

PKE and ABE with **Collusion-Resistant** Secure Key Leasing

Fuyuki Kitagawa



Ryo Nishimaki



Nikhil Pappu



PKE with **Secure Key Leasing**

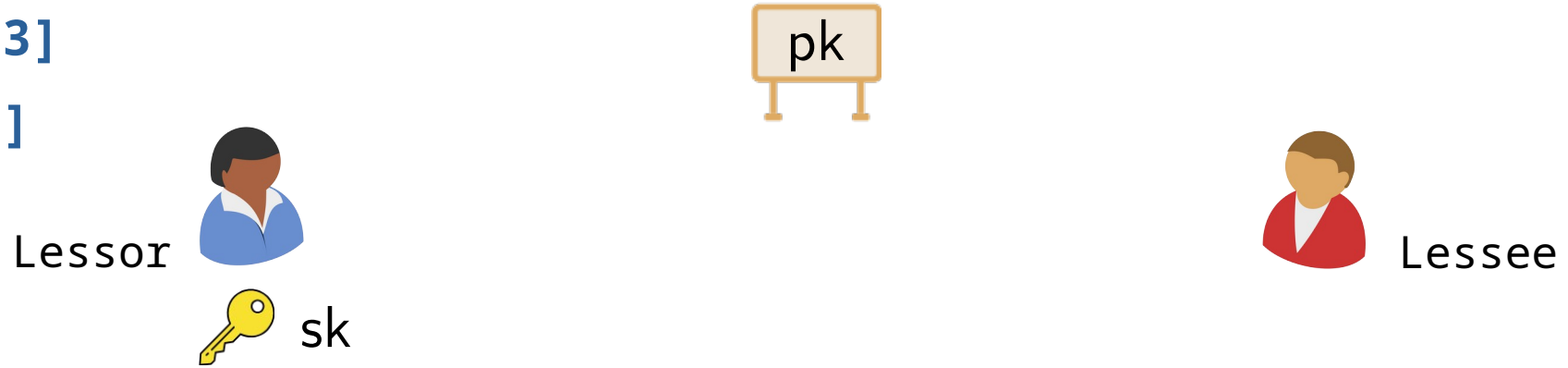
[AKN+23]

[APV23]

PKE with **Secure Key Leasing**

[AKN+23]

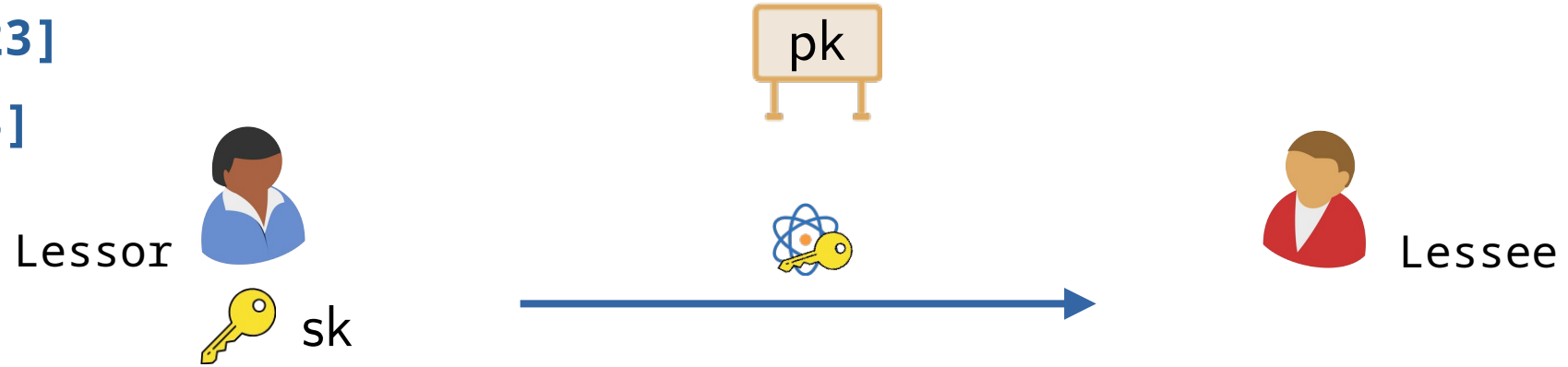
[APV23]



