LatticeFold+

Memory-Efficient Proving at Scale

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Based on joint work with Dan Boneh

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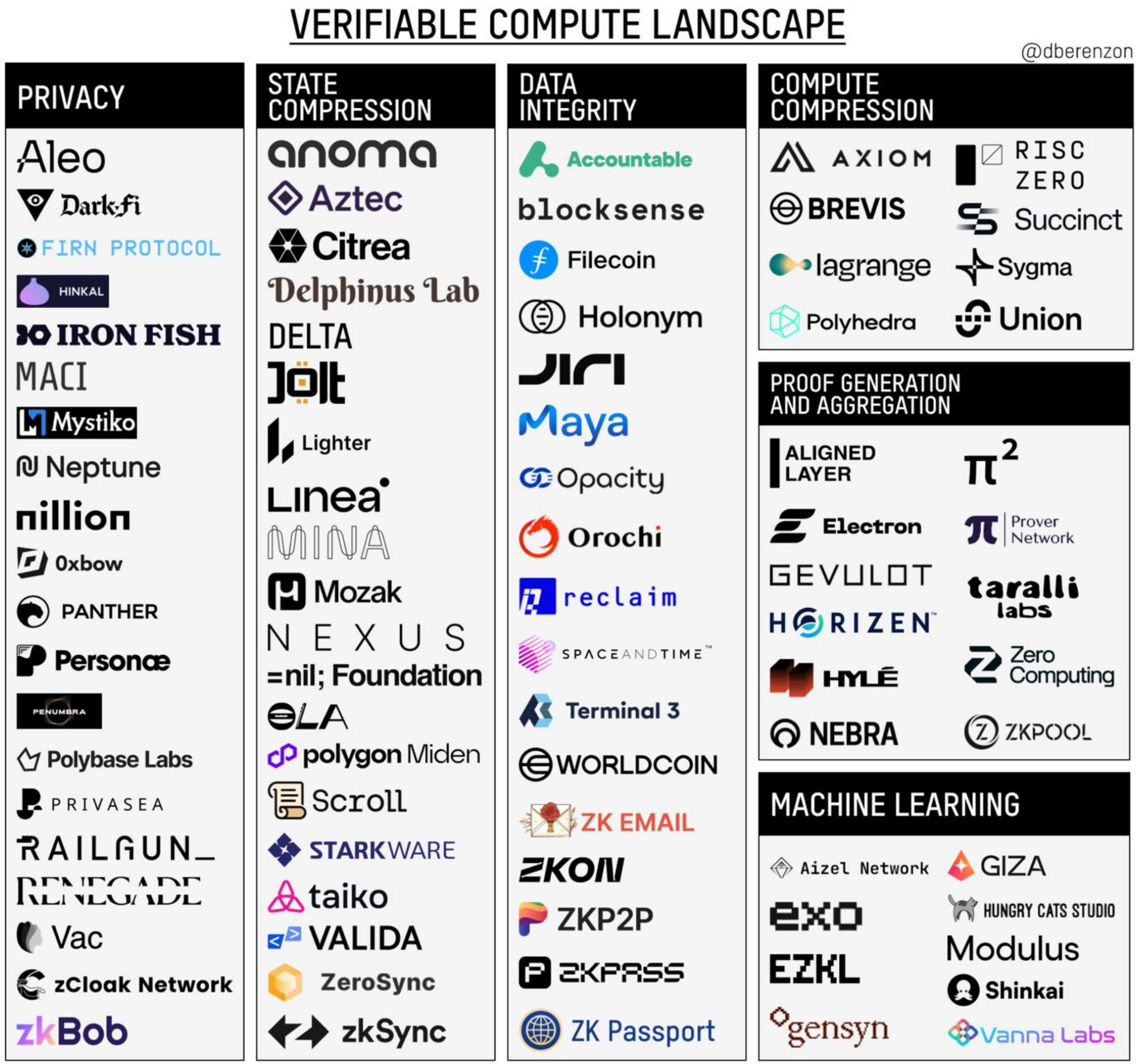
ZK-SNARKs: Advanced Applications

Applications

- zkVM/zkML
- Image/Video Provenance
- ZK Wallet/Passport
- Data Availability
- Decentralized Storage
- ……



