Crowhammer

A key-recovery attack on **FALCON**

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NTT

FALCON

Trivia

- Selected by **NIST** for **standardization** (FN-DSA)
- Efficient but complex (floating-point arithmetics, Gaussian sampler)

FALCON

Signature

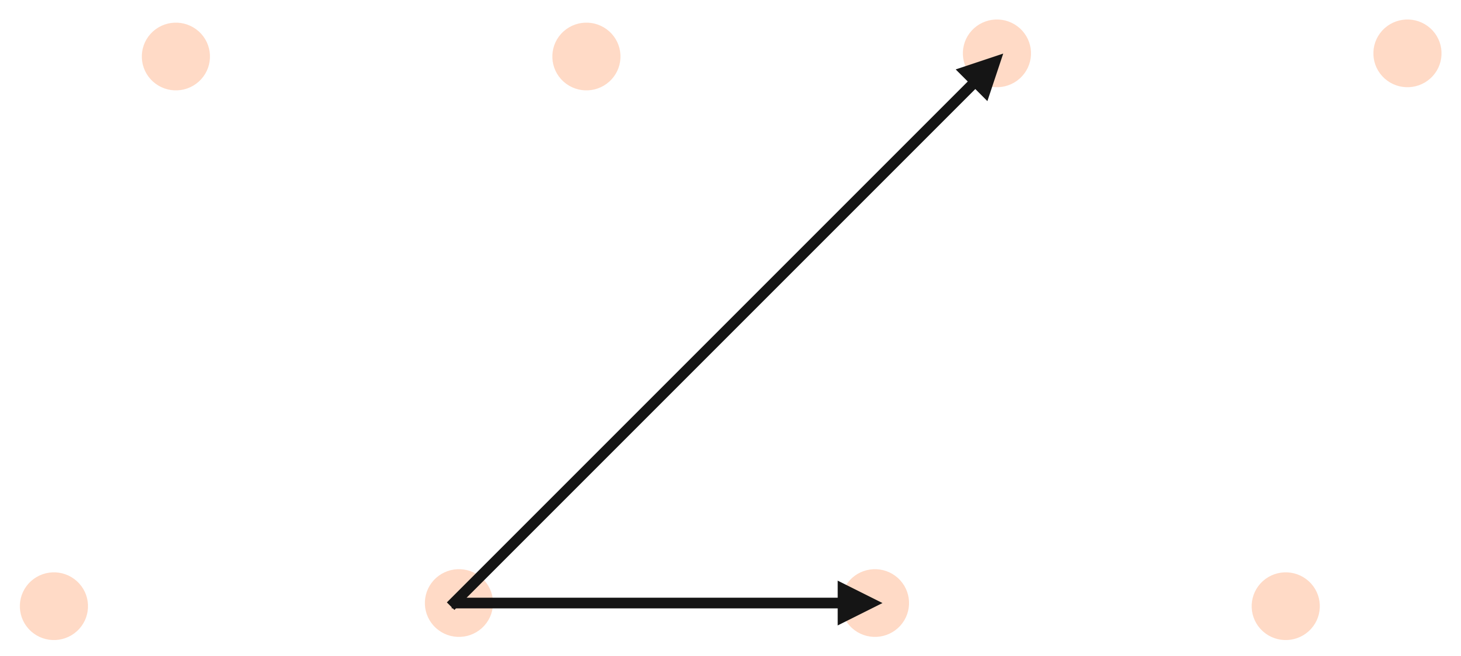
- Polynomial ring: $R = \mathbb{Z}_q[X]/(X^n + 1)$ modulo q
- NTRU Lattice: $L_h = \{(s_1, s_2) \in R^2 \mid s_2 + s_1 h = 0 \bmod q\}$ with $h = fg^{-1}$
- Secret key: small vectors $(g, -f) \in R^2, (G, -F) \in R^2$ and a "FALCON tree" T

$\begin{bmatrix} g & -f \\ G & -F \end{bmatrix}$ and $\begin{bmatrix} 1 & -h \\ 0 & q \end{bmatrix}$ are basis of L_h over R

FALCON

Signature

Nearest
Plane
Algorithm



Original basis

FALCON

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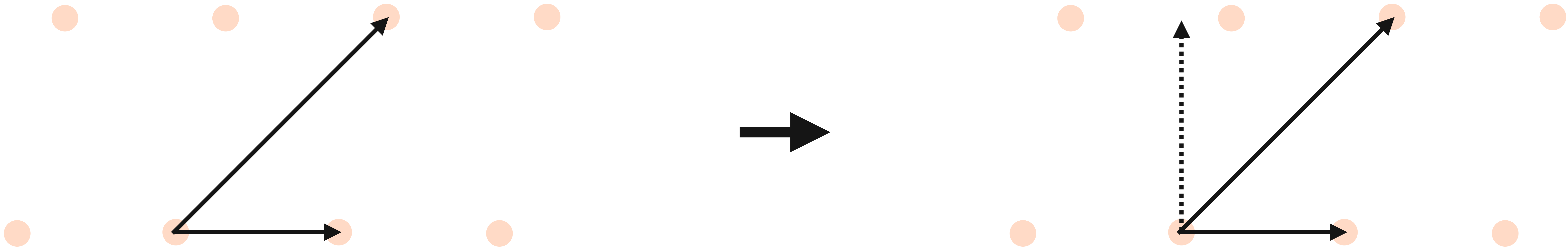

Original basis

GSO basis

FALCON

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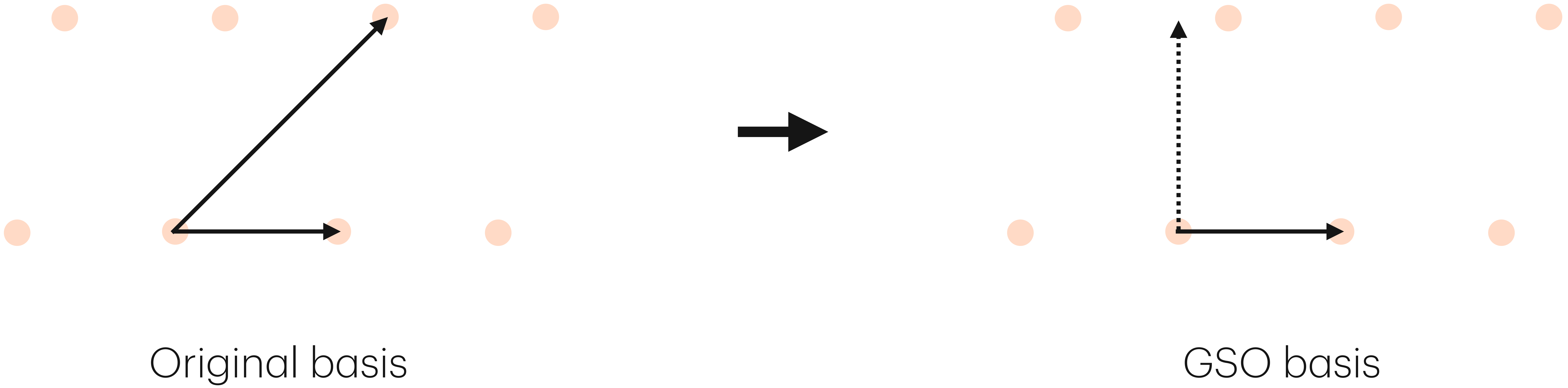

FALCON

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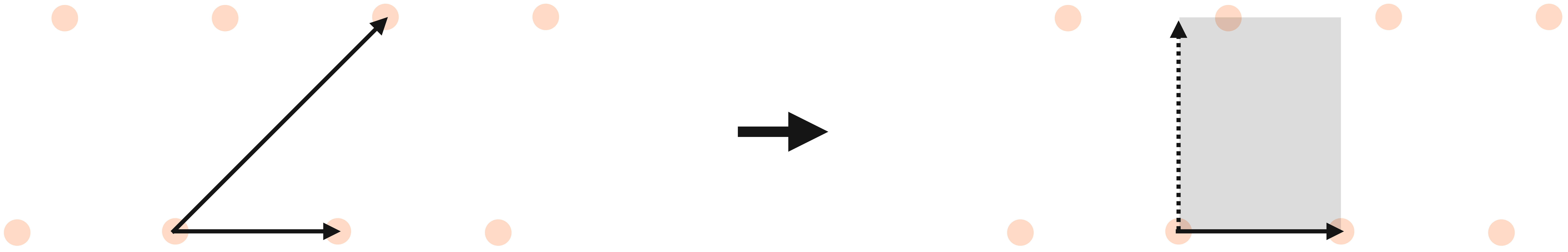

Original basis | GSO basis

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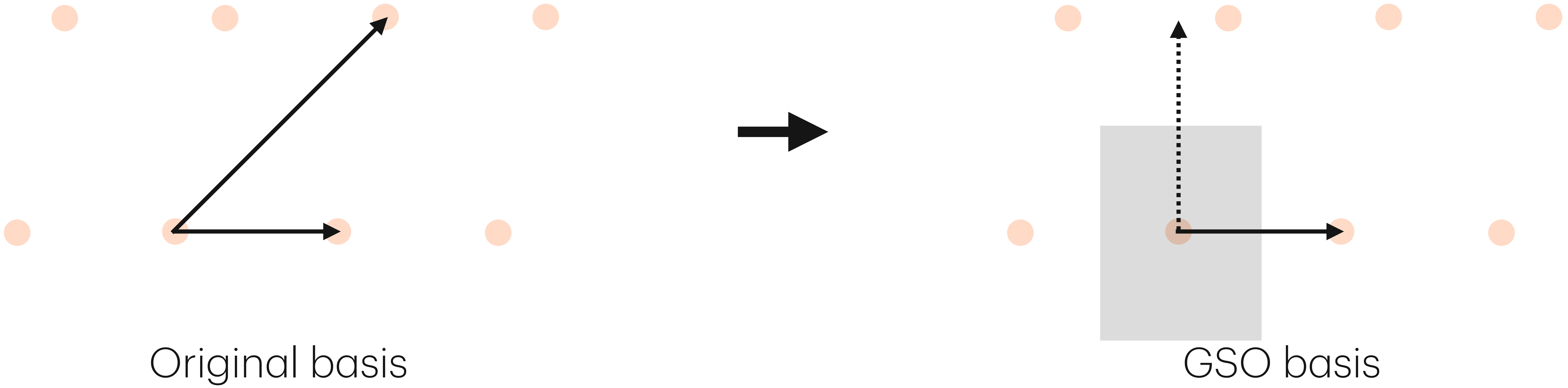

