A Note on Adaptive Security in Hierarchical Identity-Based Encryption

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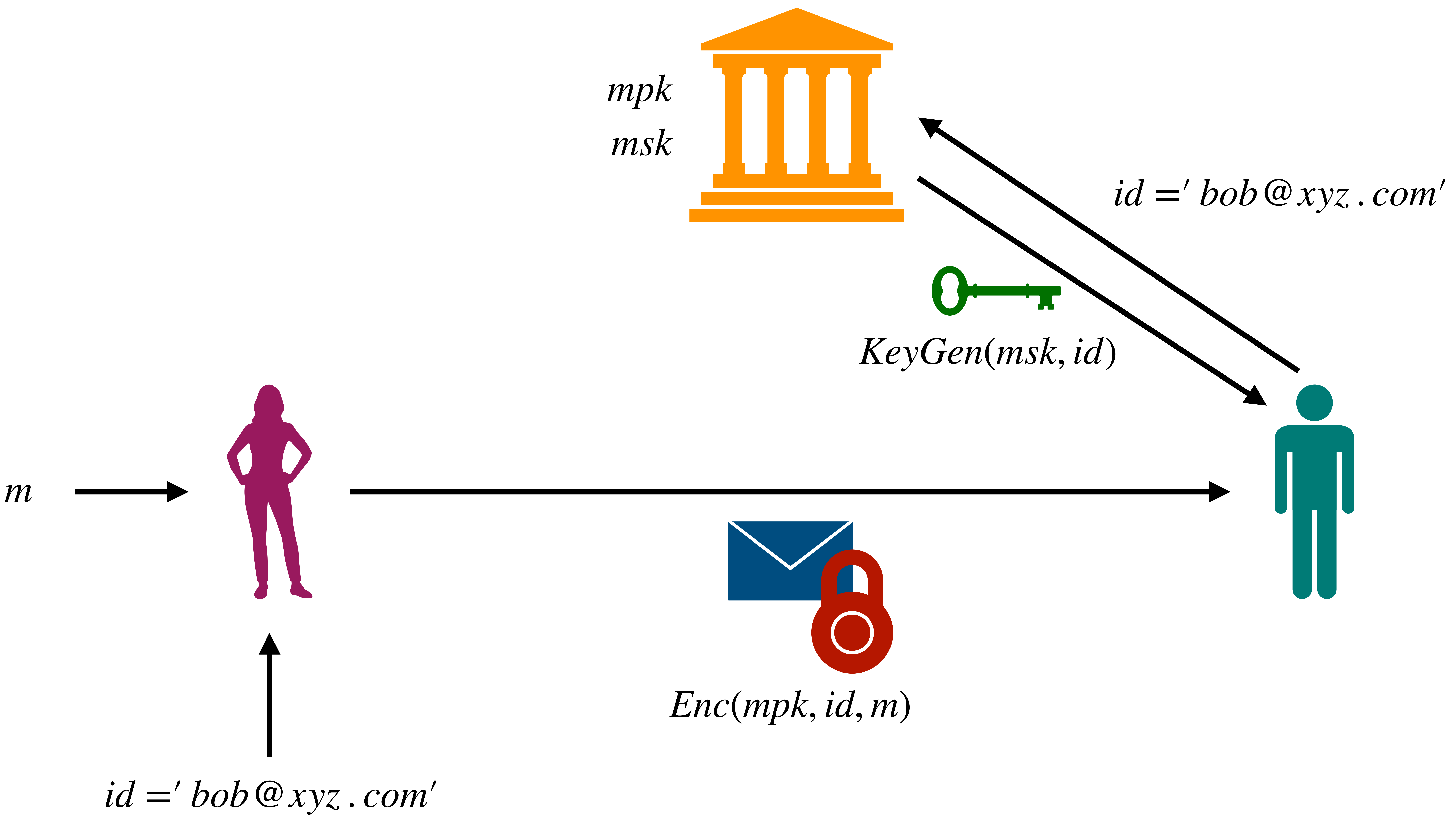
Identity Based Encryption

[Shamir'85]



Hierarchical Identity Based Encryption

[Horwitz-Lynn'02,Gentry-Silverberg'02]

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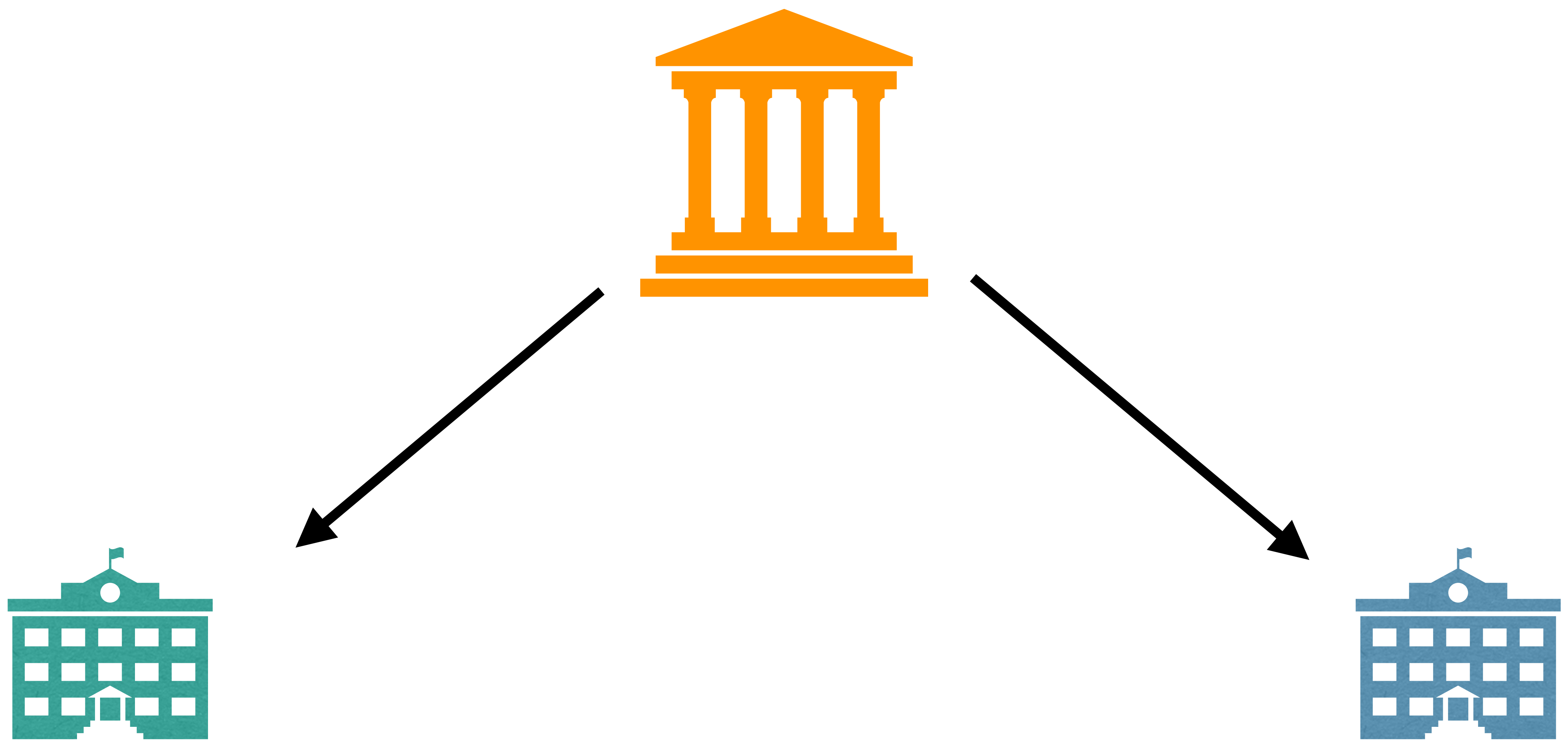

Hierarchical Identity Based Encryption

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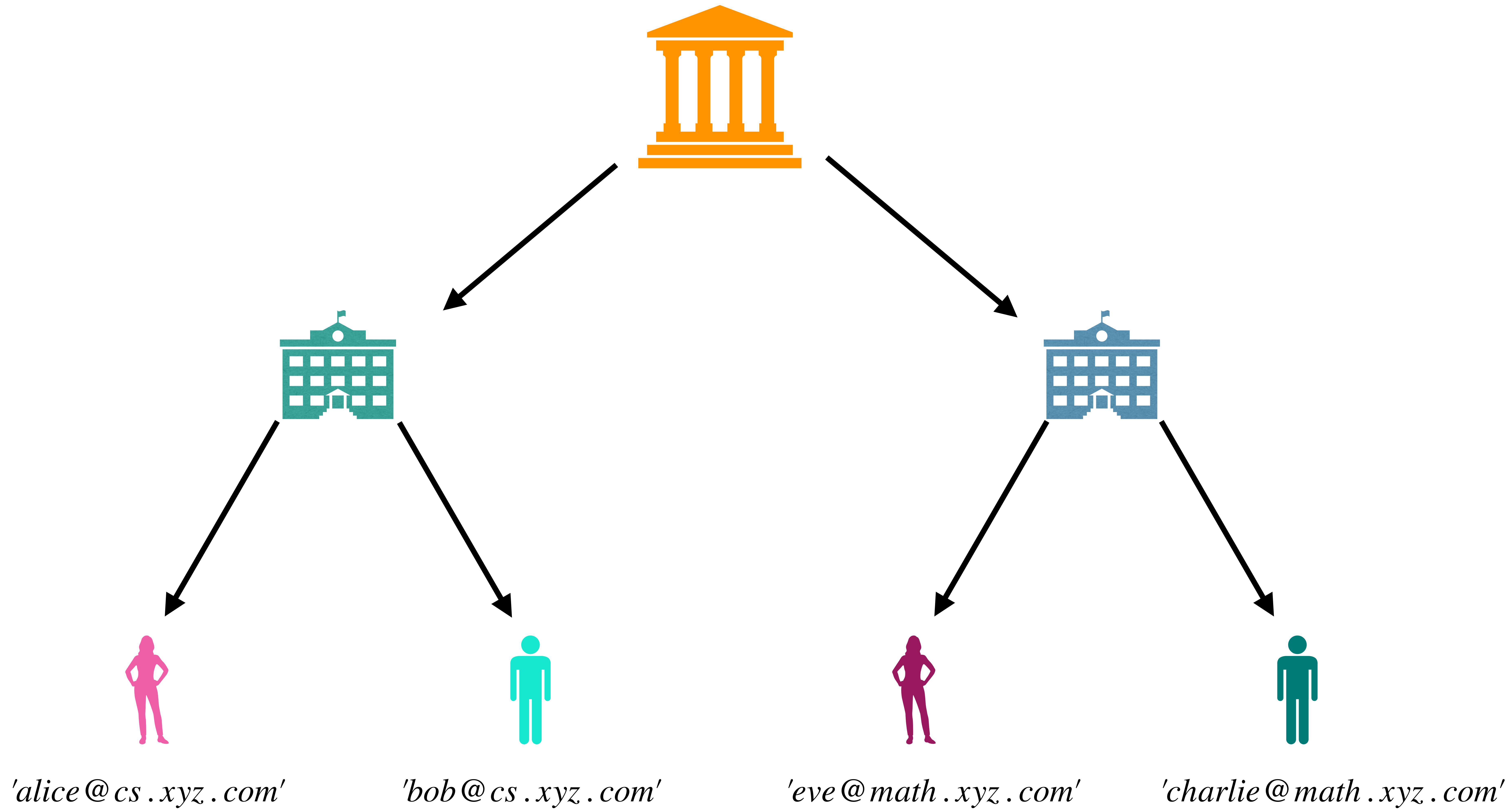

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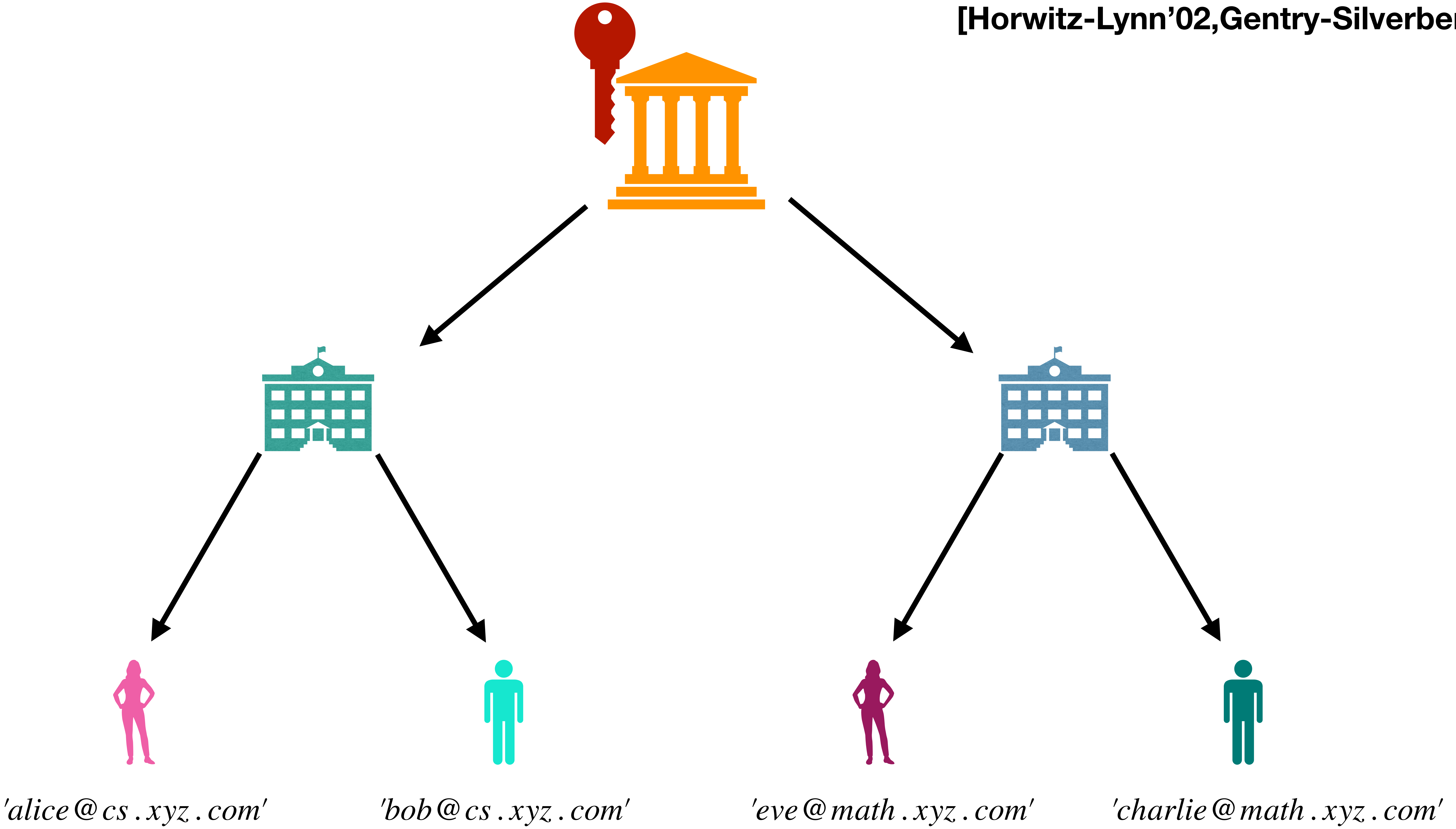

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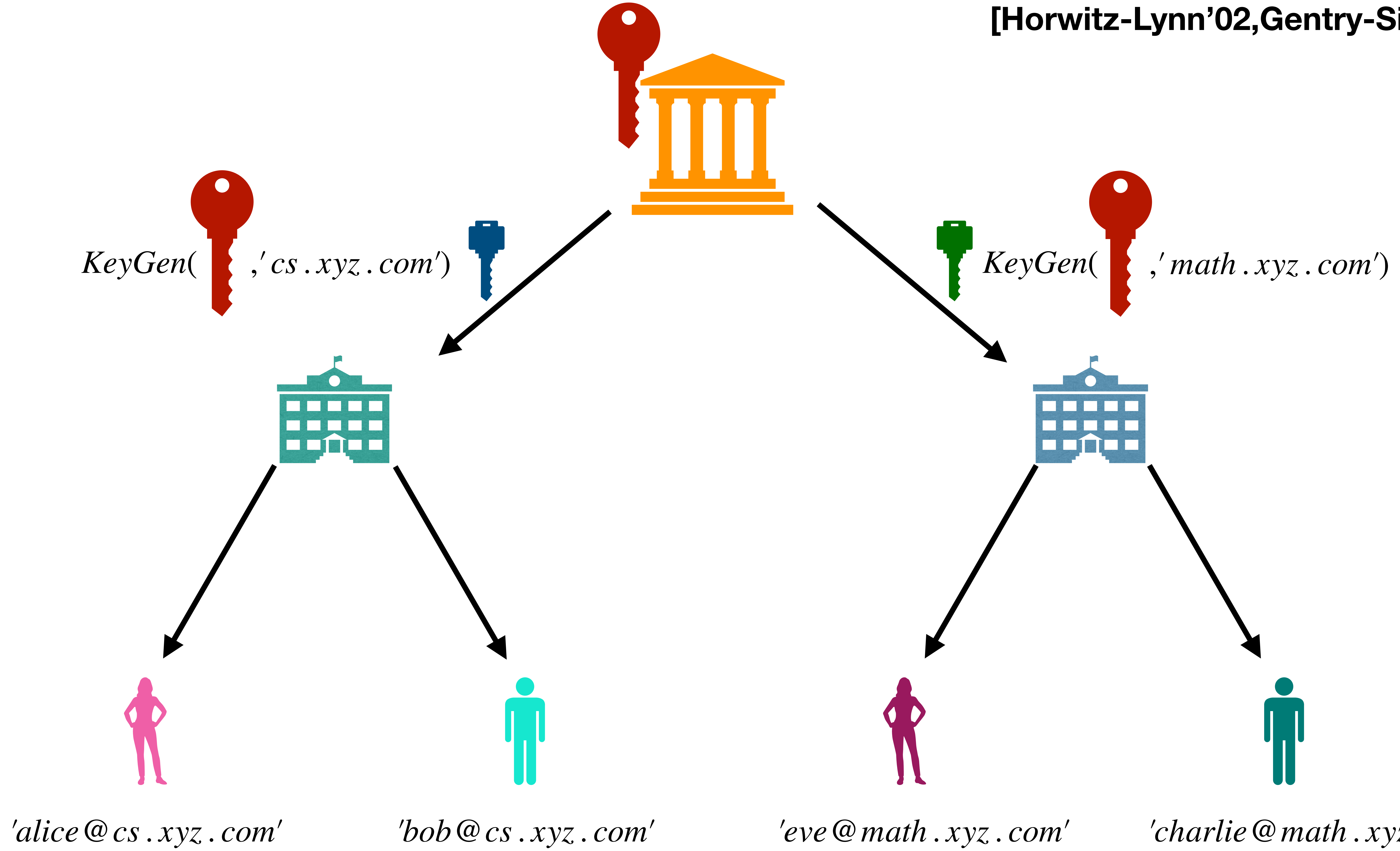

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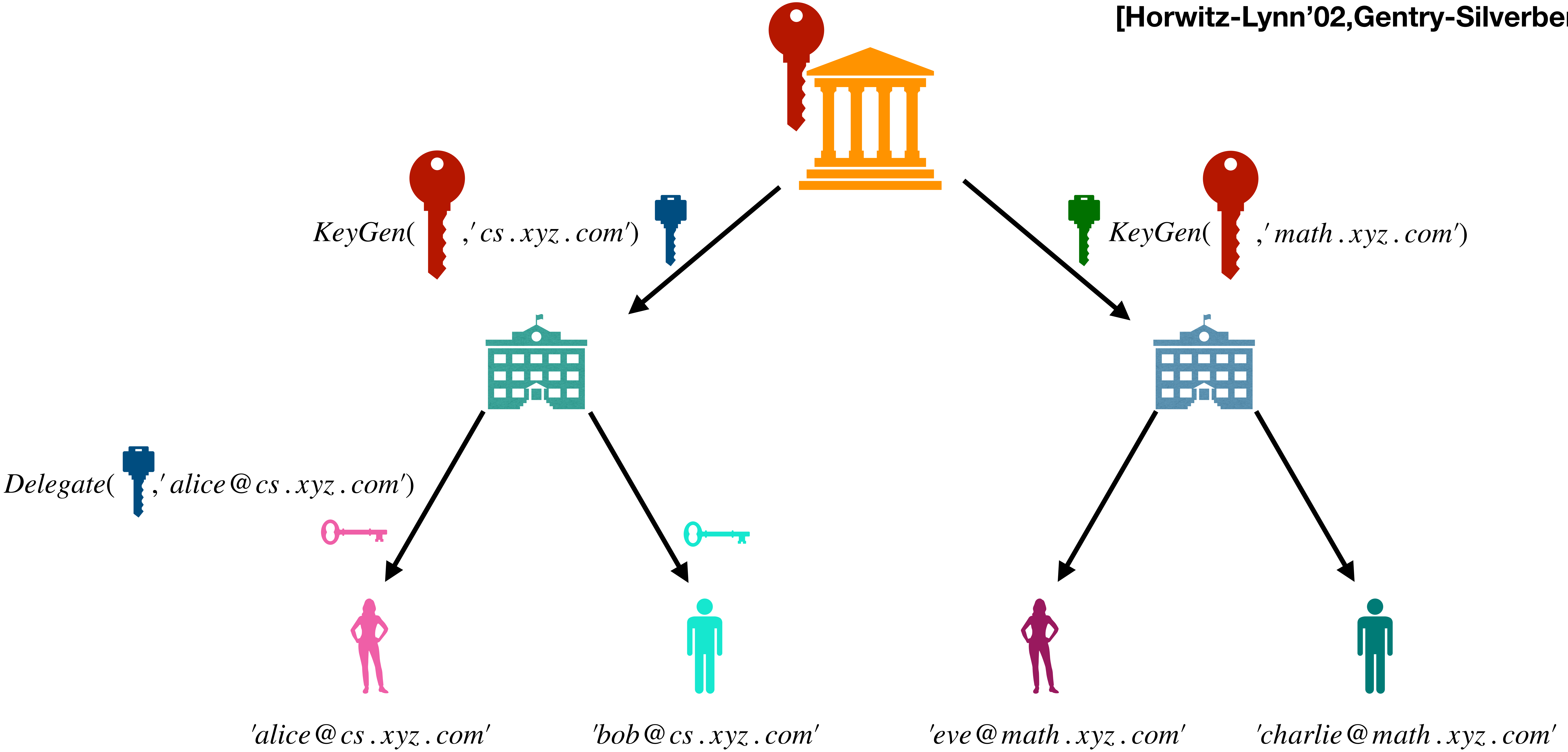

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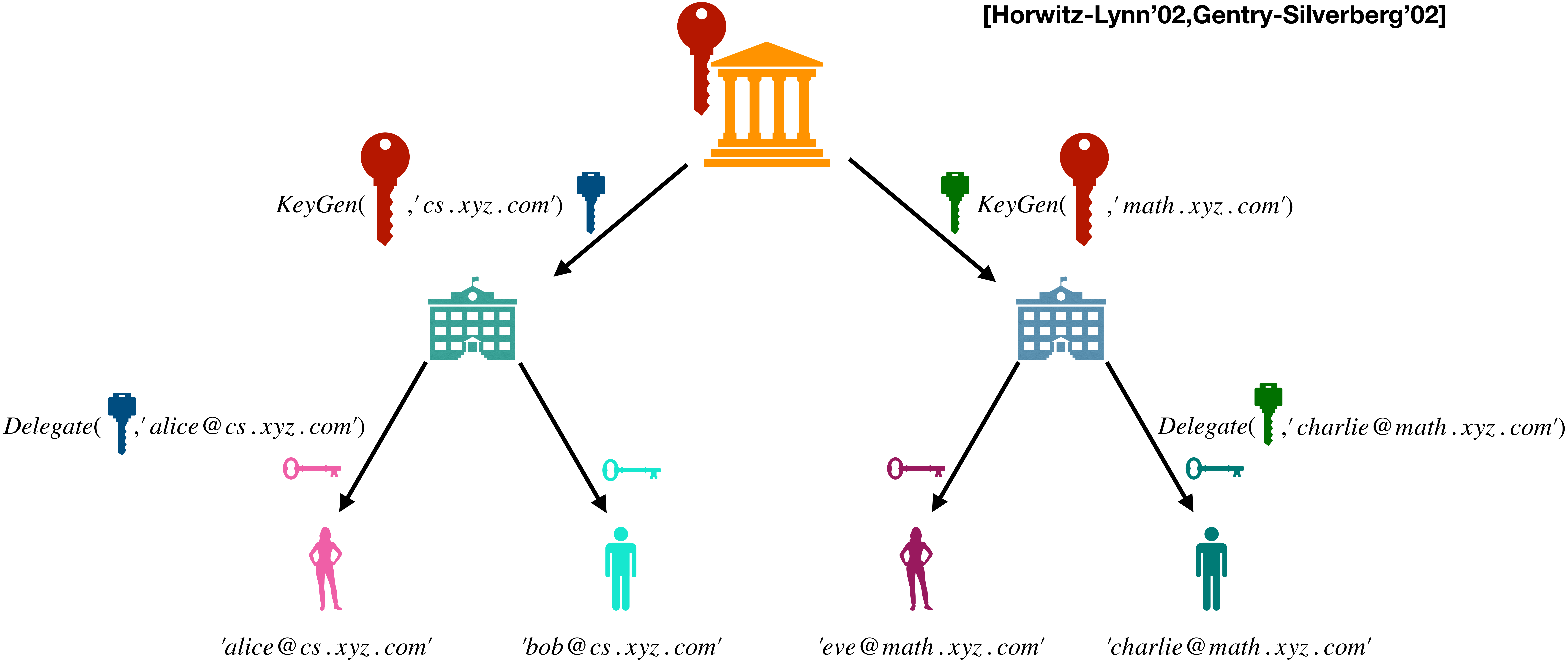

Syntax

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$(mpk, msk) \leftarrow Setup(1^\lambda)$

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$(mpk, msk) \leftarrow Setup(1^\lambda)$ $sk_{id} \leftarrow KeyGen(msk, id)$

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$(mpk, msk) \leftarrow Setup(1^\lambda)$

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$ct \leftarrow Enc(mpk, id, m)$

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$(mpk, msk) \leftarrow Setup(1^\lambda)$

$sk_{id} \leftarrow KeyGen(msk, id)$

$ct \leftarrow Enc(mpk, id, m)$

$m \leftarrow Dec(sk_{id}, ct)$

Syntax

$(mpk, msk) \leftarrow Setup(1^\lambda)$

$sk_{id} \leftarrow KeyGen(msk, id)$

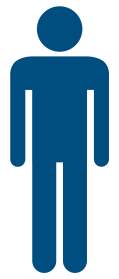
$sk_{id\|id'} \leftarrow Delegate(sk_{id}, id')$

$ct \leftarrow Enc(mpk, id, m)$

$m \leftarrow Dec(sk_{id}, ct)$

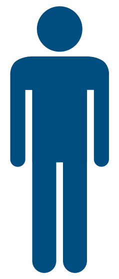
Security

Security



$(mpk, msk) \leftarrow Setup(1^{\lambda})$

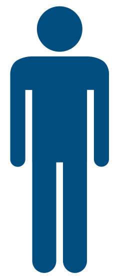
Security



$(mpk, msk) \leftarrow Setup(1^\lambda)$

$\xrightarrow{\quad\quad mpk \quad\quad}$

Security

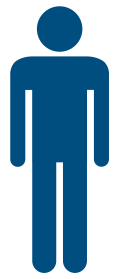


$(mpk, msk) \leftarrow Setup(1^{\lambda})$

$\xrightarrow{\quad mpk \quad}$

$\xleftarrow{\quad id \quad}$

Security



$(mpk, msk) \leftarrow Setup(1^\lambda)$

$\xrightarrow{\quad mpk \quad}$

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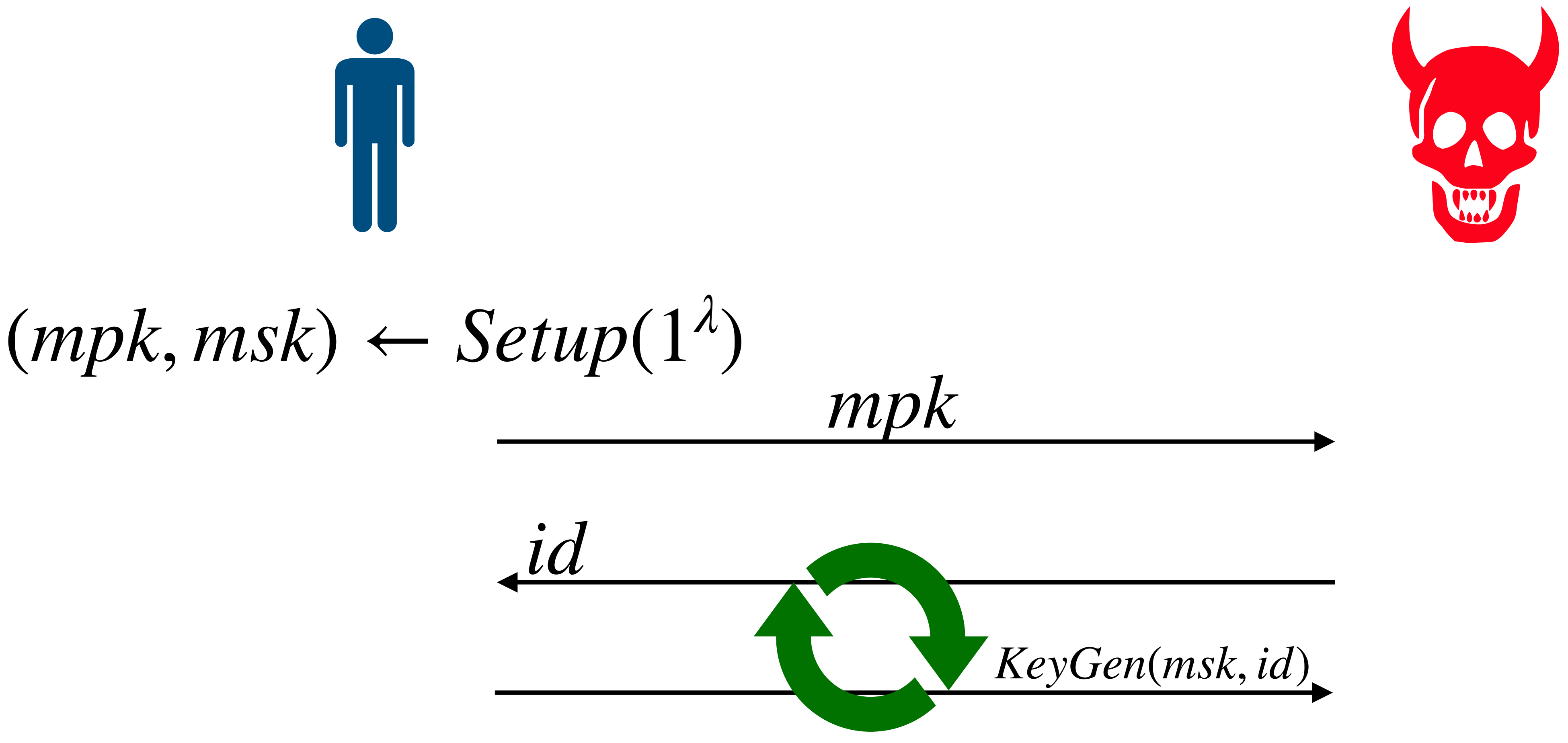
$\xrightarrow{\quad KeyGen(msk, id) \quad}$

Security



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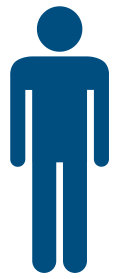

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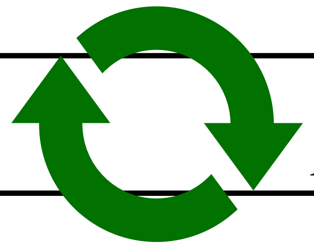
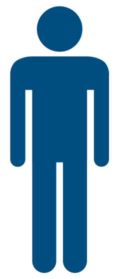


$\xrightarrow{\quad KeyGen(msk, id) \quad}$

$id \npreceq id^*$

Security



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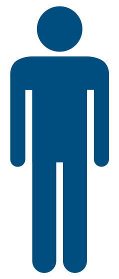
$\xleftarrow{\quad id \quad}$

$\xrightarrow{\quad KeyGen(msk, id) \quad}$

$\xleftarrow{\quad id^*, m_0, m_1 \quad}$

$id \not\preceq id^*$

Security



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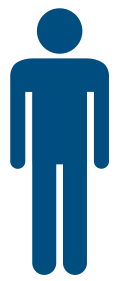
$b \leftarrow \{0,1\}$

$id \not\preceq id^*$

$ct \leftarrow Enc(mpk, id^*, m_b)$

$\xrightarrow{\quad ct^* \quad}$

Security



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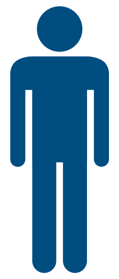
Adversary wins if $b' = b$

Security



- Weak definition - No delegation!

$(mpk, msk) \leftarrow Setup(1^\lambda)$

$\xrightarrow{\quad mpk \quad}$

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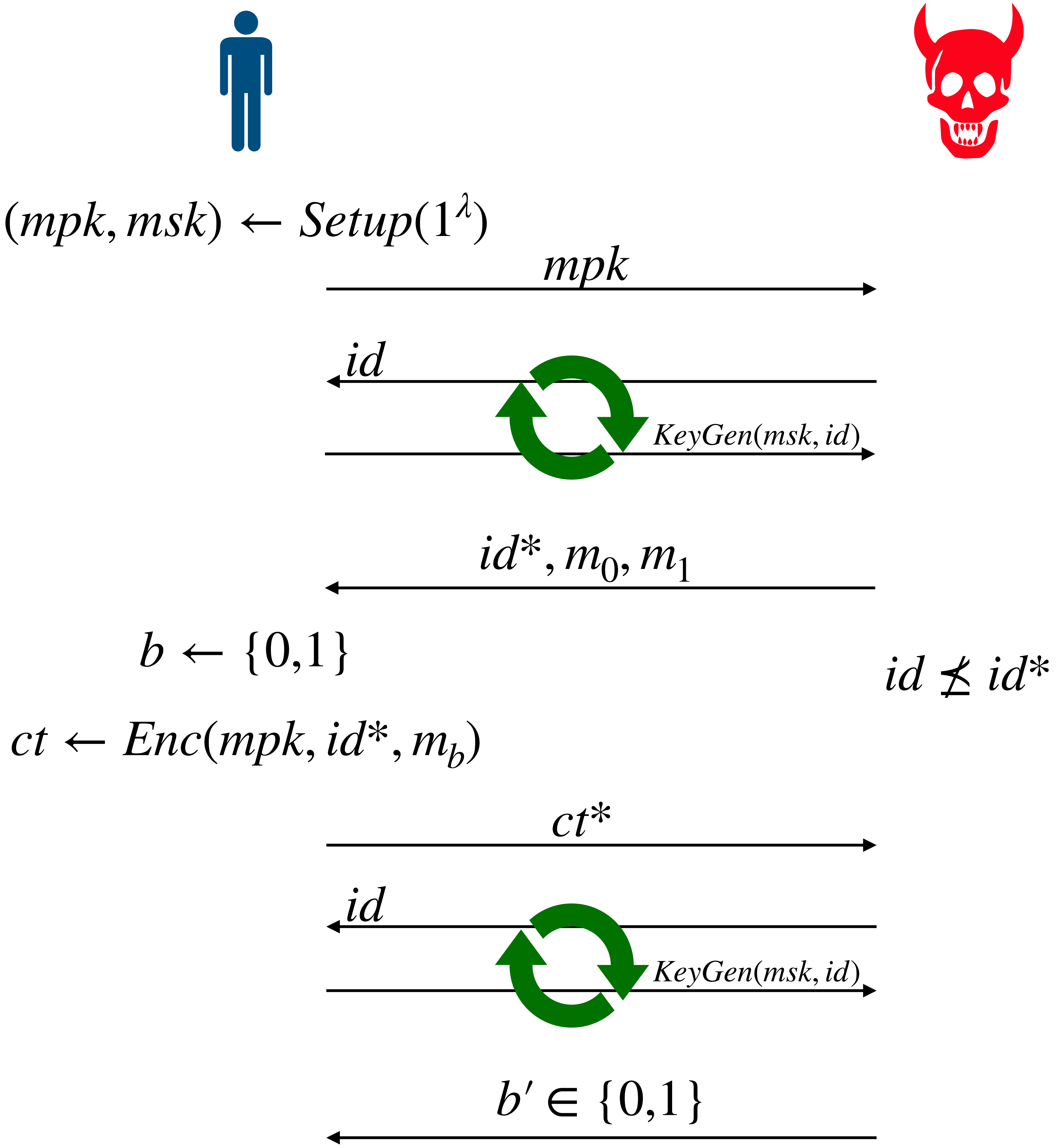

Adversary wins if $b' = b$

- Weak definition - No delegation!
- Achieved from adaptive IBE

Security

$(mpk, msk) \leftarrow Setup(1^\lambda)$

$\xrightarrow{\quad mpk \quad}$

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$ct \leftarrow Enc(mpk, id^*, m_b)$

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$\xleftarrow{\quad b' \in \{0,1\} \quad}$

Adversary wins if $b' = b$

- Weak definition - No delegation!
- Achieved from adaptive IBE
  - SETUP : $\{mpk_i, msk_i\} \leftarrow IBE.Setup(1^\lambda)$

