Breaking Verifiable Delay Functions in the Random Oracle Model

Great Random Oracle, bless me a puzzle that takes ages to solve, one glance to check, and exactly one right answer!

Ziyi Guan

Joint work with Artur Riazanov, Weiqiang Yuan

EPFL

The random oracle model

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Random oracle $\mathcal{O} := \{\mathcal{O}_\ell\}_{\ell \in \mathbb{N}}$

$\mathcal{O}_\ell$: uniform distribution over $f\colon \{0,1\}^* \to \{0,1\}^\ell$

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Algorithm A
f

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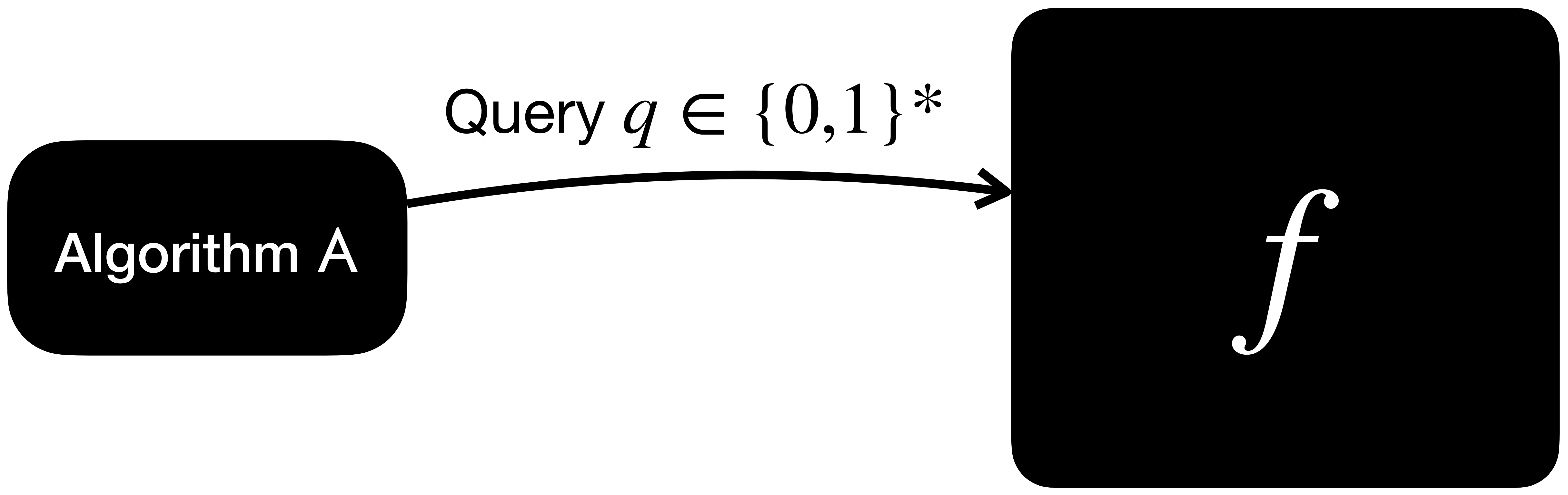

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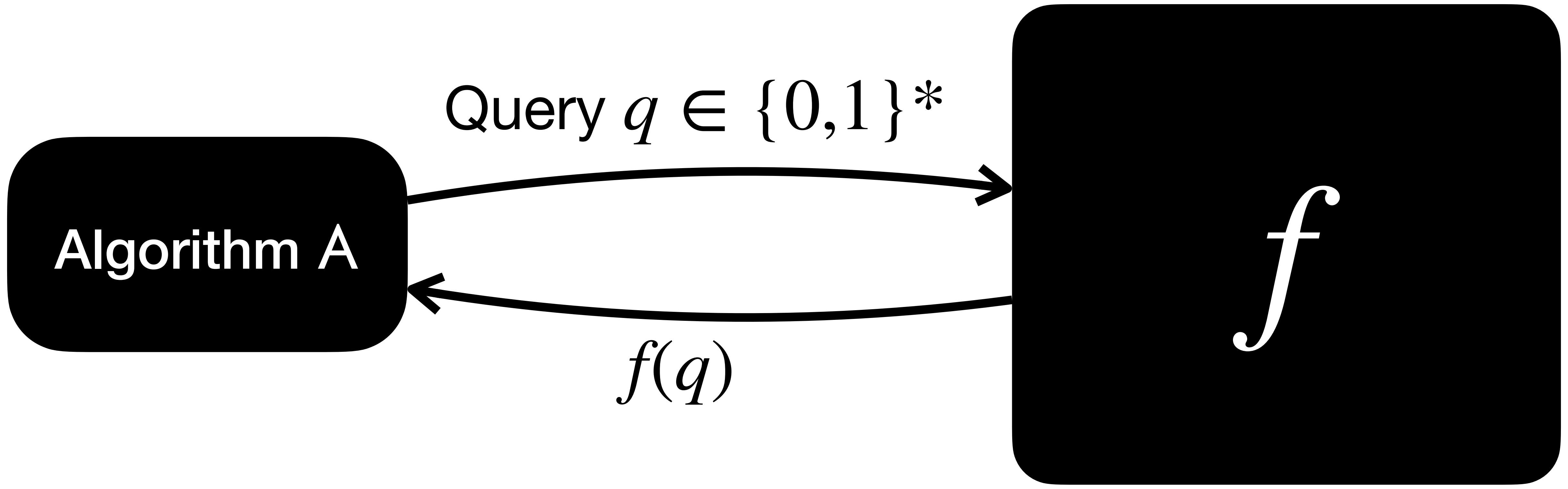

Verifiable Delay Function (VDF)

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$\mathsf{Eval}^f(x)$

$\mathsf{Verify}^f(x, y, \pi)$

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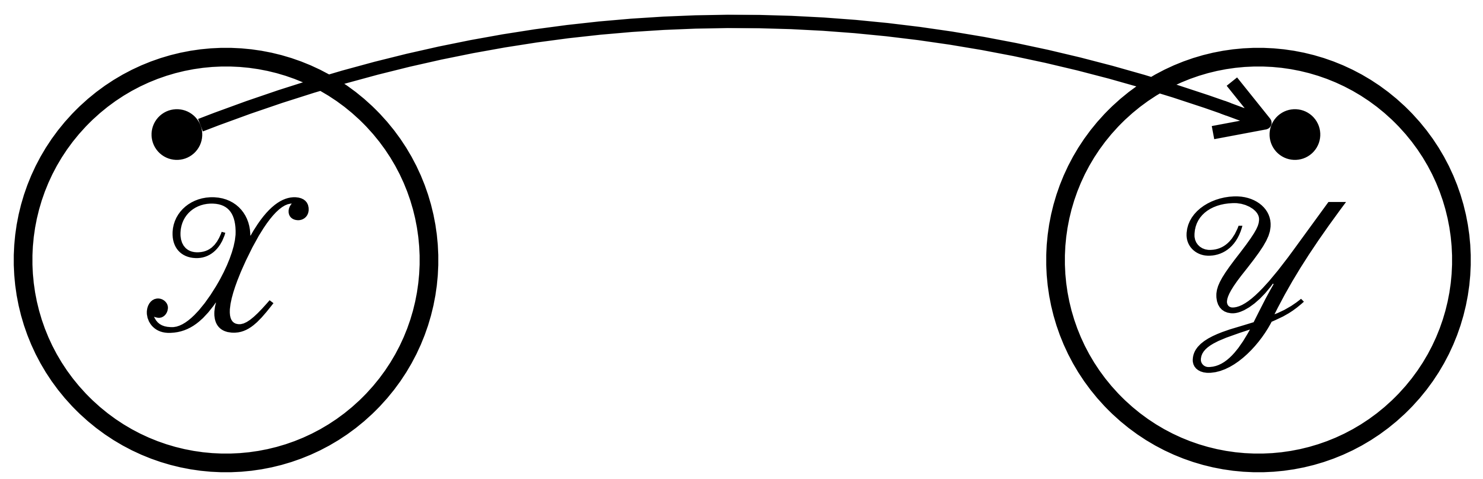

