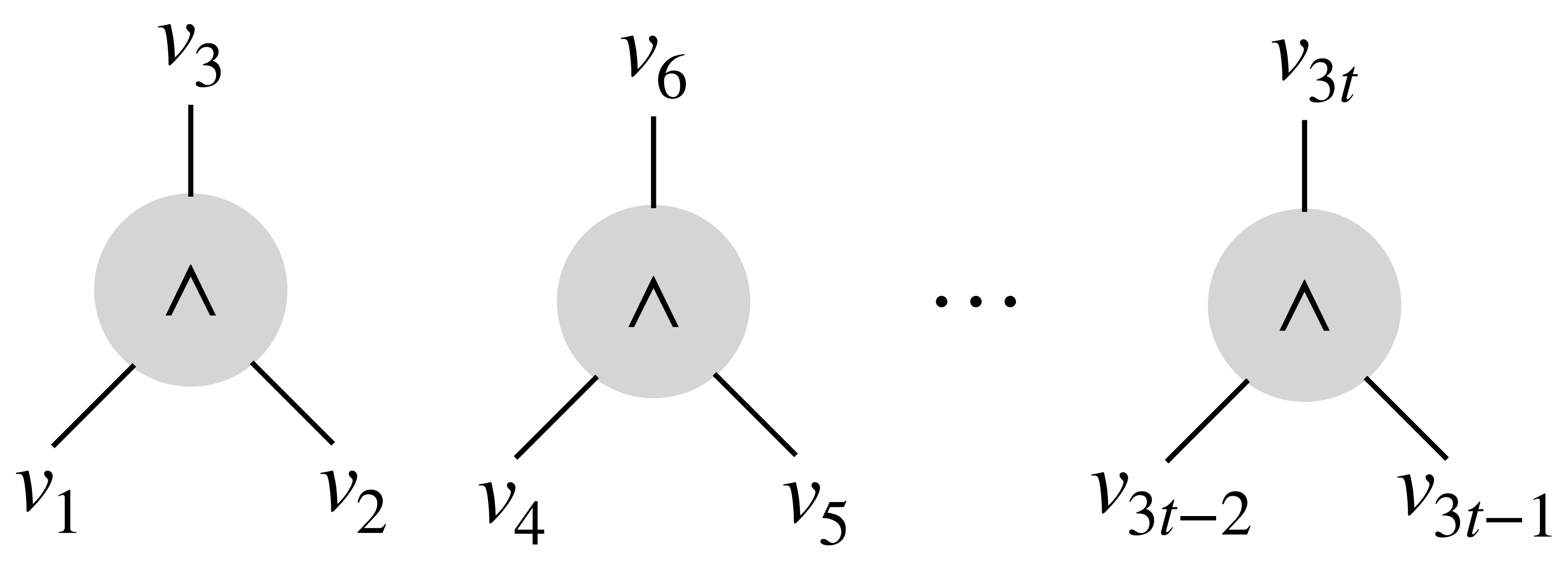
$$E = \mathsf{Pack}(v_3, \dots, v_{3t}) - G$$

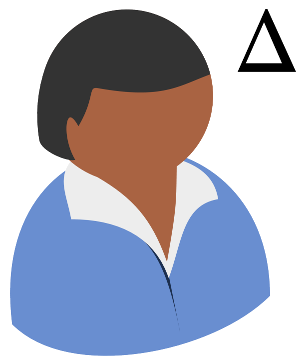
$$L = \Delta \cdot E - K \pmod{p}$$

3. Unpacking

$$(v_3, v_6, \dots, v_{3t}) = \mathsf{UnPack}(V)$$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod p$$

Evaluator

1. Packing

$$V_{\mathsf{R}} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_{\mathsf{L}} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

$$G_{\mathsf{R}} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$$

$$G_{\mathsf{L}} = \mathsf{Pack}(\mathsf{g}_1, \mathsf{g}_4, \ldots, \mathsf{g}_{3t-2})$$

$$E_{\mathsf{R}} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$$

$$E_{\mathsf{L}} = \mathsf{Pack}(\mathsf{e}_1, \mathsf{e}_4, \ldots, \mathsf{e}_{3t-2})$$

K_{L} K_{R}

$$L_{\mathsf{R}} = \Delta \cdot E_{\mathsf{R}} - K_{\mathsf{R}} \pmod p$$

$$L_{\mathsf{L}} = \Delta \cdot E_{\mathsf{L}} - K_{\mathsf{L}} \pmod p$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

G

K

$$E = \mathsf{Pack}(v_3, \ldots, v_{3t}) - G$$

$$L = \Delta \cdot E - K \pmod p$$

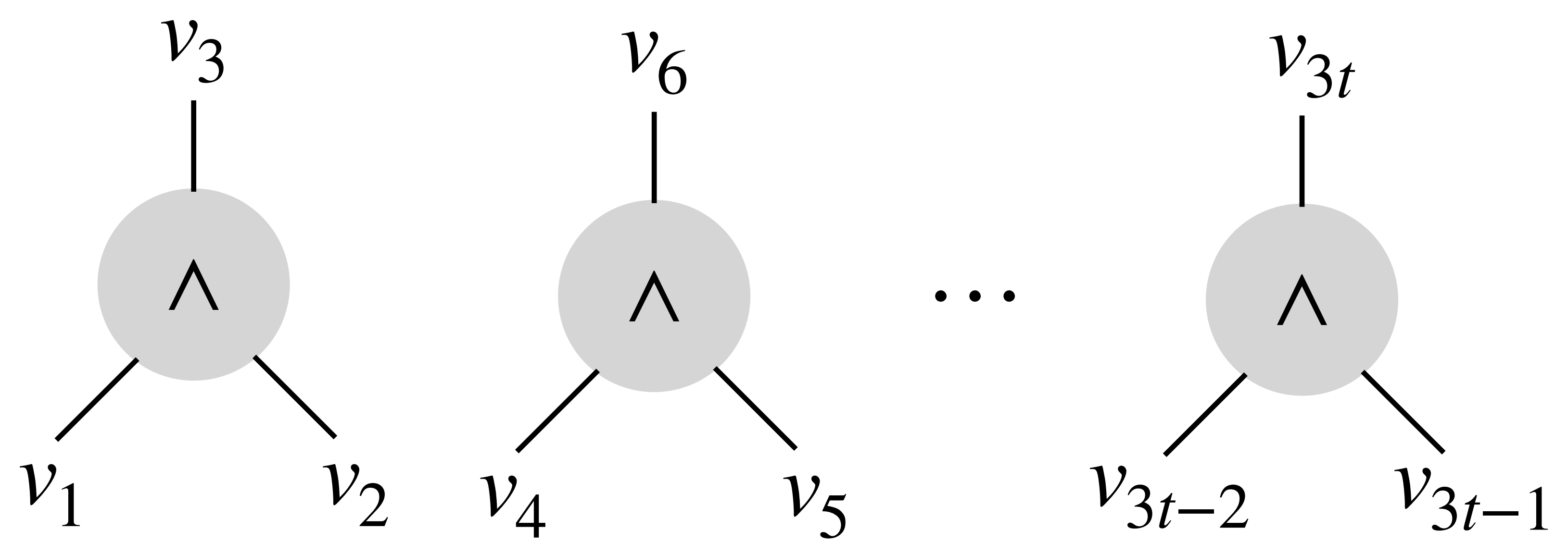
3. Unpacking

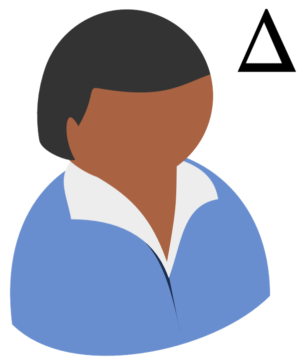
$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

$$(\mathsf{g}_3, \mathsf{g}_6, \ldots, \mathsf{g}_{3t}) = \mathsf{UnPack}(G)$$

$$(\mathsf{e}_3, \mathsf{e}_6, \ldots, \mathsf{e}_{3t}) = \mathsf{UnPack}(E)$$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$g_i \oplus e_i = v_i$$

$$k_i + \ell_i = \Delta \cdot e_i \pmod{p}$$

Evaluator

1. Packing

$$V_{\mathsf{R}} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_{\mathsf{L}} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

$$G_{\mathsf{R}} = \mathsf{Pack}(g_2, g_5, \ldots, g_{3t-1})$$

$$G_{\mathsf{L}} = \mathsf{Pack}(g_1, g_4, \ldots, g_{3t-2})$$

$$E_{\mathsf{R}} = \mathsf{Pack}(e_2, e_5, \ldots, e_{3t-1})$$

$$E_{\mathsf{L}} = \mathsf{Pack}(e_1, e_4, \ldots, e_{3t-2})$$

K_{L} K_{R}

$$L_{\mathsf{R}} = \Delta \cdot E_{\mathsf{R}} - K_{\mathsf{R}} \pmod{p}$$

$$L_{\mathsf{L}} = \Delta \cdot E_{\mathsf{L}} - K_{\mathsf{L}} \pmod{p}$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

G

K

$$E = \mathsf{Pack}(v_3, \ldots, v_{3t}) - G$$

$$L = \Delta \cdot E - K \pmod{p}$$

3. Unpacking

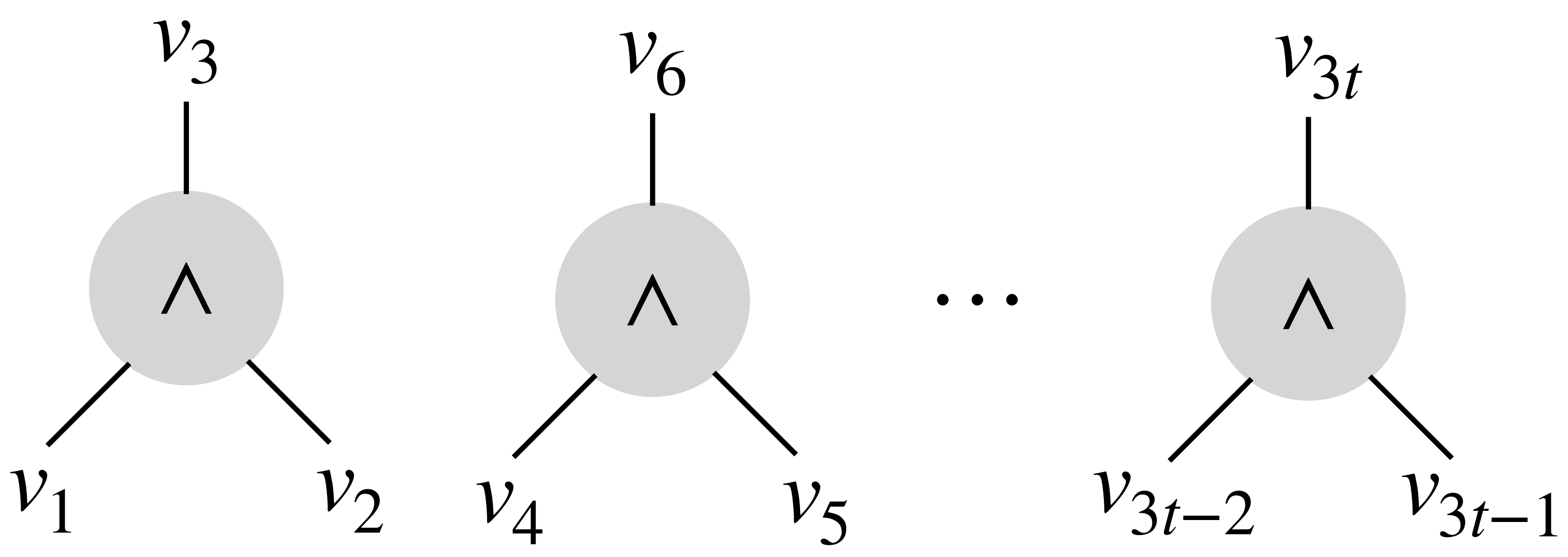
$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

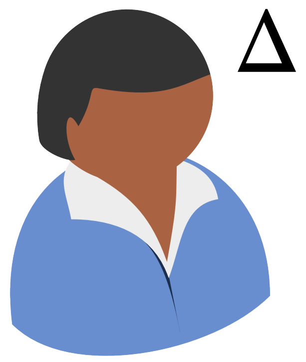
$$(g_3, g_6, \ldots, g_{3t}) = \mathsf{UnPack}(G)$$

UnPack is $\mathbb{F}_2$-linear

$$(e_3, e_6, \ldots, e_{3t}) = \mathsf{UnPack}(E)$$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$

$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod p$

Evaluator

1. Packing

$$V_{\mathsf{R}} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$
$$V_{\mathsf{L}} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

$G_{\mathsf{R}} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$

$G_{\mathsf{L}} = \mathsf{Pack}(\mathsf{g}_1, \mathsf{g}_4, \ldots, \mathsf{g}_{3t-2})$

$E_{\mathsf{R}} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$

$E_{\mathsf{L}} = \mathsf{Pack}(\mathsf{e}_1, \mathsf{e}_4, \ldots, \mathsf{e}_{3t-2})$

$K_{\mathsf{L}} \quad K_{\mathsf{R}}$

$L_{\mathsf{R}} = \Delta \cdot E_{\mathsf{R}} - K_{\mathsf{R}} \pmod p$

$L_{\mathsf{L}} = \Delta \cdot E_{\mathsf{L}} - K_{\mathsf{L}} \pmod p$

G

K

$E = \mathsf{Pack}(v_3, \ldots, v_{3t}) - G$

$L = \Delta \cdot E - K \pmod p$

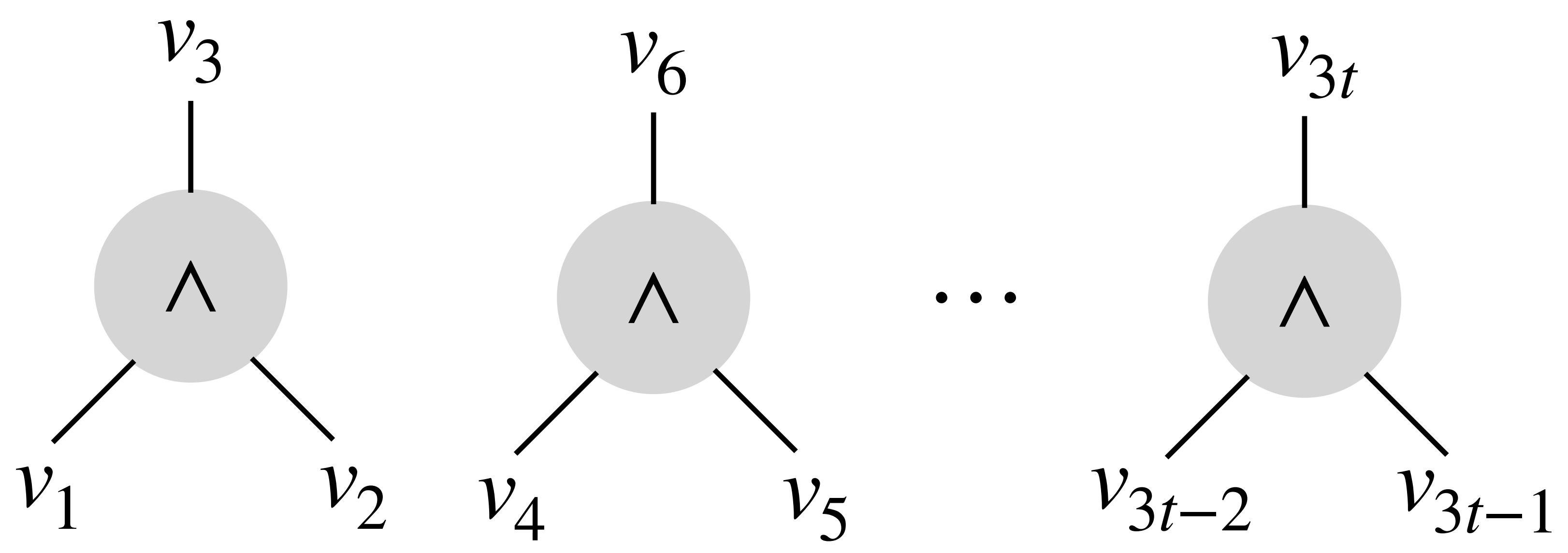
$(\mathsf{g}_3, \mathsf{g}_6, \ldots, \mathsf{g}_{3t}) = \mathsf{UnPack}(G)$

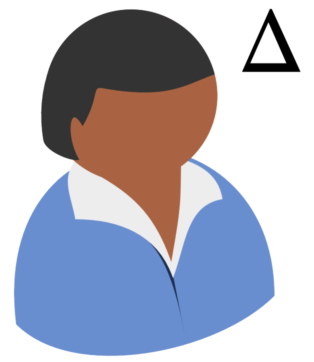
UnPack is $\mathbb{F}_2$-linear

$(\mathsf{e}_3, \mathsf{e}_6, \ldots, \mathsf{e}_{3t}) = \mathsf{UnPack}(E)$

(K, L) are shares over $\mathbb{Z}_p \Longrightarrow$ Can't use UnPack directly

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod{p}$$

Evaluator

1. Packing

$$V_\mathrm{R} = \mathrm{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_\mathrm{L} = \mathrm{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathrm{UnPack}(V)$$