Security

$(mpk, msk) \leftarrow Setup(1^\lambda)$

$\xrightarrow{mpk}$

$\xleftarrow{id}$

$\xrightarrow{KeyGen(msk, id)}$

$\xleftarrow{id^*, m_0, m_1}$

$b \leftarrow \{0,1\}$

$id \npreceq id^*$

$ct \leftarrow Enc(mpk, id^*, m_b)$

$\xrightarrow{ct^*}$

$\xleftarrow{id}$

$\xrightarrow{KeyGen(msk, id)}$

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Adversary wins if $b' = b$

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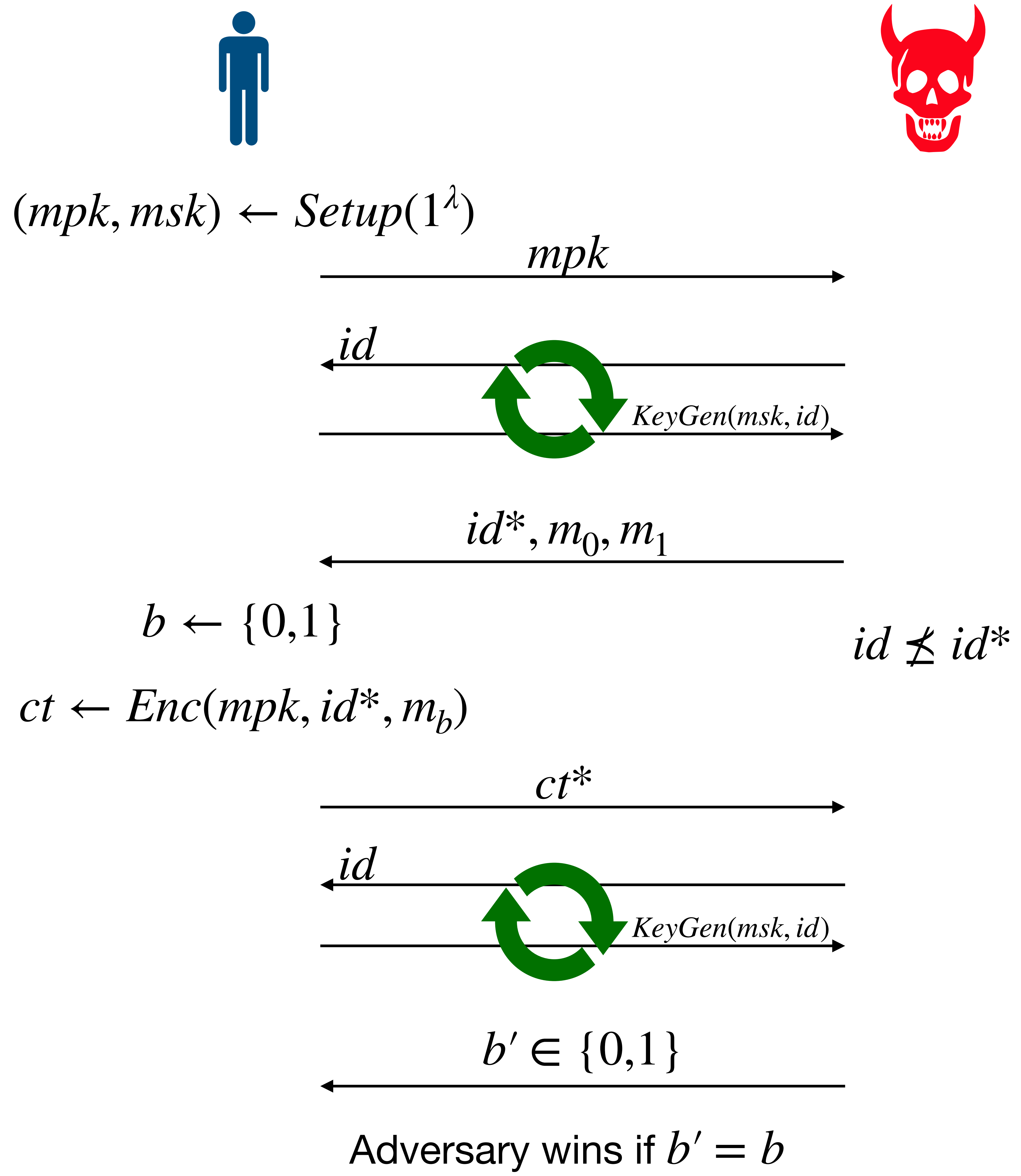
  - SETUP : $\{mpk_i, msk_i\} \leftarrow IBE.Setup(1^\lambda)$
  - KEYGEN : $sk_{id} \leftarrow IBE.KeyGen(msk_{|id|}, id)$

Security



- Weak definition - No delegation!
- Achieved from adaptive IBE
    - SETUP : $\{mpk_i, msk_i\} \leftarrow IBE.Setup(1^\lambda)$
    - KEYGEN : $sk_{id} \leftarrow IBE.KeyGen(msk_{|id|}, id)$
    - ENC : $\{ct_i \leftarrow IBE.Enc(mpk_i, id[1;i], m)\}_{i \in [|id|]}$

Security

$(mpk, msk) \leftarrow Setup(1^\lambda)$

$\xrightarrow{mpk}$

$\xleftarrow{id}$

$\xrightarrow{KeyGen(msk, id)}$

$\xleftarrow{id^*, m_0, m_1}$

$b \leftarrow \{0,1\}$

$id \npreceq id^*$

$ct \leftarrow Enc(mpk, id^*, m_b)$

$\xrightarrow{ct^*}$

$\xleftarrow{id}$

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Adversary wins if $b' = b$

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  - DELEGATE : Return sk_{id}

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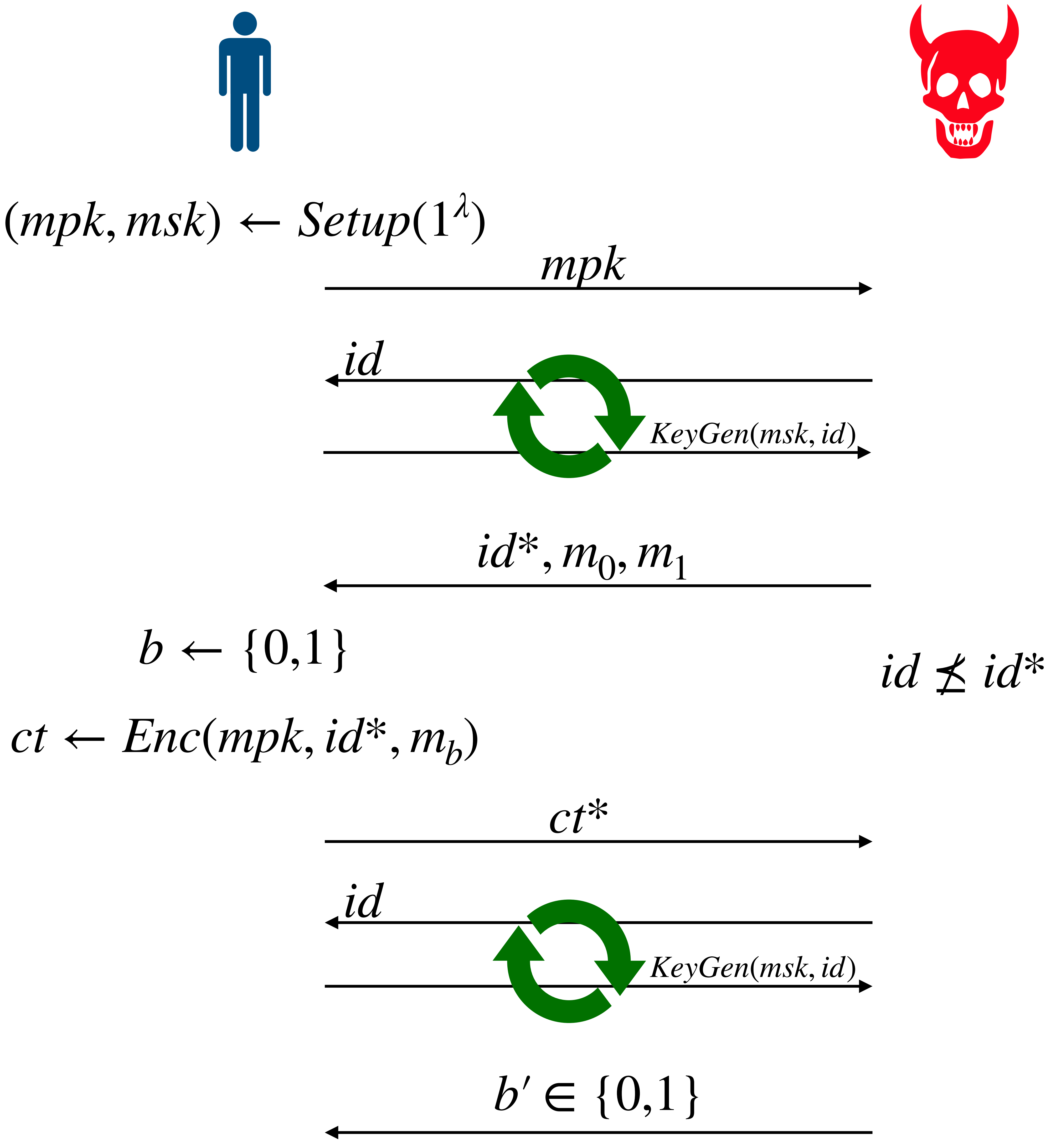

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  - SETUP : $\{mpk_i, msk_i\} \leftarrow IBE\,.\,Setup(1^\lambda)$
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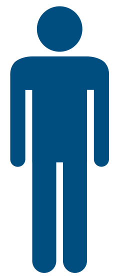
  - ENC : $\{ct_i \leftarrow IBE\,.\,Enc(mpk_i, id[1;i], m)\}_{i \in [|id|]}$
  - DELEGATE : Return sk_{id}
- Immediately broken if adversary is allowed to obtain delegated keys!

Security

[Shi-Waters'08]



$(mpk, msk) \leftarrow Setup(1^\lambda)$

$\xrightarrow{\quad mpk \quad}$

$\xleftarrow{\quad id^*, m_0, m_1 \quad}$

$b \leftarrow \{0,1\}$

$ct \leftarrow Enc(mpk, id^*, m_b)$

$\xrightarrow{\quad ct^* \quad}$

$\xleftarrow{\quad b' \in \{0,1\} \quad}$

Adversary wins if $b' = b$

Security

[Shi-Waters'08]

$(mpk, msk) \leftarrow Setup(1^\lambda)$

$\xrightarrow{\quad mpk \quad}$

query $\longleftarrow$

response $\longrightarrow$

$\xleftarrow{\quad id^*, m_0, m_1 \quad}$

$b \leftarrow \{0,1\}$

$ct \leftarrow Enc(mpk, id^*, m_b)$

$\xrightarrow{\quad ct^* \quad}$

query $\longleftarrow$

response $\longrightarrow$

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Adversary wins if $b' = b$

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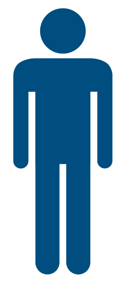


[Shi-Waters'08]

$(mpk, msk) \leftarrow Setup(1^\lambda)$

mpk →

← *query*

response →

← id^*, m_0, m_1

$b \leftarrow \{0,1\}$

$ct \leftarrow Enc(mpk, id^*, m_b)$

ct^* →

← *query*

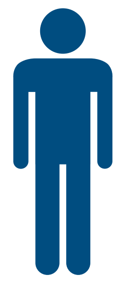
response →

← $b' \in \{0,1\}$

Adversary wins if $b' = b$

- $query = initialize(id):$
  Store $sk_{id} \leftarrow KeyGen(msk, id)$.
  No response.

Security



[Shi-Waters'08]

$(mpk, msk) \leftarrow Setup(1^\lambda)$

$\xrightarrow{\quad mpk \quad}$

$\xleftarrow{\quad query \quad}$

$\xrightarrow{\quad response \quad}$

$\xleftarrow{\quad id^*, m_0, m_1 \quad}$

$b \leftarrow \{0,1\}$

$ct \leftarrow Enc(mpk, id^*, m_b)$

$\xrightarrow{\quad ct^* \quad}$

$\xleftarrow{\quad query \quad}$

$\xrightarrow{\quad response \quad}$

$\xleftarrow{\quad b' \in \{0,1\} \quad}$

Adversary wins if $b' = b$

- $query = initialize(id)$:
  Store $sk_{id} \leftarrow KeyGen(msk, id)$.
  No response.

- $query = delegate(id_1, id_2)$:
  Store $sk_{id_1 \| id_2} \leftarrow Delegate(sk_{id_1}, id_2)$.
  No response.

Security

[Shi-Waters'08]

$(mpk, msk) \leftarrow Setup(1^\lambda)$

mpk →

← *query*

response →

← id^*, m_0, m_1

$b \leftarrow \{0,1\}$

$ct \leftarrow Enc(mpk, id^*, m_b)$

ct^* →

← *query*

response →

← $b' \in \{0,1\}$

Adversary wins if $b' = b$

- $query = initialize(id)$:
  Store $sk_{id} \leftarrow KeyGen(msk, id)$.
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- $query = delegate(id_1, id_2)$:
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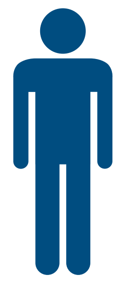
- $query = corrupt(id)$:
  Returns sk_{id}

Security

[Shi-Waters'08]



$(mpk, msk) \leftarrow Setup(1^\lambda)$

$\xrightarrow{\quad mpk \quad}$

$query$

$response$

$\xleftarrow{\quad id^*, m_0, m_1 \quad}$

$b \leftarrow \{0,1\}$

$ct \leftarrow Enc(mpk, id^*, m_b)$

$\xrightarrow{\quad ct^* \quad}$

$query$

$response$

$\xleftarrow{\quad b' \in \{0,1\} \quad}$

Adversary wins if $b' = b$

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  Store $sk_{id_1 \| id_2} \leftarrow Delegate(sk_{id_1}, id_2)$.
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- $query = corrupt(id)$:
  Returns sk_{id} $\qquad id \npreceq id^*$

Selective Security

$(mpk, msk) \leftarrow Setup(1^\lambda)$

$\xrightarrow{\quad mpk \quad}$

$\xleftarrow{\quad query \quad}$

$\xrightarrow{\quad response \quad}$

$\longleftarrow$

$b \leftarrow \{0,1\}$

$id \neq id^*$

$ct \leftarrow Enc(mpk, id^*, m_b)$

$\xrightarrow{\quad ct^* \quad}$

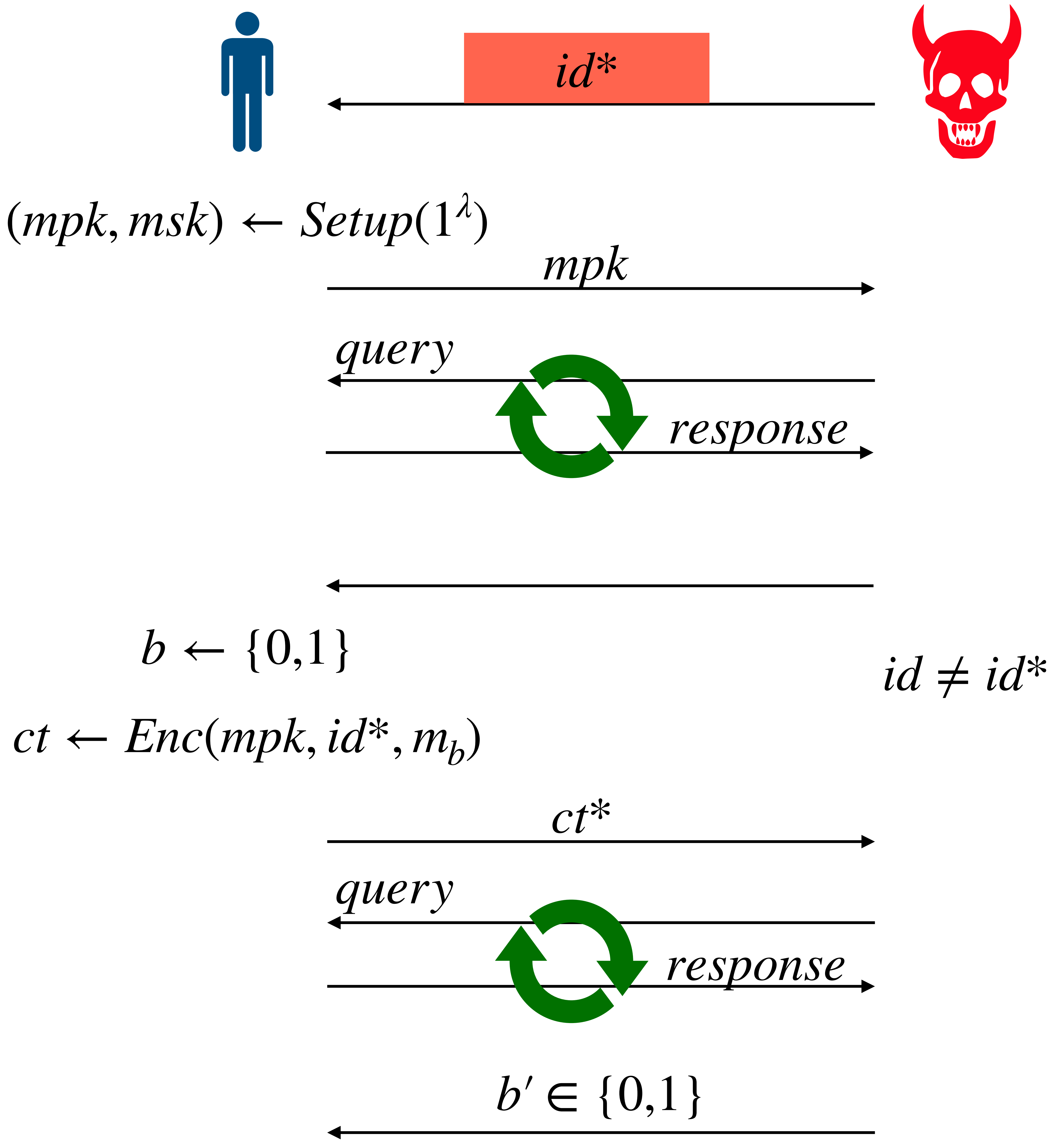
$\xleftarrow{\quad query \quad}$

$\xrightarrow{\quad response \quad}$

$\xleftarrow{\quad b' \in \{0,1\} \quad}$

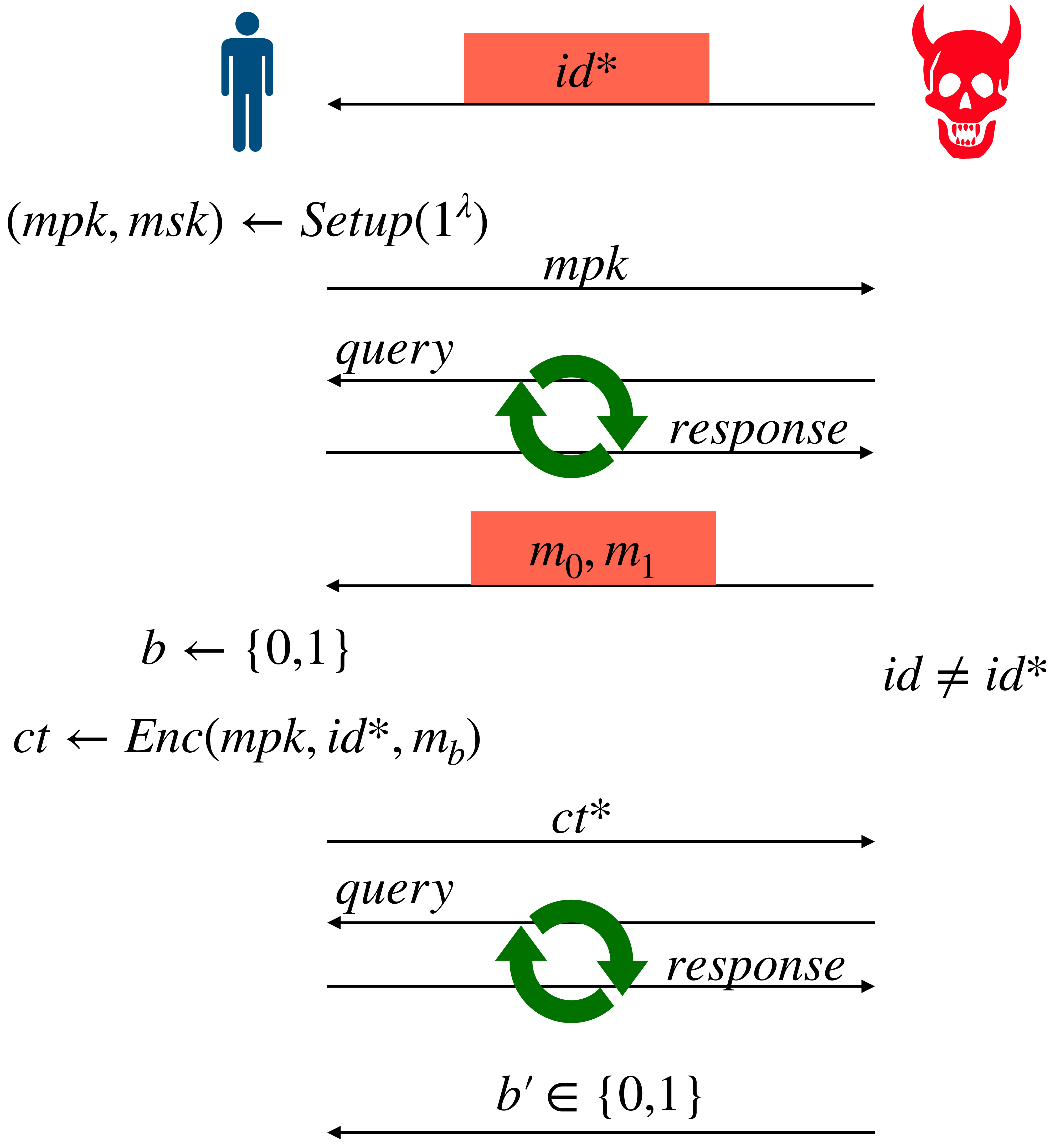
Adversary wins if $b' = b$

Selective Security



Adversary wins if $b' = b$

Selective Security



Adversary wins if $b' = b$

State of the Art

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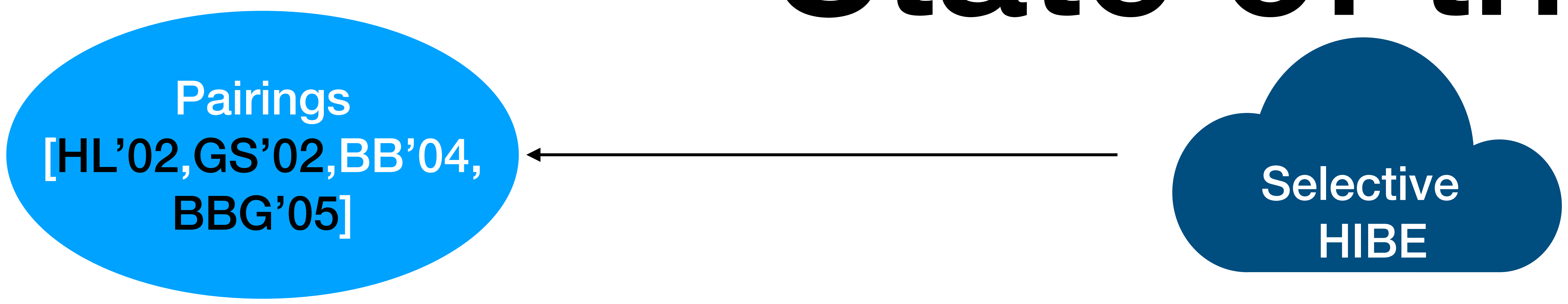

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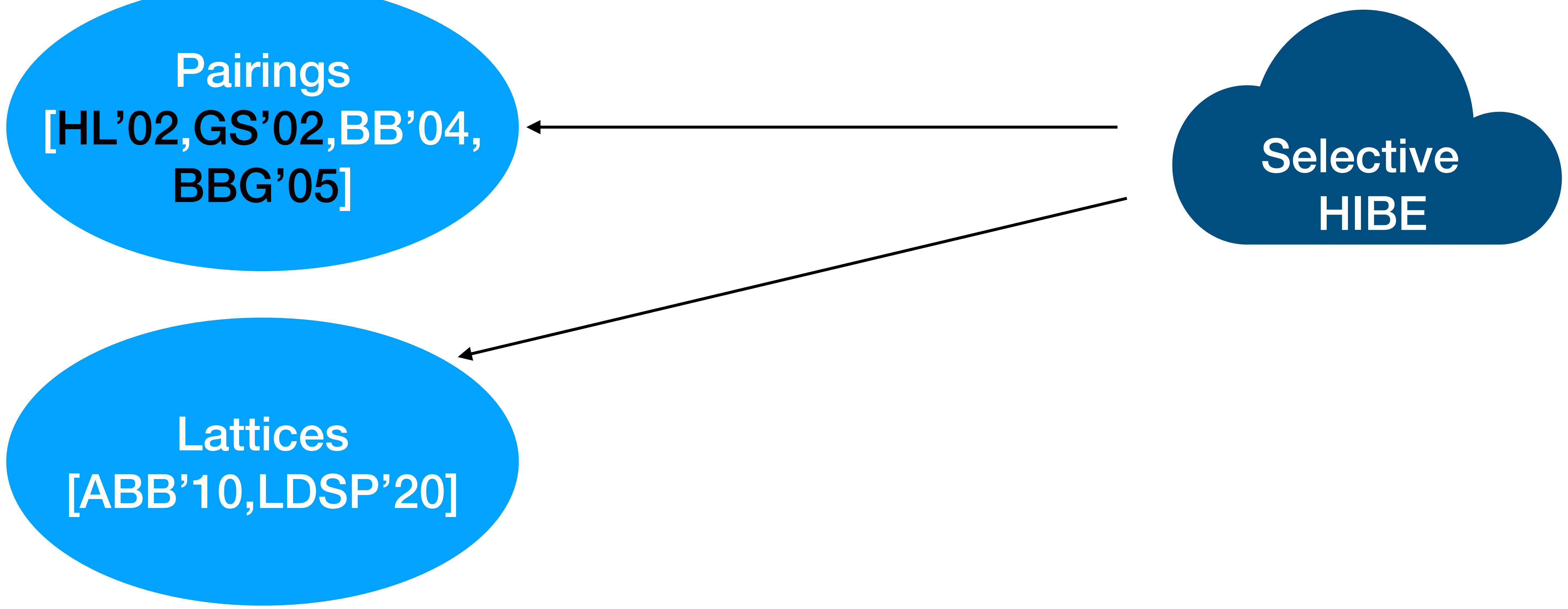

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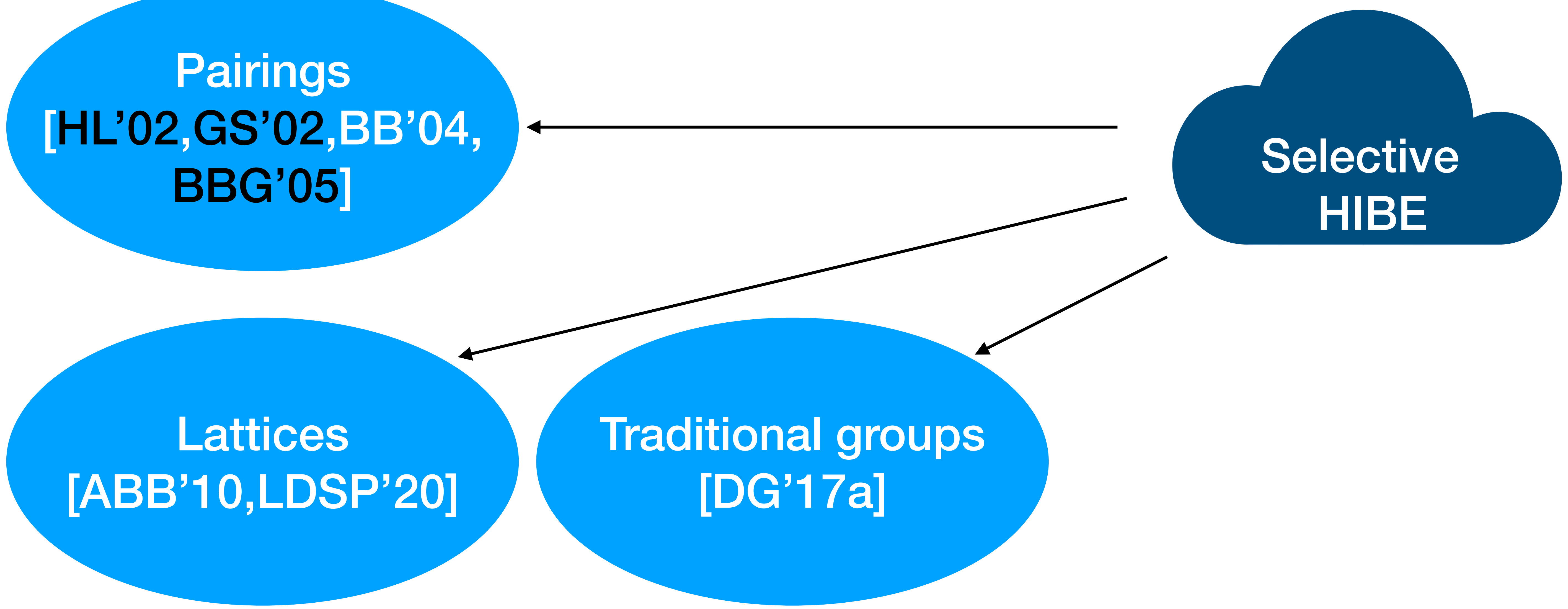

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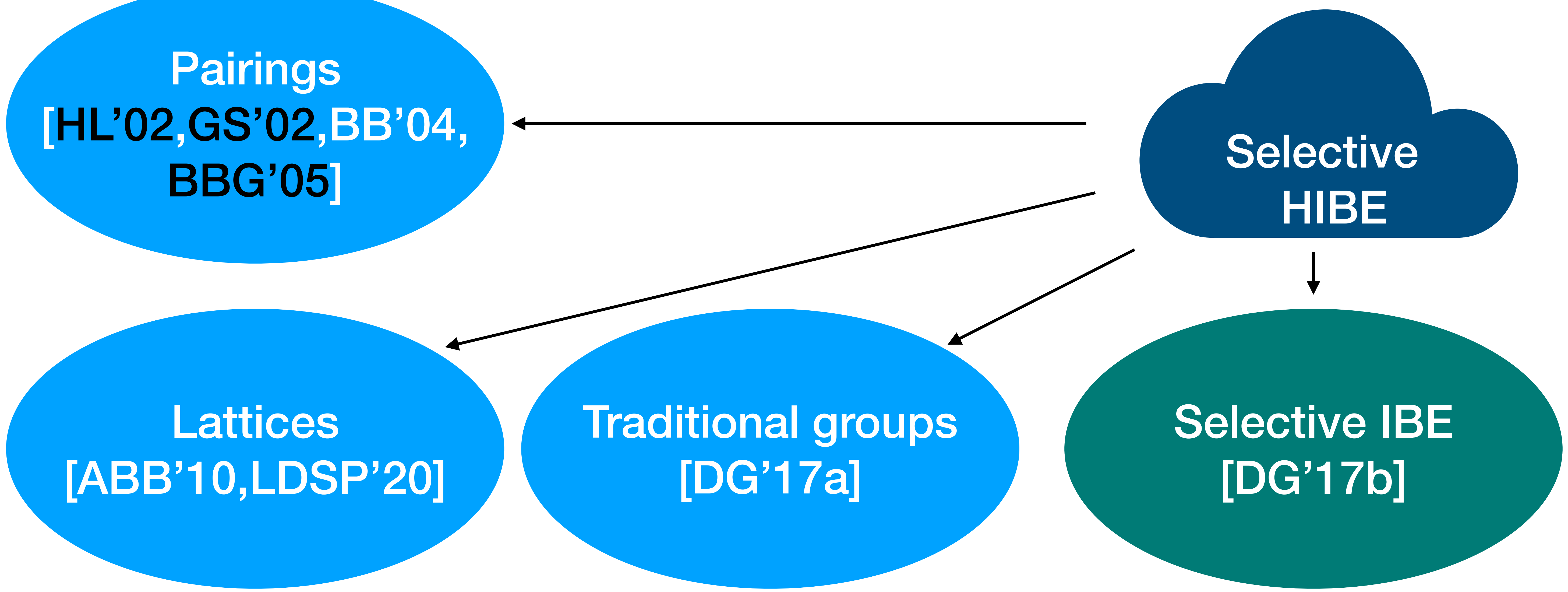

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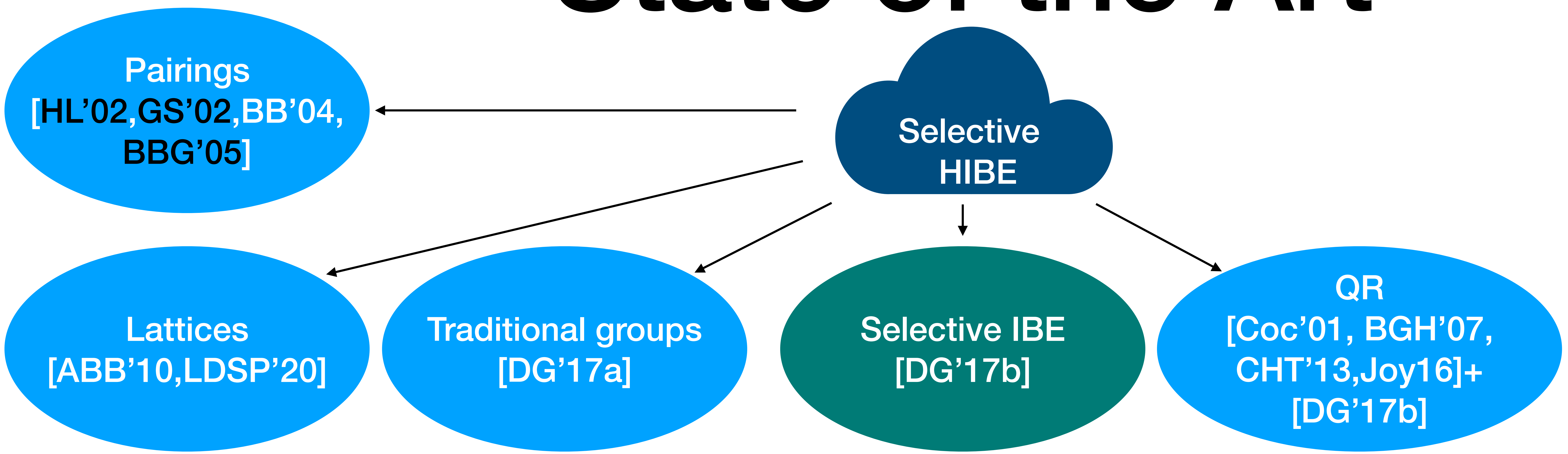

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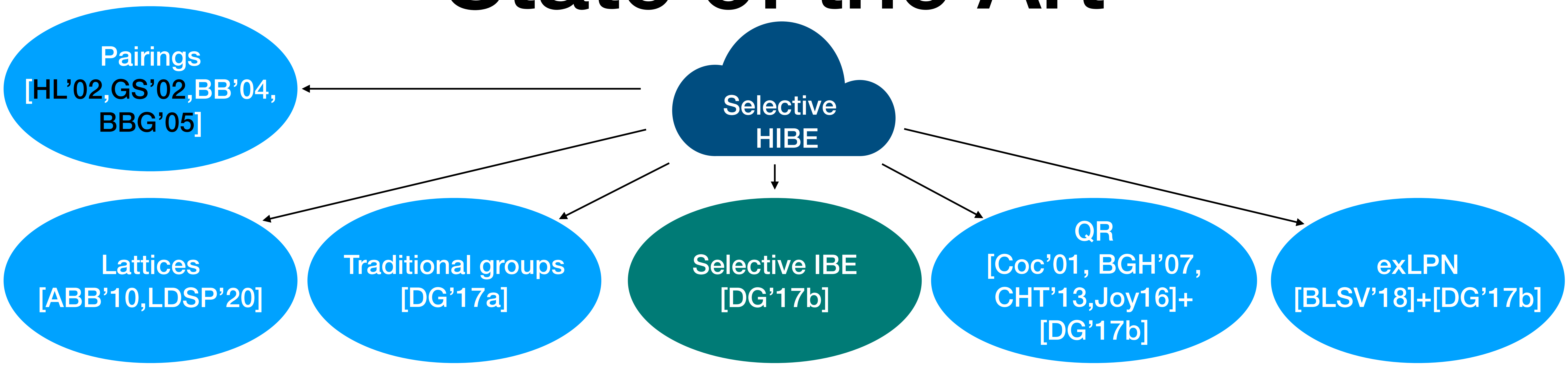

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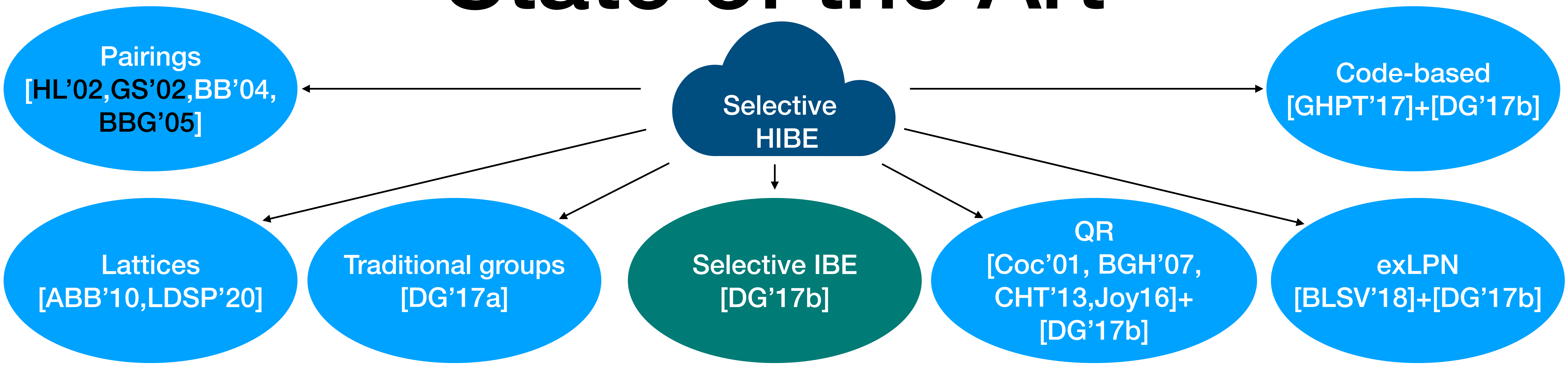

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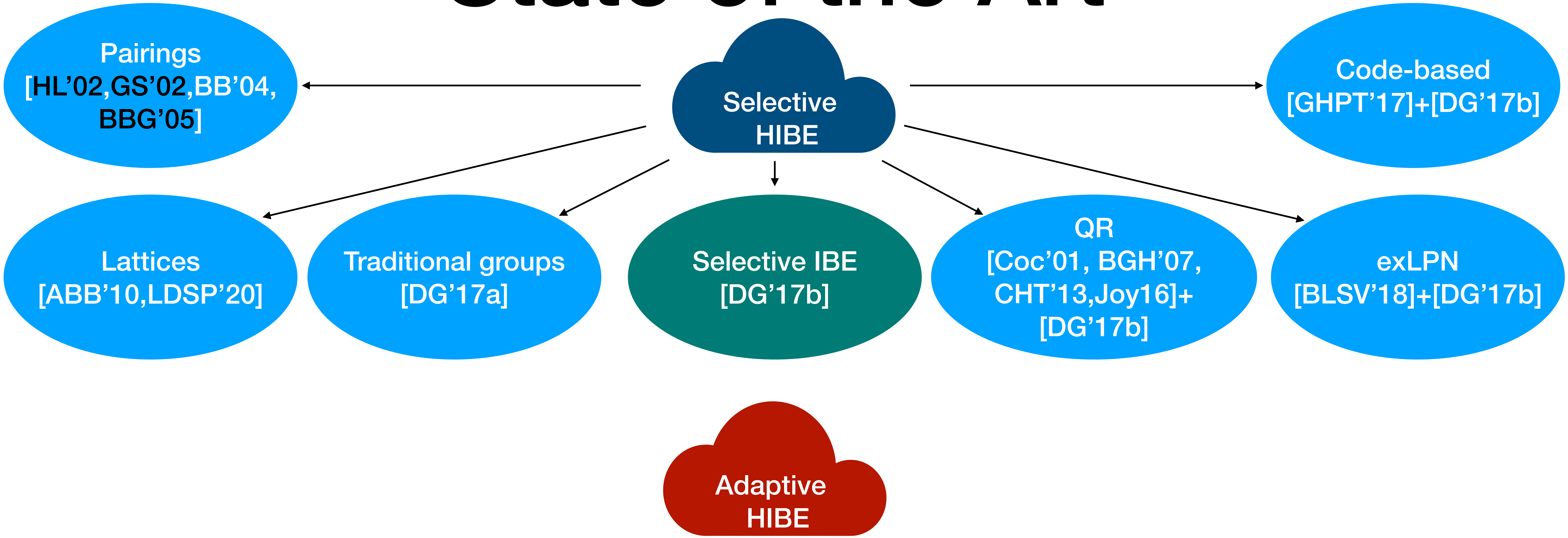

