Foundations of
Platform-Assisted Auctions

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Carnegie Mellon University

Joint work with Hao Chung and Ke Wu

Ad space on websites

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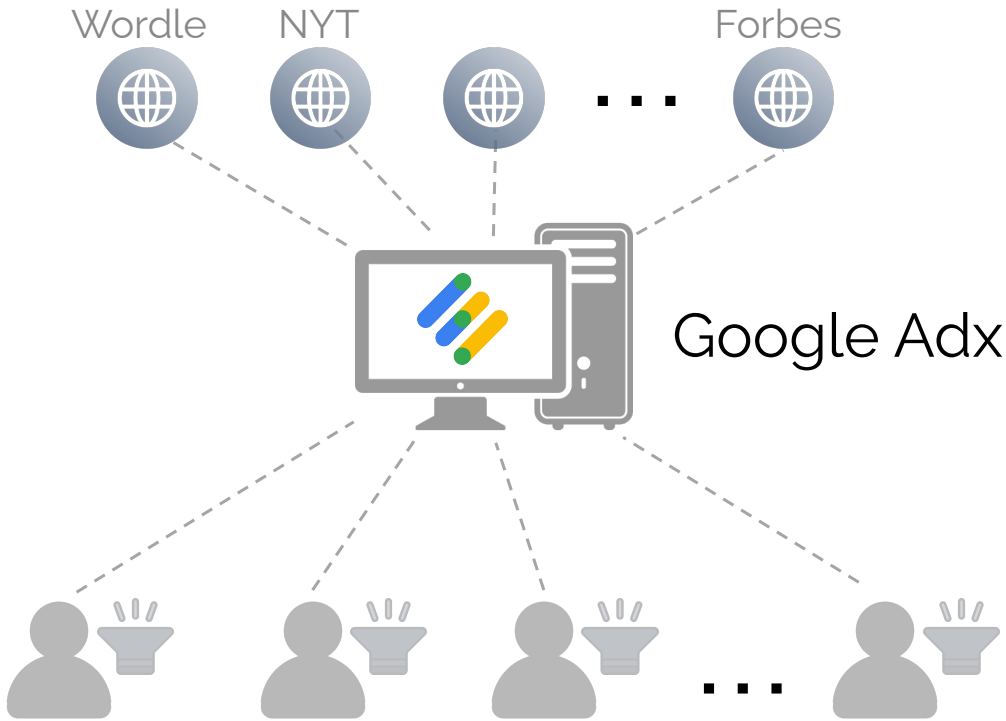
Q W E R T Y U I O P

Platform-assisted auctions

Sellers (publishers)

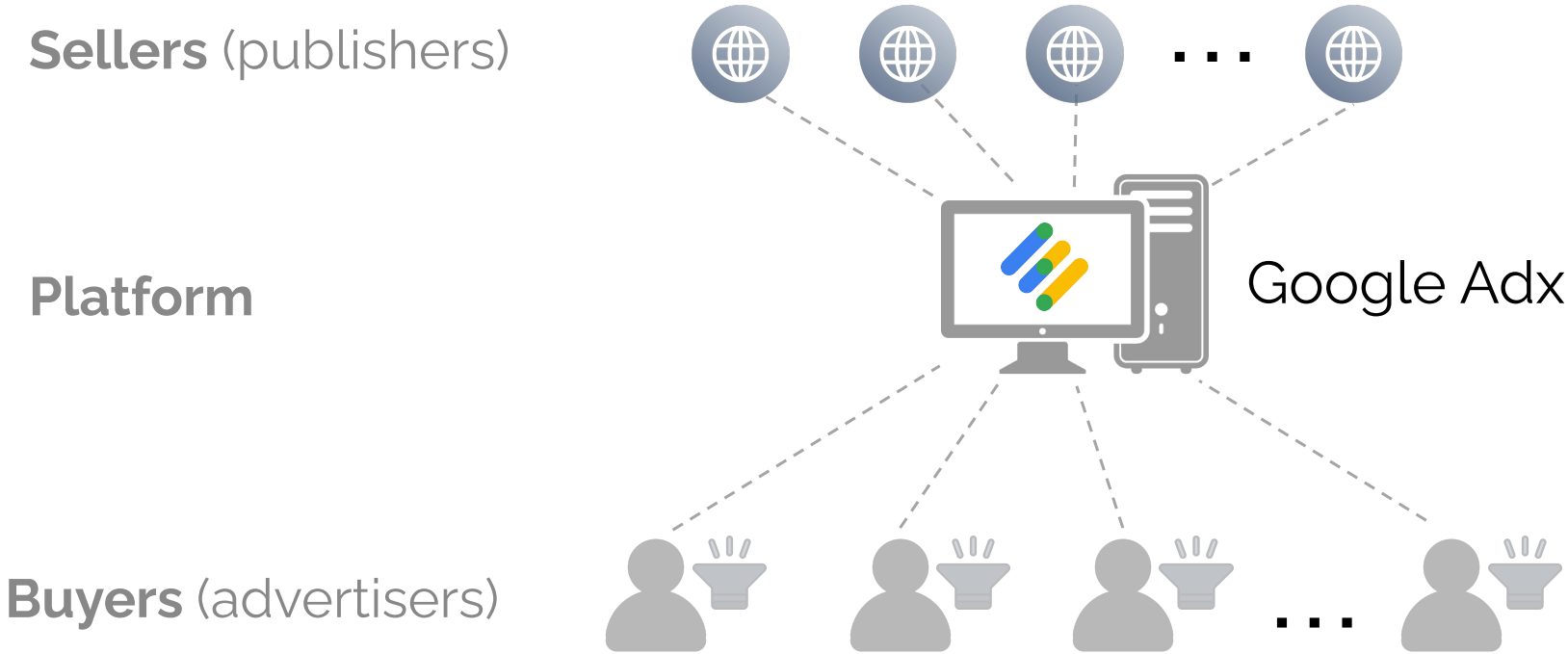
Platform

Buyers (advertisers)



Platform gets **remunerated** for **value-added services**:

↘ rendezvous, search, recommender system, payment processing



PRESS RELEASE

Justice Department Sues Google for Monopolizing Digital Advertising Technologies

Google accused of

- **withholding** seller revenue
- **injecting bids** to raise price

.....

https://www.justice.gov/archives/opa/press-release/file/1563746/dl

New theory of anti-trust auction design

New theory of anti-trust auction design

Auction literature

- Trusted auctioneer
- Assume no collusion
- Permissioned

New theory of anti-trust auction design

Auction literature

- Trusted auctioneer
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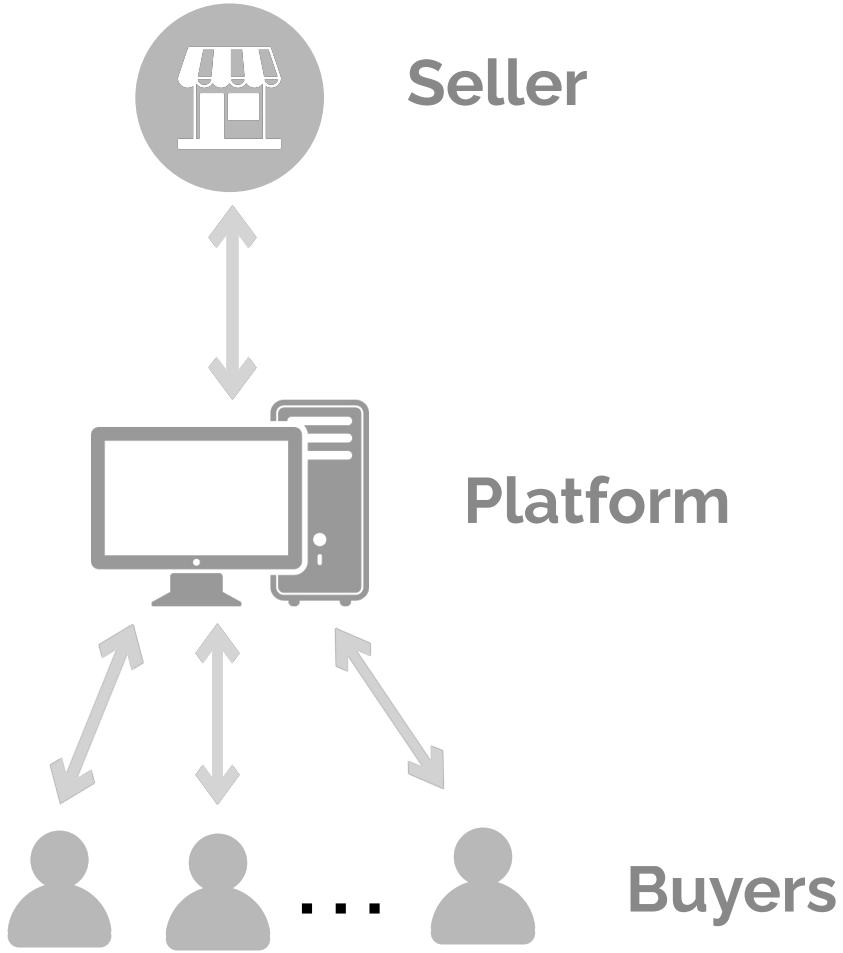
Reality