G

K

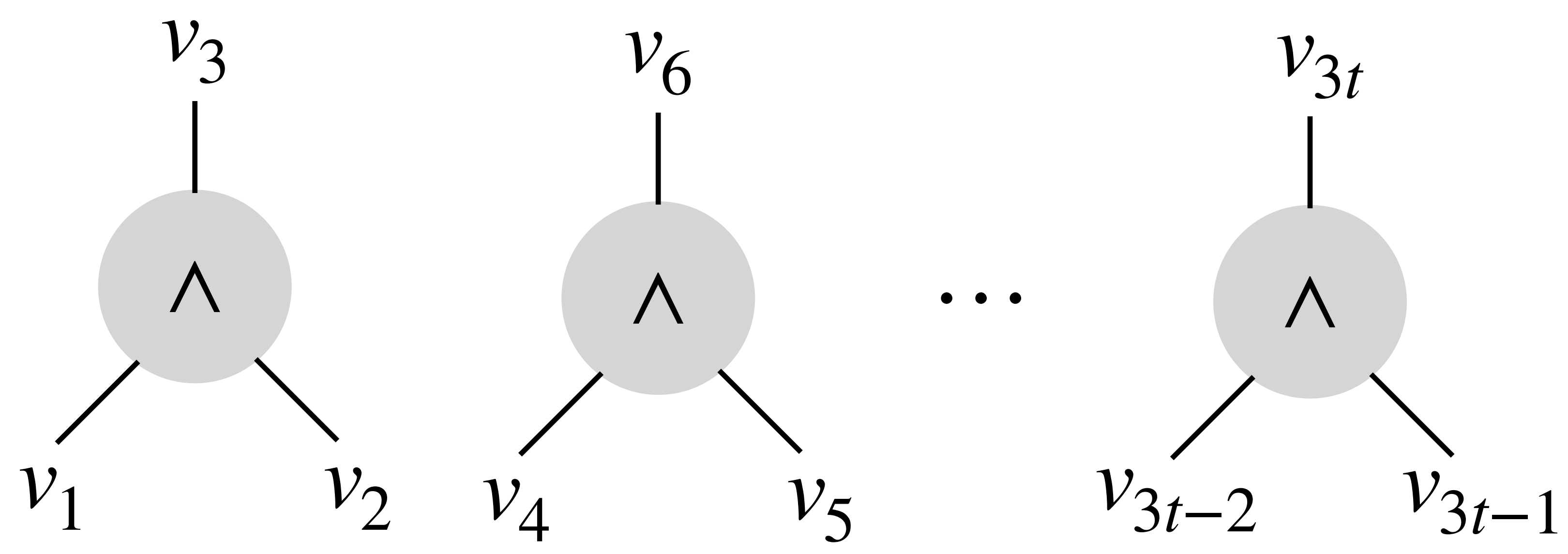
$$E = \mathrm{Pack}(v_3, \ldots, v_{3t}) - G$$

$$L = \Delta \cdot E - K \pmod{p}$$

$$(\mathsf{g}_3, \mathsf{g}_6, \ldots, \mathsf{g}_{3t}) = \mathrm{UnPack}(G)$$

$$(\mathsf{e}_3, \mathsf{e}_6, \ldots, \mathsf{e}_{3t}) = \mathrm{UnPack}(E)$$

Template for $\omega(1/\lambda)$-Rate Garbling



1. Packing

$$V_R = \text{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

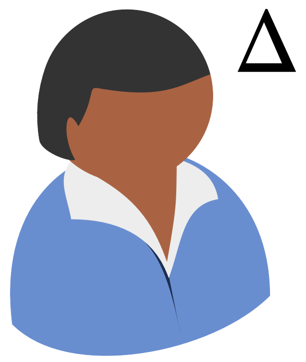
$$V_L = \text{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \text{UnPack}(V)$$



Δ

Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod p$$

Evaluator

G

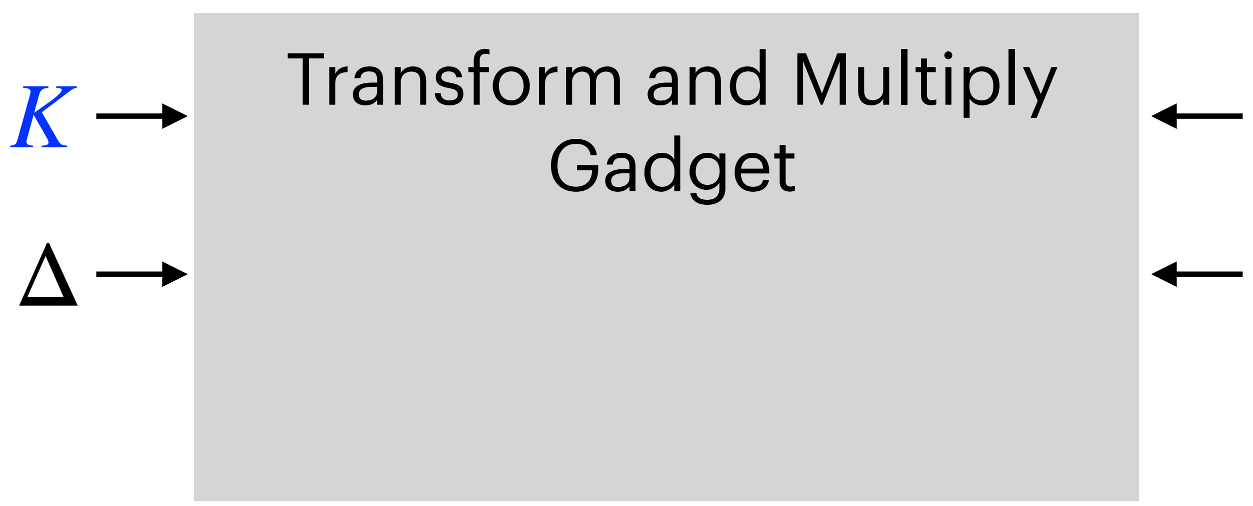
K

$E = \text{Pack}(v_3, \ldots, v_{3t}) - G$

$L = \Delta \cdot E - K \pmod p$

$(\mathsf{g}_3, \mathsf{g}_6, \ldots, \mathsf{g}_{3t}) = \text{UnPack}(G)$

$(\mathsf{e}_3, \mathsf{e}_6, \ldots, \mathsf{e}_{3t}) = \text{UnPack}(E)$



Transform and Multiply Gadget

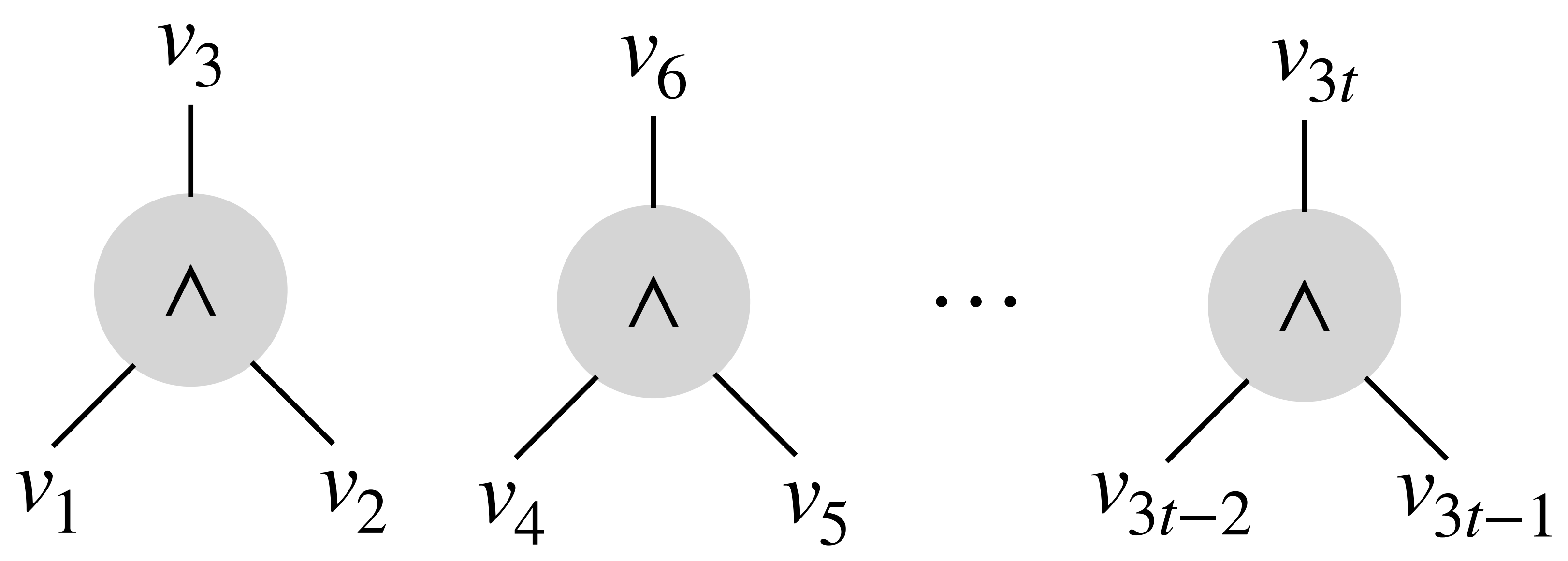
$K \rightarrow$

$\Delta \rightarrow$

$\leftarrow L = \Delta \cdot E - K \pmod p$

$\leftarrow E$

Template for $\omega(1/\lambda)$-Rate Garbling



1. Packing

$$V_{\mathrm{R}} = \mathrm{Pack}(v_2, v_5, \ldots, v_{3t-1})$$
$$V_{\mathrm{L}} = \mathrm{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathrm{UnPack}(V)$$

Garbler Δ

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$
$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod p$$

Evaluator

G

K

$$E = \mathrm{Pack}(v_3, \ldots, v_{3t}) - G$$
$$L = \Delta \cdot E - K \pmod p$$

$$(\mathsf{g}_3, \mathsf{g}_6, \ldots, \mathsf{g}_{3t}) = \mathrm{UnPack}(G)$$

$$(\mathsf{e}_3, \mathsf{e}_6, \ldots, \mathsf{e}_{3t}) = \mathrm{UnPack}(E)$$

$K \rightarrow$

$\Delta \rightarrow$

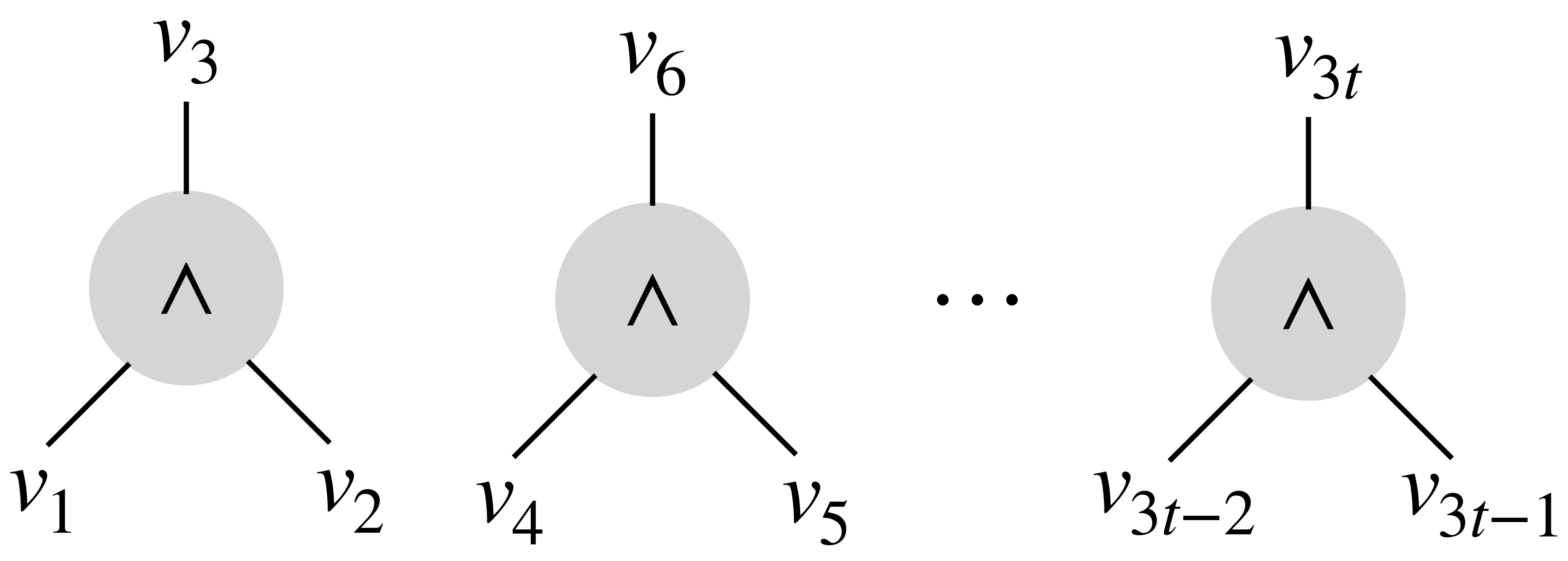
Transform and Multiply Gadget

$f_i(x) = \mathrm{Index}(i, \mathrm{UnPack}(x))$

$\leftarrow L = \Delta \cdot E - K \pmod p$

$\leftarrow E$

Template for $\omega(1/\lambda)$-Rate Garbling



1. Packing

$$V_\mathsf{R} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

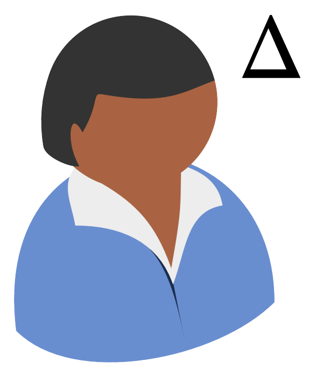
$$V_\mathsf{L} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$



Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod{p}$$

Evaluator

G

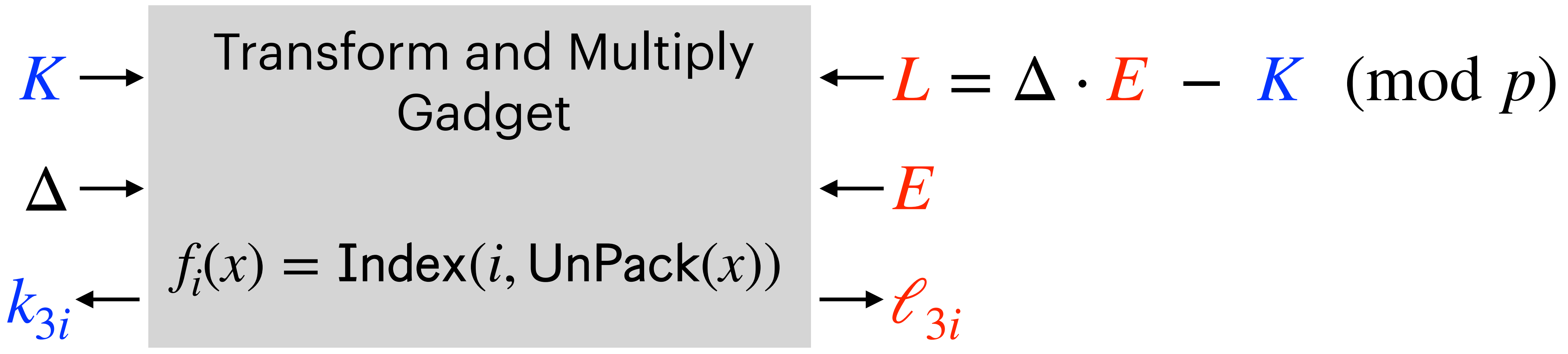
K

$$E = \mathsf{Pack}(v_3, \ldots, v_{3t}) - G$$

$$L = \Delta \cdot E - K \pmod{p}$$

$$(\mathsf{g}_3, \mathsf{g}_6, \ldots, \mathsf{g}_{3t}) = \mathsf{UnPack}(G)$$

$$(\mathsf{e}_3, \mathsf{e}_6, \ldots, \mathsf{e}_{3t}) = \mathsf{UnPack}(E)$$



Transform and Multiply Gadget

$$f_i(x) = \mathsf{Index}(i, \mathsf{UnPack}(x))$$

$K \rightarrow$

$\Delta \rightarrow$

$k_{3i} \leftarrow$

$\leftarrow L = \Delta \cdot E - K \pmod{p}$

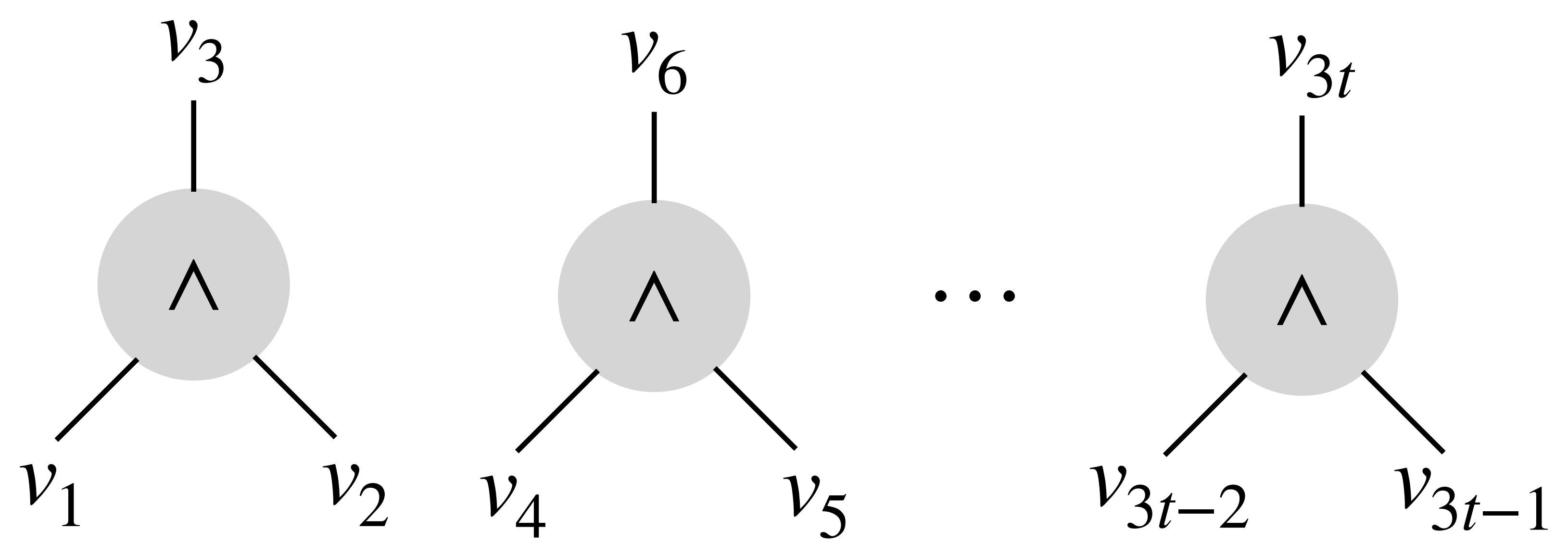
$\leftarrow E$

$\rightarrow \ell_{3i}$

$$k_{3i} + \ell_{3i} = \Delta \cdot f_i(E)$$

$$= \Delta \cdot \mathsf{e}_{3i}$$

Template for $\omega(1/\lambda)$-Rate Garbling



1. Packing

$$V_R = \text{Pack}(v_2, v_5, \ldots, v_{3t-1})$$
$$V_L = \text{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \text{UnPack}(V)$$