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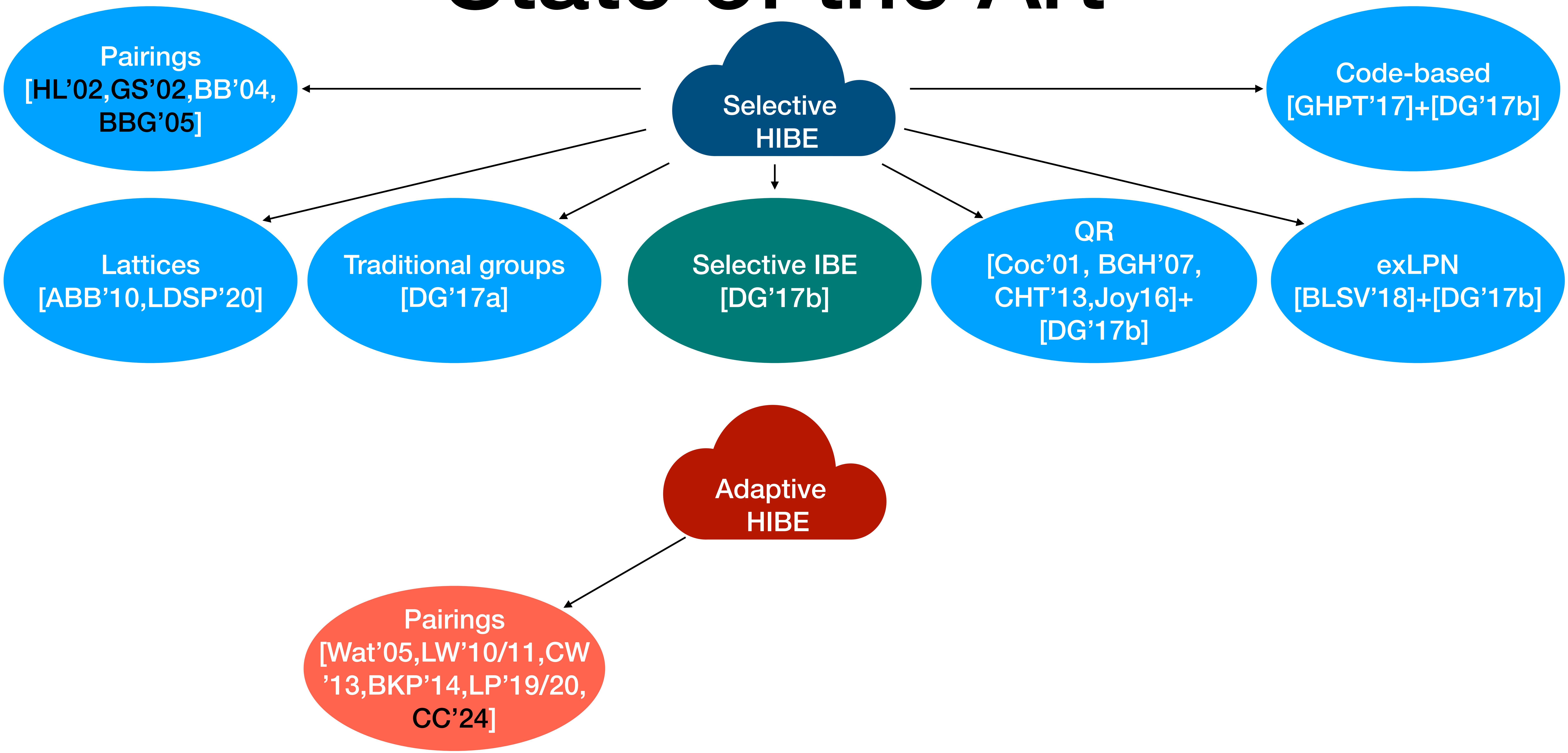

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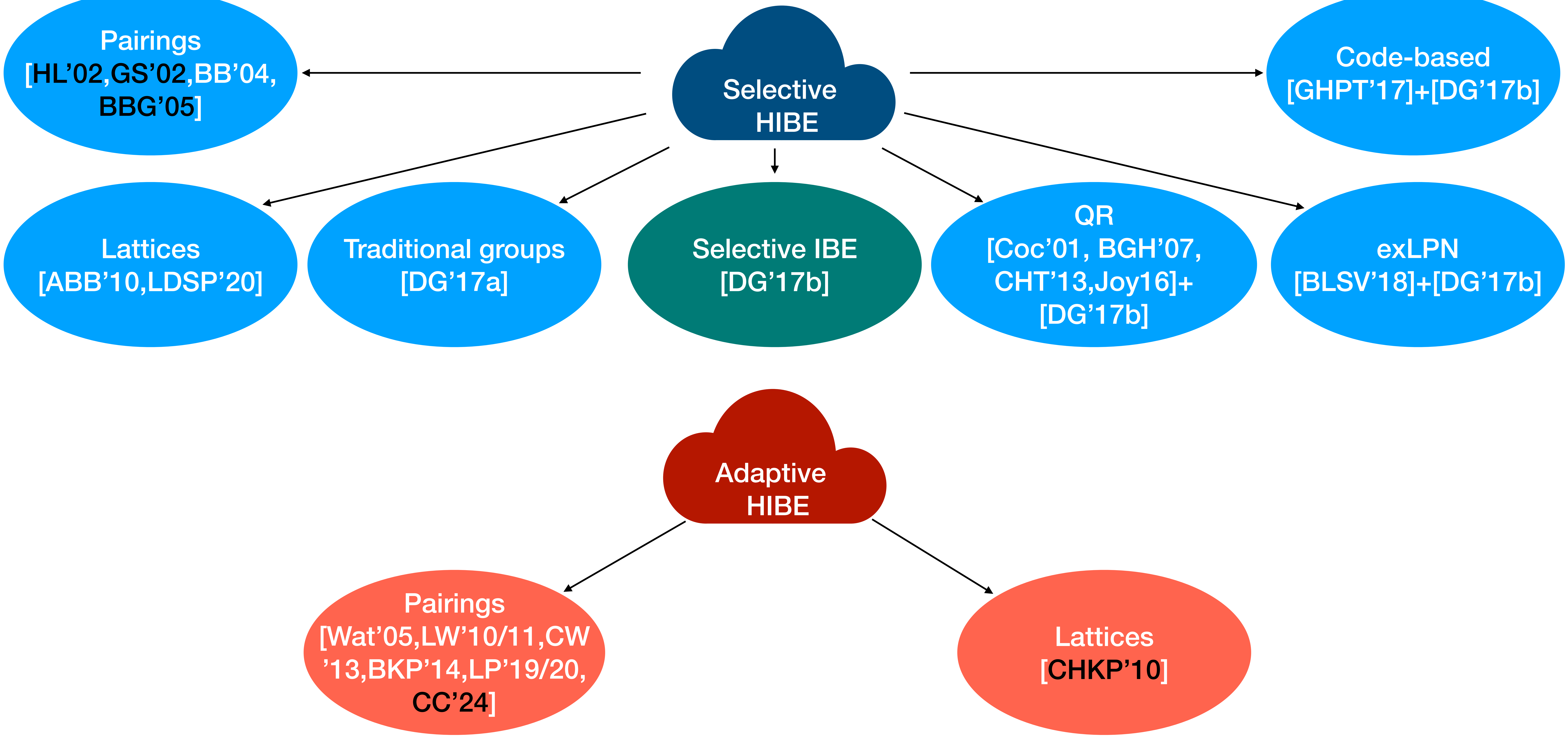

Challenges

Challenges

- Despite various heuristic approaches, achieving *adaptive secure* HIBE **in the standard model** without relying on strong or non-standard assumptions remains an open challenge.

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- Lewko-Waters'14, Brakerski-Medina'24 demonstrate that HIBE schemes satisfying "checkable" or partition-based proof conditions (and many early constructions) cannot be adapted to full security.

Security

Attacker

1. Obtains mpk
2. Adaptively makes queries
3. Sends id^*, m_0, m_1
4. Obtains challenge ciphertext ct^*
5. Adaptively makes queries
6. Outputs $b' \in \{0,1\}$

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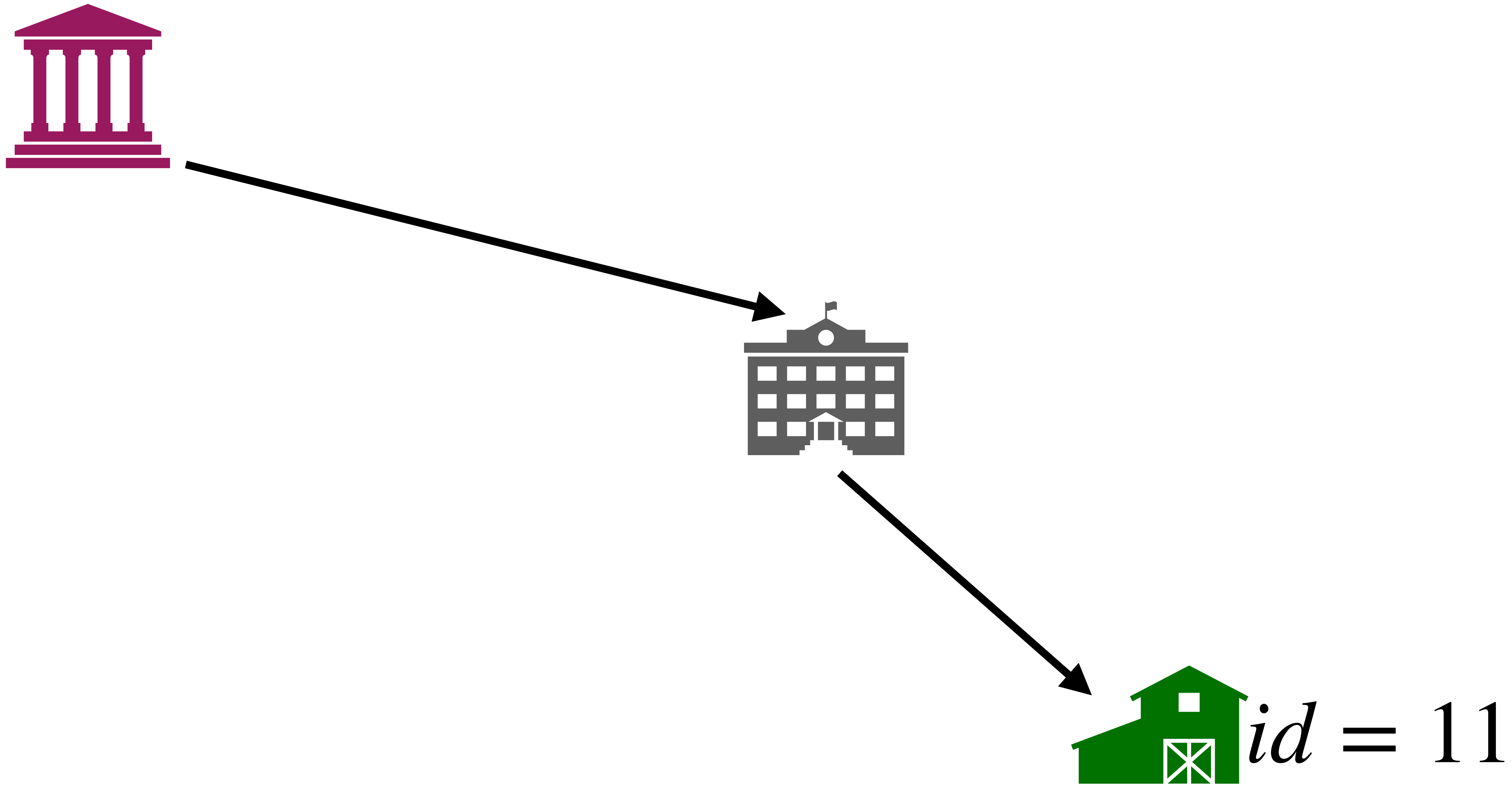

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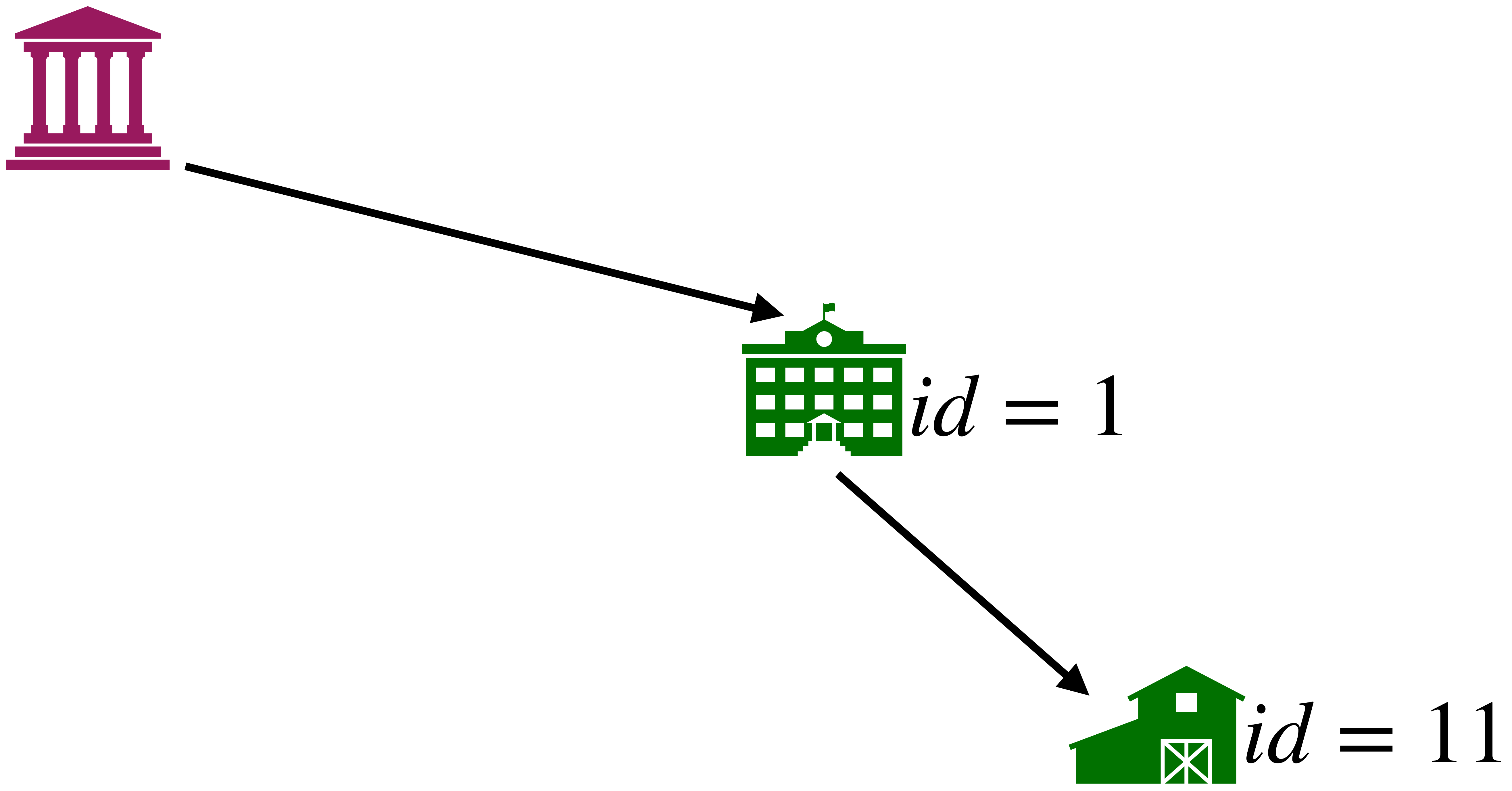

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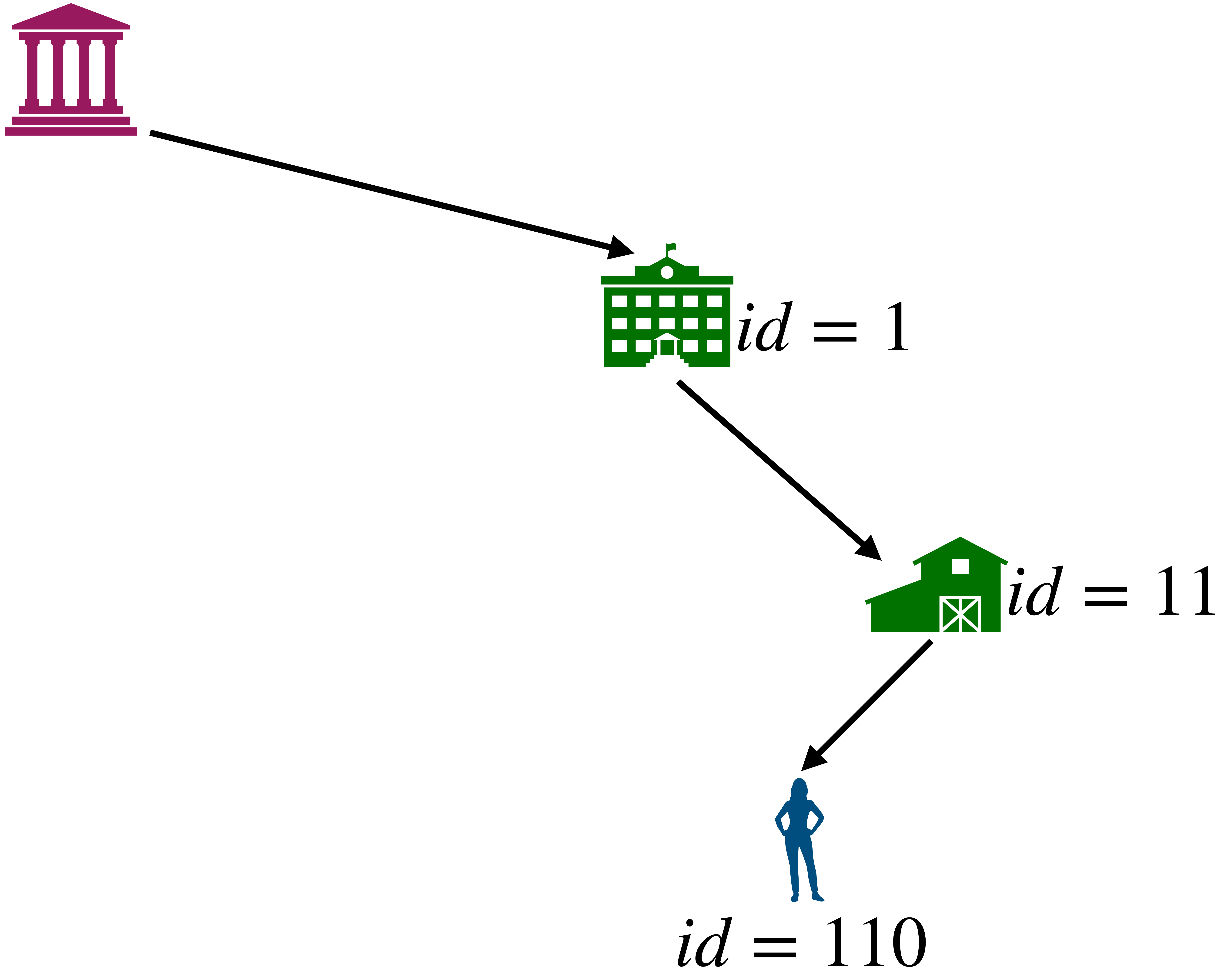

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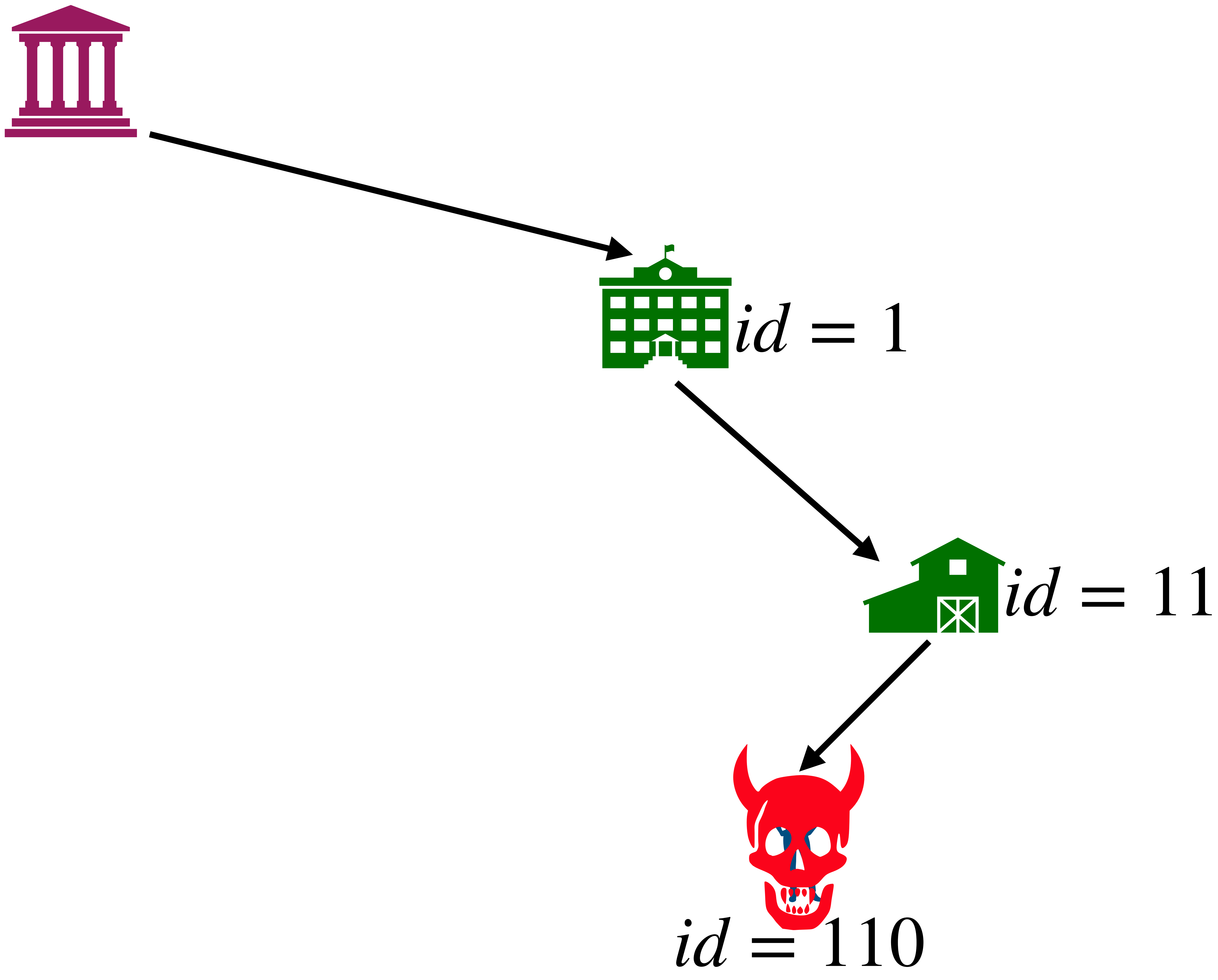

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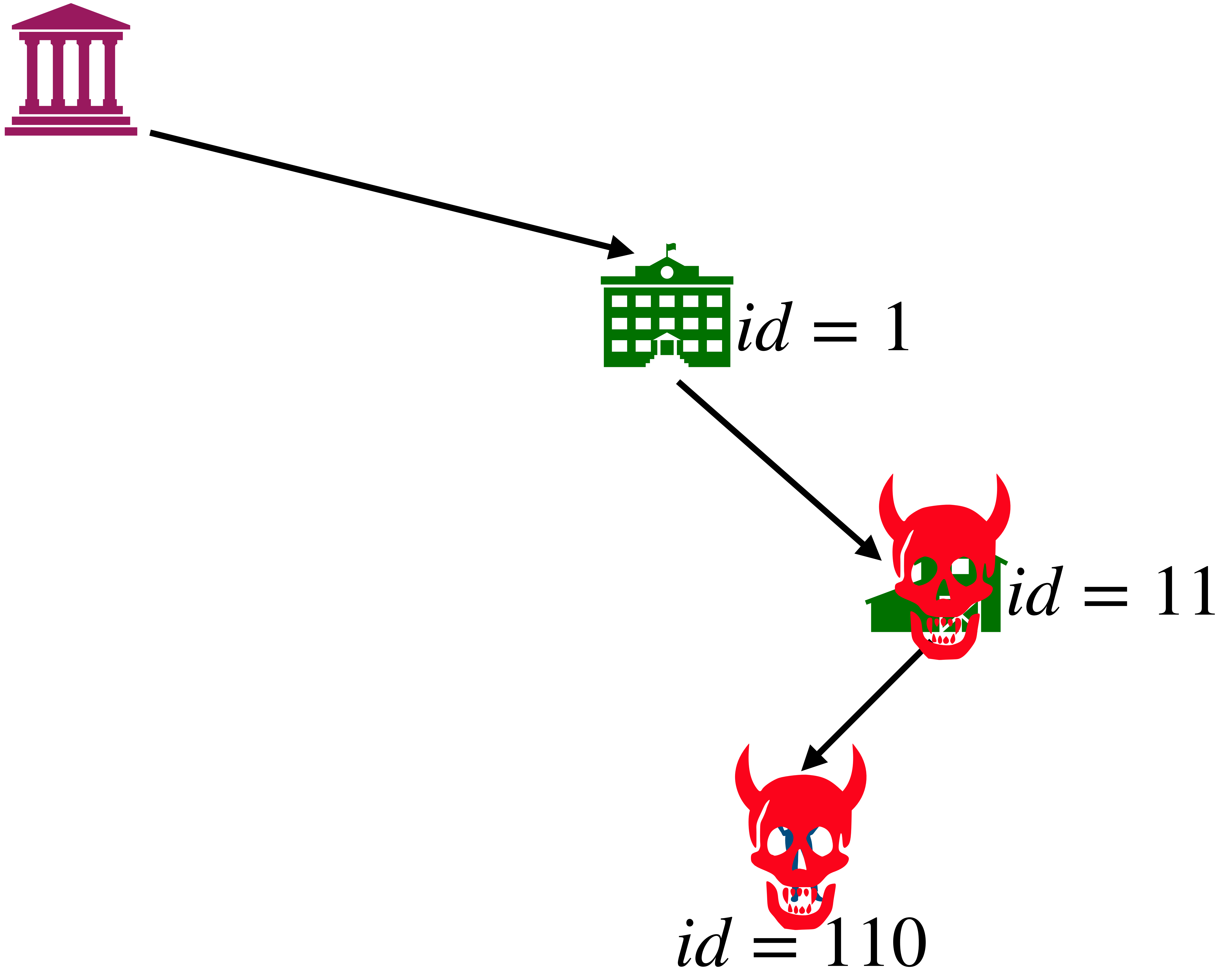

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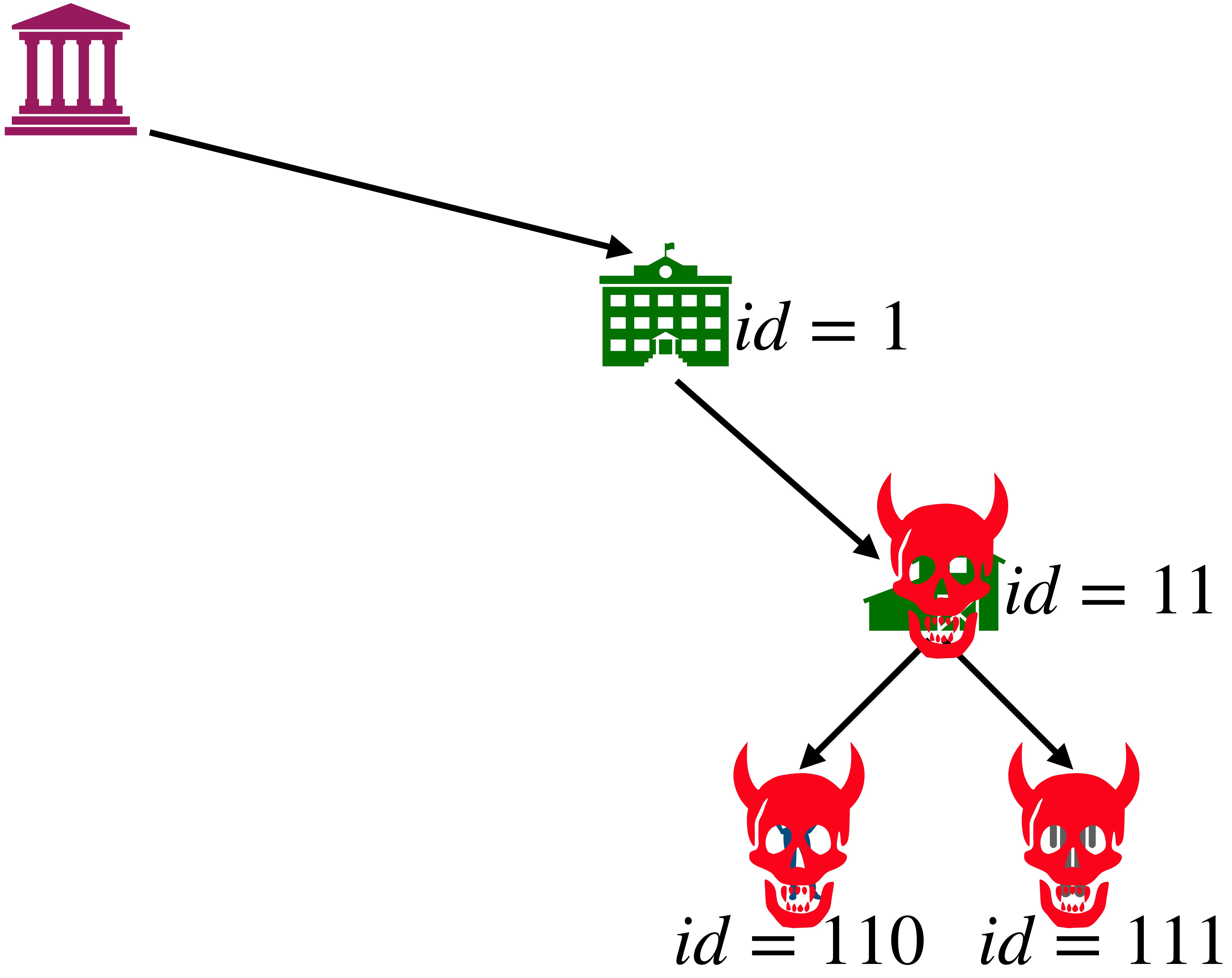

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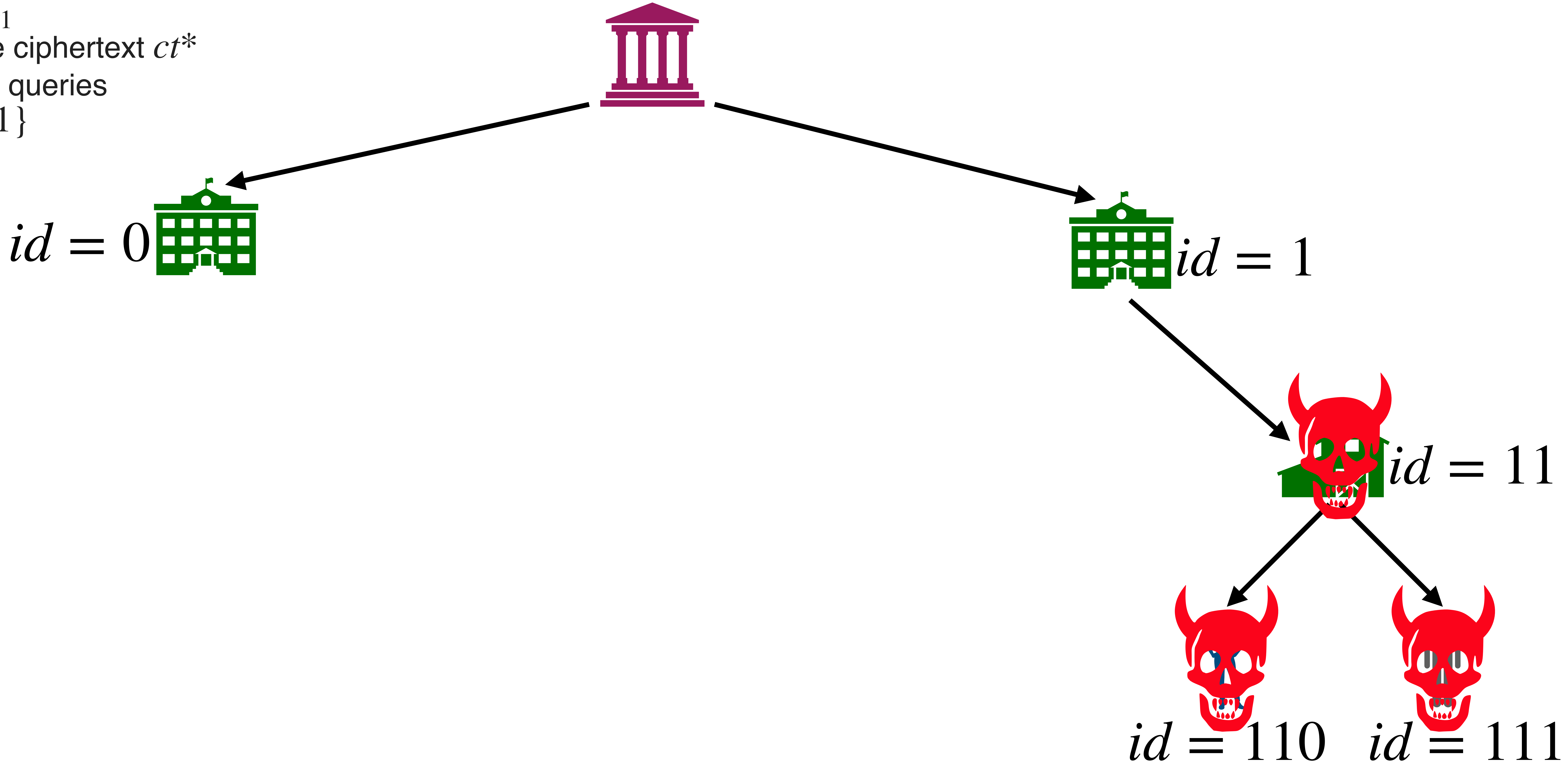

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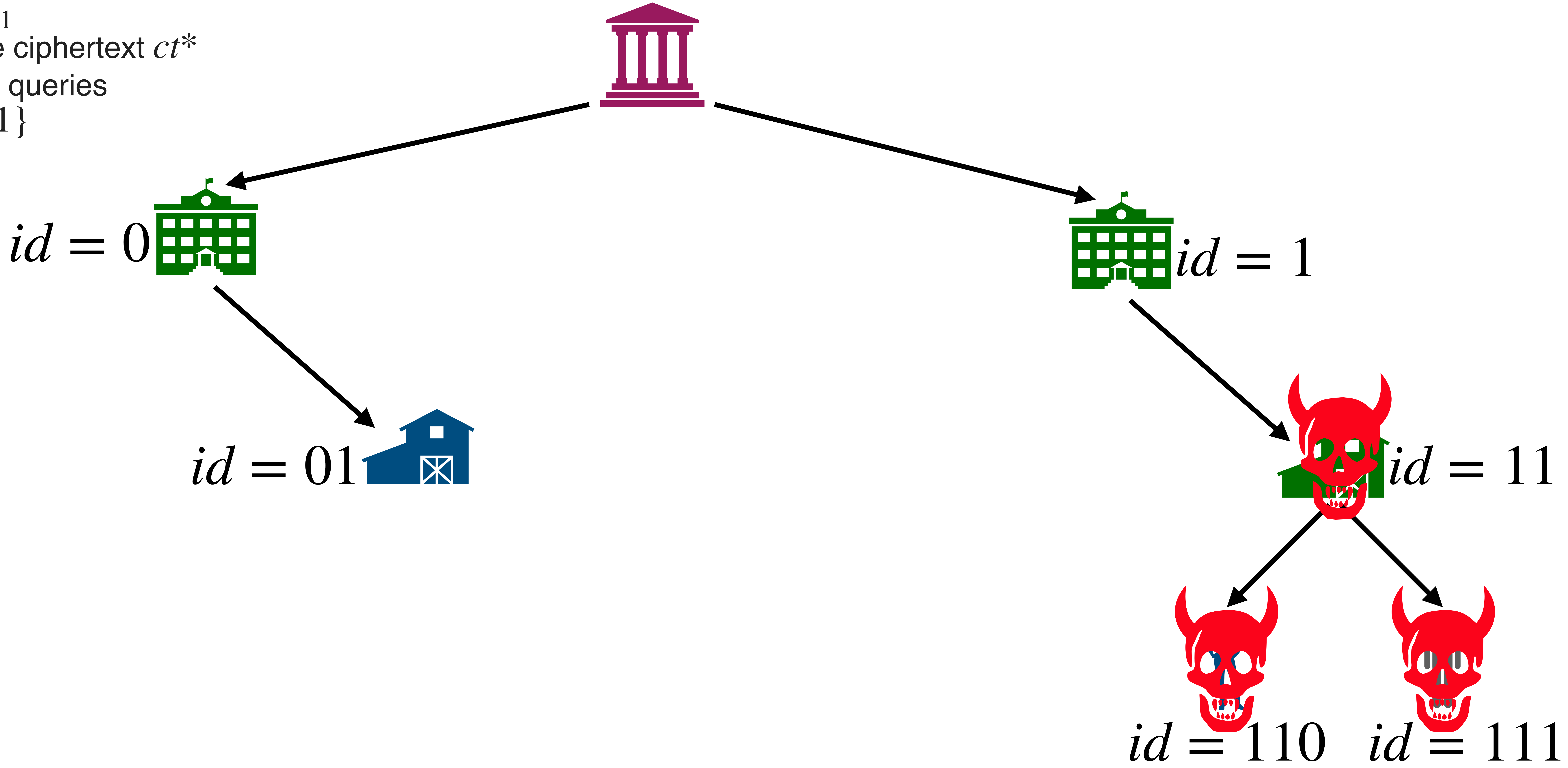

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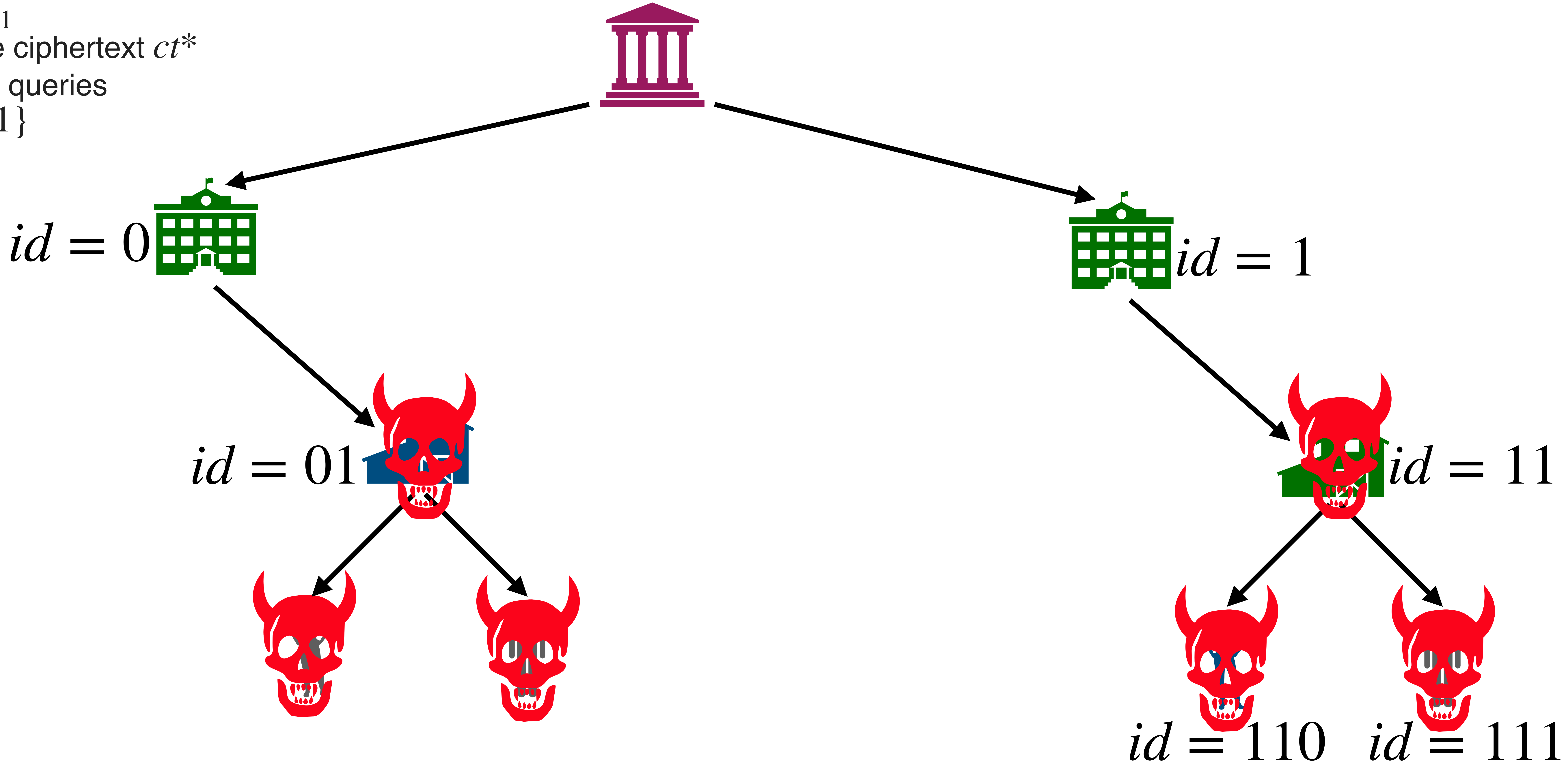

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Our Work

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Theorem 1: Assuming a ***selectively*** *secure IBE* system, there exists a ***fully*** *secure HIBE* system.