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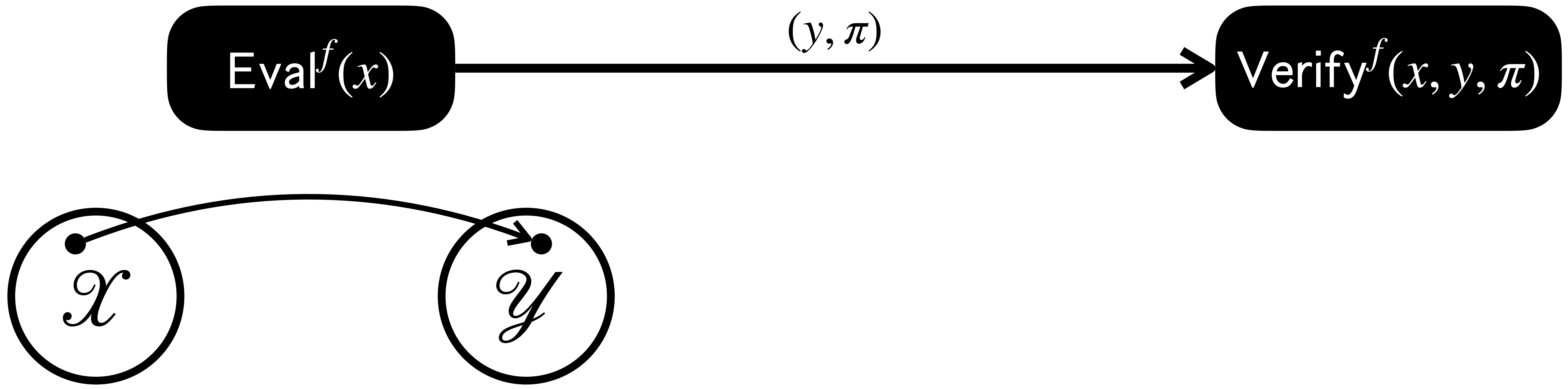
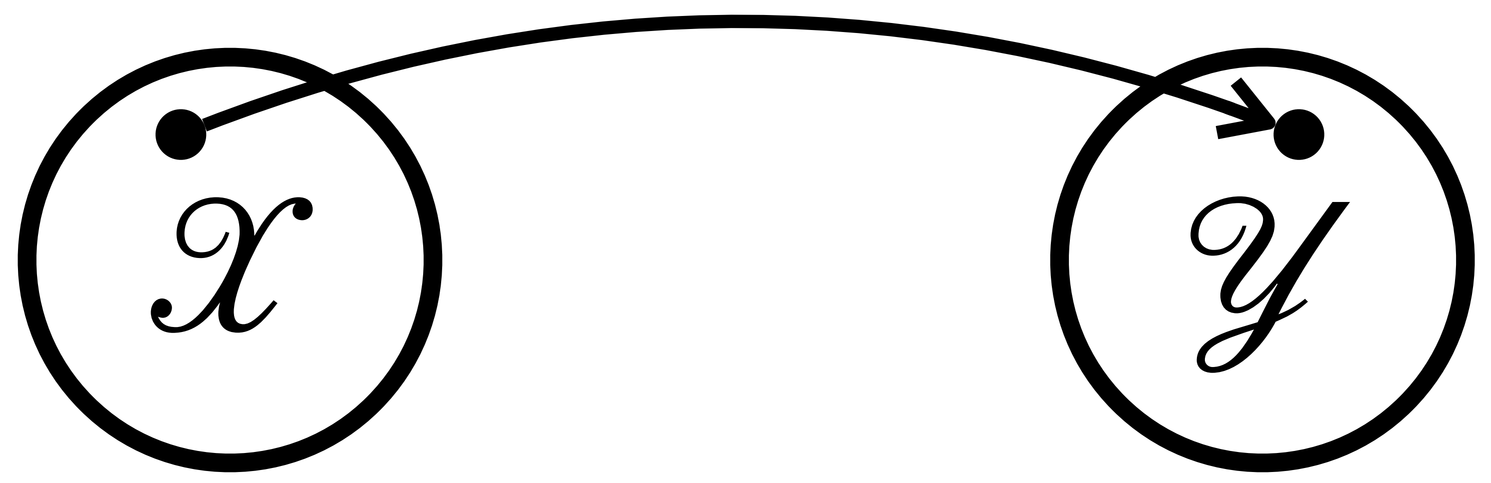


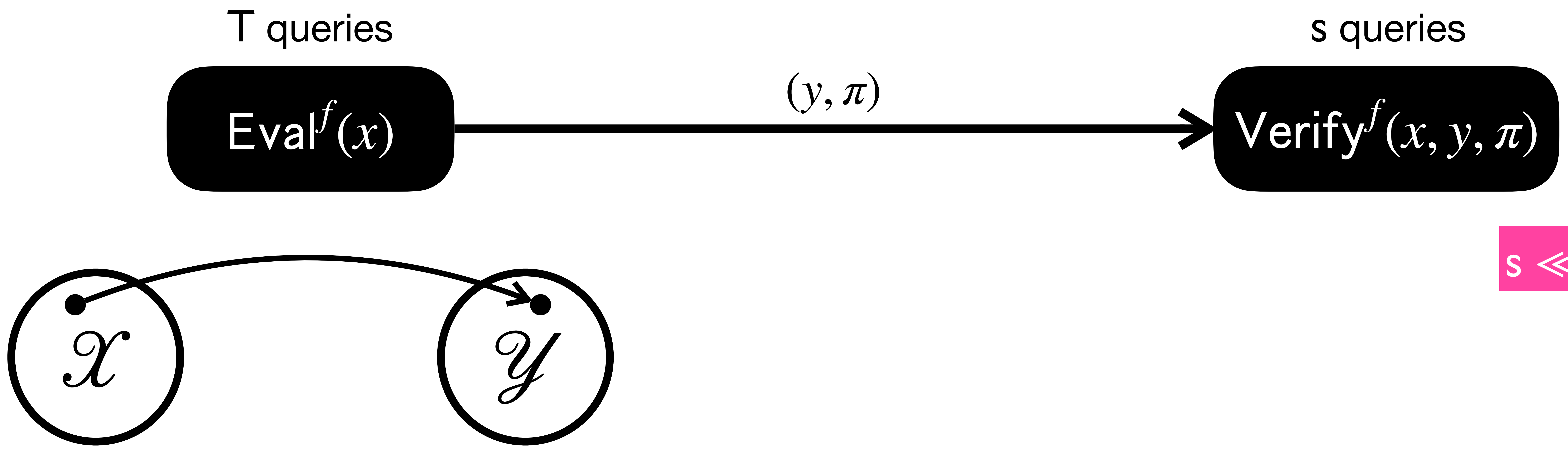
Verifiable Delay Function (VDF)

$$\boxed{\mathsf{Eval}^f(x)} \xrightarrow{(y,\pi)} \boxed{\mathsf{Verify}^f(x,y,\pi)}$$



VERIFIABLE → correctness of output efficiently publicly verifiable

Verifiable Delay Function (VDF)