Garbler Δ

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$
$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod p$$

Evaluator

G

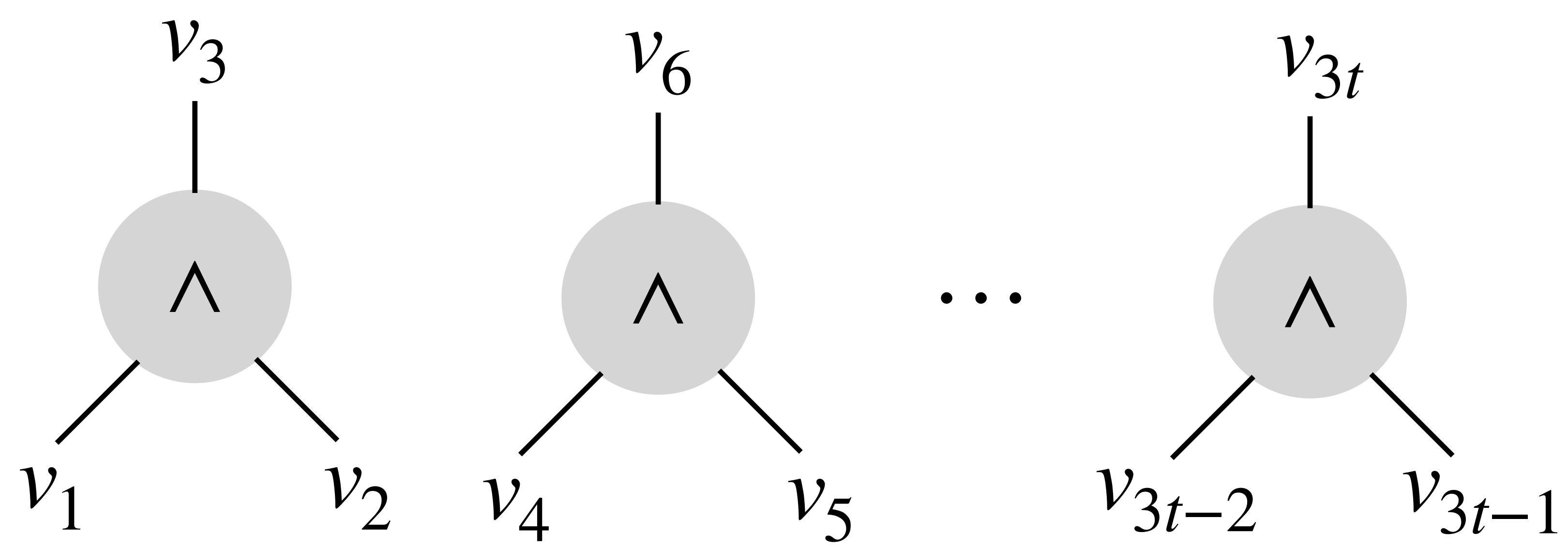
K

$$E = \text{Pack}(v_3, \ldots, v_{3t}) - G$$
$$L = \Delta \cdot E - K \pmod p$$

$$(\mathsf{g}_3, \mathsf{g}_6, \ldots, \mathsf{g}_{3t}) = \text{UnPack}(G)$$

$$(\mathsf{e}_3, \mathsf{e}_6, \ldots, \mathsf{e}_{3t}) = \text{UnPack}(E)$$

Template for $\omega(1/\lambda)$-Rate Garbling



1. Packing

$$V_R = \text{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

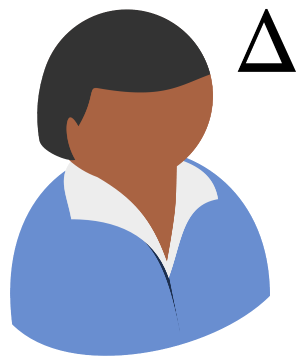
$$V_L = \text{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \text{UnPack}(V)$$



Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod{p}$$

Evaluator

G

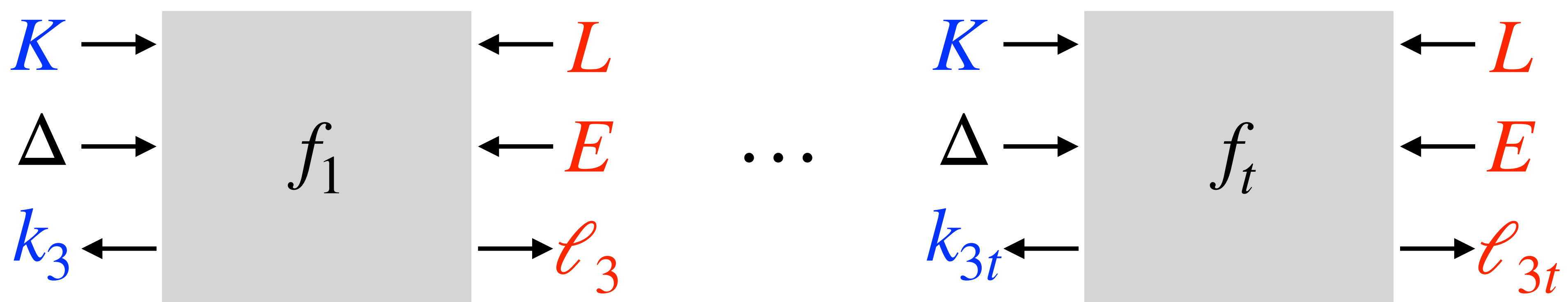
K

$$E = \text{Pack}(v_3, \ldots, v_{3t}) - G$$

$$L = \Delta \cdot E - K \pmod{p}$$

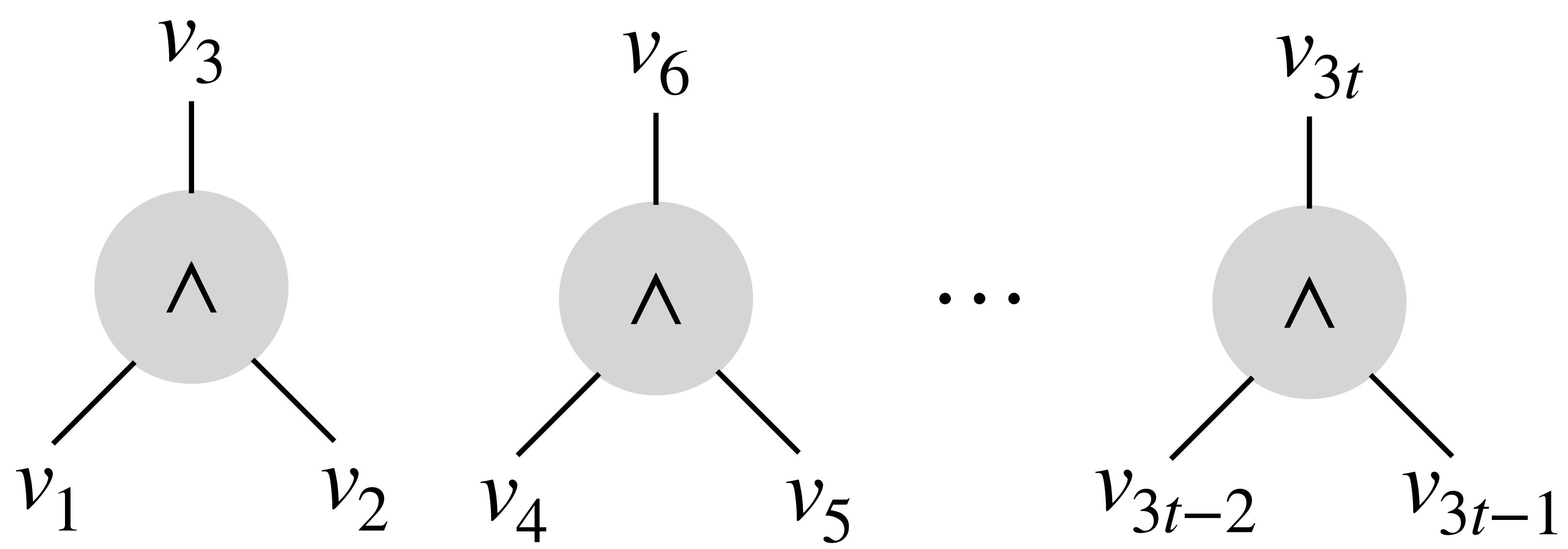
$$(\mathsf{g}_3, \mathsf{g}_6, \ldots, \mathsf{g}_{3t}) = \text{UnPack}(G)$$

$$(\mathsf{e}_3, \mathsf{e}_6, \ldots, \mathsf{e}_{3t}) = \text{UnPack}(E)$$



t invocations $\Longrightarrow$ $O(\lambda)$-bit communication per gate

Template for $\omega(1/\lambda)$-Rate Garbling



1. Packing

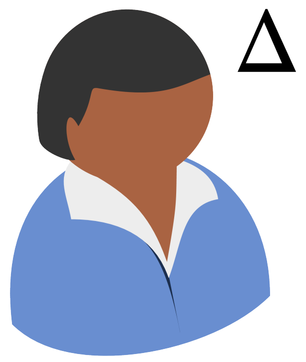
$$V_R = \text{Pack}(v_2, v_5, \ldots, v_{3t-1})$$
$$V_L = \text{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \text{UnPack}(V)$$



Garbler

Invariant

$$g_i \oplus e_i = v_i$$
$$k_i + \ell_i = \Delta \cdot e_i \pmod p$$

Evaluator

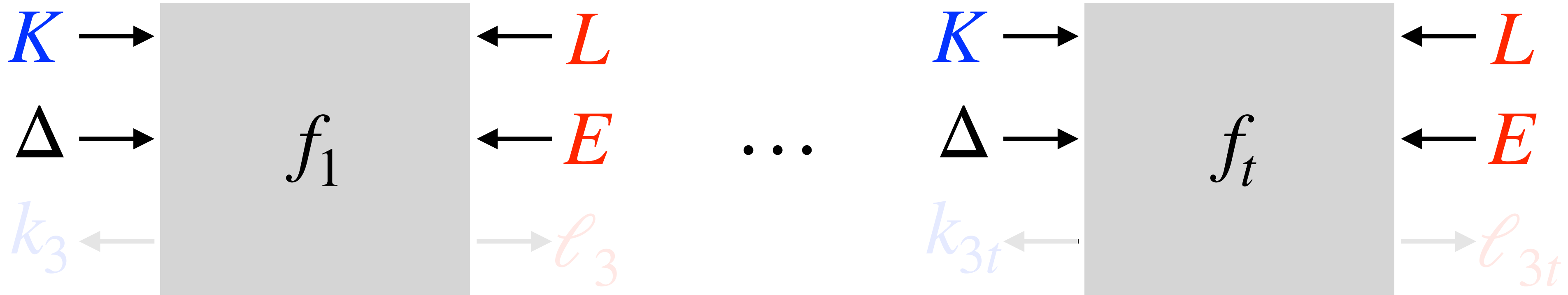
G

K

$$E = \text{Pack}(v_3, \ldots, v_{3t}) - G$$
$$L = \Delta \cdot E - K \pmod p$$

$$(g_3, g_6, \ldots, g_{3t}) = \text{UnPack}(G)$$

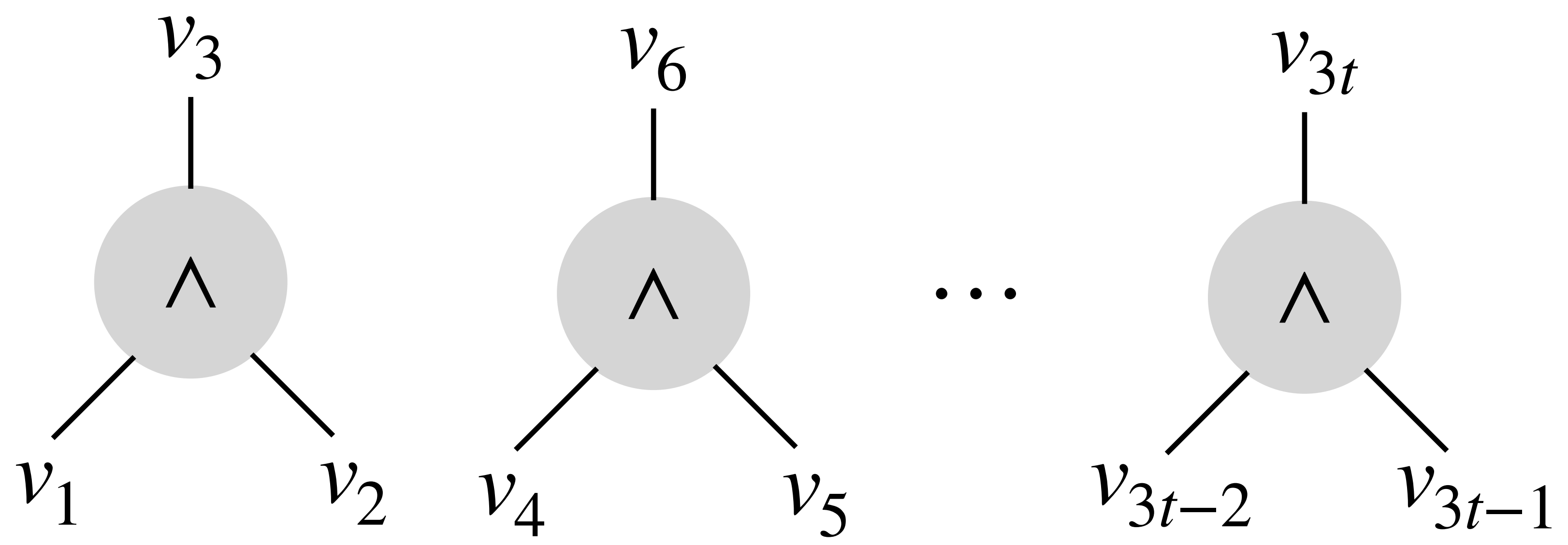
$$(e_3, e_6, \ldots, e_{3t}) = \text{UnPack}(E)$$

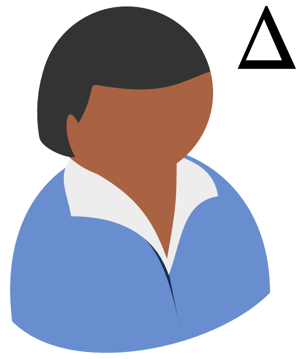


t invocations $\implies$ $O(\lambda)$-bit communication per gate

Observation: Different functions on same inputs $\implies$ $O(\lambda)$ communication across all invocations