PKE with **Secure Key Leasing**

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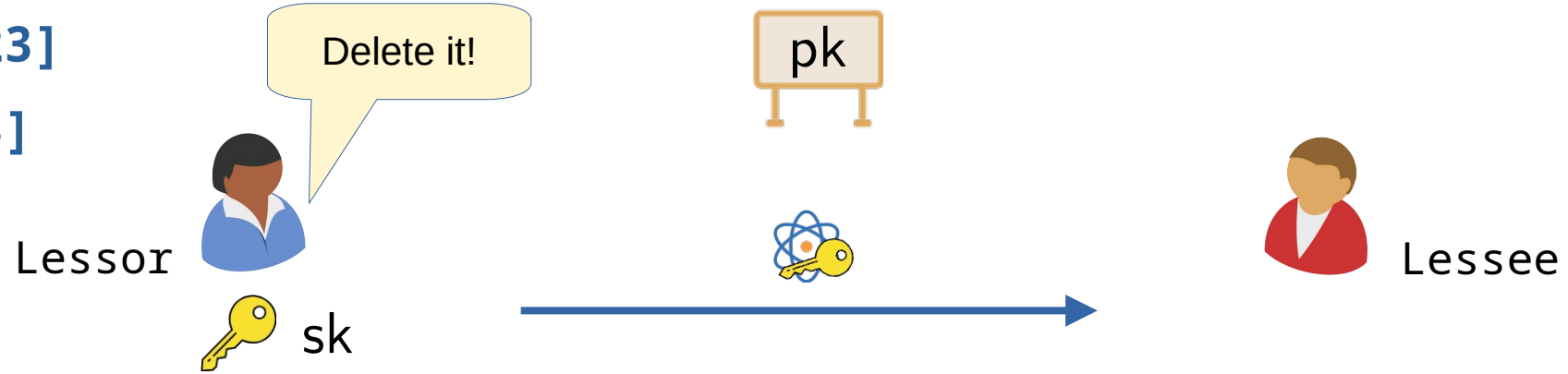
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PKE with Secure Key Leasing

[AKN+23]

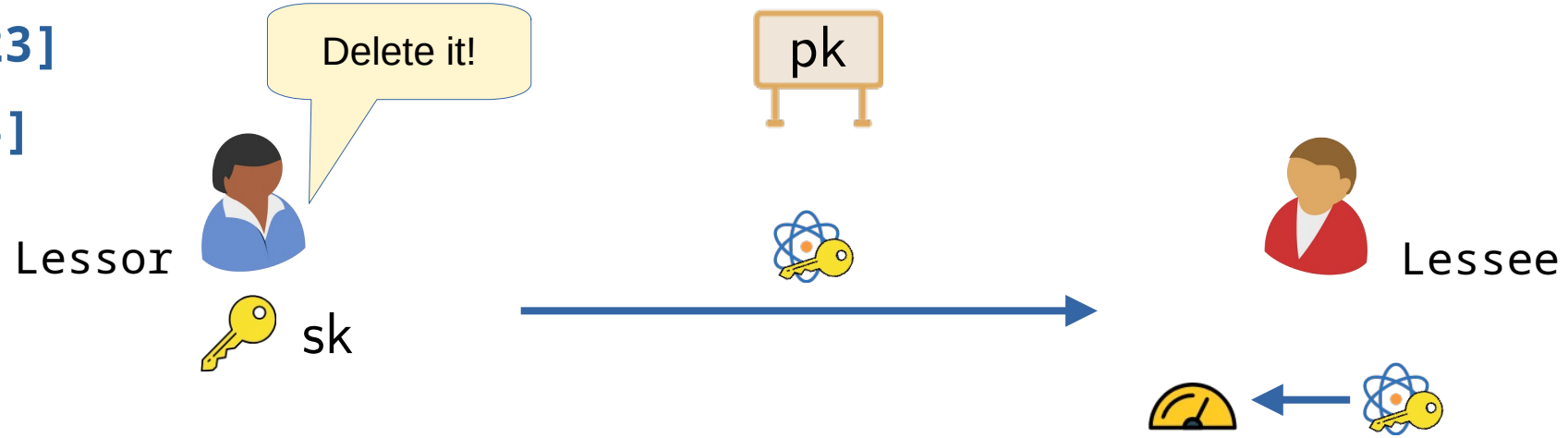
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PKE with **Secure Key Leasing**