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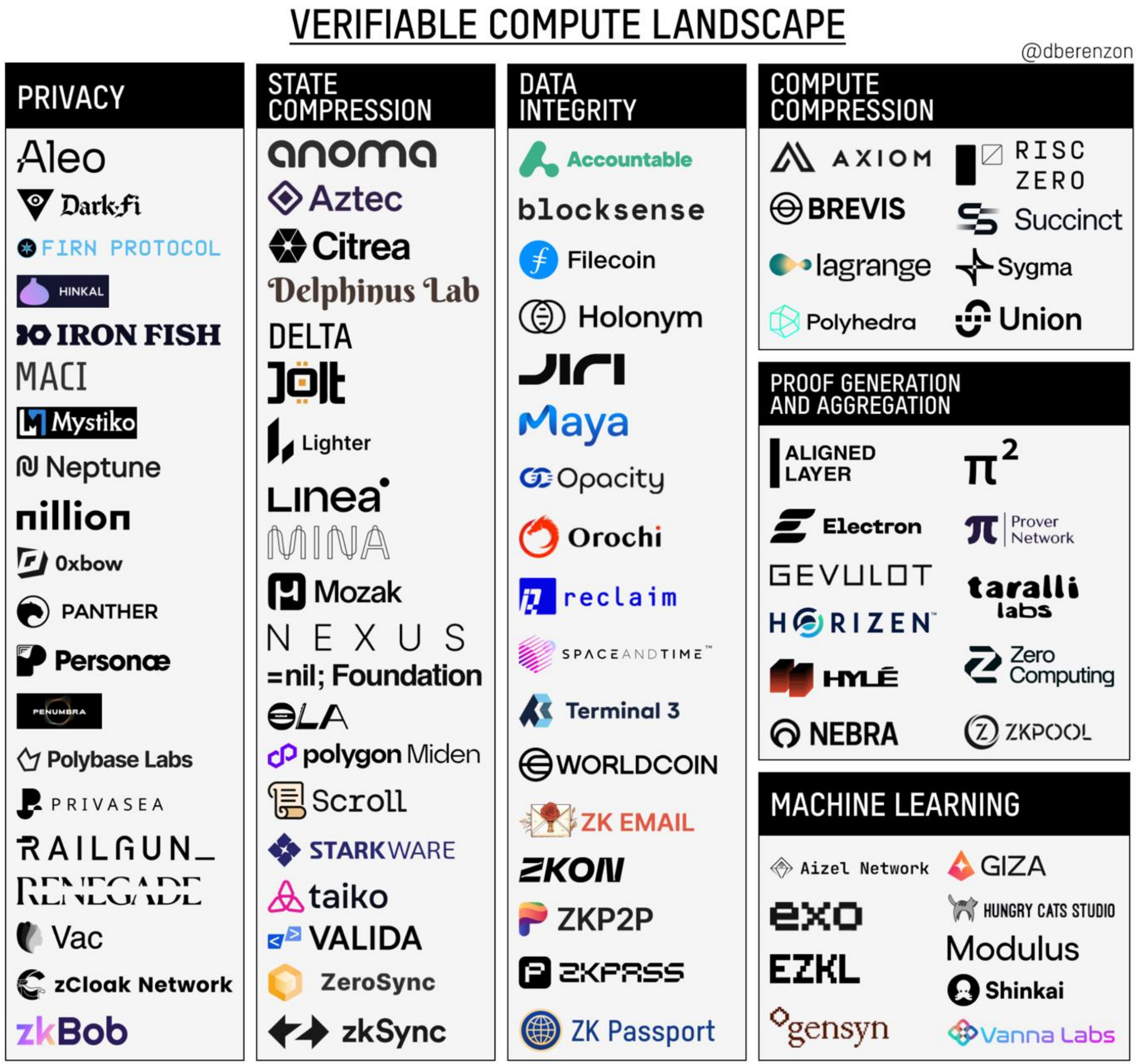
Common Theme

Large-scale computation



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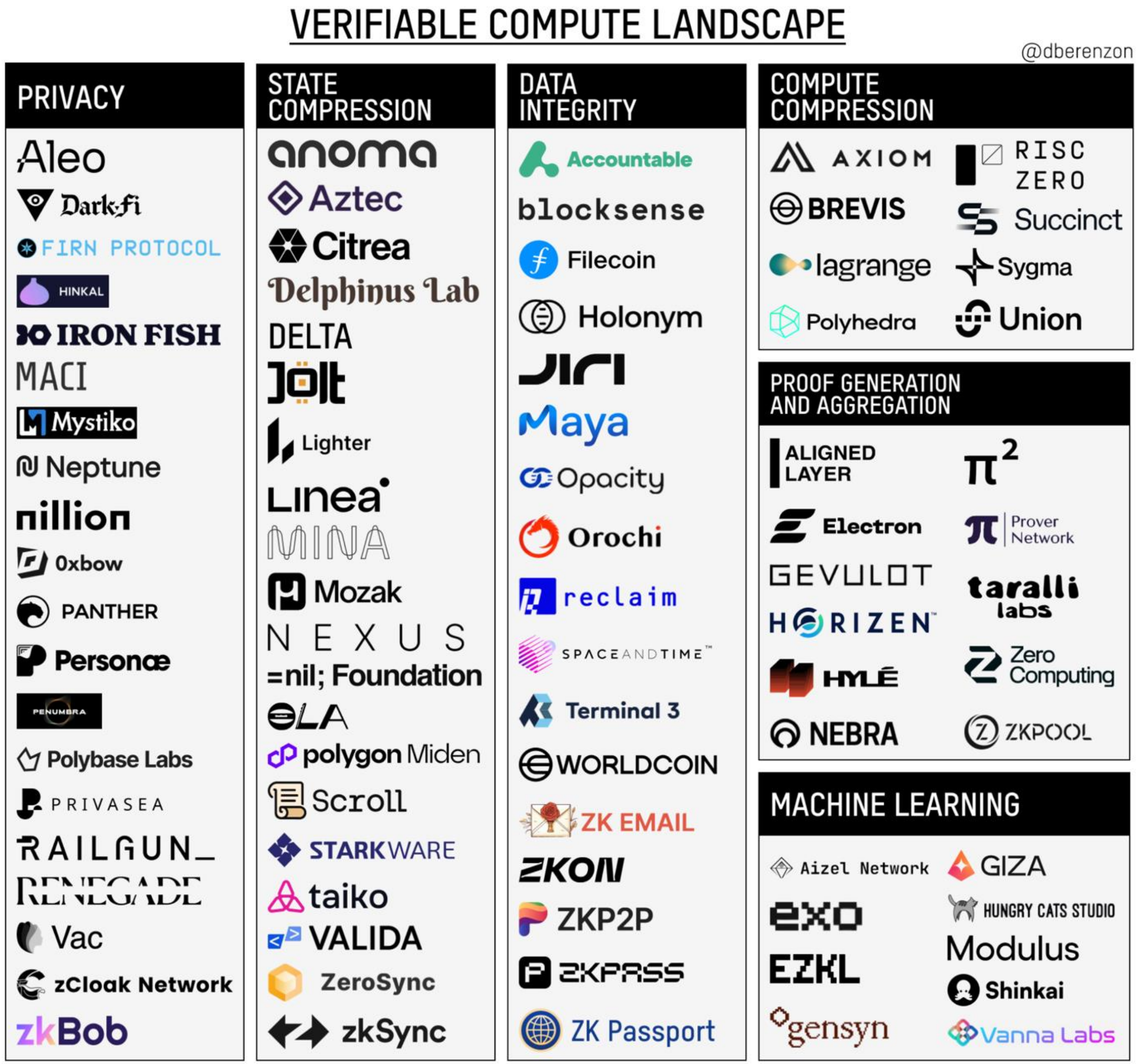
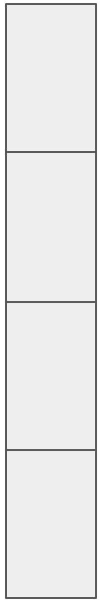
Design Requirements