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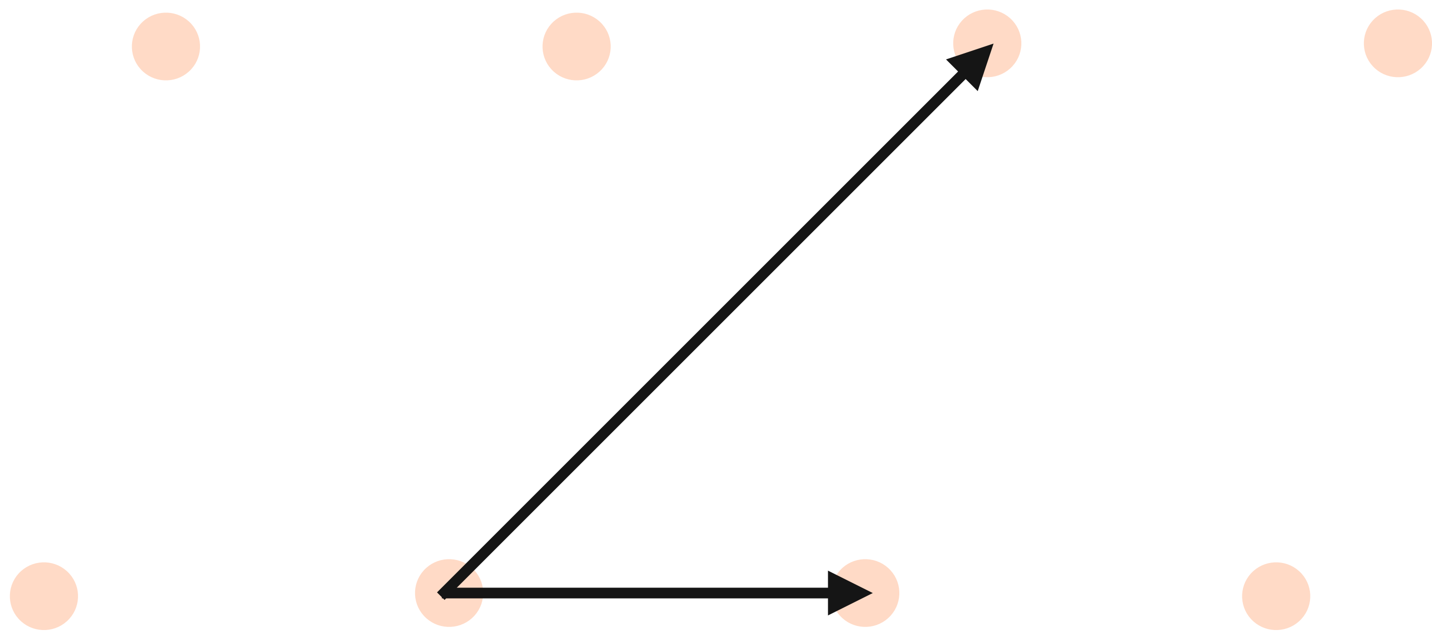

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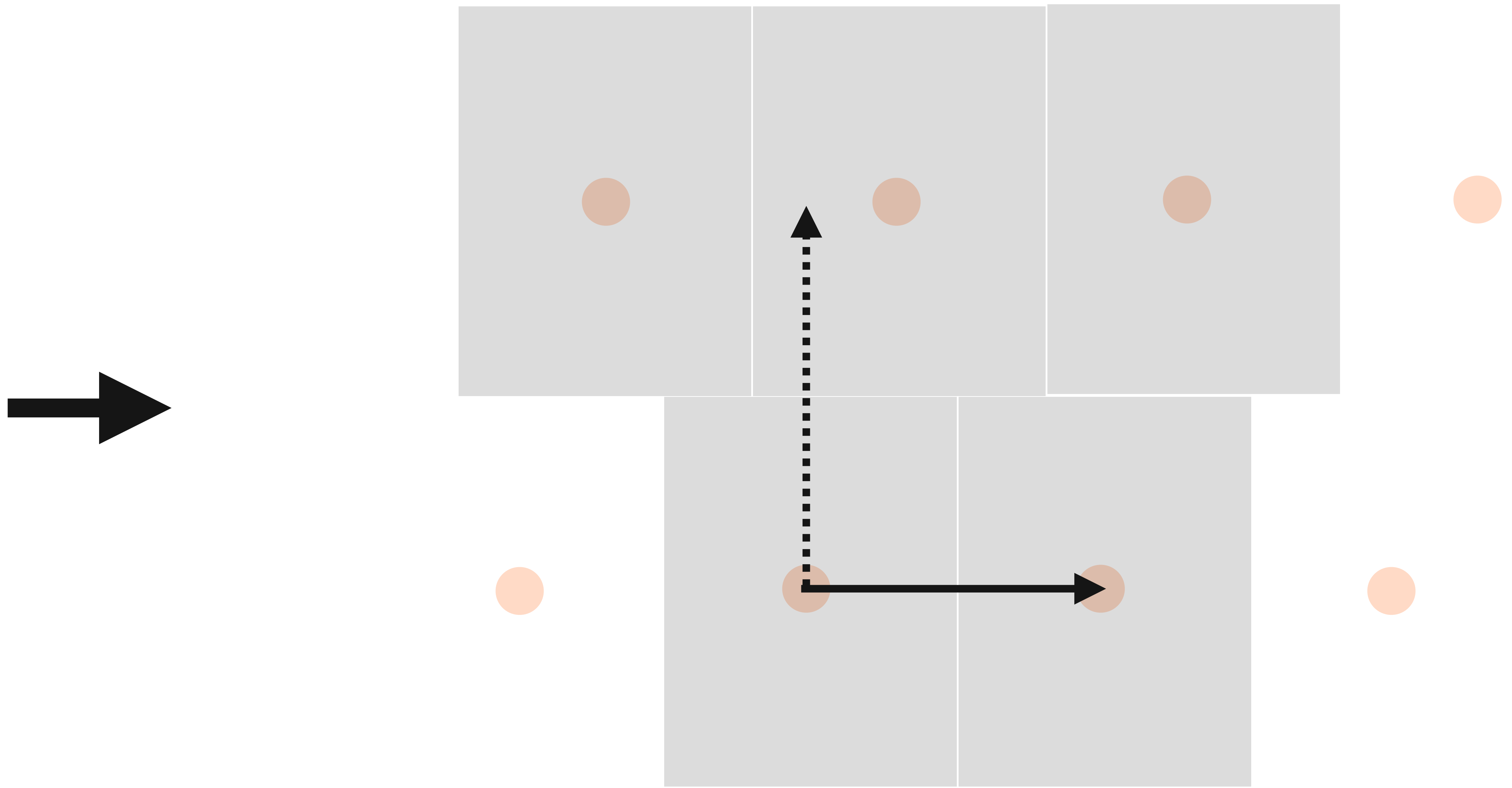

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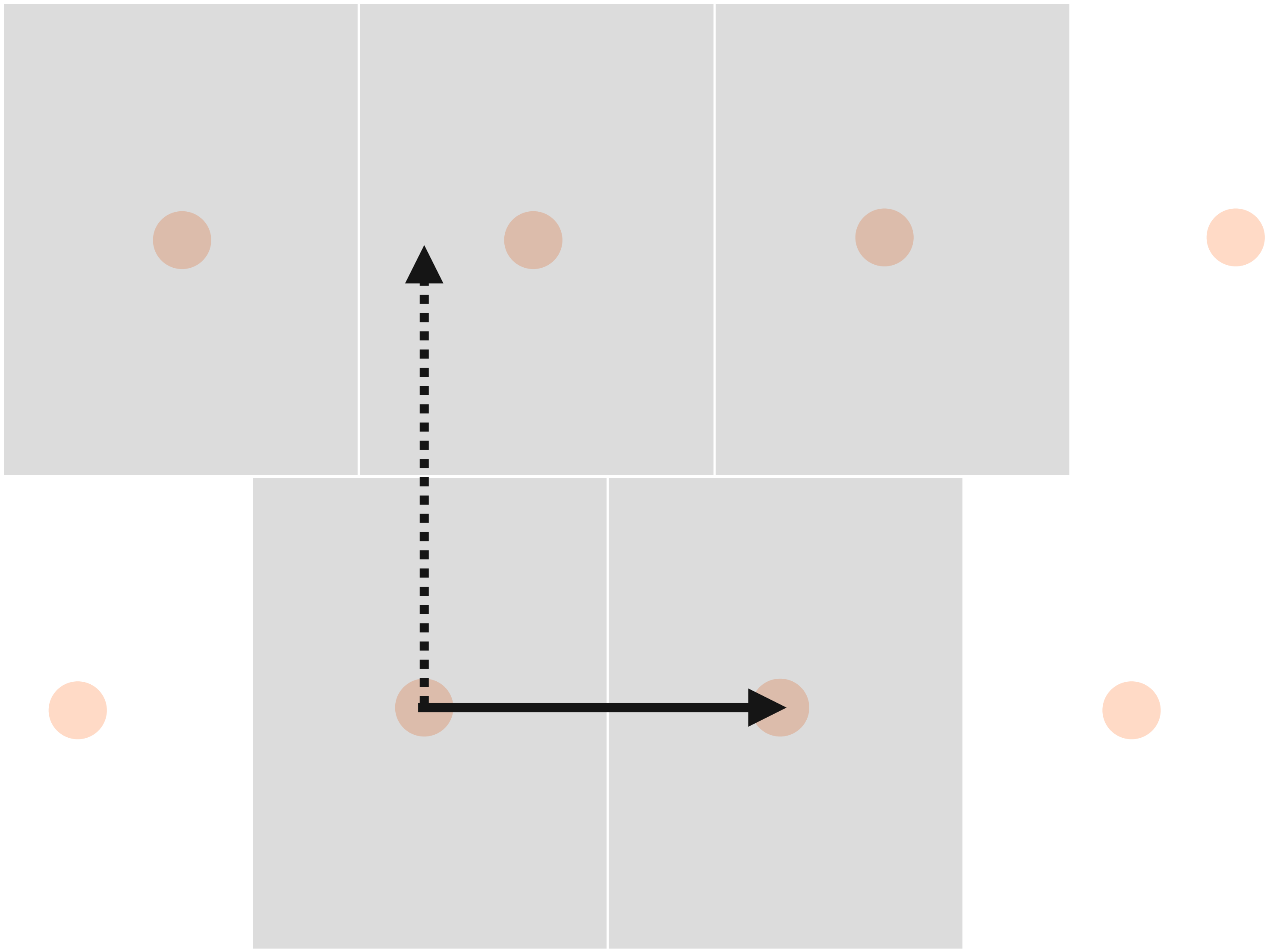