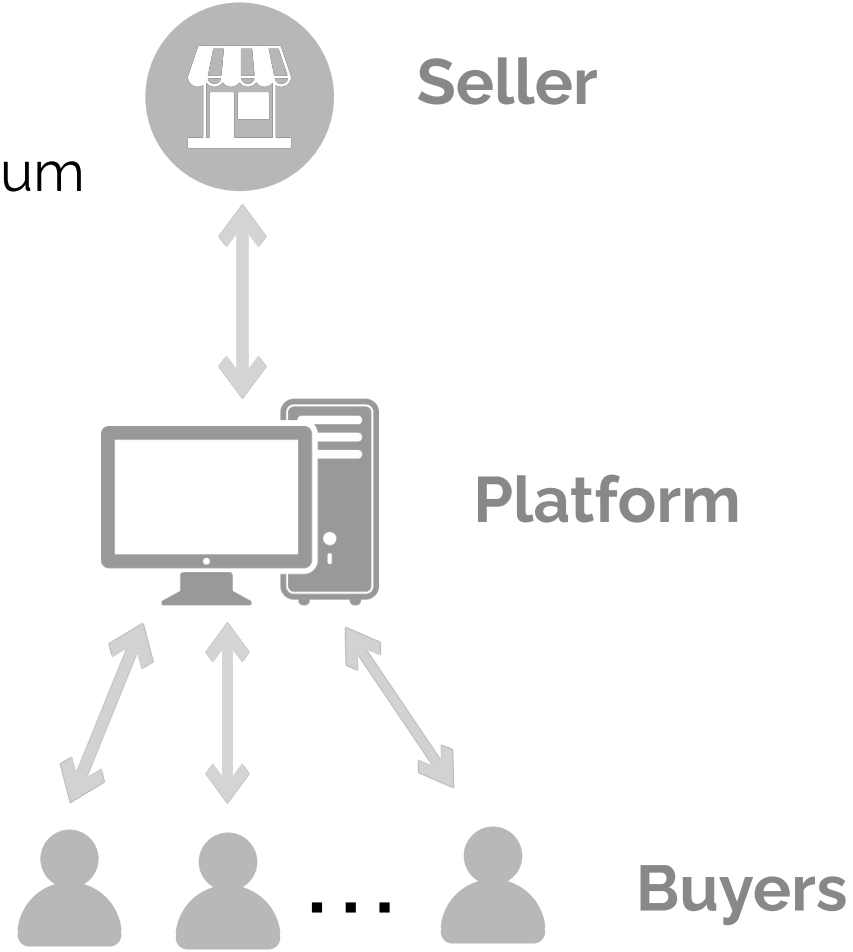
- Trustless environment
- Collusion made easy
- Permissionless

What is a **dream** platform-assisted auction?



Incentive compatible (IC)

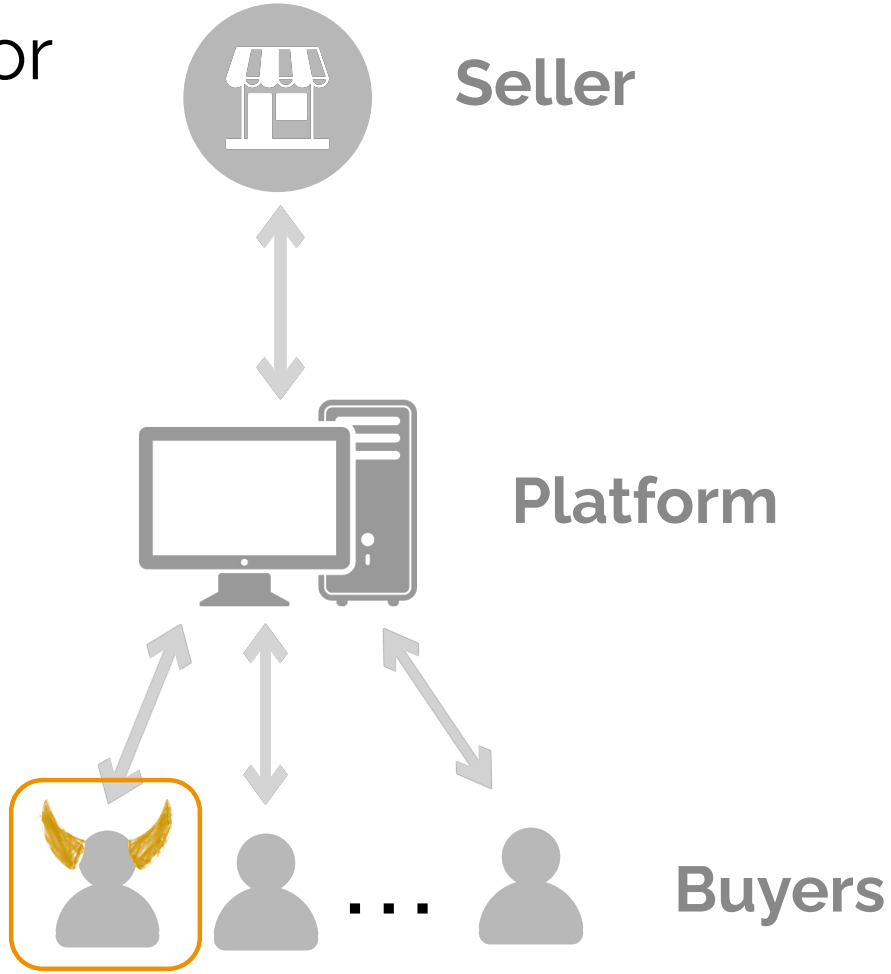
↳ **Honest** is best response/equilibrium



What is a **dream** platform-assisted auction?

Incentive compatible (IC) for

- a buyer



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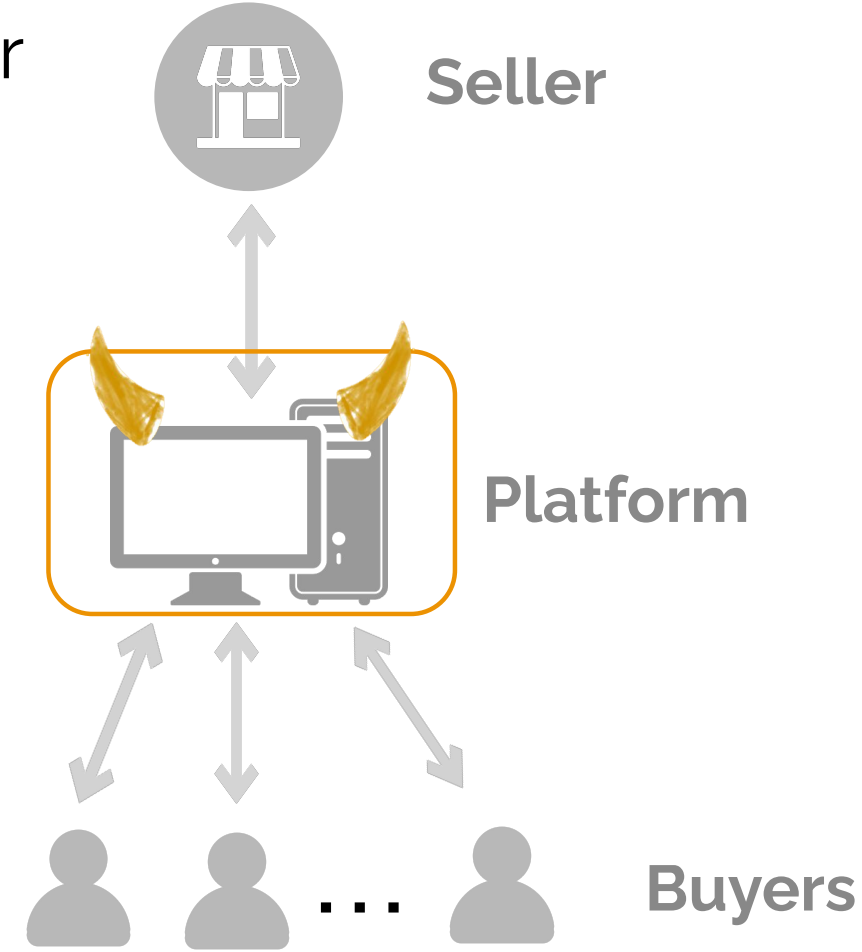

What is a **dream** platform-assisted auction?