Scalable prover + post-quantum security



Piecemeal SNARKs [Val'08, BCCT'12, BGH'19, COS'20, NDCTB'24…]

Computation



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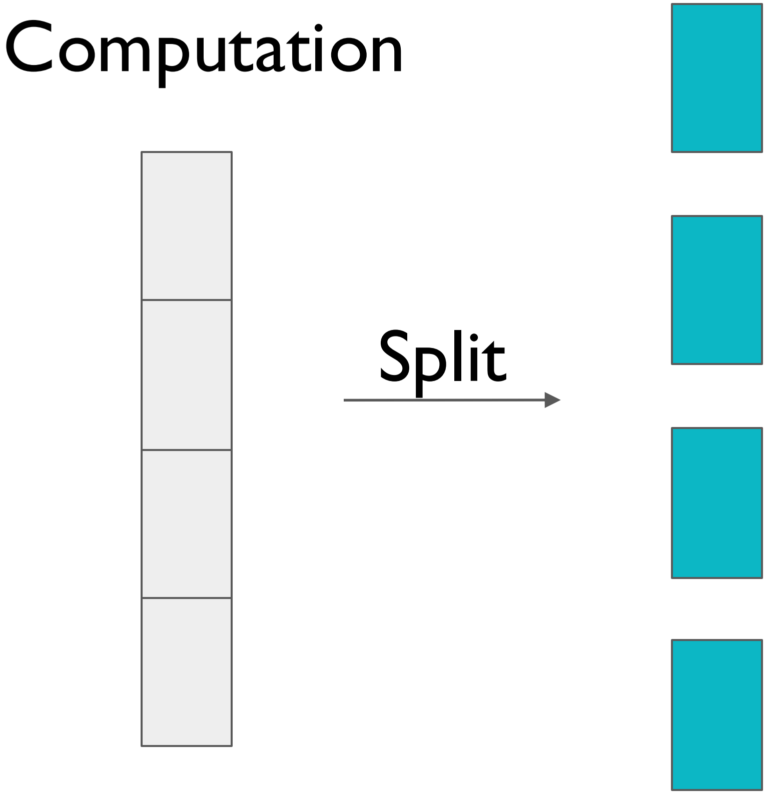

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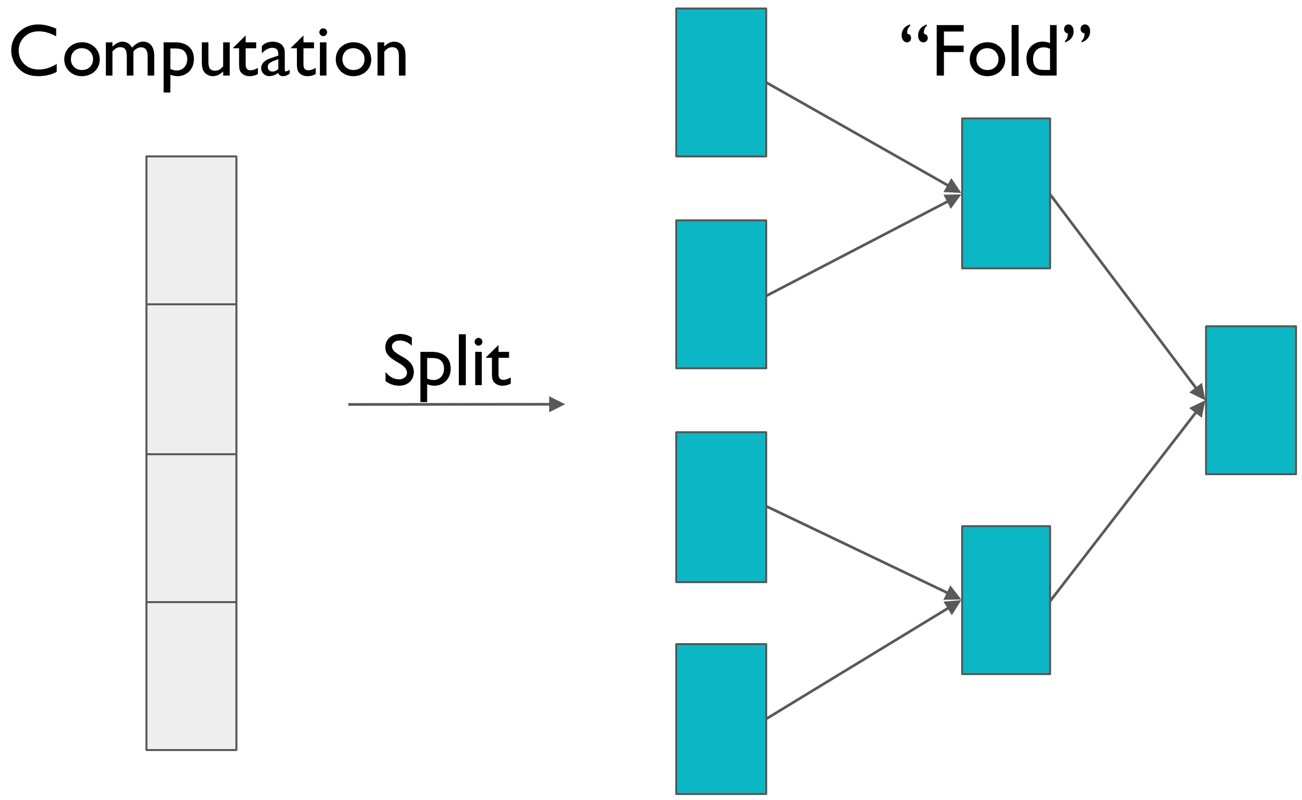