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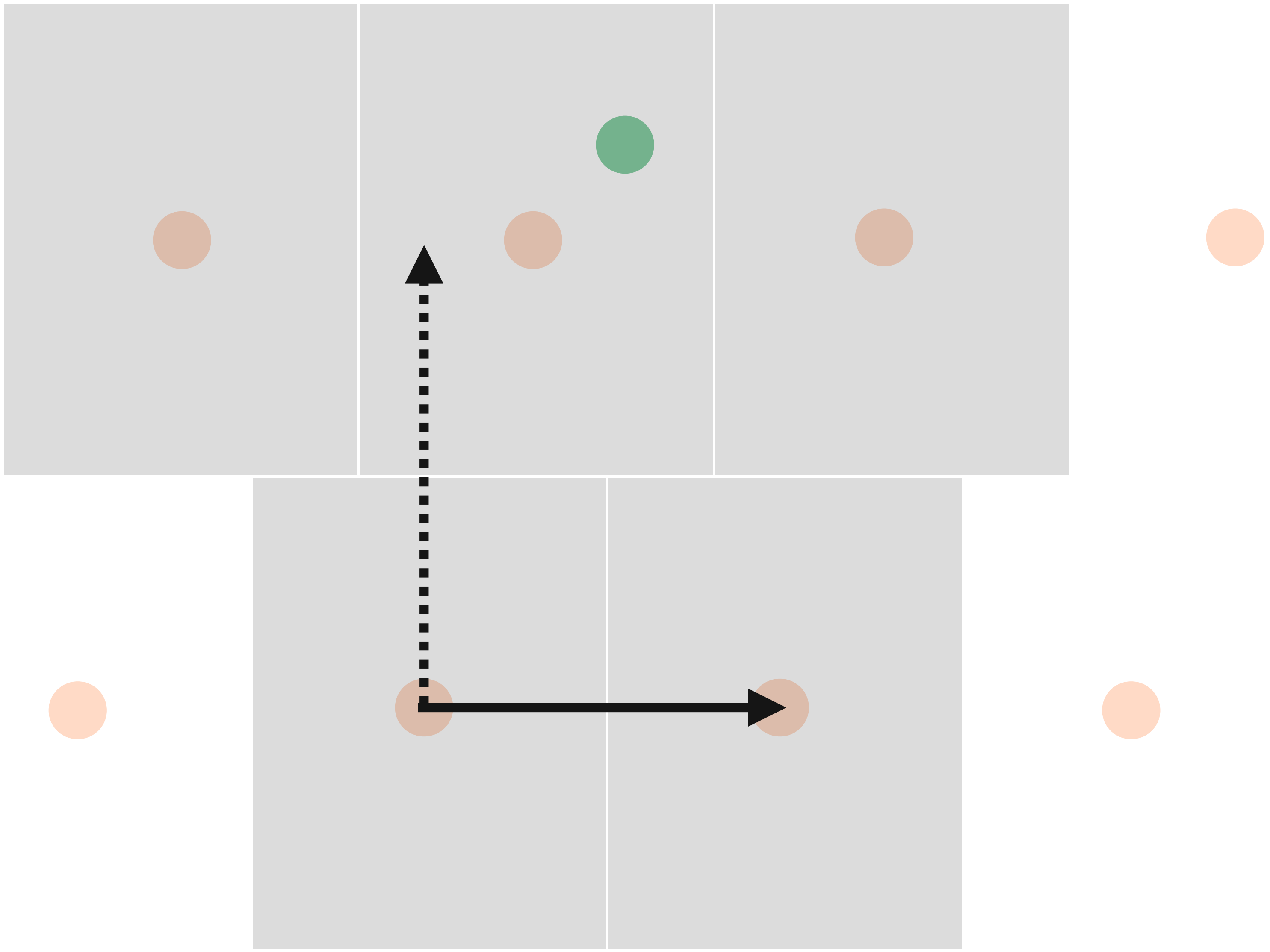
Approximate
Closest Vector Problem
Find ● "close" to ●



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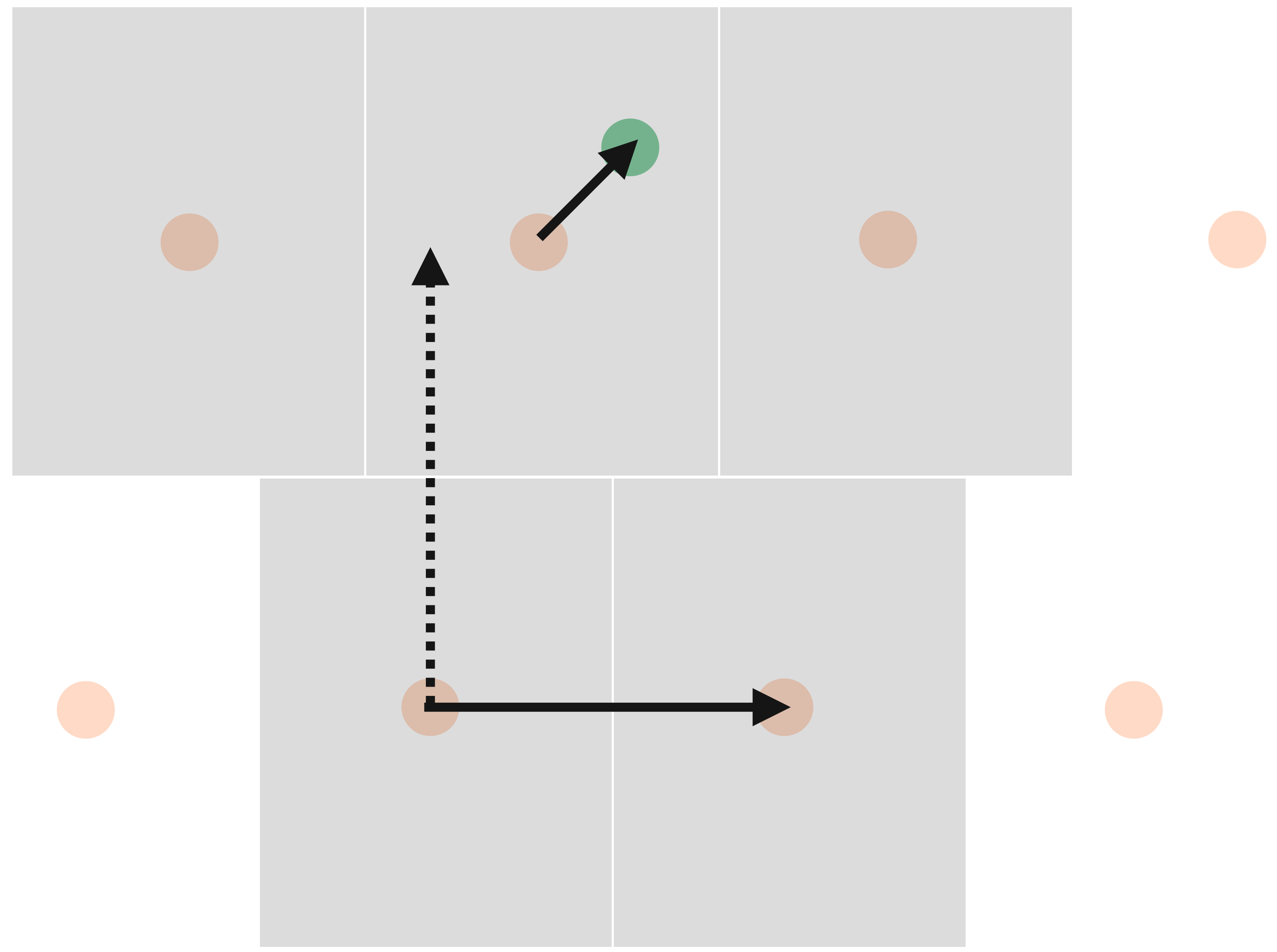

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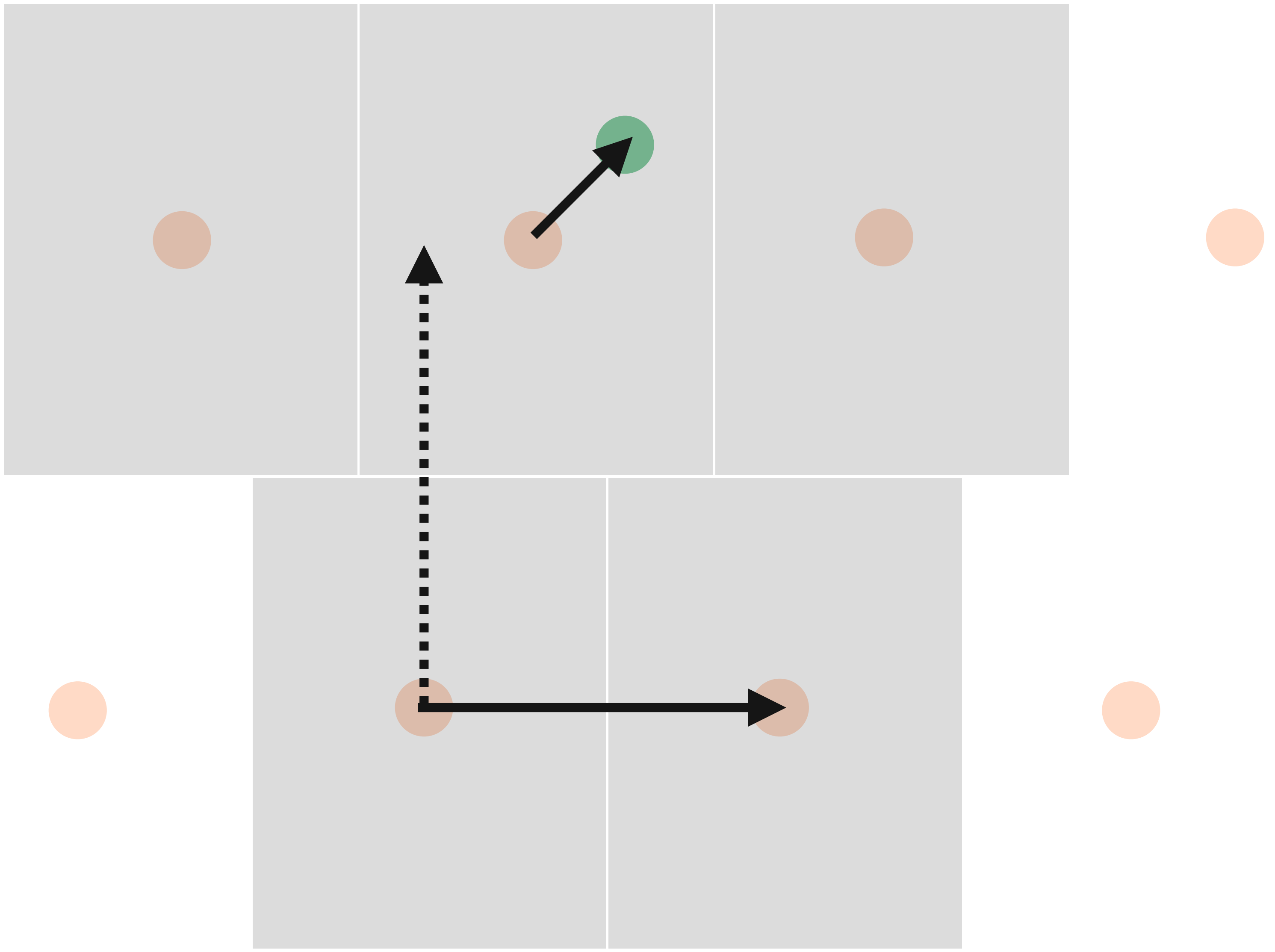