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod{p}$$

Evaluator

1. Packing

$$V_\mathsf{R} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_\mathsf{L} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

$$G_\mathsf{R} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$$

$$G_\mathsf{L} = \mathsf{Pack}(\mathsf{g}_1, \mathsf{g}_4, \ldots, \mathsf{g}_{3t-2})$$

$$K_\mathsf{L} \quad K_\mathsf{R}$$

$$G$$

$$K$$

$$E_\mathsf{R} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$$

$$E_\mathsf{L} = \mathsf{Pack}(\mathsf{e}_1, \mathsf{e}_4, \ldots, \mathsf{e}_{3t-2})$$

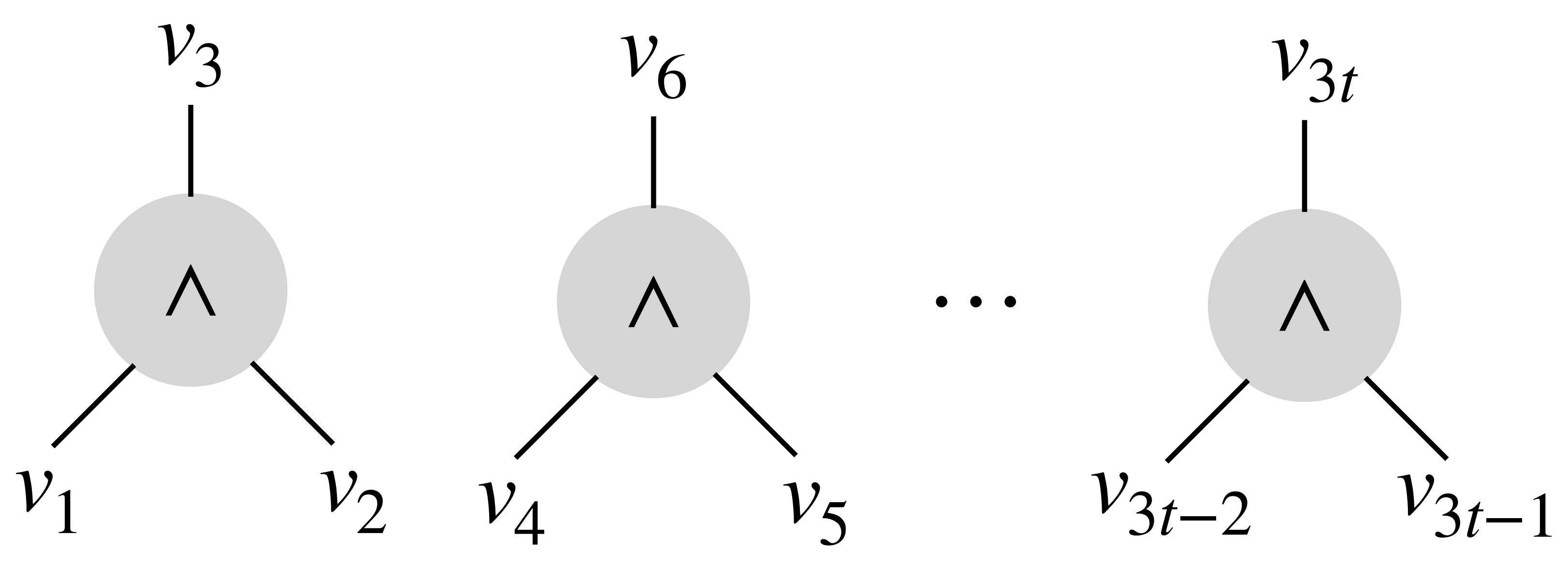
$$L_\mathsf{R} = \Delta \cdot E_\mathsf{R} - K_\mathsf{R} \pmod{p}$$

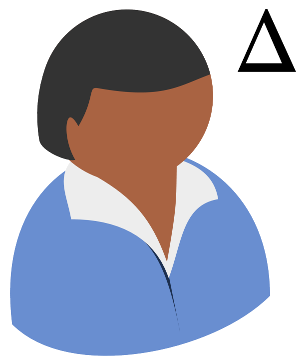
$$L_\mathsf{L} = \Delta \cdot E_\mathsf{L} - K_\mathsf{L} \pmod{p}$$

$$E = \mathsf{Pack}(v_3, \ldots, v_{3t}) - G$$

$$L = \Delta \cdot E - K \pmod{p}$$

Template for $\omega(1/\lambda)$-Rate Garbling




Δ

Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$
$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod p$$

Evaluator

1. Packing

$$V_\mathsf{R} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$
$$V_\mathsf{L} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

$$G_\mathsf{R} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$$
$$G_\mathsf{L} = \mathsf{Pack}(\mathsf{g}_1, \mathsf{g}_4, \ldots, \mathsf{g}_{3t-2})$$

$$E_\mathsf{R} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$$
$$E_\mathsf{L} = \mathsf{Pack}(\mathsf{e}_1, \mathsf{e}_4, \ldots, \mathsf{e}_{3t-2})$$

K_L K_R

$$L_\mathsf{R} = \Delta \cdot E_\mathsf{R} - K_\mathsf{R} \pmod p$$
$$L_\mathsf{L} = \Delta \cdot E_\mathsf{L} - K_\mathsf{L} \pmod p$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

G

K

$$E = \mathsf{Pack}(v_3, \ldots, v_{3t}) - G$$
$$L = \Delta \cdot E - K \pmod p$$

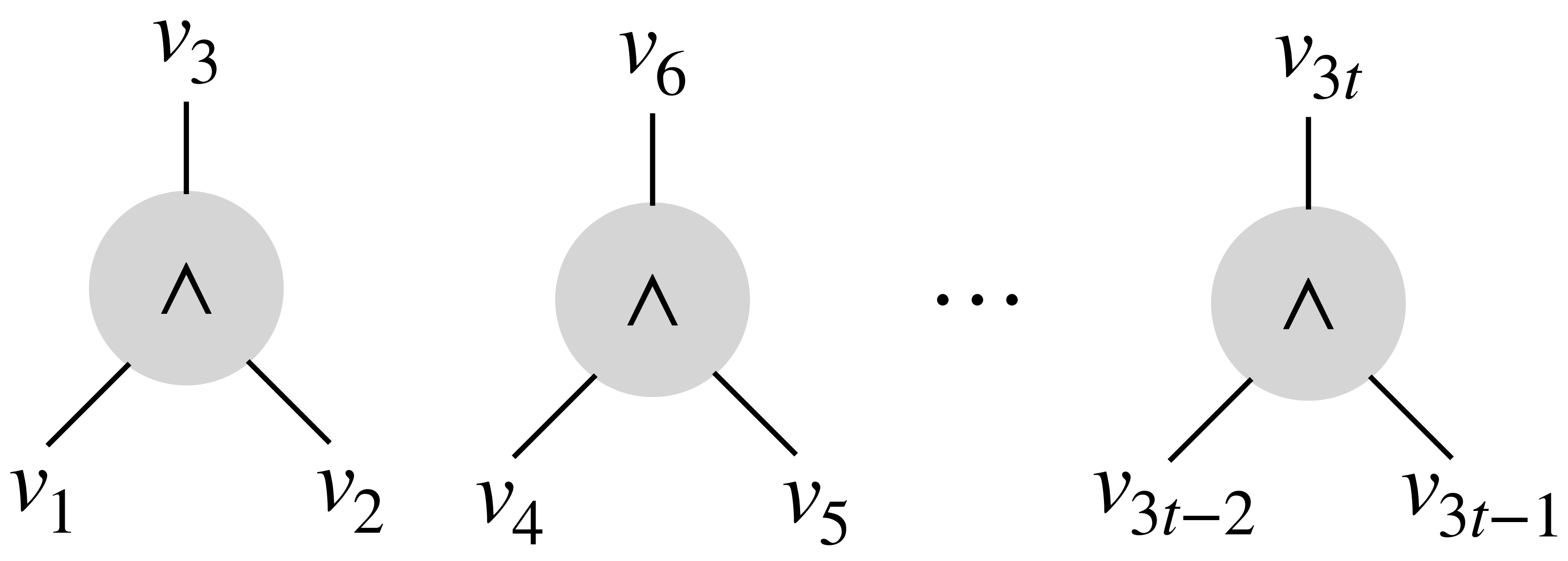
3. Unpacking

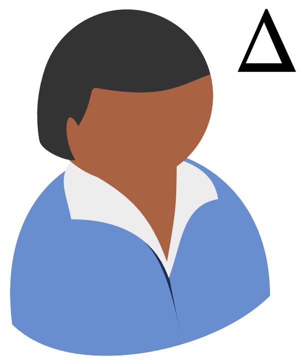
$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

$$(\mathsf{g}_3, \mathsf{g}_6, \ldots, \mathsf{g}_{3t}) = \mathsf{UnPack}(G)$$

$$(\mathsf{e}_3, \mathsf{e}_6, \ldots, \mathsf{e}_{3t}) = \mathsf{UnPack}(E)$$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod p$$

Evaluator

1. Packing

$$V_{\mathsf{R}} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_{\mathsf{L}} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

$$G_{\mathsf{R}} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$$

$$G_{\mathsf{L}} = \mathsf{Pack}(\mathsf{g}_1, \mathsf{g}_4, \ldots, \mathsf{g}_{3t-2})$$

$$E_{\mathsf{R}} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$$

$$E_{\mathsf{L}} = \mathsf{Pack}(\mathsf{e}_1, \mathsf{e}_4, \ldots, \mathsf{e}_{3t-2})$$

$$K_{\mathsf{L}} \quad K_{\mathsf{R}}$$

$$L_{\mathsf{R}} = \Delta \cdot E_{\mathsf{R}} - K_{\mathsf{R}} \pmod p$$

$$L_{\mathsf{L}} = \Delta \cdot E_{\mathsf{L}} - K_{\mathsf{L}} \pmod p$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

$$G$$

$$K$$

$$E = \mathsf{Pack}(v_3, \ldots, v_{3t}) - G$$

$$L = \Delta \cdot E - K \pmod p$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

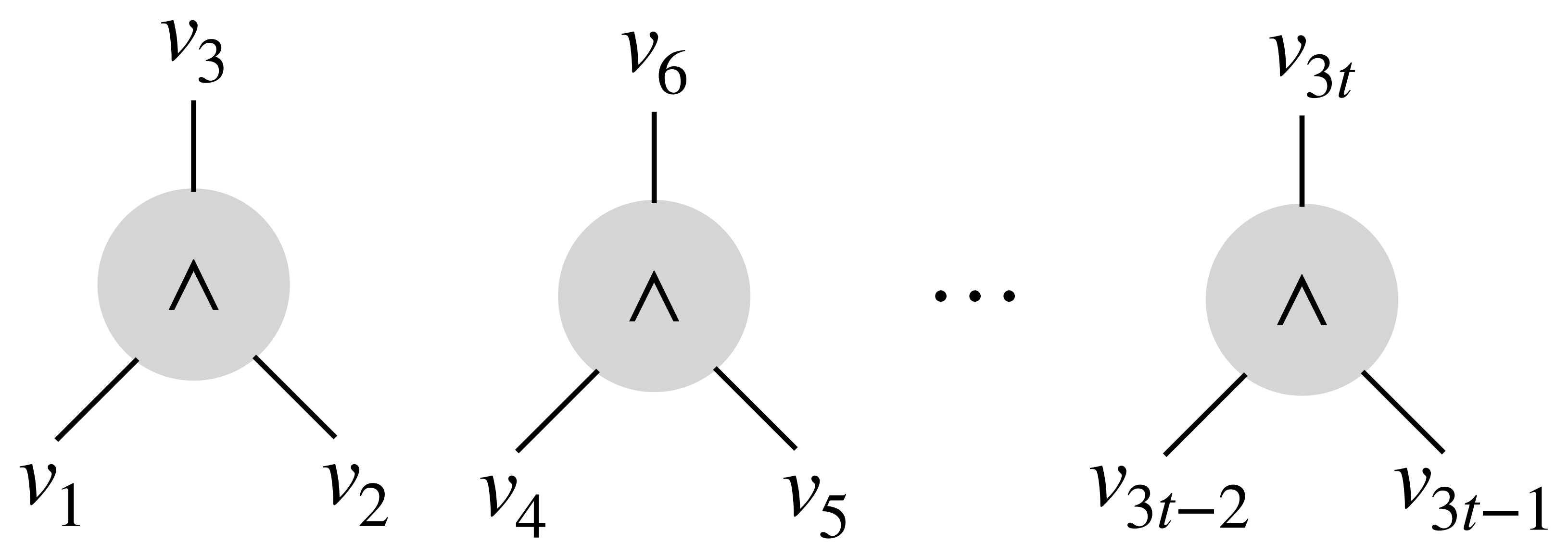
$$(\mathsf{g}_3, \mathsf{g}_6, \ldots, \mathsf{g}_{3t}) = \mathsf{UnPack}(G)$$

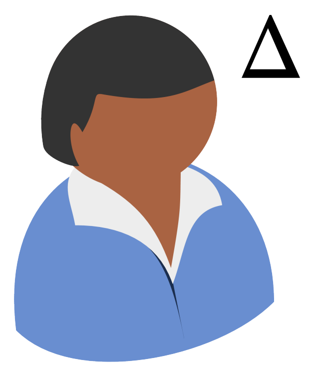
$$(k_3, \ldots, k_{3t})$$

$$(\mathsf{e}_3, \mathsf{e}_6, \ldots, \mathsf{e}_{3t}) = \mathsf{UnPack}(E)$$

$$\ell_i = \Delta \cdot \mathsf{e}_i - k_i \pmod p$$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod p$$

Evaluator

1. Packing

$$V_\mathsf{R} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_\mathsf{L} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

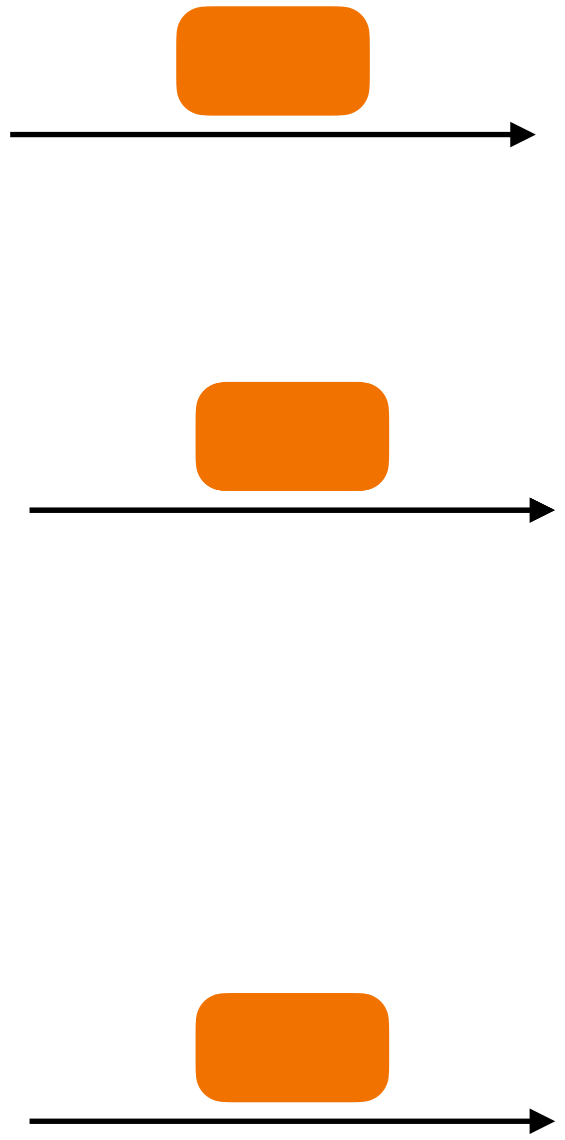
$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

$$G_\mathsf{R} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$$

$$G_\mathsf{L} = \mathsf{Pack}(\mathsf{g}_1, \mathsf{g}_4, \ldots, \mathsf{g}_{3t-2})$$

$$E_\mathsf{R} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$$

$$E_\mathsf{L} = \mathsf{Pack}(\mathsf{e}_1, \mathsf{e}_4, \ldots, \mathsf{e}_{3t-2})$$