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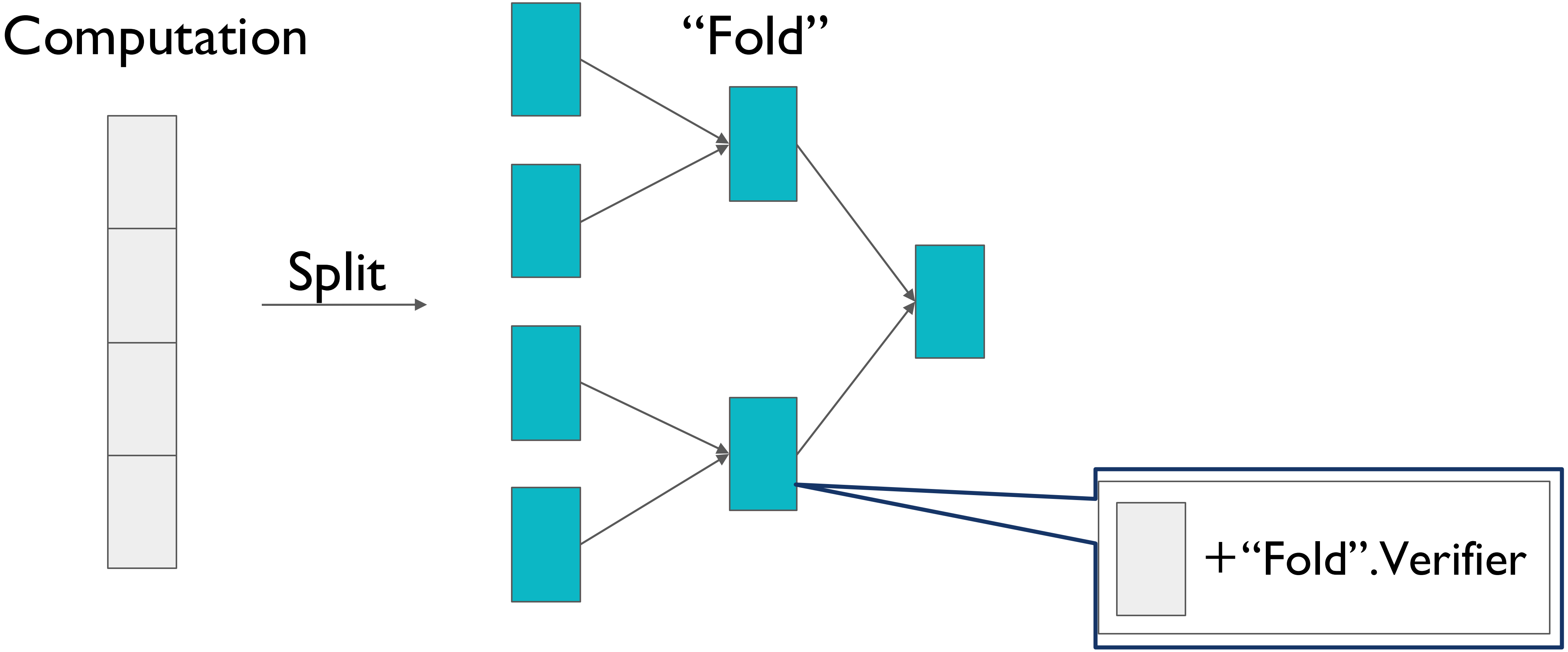

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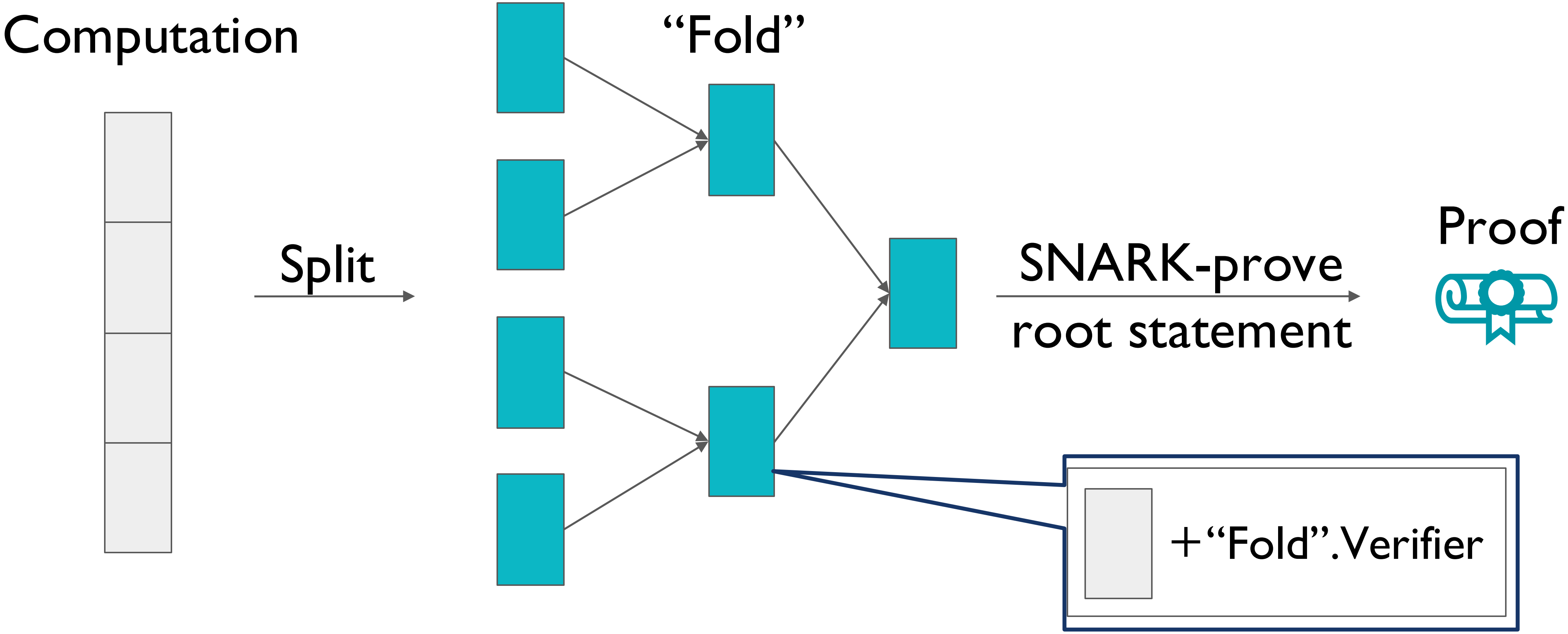