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Broken by NR06

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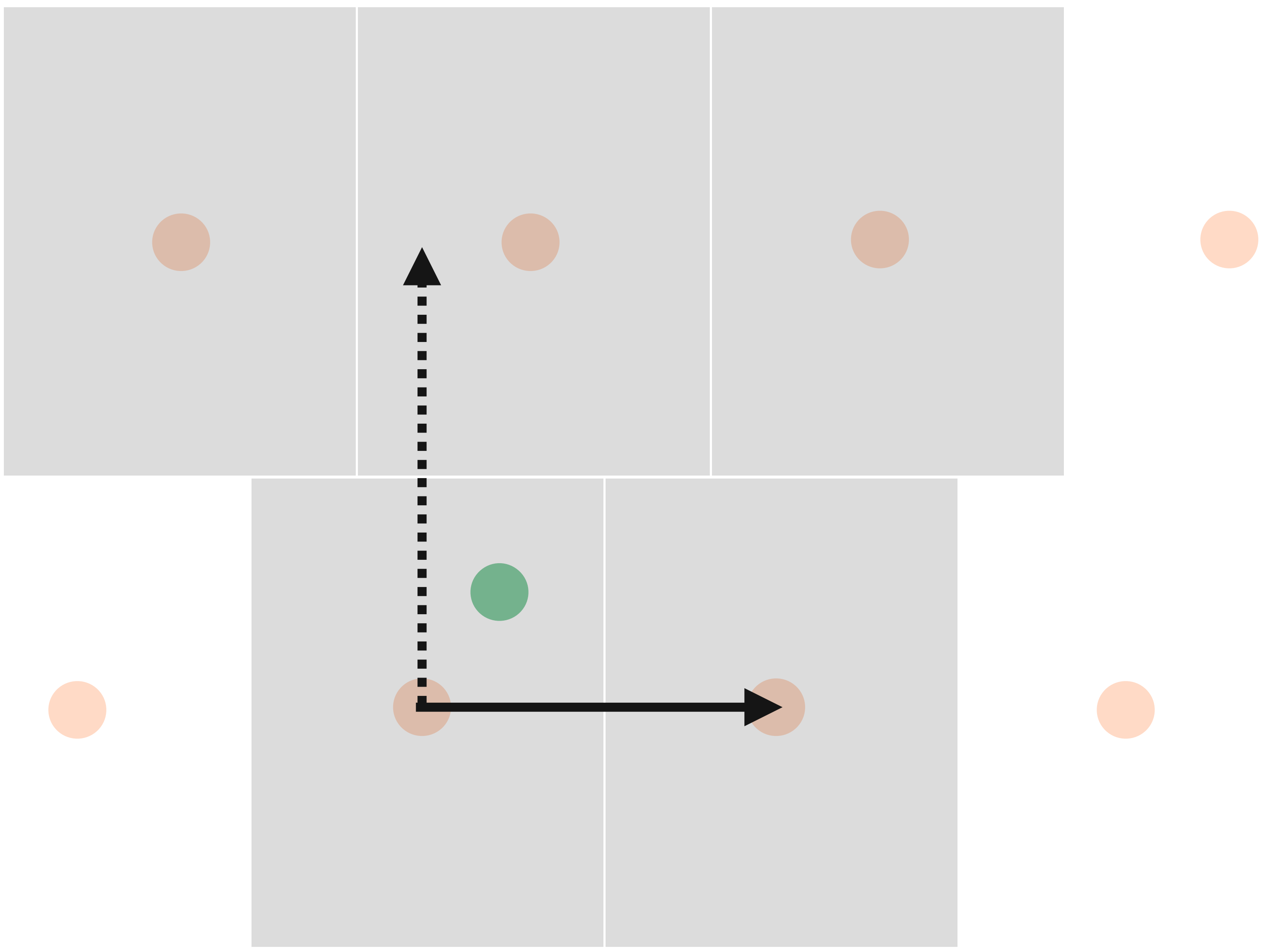

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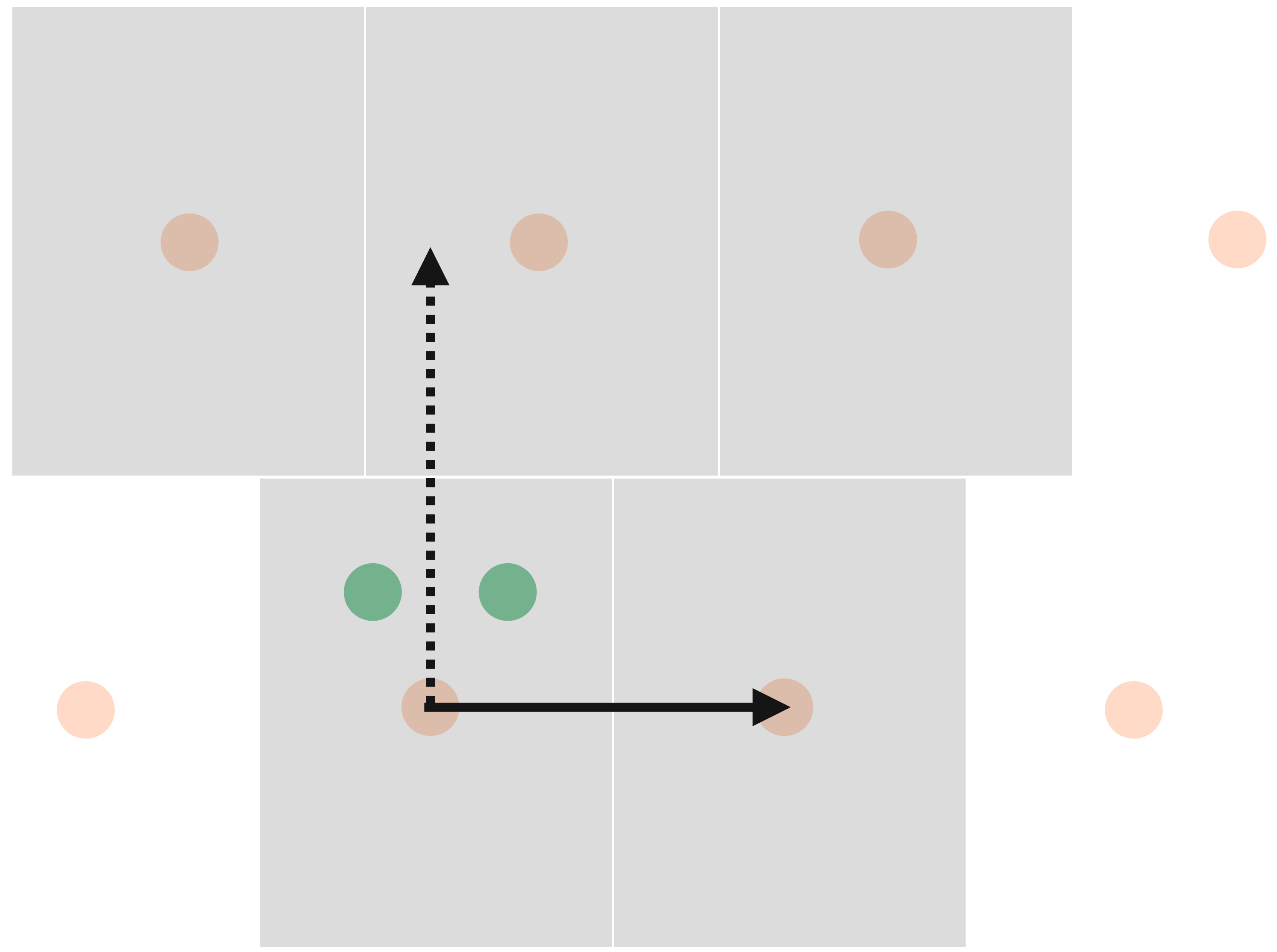