$s \ll T$

VERIFIABLE → correctness of output efficiently publicly verifiable

Verifiable Delay Function (VDF)

T queries s queries

$$\begin{tikzcd}
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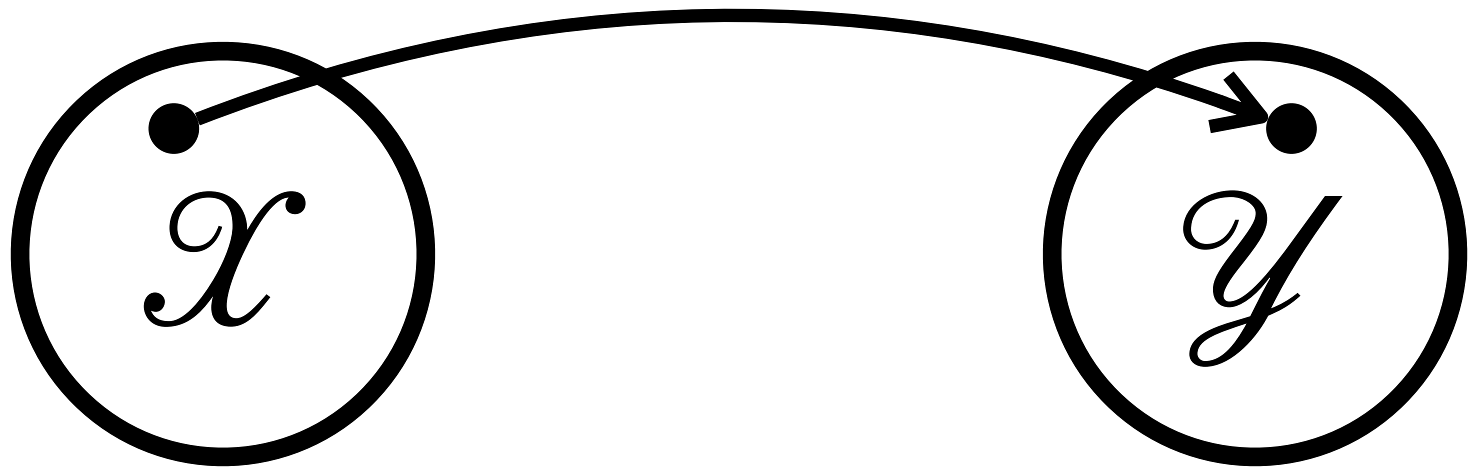

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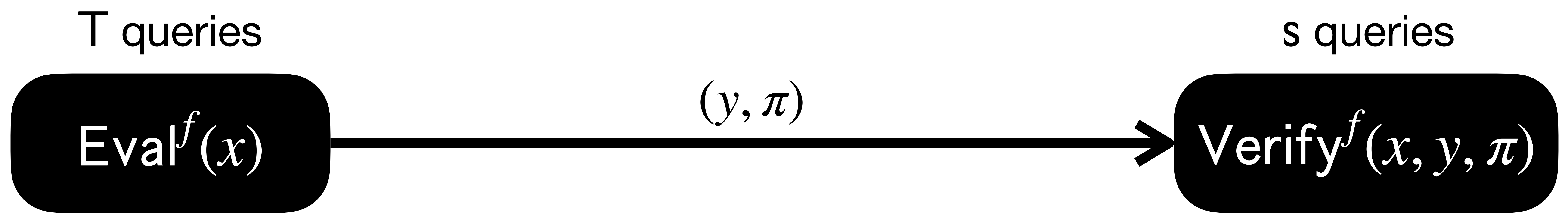

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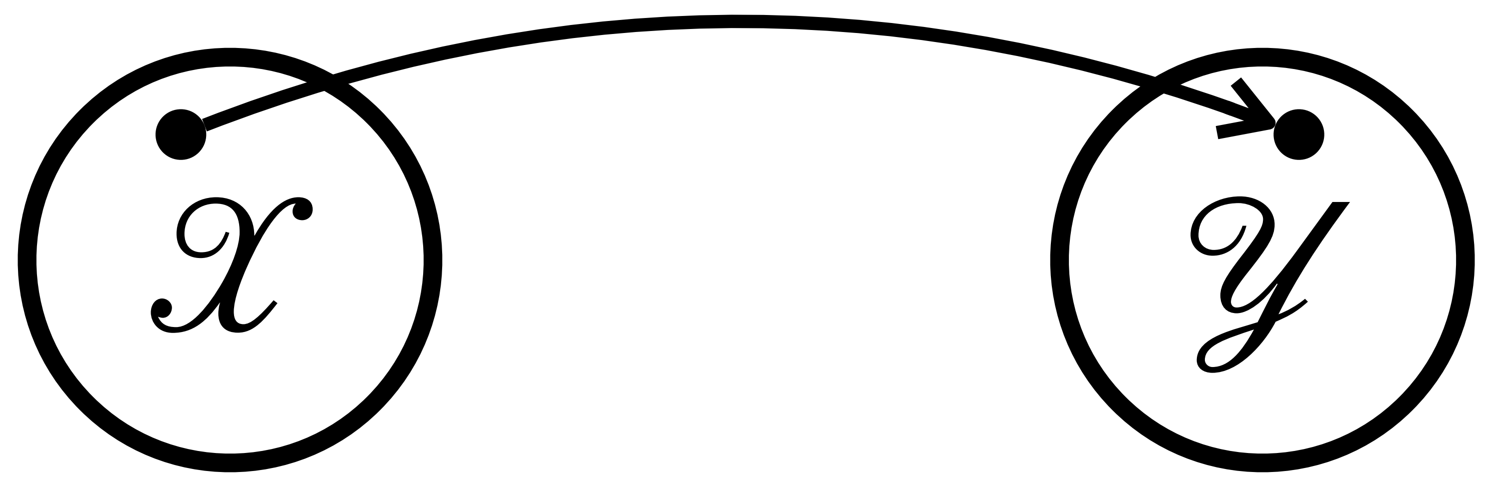

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→ Cannot be evaluated in $o(T)$ rounds of queries

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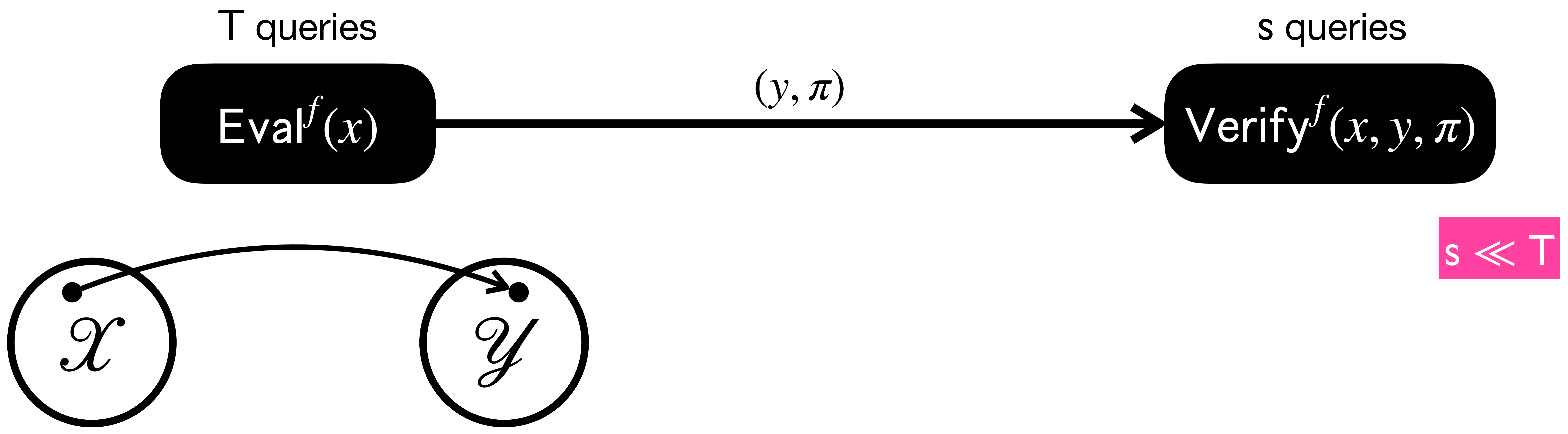