K_L K_R

$$L_\mathsf{R} = \Delta \cdot E_\mathsf{R} - K_\mathsf{R} \pmod p$$

$$L_\mathsf{L} = \Delta \cdot E_\mathsf{L} - K_\mathsf{L} \pmod p$$

G

K

$$E = \mathsf{Pack}(v_3, \ldots, v_{3t}) - G$$

$$L = \Delta \cdot E - K \pmod p$$

$$(\mathsf{g}_3, \mathsf{g}_6, \ldots, \mathsf{g}_{3t}) = \mathsf{UnPack}(G)$$

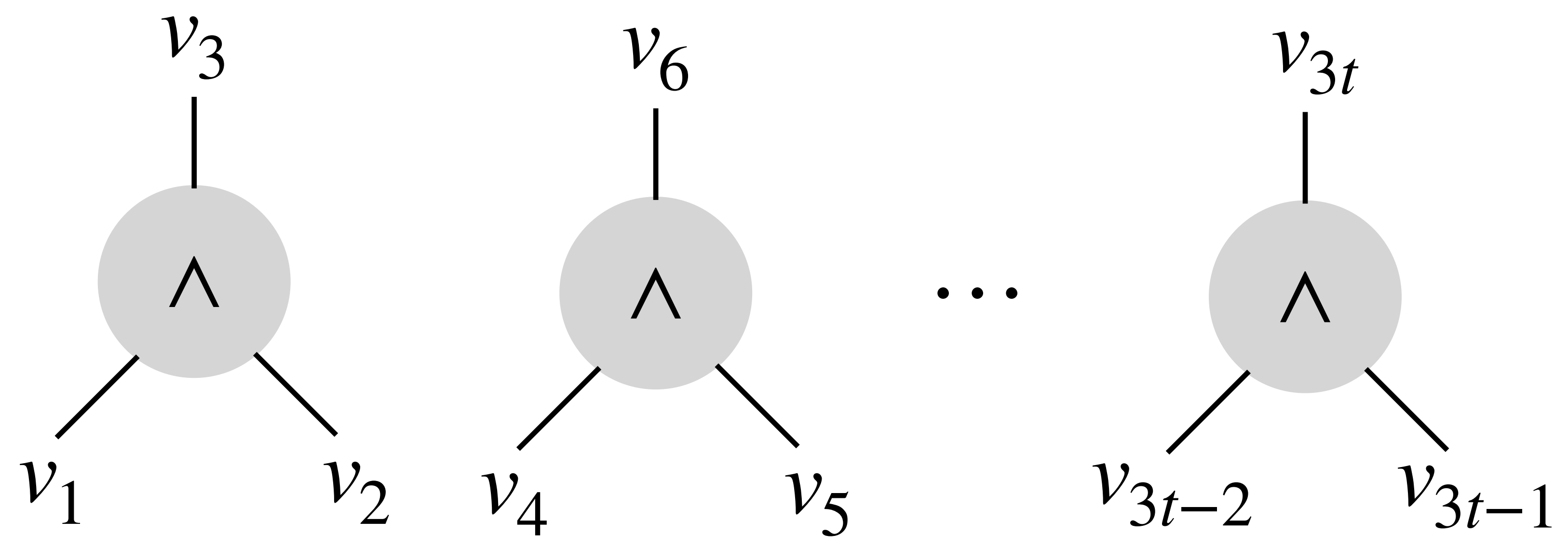
$$(\mathsf{e}_3, \mathsf{e}_6, \ldots, \mathsf{e}_{3t}) = \mathsf{UnPack}(E)$$

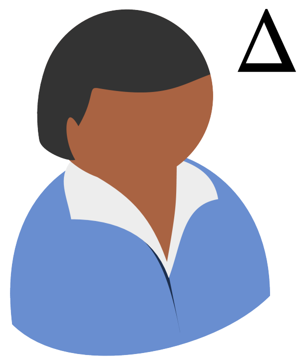
$$(k_3, \ldots, k_{3t})$$

$$\ell_i = \Delta \cdot \mathsf{e}_i - k_i \pmod p$$

Invariant maintained!

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$g_i \oplus e_i = v_i$$

$$k_i + \ell_i = \Delta \cdot e_i \pmod p$$

Evaluator

1. Packing

$$V_{\mathsf{R}} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$$

$$V_{\mathsf{L}} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$$

2. Gate Evaluation

$$V = V_1 \cdot V_2$$

3. Unpacking

$$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$$

$$G_{\mathsf{R}} = \mathsf{Pack}(g_2, g_5, \ldots, g_{3t-1})$$

$$G_{\mathsf{L}} = \mathsf{Pack}(g_1, g_4, \ldots, g_{3t-2})$$

K_{L} K_{R}

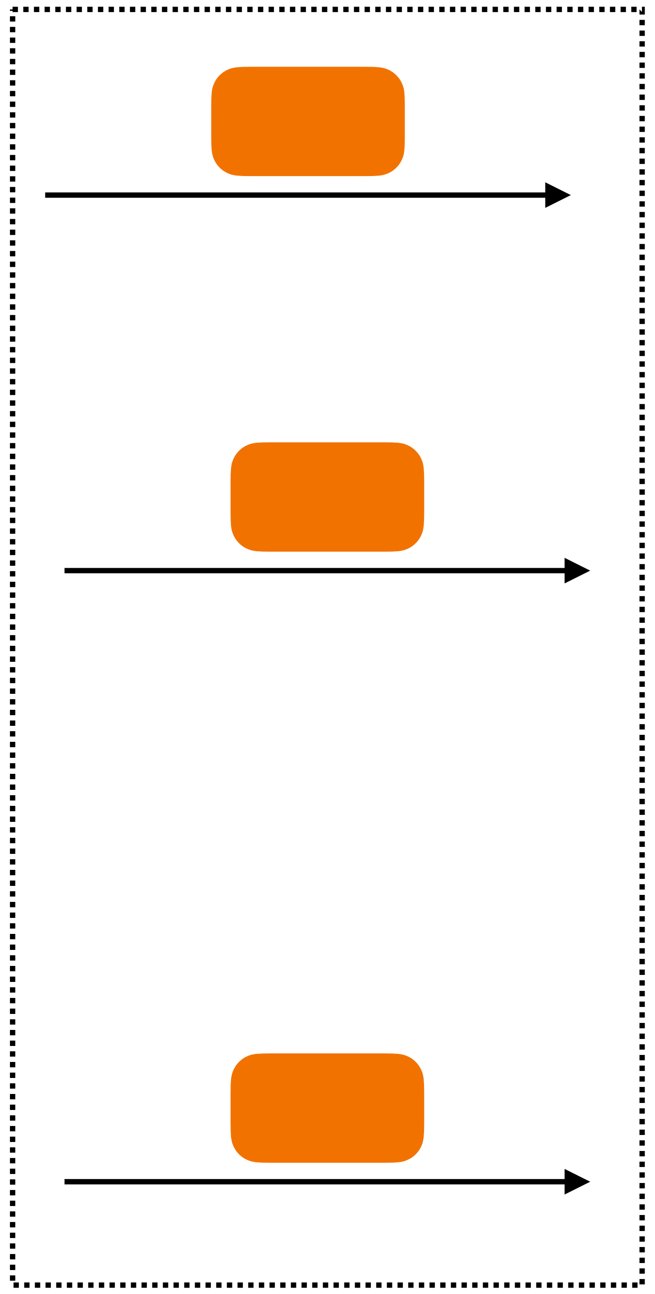
G

K

$$(g_3, g_6, \ldots, g_{3t}) = \mathsf{UnPack}(G)$$

$$(k_3, \ldots, k_{3t})$$

$O(\lambda)$ bits



$$E_{\mathsf{R}} = \mathsf{Pack}(e_2, e_5, \ldots, e_{3t-1})$$

$$E_{\mathsf{L}} = \mathsf{Pack}(e_1, e_4, \ldots, e_{3t-2})$$

$$L_{\mathsf{R}} = \Delta \cdot E_{\mathsf{R}} - K_{\mathsf{R}} \pmod p$$

$$L_{\mathsf{L}} = \Delta \cdot E_{\mathsf{L}} - K_{\mathsf{L}} \pmod p$$

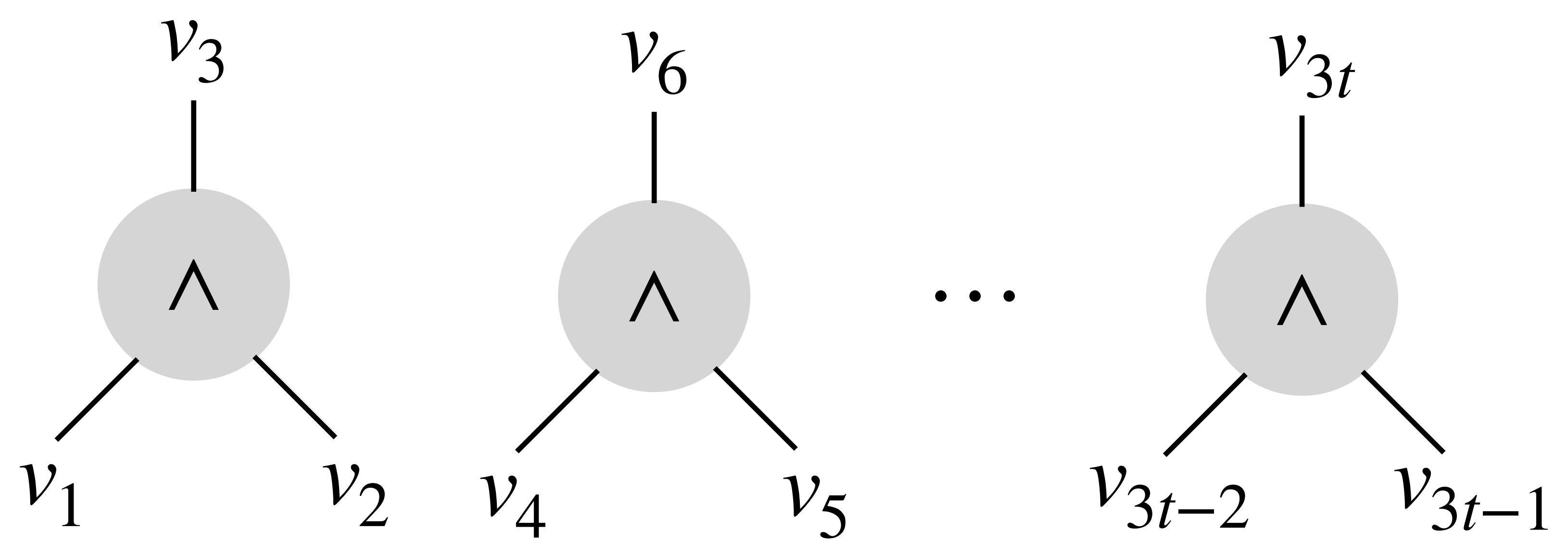
$$E = \mathsf{Pack}(v_3, \ldots, v_{3t}) - G$$

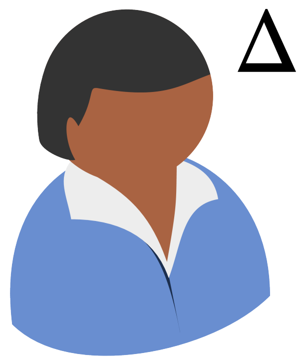
$$L = \Delta \cdot E - K \pmod p$$

$$(e_3, e_6, \ldots, e_{3t}) = \mathsf{UnPack}(E)$$

$$\ell_i = \Delta \cdot e_i - k_i \pmod p$$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$
$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod p$$

Evaluator

1. Packing

Rate: $O\left(\frac{t}{\lambda}\right)$

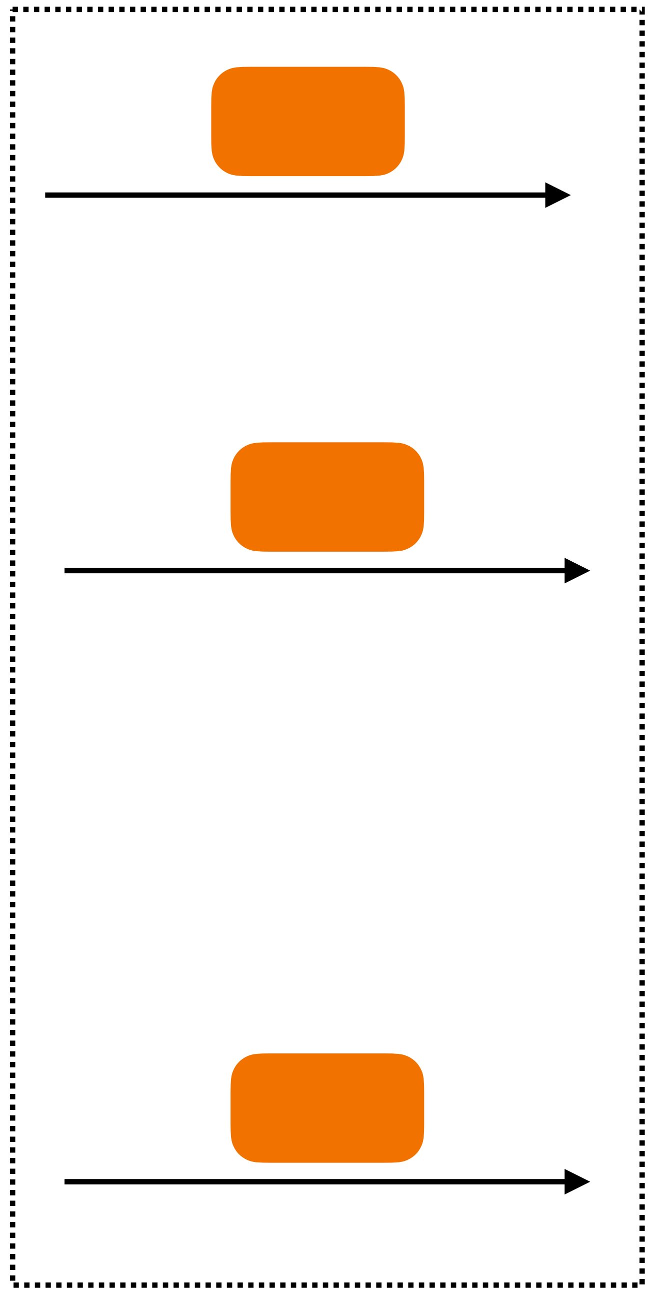
$V_{\mathsf{R}} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$

$V_{\mathsf{L}} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$

$G_{\mathsf{R}} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$

$G_{\mathsf{L}} = \mathsf{Pack}(\mathsf{g}_1, \mathsf{g}_4, \ldots, \mathsf{g}_{3t-2})$

$O(\lambda)$ bits



$E_{\mathsf{R}} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$

$E_{\mathsf{L}} = \mathsf{Pack}(\mathsf{e}_1, \mathsf{e}_4, \ldots, \mathsf{e}_{3t-2})$

$K_{\mathsf{L}} \quad K_{\mathsf{R}}$

$L_{\mathsf{R}} = \Delta \cdot E_{\mathsf{R}} - K_{\mathsf{R}} \pmod p$
$L_{\mathsf{L}} = \Delta \cdot E_{\mathsf{L}} - K_{\mathsf{L}} \pmod p$

2. Gate Evaluation

$V = V_1 \cdot V_2$

G

K

$E = \mathsf{Pack}(v_3, \ldots, v_{3t}) - G$

$L = \Delta \cdot E - K \pmod p$

3. Unpacking

$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$

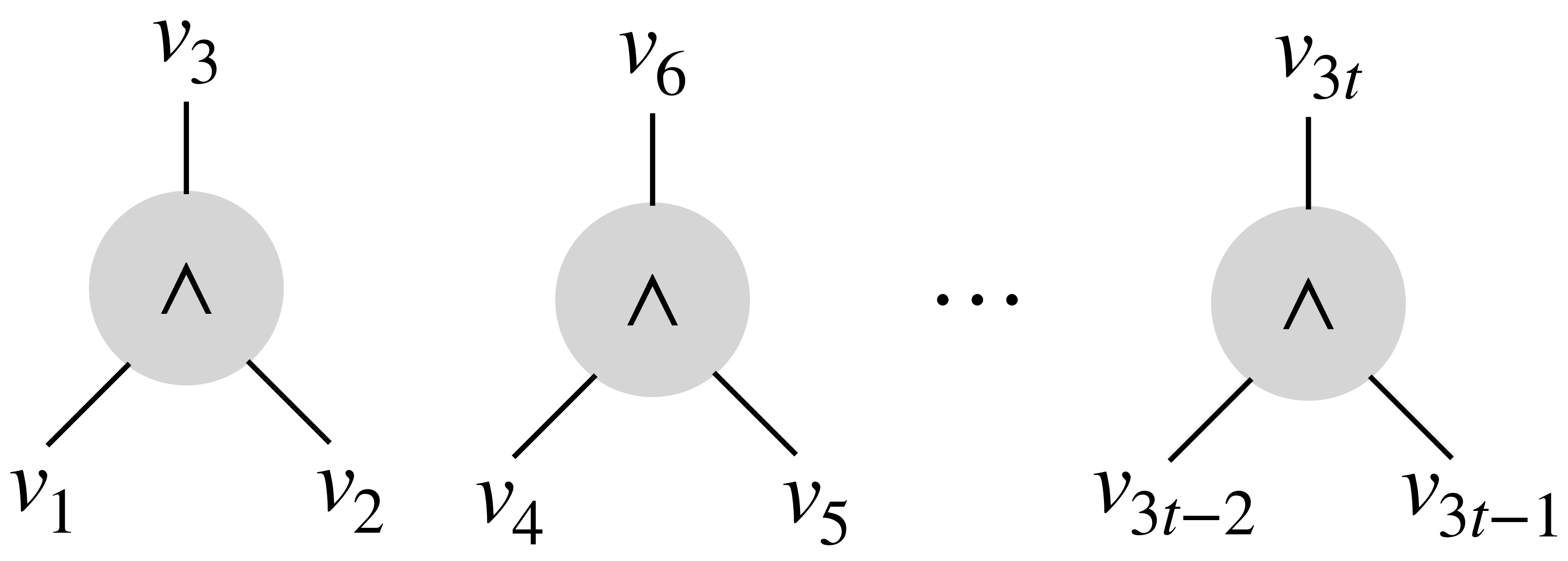
$(\mathsf{g}_3, \mathsf{g}_6, \ldots, \mathsf{g}_{3t}) = \mathsf{UnPack}(G)$