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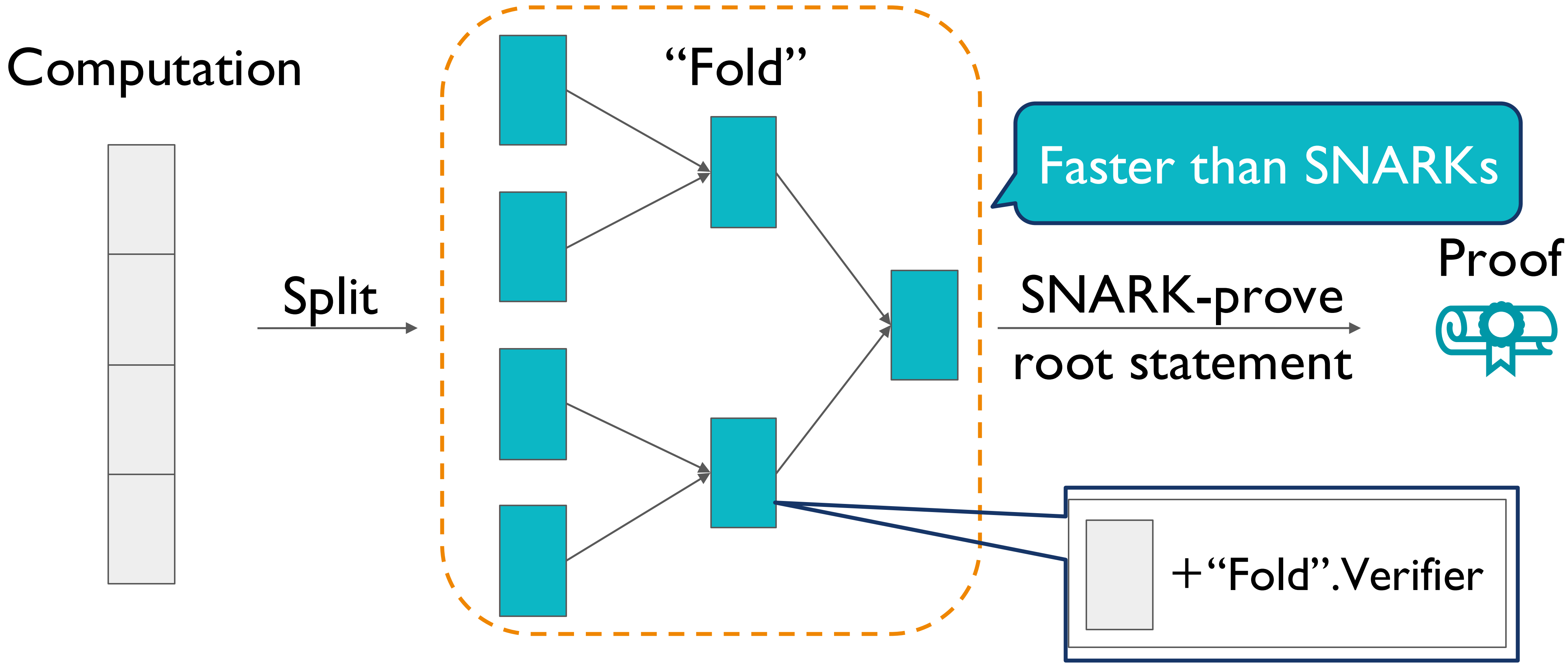