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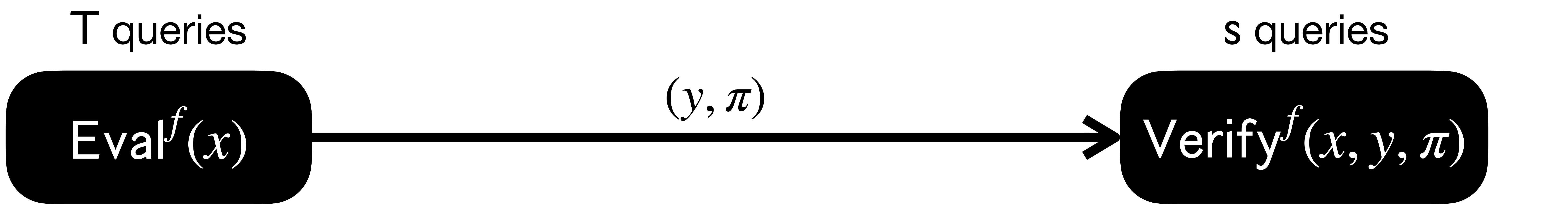

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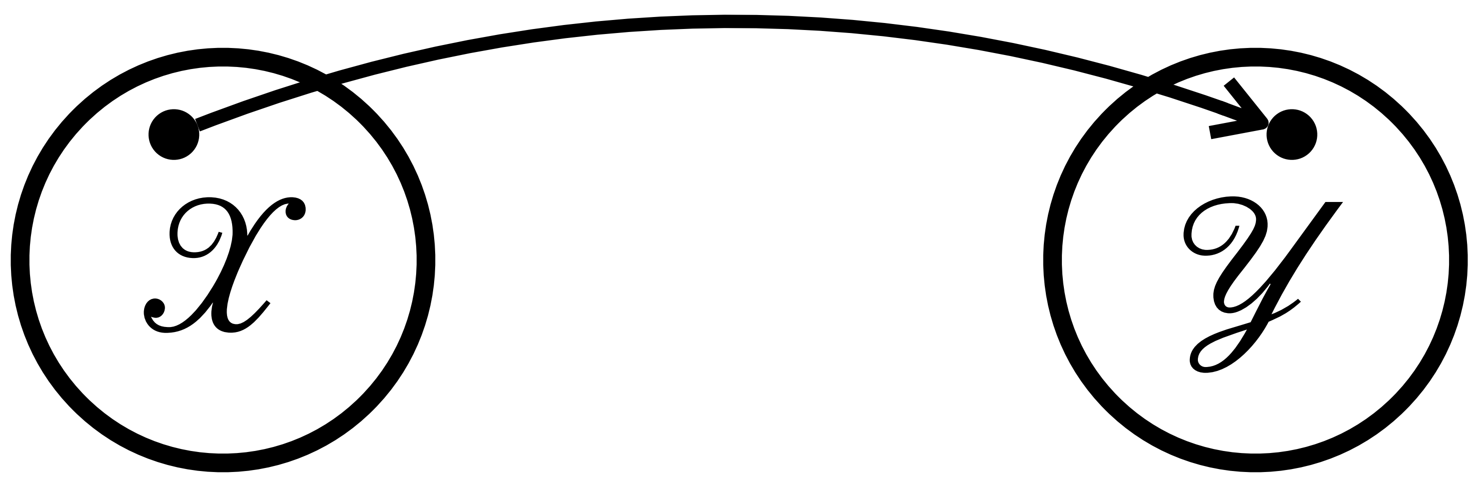

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FUNCTION → one **unique** output

Why study VDF?

Randomness beacon

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- Publish randomness regularly
- Cannot predict/manipulate

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Stock Prices (public source of randomness)

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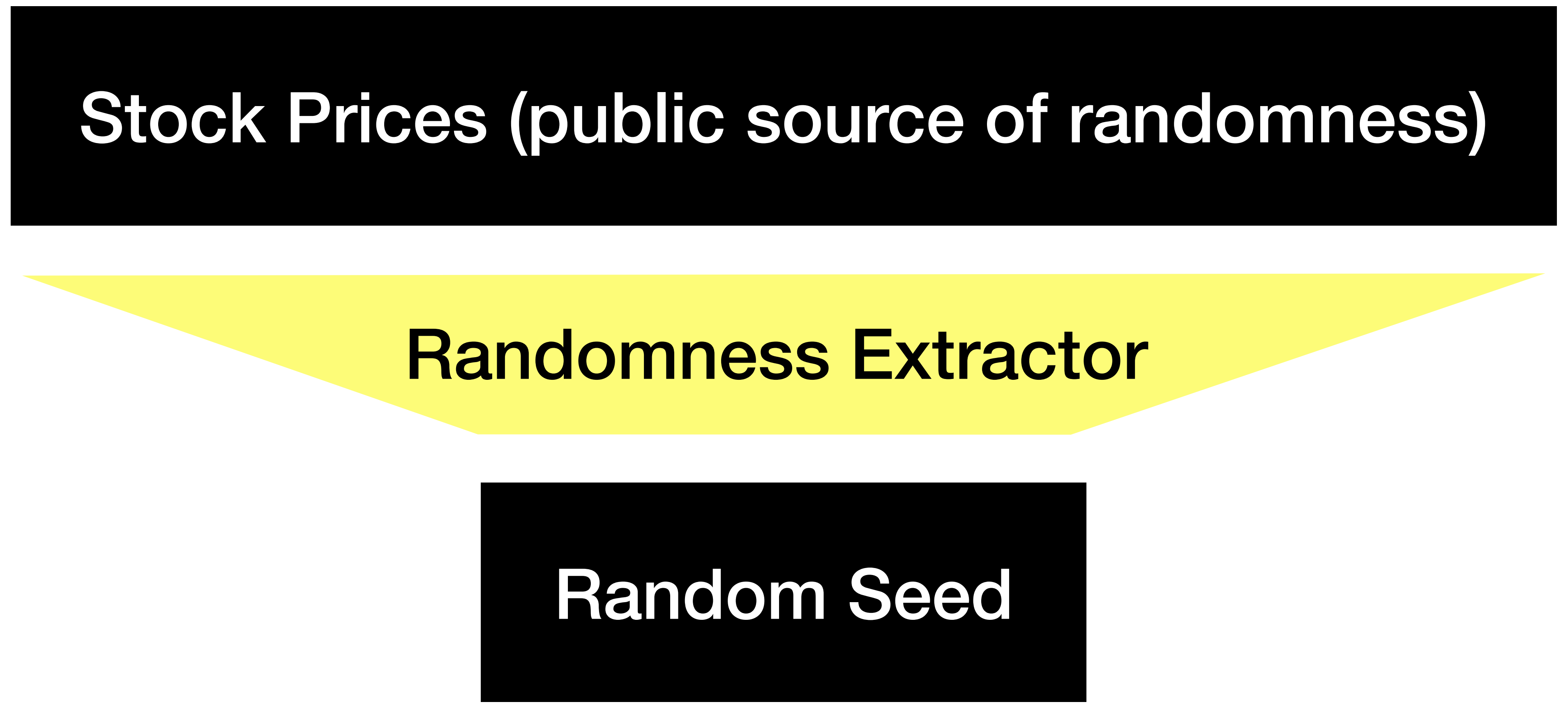