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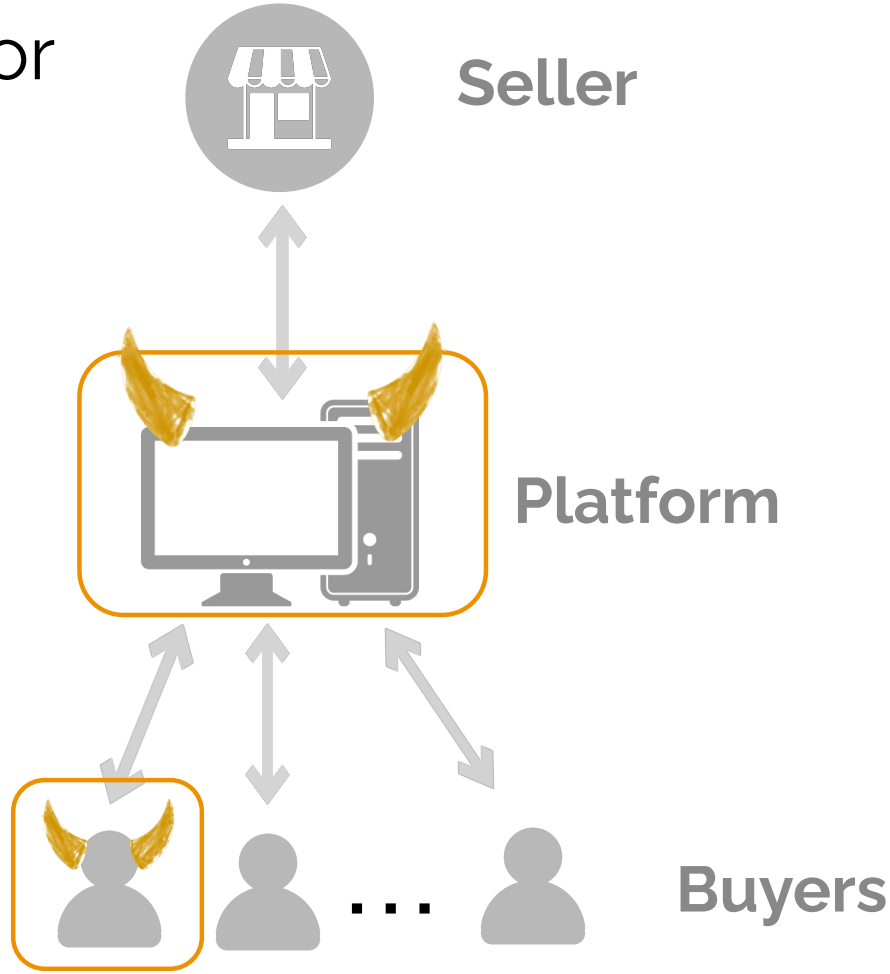
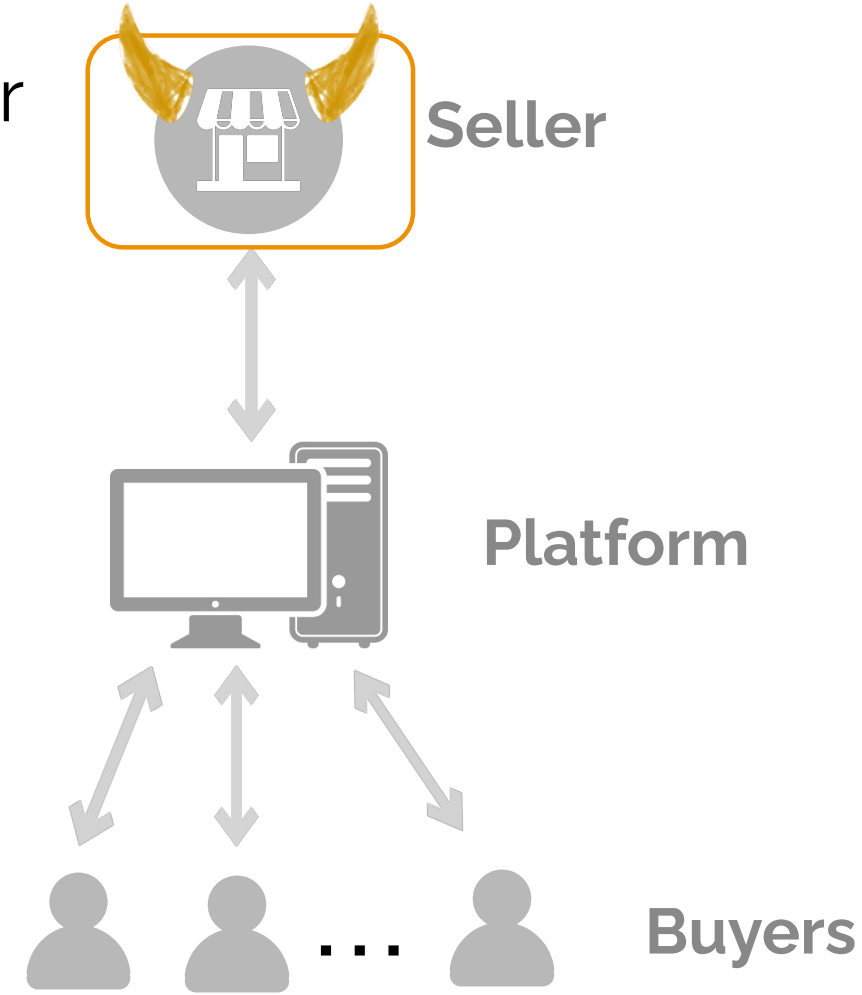
- platform-buyer coalition



What is a **dream** platform-assisted auction?

Incentive compatible (IC) for

- a buyer
- the platform
- platform-buyer coalition
- the seller



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Incentive compatible (IC) for

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- platform-buyer coalition
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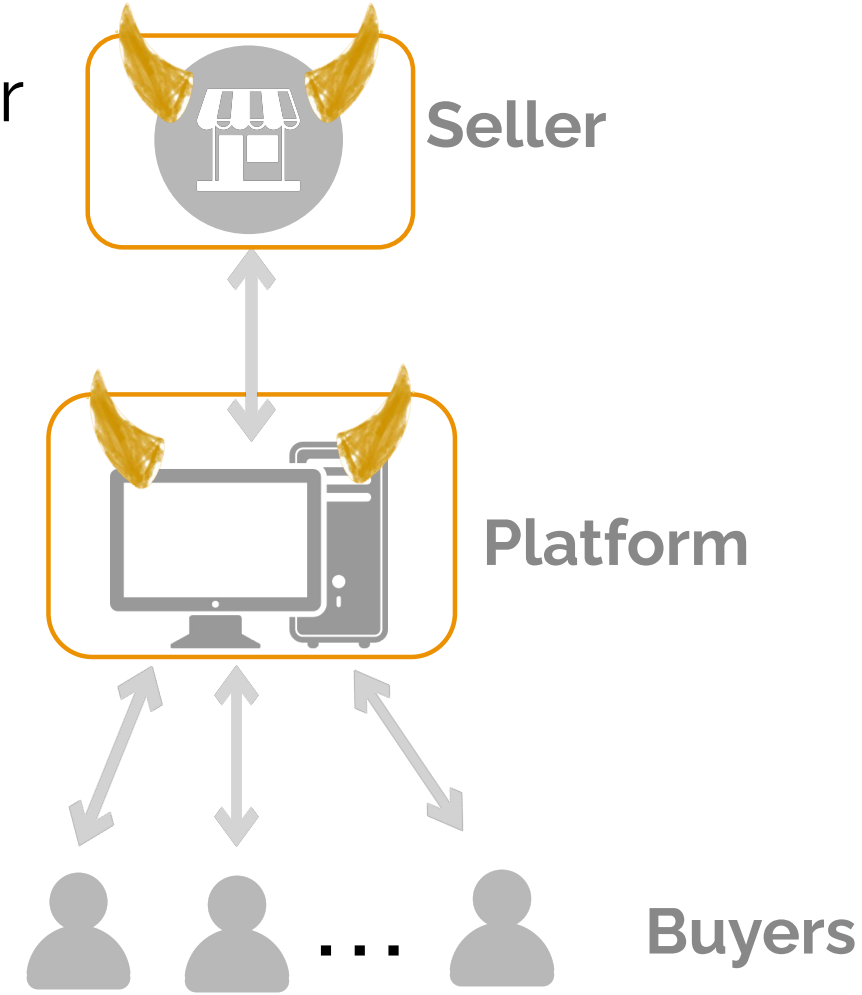
- platform-seller coalition



What is a **dream** platform-assisted auction?

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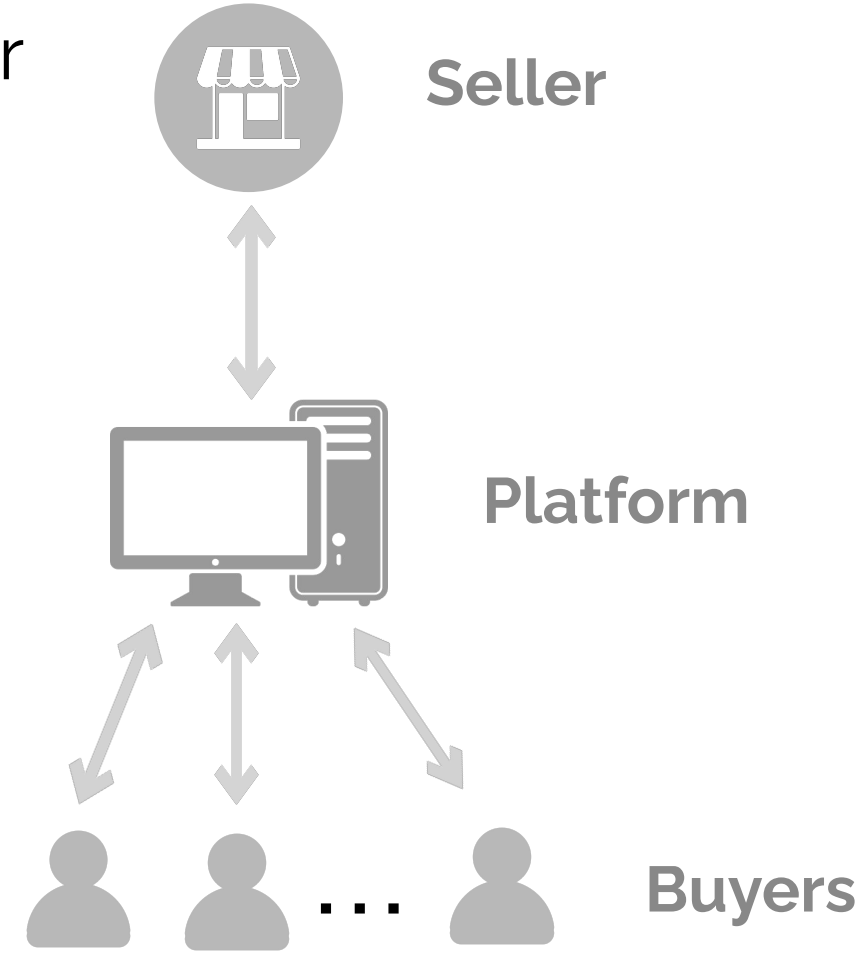
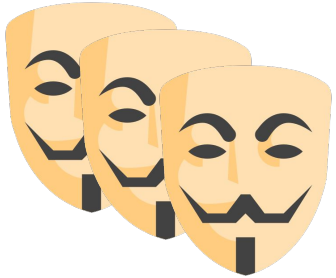
What is a **dream** platform-assisted auction?