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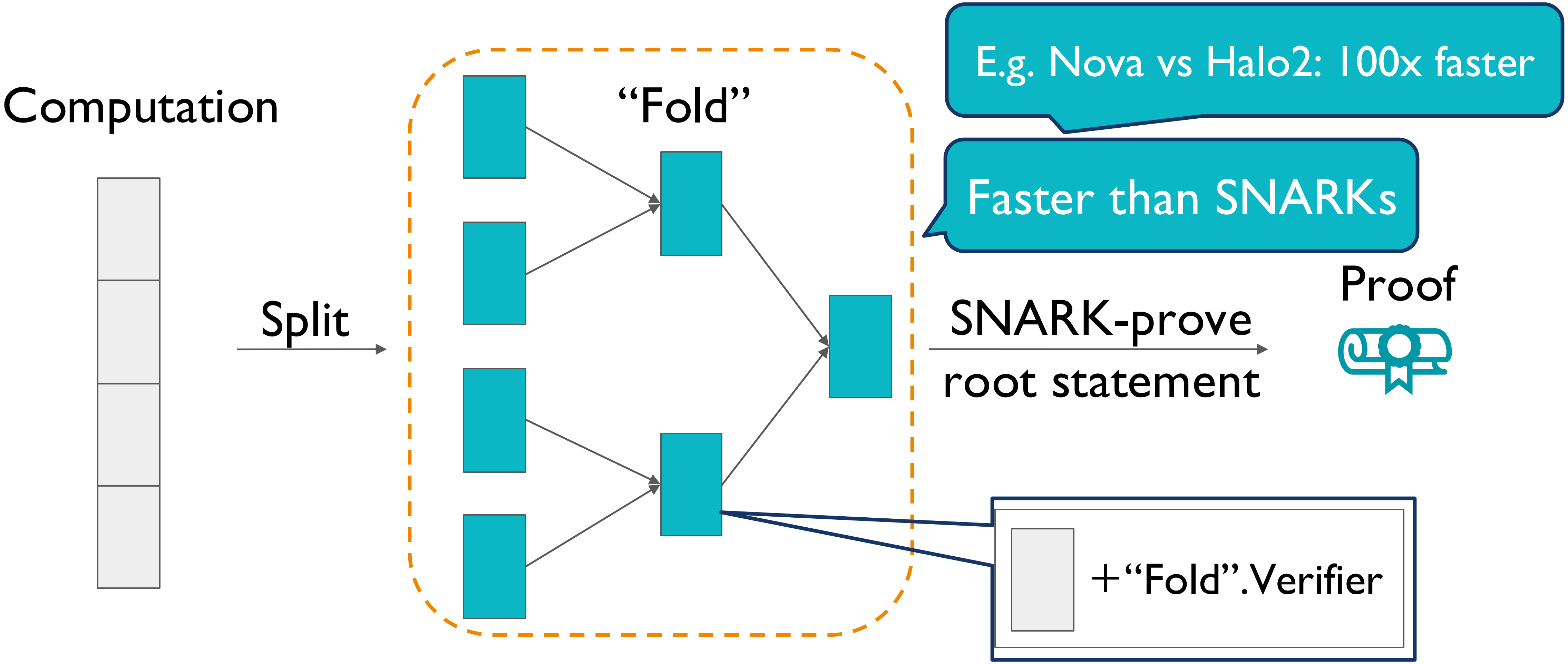

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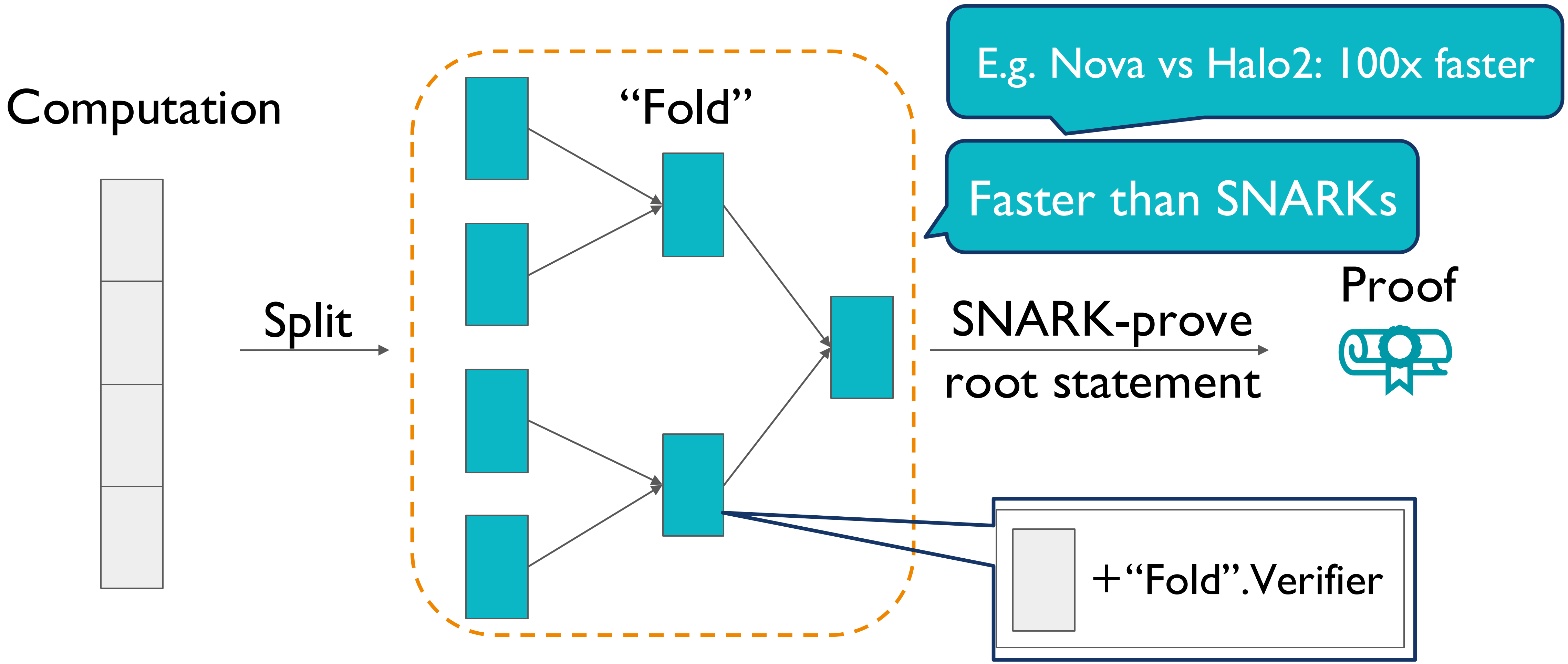