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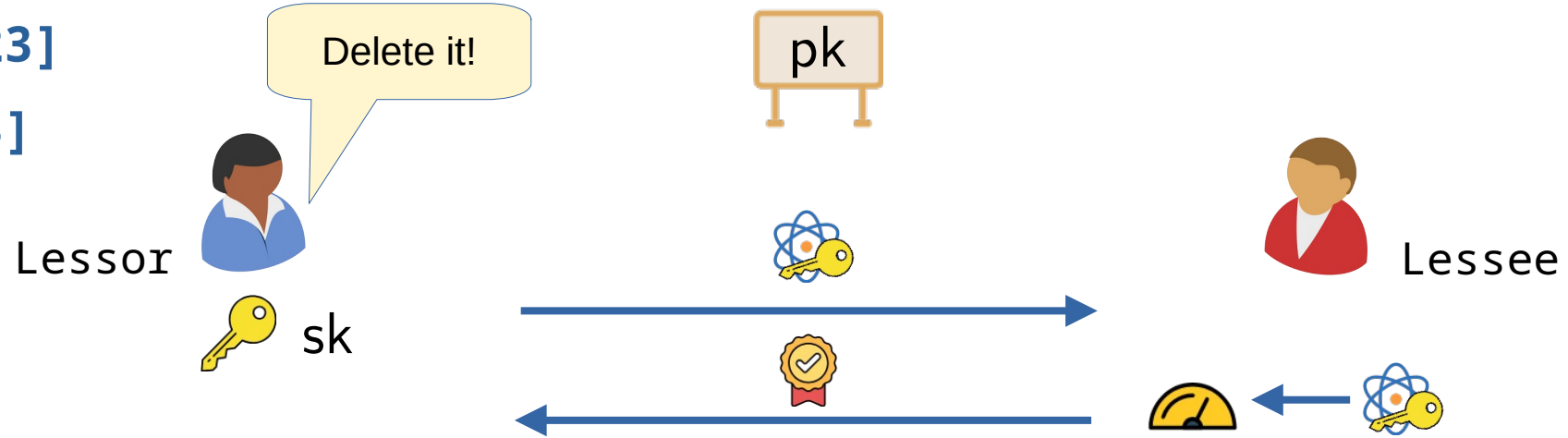
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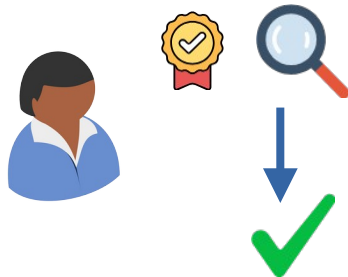
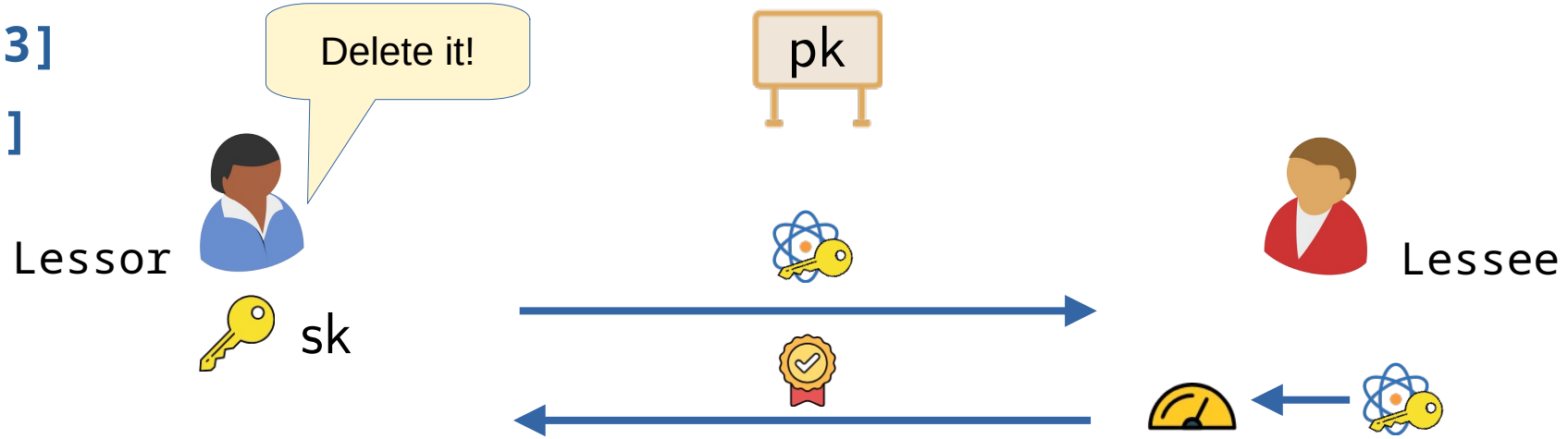
[APV23]