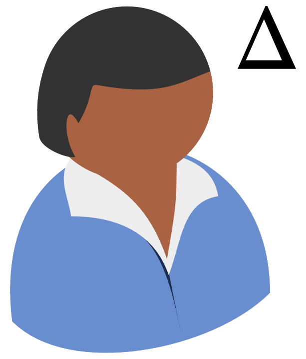
$(\mathsf{e}_3, \mathsf{e}_6, \ldots, \mathsf{e}_{3t}) = \mathsf{UnPack}(E)$

$(k_3, \ldots, k_{3t})$

$\ell_i = \Delta \cdot \mathsf{e}_i - k_i \pmod p$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$g_i \oplus e_i = v_i$$

$$k_i + \ell_i = \Delta \cdot e_i \pmod{p}$$

Evaluator

1. Packing

$V_R = \text{Pack}(v_2, v_5, \ldots, v_{3t-1})$

$V_L = \text{Pack}(v_1, v_4, \ldots, v_{3t-2})$

$G_R = \text{Pack}(g_2, g_5, \ldots, g_{3t-1})$

$G_L = \text{Pack}(g_1, g_4, \ldots, g_{3t-2})$

$E_R = \text{Pack}(e_2, e_5, \ldots, e_{3t-1})$

$E_L = \text{Pack}(e_1, e_4, \ldots, e_{3t-2})$

Rate: $O\left(\frac{t}{\lambda}\right)$

$$t = O\left(\sqrt{\log \lambda}\right)$$

$K_L \quad K_R$

$L_R = \Delta \cdot E_R - K_R \pmod{p}$

$L_L = \Delta \cdot E_L - K_L \pmod{p}$

2. Gate Evaluation

$V = V_1 \cdot V_2$

G

K

$E = \text{Pack}(v_3, \ldots, v_{3t}) - G$

$L = \Delta \cdot E - K \pmod{p}$

3. Unpacking

$(v_3, v_6, \ldots, v_{3t}) = \text{UnPack}(V)$

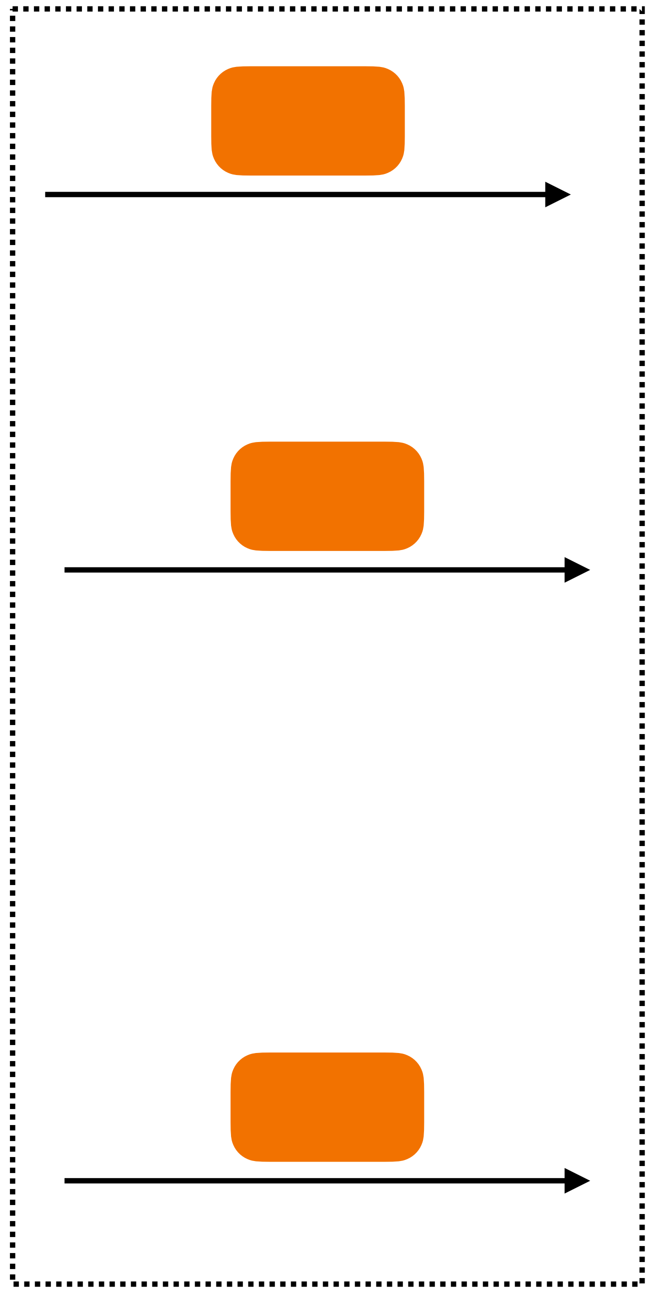
$(g_3, g_6, \ldots, g_{3t}) = \text{UnPack}(G)$

$(e_3, e_6, \ldots, e_{3t}) = \text{UnPack}(E)$

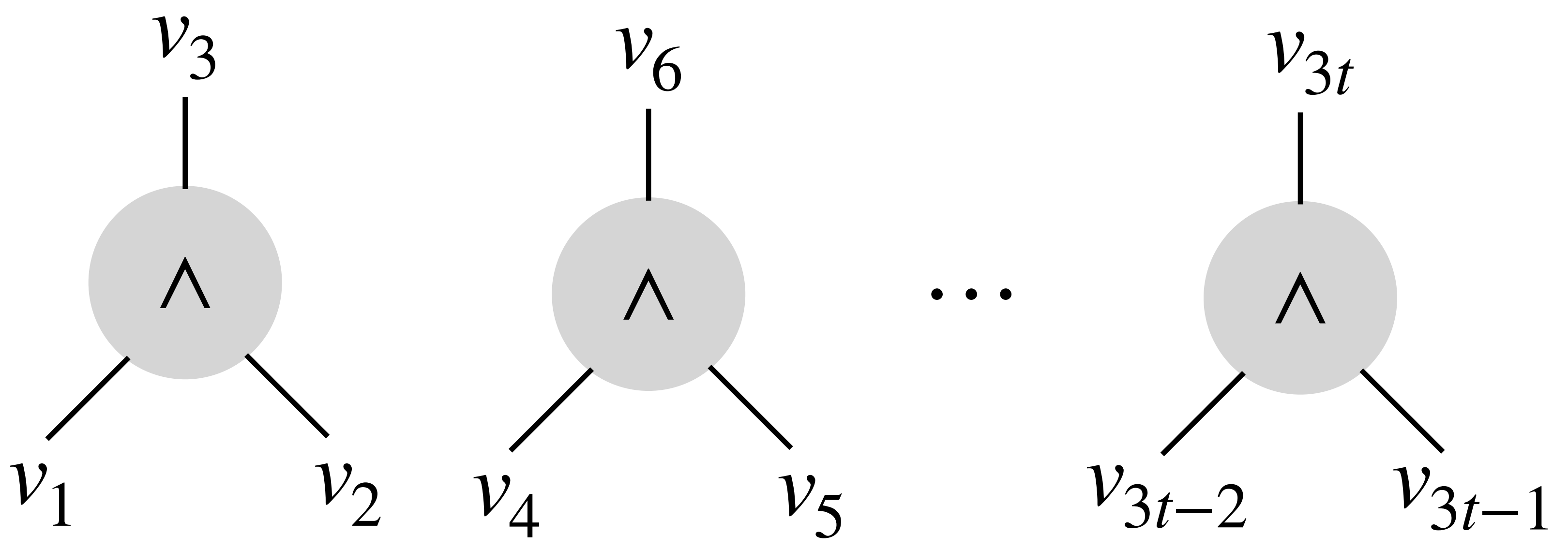
$(k_3, \ldots, k_{3t})$

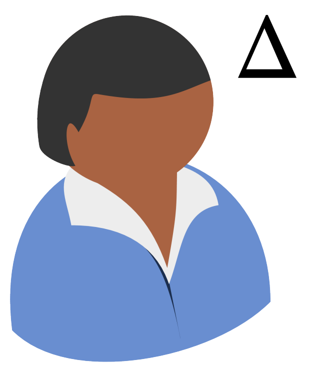
$\ell_i = \Delta \cdot e_i - k_i \pmod{p}$

$O(\lambda)$ bits



Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$

$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod{p}$

Evaluator

1. Packing

$V_{\mathsf{R}} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$

$V_{\mathsf{L}} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$

Rate: $O\left(\frac{t}{\lambda}\right) = O\left(\frac{\sqrt{\log \lambda}}{\lambda}\right)$

$t = O\left(\sqrt{\log \lambda}\right)$

$G_{\mathsf{R}} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$

$G_{\mathsf{L}} = \mathsf{Pack}(\mathsf{g}_1, \mathsf{g}_4, \ldots, \mathsf{g}_{3t-2})$

$E_{\mathsf{R}} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$

$E_{\mathsf{L}} = \mathsf{Pack}(\mathsf{e}_1, \mathsf{e}_4, \ldots, \mathsf{e}_{3t-2})$

$K_{\mathsf{L}} \quad K_{\mathsf{R}}$

$L_{\mathsf{R}} = \Delta \cdot E_{\mathsf{R}} - K_{\mathsf{R}} \pmod{p}$

$L_{\mathsf{L}} = \Delta \cdot E_{\mathsf{L}} - K_{\mathsf{L}} \pmod{p}$

2. Gate Evaluation

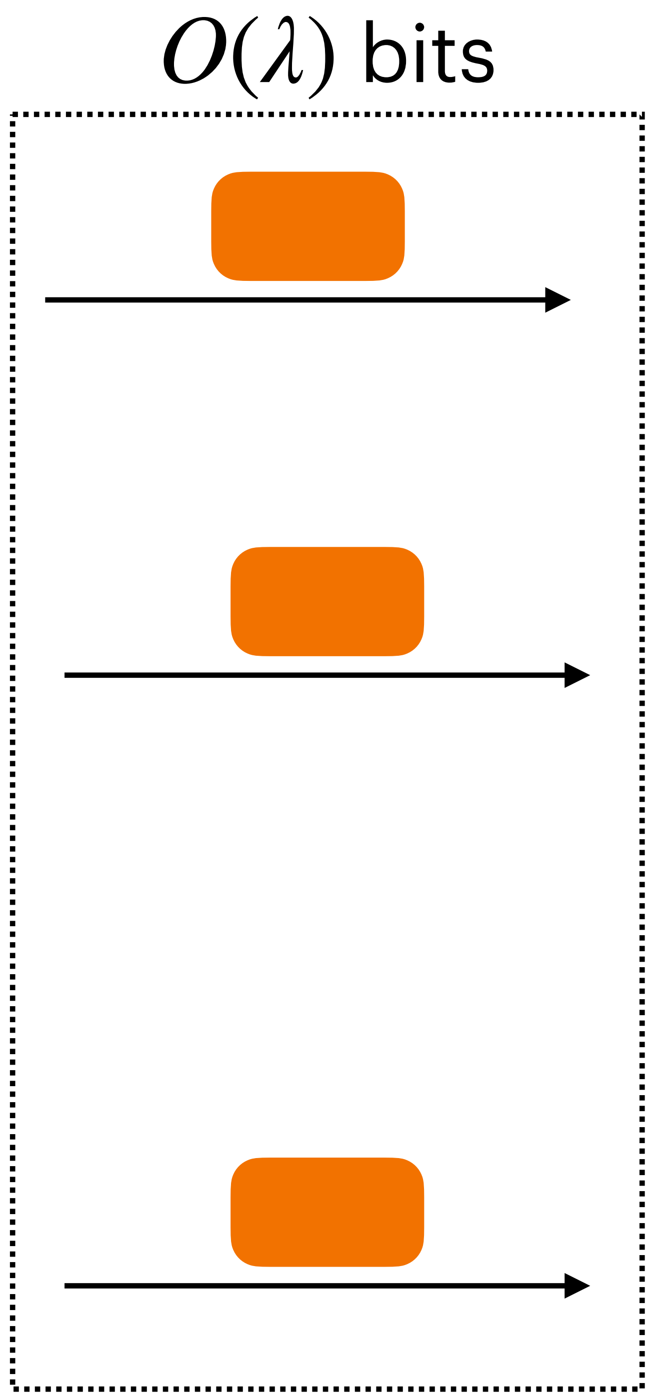
$V = V_1 \cdot V_2$

G

K

$E = \mathsf{Pack}(v_3, \ldots, v_{3t}) - G$

$L = \Delta \cdot E - K \pmod{p}$



3. Unpacking

$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$

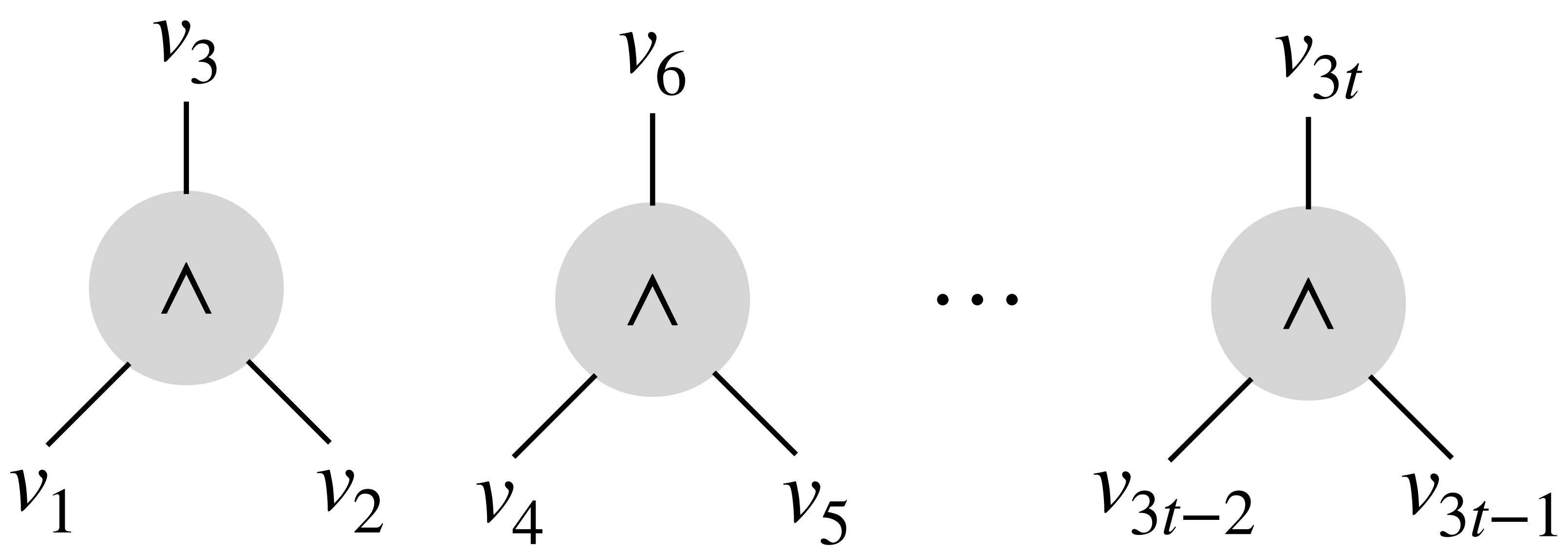
$(\mathsf{g}_3, \mathsf{g}_6, \ldots, \mathsf{g}_{3t}) = \mathsf{UnPack}(G)$

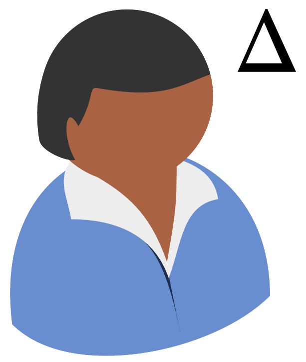
$(\mathsf{e}_3, \mathsf{e}_6, \ldots, \mathsf{e}_{3t}) = \mathsf{UnPack}(E)$

$(k_3, \ldots, k_{3t})$

$\ell_i = \Delta \cdot \mathsf{e}_i - k_i \pmod{p}$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod p$$

Evaluator

Rate: $O\left(\frac{t}{\lambda}\right) = O\left(\frac{\sqrt{\log \lambda}}{\lambda}\right)$

Why $t = O\left(\sqrt{\log \lambda}\right)$?