Our focus: Public-coin interactive folding schemes

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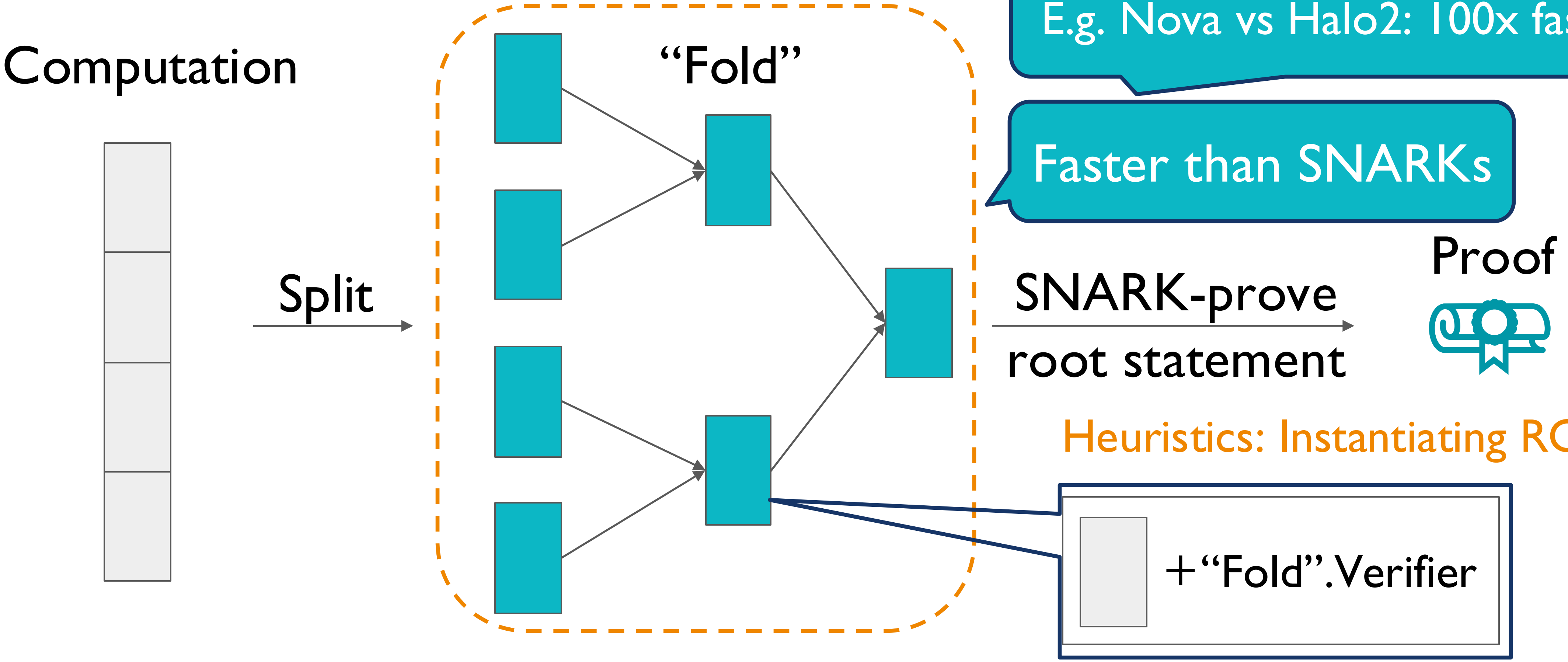

E.g. Nova vs Halo2: 100x faster

Faster than SNARKs

Heuristics: Instantiating RO

Our focus: Public-coin interactive folding schemes

Folding Schemes [BGH'19, BCLMS'20, KST'21]

$(x_1, w_1) \in_? R_{NP}$

$(x_2, w_2) \in_? R_{acc}$

$$\Pi_{\mathrm{fd}}[P, V]$$

Folding Schemes [BGH'19, BCLMS'20, KST'21]

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Properties:

- Completeness + Knowledge soundness + Succinctness
- Advantage: faster provers & verifiers than SNARKs

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$|R_{NP}| \approx |R_{acc}|$

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Lattice-Based Folding

Nice features

- Plausible post-quantum security
- Fast prover + succinct verifier

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Much simpler than Lattice SNARKs

[BS'23], [NS'25], [CMNW'24], [KLNO'24], [HSS'24], [FLV'23, [CLM'23], [KLNO'25]

Lattice-Based Folding

Nice features

- Plausible post-quantum security
- Fast prover + succinct verifier

Prior/Concurrent Work

- LatticeFold [BC'24] / Neo [NS'25] :
  - MSIS [LS'15] + Sumcheck-based norm proof
- Lova [FKNP'24] :
  - SIS [Ajtai'96] + Euclidean norm proof
  - Less concretely efficient

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Prover: Require computing *many* helper commitments

Our Contribution

$R_q = \mathbb{Z}_q[X]/(X^d + 1)$ (e.g. q = 64-bit prime, $d = 64$)
$\kappa \cdot d$ = Lattice dimension
n = # of ring elems in input witness

LatticeFold+:

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LatticeFold+:

- Security assumption: Module-SIS [LS'15]

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Highly parallelizable

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- Prover cost per $\mathbb{Z}_q$: $O(\kappa \log d)$ $\mathbb{Z}_q$-muls + $O(\kappa d)$ $\mathbb{Z}_q$-adds
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- Verifier: $O(\kappa d + \log n)$ $\mathbb{F}_q$-hashes + $O(d)$ R_q-ops
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Key technique