PKE with Secure Key Leasing

[AKN+23]

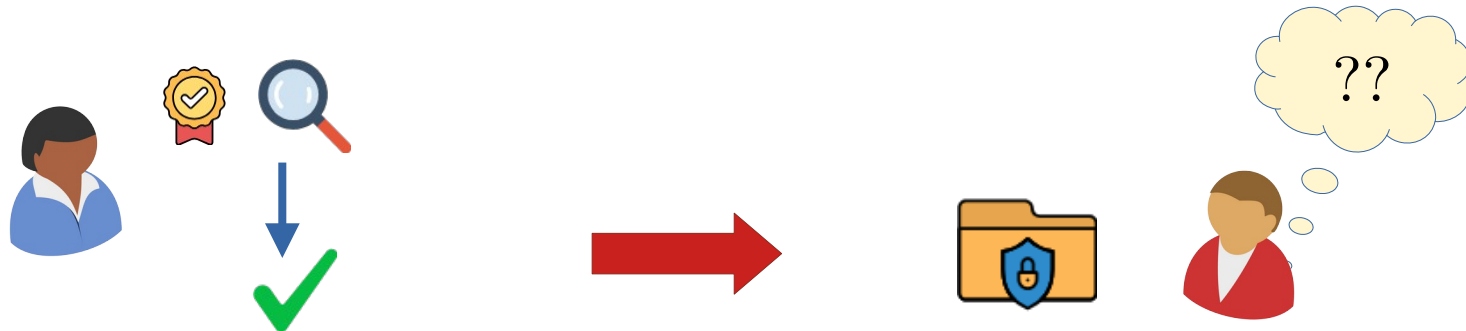
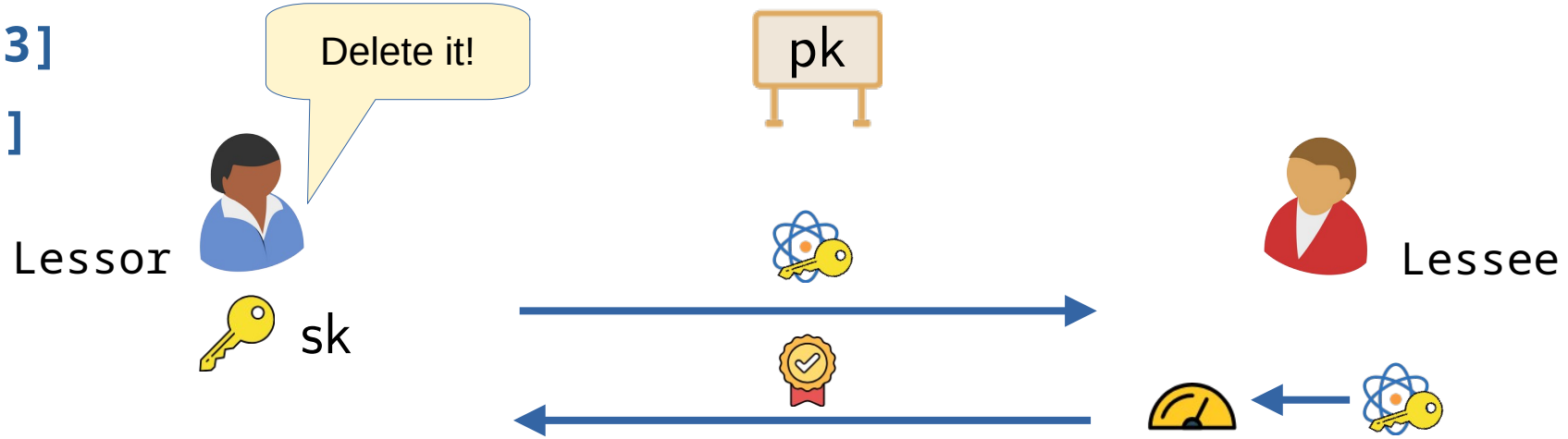
[APV23]