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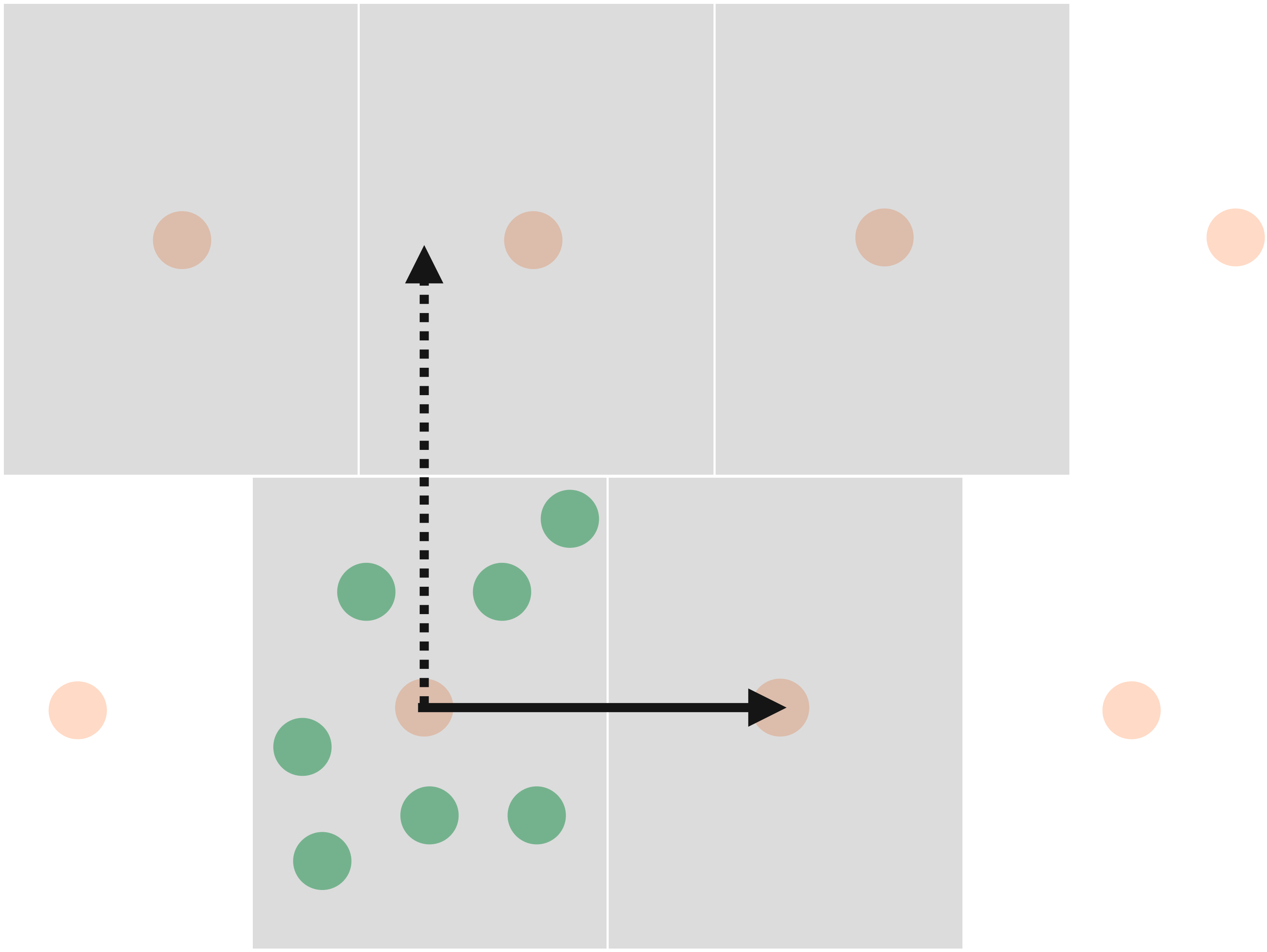

GSO basis

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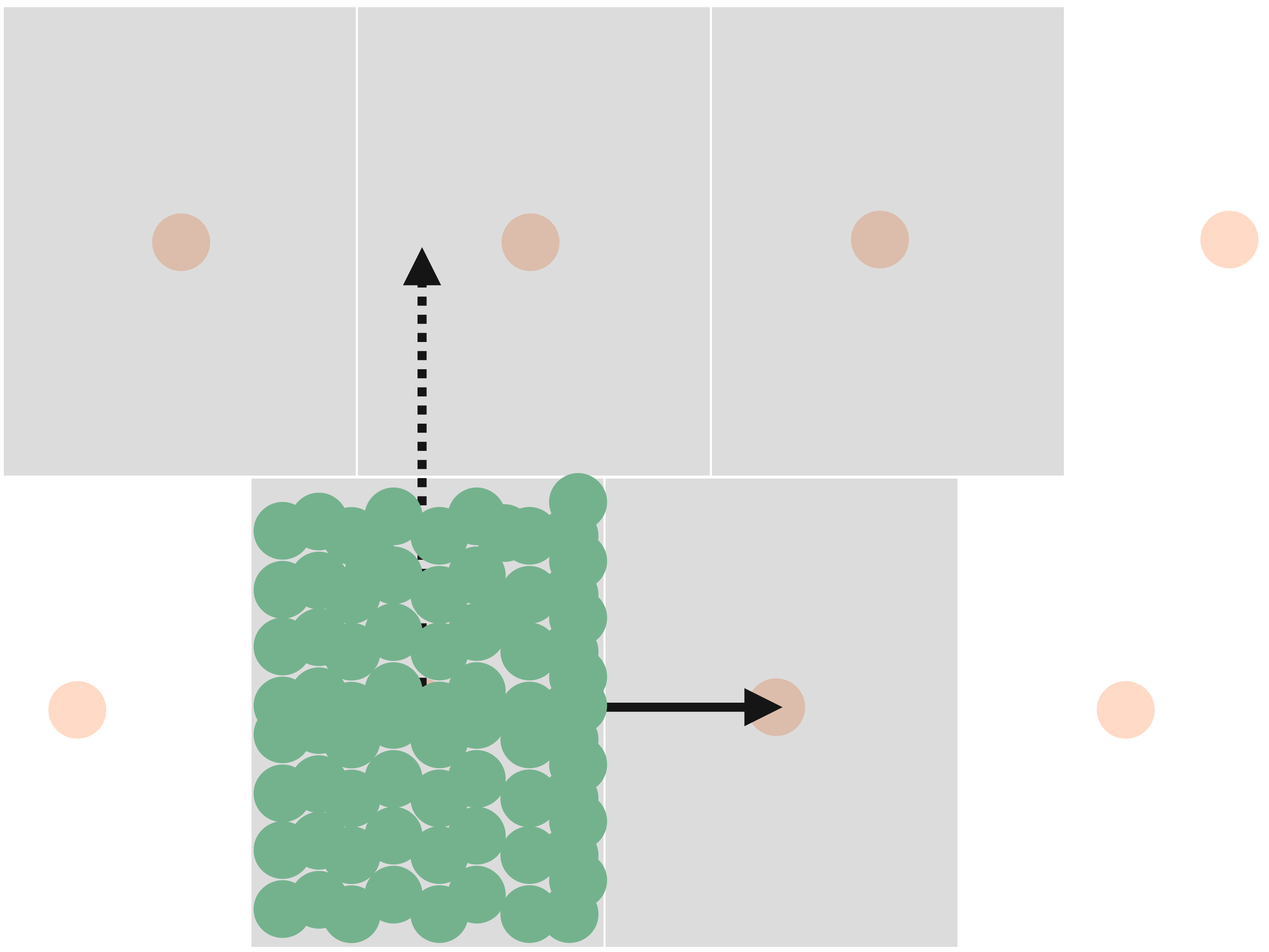
Approximate **Closest Vector Problem**

Find "close" to



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FALCON

Distribution of signatures

FALCON signatures follow a discrete Gaussian distribution $\sim \mathscr{D}_{B^T \Sigma B}$

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GSO decomposition $B = LD^{1/2}U$

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If $\Sigma = L^{-T}\sigma^2 D^{-1}L^{-1}$ then $B^T\Sigma B = \sigma^2 Id$

FALCON

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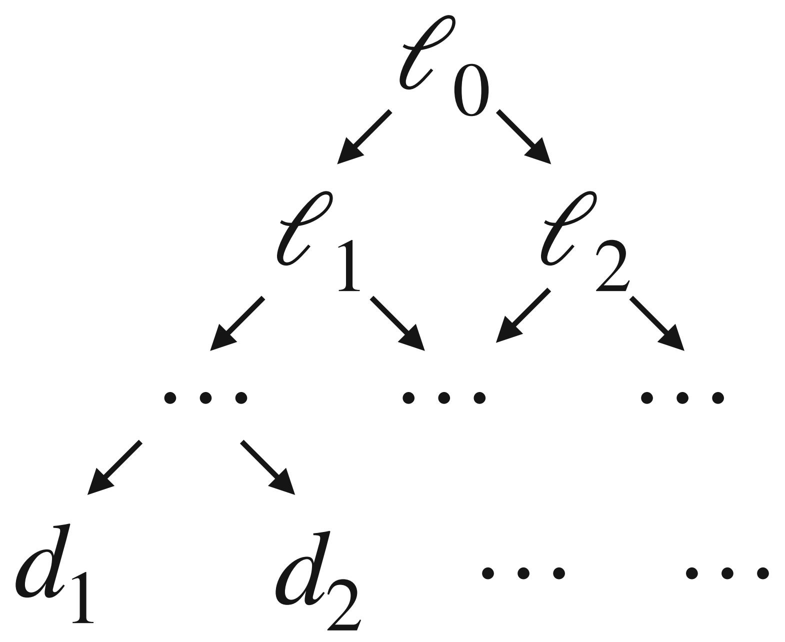
FALCON Tree



to sample Σ

FALCON

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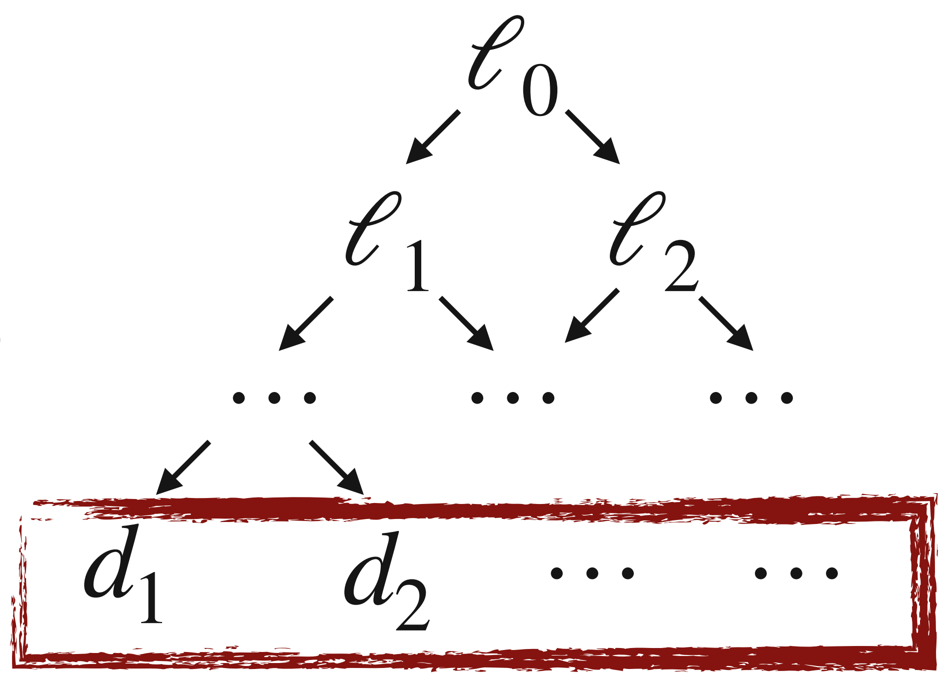