- An algebraic norm proof without high-arity decomposition
- $\log(d)$-factor fewer helper commitments than prior work

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Technical Overview

A Simpler Goal

$$R_{\mathrm{cm},B} = \left\{ \begin{array}{c} \mathbb{x} = (\mathrm{cm}), \mathbb{w} = (f \in R^n) \ \text{s.t.} \\ \mathrm{cm} = \mathbf{A} f \bmod q \wedge \|f\| < B \end{array} \right\}$$

$\mathbf{A} \leftarrow_{\$} R_q^{\kappa \times n}$

$\|\cdot\|$: infinity norm

A Simpler Goal

$(x_1, w_1) \in_? R_{cm,B}$

$(x_2, w_2) \in_? R_{acc,B}$

$$\Pi_{\mathrm{fd}}[P, V]$$

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$R_{acc,B} \approx R_{cm,B}$

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$\Pi_{\mathrm{fd}}[P, V]$

Idea: random linear combination

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A Simpler Goal

Challenge: Norm control

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Norm Control Techniques

$\|\cdot\|$: infinity norm

Low norm challenges: $S := \{g \in R, \|g\| \leq 1\}$

$[\mathrm{cm}_1, f_1]$

$[\mathrm{cm}_2, f_2]$

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$$f \in R^n \xrightarrow{\mathrm{Decomp}_{b,k}(f)} f_1, f_2, \ldots, f_k \in R^n \begin{cases} f = \sum_i b^{i-1} \cdot f_i \\ \|f_1\|, \ldots, \|f_k\| < b \end{cases}$$

s.t. $\|f\| < b^k$

Decompose-and-Fold [BC'24]

$|| \cdot ||$: infinity norm

$S := \{g \in R, ||g|| \leq 1\}$

Norm upper-bound

$[\mathrm{cm}_1, f_1, b^k]$

$[\mathrm{cm}_2, f_2, b^k]$

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$$[\mathrm{cm}_1, f_1, b^k], \quad [\mathrm{cm}_2, f_2, b^k]$$

$$\xrightarrow[\text{Decompose}]{\text{Prover}}$$

$$[\overline{\mathrm{cm}}_1, \bar{f}_1, b], \quad \dots \dots, \quad \dots \dots, \quad [\overline{\mathrm{cm}}_{2k}, \bar{f}_{2k}, b]$$

$$\xrightarrow[\vec{\alpha} \leftarrow_R S^{2k}]{\text{Verifier}}$$

$$[\overline{\mathrm{cm}}^*, \bar{f}^*] = \sum \vec{\alpha}_i \cdot [\overline{\mathrm{cm}}_i, \bar{f}_i]$$

$$\approx 2k\lambda b$$

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Warning: Satisfies completeness but not knowledge soundness

Decompose-and-Fold [BC'24]

$\|\cdot\|$: infinity norm

$S := \{g \in R, \|g\| \le 1\}$

Need a batch-PoK of norms

$[\mathrm{cm}_1, f_1, b^k]$

$[\mathrm{cm}_2, f_2, b^k]$

$\xrightarrow[\text{Decompose}]{\text{Prover}}$

$[\overline{\mathrm{cm}}_1, \bar{f}_1, b]$

… …

… …

$[\overline{\mathrm{cm}}_{2k}, \bar{f}_{2k}, b]$

$\xrightarrow[\vec{\alpha} \leftarrow_R S^{2k}]{\text{Verifier}}$

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LatticeFold Norm Proofs [BC'24]

$$[\overline{\mathrm{cm}}_1, \bar{f}_1] \in_? R_{\mathrm{cm},\, b}$$

… …

… …

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A batched Sumcheck of degree $2b$

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A batched Sumcheck of degree $2b$ $\xrightarrow[\text{[LFKN'90]}]{\text{Sumcheck Protocol}}$ $\left(x = [\overline{\mathrm{cm}}^*, r, v], \bar{f}^*\right) \in_? R_{\mathrm{acc},\, B}$

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Sumcheck Protocol [LFKN'90]

$\left(x = [\overline{\mathrm{cm}}^*, \ldots], \bar{f}^*\right) \in_? R_{\mathrm{acc},\, B}$

$R_{\mathrm{acc},\, B} \approx (R_{\mathrm{cm},\, B} +$ "an eval stmnt"$)$

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$\xrightarrow[\text{[LFKN'90]}]{\text{Sumcheck Protocol}}$

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Pros: Polylogarithmic verifier time

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Prover overhead: Fix $B = b^k$

- Small b & Large k: Expensive computation to $\overline{\mathrm{cm}}_1, \ldots, \overline{\mathrm{cm}}_{2k}$
- Small k & Large b: Expensive high-degree Sumcheck

An Algebraic Norm Proof over Rq

Over $\mathbb{Z}_q$:

$$a \in [0, d) \cap \mathbb{Z} \iff \prod_{i=[0,d)} (a - i) = 0 \bmod q$$

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$$T(X) = \sum_{i \in [0,d)} -i \cdot X^{d-i}$$

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$\downarrow$

$(\mathbb{x}' = [\overline{\mathrm{cm}}^*, r, v], \bar{f}^*) \in_? R_{\mathrm{acc}}$

Extensions & Summary

The actual scheme:

- Support larger norm ranges than $[0, d)$
- Support smaller modulus q via tensor-of-rings [NS'25, BC'25]
- Folding committed R1CS relations

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Summary

- Lattice-based folding is easier to build than lattice SNARKs
- Faster & parallelizable provers + succinct verifiers
- Core technique: A new lattice-based algebraic range proof

Future Work

Non-power-of-2 cyclotomic rings?

Non-prime modulus q?

Efficient ℓ_2-norm range proofs?

QROM analysis?

THANK YOU

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