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Theorem 2: Assuming hardness of CDH or LWE, there exists a ***fully*** *secure* ***anonymous*** *HIBE* system.

DG17 Construction

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Building blocks

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- Delegatable PRF (S, E, D)

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- Garbling scheme $(Garb, Eval)$
- IBE OTSE (Gen, KG, Enc, Dec)

Delegatable PRF

Delegatable PRF

$Gen(1^{\lambda}) \rightarrow$ key k

Delegatable PRF

$Gen(1^\lambda) \to$ key k

$Eval(k, x) \to y_x$

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Correctness: For all x, x'

$Eval(k, x \| x') = Eval(k_x, x')$

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Security

$(x^*, \{Eval(k, x')\}_{x' \leq x^*},$ $\qquad (x^*,$

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Delegatable PRF

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$(x^*, \{Eval(k, x')\}_{x' \preceq x^*}, \{x, Eval(k, x)\},$

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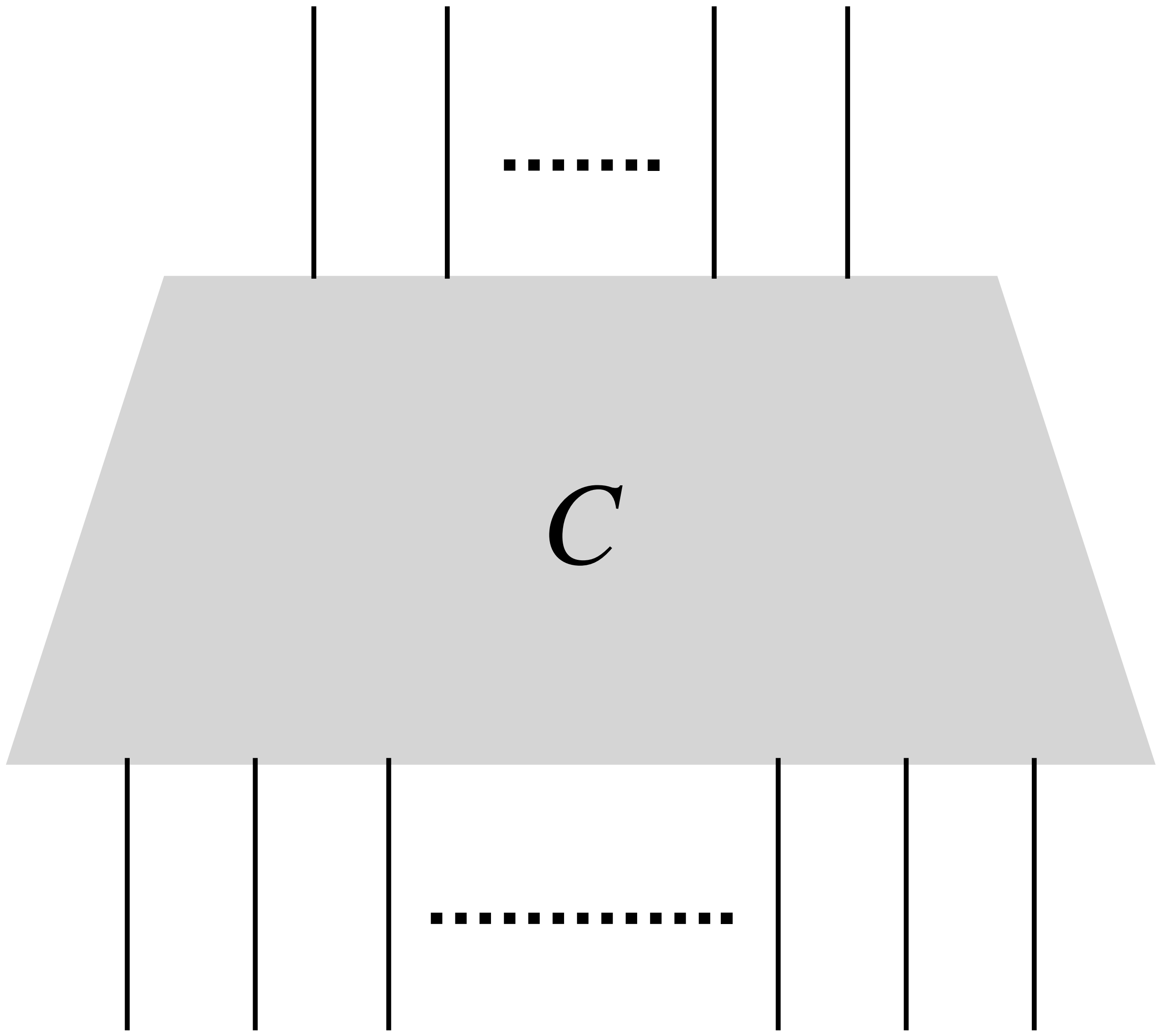
Security

$$(x^*, \{Eval(k, x')\}_{x' \leq x^*}, \{x, Eval(k, x)\}, \{x, Delegate(k, x)\}) \approx_c (x^*, \{y_i^*\}_{i \in [|x^*|]}, \{x, Eval(k, x)\}, \{x, Delegate(k, x)\})$$

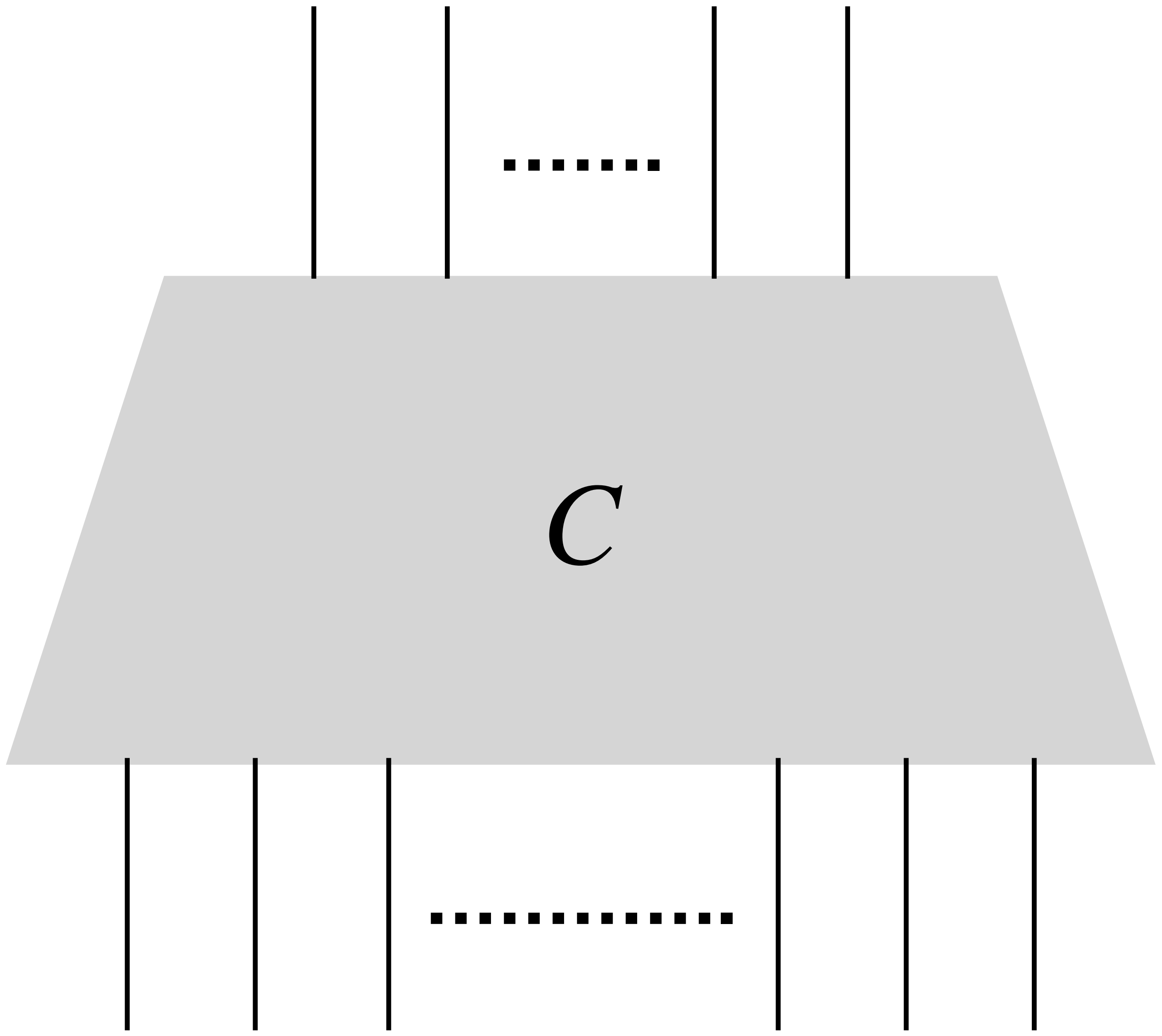
Garbling Scheme

Garbling Scheme



$C : \{0,1\}^k \rightarrow \{0,1\}^n$

Garbling Scheme



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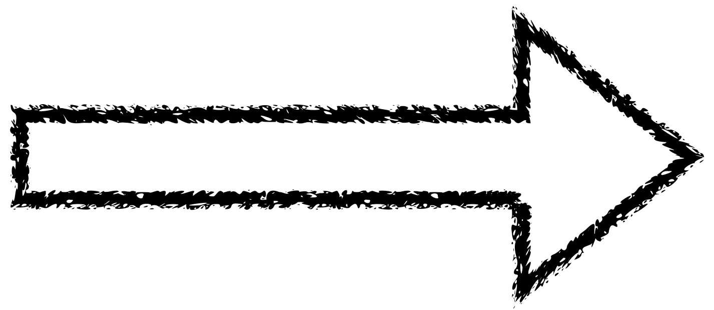

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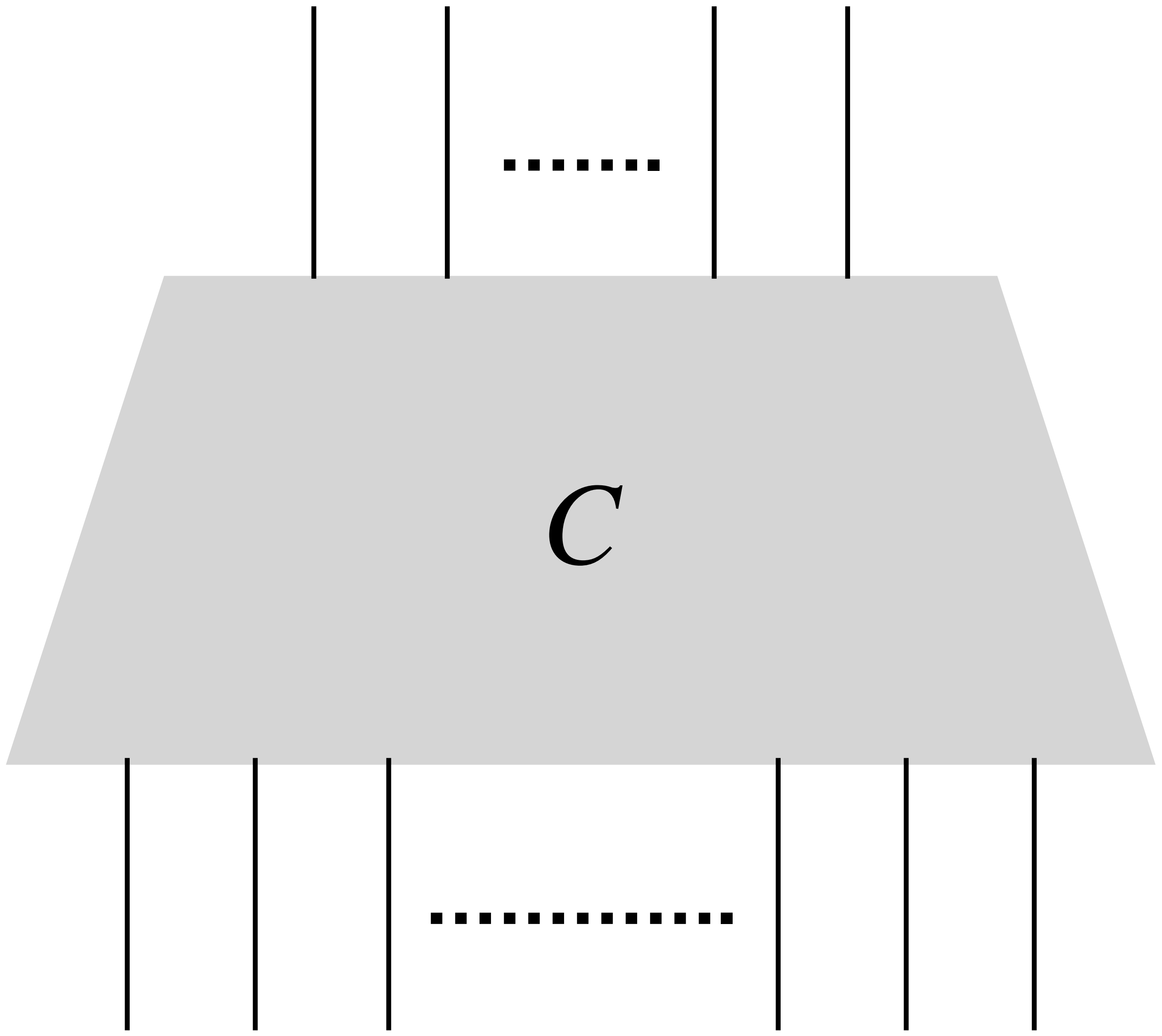

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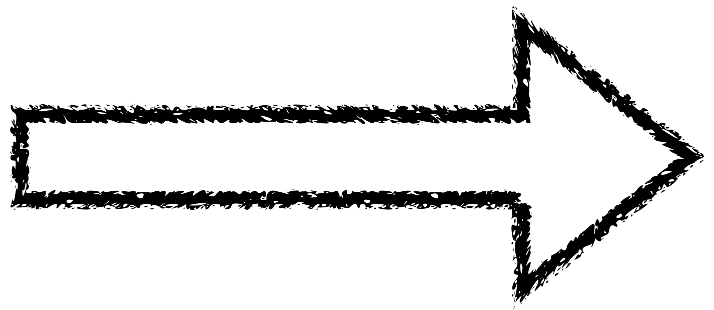

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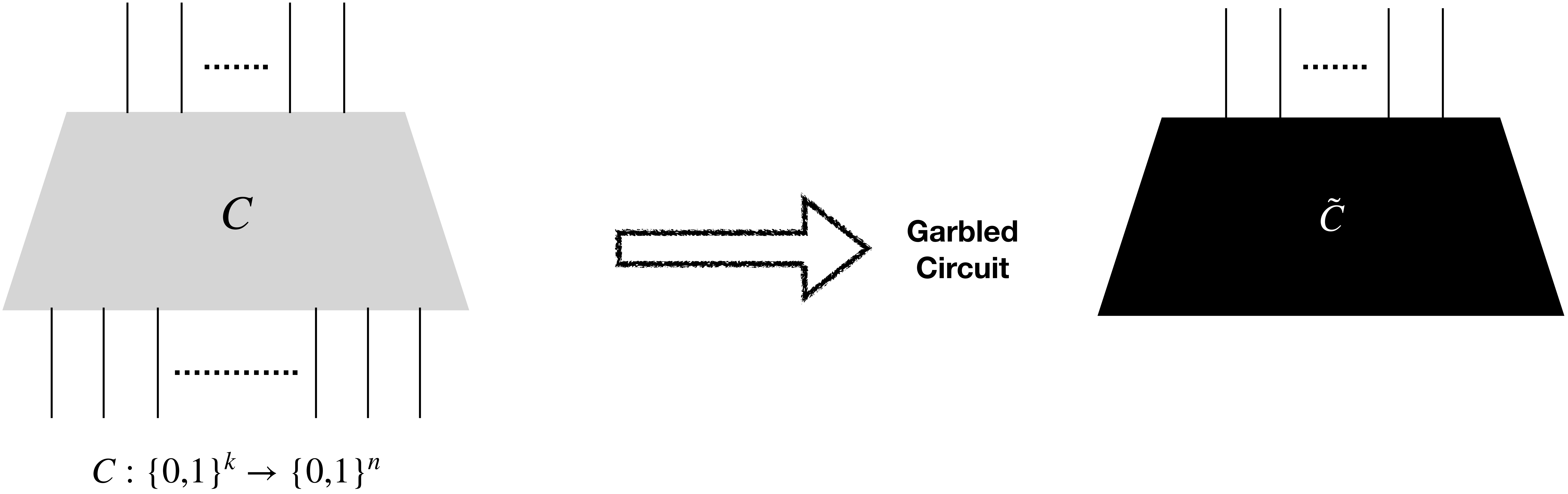

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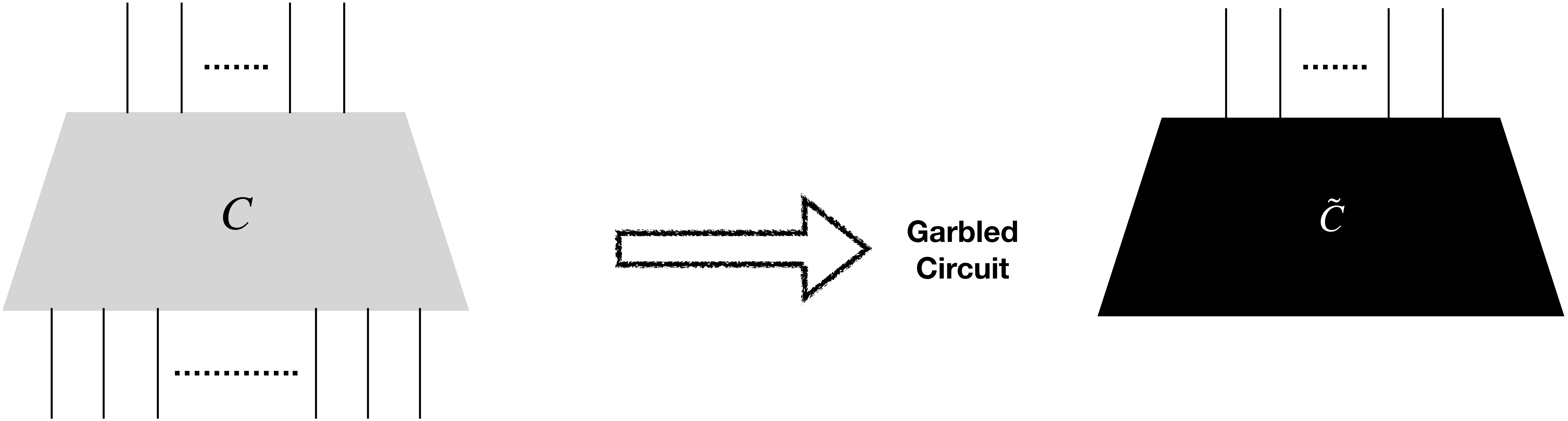

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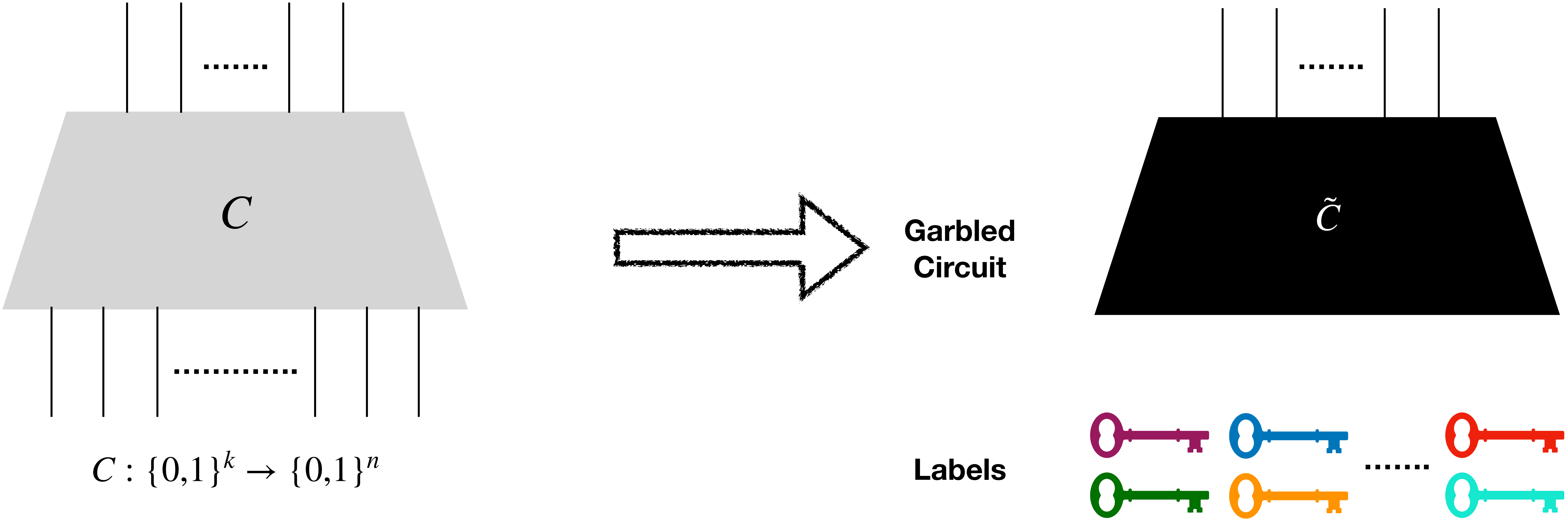
Labels

Garbling Scheme



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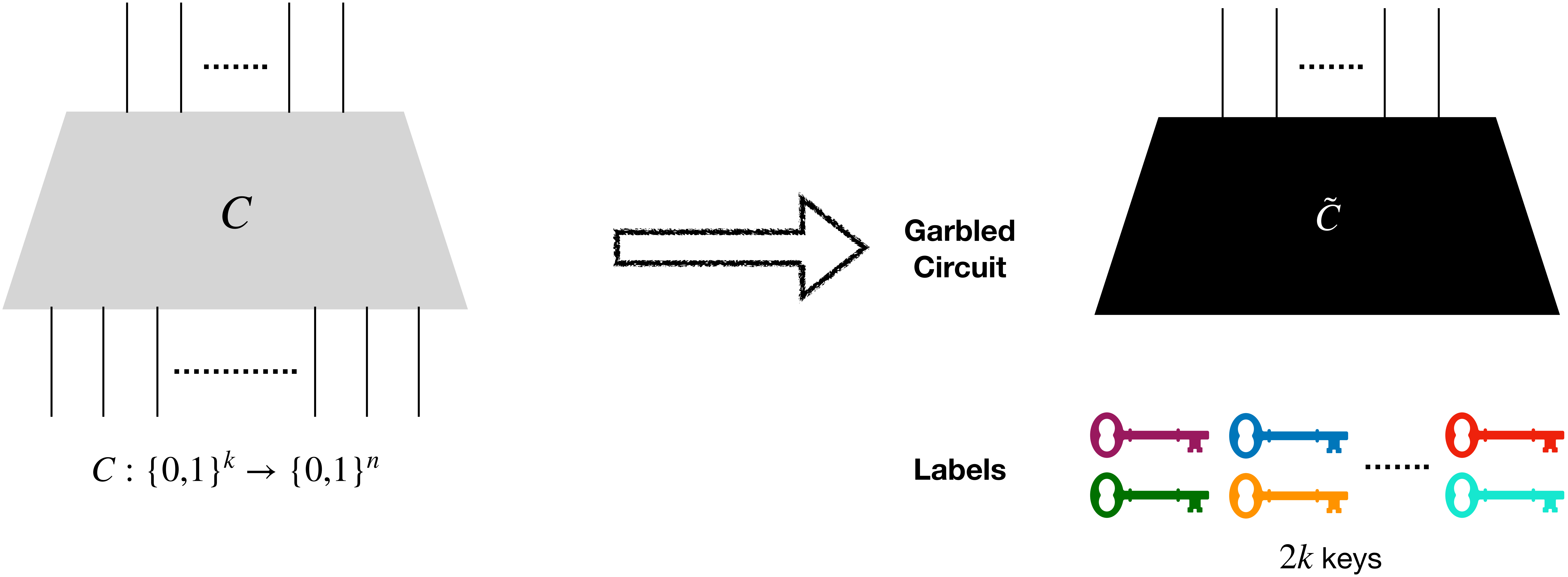

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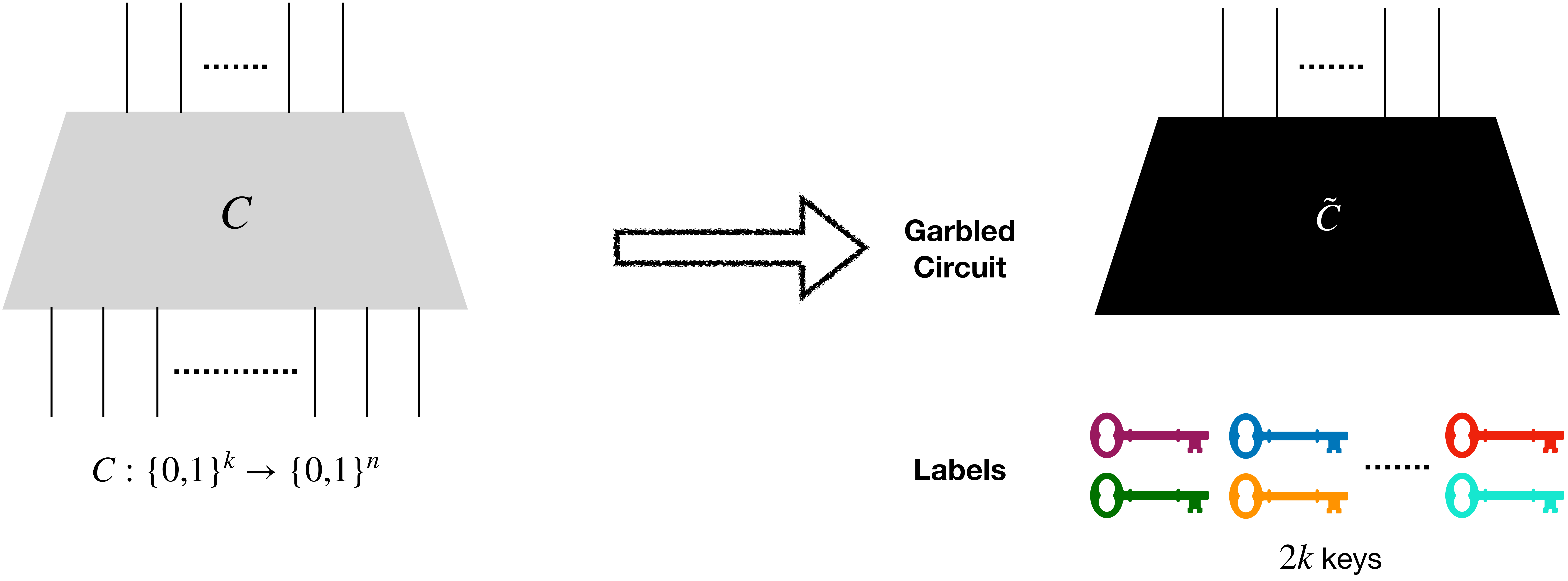

Correctness - For any x, $C(x) = \tilde{C}($ .... $)$

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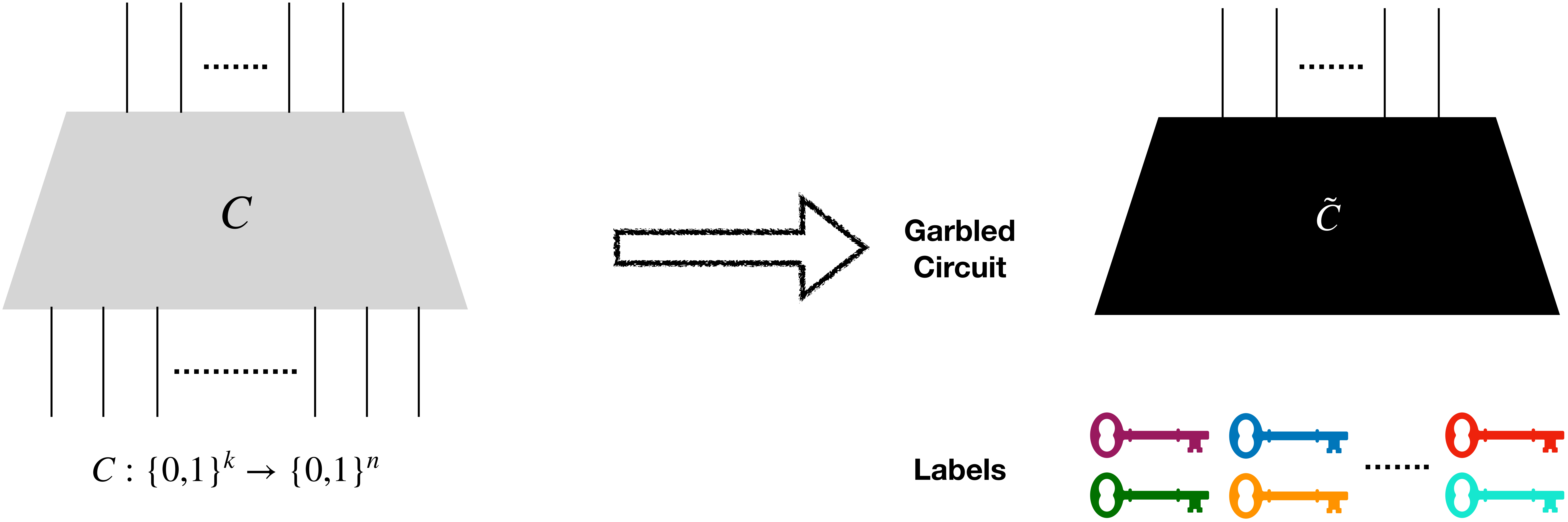

$C : \{0,1\}^k \to \{0,1\}^n$

$2k$ keys

Correctness - For any x, $C(x) = \tilde{C}($ .... $)$

Security - $\tilde{C}$ and $\{$ .... $\}$ reveals $C(x)$ and nothing more.