Overbid, underbid

Fake bids

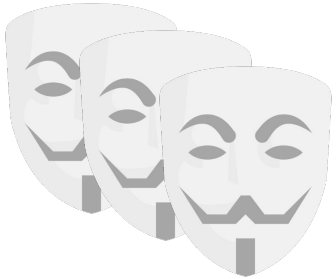


Arbitrarily deviate from protocol

Strategy space

Overbid, underbid

Fake bids



Arbitrarily deviate from protocol

Assumption:

✓ cares about reputation

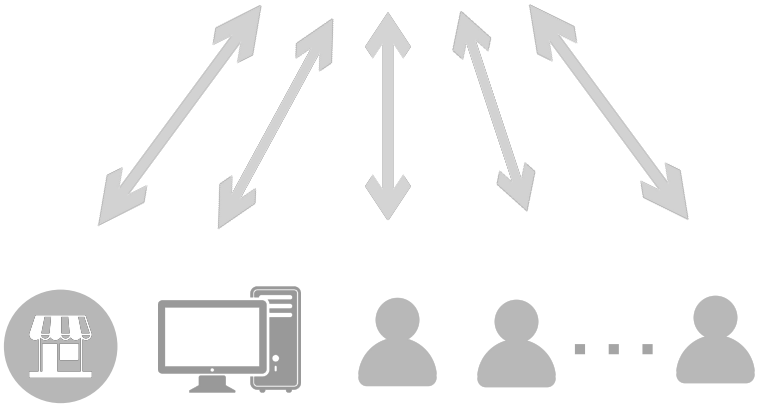
✓ adopts only **safe** strategies that do not risk detection

[Chung, Shi, SODA'23]

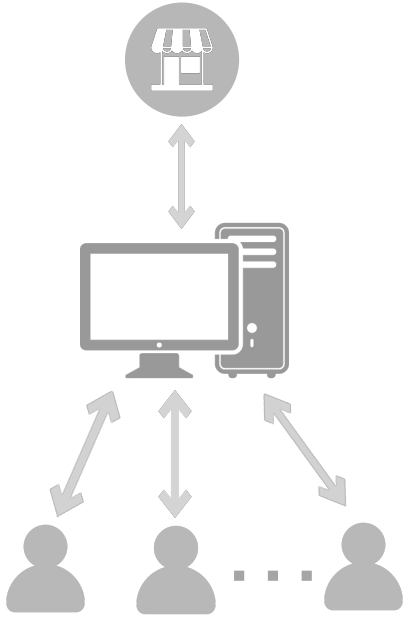
Why not just use MPC?

Ideal world

Ideal functionality

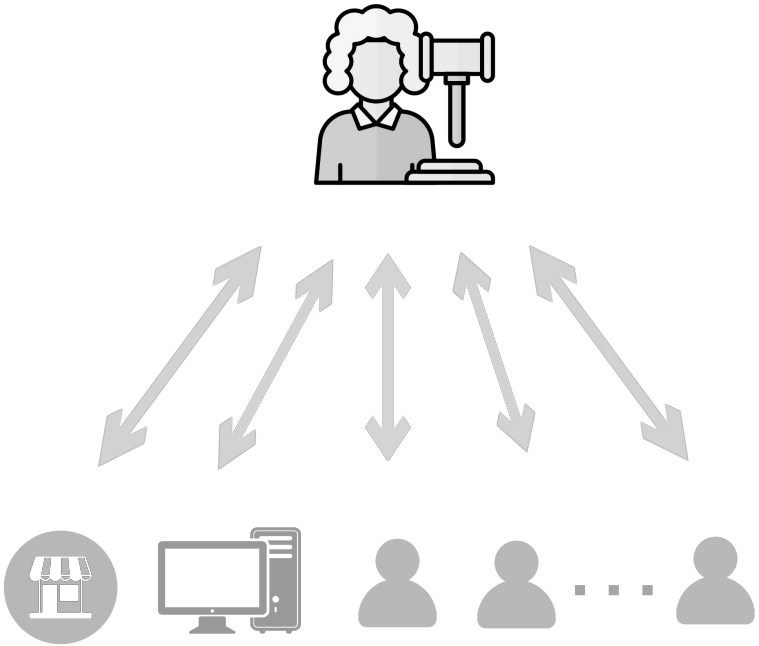


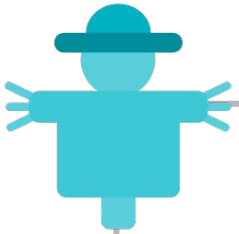
Real world



MPC

Ideal world

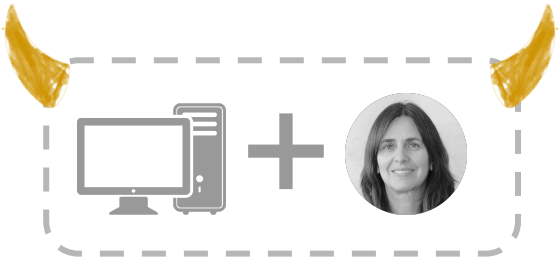




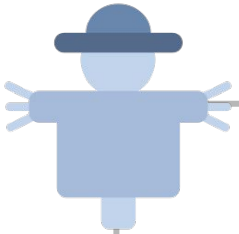
2nd price auction

- Allocate to top bidder
- Each pays 2nd price
- Platform gets 10% of revenue, seller gets the rest

[Vickerey61, Bogetoft08]



benefit from overbidding



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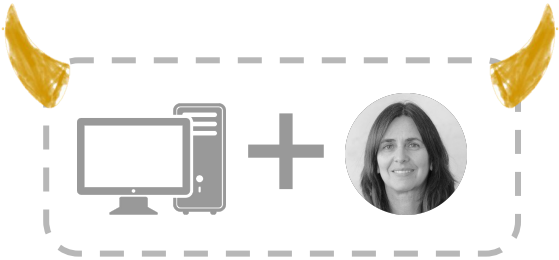

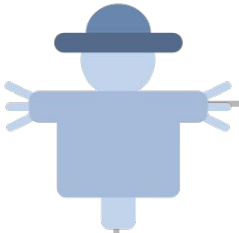

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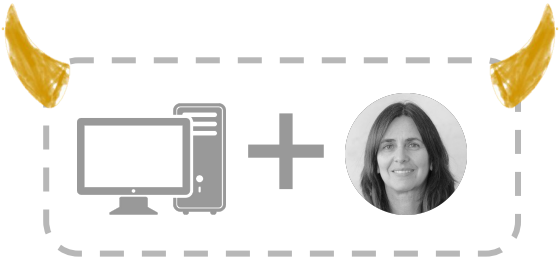

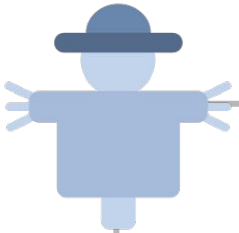

2nd price auction

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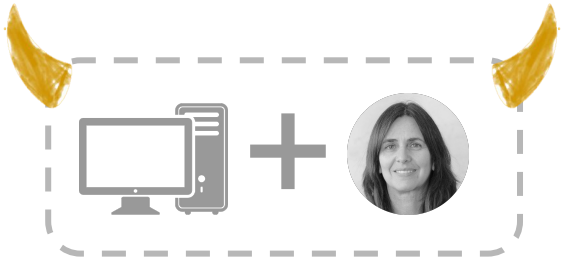
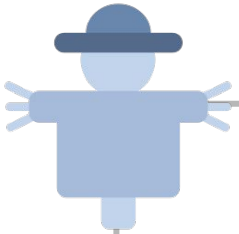
Example: 2 buyers

value = 5

value = 8





2nd price auction

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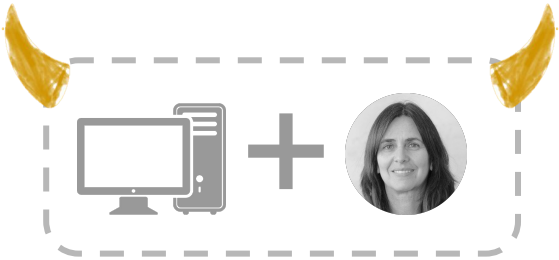
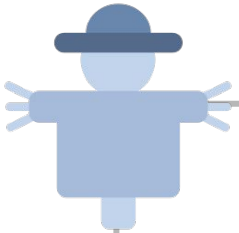
should bid **8 - ε**





2nd price auction

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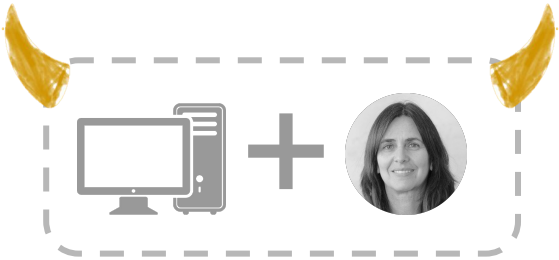
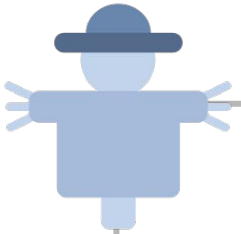
Example: 2 buyers

value = 5

value $\overset{\$}{\leftarrow}$ [0, 10]





2nd price auction

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Example: 2 buyers

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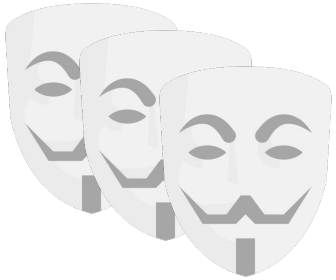
value $\overset{\$}{\leftarrow}$ [0, 10]

should bid **5.45**

Overbid, underbid

Fake bids



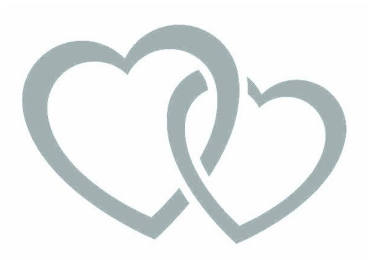
Arbitrarily deviate from protocol

No protection

MPC dos and don’ts

Can we have a **dream** platform-assisted auction?