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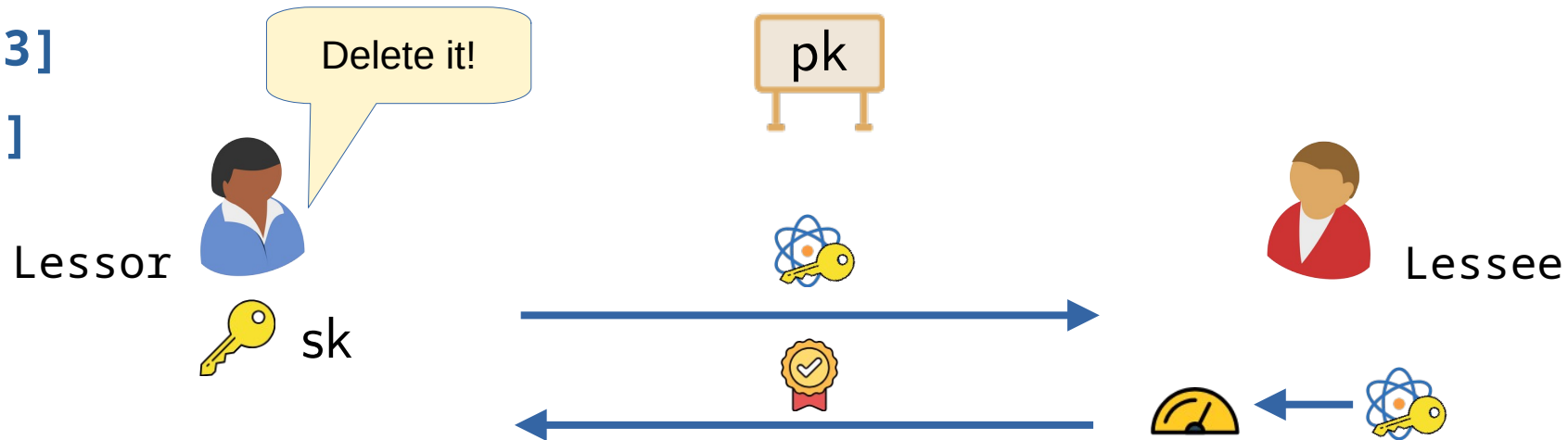
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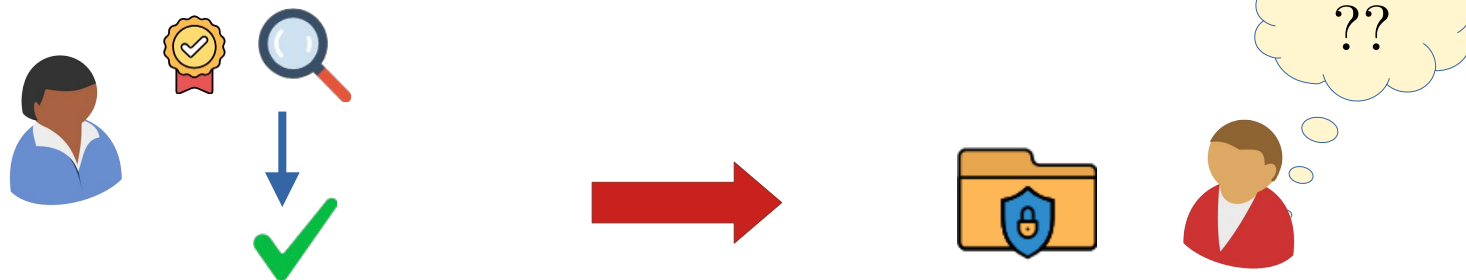
PKE with Secure Key Leasing

[AKN+23]

[APV23]