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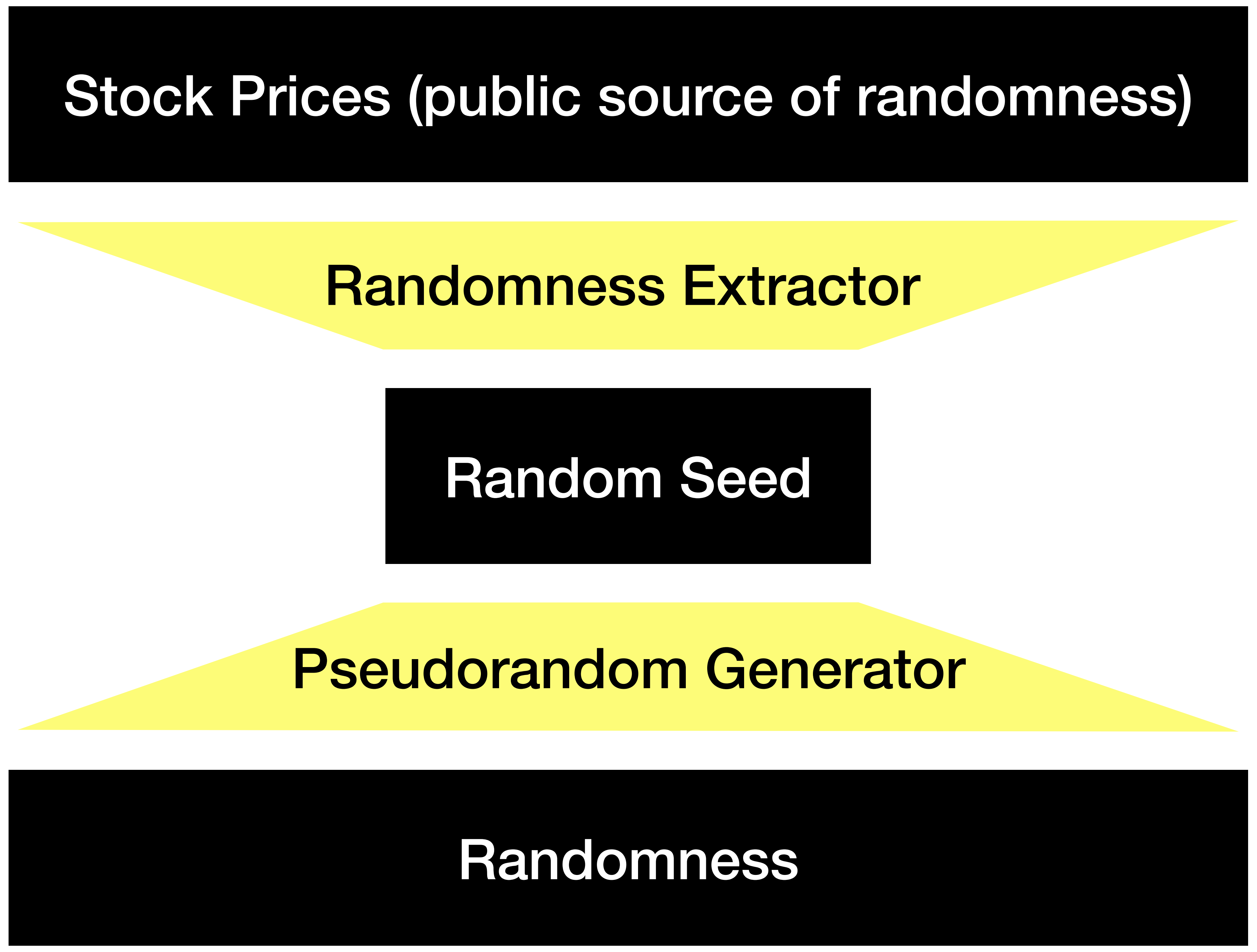

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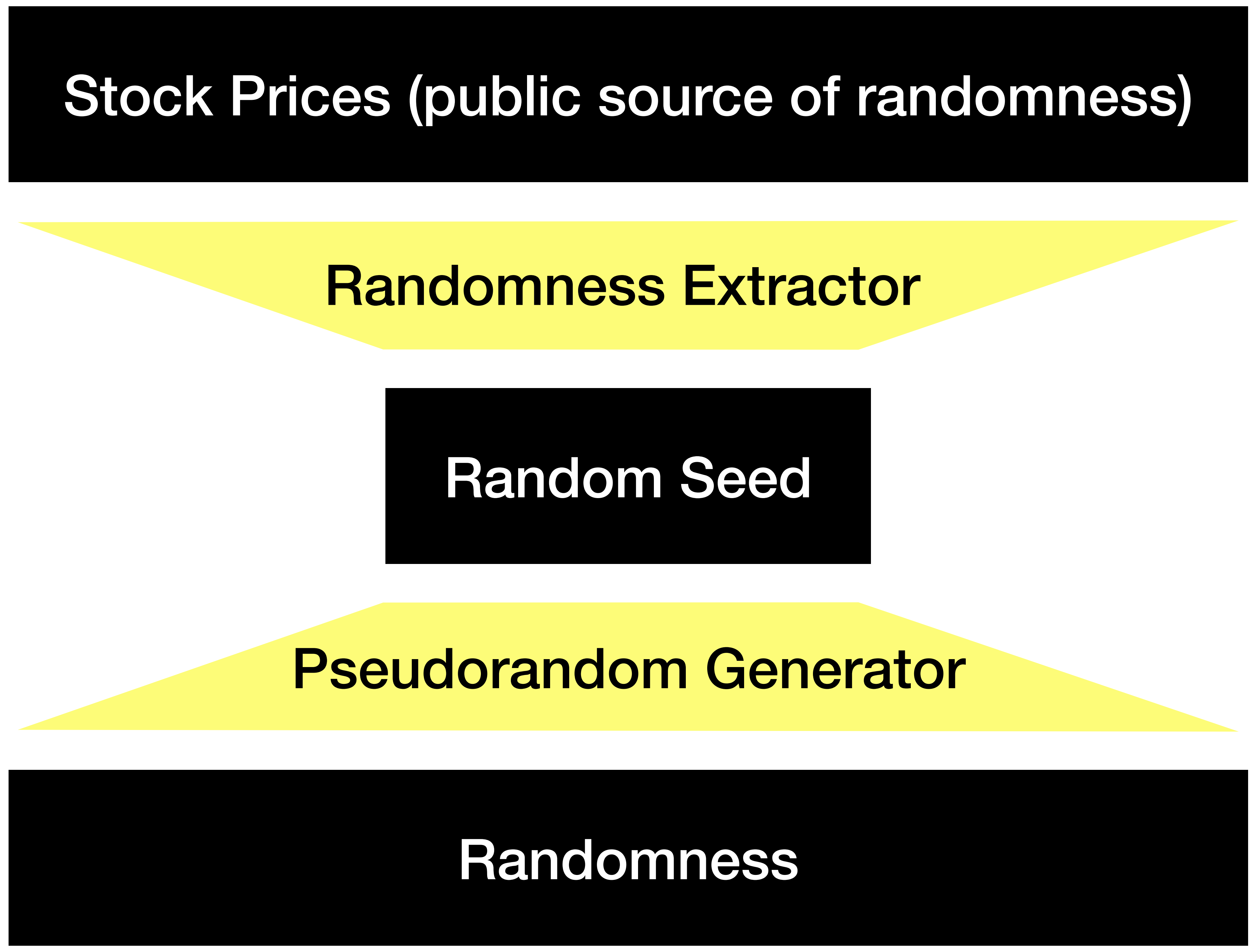

