Integral Resistance of Block Ciphers with Key Whitening by Modular Addition

CRYPTO 2025 ,

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Motivation

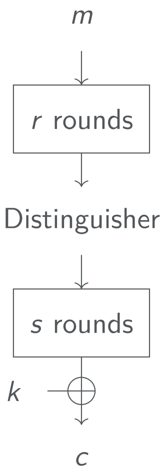
erc SymTrust

Focus on Security Arguments

Give strong security arguments for symmetric cryptographic primitives

State-of-the-art: Many arguments for linear and differential attacks. Few for integral cryptanalysis

Distinguisher and Key Recovery



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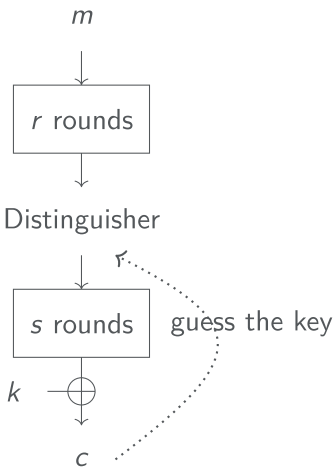

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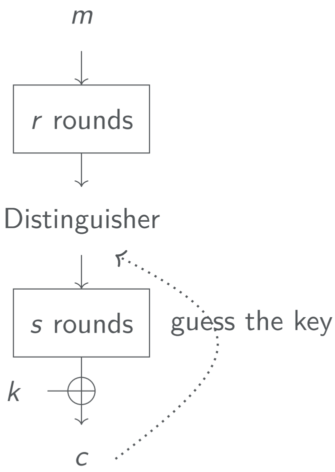

Here:

- Integral distinguisher
- Aim: Argue the non-existence
- Ignore the key-guessing

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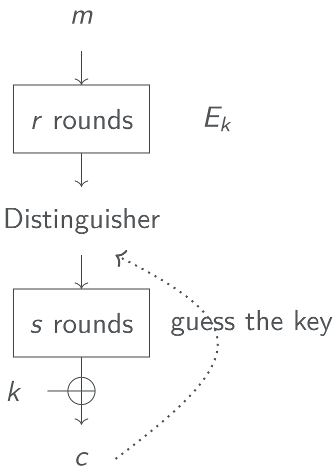

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Integral Distinguisher

Lars Ramkilde Knudsen

- Invented by Lars Knudsen
- Originally on AES-like designs
- Many improvements since then: e.g. division property, monomial prediction, geometric approach, ...

General Setting

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Zero-Sum

Given a block cipher $E_k : \mathbb{F}_2^n \to \mathbb{F}_2^n$ find a set $M \subseteq \mathbb{F}_2^n$ s.t.

$$\sum_{x \in M} E_k(x) = 0$$

- Enough if it happens on some bits

To simplify we consider only Boolean functions

$$f_k : \mathbb{F}_2^n \to \mathbb{F}_2$$

(think of one bit of the cipher-text)

Security Argument

Given $f_k : \mathbb{F}_2^n \to \mathbb{F}_2$

Goal

Show that for any (non trivial) set $M \subseteq \mathbb{F}_2^n$ it holds

$$\sum_{x \in M} f_k(x) \neq 0$$

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This is the same as linear independence of the functions $k \mapsto f_k(x)$.

Goal

Show that the functions $(k \mapsto f_k(x))_{x \in \mathbb{F}_2^n}$ are linear independent.

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Hebborn et al (AC21)

Can be drastically simplified by two ingredients:

- Look at the ANF
- Use pre-whitening keys

ANF

Every function can be written in its algebraic normal form

$$f_k(x) = \sum_{u \in \mathbb{F}_2^n} p_u(k) x^u$$

where

$$x^u = \prod_i x_i^{u_i} \text{ and } p_u : \mathbb{F}_2^\kappa \to \mathbb{F}_2$$

p_u can be computed *linearly* from $(k \mapsto f_k(x))$ (*and vice versa*)

Goal for ANF

$(k \mapsto f_k(x))_{x \in \mathbb{F}_2^n}$ are linear independent $\Leftrightarrow$ $(k \mapsto p_u(k))_{u \in \mathbb{F}_2^n}$ are linear independent.

ANF

$$f_k(x) = \sum_{u \in \mathbb{F}_2^n} p_u(k) x^u$$

Goal for ANF

$(k \mapsto p_u(k))_{u \in \mathbb{F}_2^n}$ are linear independent.

Each p_u can be written as

$$p_u(k) = \sum_{v \in \mathbb{F}_2^\kappa} \lambda_v^{(u)} k^v$$

Using division property/ monomial prediction we can compute (some!) $\lambda_v^{(u)}$

Whitening Keys (2^n becomes n)

$$f_k(x) = \sum_{u \in \mathbb{F}_2^n} p_u(k) x^u \text{ with } p_u(k) = \sum_{v \in \mathbb{F}_2^\kappa} \lambda_v^{(u)} k^v$$

Still 2^n (unstructured, hard to evaluate) functions 🤯

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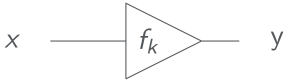

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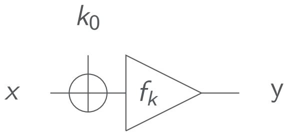

$$\widehat{f}_{k_0,k}(x) = f_k(x + k_0) = \sum_u p_u(k)(x + k_0)^u$$

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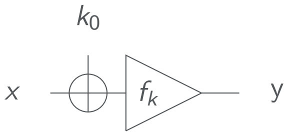

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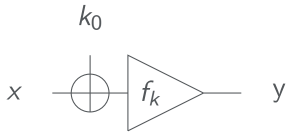

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- Still n functions
- requires computation of n^2 values $\lambda_v^{(u)}$
- Only XOR-whitening keys handled

$\Rightarrow$ Practically expensive and limited scope.