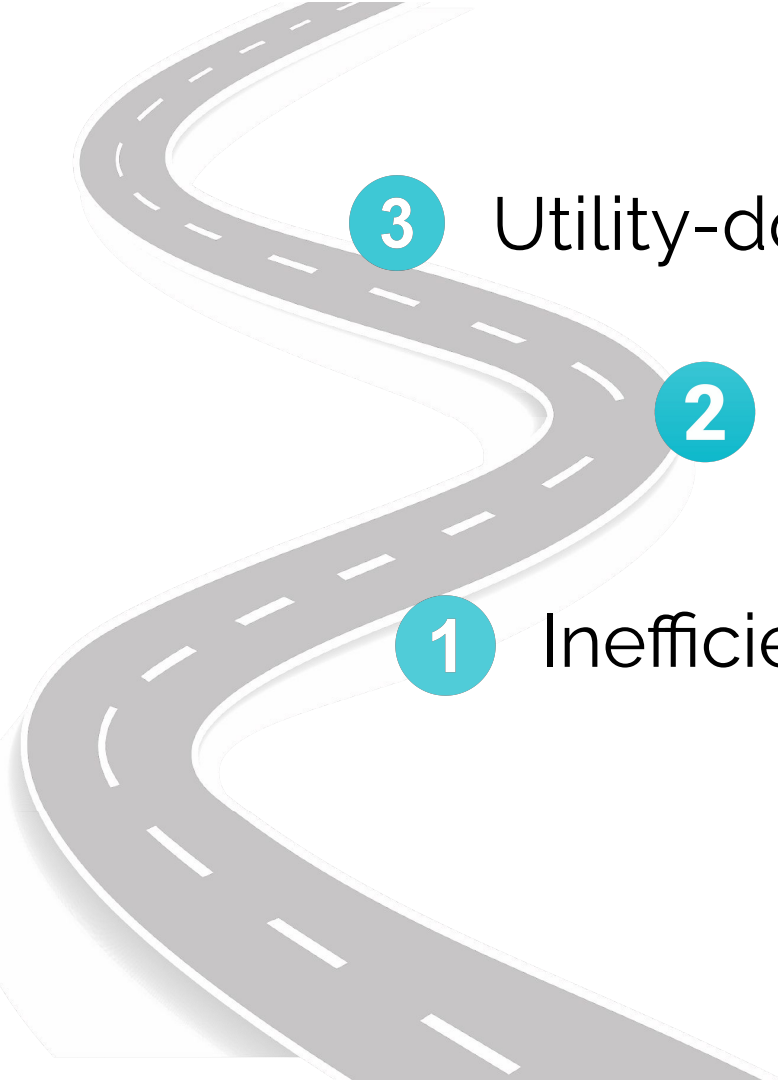
Crypto



Mechanism design

"**Decentralized** mechanism design"

Our Results



3 Utility-dominated emulation

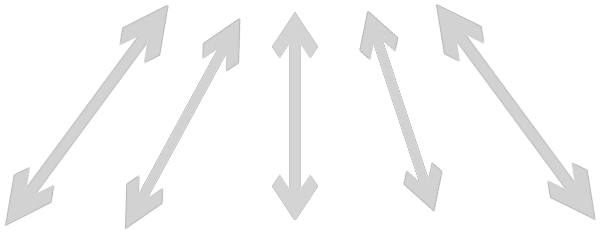
2 Fundamental limitations

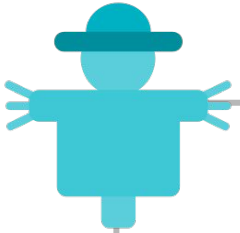
1 Inefficient MPC-based auction

Our Results

Recall the strawman MPC protocol

Ideal world



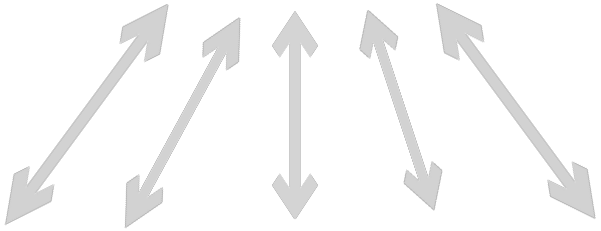


2nd price auction

- Allocate to top k bidders
- Sale price = (k+1)-st price
- Platform gets 10%
- Everyone learns their private outcome

The fix

Ideal world



2nd price with reserve R

- Allocate to top k bidders **who bid ≥R** 1
- Sale price = **(k+1)-st** price or **R, whichever greater**
- Platform gets **nothing** 2
- **Broadcast final price** to all 3

IC for:

✓ buyer

2nd price with reserve R

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IC for:

✓ buyer ✓ platform

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What can the platform do that the buyer cannot on its own?

2nd price with reserve R

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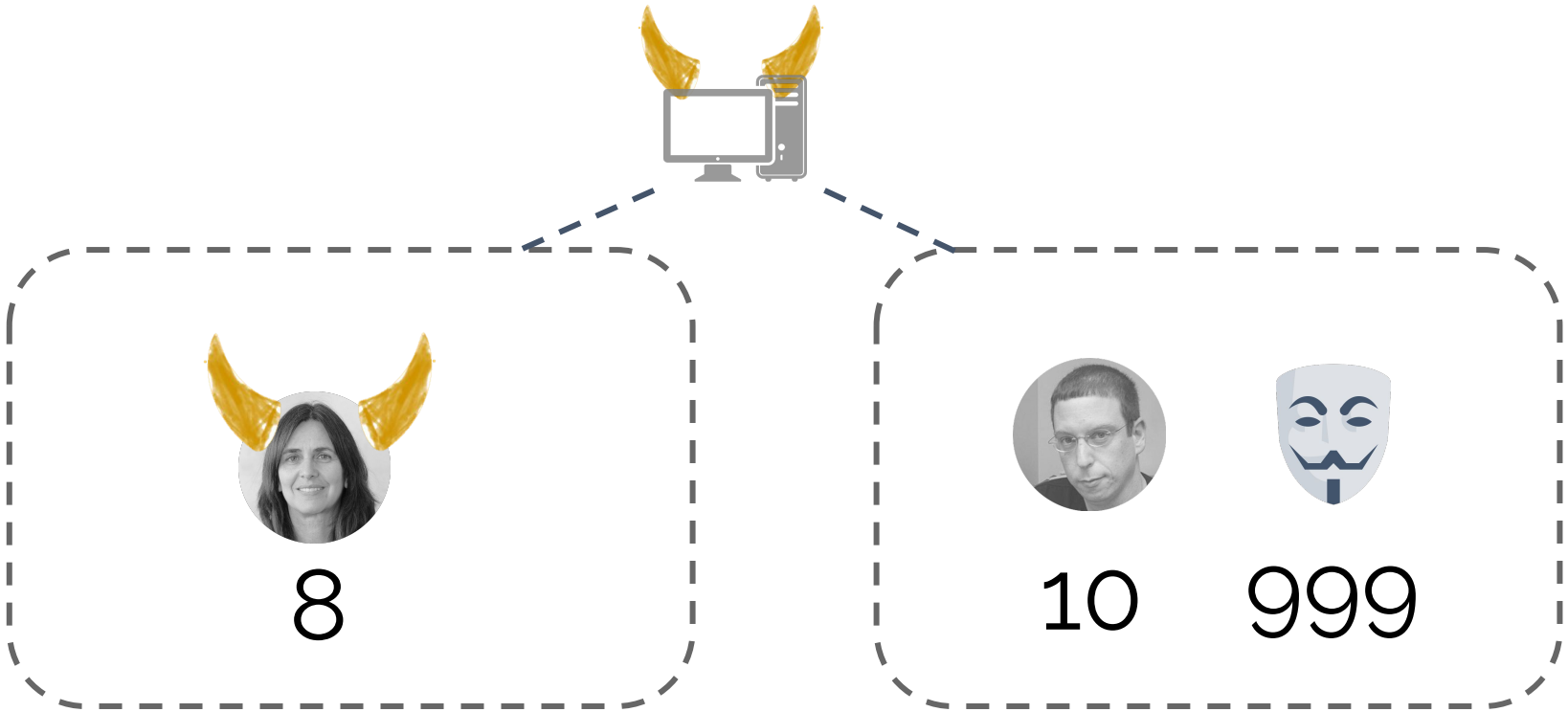
Broadcast prevents the “partitioned world” attack

8

10

1 item, reserve = 0

Broadcast prevents the “partitioned world” attack



1 item, reserve = 0

IC for:

✓ buyer ✓ platform

✓ platform-buyer coalition

Bayesian IC for:

✓ seller

✓ platform-seller coalition

assume: **suitable reserve**

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[AL'20]

IC for:

✓ buyer ✓ platform

✓ platform-buyer coalition

Bayesian IC for:

✓ seller

✓ platform-seller coalition

Bake optimal price floor into mechanism itself

assume: **suitable reserve**

2nd price with reserve R

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IC for:

✓ buyer ✓ platform

✓ platform-buyer coalition

Bayesian IC for:

✓ seller

✓ platform-seller coalition

Revenue optimal!

assume: **suitable reserve**

2nd price with reserve R

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Privacy of MPC is important!

Bayesian IC for:

- ✓ seller
- ✓ platform-seller coalition

Revenue optimal!

assume: **suitable reserve**

2nd price with reserve R

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Summary

IC for:

- ✓ buyer
- ✓ platform
- ✓ platform-buyer coalition

Bayesian IC for:

- ✓ seller
- ✓ platform-seller coalition

Revenue optimal!

assume: **suitable reserve**

2nd price with reserve R

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Limitations

2nd price with reserve R

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Limitations

Different fee structure

2nd price with reserve R

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Limitations

Different fee structure

Avoid the broadcast

2nd price with reserve R

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Limitations

Different fee structure

Avoid the broadcast

Improve efficiency

2nd price with reserve R

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Different fee structure

Avoid the broadcast

Improve efficiency

Impossible

2nd price with reserve R

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Different fee structure

Avoid the broadcast

Improve efficiency

Utility dominated emulation

2nd price with reserve R

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- Platform gets **nothing**
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Generic MPC incurs n^2 cost!

Generic MPC incurs n^2 cost!