1. Packing

$V_{\mathsf{R}} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$

$V_{\mathsf{L}} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$

$G_{\mathsf{R}} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$

$G_{\mathsf{L}} = \mathsf{Pack}(\mathsf{g}_1, \mathsf{g}_4, \ldots, \mathsf{g}_{3t-2})$

$E_{\mathsf{R}} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$

$E_{\mathsf{L}} = \mathsf{Pack}(\mathsf{e}_1, \mathsf{e}_4, \ldots, \mathsf{e}_{3t-2})$

$K_{\mathsf{L}} \quad K_{\mathsf{R}}$

$L_{\mathsf{R}} = \Delta \cdot E_{\mathsf{R}} - K_{\mathsf{R}} \pmod p$

$L_{\mathsf{L}} = \Delta \cdot E_{\mathsf{L}} - K_{\mathsf{L}} \pmod p$

2. Gate Evaluation

$V = V_1 \cdot V_2$

G

K

$E = \mathsf{Pack}(v_3, \ldots, v_{3t}) - G$

$L = \Delta \cdot E - K \pmod p$

3. Unpacking

$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$

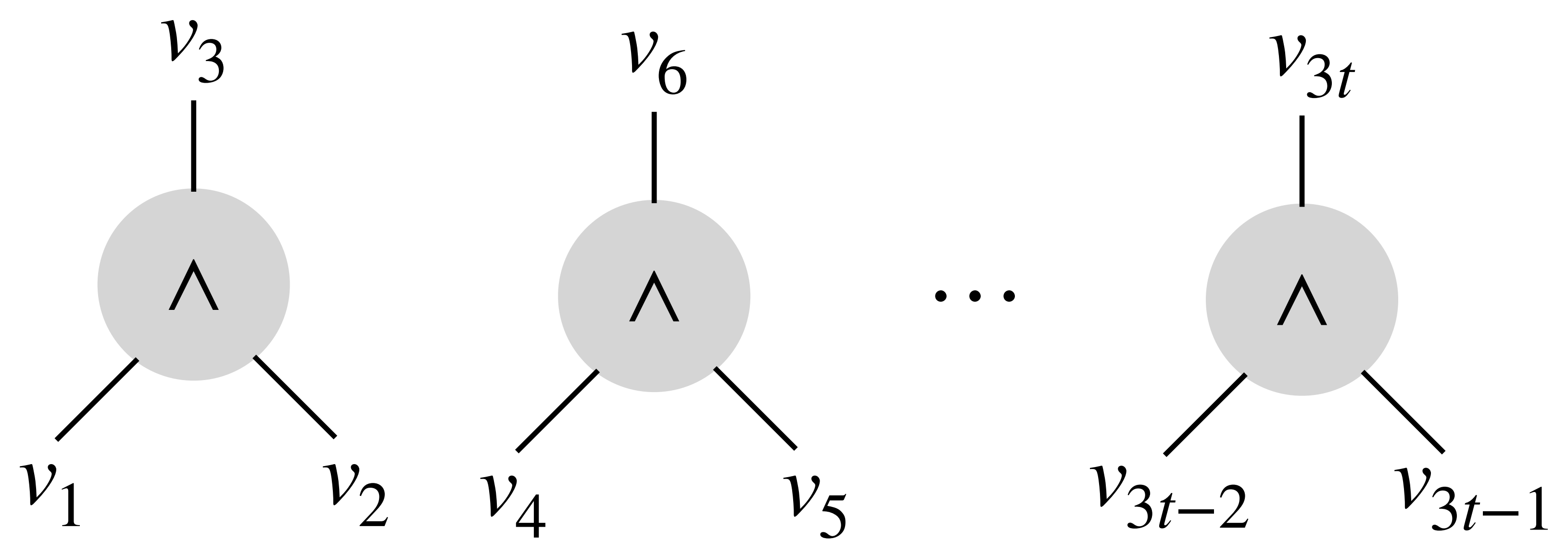
$(\mathsf{g}_3, \mathsf{g}_6, \ldots, \mathsf{g}_{3t}) = \mathsf{UnPack}(G)$

$(k_3, \ldots, k_{3t})$

$(\mathsf{e}_3, \mathsf{e}_6, \ldots, \mathsf{e}_{3t}) = \mathsf{UnPack}(E)$

$\ell_i = \Delta \cdot \mathsf{e}_i - k_i \pmod p$

Template for $\omega(1/\lambda)$-Rate Garbling



1. Packing

$V_\mathsf{R} = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$

$V_\mathsf{L} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$

Rate: $O\left(\frac{t}{\lambda}\right) = O\left(\frac{\sqrt{\log \lambda}}{\lambda}\right)$

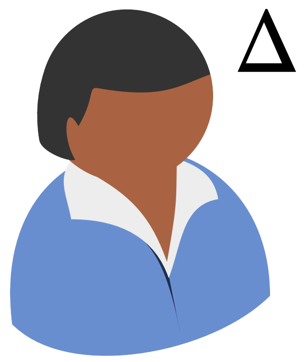
Why $t = O\left(\sqrt{\log \lambda}\right)$?

2. Gate Evaluation

$V = V_1 \cdot V_2$

3. Unpacking

$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$



Garbler

Invariant

$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$

$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod p$

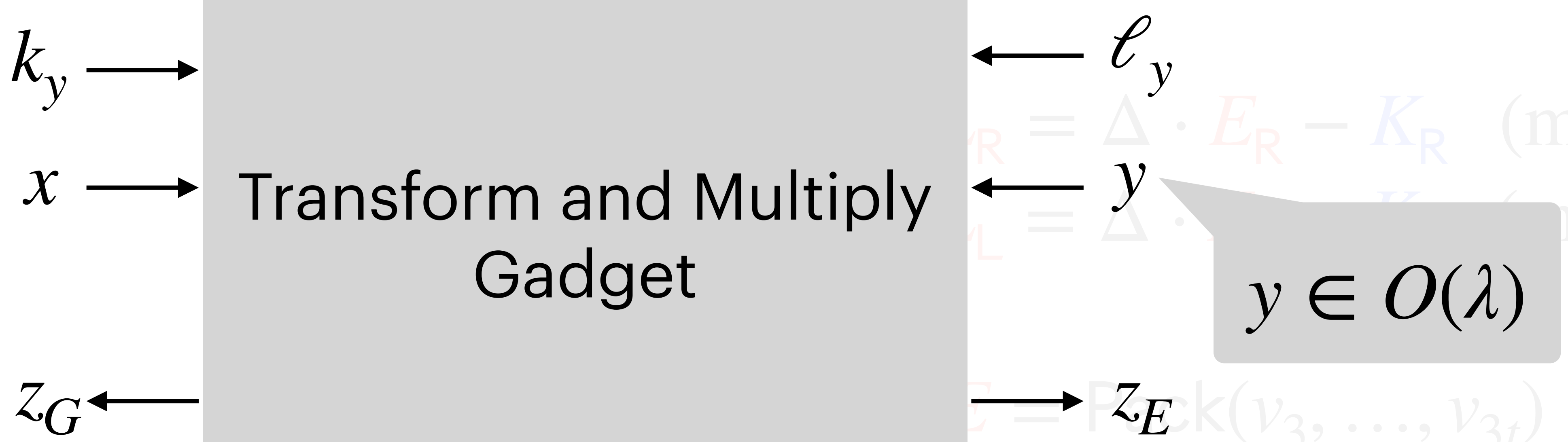
Evaluator

$G_\mathsf{R} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$

$G_\mathsf{L} = \mathsf{Pack}(\mathsf{g}_1, \mathsf{g}_4, \ldots, \mathsf{g}_{3t-2})$

$E_\mathsf{R} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$

$E_\mathsf{L} = \mathsf{Pack}(\mathsf{e}_1, \mathsf{e}_4, \ldots, \mathsf{e}_{3t-2})$



$z_G + z_E = x \cdot f(y)$

K_L K_R

$= \Delta \cdot E_\mathsf{R} - K_\mathsf{R} \pmod p$

$\pmod p$

G

K

$\mathsf{Pack}(v_3, \ldots, v_{3t}) - G$

$L = \Delta \cdot E - K \pmod p$

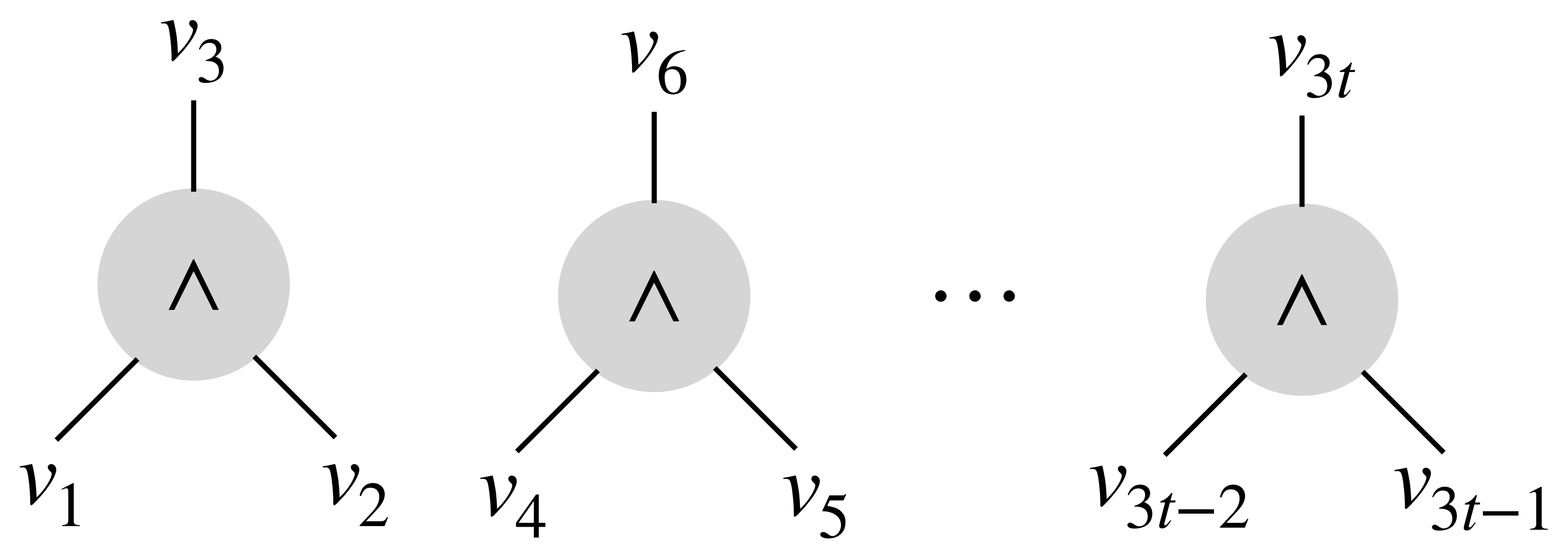
$(\mathsf{g}_3, \mathsf{g}_6, \ldots, \mathsf{g}_{3t}) = \mathsf{UnPack}(G)$

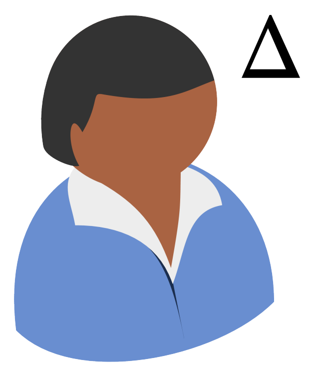
$(\mathsf{e}_3, \mathsf{e}_6, \ldots, \mathsf{e}_{3t}) = \mathsf{UnPack}(E)$

$(k_3, \ldots, k_{3t})$

$\ell_i = \Delta \cdot \mathsf{e}_i - k_i \pmod p$

Template for $\omega(1/\lambda)$-Rate Garbling




Δ

Garbler

Invariant

$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$

$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod p$


Evaluator

1. Packing

$V_R = \mathsf{Pack}(v_2, v_5, \ldots, v_{3t-1})$

$V_L = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$

$G_R = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$

$G_L = \mathsf{Pack}(\mathsf{g}_1, \mathsf{g}_4, \ldots, \mathsf{g}_{3t-2})$

$E_R = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$

$E_L = \mathsf{Pack}(\mathsf{e}_1, \mathsf{e}_4, \ldots, \mathsf{e}_{3t-2})$

Rate: $O\left(\frac{t}{\lambda}\right) = O\left(\frac{\sqrt{\log \lambda}}{\lambda}\right)$

Why $t = O\left(\sqrt{\log \lambda}\right)$?

- $t \in O(\log \lambda)$

2. Gate Evaluation

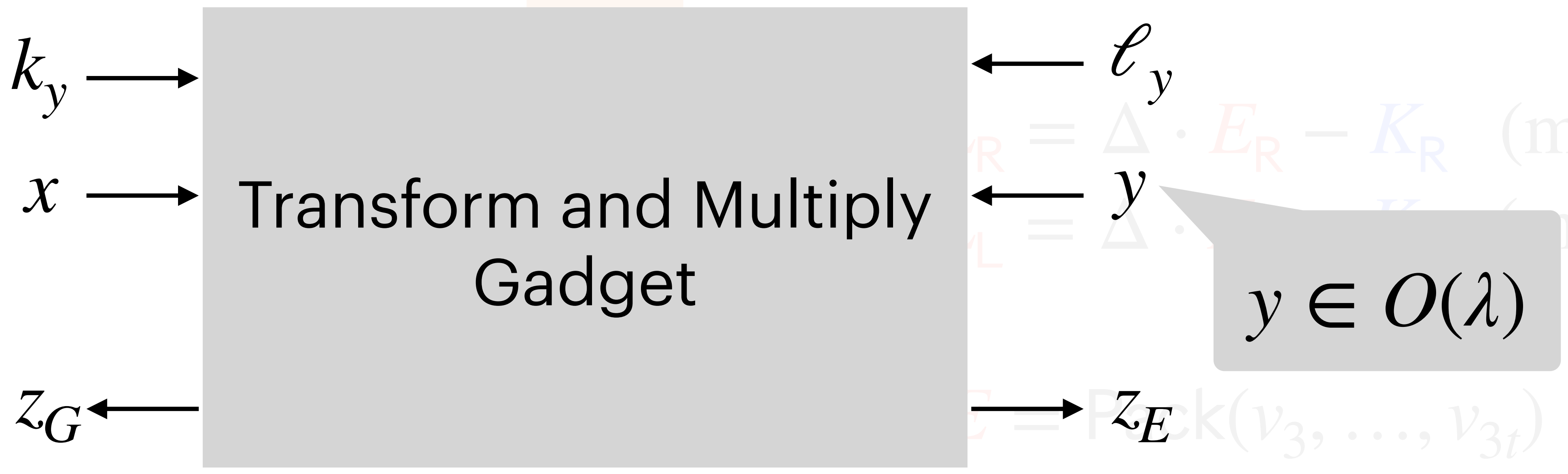
$K_L \quad K_R$

G

K

$= \Delta \cdot E_R - K_R \pmod p$

$L = \Delta \cdot E - K \pmod p$



$z_G + z_E = x \cdot f(y)$

3. Unpacking

$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$

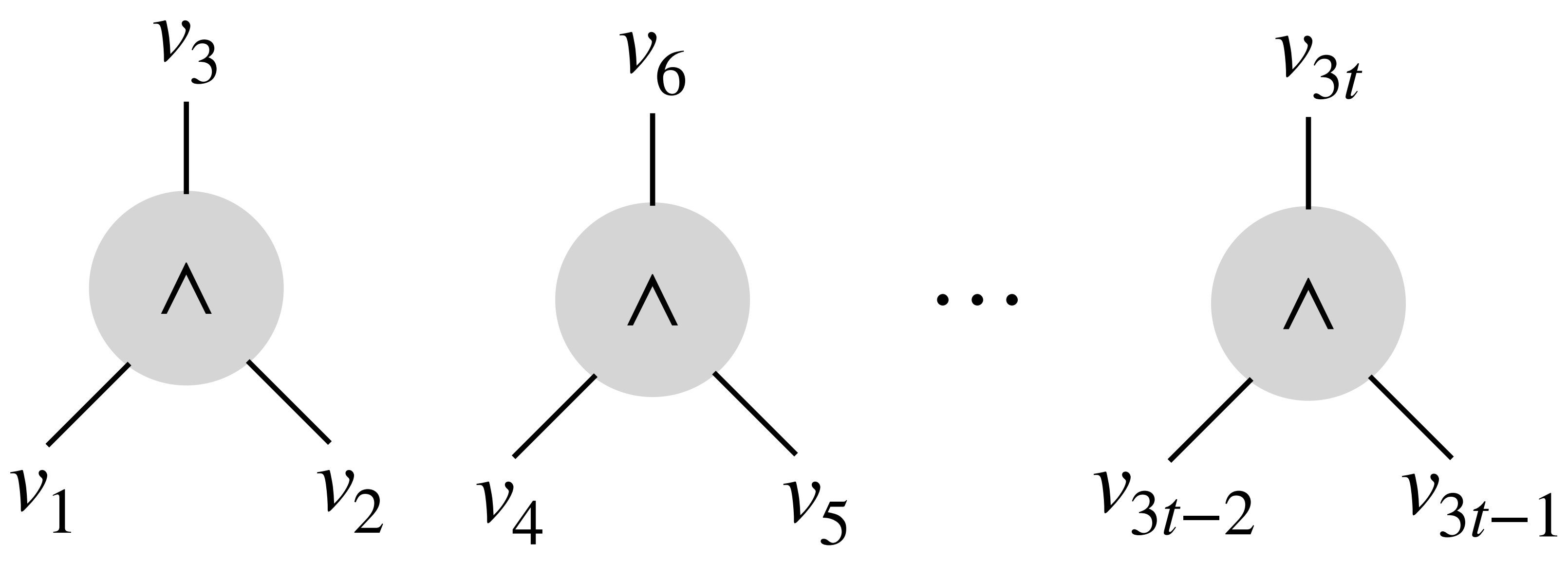
$(\mathsf{g}_3, \mathsf{g}_6, \ldots, \mathsf{g}_{3t}) = \mathsf{UnPack}(G)$

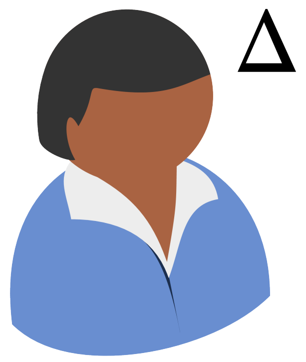
$(\mathsf{e}_3, \mathsf{e}_6, \ldots, \mathsf{e}_{3t}) = \mathsf{UnPack}(E)$

$(k_3, \ldots, k_{3t})$

$\ell_i = \Delta \cdot \mathsf{e}_i - k_i \pmod p$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$

$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod{p}$

Evaluator

1. Packing

$G_\mathsf{R} = \mathsf{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$

$E_\mathsf{R} = \mathsf{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$

$V_\mathsf{L} = \mathsf{Pack}(v_1, v_4, \ldots, v_{3t-2})$

Rate: $O\left(\frac{t}{\lambda}\right) = O\left(\frac{\sqrt{\log \lambda}}{\lambda}\right)$

- Gadget adds noise to ensure privacy, which increases magnitude of shares
- Retaining homomorphism over these shares requires packing fewer bits

Why $t = O\left(\sqrt{\log \lambda}\right)$?