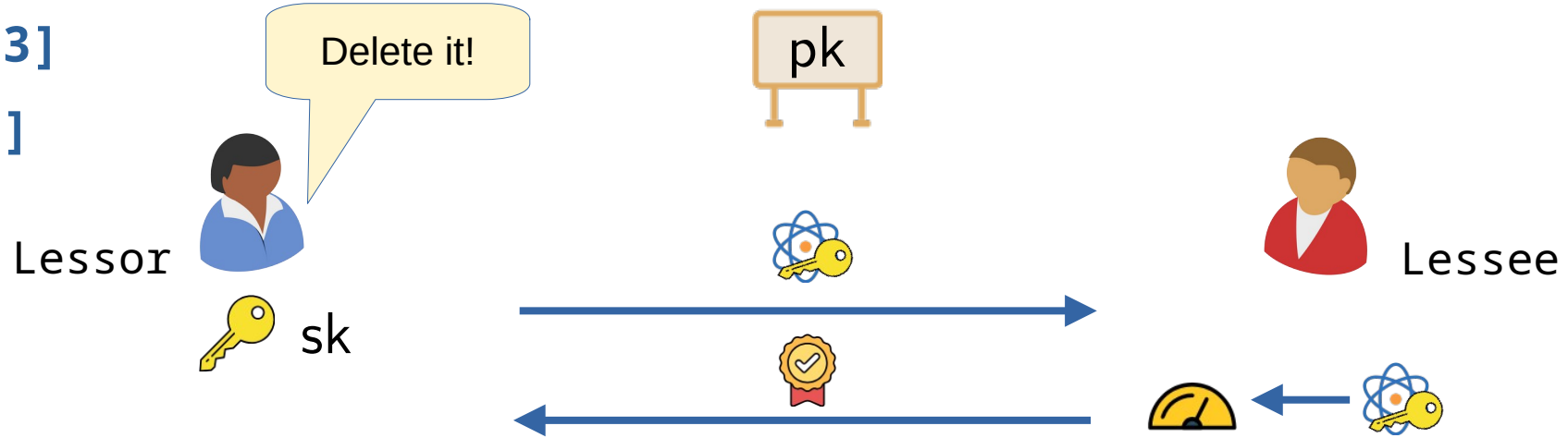
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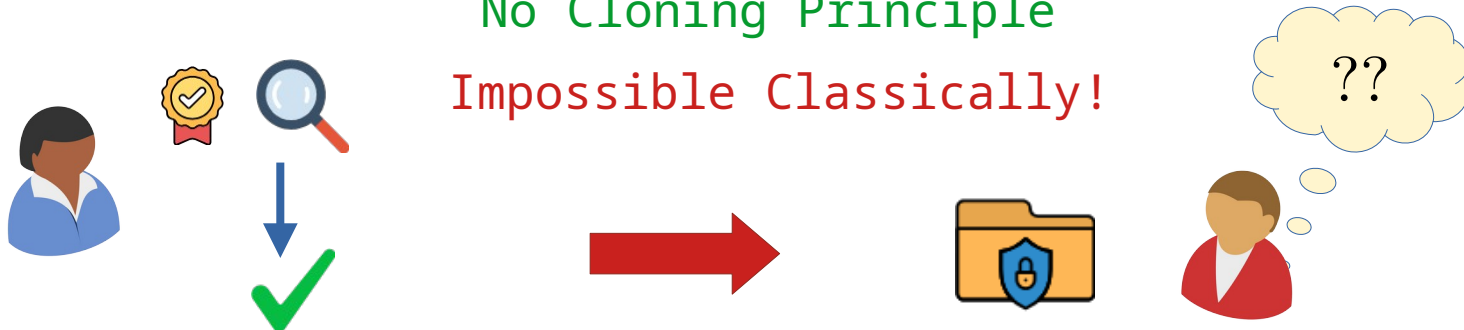
PKE with Secure Key Leasing

[AKN+23]

[APV23]



No Cloning Principle
Impossible Classically!