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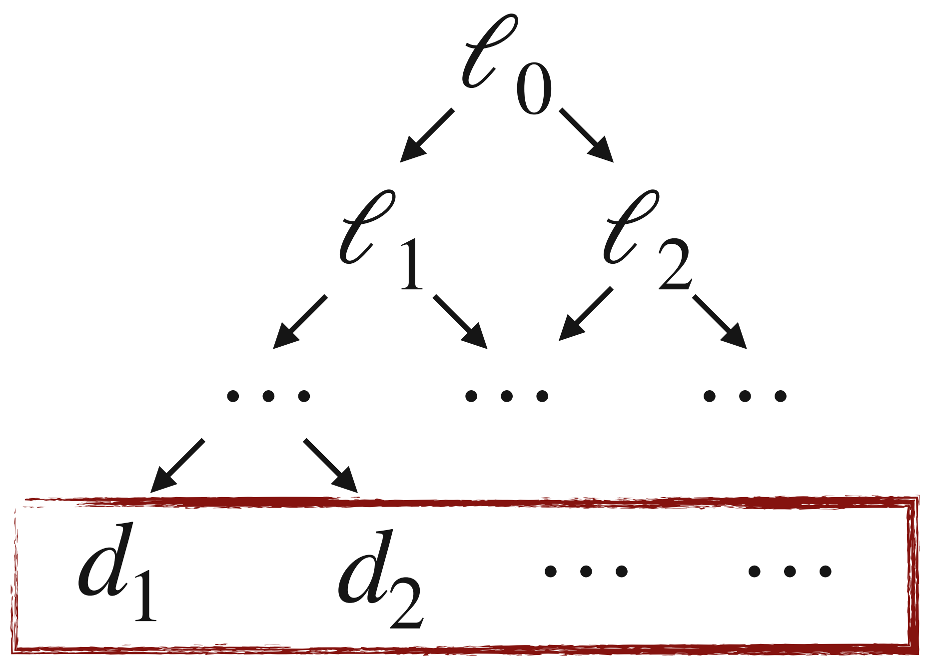
FALCON

Gaussian Sampling



Strategy: Perform (half) Gaussian sampling over $\mathbb{Z}$ then merge

Implementation: constant-time, linear scan over a table

FALCON

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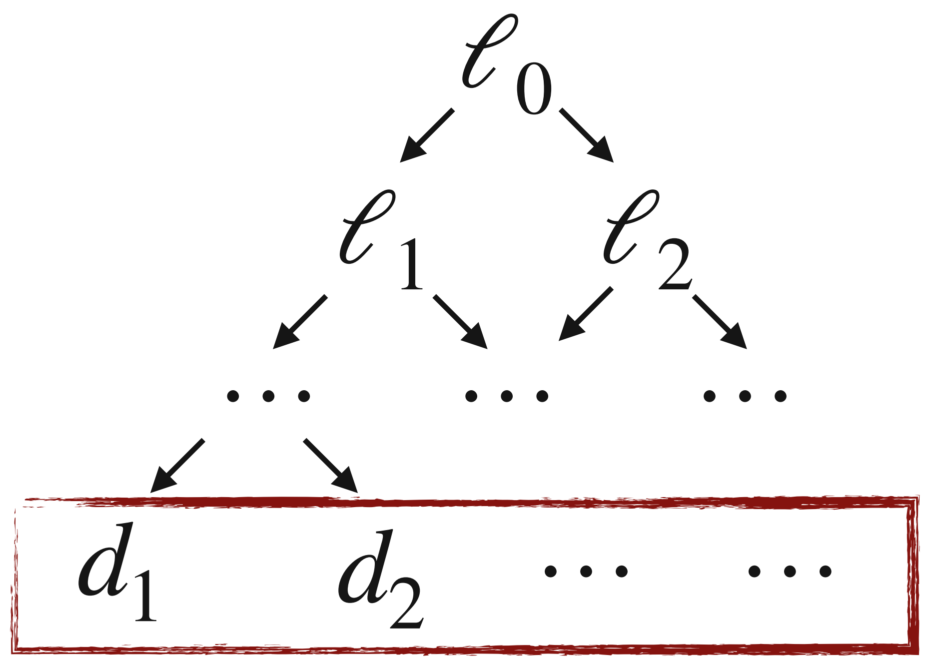

Strategy: Perform (half) Gaussian sampling over $\mathbb{Z}$ then merge

Implementation: constant-time, linear scan over a table

```
def sampleZ():
    u = sampleUniform(72)
    s = 0
    for i in range(N):
        if RCDT[i] > u:
            s += 1
    return s
```

FALCON

Gaussian Sampling

Smaller `RCDT[*]` $\Longrightarrow$ **Smaller Gaussian**

```
def sampleZ():
    u = sampleUniform()
    s = 0
    for i in range(N):
        if RCDT[i] > u:
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    return s
```
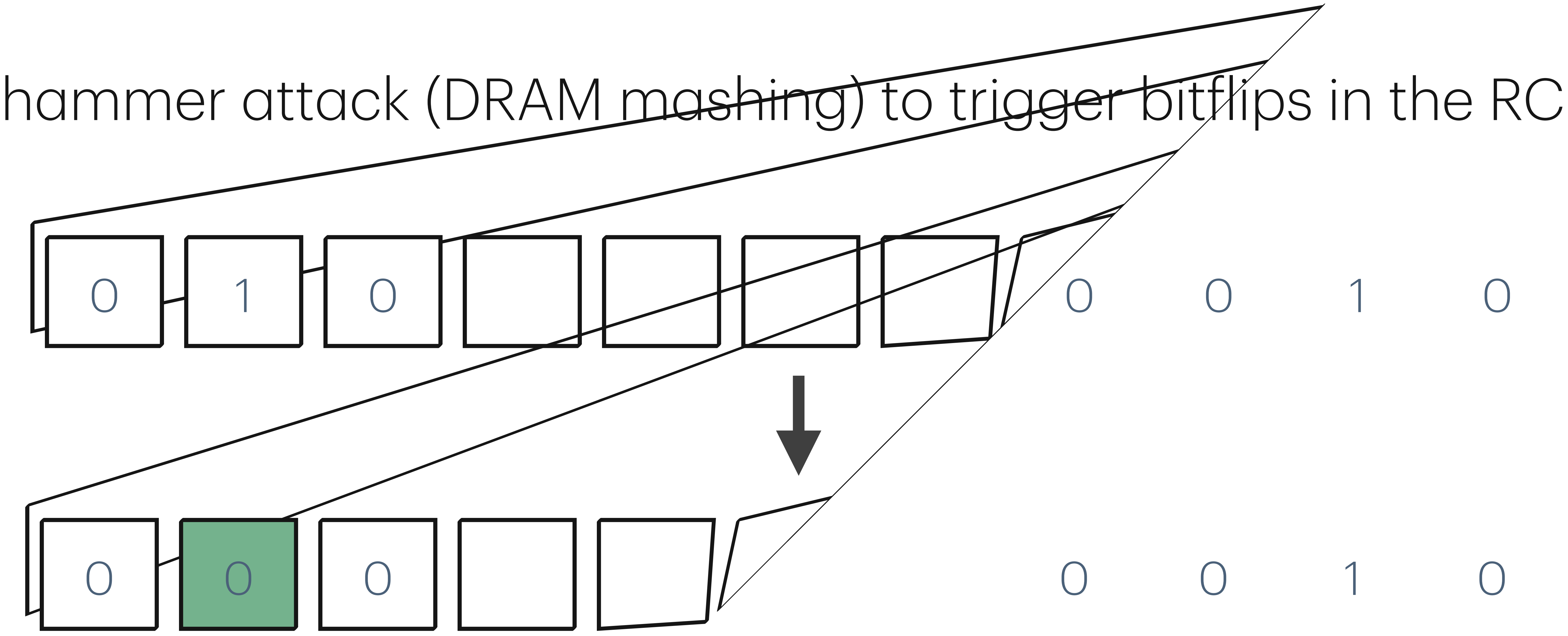

Rowhammer

Attacking DRAM

Idea: Attack `RCDT[*]` to lower its values and cause statistical leakage

Tool: Rowhammer attack (DRAM mashing) to trigger bitflips in the RCDT



Attack

Nguyen-Regev

How many bitflips to work?