- $t \in O(\log \lambda)$

2. Gate Evaluation

$E = \mathsf{Pack}(v_3, \ldots, v_{3t}) - G$

$L = \Delta \cdot E - K \pmod{p}$

$V = V_1 \cdot V_2 = (G_\mathsf{L} + E_\mathsf{L}) \cdot (G_\mathsf{R} + E_\mathsf{R})$

$= G_\mathsf{L} \cdot G_\mathsf{R} + G_\mathsf{L} \cdot E_\mathsf{R} + E_\mathsf{L} \cdot G_\mathsf{R} + E_\mathsf{L} \cdot E_\mathsf{R}$

Use multiplication gadget

3. Unpacking

$(\mathsf{g}_3, \mathsf{g}_6, \ldots, \mathsf{g}_{3t}) = \mathsf{UnPack}(G)$

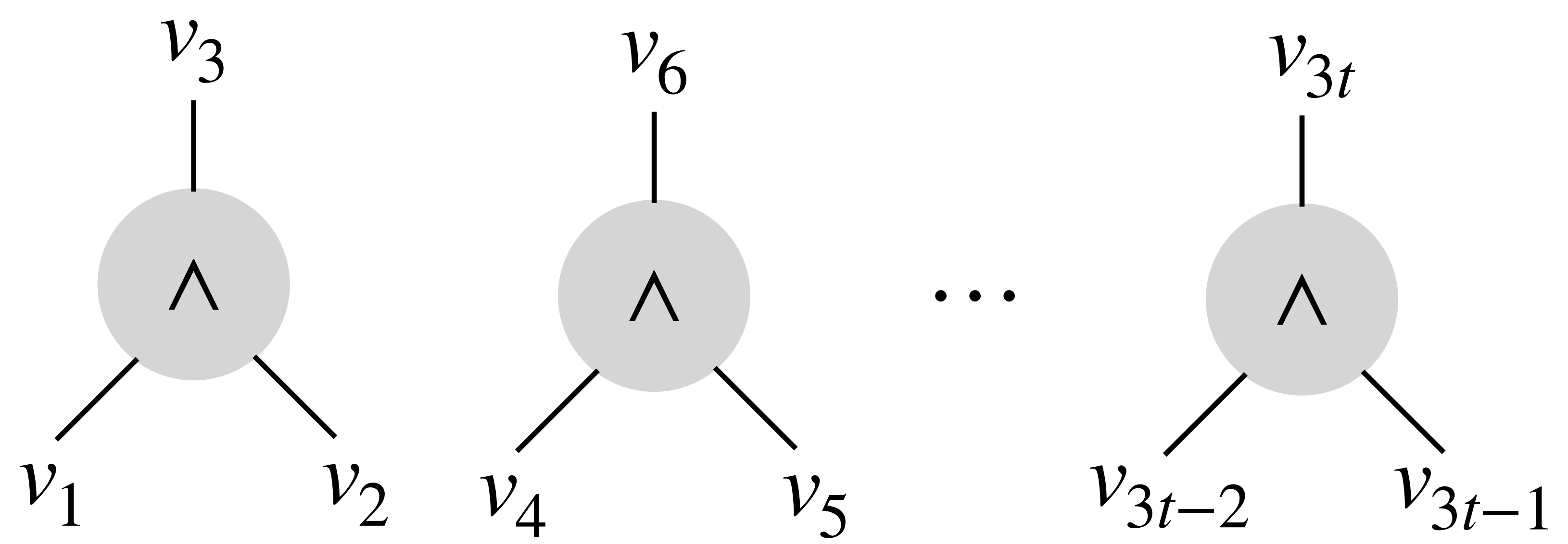
$(\mathsf{e}_3, \mathsf{e}_6, \ldots, \mathsf{e}_{3t}) = \mathsf{UnPack}(E)$

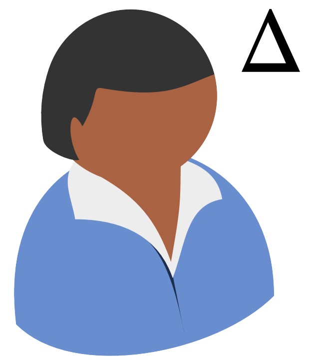
$(v_3, v_6, \ldots, v_{3t}) = \mathsf{UnPack}(V)$

$(k_3, \ldots, k_{3t})$

$= \Delta \cdot \mathsf{e}_i - k_i \pmod{p}$

Template for $\omega(1/\lambda)$-Rate Garbling





Garbler

Invariant

$$\mathsf{g}_i \oplus \mathsf{e}_i = v_i$$

$$k_i + \ell_i = \Delta \cdot \mathsf{e}_i \pmod p$$

Evaluator

1. Packing

$V_R = \text{Pack}(v_2, v_5, \ldots, v_{3t-1})$

$V_L = \text{Pack}(v_1, v_4, \ldots, v_{3t-2})$

Rate: $O\left(\frac{t}{\lambda}\right) = O\left(\frac{\sqrt{\log \lambda}}{\lambda}\right)$

Why $t = O\left(\sqrt{\log \lambda}\right)$?

- $t \in O(\log \lambda)$
- $t \in O\left(\sqrt{\log \lambda}\right)$

2. Gate Evaluation

$V = V_1 \cdot V_2$

3. Unpacking

$(v_3, v_6, \ldots, v_{3t}) = \text{UnPack}(V)$

$G_R = \text{Pack}(\mathsf{g}_2, \mathsf{g}_5, \ldots, \mathsf{g}_{3t-1})$

$E_R = \text{Pack}(\mathsf{e}_2, \mathsf{e}_5, \ldots, \mathsf{e}_{3t-1})$

- Gadget adds noise to ensure privacy, which increases magnitude of shares
- Retaining homomorphism over these shares requires packing fewer bits

$E = \text{Pack}(v_3, \ldots, v_{3t}) - G$

$L = \Delta \cdot E - K \pmod p$

$$V = V_1 \cdot V_2 = (G_L + E_L) \cdot (G_R + E_R)$$

$$= G_L \cdot G_R + G_L \cdot E_R + E_L \cdot G_R + E_L \cdot E_R$$

Use multiplication gadget

$(\mathsf{g}_3, \mathsf{g}_6, \ldots, \mathsf{g}_{3t}) = \text{UnPack}(G)$

$(\mathsf{e}_3, \mathsf{e}_6, \ldots, \mathsf{e}_{3t}) = \text{UnPack}(E)$

$(k_3, \ldots, k_{3t})$

$= \Delta \cdot \mathsf{e}_i - k_i \pmod p$

Packing Scheme

Packing Scheme

Packing from $\mathbb{F}_2^t$ to $\mathbb{Z}$: 1) Pack and UnPack are $\mathbb{F}_2$-linear 2) Homomorphic for sum of products

Packing Scheme

Packing from $\mathbb{F}_2^t$ to $\mathbb{Z}$:

1) Pack and UnPack are $\mathbb{F}_2$-linear

2) Homomorphic for sum of products

$$\begin{tikzcd}
\mathbb{F}_2^t \arrow[r, bend left, dotted] & \mathbb{F}_{2^m} \arrow[l, bend left, dotted] \arrow[r, bend left, dotted] & \mathbb{Z} \arrow[l, bend left, dotted]
\end{tikzcd}$$

Packing Scheme

Packing from $\mathbb{F}_2^t$ to $\mathbb{Z}$: 1) Pack and UnPack are $\mathbb{F}_2$-linear 2) Homomorphic for sum of products

Reverse Multiplication-Friendly Embeddings

[Cascudo-Cramer-Xing-Yuan’18]

$$\begin{tikzcd}
\mathbb{F}_2^t \arrow[r, bend left, "\text{RMFE}"] & \mathbb{F}_{2^m} \arrow[l, bend left, "\text{RMFE}^{-1}"] \arrow[r, bend left] & \mathbb{Z} \arrow[l, bend left]
\end{tikzcd}$$

Packing Scheme

Packing from $\mathbb{F}_2^t$ to $\mathbb{Z}$: 1) **Pack** and **UnPack** are $\mathbb{F}_2$-linear 2) Homomorphic for sum of products

Reverse Multiplication-Friendly Embeddings
[Cascudo-Cramer-Xing-Yuan'18]

$$\begin{tikzcd}
\mathbb{F}_2^t \arrow[r, bend left, dotted, "\text{RMFE}"] & \mathbb{F}_{2^m} \arrow[l, bend left, dotted, "\text{RMFE}^{-1}"] \arrow[r, bend left, dotted] & \mathbb{Z} \arrow[l, bend left, dotted]
\end{tikzcd}$$

RMFE and RMFE^{-1} are $\mathbb{F}_2$-linear

Homomorphic for sum of products

$$m = \Theta(t)$$

Packing Scheme

Packing from $\mathbb{F}_2^t$ to $\mathbb{Z}$: 1) Pack and UnPack are $\mathbb{F}_2$-linear 2) Homomorphic for sum of products

Reverse Multiplication-Friendly Embeddings
[Cascudo-Cramer-Xing-Yuan'18]

Eval(x) : Lift $x \in \mathbb{F}_2[X]$ to $\mathbb{Z}[X]$ and
evaluate at N

$$\begin{tikzcd}
\mathbb{F}_2^t \arrow[r, bend left, "\text{RMFE}"] & \mathbb{F}_{2^m} \arrow[l, bend left, "\text{RMFE}^{-1}"] \arrow[r, bend left] & \mathbb{Z} \arrow[l, bend left]
\end{tikzcd}$$

RMFE and RMFE^{-1} are $\mathbb{F}_2$-linear

Homomorphic for sum of products

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Packing Scheme

Packing from $\mathbb{F}_2^t$ to $\mathbb{Z}$: 1) Pack and UnPack are $\mathbb{F}_2$-linear 2) Homomorphic for sum of products

Reverse Multiplication-Friendly Embeddings
[Cascudo-Cramer-Xing-Yuan'18]

Eval(x) : Lift $x \in \mathbb{F}_2[X]$ to $\mathbb{Z}[X]$ and evaluate at N

$$\begin{tikzcd}
\mathbb{F}_2^t \arrow[r, bend left, dotted, "\mathsf{RMFE}"] & \mathbb{F}_{2^m} \arrow[l, bend left, dotted, "\mathsf{RMFE}^{-1}"] \arrow[r, bend left, dotted] & \mathbb{Z} \arrow[l, bend left, dotted]
\end{tikzcd}$$

ToPoly(x) :
1. N-ary decomposition $x = \sum_{i=1}^{m} x_i \cdot N^{i-1}$
2. Output $\sum_{i=1}^{m} x_i \cdot X^{i-1} \pmod 2 \in \mathbb{F}_{2^m}$

RMFE and RMFE^{-1} are $\mathbb{F}_2$-linear

Homomorphic for sum of products

$$m = \Theta(t)$$

Packing Scheme

Packing from $\mathbb{F}_2^t$ to $\mathbb{Z}$: 1) Pack and UnPack are $\mathbb{F}_2$-linear 2) Homomorphic for sum of products

Reverse Multiplication-Friendly Embeddings
[Cascudo-Cramer-Xing-Yuan'18]

Eval(x) : Lift $x \in \mathbb{F}_2[X]$ to $\mathbb{Z}[X]$ and evaluate at N

$$\begin{tikzcd}
\mathbb{F}_2^t \arrow[r, bend left, dotted, "\text{RMFE}"] & \mathbb{F}_{2^m} \arrow[l, bend left, dotted, "\text{RMFE}^{-1}"] \arrow[r, bend left, dotted] & \mathbb{Z} \arrow[l, bend left, dotted]
\end{tikzcd}$$

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1. N-ary decomposition $x = \sum_{i=1}^{m} x_i \cdot N^{i-1}$
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RMFE and RMFE^{-1} are $\mathbb{F}_2$-linear

Homomorphic for sum of products

$m = \Theta(t)$

Eval and ToPoly are $\mathbb{F}_2$-linear

Homomorphic for sum of k products when $N > k \cdot m$

Conclusion

Rate-$\Omega(1/\lambda)$

Symmetric-key crypto Black-box use of crypto

[Yao'86] [Lindell-Pinkas'07] [Koleshnikov-Schneider'08] [Zahur-Rosulek-Evans'15] [Rosulek-Roy'21]

Rate-1

Ring LWE / NTRU Non-black-box use of crypto

[Liu-Wang-Yang-Yu'24]

Rate-$\omega(1)$

ABE + FHE / iO + OWF Non-black-box use of crypto

[Boneh-Gentry-Grobunov-Halevi-Nikolaenko-Segev-Vaikuntanathan-Vinayagamurthy'14] [Lin-Pass'14]
[Goldwasser-Kalai-Popa-Vaikuntanathan-Zeldovich'13] [Koppula-Lewko-Waters'15] [Hsieh-Lin-Luo'23]

Conclusion

Rate-$\Omega(1/\lambda)$

Symmetric-key crypto Black-box use of crypto

[Yao'86] [Lindell-Pinkas'07] [Koleshnikov-Schneider'08] [Zahur-Rosulek-Evans'15] [Rosulek-Roy'21]

Rate-$O(\sqrt{\log \lambda} \ / \ \lambda)$

Generic Group Model Black-box use of crypto

This Work

Rate-1

Ring LWE / NTRU Non-black-box use of crypto

[Liu-Wang-Yang-Yu'24]

Rate-$\omega(1)$

ABE + FHE / iO + OWF Non-black-box use of crypto

[Boneh-Gentry-Grobunov-Halevi-Nikolaenko-Segev-Vaikuntanathan-Vinayagamurthy'14] [Lin-Pass'14]
[Goldwasser-Kalai-Popa-Vaikuntanathan-Zeldovich'13] [Koppula-Lewko-Waters'15] [Hsieh-Lin-Luo'23]

Conclusion

Rate-$\Omega(1/\lambda)$

Symmetric-key crypto Black-box use of crypto

[Yao'86] [Lindell-Pinkas'07] [Koleshnikov-Schneider'08] [Zahur-Rosulek-Evans'15] [Rosulek-Roy'21]

Rate-$O(\sqrt{\log \lambda}\ /\ \lambda)$

Generic Group Model Black-box use of crypto

This Work

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Ring LWE / NTRU

DDH / DCR

Non-black-box use of crypto

[Liu-Wang-Yang-Yu'24] **[Meyer-Orlandi-Roy-Scholl'25]** **[Ishai-Li-Lin'25]**

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Rate-$O(\sqrt{\log \lambda}\ /\ \lambda)$

Generic Group Model Black-box use of crypto

This Work

Can we get better rate while being black-box in crypto?

Rate-1

Ring LWE / NTRU

DDH / DCR

Non-black-box use of crypto

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Conclusion

Rate-$\Omega(1/\lambda)$

Symmetric-key crypto   Black-box use of crypto

[Yao'86] [Lindell-Pinkas'07] [Koleshnikov-Schneider'08] [Zahur-Rosulek-Evans'15] [Rosulek-Roy'21]

Can we get $\omega(1/\lambda)$-rate from other assumptions?

Rate-$O(\sqrt{\log \lambda}\ /\ \lambda)$

Generic Group Model   Black-box use of crypto

This Work

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Ring LWE / NTRU

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Non-black-box use of crypto

[Liu-Wang-Yang-Yu'24] **[Meyer-Orlandi-Roy-Scholl'25]** **[Ishai-Li-Lin'25]**

Rate-$\omega(1)$

ABE + FHE / iO + OWF   Non-black-box use of crypto

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Conclusion

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Rate-$O(\sqrt{\log \lambda}\ /\ \lambda)$

Generic Group Model   Black-box use of crypto

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[Liu-Wang-Yang-Yu'24] **[Meyer-Orlandi-Roy-Scholl'25]** **[Ishai-Li-Lin'25]**

Rate-$\omega(1)$

ABE + FHE / iO + OWF   Non-black-box use of crypto

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Can we get $\omega(1/\lambda)$-rate from other assumptions?

Can we get better rate while being black-box in crypto?

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Thank You