- Each player has a different output
- indistinguishability obfuscation

  communication $\Longrightarrow$ **~O(n)**

  compute: still n^2

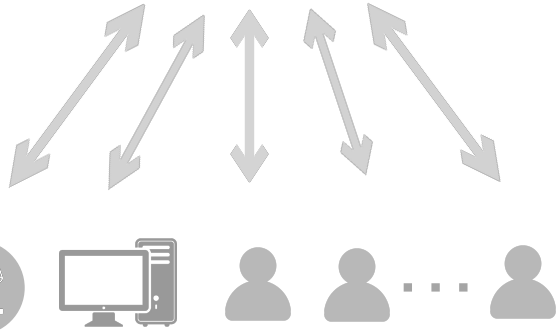
[HW15]

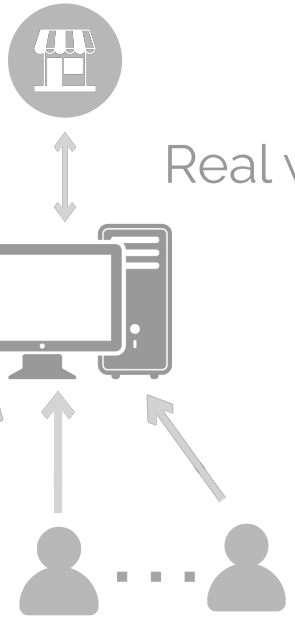
Design paradigm of MPC

Ideal world

Compiler

Real world

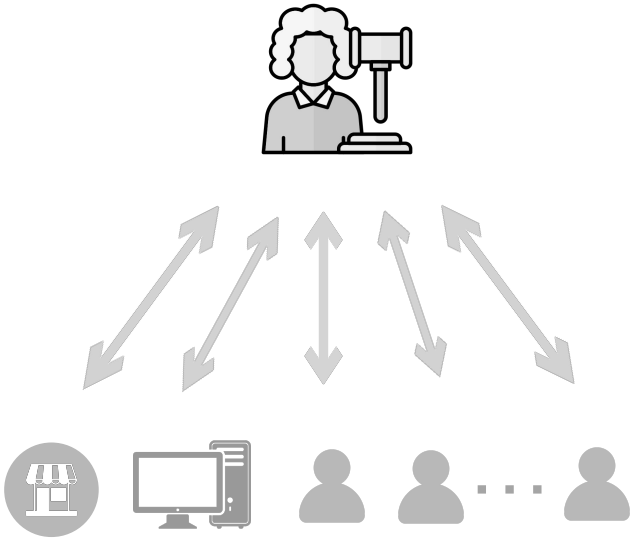


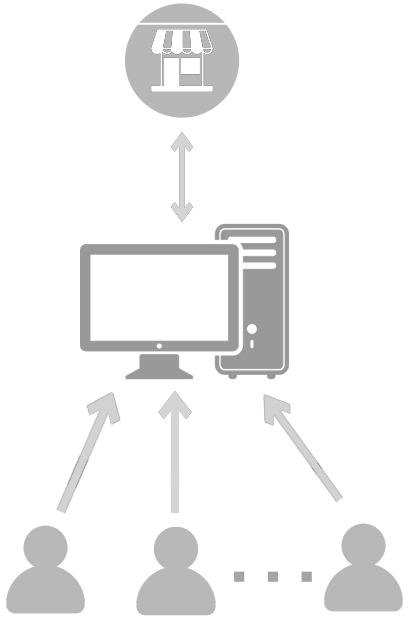


Mechanism designer

Cryptographer

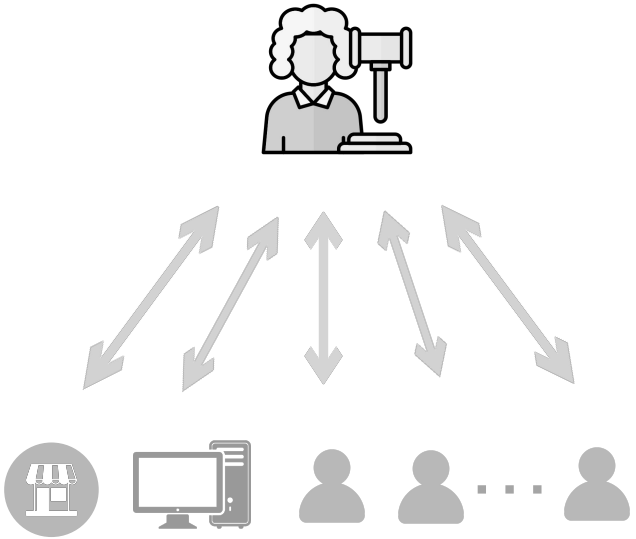
Can we improve the **efficiency** but preserve the **design paradigm?**

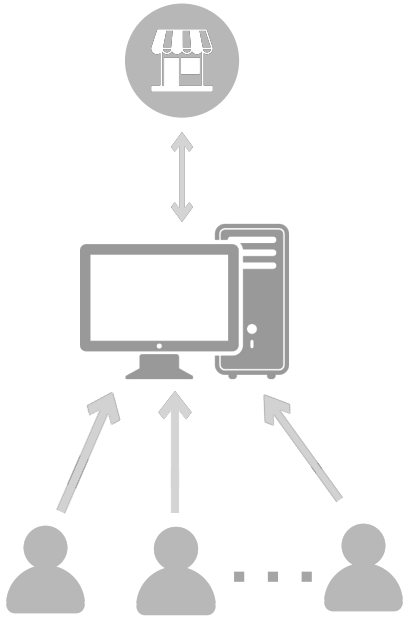




Utility-dominated emulation:

Any **real-world** strategy is **utility-dominated** by an **ideal-world** strategy





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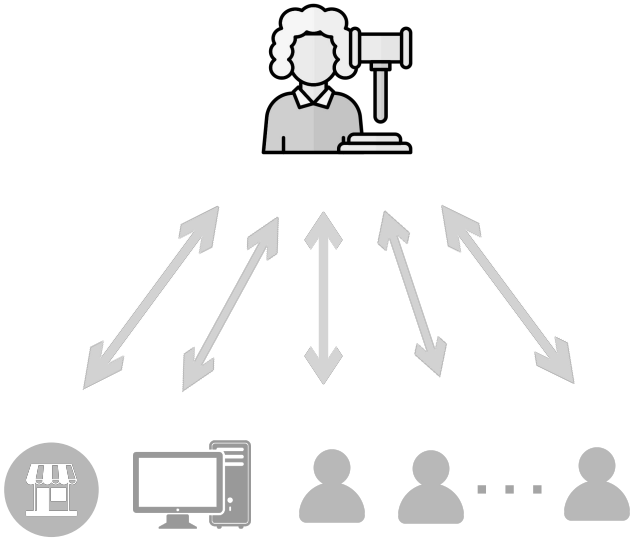
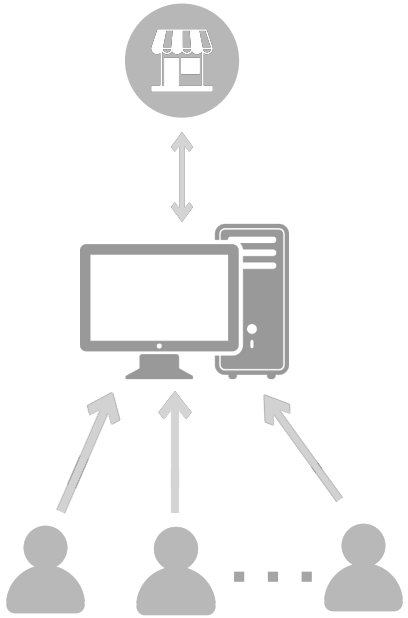




Thm: Ideal is IC + util-dominated emulation $\Longrightarrow$ Real is IC

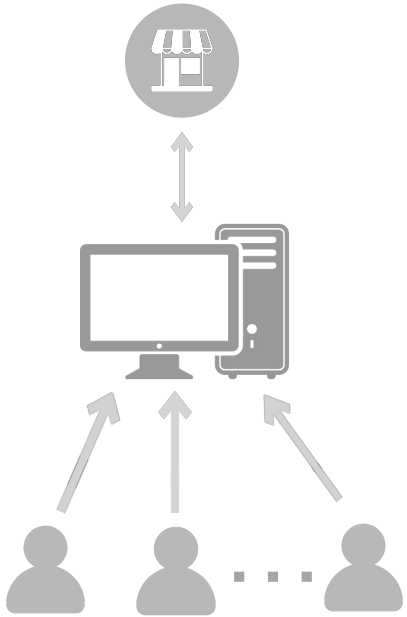
Our protocol

~O(n) cost

O(1) rounds

O(1) broadcast

broadcast necessary due to permissionless



See our paper for more

- More impossibilities & structural characterizations
- Efficient cryptography construction using ZK
- Computationally sound defn of “safe deviation”
- Proofs

https://eprint.iacr.org/2025/019

Decentralized mechanism design:

a **goldmine** of open questions

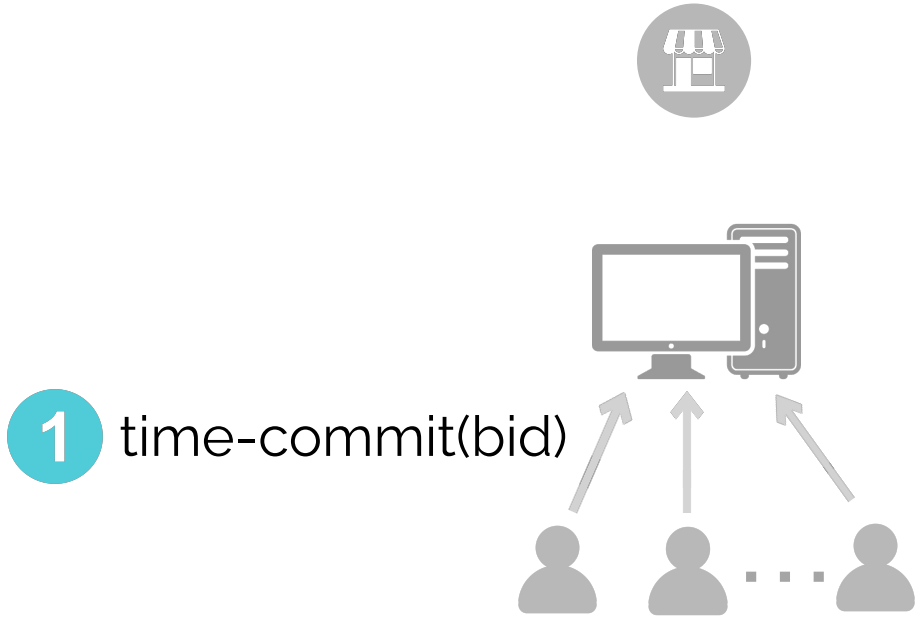
- Biggest challenge for **blockchains**
- **Heuristic** protocols used in practice
- What's the **right game-theoretic notion**?
- **Crypto** meets mechanism design