	Nguyen-Regev Attack
Full flip (Empty table)	✔
8 bitflips	✘
1 bitflip	✘

Not realistic

Scenario of Fahr et al. on FrodoKEM (CCS2022)

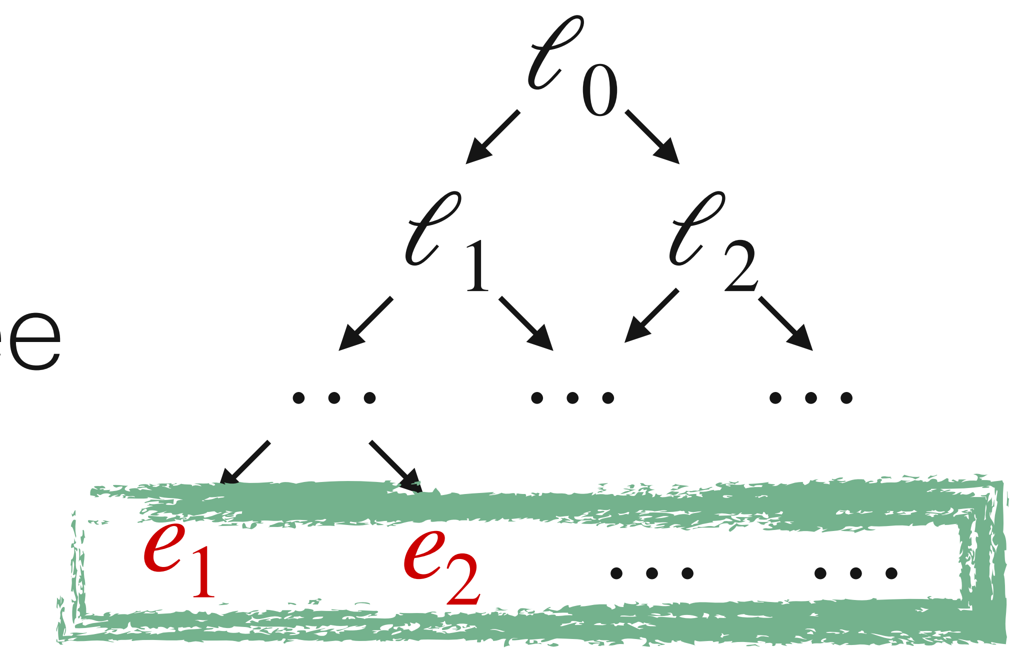
Attack

Effects of bitflips

Question: How does the signature distribution behave with bitflips?

If $\Sigma = L^{-T}\sigma^2 D^{-1} L^{-1}$ then $B^T \Sigma B = \sigma^2 Id$

FALCON Tree



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$$\text{If } \Sigma = L^{-T}\sigma^2 E^{-1} L^{-1} \text{ then } B^T \Sigma B = \sigma^2 U^T D E^{-1} U = \tilde{\Sigma}$$

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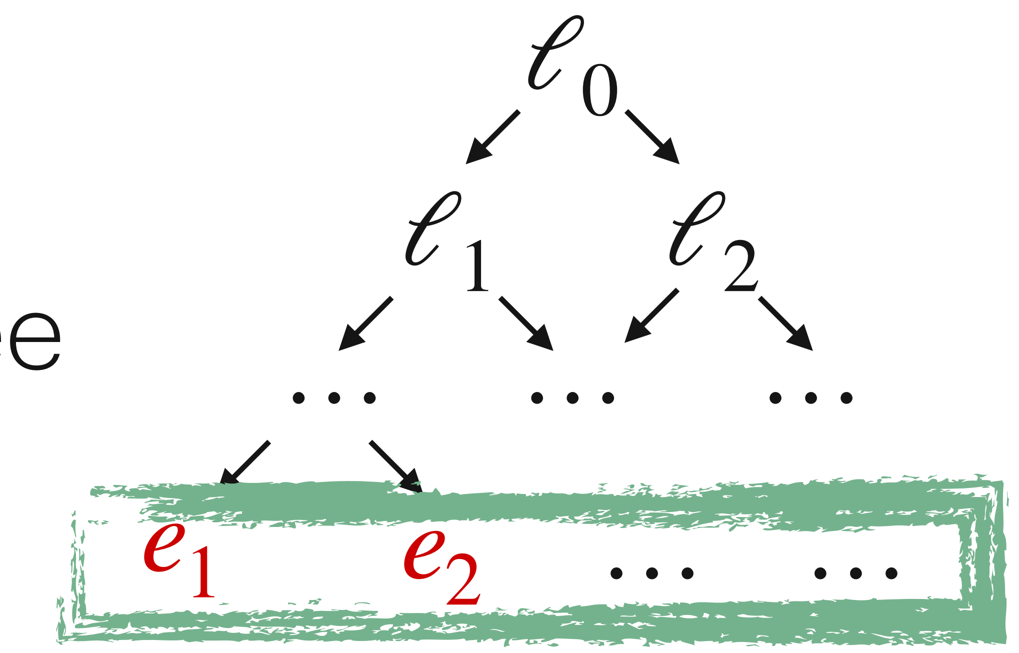

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Observation: the vectors of the normalized GSO U are eigenvectors of $\tilde{\Sigma}$

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Idea: get a good approximation of $\tilde{\Sigma}$ and compute its eigenvectors

Attack

Eigenvalue attack

Idea: get a good approximation of $\tilde{\Sigma}$ and compute its eigenvectors

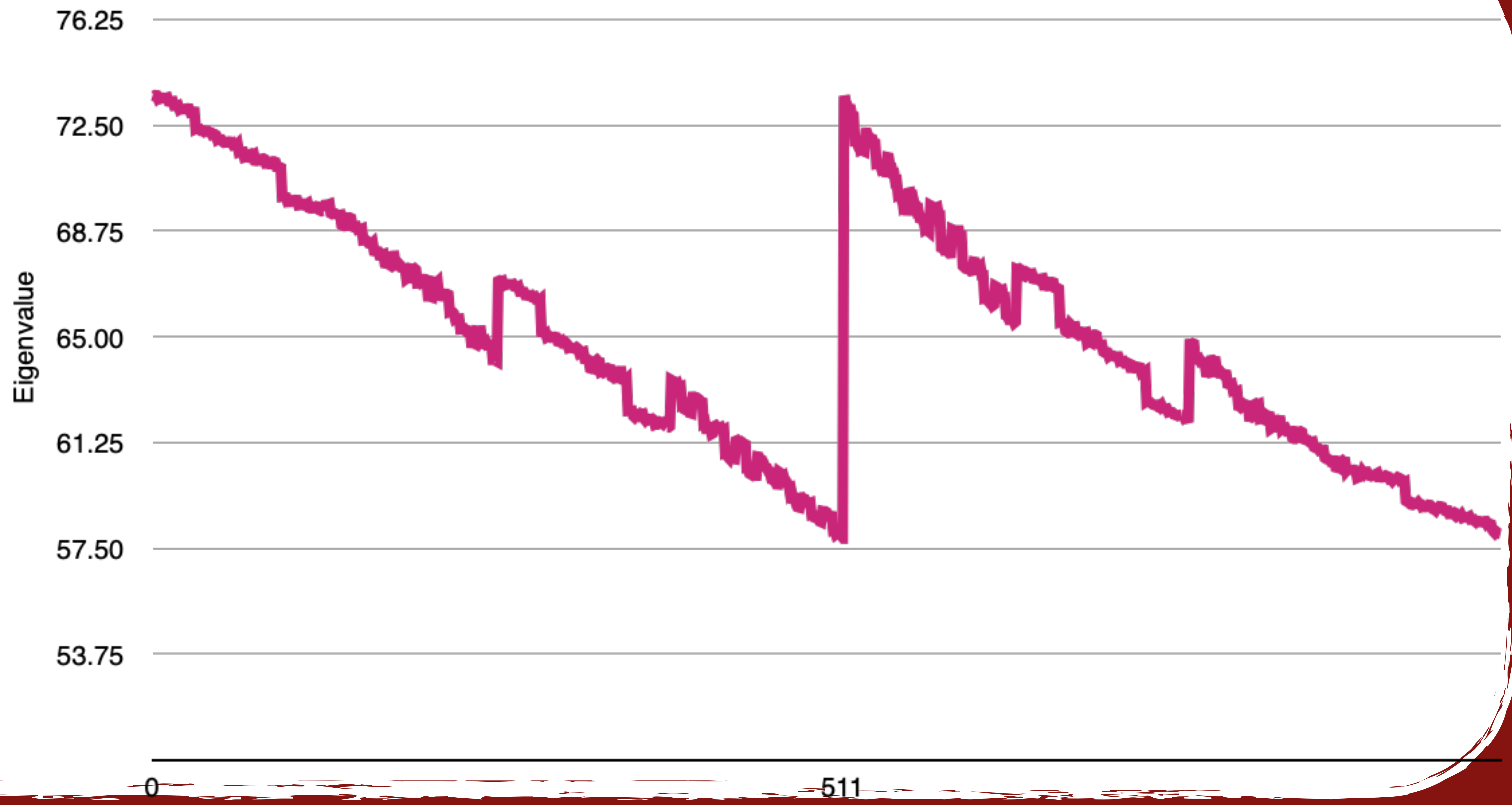
Advantage: Memory + CPU efficient (Billions of signatures can be processed)

Drawback: Does not work alone, eigenspaces are of dimension 2

Attack

Eigenvalue attack

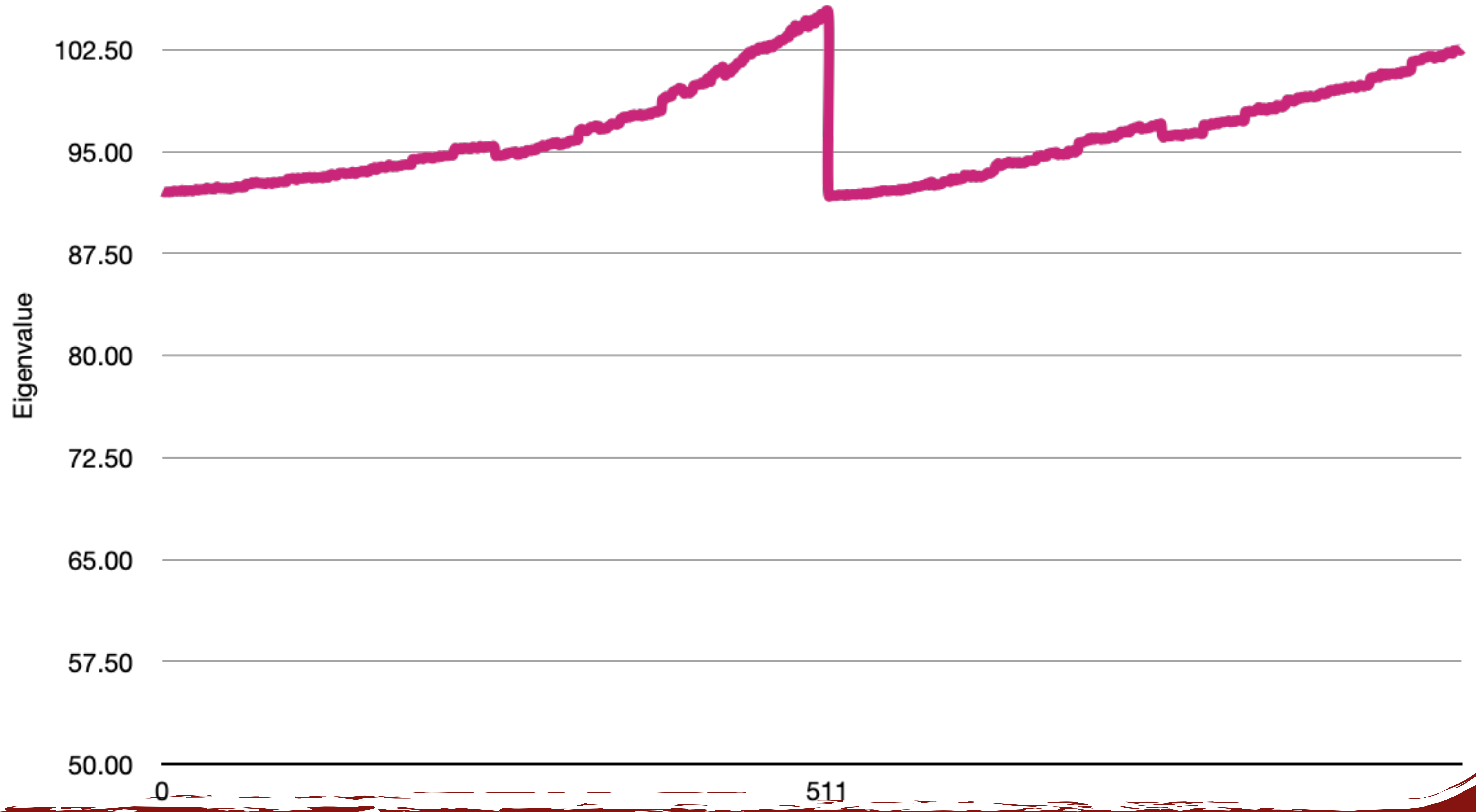
Distribution of **eigenvalues** of the **GSO vectors** for **8 bitflips**