Setup

[**Döttling-Garg'17**]

Delegatable PRF (S, E, D)

Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)

[**Döttling-Garg'17**]

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$\leftarrow S(1^\lambda)$

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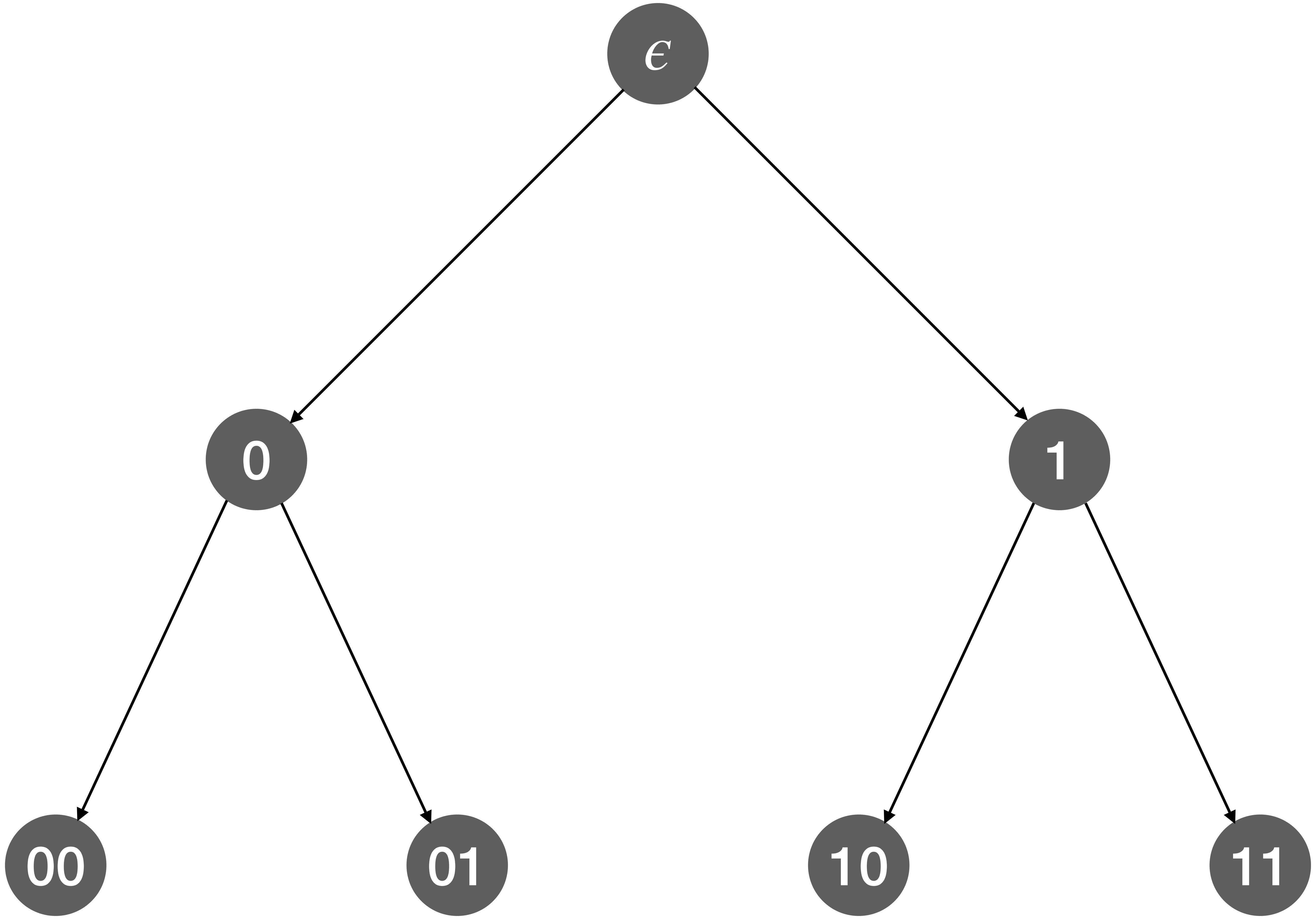

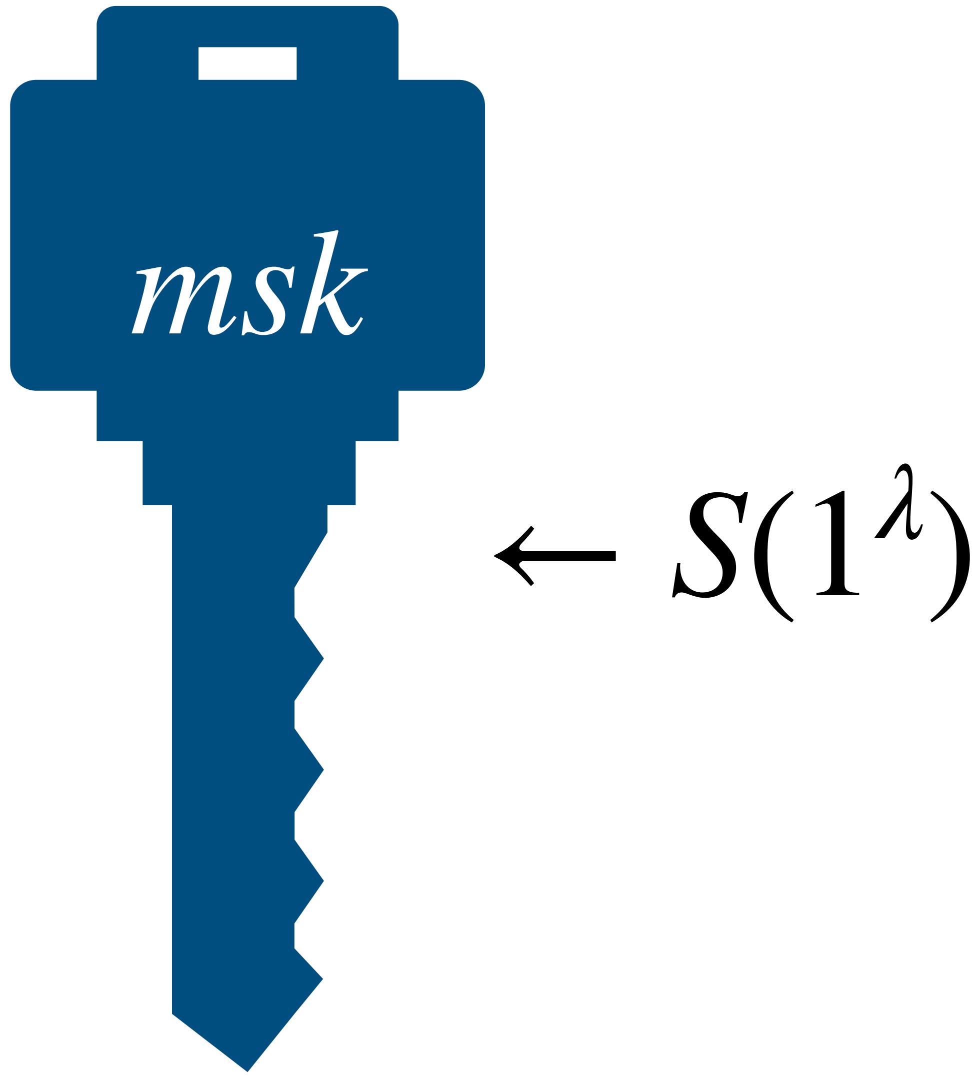

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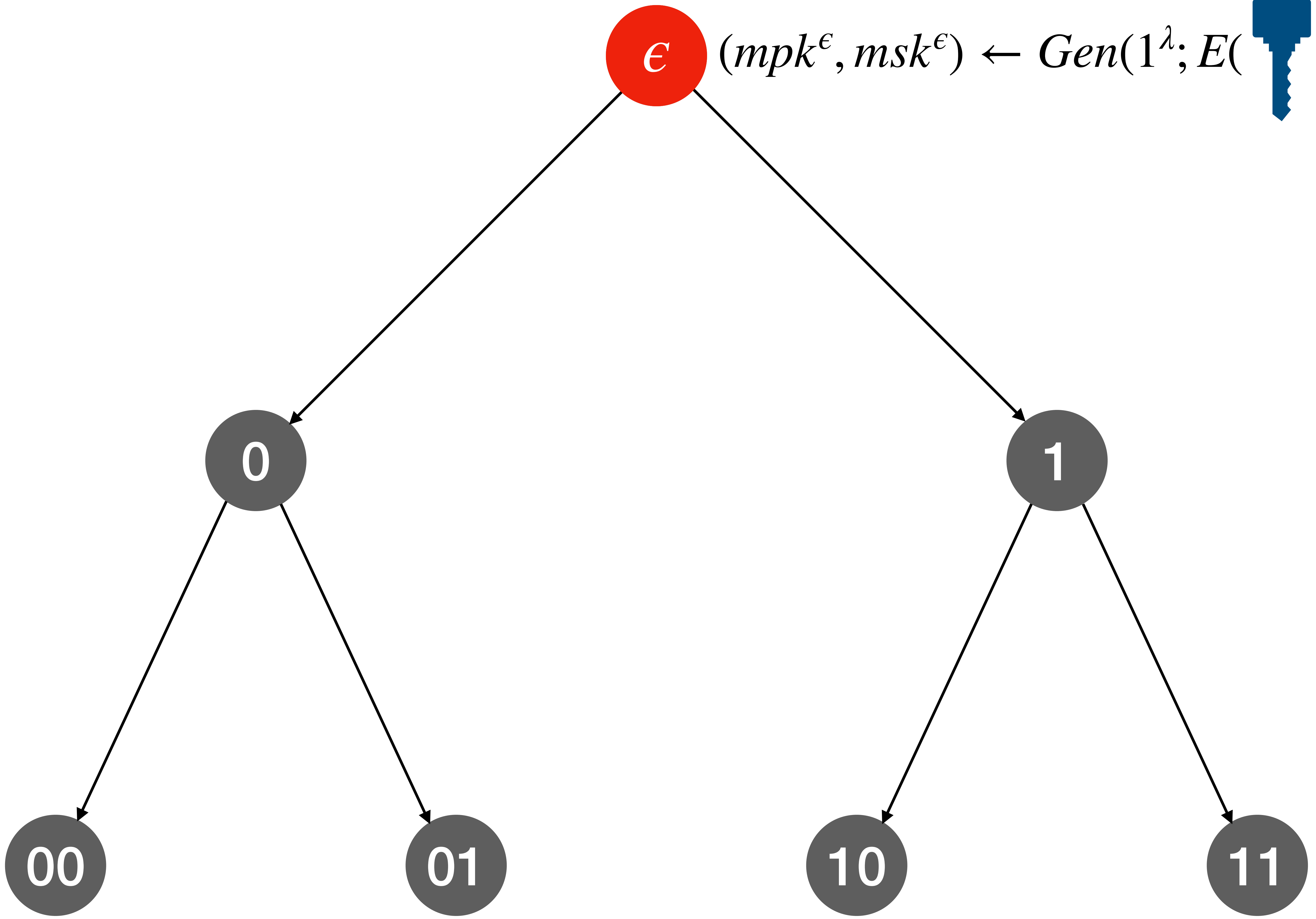

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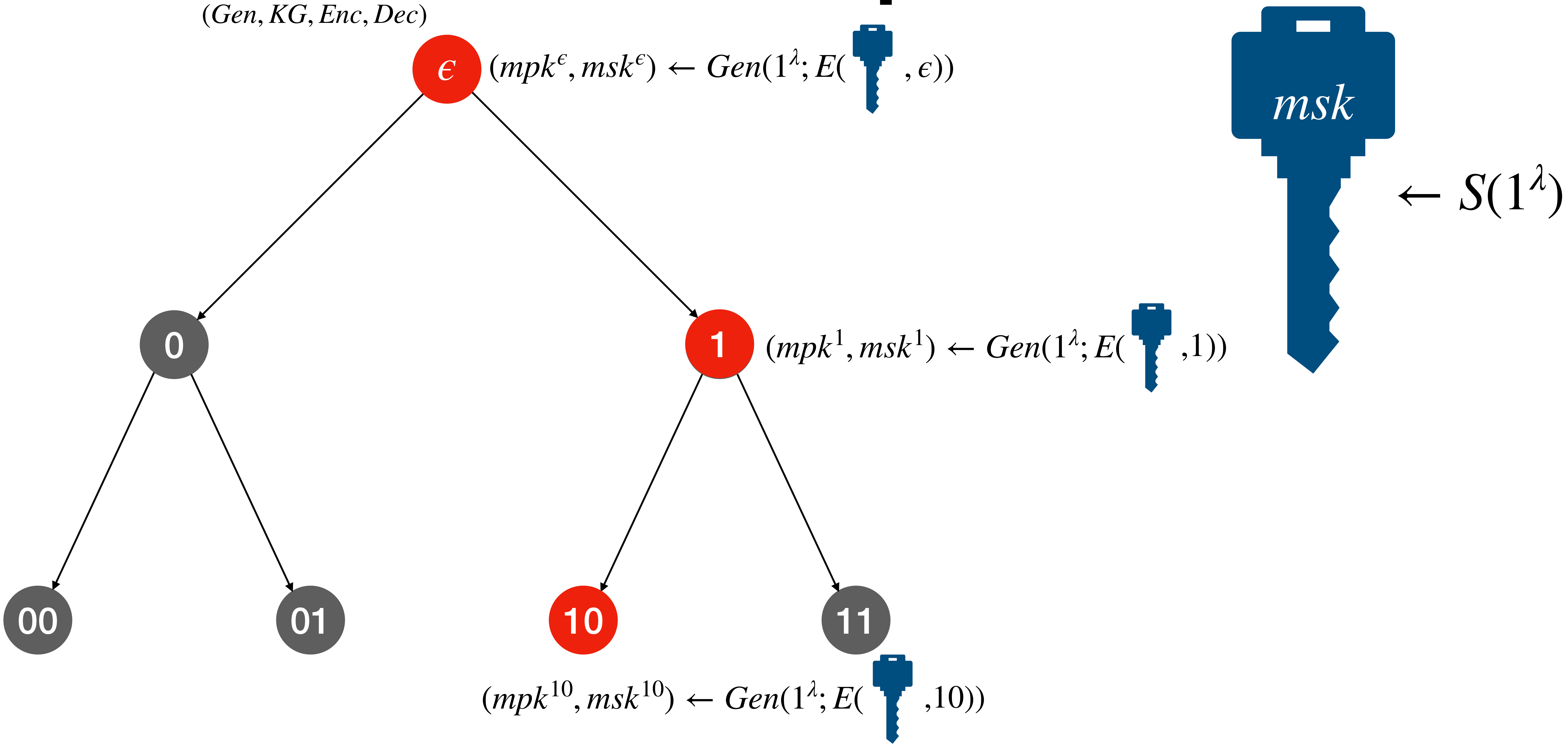

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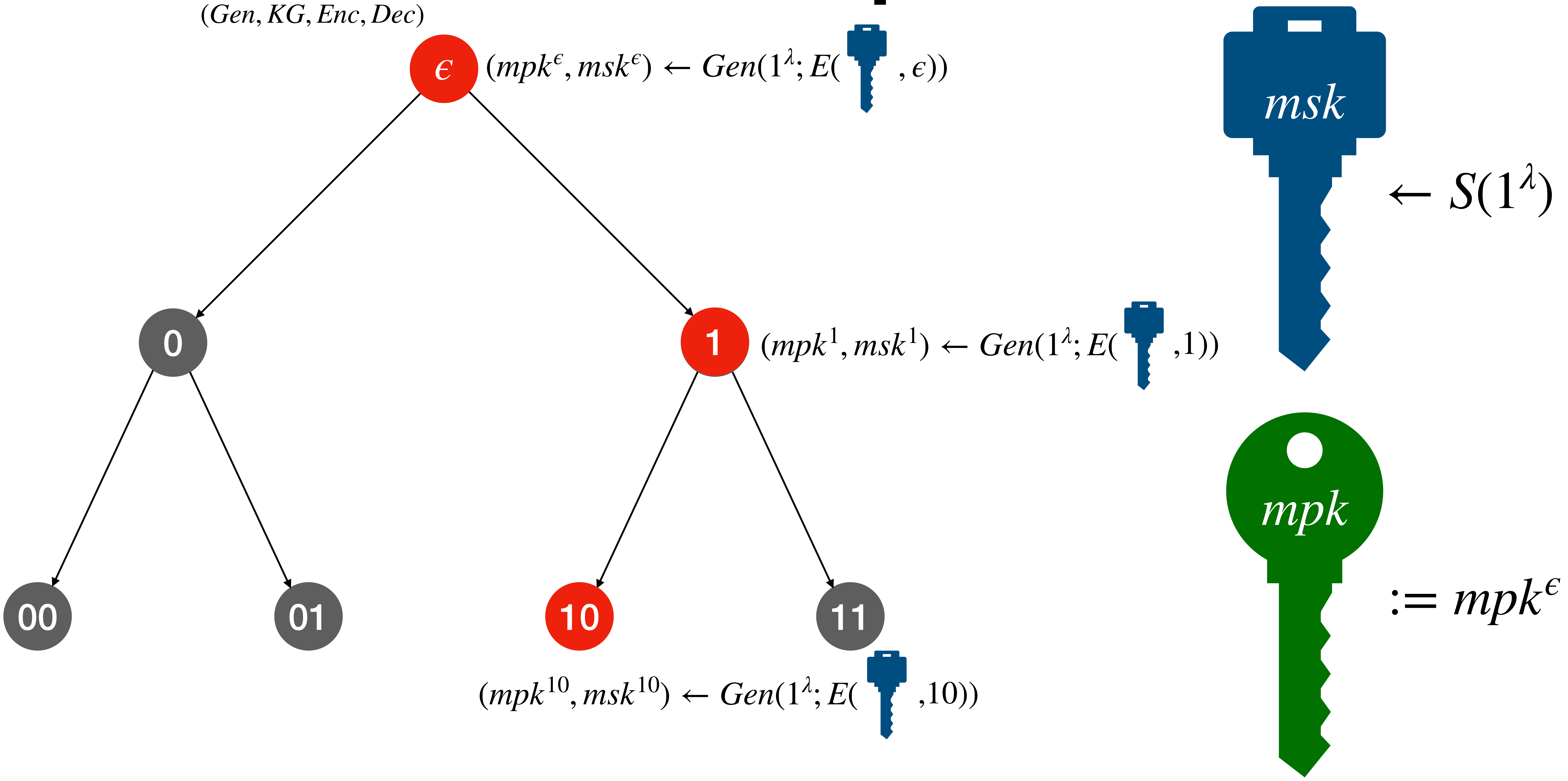

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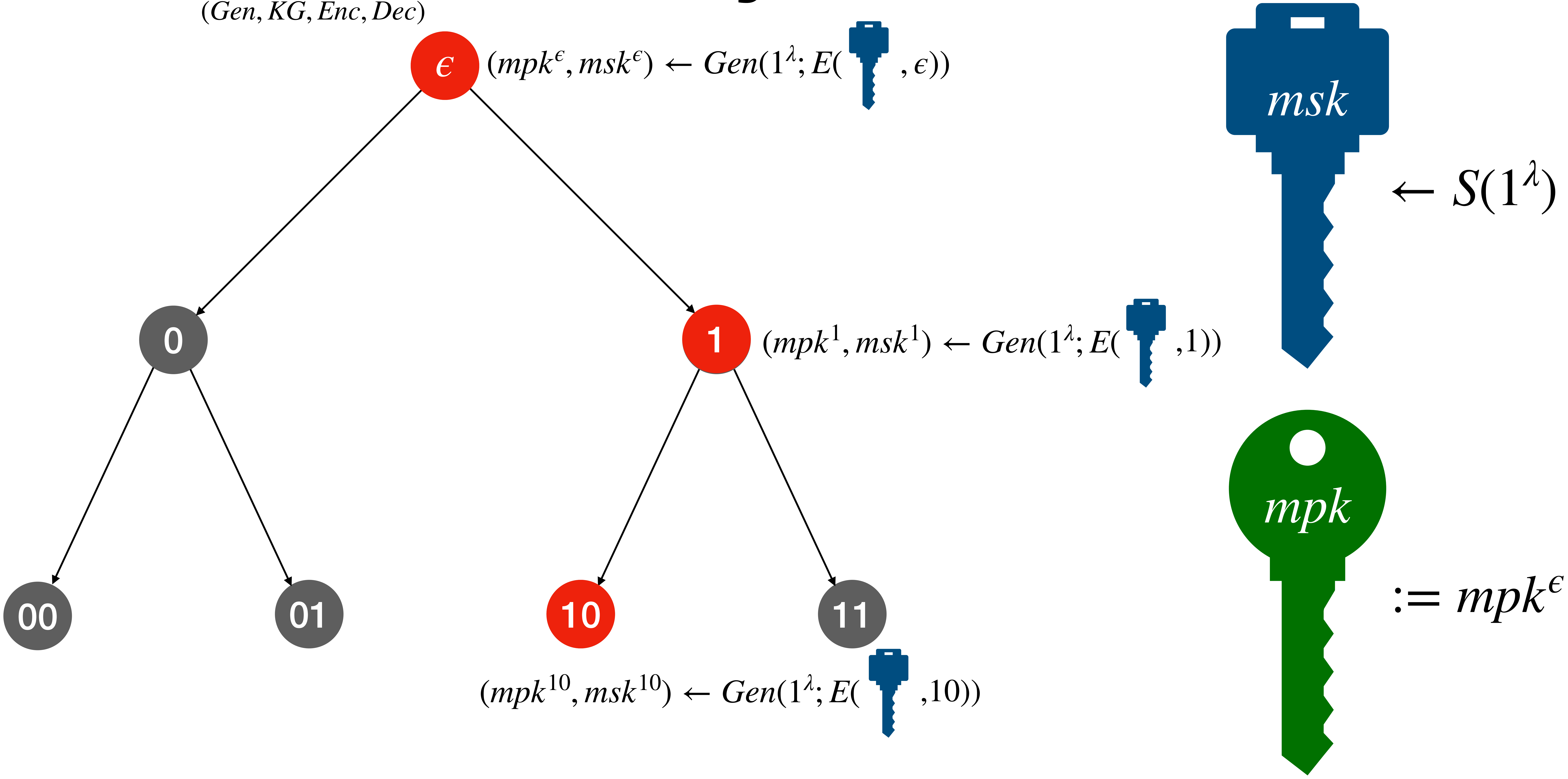
KeyGen



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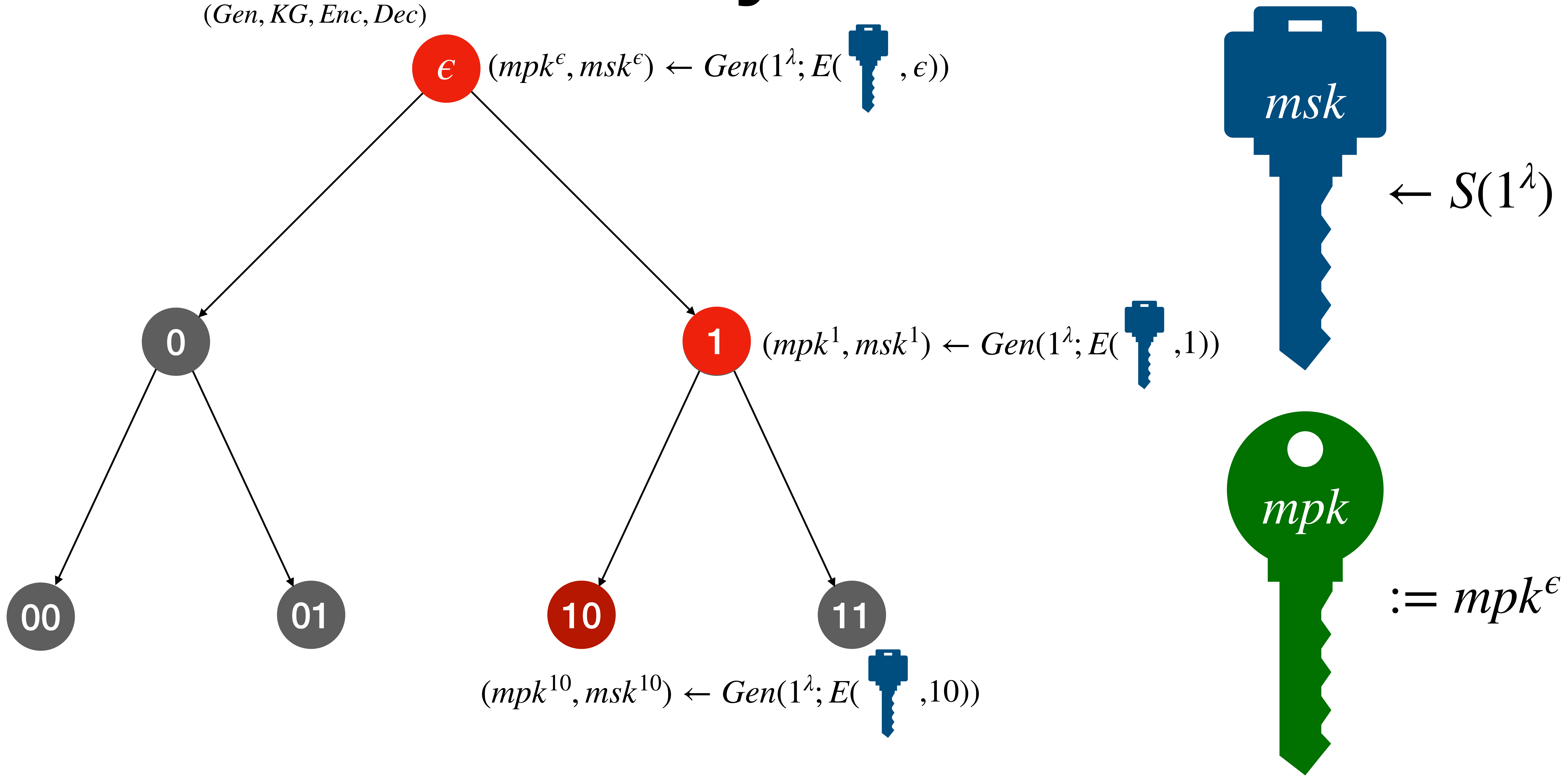

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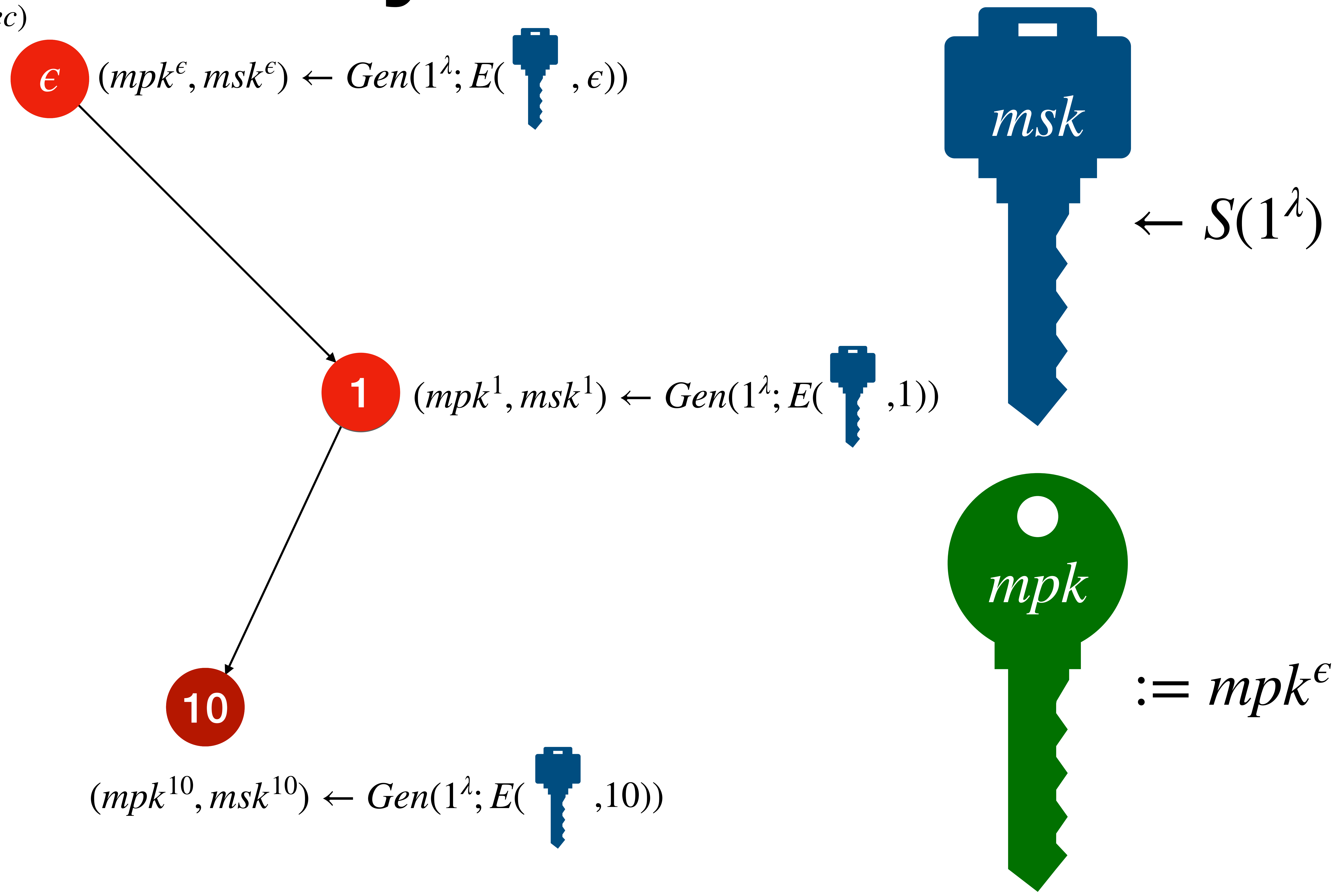

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$(mpk^{10}, msk^{10}) \leftarrow Gen(1^{\lambda}; E($  $, 10))$ 10

$\leftarrow S(1^{\lambda})$

$:= mpk^{\epsilon}$

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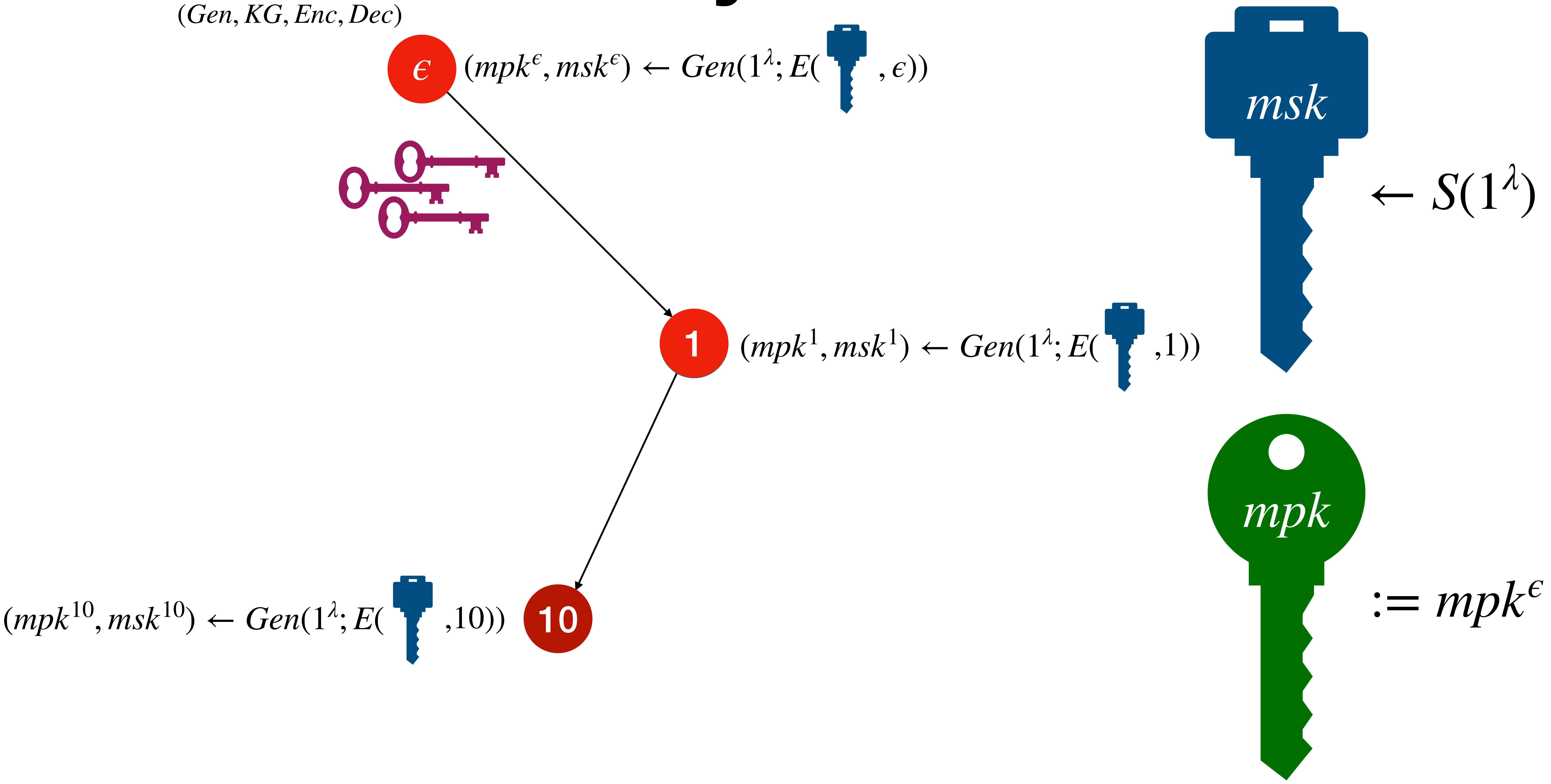

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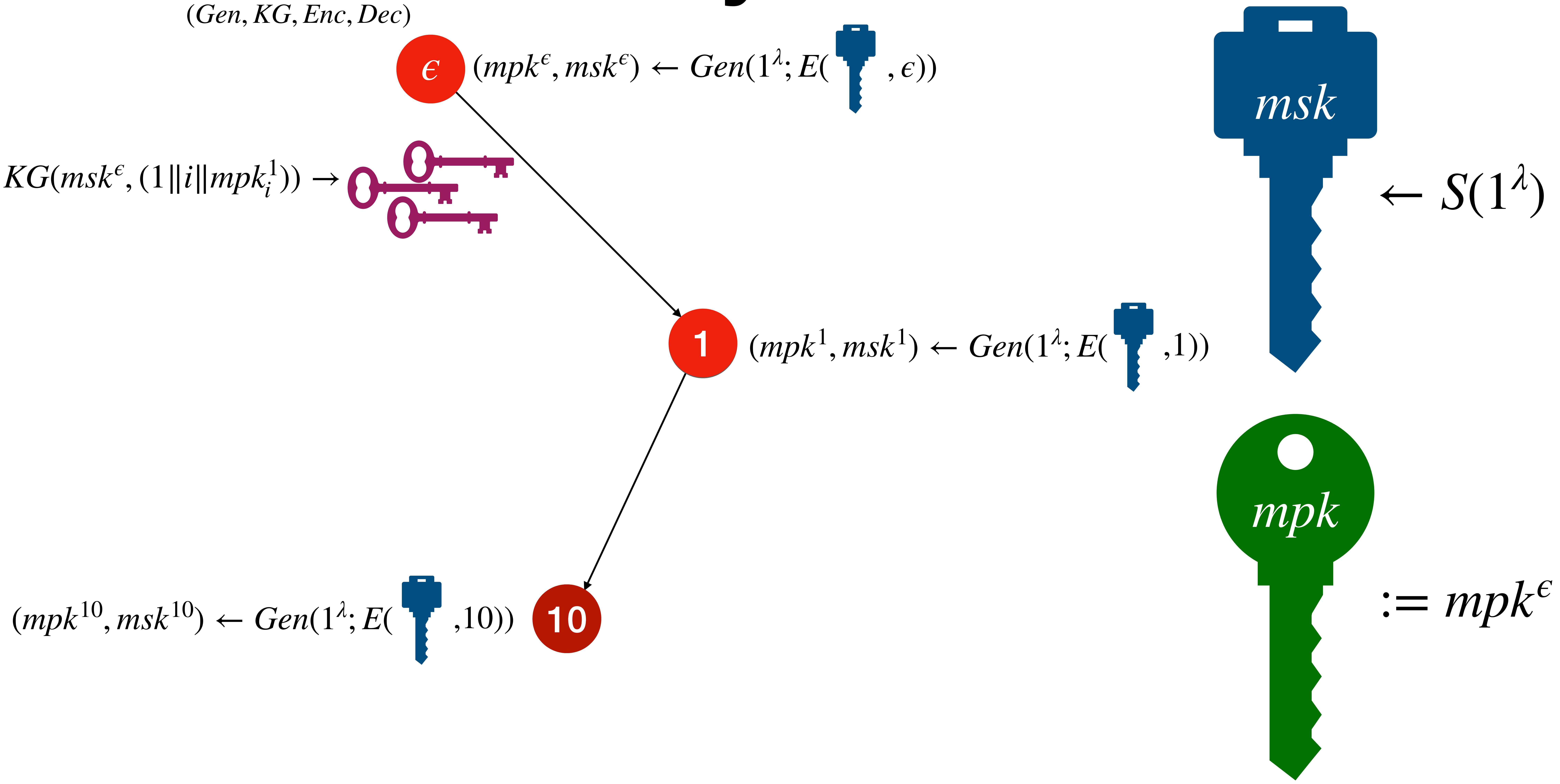

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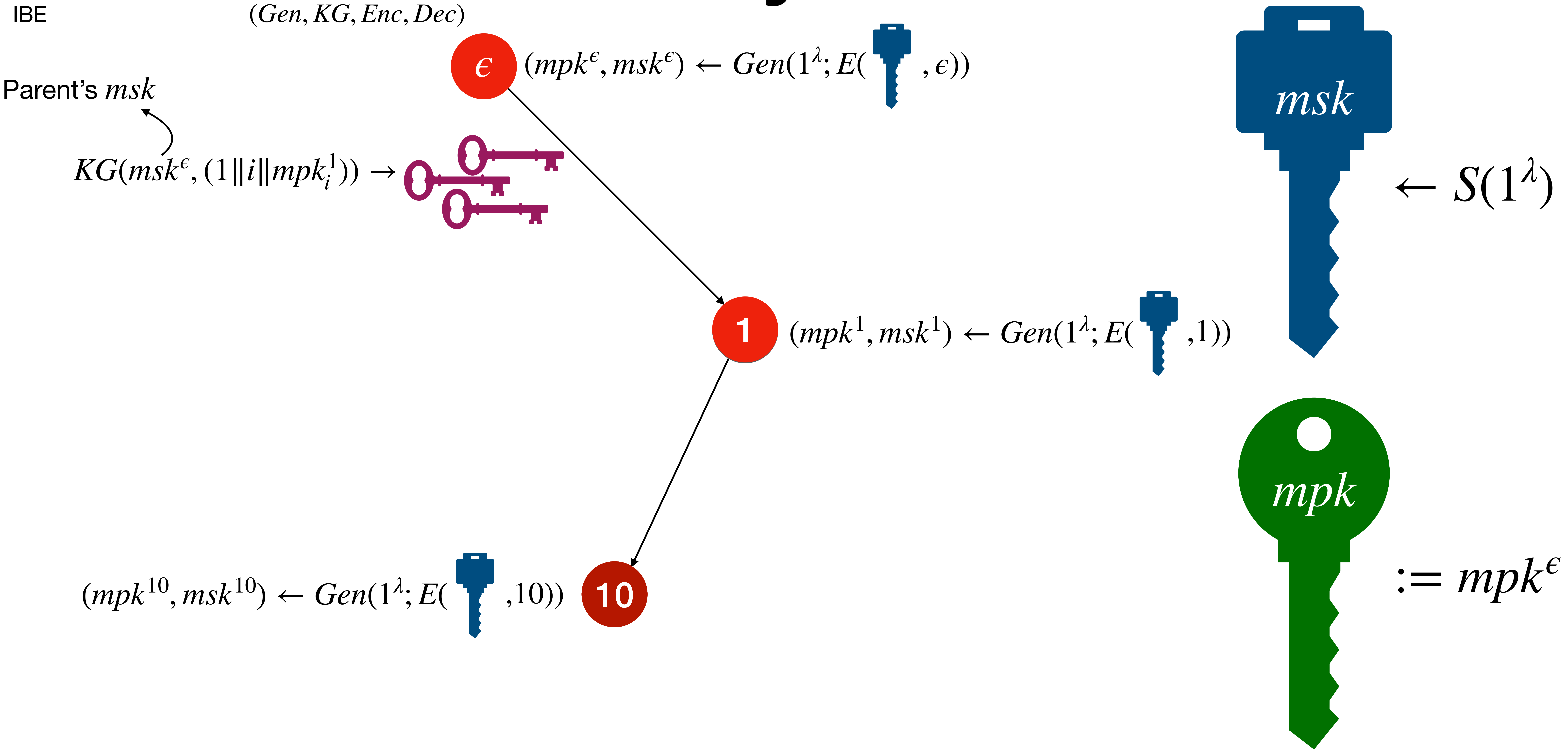

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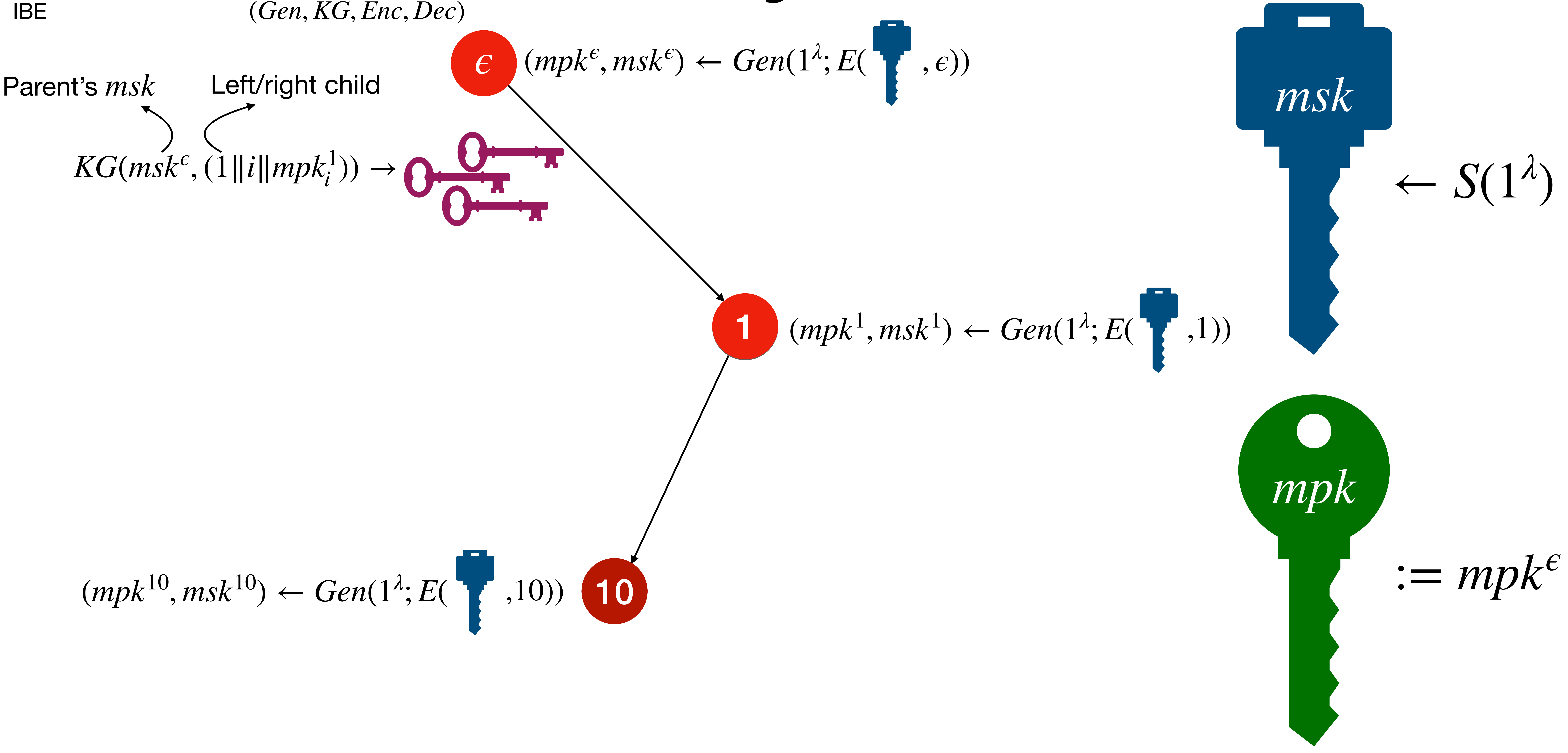