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(Stock prices can be biased/manipulated)

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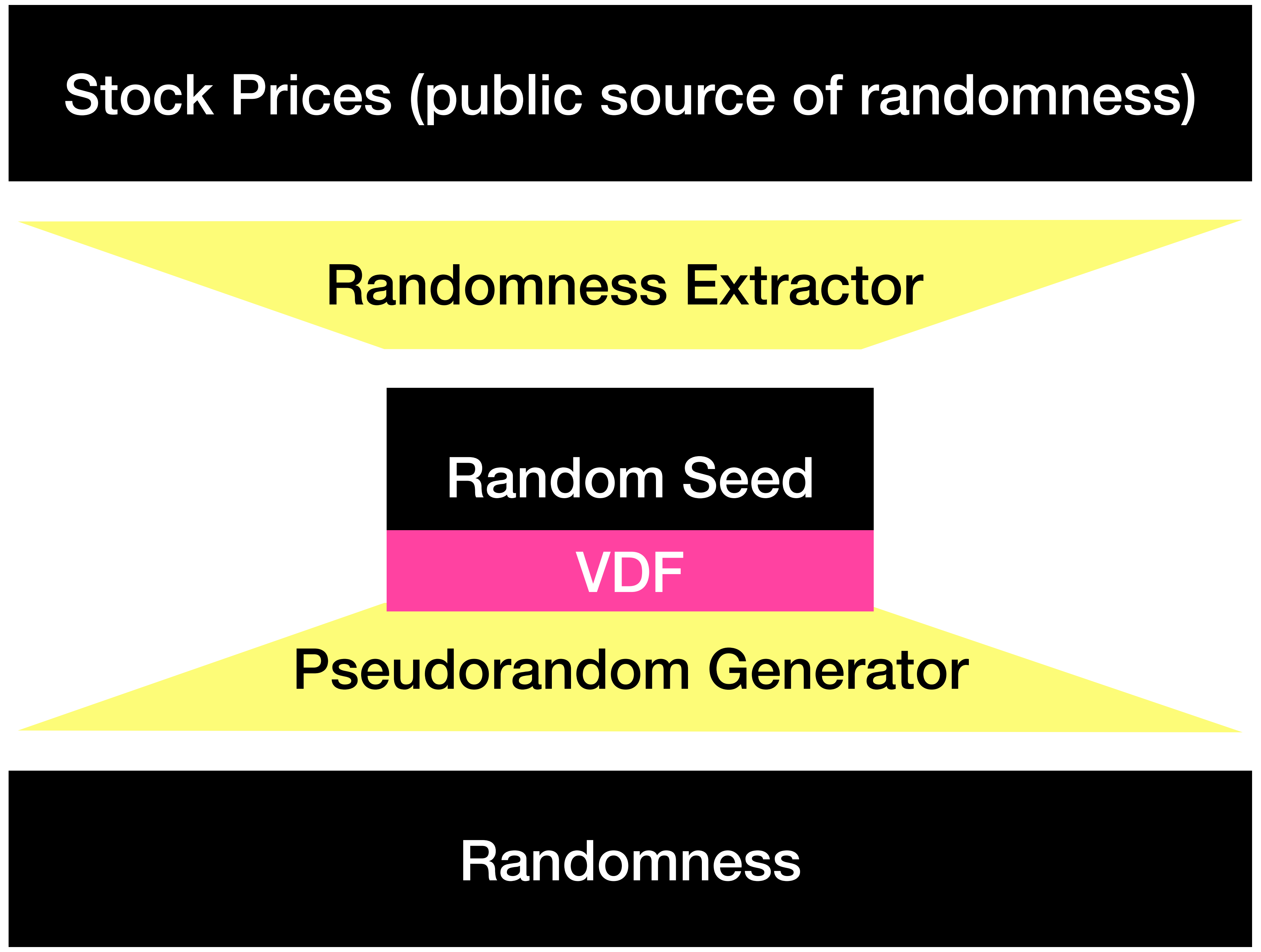

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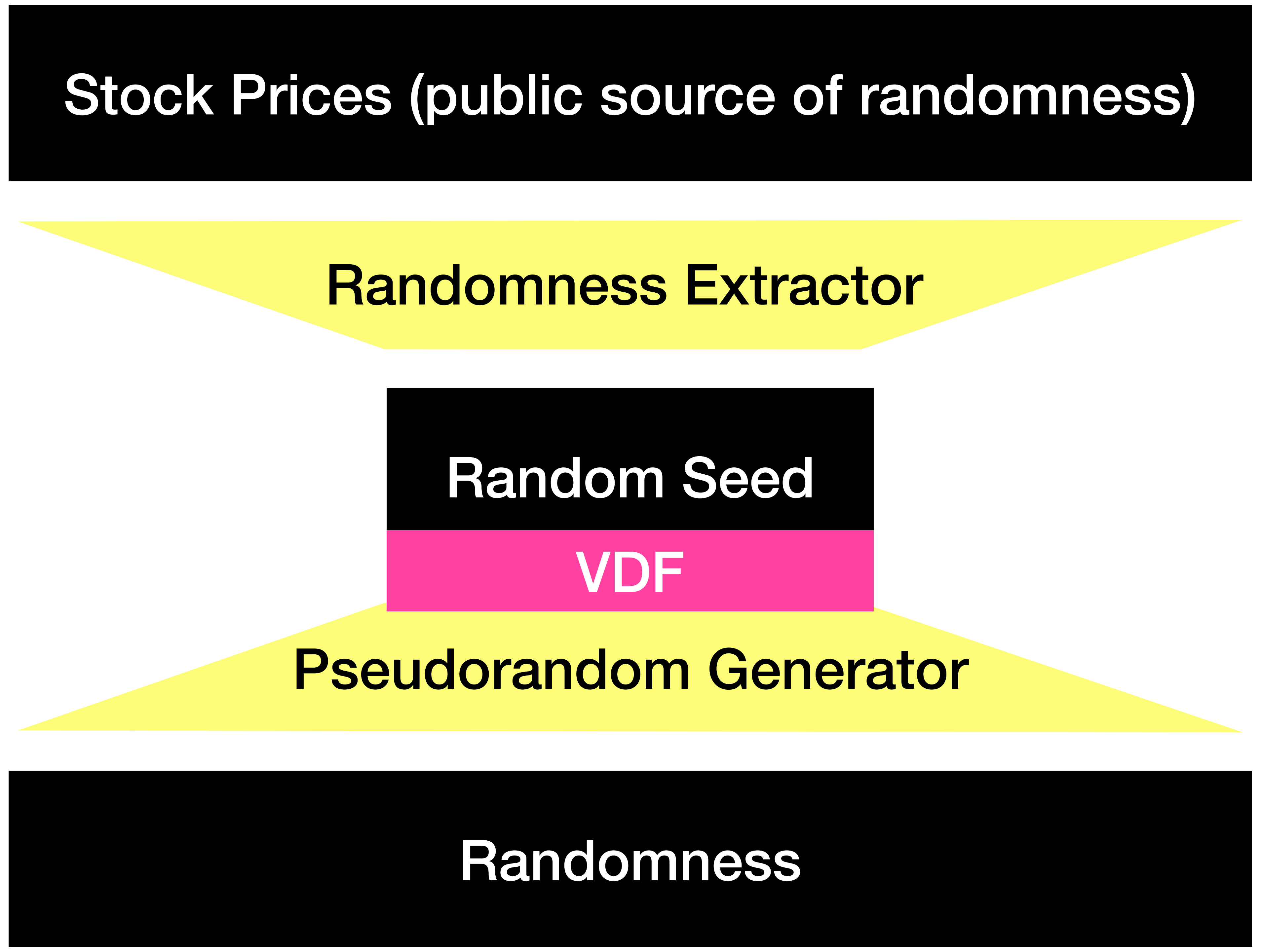