Attack

Eigenvalue attack

Distribution of **eigenvalues** of the **GSO vectors** for **1 bitflip**



Attack

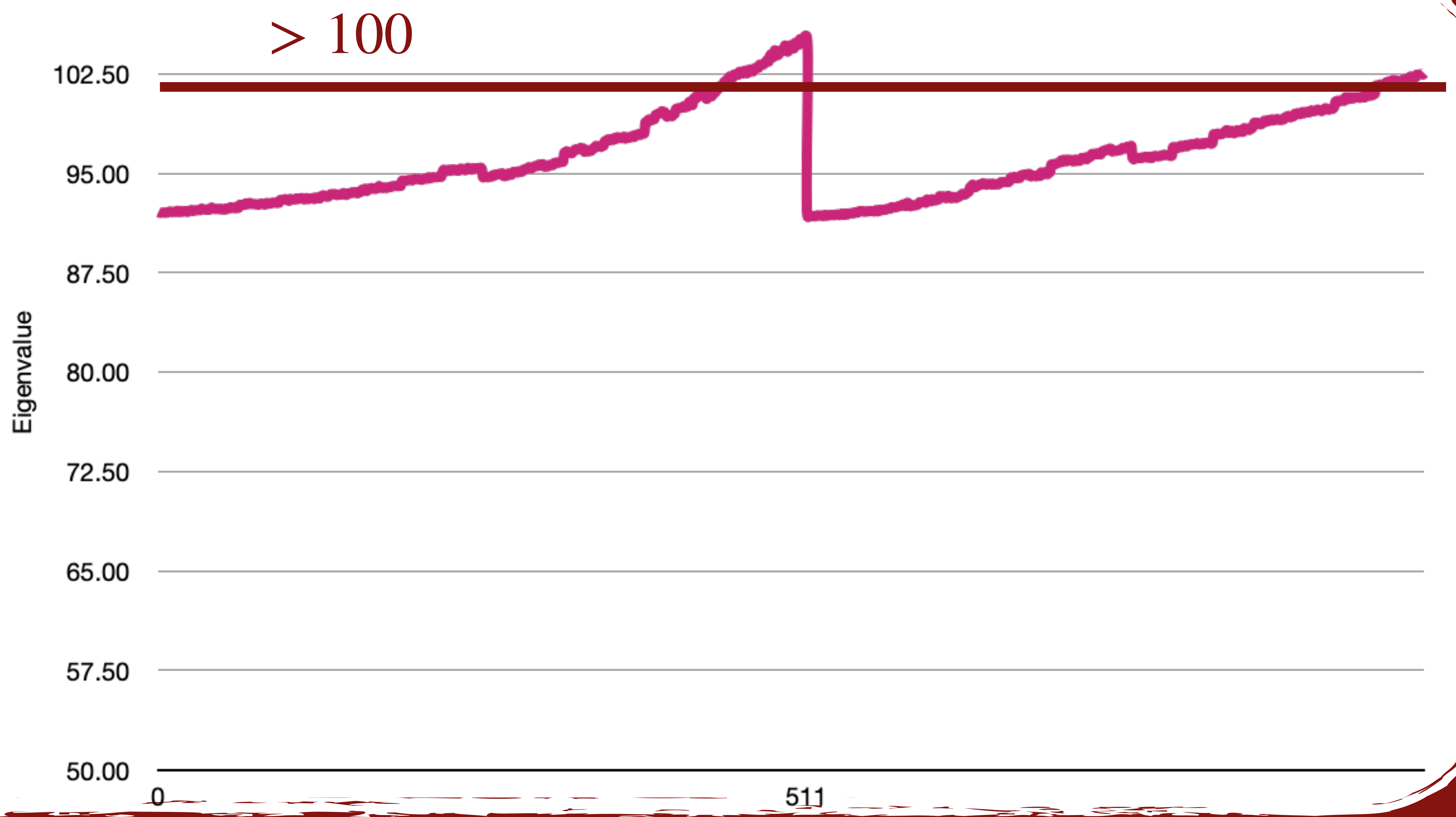
Summary: Shortcomings of NR06

- Bad results for full dimension (1024)
- **Observation**: Relevant eigenvectors live in small Subspace
- **Idea**: Perform search on Subspace
- **Problem**: How to find this subspace?

Attack

Eigenvalue attack

Distribution of **eigenvalues** of the **GSO vectors** for **1 bitflip**



Attack

Finding a good Subspace of dimension k

<u>Real Covariance</u>

$\Sigma, V = \text{Eigenspace}(\lambda_1, \ldots, \lambda_k)$

<u>Approximation</u>

$\hat{\Sigma}, \hat{V} = \text{Eigenspace}(\hat{\lambda}_1, \ldots, \hat{\lambda}_k)$

If $V = \hat{V}$, **project** signatures on $\hat{V}$ and perform NR06

Attack

Finding a good Subspace of dimension k

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But $V \neq \hat{V}$ in practice...

Attack

Finding a good Subspace of dimension k

Real Covariance

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Approximation

$\hat{\Sigma}, \hat{V} = \text{Eigenspace}(\hat{\lambda}_1, \ldots, \hat{\lambda}_k)$

If $V = \hat{V}$, **project** signatures on $\hat{V}$ and perform NR06

How "**close**" does it have to be?

Attack

(Variant of) Davis-Kahan theorem

$$\|\sin\Theta(\hat{V}, V)\|_{\mathrm{F}} \leq \frac{2\min(d^{1/2}\|\hat{\Sigma} - \Sigma\|_{\mathrm{op}}, \|\hat{\Sigma} - \Sigma\|_{\mathrm{F}})}{\min(\lambda_{r-1} - \lambda_r, \lambda_s - \lambda_{s+1})}.$$

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$V = \text{Eigenspace}(\lambda_1, \ldots, \lambda_k)$

$\hat{V} = \text{Eigenspace}(\hat{\lambda}_1, \ldots, \hat{\lambda}_k)$

Davis-Kahan: subspaces are $\dfrac{\|\Sigma - \hat{\Sigma}\|}{\lambda_k - \lambda_{k+1}}$-close

Attack

(Variant of) Davis-Kahan theorem

$$\|\sin\Theta(\hat{V}, V)\|_F \le \frac{2\min(d^{1/2}\|\hat{\Sigma}-\Sigma\|_{op}, \|\hat{\Sigma}-\Sigma\|_F)}{\min(\lambda_{r-1}-\lambda_r, \lambda_s-\lambda_{s+1})}.$$

$V = \text{Eigenspace}(\lambda_1, \ldots, \lambda_k)$

$\hat{V} = \text{Eigenspace}(\hat{\lambda}_1, \ldots, \hat{\lambda}_k)$

But we only need **one** GSO vector to be in $\hat{V}$...

Attack

(Variant of) Davis-Kahan theorem

$$\|\sin\Theta(\hat{V}, V)\|_{\mathrm{F}} \le \frac{2\min(d^{1/2}\|\hat{\Sigma}-\Sigma\|_{\mathrm{op}}, \|\hat{\Sigma}-\Sigma\|_{\mathrm{F}})}{\min(\lambda_{r-1}-\lambda_r, \lambda_s-\lambda_{s+1})}.$$

$V = \text{Eigenspace}(\lambda_1, \ldots, \lambda_k)$

$\hat{V} = \text{Eigenspace}(\hat{\lambda}_1, \ldots, \hat{\lambda}_k)$

Our result: eigenvector v_1 is $\frac{\|\Sigma - \hat{\Sigma}\|}{\lambda_1 - \lambda_{k+1}}$-close to $\hat{V}$

Attack

Summary

2-step attack:

Compute a good approximation $\hat{\Sigma}$

Project signatures on $\hat{V}$ to perform NR

Results

Eigenvalue attack

How many bitflips to work?

	Nguyen-Regev Attack	**This work**
Full flip (Empty table)	✓	✓
8 bitflips	✗	✓
1 bitflip	✗	✓

Results

Eigenvalue attack

Efficiency?

	Nguyen-Regev Attack	**This work**
Full flip (Empty table)	**2M**	✓
8 bitflips	✗	**20M + 2M ($k < 16$)**
1 bitflip	✗	**300M + 20M ($k < 64$)**

Results

Countermeasures

- Bitflips **reduce** signature sizes
- Bitflips are **permanent** in RAM

Results

Countermeasures

- Bitflips **reduce** signature sizes
- Bitflips are **permanent** in RAM

Lower Bound Rejection

Integrity Check

Conclusion

What next?

- Extend the attack to other RCDT-based schemes (Hawk)
- Find other ways to finish the attack

Thanks for watching!