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Parent's msk Left/right child

$KG(msk^\epsilon, (1\|\underbrace{i}_{i^{th} \text{ bit of child's } mpk}\|mpk_i^1)) \to$

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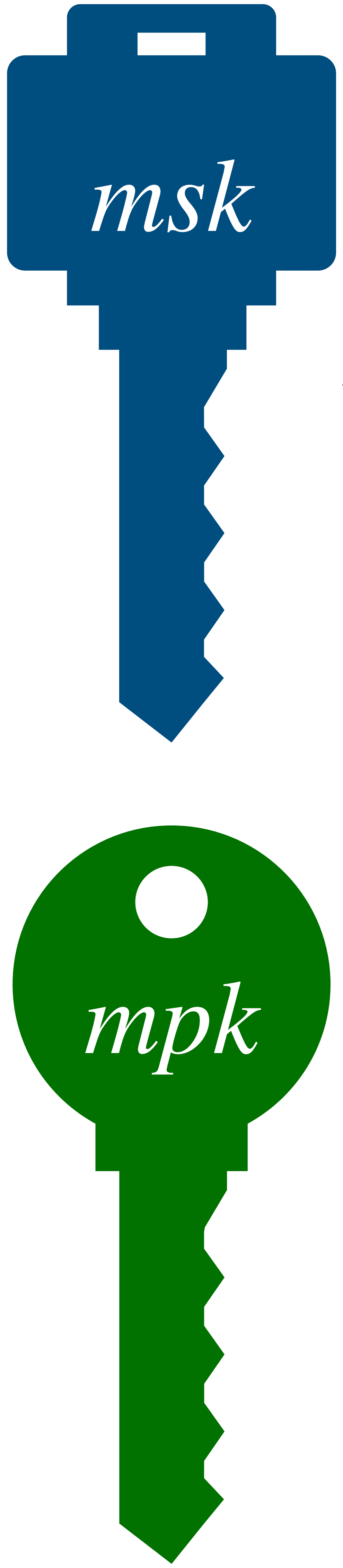

$\leftarrow S(1^\lambda)$

$:= mpk^\epsilon$

KeyGen

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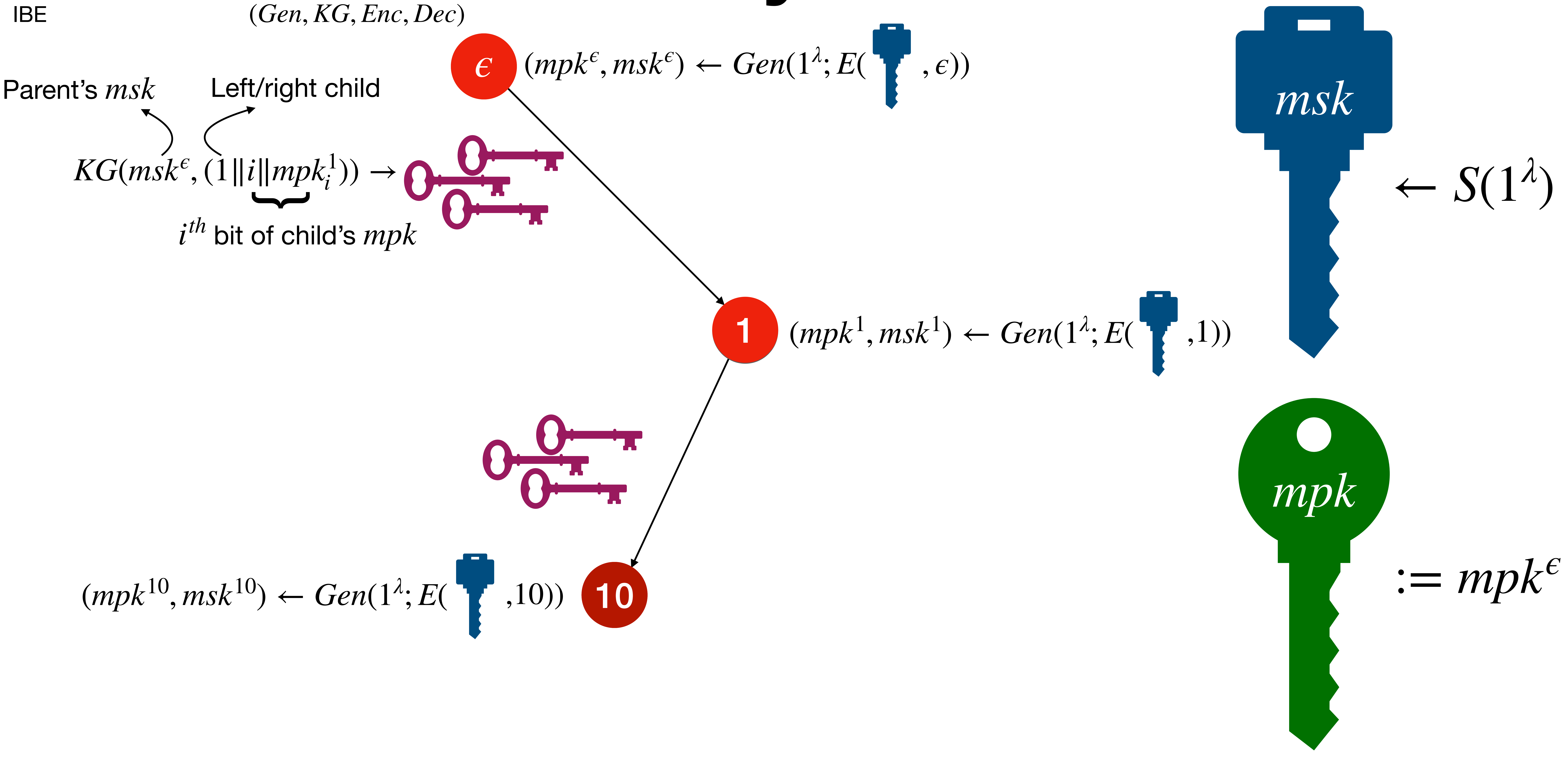

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ϵ $(mpk^{\epsilon}, msk^{\epsilon}) \leftarrow Gen(1^{\lambda}; E(\quad, \epsilon))$

Parent's msk Left/right child

$KG(msk^{\epsilon}, (1 \| \underbrace{i \| mpk_i^1}))\rightarrow$

i^{th} bit of child's mpk

1 $(mpk^1, msk^1) \leftarrow Gen(1^{\lambda}; E(\quad, 1))$

$KG(msk^1, (1 \| i \| mpk_i^{10}))\rightarrow$

$(mpk^{10}, msk^{10}) \leftarrow Gen(1^{\lambda}; E(\quad, 10))$ 10

msk $\leftarrow S(1^{\lambda})$

mpk $:= mpk^{\epsilon}$

KeyGen

[Döttling-Garg'17]

Delegatable PRF	(S, E, D)
Garbling scheme	$(Garb, Eval)$
IBE	(Gen, KG, Enc, Dec)

ϵ $(mpk^\epsilon, msk^\epsilon) \leftarrow Gen(1^\lambda; E(\;, \epsilon))$

Parent's msk Left/right child

$KG(msk^\epsilon, (1\|\underbrace{i\|mpk_i^1}_{i^{th} \text{ bit of child's } mpk})) \rightarrow$

1 $(mpk^1, msk^1) \leftarrow Gen(1^\lambda; E(\;, 1))$

$KG(msk^1, (1\|i\|mpk_i^{10})) \rightarrow$

$(mpk^{10}, msk^{10}) \leftarrow Gen(1^\lambda; E(\;, 10))$ 10

$msk \leftarrow S(1^\lambda)$

$mpk := mpk^\epsilon$

KeyGen

[Döttling-Garg'17]

Delegatable PRF	(S, E, D)
Garbling scheme	$(Garb, Eval)$
IBE	(Gen, KG, Enc, Dec)

ϵ $(mpk^{\epsilon}, msk^{\epsilon}) \leftarrow Gen(1^{\lambda}; E(\quad, \epsilon))$

Parent's msk Left/right child

$KG(msk^{\epsilon}, (1 \| \underbrace{i \|}_{} mpk_i^{1})) \rightarrow$

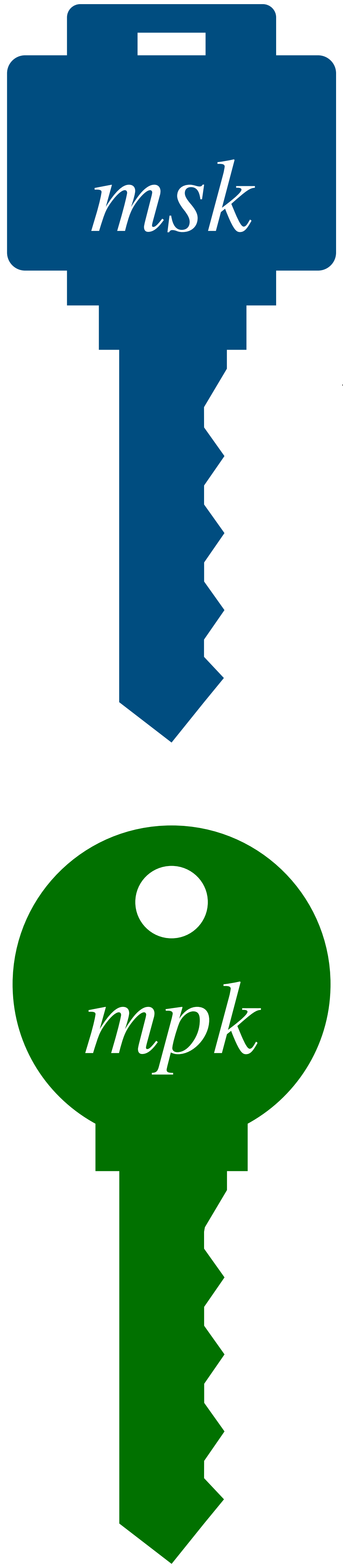
i^{th} bit of child's mpk

1 $(mpk^{1}, msk^{1}) \leftarrow Gen(1^{\lambda}; E(\quad, 1))$

$KG(msk^{1}, (1 \| i \| mpk_i^{10})) \rightarrow$

$(mpk^{10}, msk^{10}) \leftarrow Gen(1^{\lambda}; E(\quad, 10))$ 10

$KG(msk^{10}, \epsilon) \rightarrow$



$\leftarrow S(1^{\lambda})$

$:= mpk^{\epsilon}$

KeyGen

[Döttling-Garg'17]

Delegatable PRF (S, E, D)

Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)

ϵ $(mpk^{\epsilon}, msk^{\epsilon}) \leftarrow Gen(1^{\lambda}; E(\quad, \epsilon))$

Parent's msk

Left/right child

$KG(msk^{\epsilon}, (1 \| \underbrace{i}_{i^{th} \text{ bit of child's } mpk} \| mpk_i^1)) \rightarrow$

1 $(mpk^1, msk^1) \leftarrow Gen(1^{\lambda}; E(\quad, 1))$

$KG(msk^1, (1 \| i \| mpk_i^{10})) \rightarrow$

$(mpk^{10}, msk^{10}) \leftarrow Gen(1^{\lambda}; E(\quad, 10))$ 10

$KG(msk^{10}, \epsilon) \rightarrow$

$msk \leftarrow S(1^{\lambda})$

$mpk := mpk^{\epsilon}$

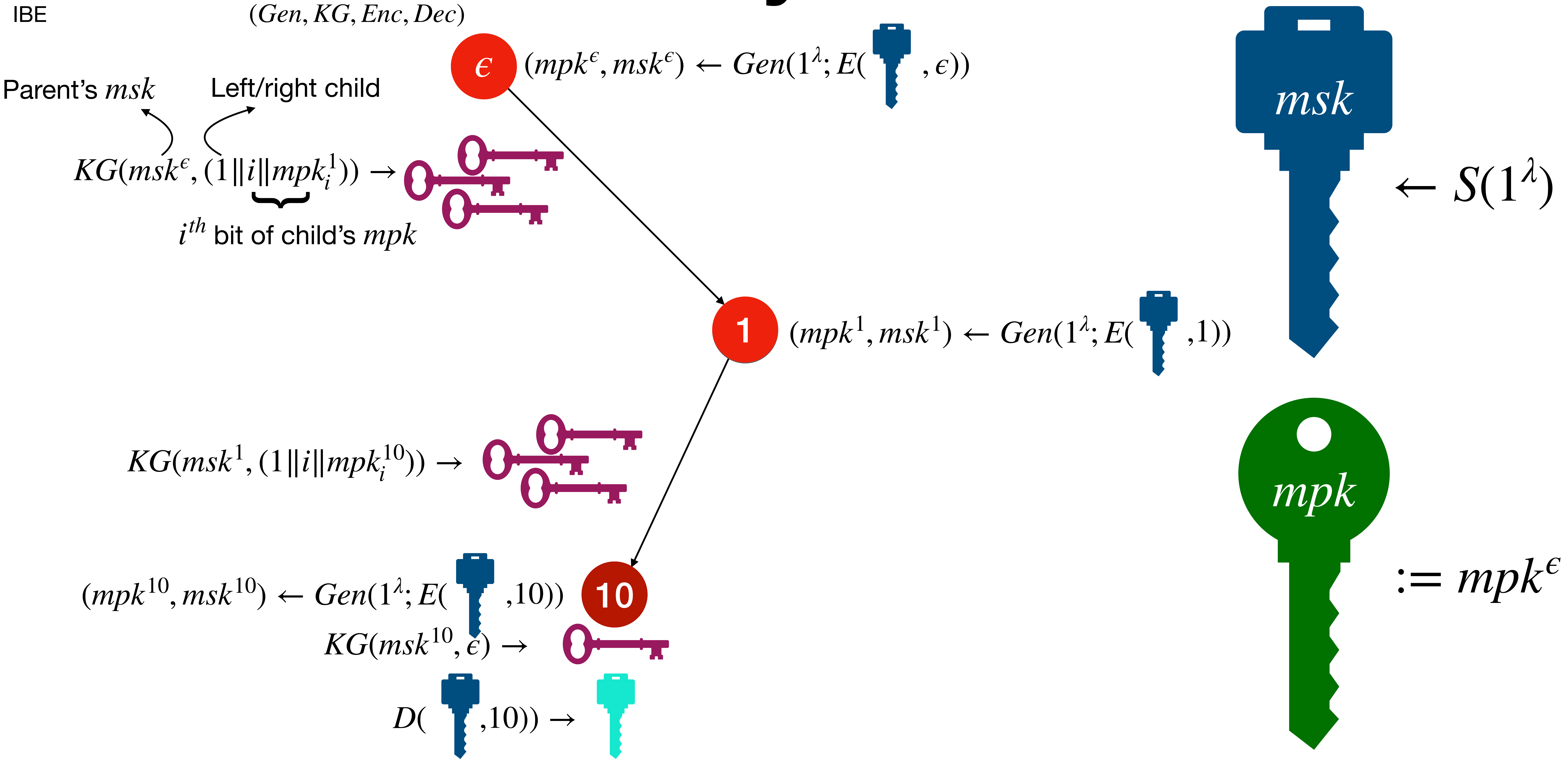
KeyGen

[Döttling-Garg'17]

Delegatable PRF (S, E, D)

Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)



Encryption

[**Döttling-Garg'17**]

Delegatable PRF	(S, E, D)
Garbling scheme	$(Garb, Eval)$
IBE	(Gen, KG, Enc, Dec)

[**Döttling-Garg'17**]

Delegatable PRF (S, E, D)

Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)

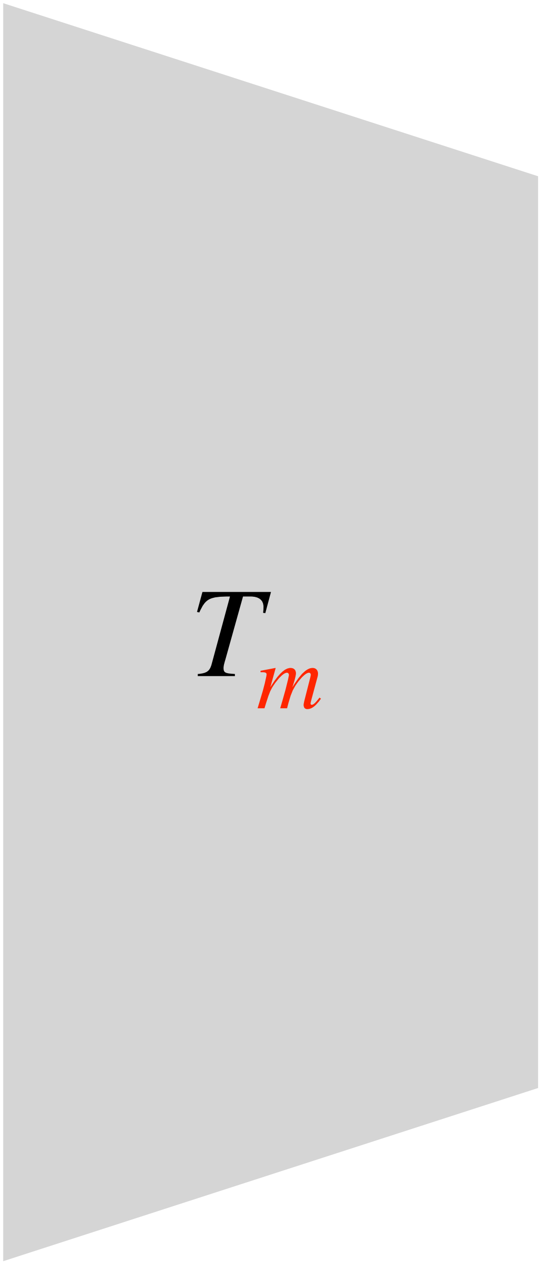
Encryption

 $:= mpk^{\epsilon}$

[Döttling-Garg'17]

Delegatable PRF	(S, E, D)
Garbling scheme	$(Garb, Eval)$
IBE	(Gen, KG, Enc, Dec)

Encryption

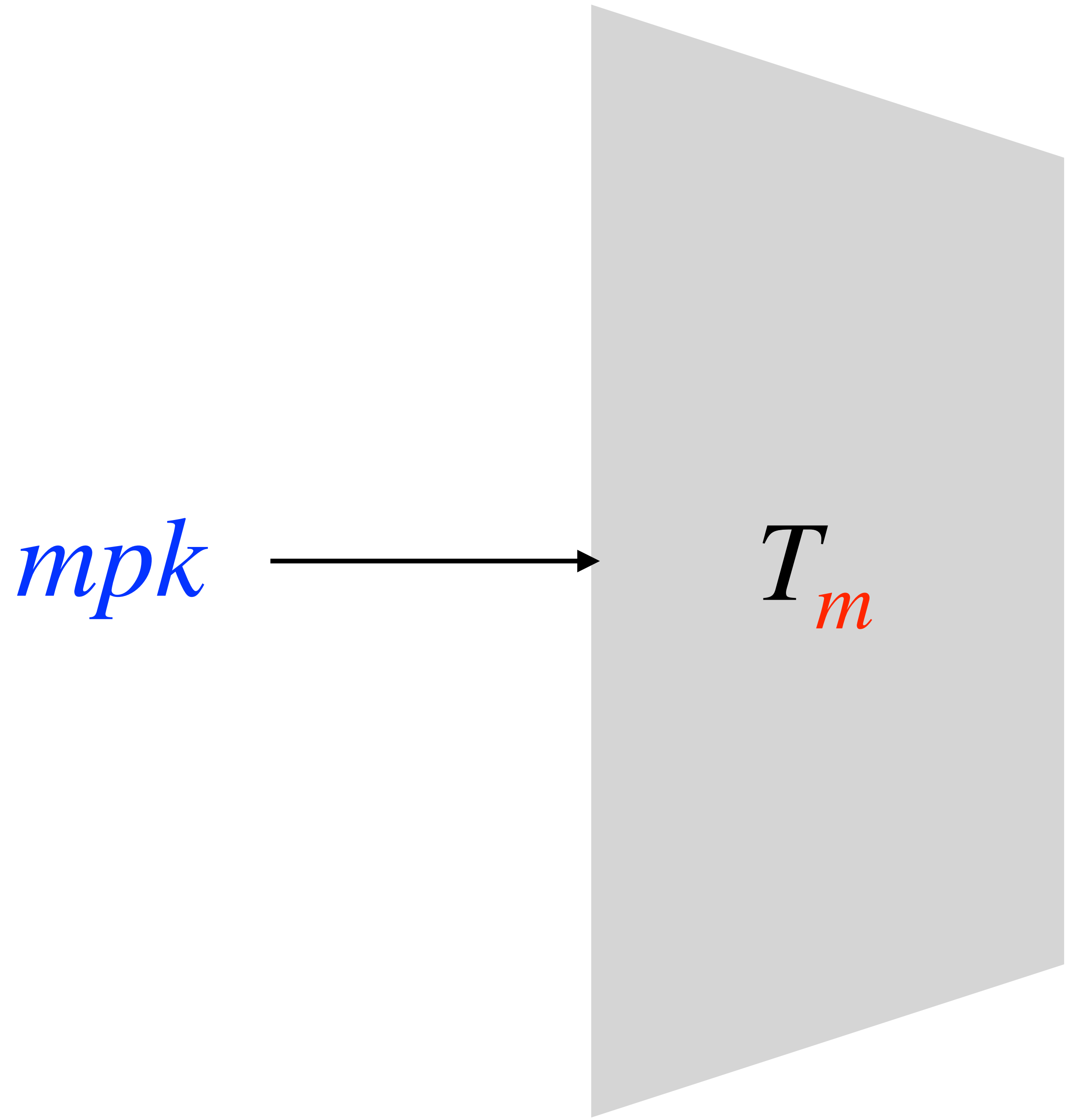


 $:= mpk^{\epsilon}$

[**Döttling-Garg’17**]

Delegatable PRF	(S, E, D)
Garbling scheme	$(Garb, Eval)$
IBE	(Gen, KG, Enc, Dec)

Encryption



[Döttling-Garg'17]

Delegatable PRF	(S, E, D)
Garbling scheme	$(Garb, Eval)$
IBE	(Gen, KG, Enc, Dec)

Encryption

$$mpk \longrightarrow T_m \longrightarrow Enc(mpk, \epsilon, m)$$

 $:= mpk^{\epsilon}$

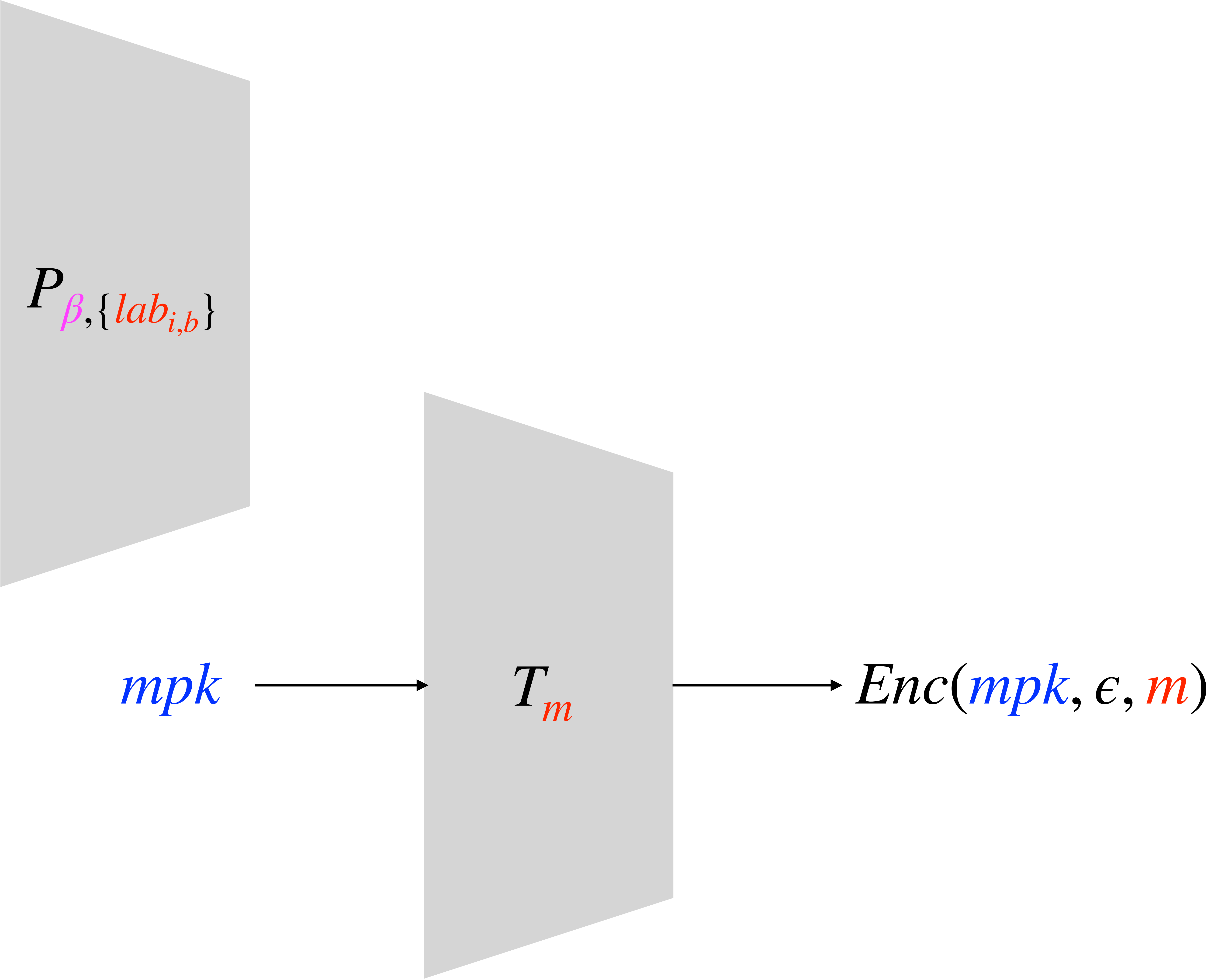
[**Döttling-Garg’17**]

Delegatable PRF (S, E, D)

Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)

Encryption

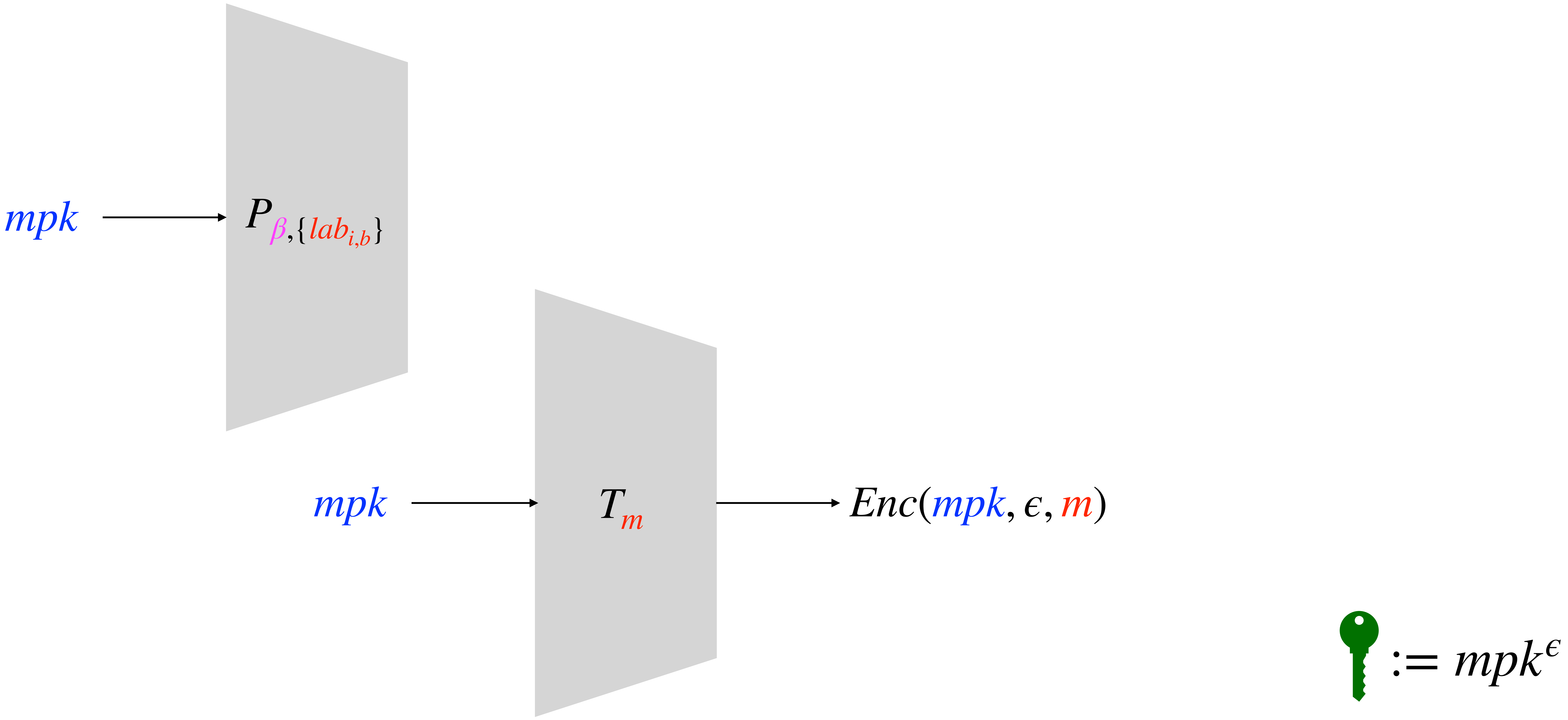


$:= mpk^{\epsilon}$

[**Döttling-Garg'17**]

Delegatable PRF	(S, E, D)
Garbling scheme	$(Garb, Eval)$
IBE	(Gen, KG, Enc, Dec)

Encryption



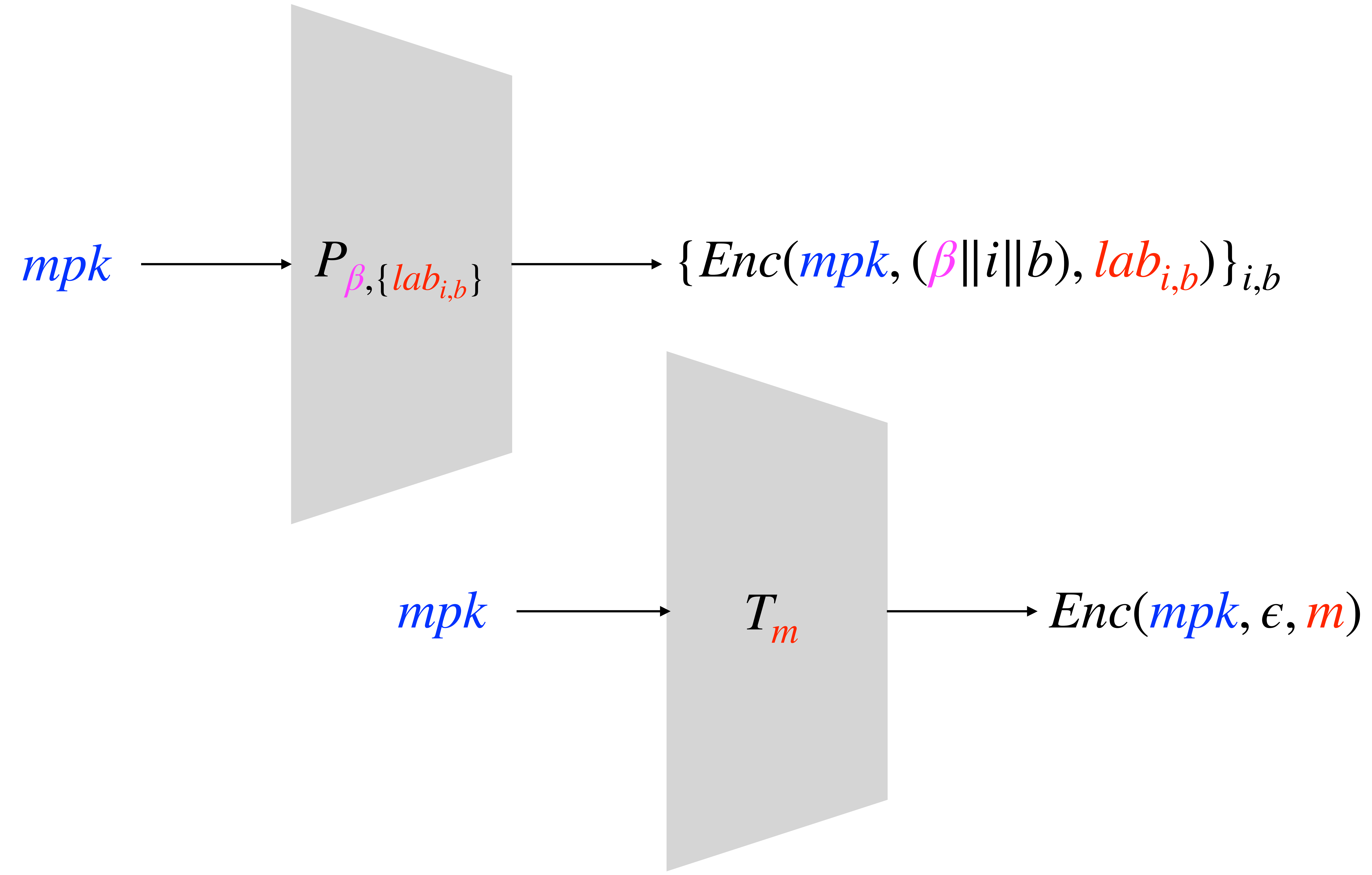
[**Döttling-Garg'17**]

Delegatable PRF (S, E, D)

Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)

Encryption



 $:= mpk^{\epsilon}$

[Döttling-Garg'17]

Delegatable PRF (S, E, D)

Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)

Encryption

$$mpk \longrightarrow P_{\beta,\{lab_{i,b}\}} \longrightarrow \{Enc(mpk, (\beta \| i \| b), lab_{i,b})\}_{i,b}$$

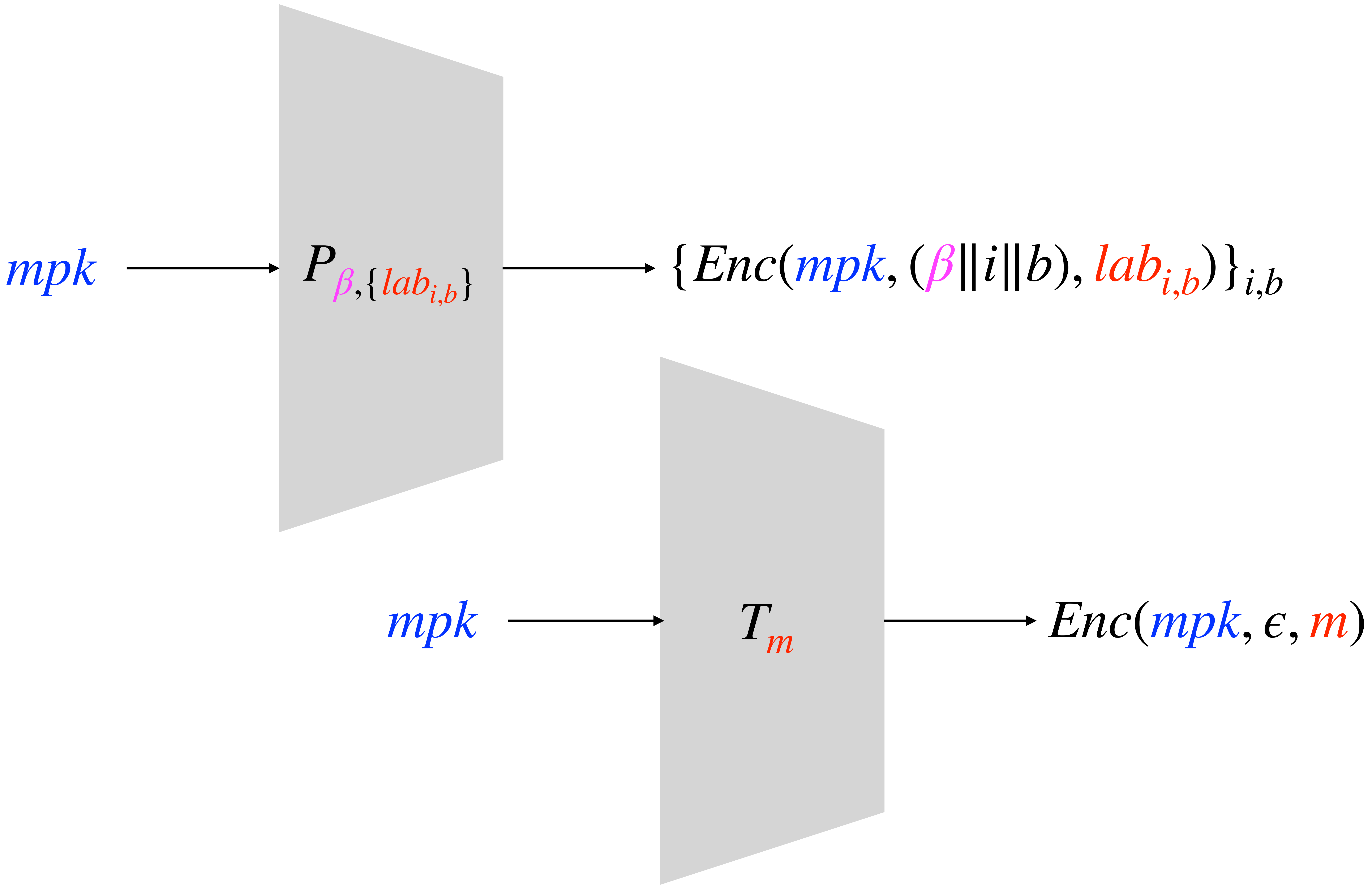
$$mpk \longrightarrow T_m \longrightarrow Enc(mpk, \epsilon, m)$$

$id = 10,$  $:= mpk^{\epsilon}$

Encryption

[Döttling-Garg'17]

Delegatable PRF	(S, E, D)
Garbling scheme	$(Garb, Eval)$
IBE	(Gen, KG, Enc, Dec)



$\tilde{T}, \{lab_{i,b}\} \leftarrow Garb(T_m)$

$id = 10,$  $:= mpk^{\epsilon}$

[**Döttling-Garg'17**]

Delegatable PRF	(S, E, D)
Garbling scheme	$(Garb, Eval)$
IBE	(Gen, KG, Enc, Dec)