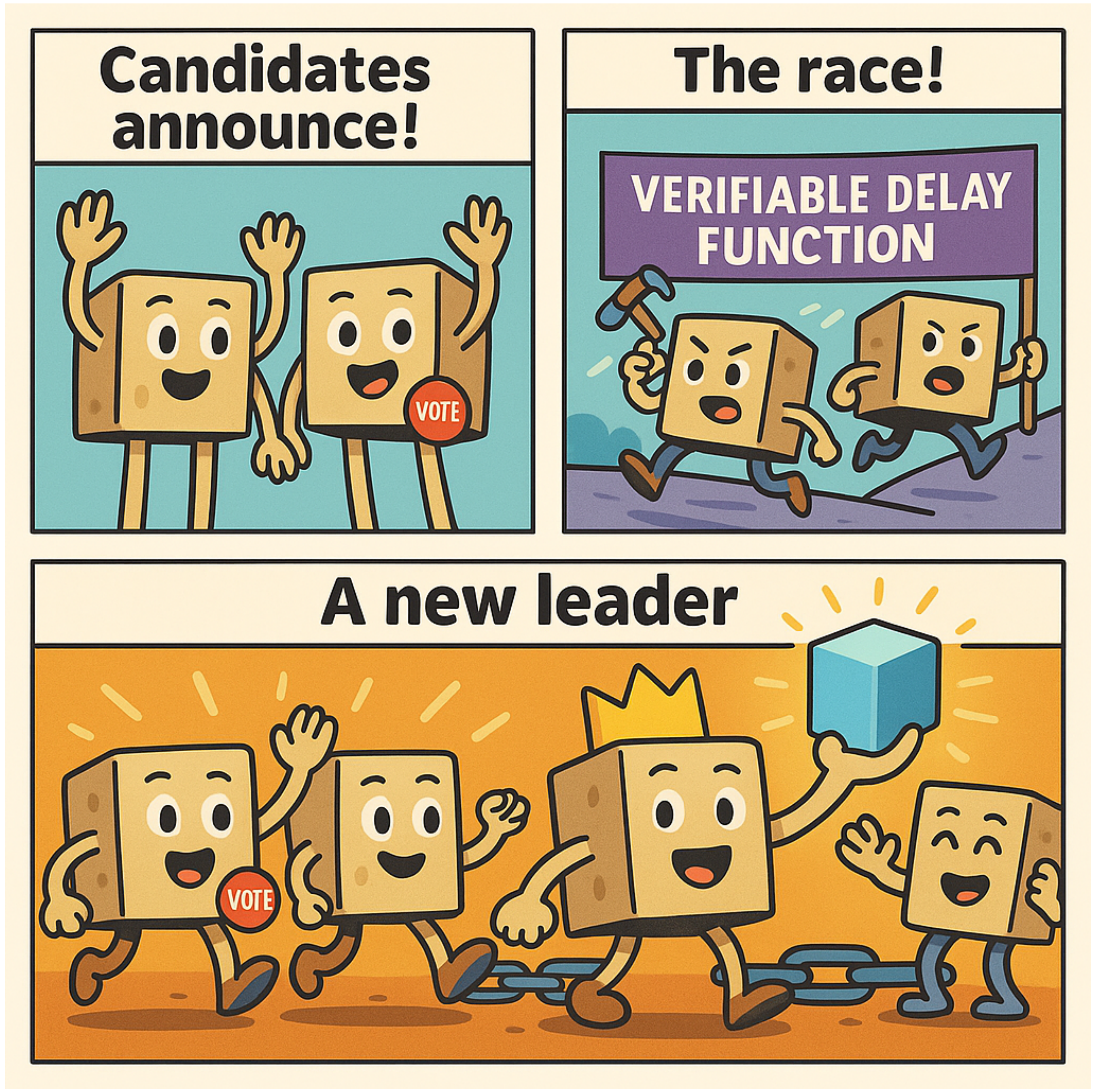
ISSUE: final randomness easy to compute & manipulate
(Stock prices can be biased/manipulated)

DELAY: not computable before market closes

UNIQUENESS: no ambiguity on output

Why study VDF?

Blockchain: leader election



UNIQUENESS → one **unique** leader

DELAY → cannot predict the next leader until shortly before the announcement

SOLANA Tezos chia IOTA

Verifiable Delay Functions Do Not Exist in the Random Oracle Model!!!!!!

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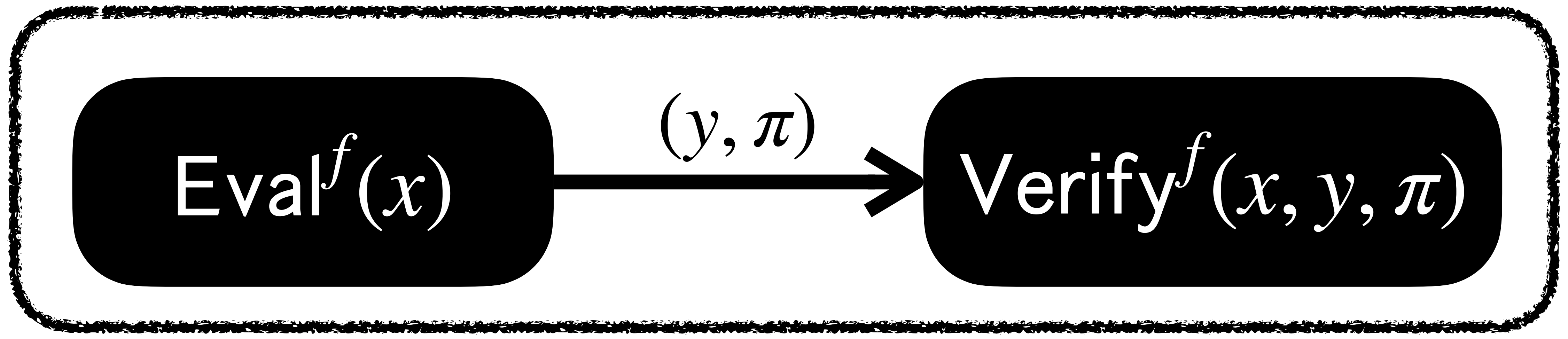