Thank you !

elainershi@gmail.com

The protocol

1 Buyers send timed commitments of bids

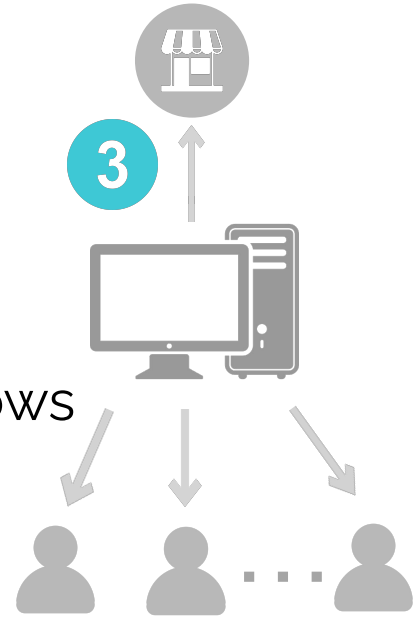


1 Buyers send timed commitments of bids

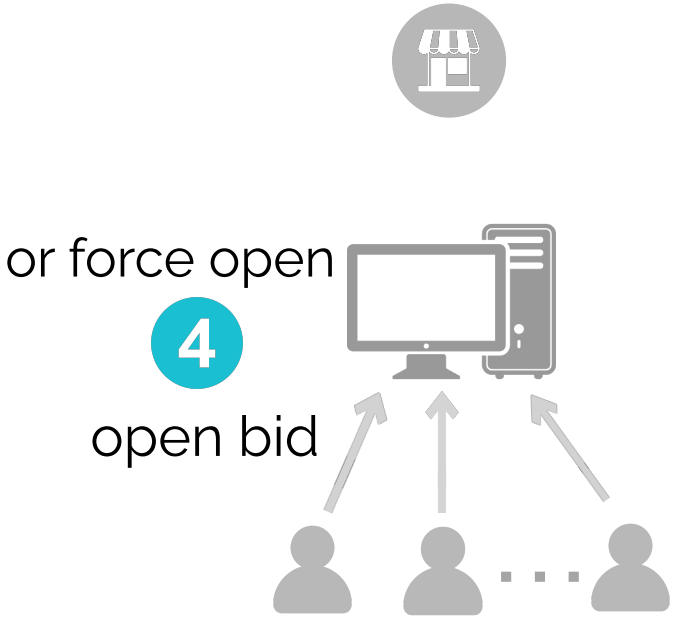
2 Platform broadcasts hash of commitments

1. Buyers send timed commitments of bids
2. Platform broadcasts hash of commitments
3. Platform proves it knows opening of hash

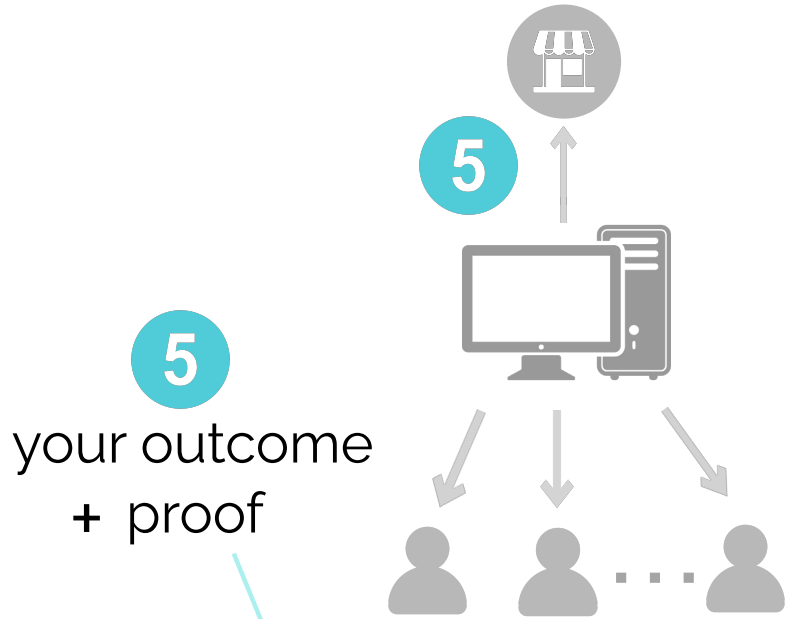
3 proof that it knows the opening



1. Buyers send timed commitments of bids
2. Platform broadcasts hash of commitments
3. Platform proves it knows opening of hash
4. Open or force-open bids



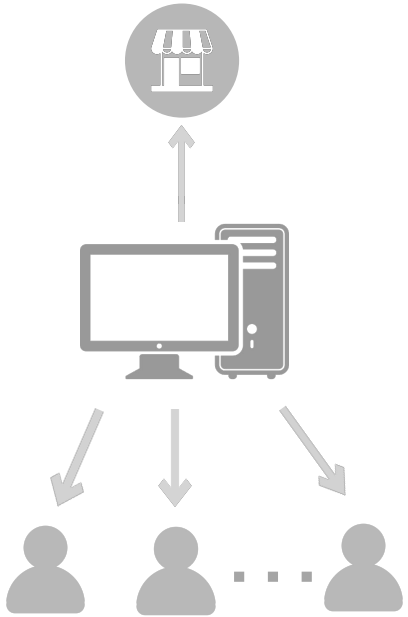
1. Buyers send timed commitments of bids
2. Platform broadcasts hash of commitments
3. Platform proves it knows opening of hash
4. Open or force-open bids
5. Platform sends everyone its outcome + proof



- outcome correct w.r.t. hash
- buyer's bid is included once

1. Buyers send timed commitments of bids
2. Platform broadcasts hash of commitments
3. Platform proves it knows opening of hash
4. Open or force-open bids
5. Platform sends everyone its outcome + proof

O(1) rounds
O(1) bcast
~O(n) cost



Thank you !

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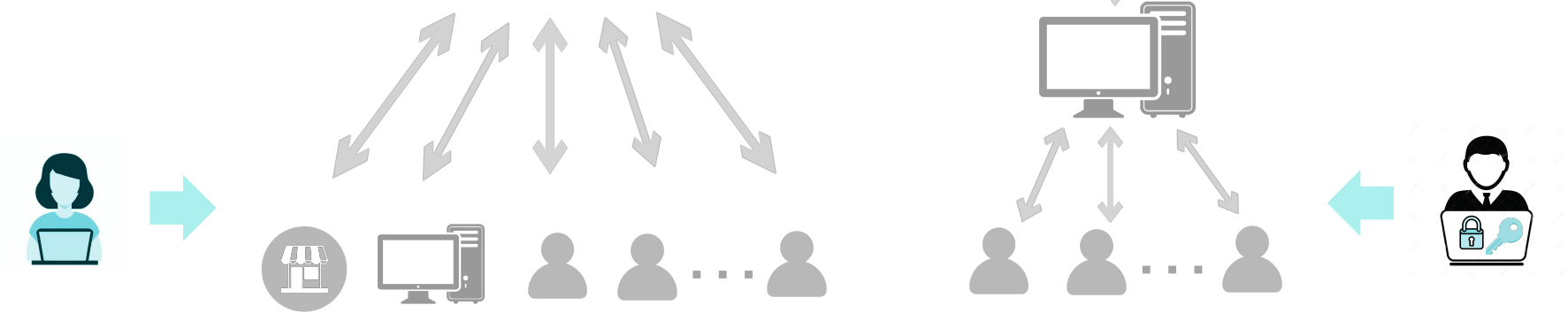
Design paradigm of MPC

Ideal world

Compiler

Real world

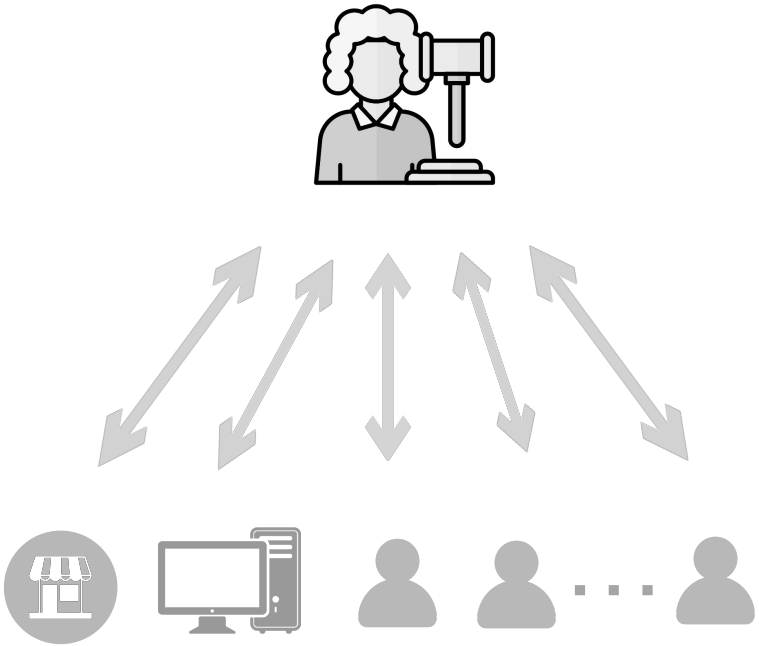


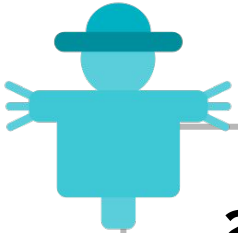
Mechanism designer

Cryptographer

Summary

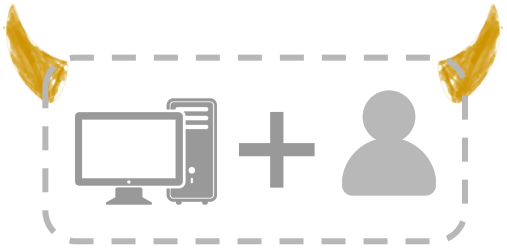
Ideal world





2nd price with reserve R

- Ignore bids under R
- Allocate to top k bidders
- Each pays (k+1)-st price or R, whichever greater
- Platform gets 10% of revenue, seller gets the rest