Encryption

$mpk \longrightarrow P_{\beta,\{lab_{i,b}\}} \longrightarrow \{Enc(mpk, (\beta \| i \| b), lab_{i,b})\}_{i,b}$

$mpk \longrightarrow T_m \longrightarrow Enc(mpk, \epsilon, m)$

$\tilde{T}, \{lab_{i,b}\} \leftarrow Garb(T_m)$

$id = 10,$  $:= mpk^{\epsilon}$

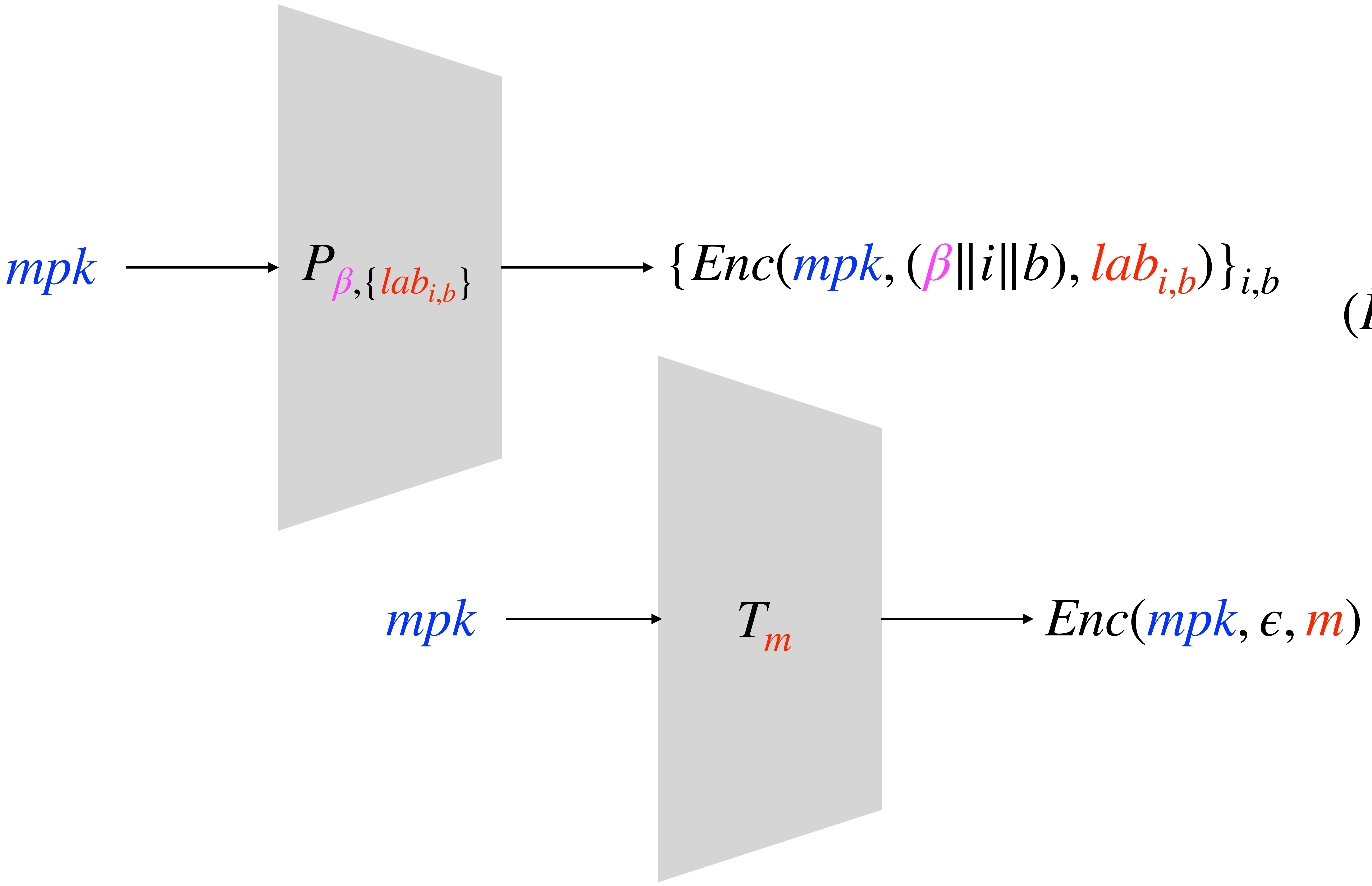
[**Döttling-Garg’17**]

Delegatable PRF (S, E, D)

Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)

Encryption



$(\tilde{P}, Lab_{i,b}) \leftarrow Garb(P_{'0',\{lab_{i,b}\}})$

$\tilde{T}, \{lab_{i,b}\} \leftarrow Garb(T_m)$

$id = 10,$  $:= mpk^{\epsilon}$

[**Döttling-Garg'17**]

Delegatable PRF (S, E, D)

Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)

Encryption

$mpk \longrightarrow P_{\beta,\{lab_{i,b}\}} \longrightarrow \{Enc(mpk, (\beta \| i \| b), lab_{i,b})\}_{i,b}$

$(\tilde{P}, Lab_{i,b}) \leftarrow Garb(P_{'0',\{lab_{i,b}\}})$

$mpk \longrightarrow T_m \longrightarrow Enc(mpk, \epsilon, m)$

$\tilde{T}, \{lab_{i,b}\} \leftarrow Garb(T_m)$

$id = 10,$ $:= mpk^{\epsilon}$

[**Döttling-Garg'17**]

Delegatable PRF (S, E, D)

Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)

Encryption

$(\tilde{Q}, LAB_{i,b}) \leftarrow Garb(P_{'1', \{Lab_{i,b}\}})$

$mpk \longrightarrow P_{\beta, \{lab_{i,b}\}} \longrightarrow \{Enc(mpk, (\beta \| i \| b), lab_{i,b})\}_{i,b}$

$(\tilde{P}, Lab_{i,b}) \leftarrow Garb(P_{'0', \{lab_{i,b}\}})$

$mpk \longrightarrow T_m \longrightarrow Enc(mpk, \epsilon, m)$

$\tilde{T}, \{lab_{i,b}\} \leftarrow Garb(T_m)$

$id = 10,$  $:= mpk^{\epsilon}$

[**Döttling-Garg'17**]

Delegatable PRF (S, E, D)

Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)

Encryption

$Return(\tilde{T}, \tilde{P}, \tilde{Q}, \{LAB_{i,mpk_i^{\epsilon}}\})$

$(\tilde{Q}, LAB_{i,b}) \leftarrow Garb(P_{'1',\{Lab_{i,b}\}})$

$mpk \longrightarrow P_{\beta,\{lab_{i,b}\}} \longrightarrow \{Enc(mpk, (\beta\|i\|b), lab_{i,b})\}_{i,b}$

$(\tilde{P}, Lab_{i,b}) \leftarrow Garb(P_{'0',\{lab_{i,b}\}})$

$mpk \longrightarrow T_m \longrightarrow Enc(mpk, \epsilon, m)$

$\tilde{T}, \{lab_{i,b}\} \leftarrow Garb(T_m)$

$id = 10,$ $:= mpk^{\epsilon}$

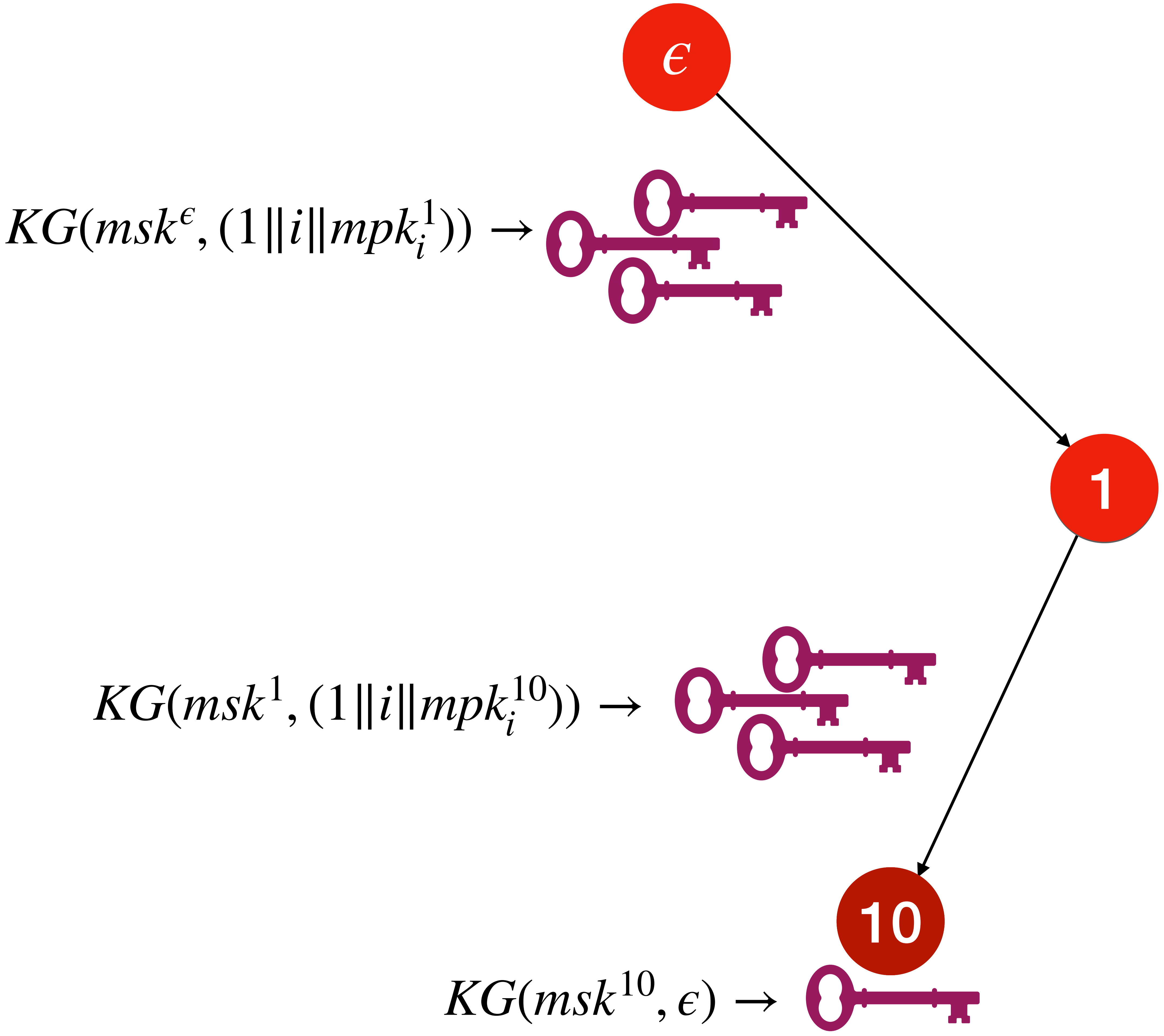
[**Döttling-Garg'17**]

Delegatable PRF (S, E, D)

Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)

Decryption

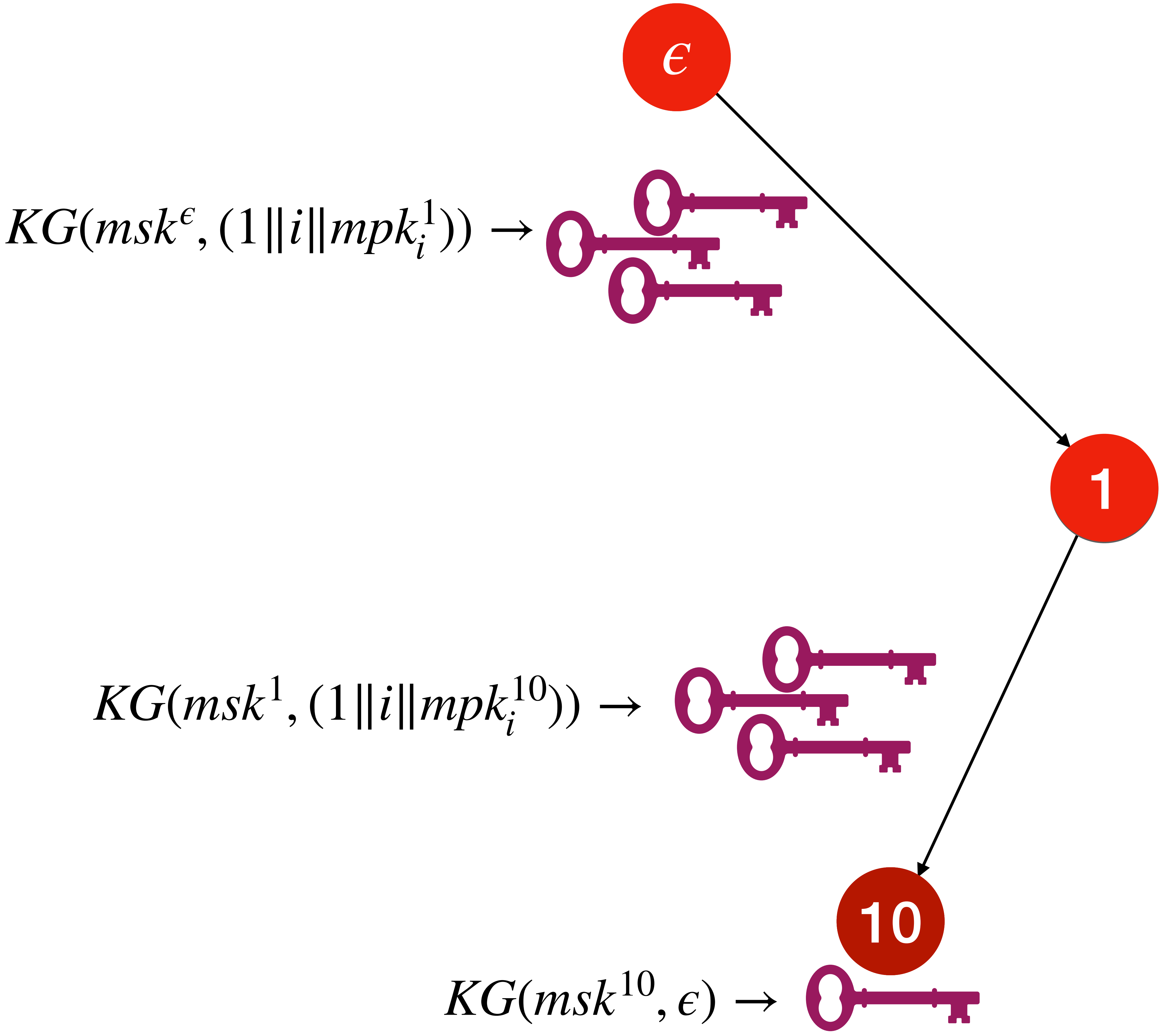


[**Döttling-Garg’17**]

Delegatable PRF	(S, E, D)
Garbling scheme	$(Garb, Eval)$
IBE	(Gen, KG, Enc, Dec)

Decryption

$(\tilde{T}, \tilde{P}, \tilde{Q}, \{LAB_{i,mpk_i^{\epsilon}}\})$

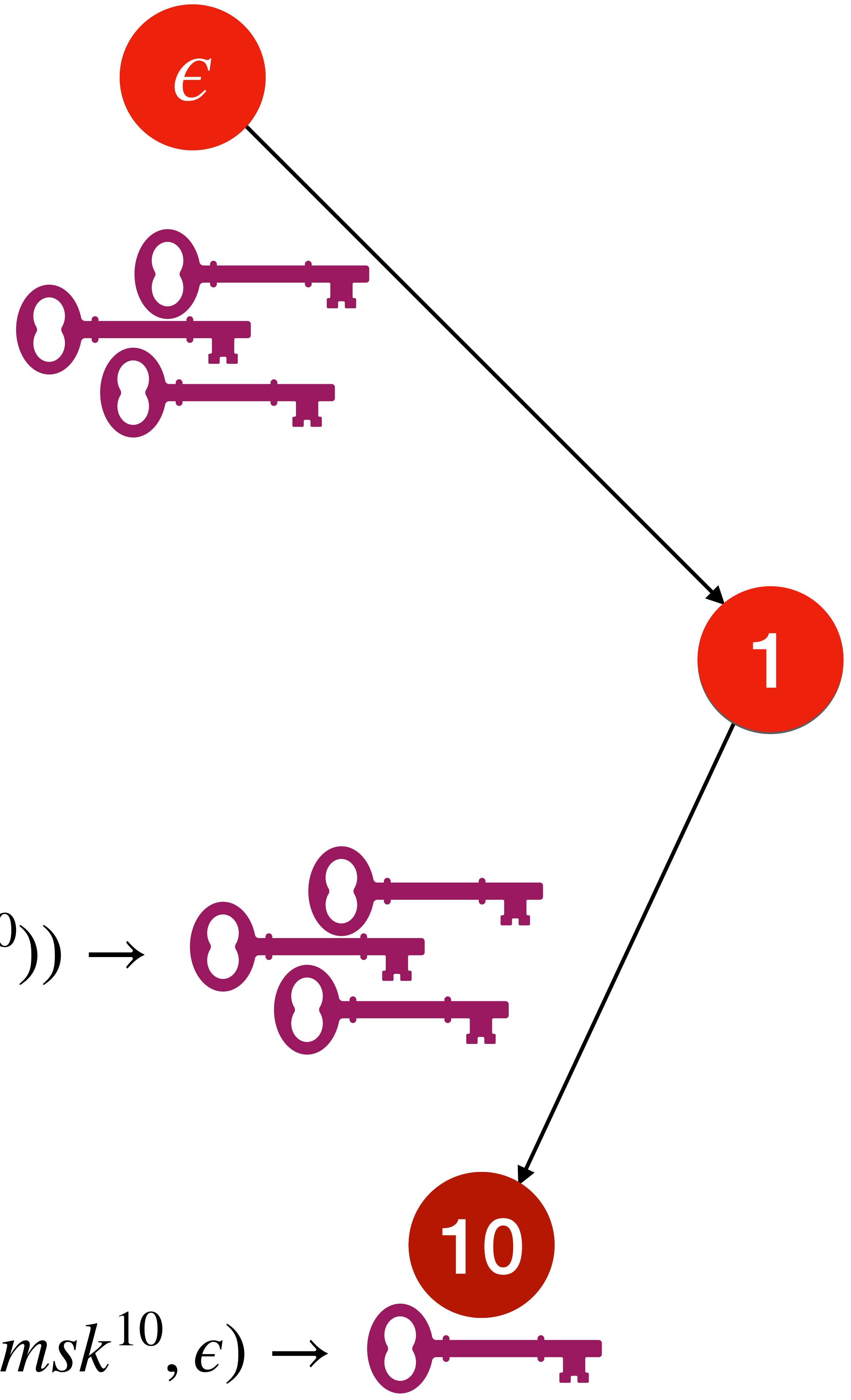


[Döttling-Garg'17]

Delegatable PRF	(S, E, D)
Garbling scheme	$(Garb, Eval)$
IBE	(Gen, KG, Enc, Dec)

Decryption

$(\tilde{T}, \tilde{P}, \tilde{Q}, \{LAB_{i,mpk_i^{\epsilon}}\})$



$KG(msk^{\epsilon}, (1\|i\|mpk_i^1)) \rightarrow$

$KG(msk^{1}, (1\|i\|mpk_i^{10})) \rightarrow$

$KG(msk^{10}, \epsilon) \rightarrow$

[**Döttling-Garg’17**]

Delegatable PRF (S, E, D)

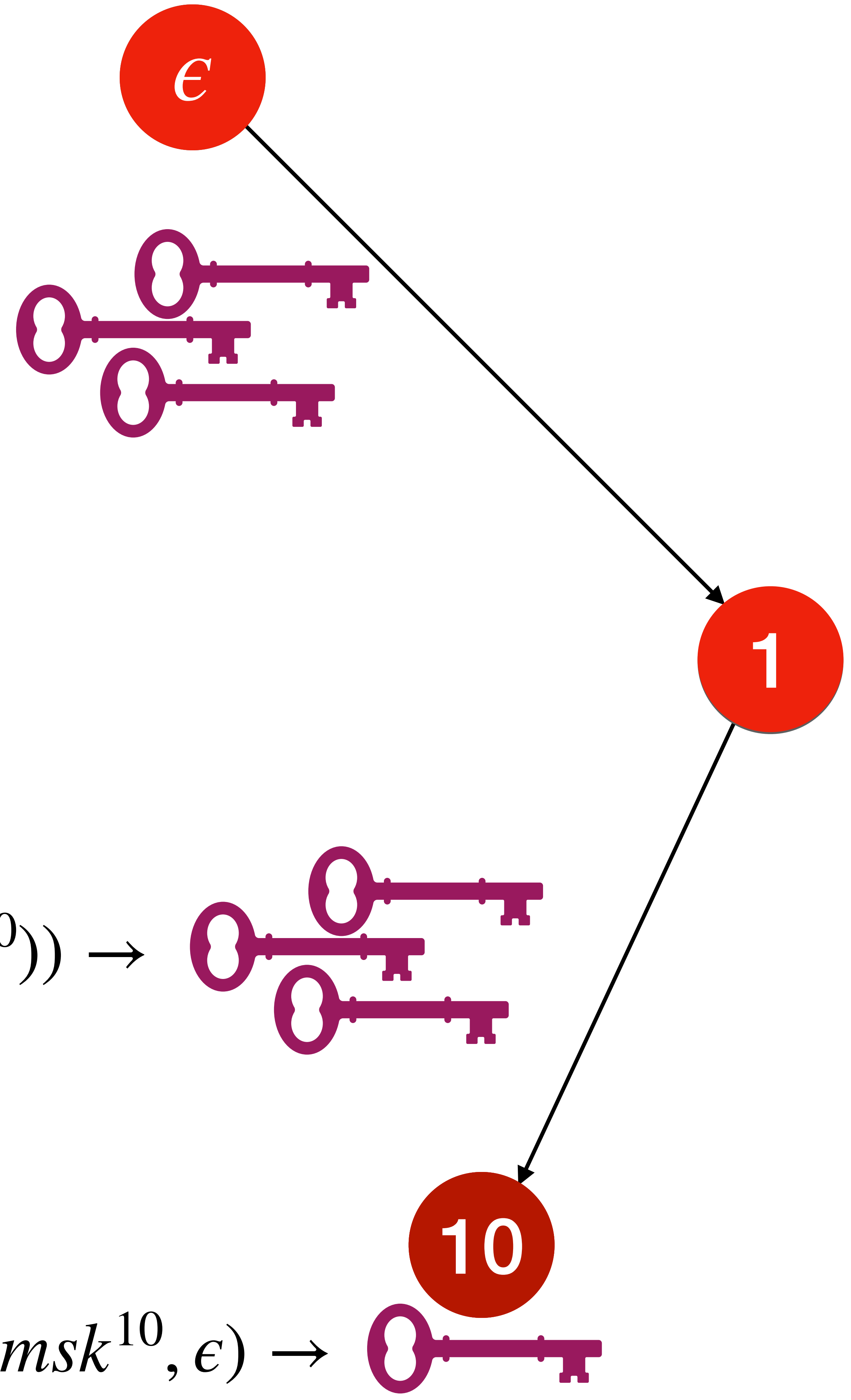
Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)

Decryption

$$(\tilde{T}, \tilde{P}, \tilde{Q}, \{LAB_{i,mpk_i^\epsilon}\})$$

$Eval$



$KG(msk^\epsilon, (1\|i\|mpk_i^1)) \rightarrow$

$KG(msk^1, (1\|i\|mpk_i^{10})) \rightarrow$

$KG(msk^{10}, \epsilon) \rightarrow$

[**Döttling-Garg'17**]

Delegatable PRF	(S, E, D)
Garbling scheme	$(Garb, Eval)$
IBE	(Gen, KG, Enc, Dec)

Decryption

$(\tilde{T}, \tilde{P}, \tilde{Q}, \{LAB_{i,mpk_i^\epsilon}\})$

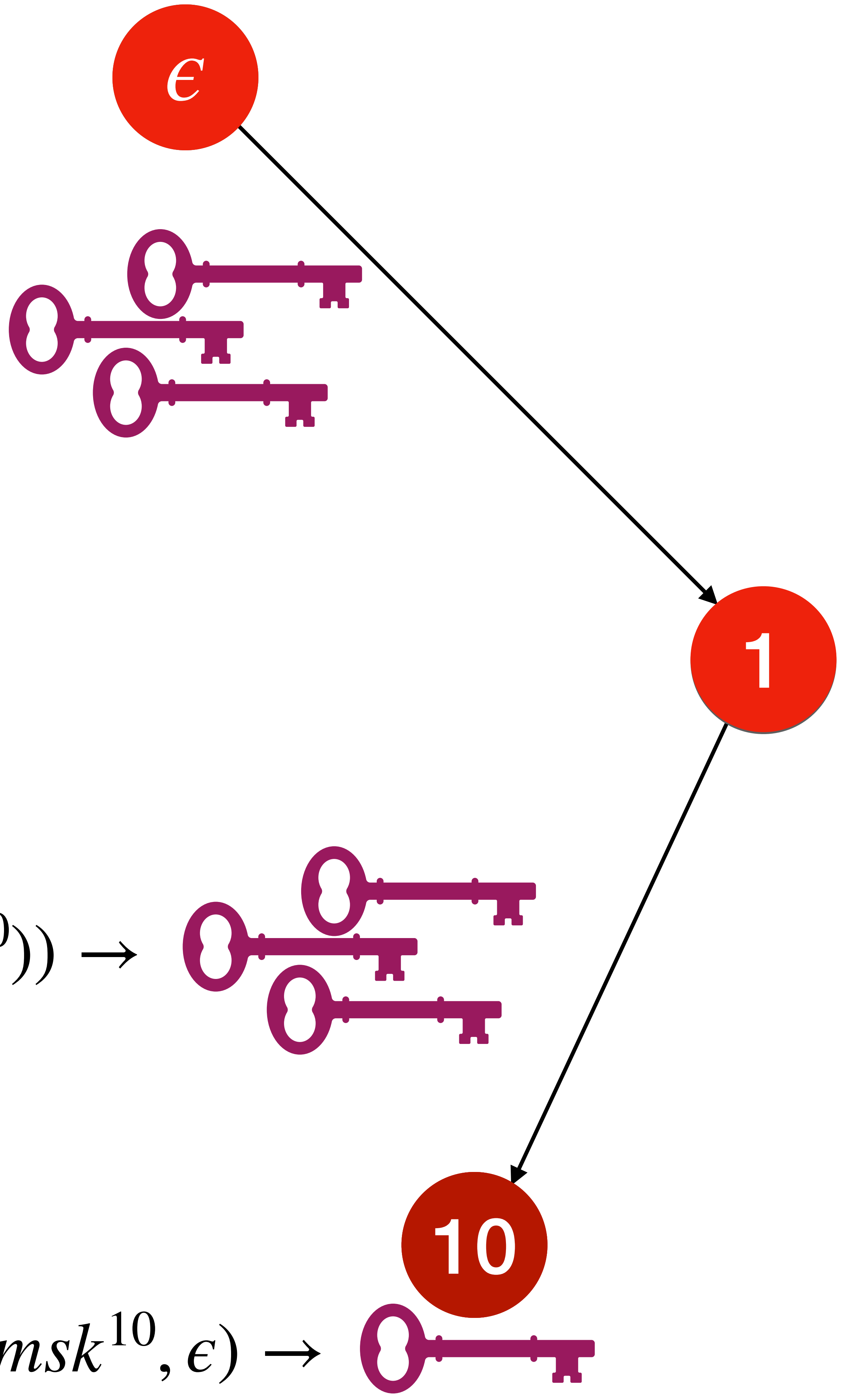
Eval

$CT = Enc(mpk^\epsilon, 1\|i\|b, Lab_{i,b})$

$KG(msk^\epsilon, (1\|i\|mpk_i^1)) \to$

$KG(msk^1, (1\|i\|mpk_i^{10})) \to$

$KG(msk^{10}, \epsilon) \to$



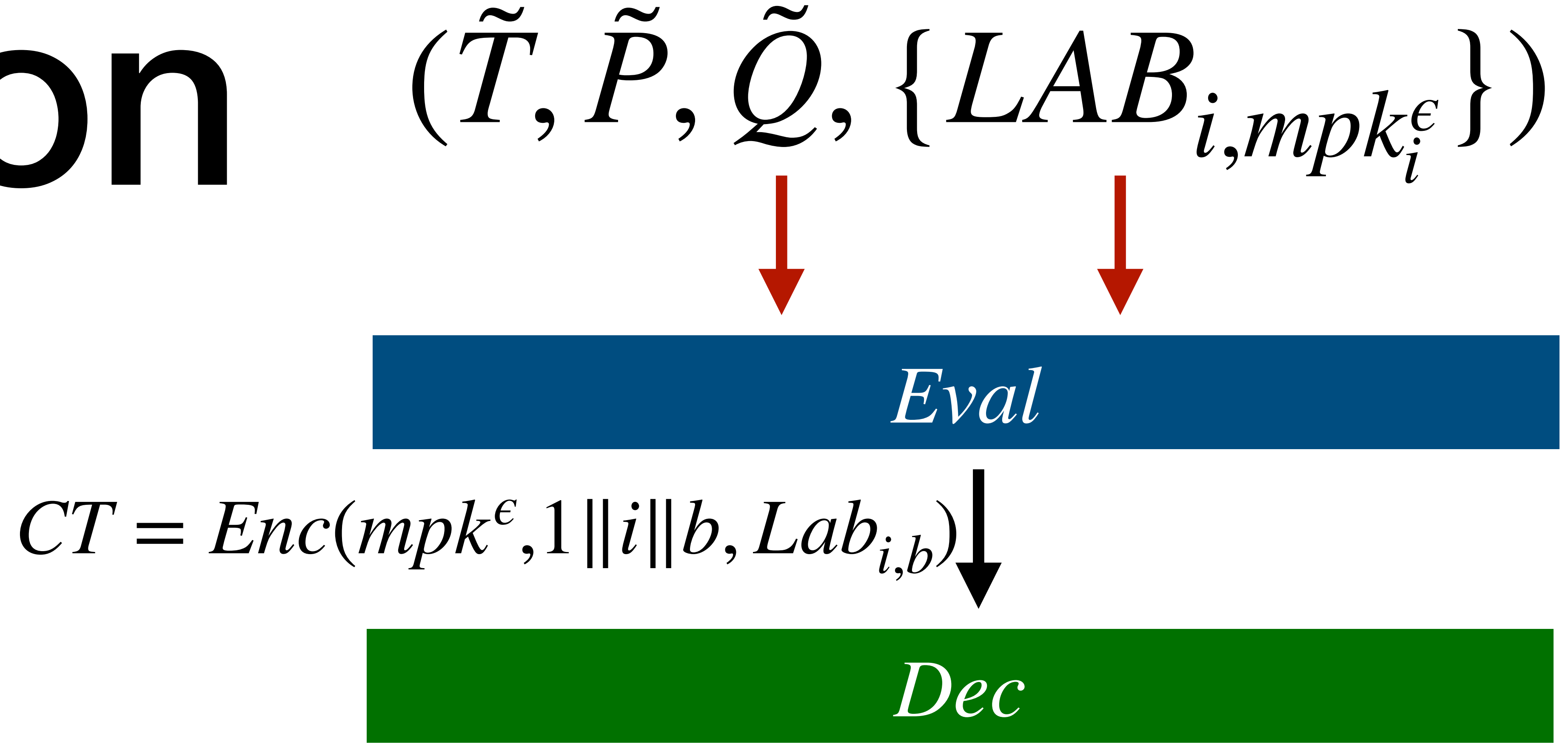
[**Döttling-Garg'17**]

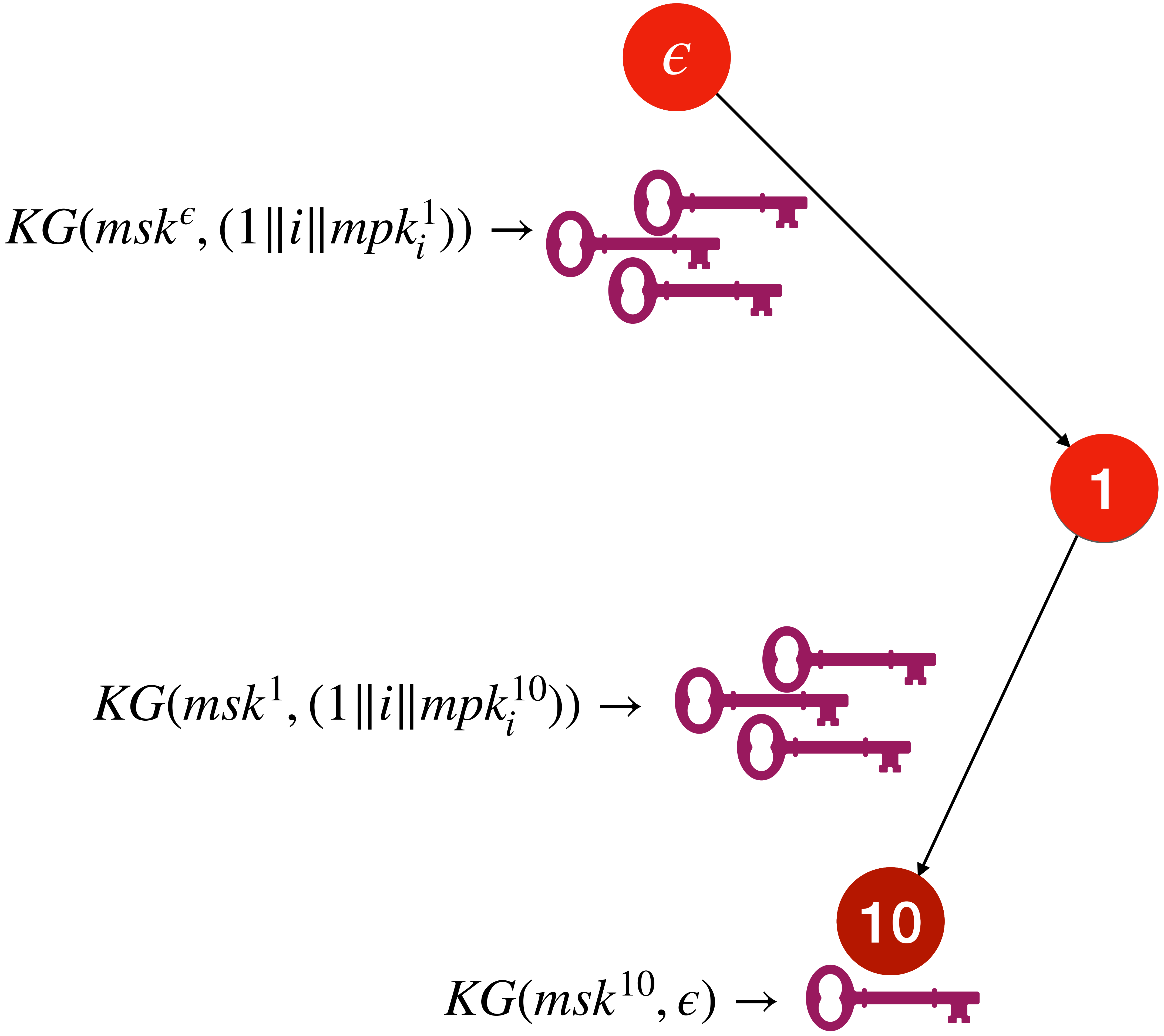
Delegatable PRF (S, E, D)

Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)

Decryption





[Döttling-Garg'17]

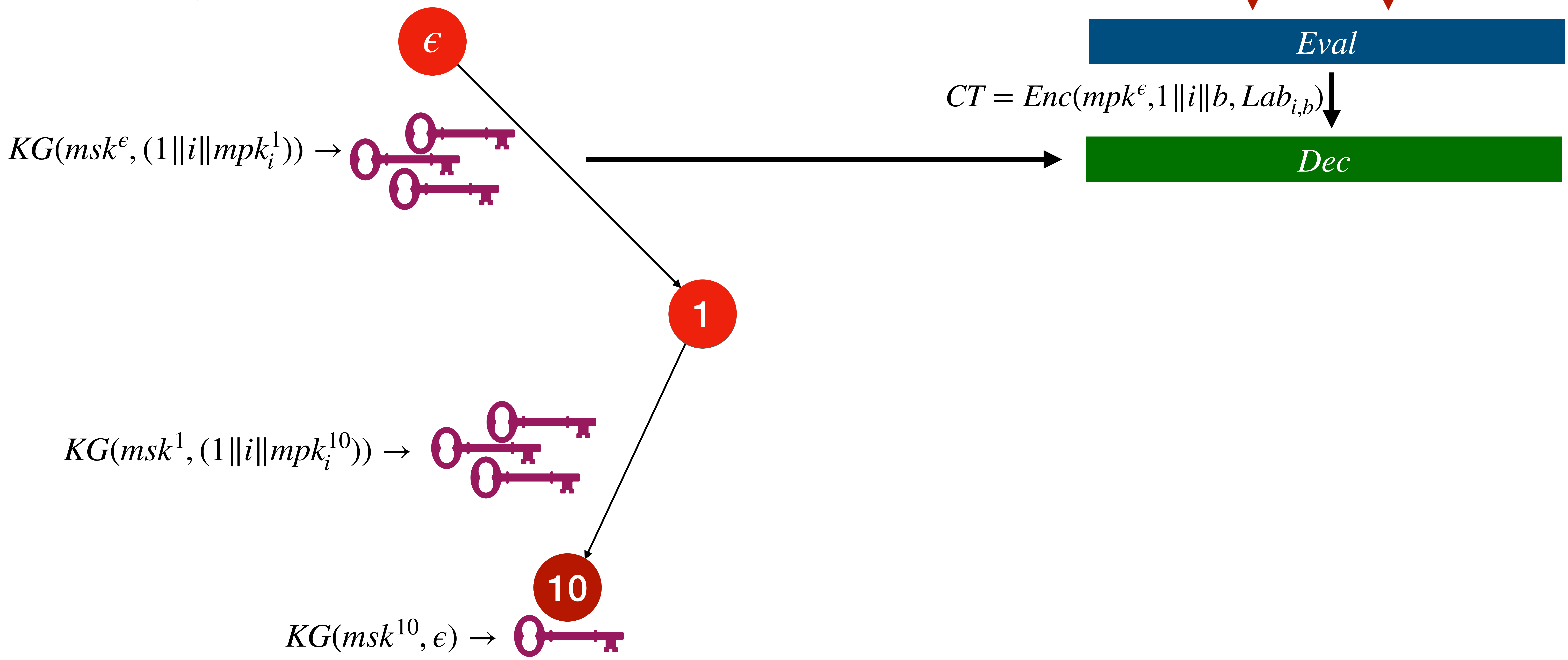
Delegatable PRF (S, E, D)

Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)