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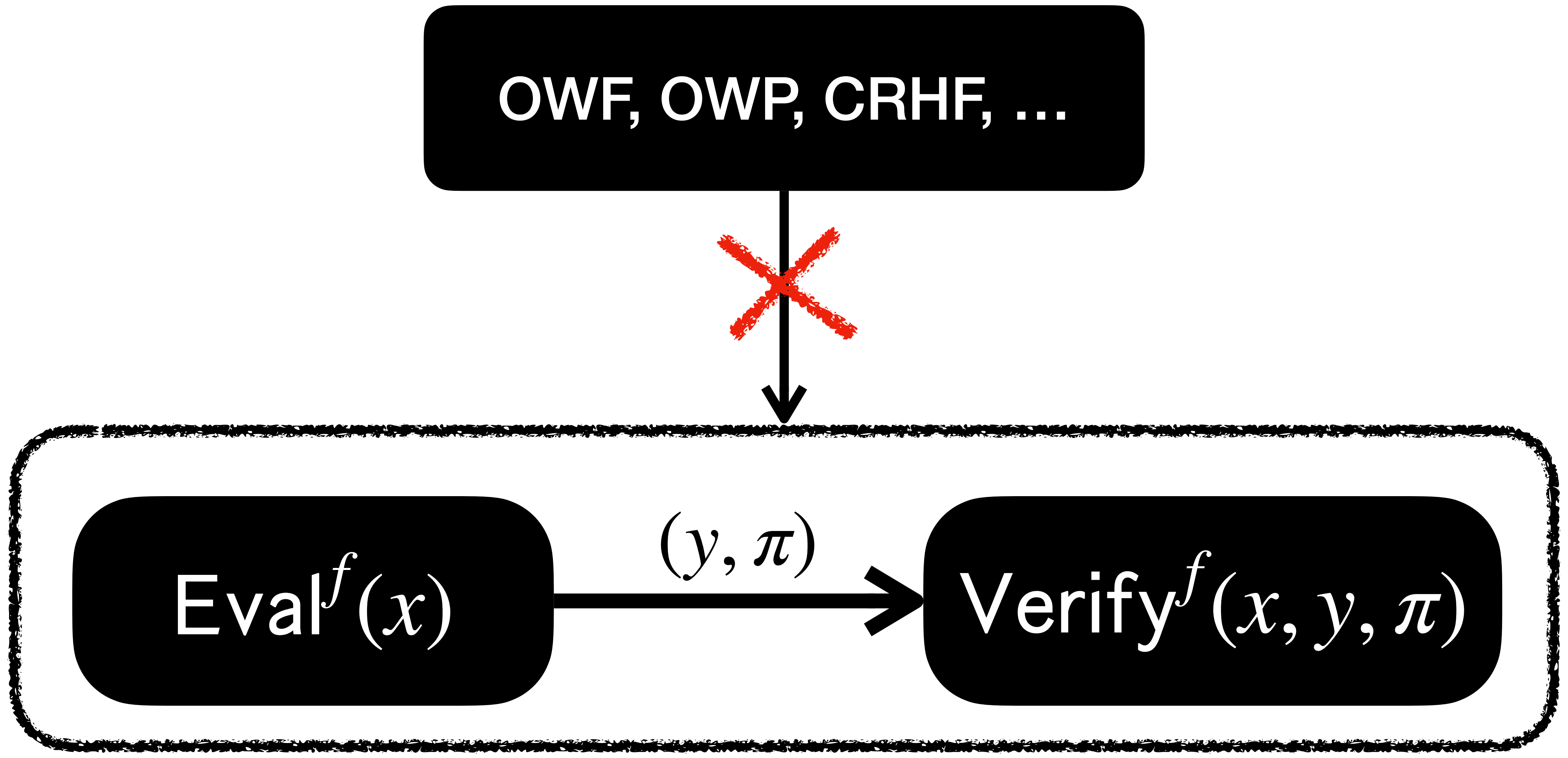

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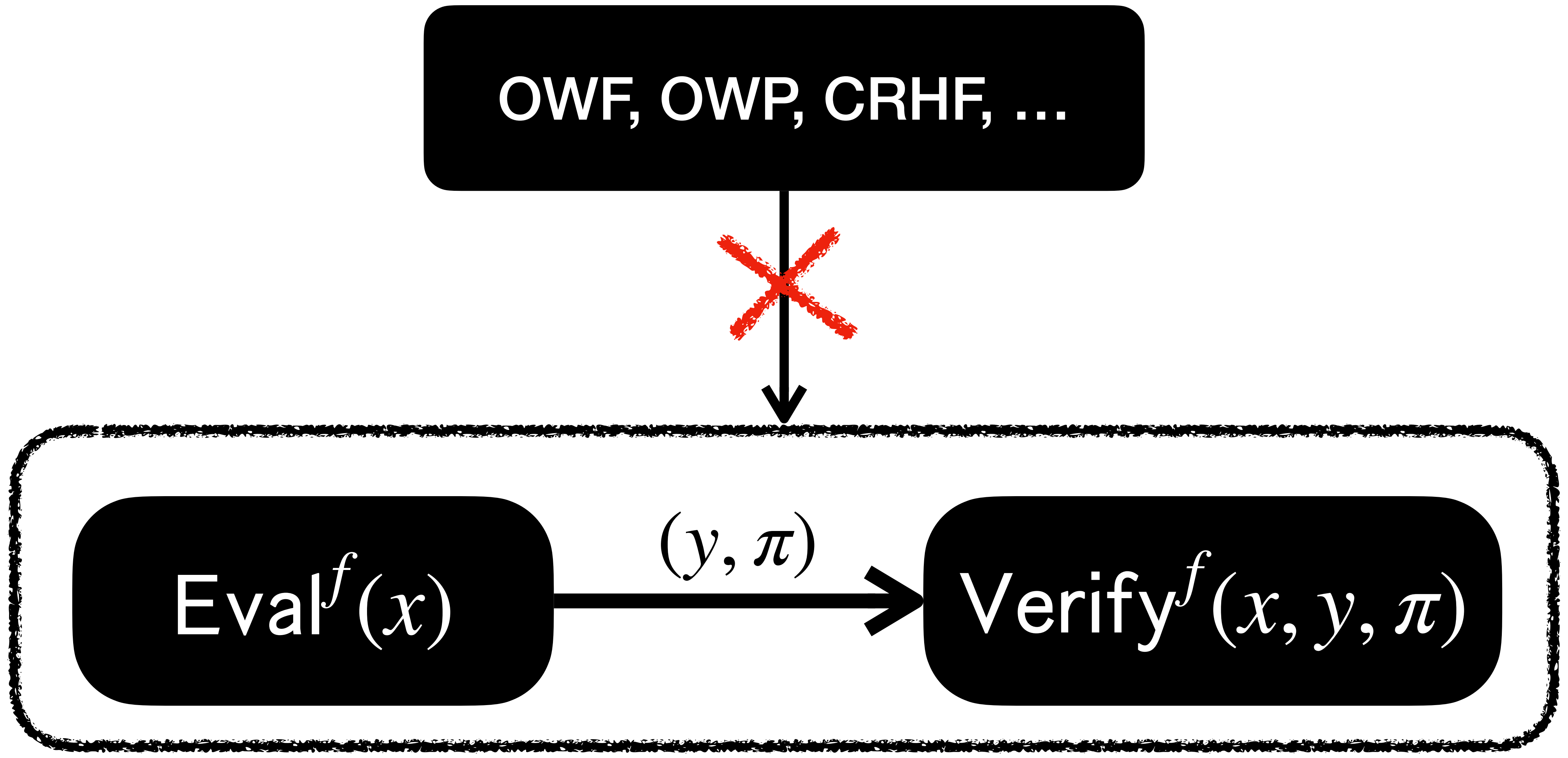

Complex assumptions (e.g. lattice) necessary for post-quantum VDF

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Similar approach for VDFs?

Let’s get a little bit technical…

Query complexity vs. certificate complexity

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Query complexity $D(G)$: min # bits in a needed to compute G

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$$D(G) \leq C(G)^2$$

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Challenge: VDF only has cryptographic correctness, above only works for statistical correctness…

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Thank you!