can increase their expected gain by overbidding

2nd price with reserve R

- Ignore bids under R
- Allocate to top k bidders
- Each pays (k+1)-st price or R, whichever greater
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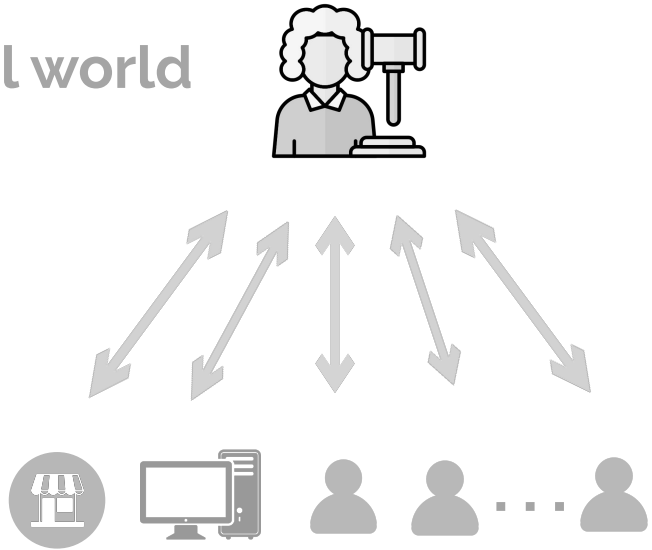
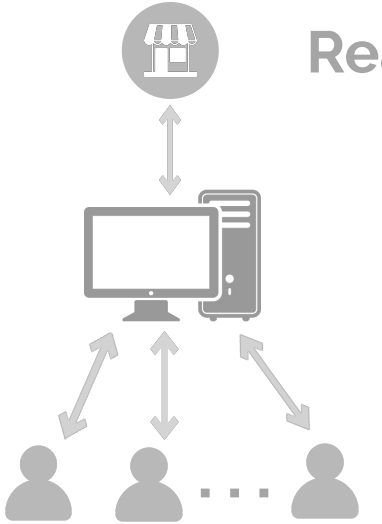
Can we improve the efficiency but preserve the design paradigm?

Ideal world



Real world



Design paradigm of MPC

Can we preserve the efficiency but preserve the design paradigm?

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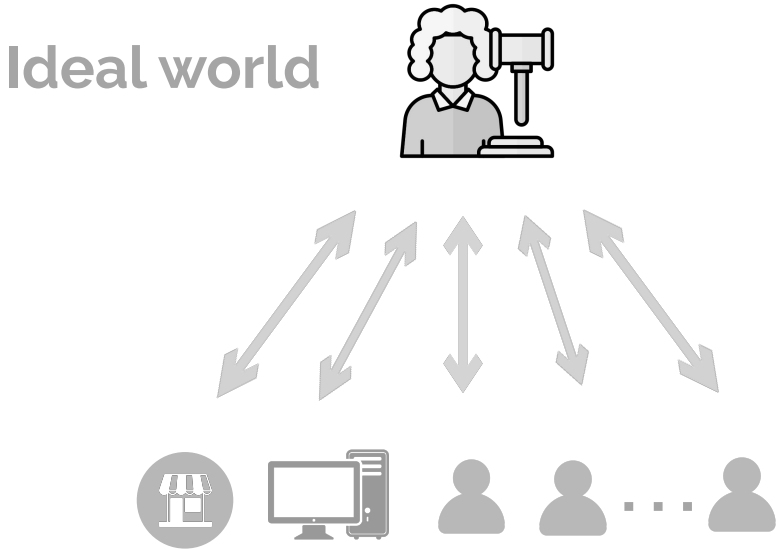
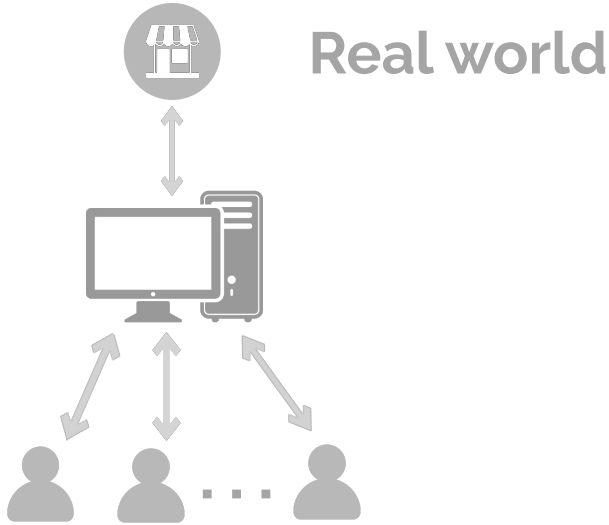
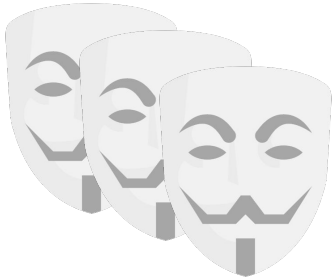




Overbid, underbid

Fake bids



Arbitrarily deviate from protocol

Assumption:

✓ cares about reputation

✓ adopts only **safe** strategies that do not risk detection

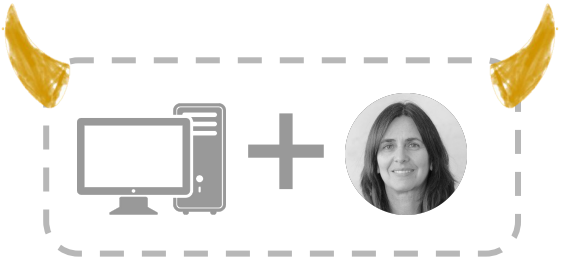
[Chung, Shi, SODA'23]

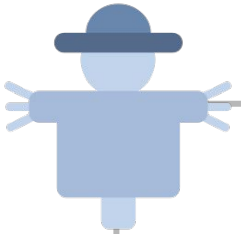
Crypto

ensures **faithful** impl.
of auction rules

cannot prevent

- misreporting input
- injecting fake bids





2nd price auction

- Allocate to top k buyers
- Each pays (k+1)-st price
- Platform gets 10% of revenue, seller gets the rest

Example: 1 item 2 buyers

$$\text{value} = \frac{1}{2}$$

$$\text{value} \xleftarrow{\$} [0,1]$$

$$\text{should bid } \frac{1}{2} + \frac{1}{22}$$

IC for:

✓ buyer ✓ platform

2nd price with reserve R

- Allocate to top k bidders **who bid ≥R**
- Sale price = **(k+1)-st** price or **R, whichever greater**
- Platform gets **nothing**
- **Broadcast final price** to all

Can we have a **dream** platform-assisted auction?

- What **fee structure** should we use?
- How does **cryptography** help?
- What **comm. structure** is needed?
- How many **rounds** do we need?

IC for:

✓ buyer ✓ platform

✓ platform-buyer coalition

2nd price with reserve R

- Allocate to top k bidders **who bid ≥R**
- Sale price = **(k+1)-st** price or **R, whichever greater**
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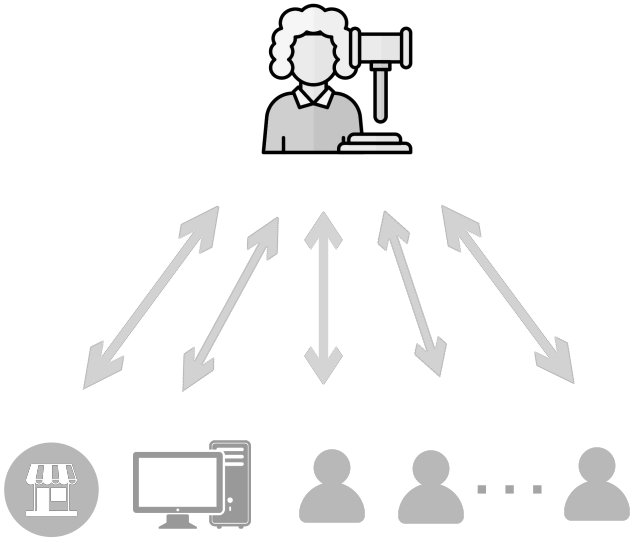

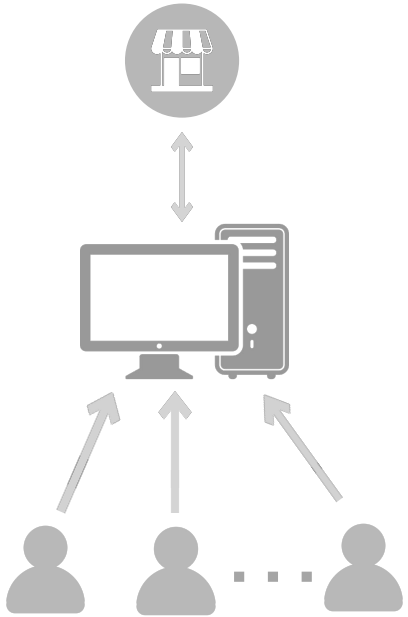

Strategic util in Real ≤ Strategic util in Ideal ≤ Honest util in Ideal

= Honest util in Real