Decryption

$(\tilde{T}, \tilde{P}, \tilde{Q}, \{LAB_{i,mpk_i^\epsilon}\})$



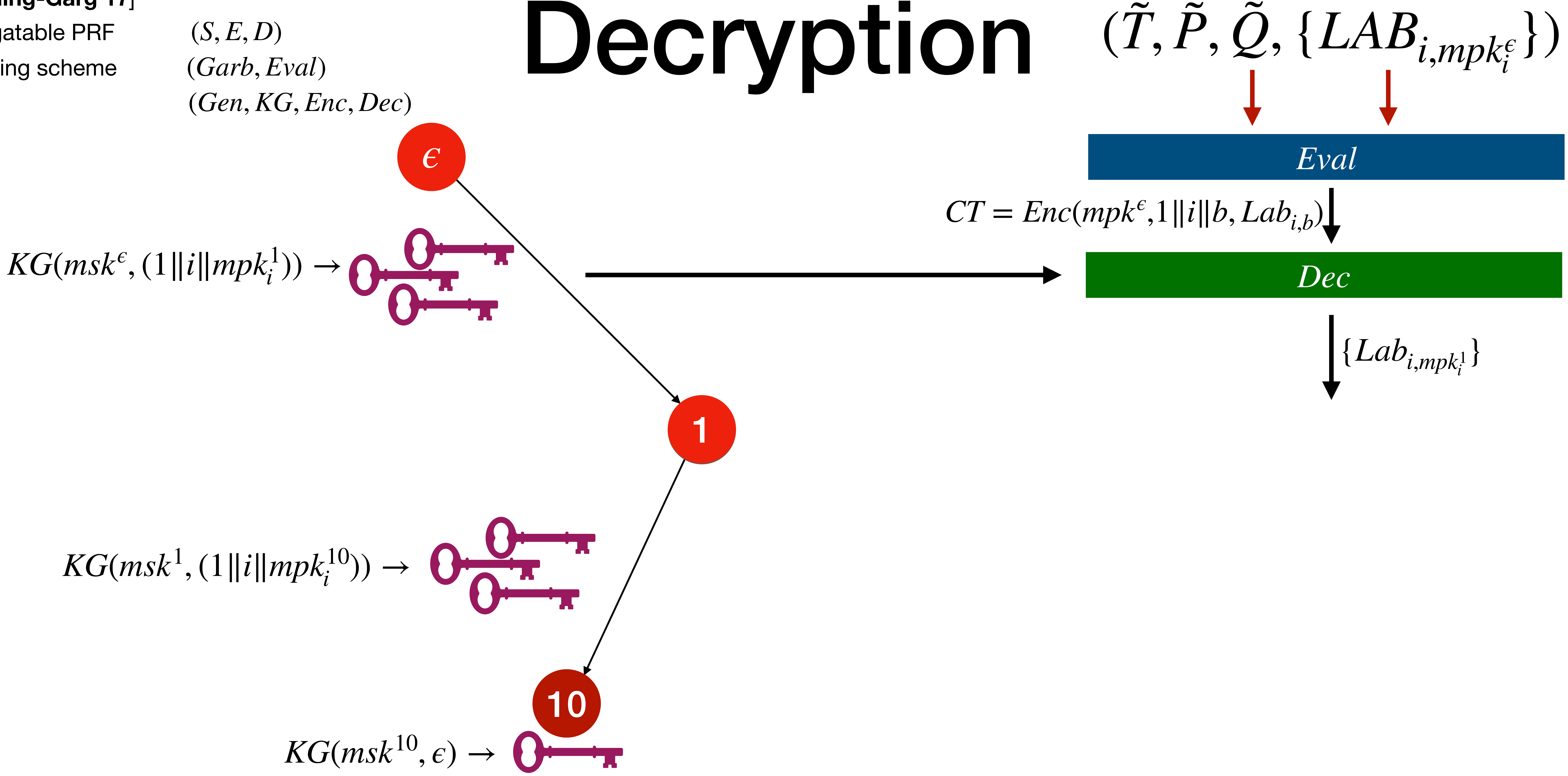
Decryption

[Döttling-Garg'17]

Delegatable PRF (S, E, D)

Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)

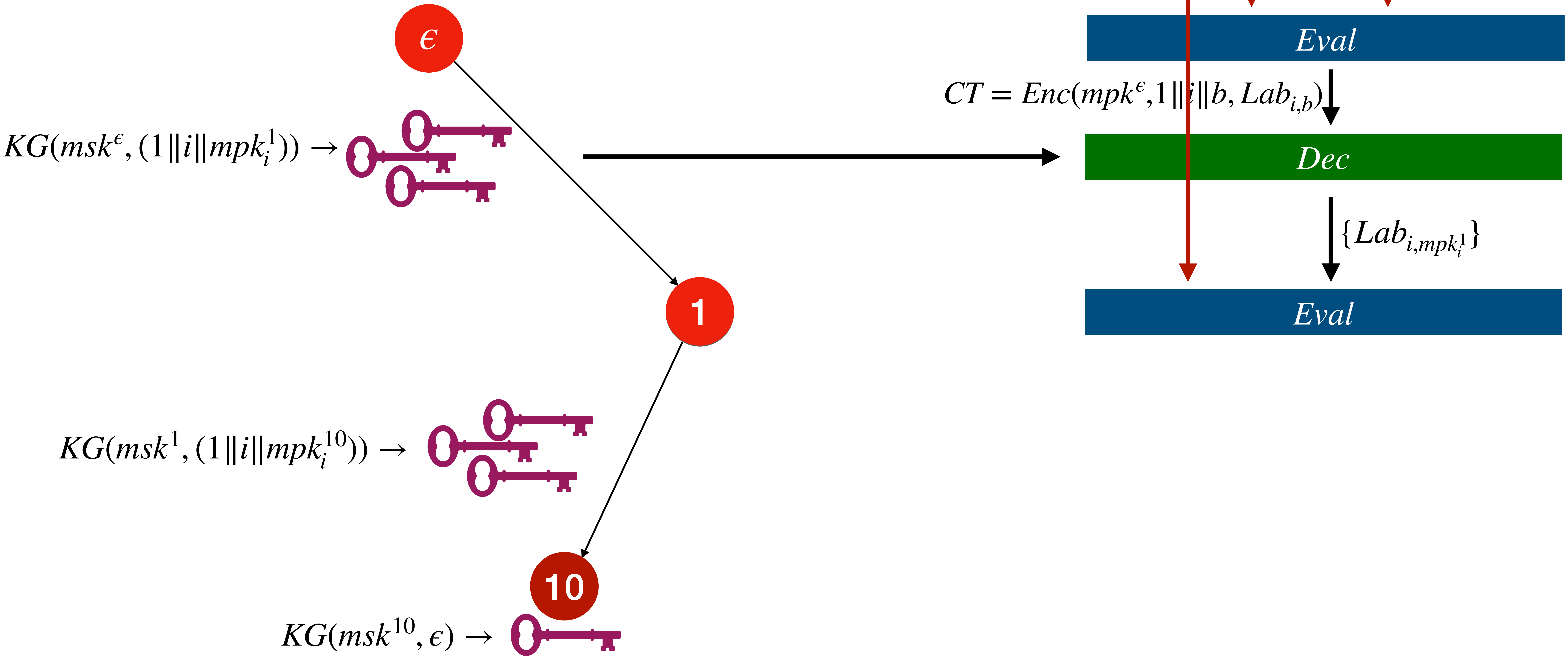


[**Döttling-Garg'17**]

Delegatable PRF	(S, E, D)
Garbling scheme	$(Garb, Eval)$
IBE	(Gen, KG, Enc, Dec)

Decryption

$(\tilde{T}, \tilde{P}, \tilde{Q}, \{LAB_{i,mpk_i^\epsilon}\})$



Decryption

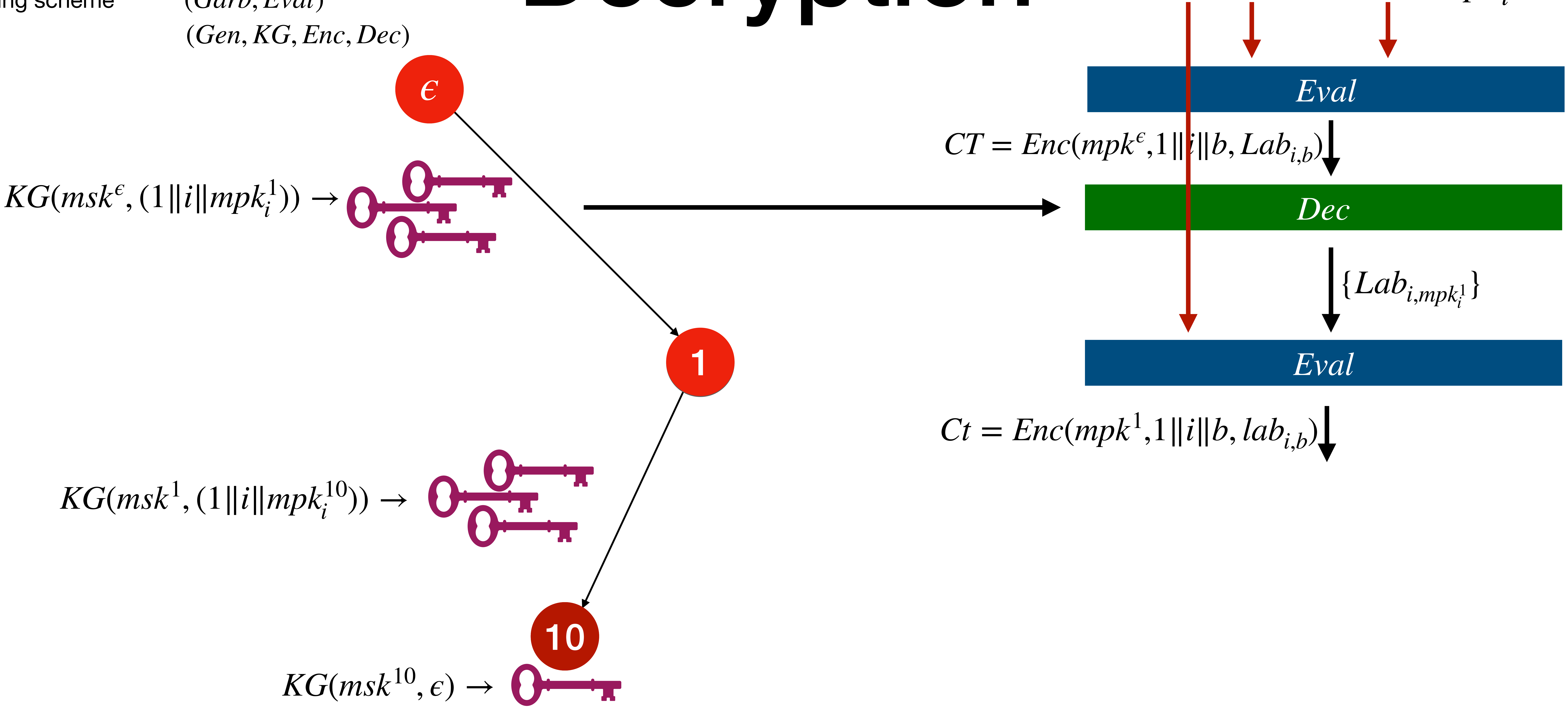
[**Döttling-Garg'17**]

Delegatable PRF (S, E, D)

Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)

$(\tilde{T}, \tilde{P}, \tilde{Q}, \{LAB_{i,mpk_i^{\epsilon}}\})$

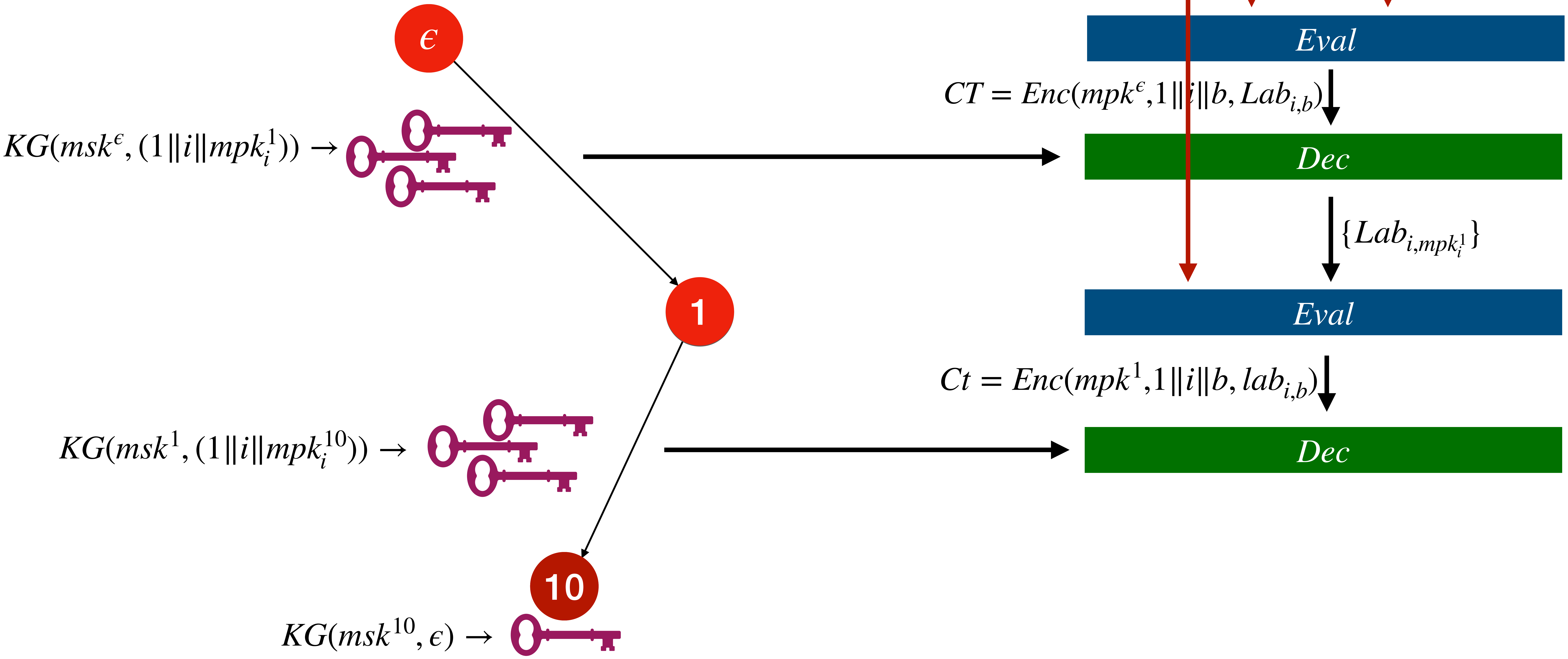


[**Döttling-Garg’17**]

Delegatable PRF	(S, E, D)
Garbling scheme	$(Garb, Eval)$
IBE	(Gen, KG, Enc, Dec)

Decryption

$(\tilde{T}, \tilde{P}, \tilde{Q}, \{LAB_{i,mpk_i^{\epsilon}}\})$

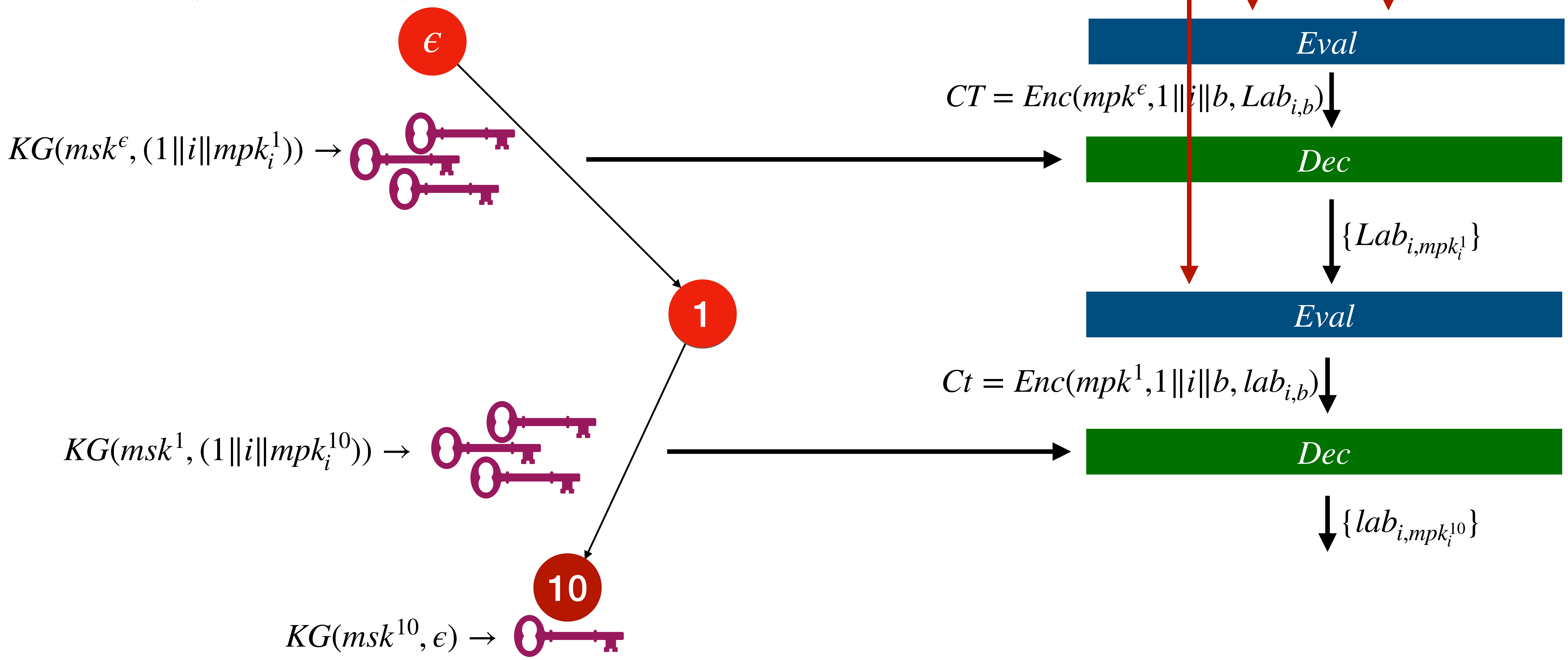


[**Döttling-Garg'17**]

Delegatable PRF	(S, E, D)
Garbling scheme	$(Garb, Eval)$
IBE	(Gen, KG, Enc, Dec)

Decryption

$(\tilde{T}, \tilde{P}, \tilde{Q}, \{LAB_{i,mpk_i^{\epsilon}}\})$



[**Döttling-Garg’17**]

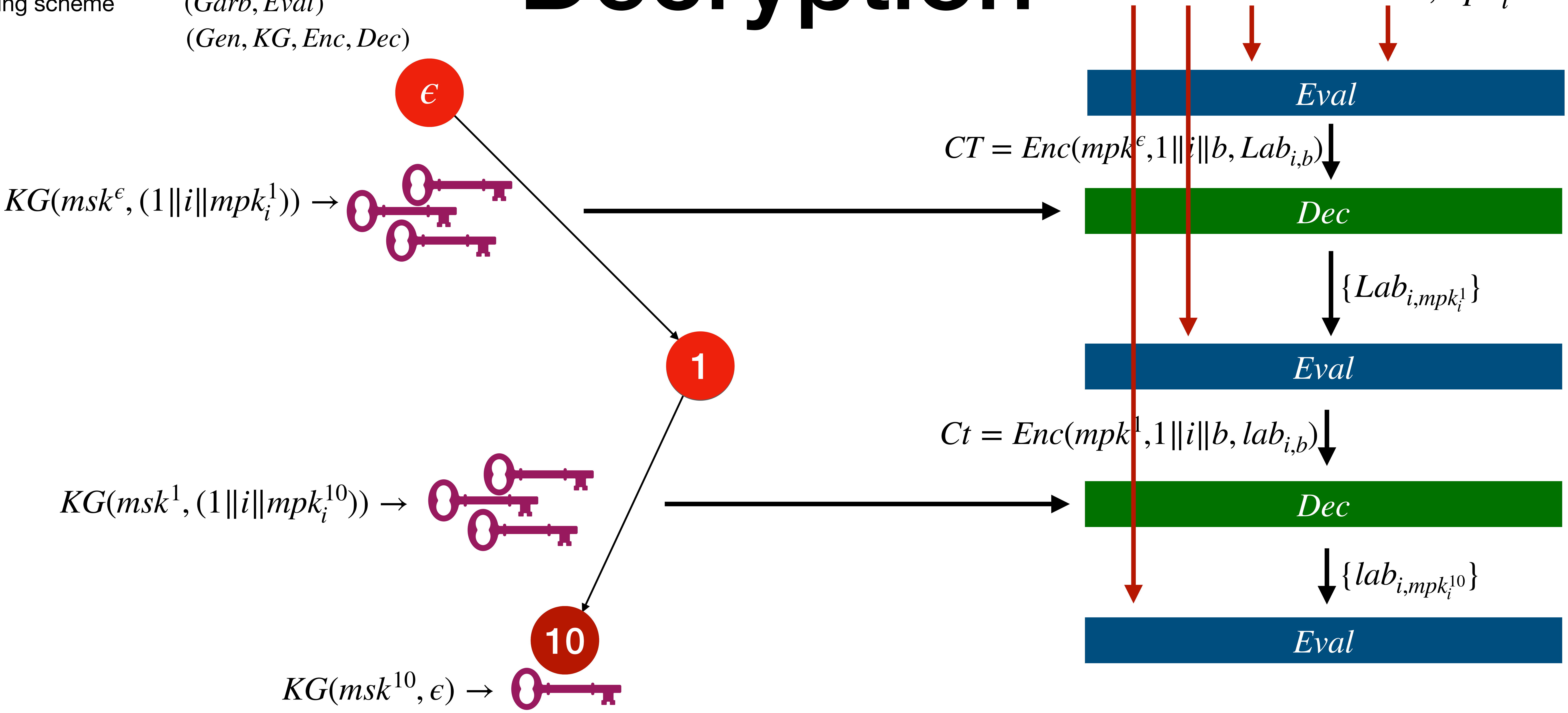
Delegatable PRF (S, E, D)

Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)

Decryption

$(\tilde{T}, \tilde{P}, \tilde{Q}, \{LAB_{i,mpk_i^\epsilon}\})$

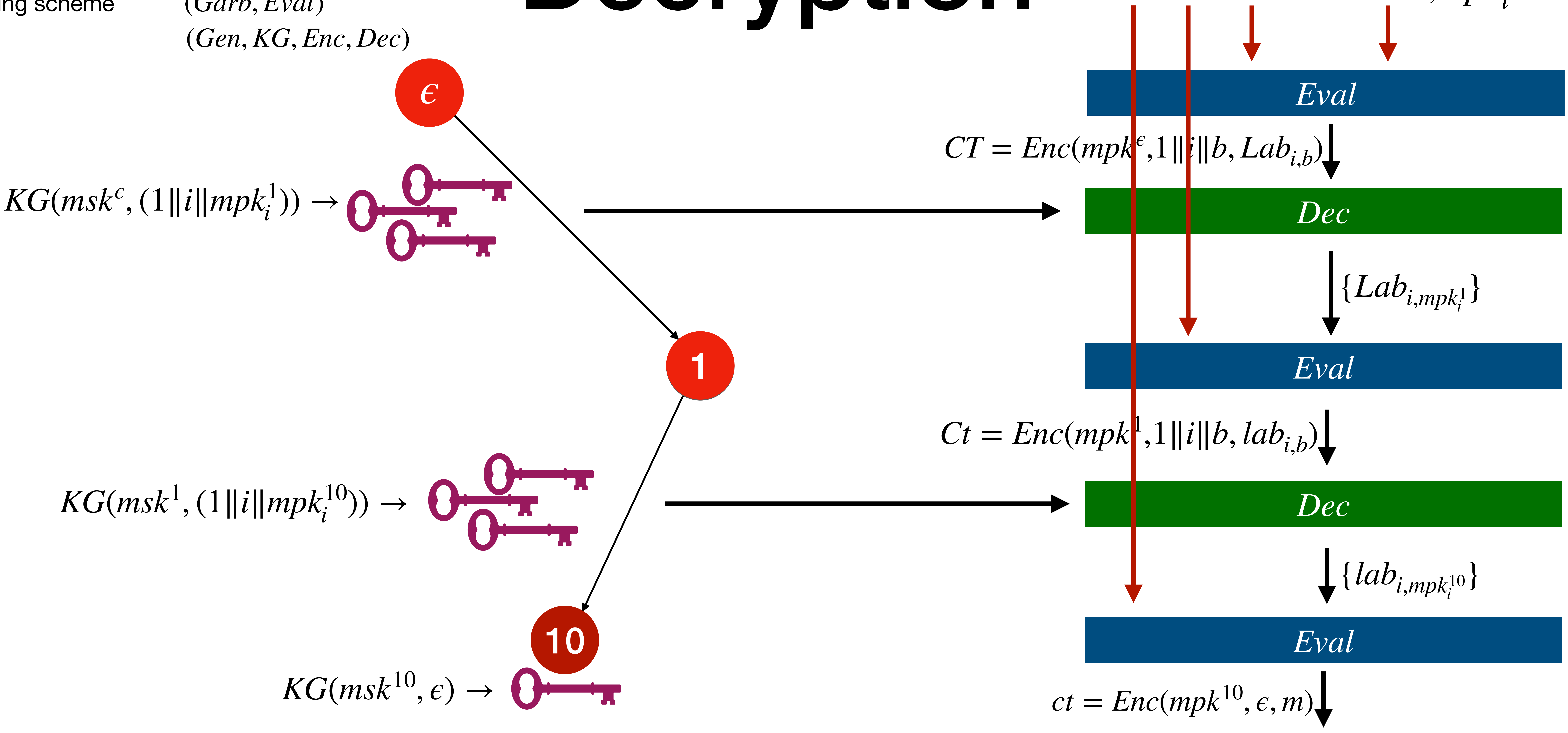


[**Döttling-Garg’17**]

Delegatable PRF	(S, E, D)
Garbling scheme	$(Garb, Eval)$
IBE	(Gen, KG, Enc, Dec)

Decryption

$(\tilde{T}, \tilde{P}, \tilde{Q}, \{LAB_{i,mpk_i^\epsilon}\})$



[**Döttling-Garg'17**]

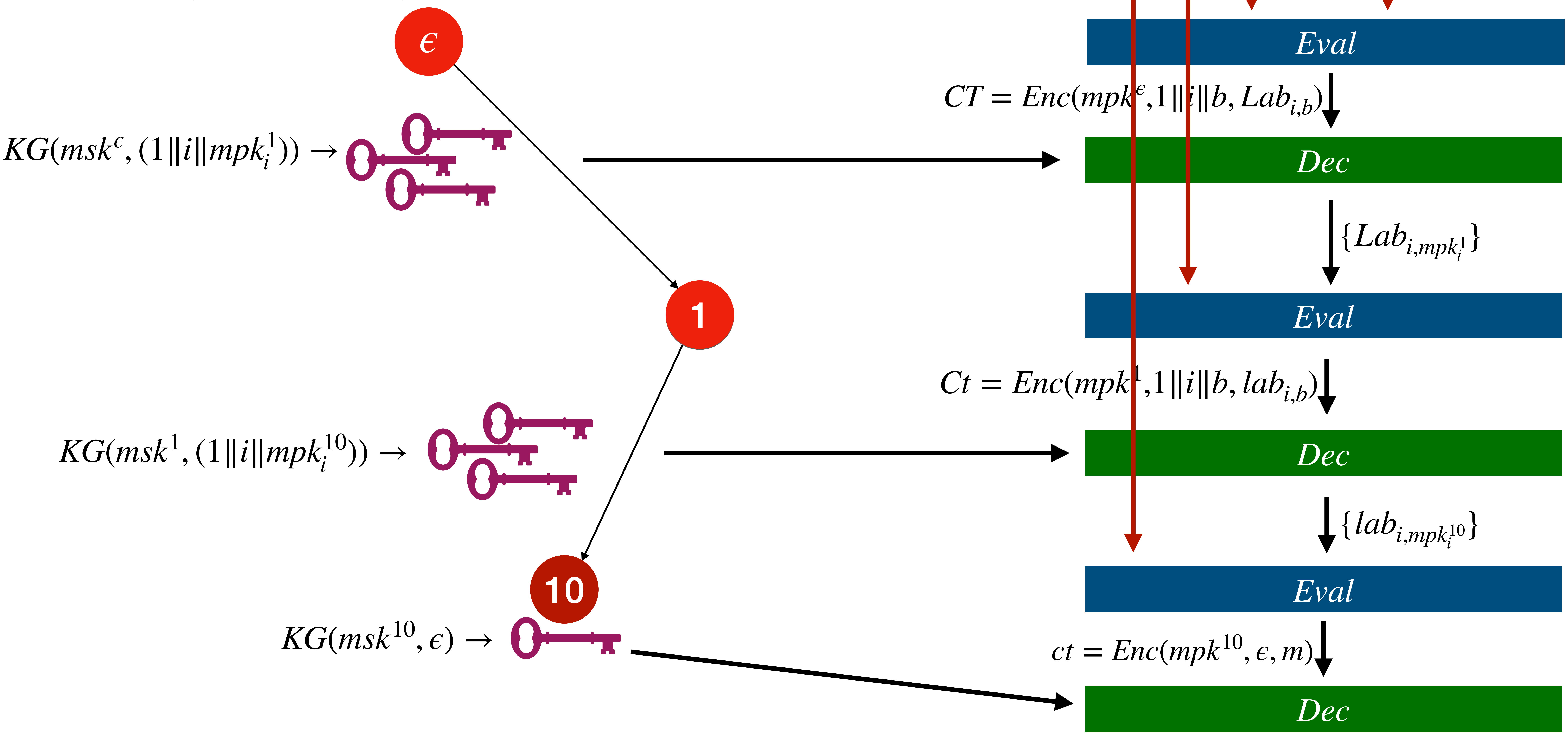
Delegatable PRF (S, E, D)

Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)

Decryption

$(\tilde{T}, \tilde{P}, \tilde{Q}, \{LAB_{i,mpk_i^\epsilon}\})$



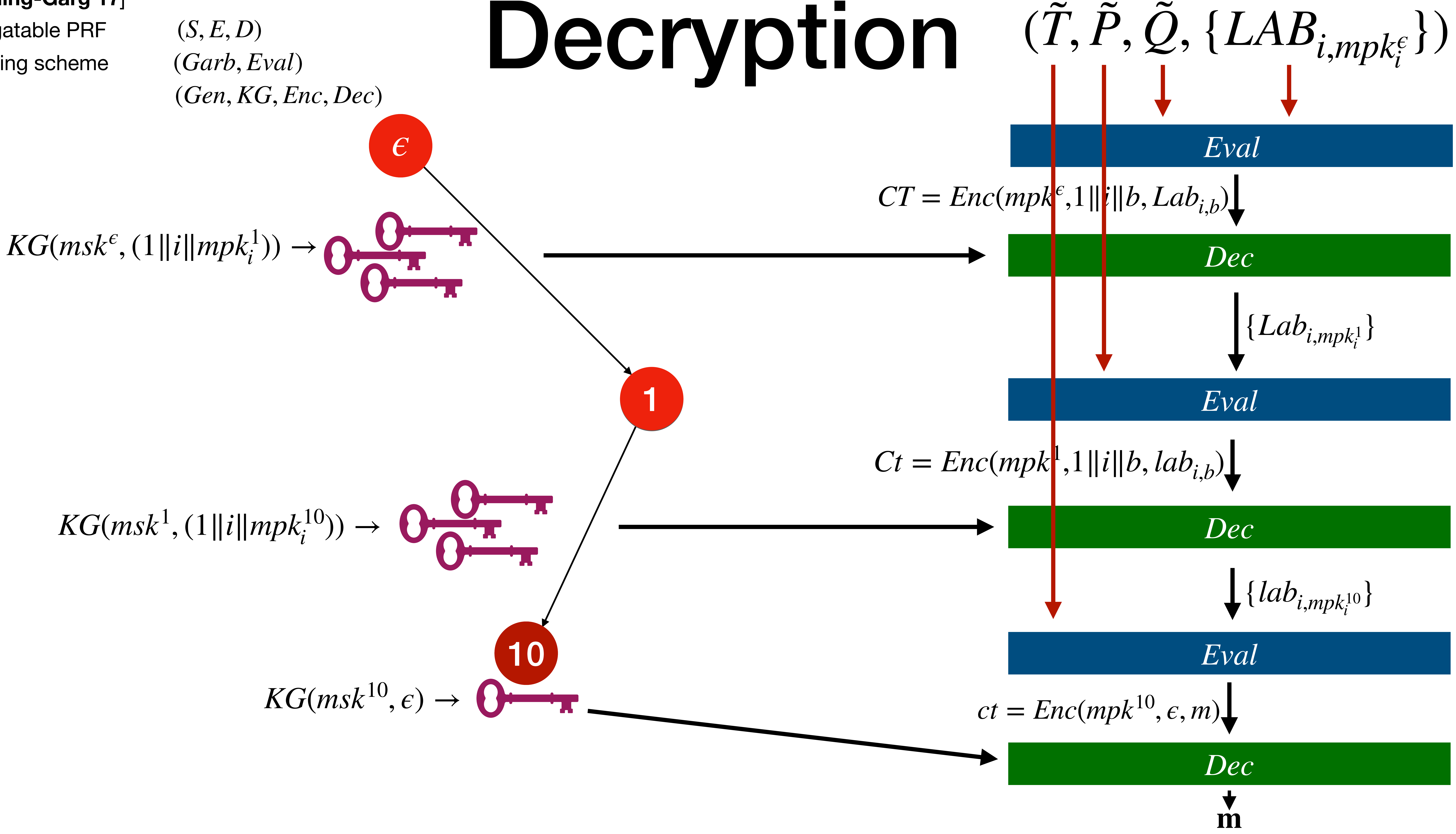
[**Döttling-Garg'17**]

Delegatable PRF (S, E, D)

Garbling scheme $(Garb, Eval)$

IBE (Gen, KG, Enc, Dec)

Decryption



Selective Security of DG17

[Döttling-Garg'17]

Selective Security of DG17

[Döttling-Garg'17]

$(\tilde{Q}, LAB_{i,b}) \leftarrow Garb(P_{'1',\{Lab_{i,b}\}})$

$(\tilde{P}, Lab_{i,b}) \leftarrow Garb(P_{'0',\{lab_{i,b}\}})$

$\tilde{T}, \{lab_{i,b}\} \leftarrow Garb(T_m)$

$(\tilde{T}, \tilde{P}, \tilde{Q}, \{LAB_{i,mpk_i^{\epsilon}}\})$

Selective Security of DG17

[Döttling-Garg'17]

$(\tilde{Q}, LAB_{i,b}) \leftarrow Garb(P_{'1',\{Lab_{i,b}\}})$

1. Replace PRF evaluations with truly random

$(\tilde{P}, Lab_{i,b}) \leftarrow Garb(P_{'0',\{lab_{i,b}\}})$

$\tilde{T}, \{lab_{i,b}\} \leftarrow Garb(T_m)$

$$(\tilde{T}, \tilde{P}, \tilde{Q}, \{LAB_{i,mpk_i^{\epsilon}}\})$$

Selective Security of DG17

[Döttling-Garg'17]

$$(\tilde{Q}, LAB_{i,b}) \leftarrow Garb(P_{'1',\{Lab_{i,b}\}})$$

1. Replace PRF evaluations with truly random
    1. Use simulators of GC

$$(\tilde{P}, Lab_{i,b}) \leftarrow Garb(P_{'0',\{lab_{i,b}\}})$$

$$\tilde{T}, \{lab_{i,b}\} \leftarrow Garb(T_m)$$

$$(\tilde{T}, \tilde{P}, \tilde{Q}, \{LAB_{i,mpk_i^{\epsilon}}\})$$

Selective Security of DG17

[Döttling-Garg'17]

$(\tilde{Q}, LAB_{i,mpk_i^{\epsilon}}) \leftarrow Sim(\{CT_{i,b}\})$

1. Replace PRF evaluations with truly random
    1. Use simulators of GC

$(\tilde{P}, Lab_{i,b}) \leftarrow Garb(P_{'0',\{lab_{i,b}\}})$

$\tilde{T}, \{lab_{i,b}\} \leftarrow Garb(T_m)$

$$(\tilde{T}, \tilde{P}, \tilde{Q}, \{LAB_{i,mpk_i^{\epsilon}}\})$$

Selective Security of DG17

[Döttling-Garg'17]

1. Replace PRF evaluations with truly random
    1. Use simulators of GC
    2. Replace IBE ciphertexts

$$(\tilde{Q}, LAB_{i,mpk_i^\epsilon}) \leftarrow Sim(\{CT_{i,b}\})$$

$$(\tilde{P}, Lab_{i,b}) \leftarrow Garb(P_{'0',\{lab_{i,b}\}})$$

$$\tilde{T}, \{lab_{i,b}\} \leftarrow Garb(T_m)$$

$$(\tilde{T}, \tilde{P}, \tilde{Q}, \{LAB_{i,mpk_i^\epsilon}\})$$

Selective Security of DG17

[Döttling-Garg'17]

$(\tilde{Q}, LAB_{i,mpk_i^\epsilon}) \leftarrow Sim(\{CT_{i,b}\})$

1. Replace PRF evaluations with truly random

$CT_{i,1-mpk_i^1} = Enc(mpk^\epsilon, (1\|i\|1-mpk_i^1), \mathbf{0})$

   1. Use simulators of GC
   2. Replace IBE ciphertexts

$(\tilde{P}, Lab_{i,b}) \leftarrow Garb(P_{'0',\{lab_{i,b}\}})$

$\tilde{T}, \{lab_{i,b}\} \leftarrow Garb(T_m)$

$$(\tilde{T}, \tilde{P}, \tilde{Q}, \{LAB_{i,mpk_i^\epsilon}\})$$

Selective Security of DG17

[Döttling-Garg'17]

1. Replace PRF evaluations with truly random
    1. Use simulators of GC
    2. Replace IBE ciphertexts

$$(\tilde{Q}, LAB_{i,mpk_i^{\epsilon}}) \leftarrow Sim(\{CT_{i,b}\})$$

$$CT_{i,1-mpk_i^1} = Enc(mpk^{\epsilon}, (1\|i\|1 - mpk_i^1), \mathbf{0})$$

$$(\tilde{P}, Lab_{i,mpk_i^1}) \leftarrow Sim(\{Ct_{i,b}\})$$

$$Ct_{i,1-mpk_i^{10}} = Enc(mpk^{\epsilon}, (1\|i\|1 - mpk_i^{10}), \mathbf{0})$$

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Selective Security of DG17

[Döttling-Garg'17]

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    1. Use simulators of GC
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Does this/similar techniques help us in building HIBE?

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Nested Hybrids & Pebbling-style strategy

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Pebbling strategy
[Hemenway-Jafargholi-Ostrovsky-Scafuro-Wichs'16]
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[Hohenberger-Waters’10,Fuchsbauer-Konstantinov-Pietrzak-Rao’14,Fuchsbauer-Jafargholi-Pietrzak’15]

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Nested Hybrid → Depends on q - first key query that overlaps with $id^*[1:i]$

[Hohenberger-Waters'10,Fuchsbauer-Konstantinov-Pietrzak-Rao'14,Fuchsbauer-Jafargholi-Pietrzak'15]

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  - Tighter security loss? Quadratic
  - Practically efficient constructions? Avoid garbling schemes
  - Can we use these techniques in other settings such as Delegatable ABE? Current work by [Goyal-Yadugiri'25]

Thank You

https://mahe94.github.io

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