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Our work

Generalize to include modular addition of whitening keys and reduce computational complexity.

Notions of Order: The Influence of Pre-whitening Keys

XOR

$$(x \oplus k_0)^u = \sum_{v \leq u} k_0^{u \oplus v} x^v$$

$$v \leq u \Leftrightarrow \; v_i \leq u_i$$

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Modular-Add-Case (Braeken, Semaev)

$$(x \boxplus k_0)^u = \sum_{v \leq u} k_0^{u \boxminus v} x^v.$$

$v \leq u$ as integers

Notions of Order: The Influence of Pre-whitening Keys

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XOR	Modular-Add-Case (Braeken, Semaev)
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$v \leq u$ as integers

In a nutshell: Every v that is influenced becomes linear independent

Notions of Order: The Influence of Pre-whitening Keys

- Everything that is influenced becomes linear independent
- f_k balanced $\Rightarrow u = (1...1) = 2^n - 1$ is excluded.

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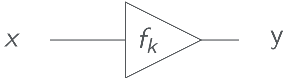
n elements of wt $= n - 1$ needed

$u = 2^n - 2$ alone is sufficient.

Condition gets much weaker + computationally cheaper

Mod Add Whitening Keys (2^n becomes 1!)

$$f_k(x) = \sum_{u \in \mathbb{F}_2^n} p_u(k) x^u \text{ with } p_u(k) = \sum_{v \in \mathbb{F}_2^\kappa} \lambda_v^{(u)} k^v$$



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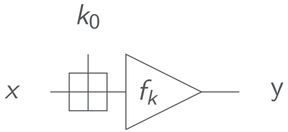

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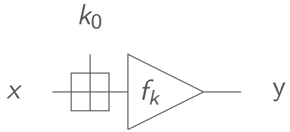

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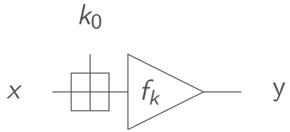

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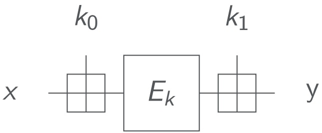
Theorem 2

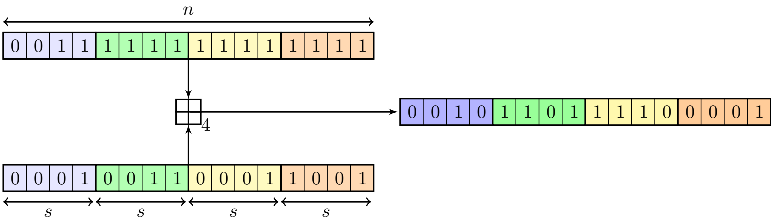
If $p_{2^n-2} \neq 0$ all q_u are linear independent.

What else (I/III): Post-whitening Keys



- Allows to lift the idea to vectorial version
- Still enough to compute one coefficient

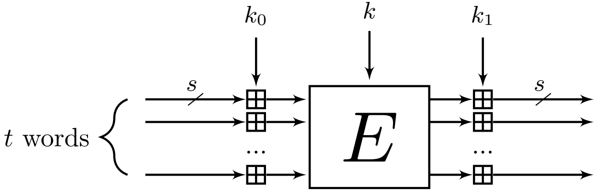
What else (II/III): Word-wise Addition



- Used for better performance
- ARX ciphers
- Give a unified view

What else (II/III): A Unified Framework

General Theorem to handle all those cases.



- $t = n$: XOR-whitening keys
- $t = 1$: Mod-Add-whitening keys

What else (III/III): d-th Order Integral Resistance

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Zero-Sum

Given a block cipher $E_k : \mathbb{F}_2^n \to \mathbb{F}_2^n$ find a set $M \subseteq \mathbb{F}_2^n$ s.t.

$$\sum_{x \in M} E_k(x) = 0$$

- Enough if it happens on some bits ✓
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We introduce d-th order integral resistance to capture that.

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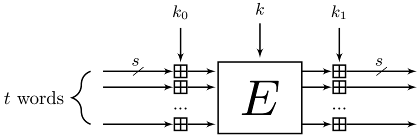
The End!



- No surprise that modular key addition makes it more resistant
- But: surprise how nice everything works out
- More in the paper: full proof, concrete examples, link to data, inverse cipher

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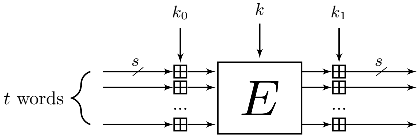

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Thank you for your attention!