Collusion-Resistance

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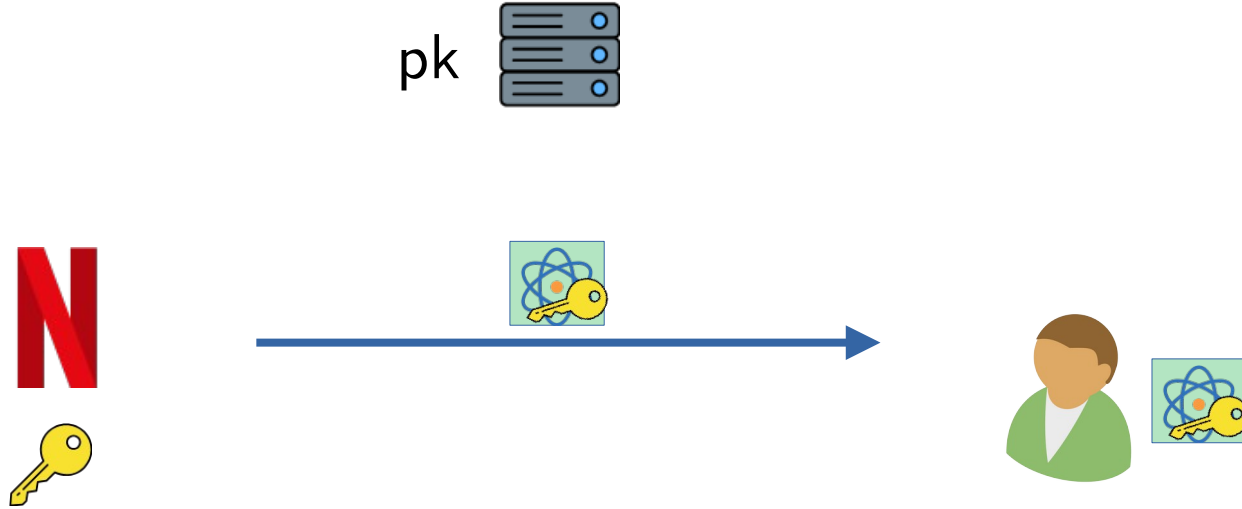
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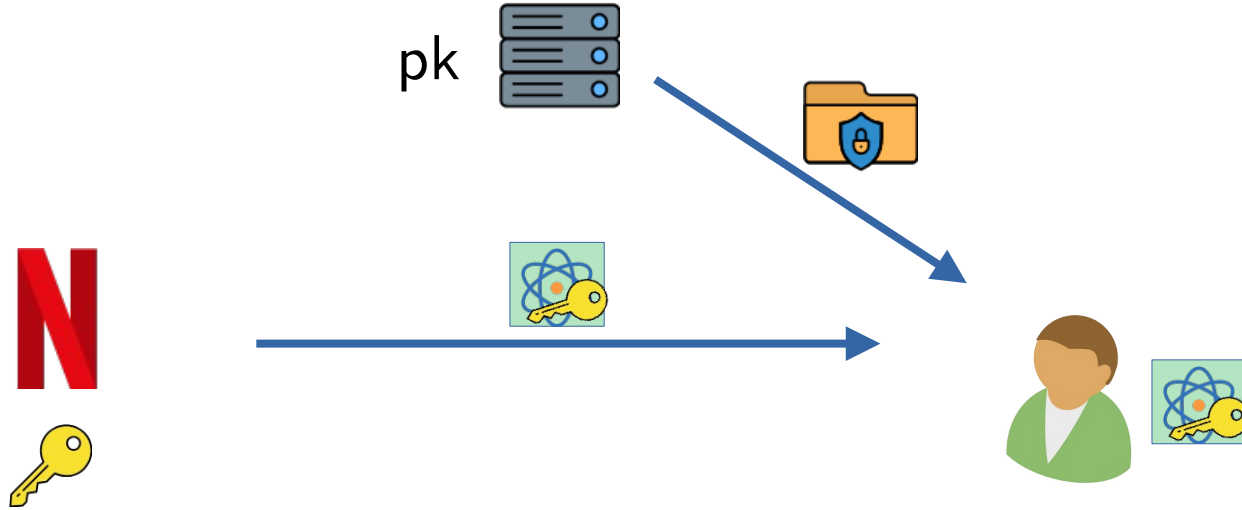
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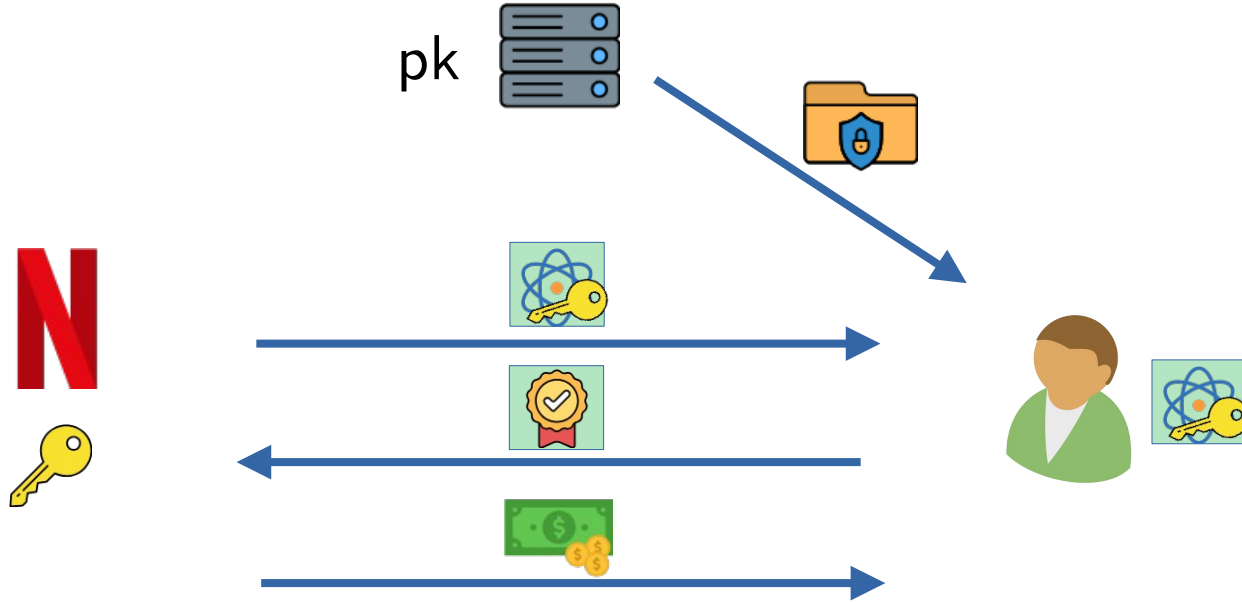
Collusion-Resistance



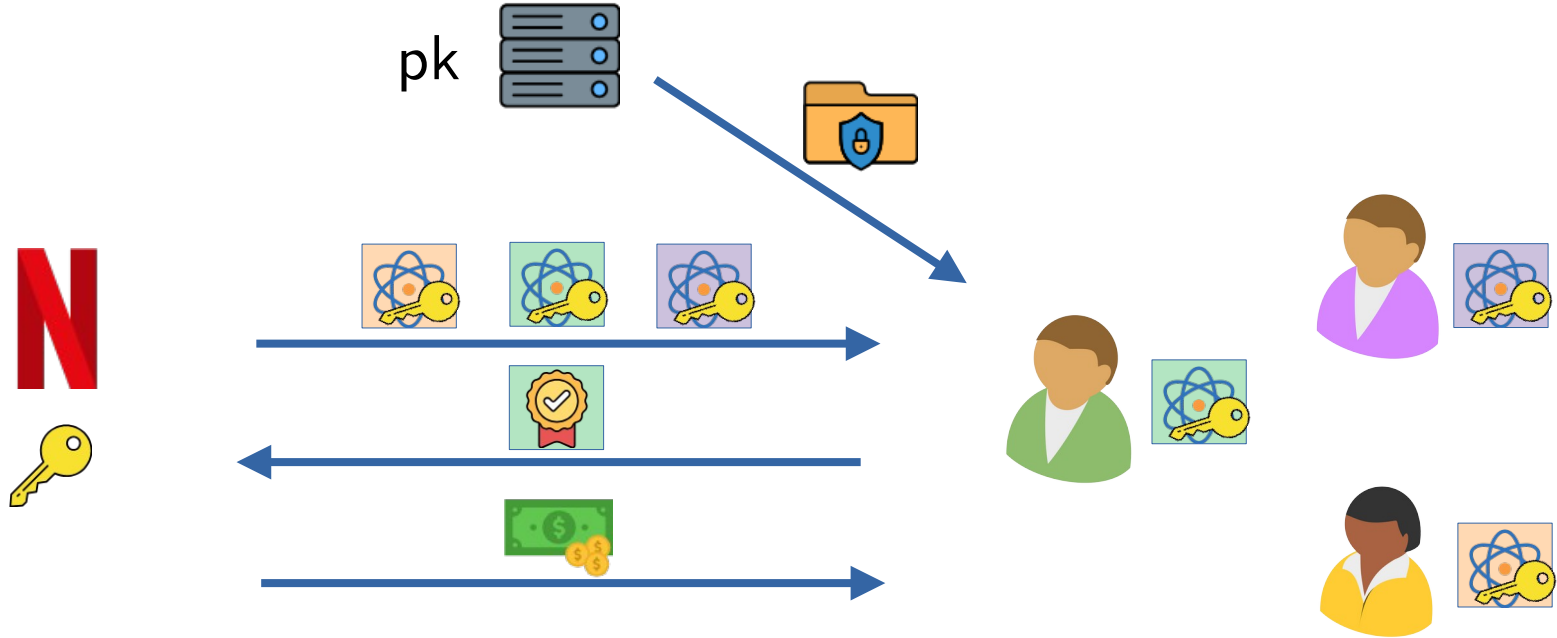
Collusion-Resistance



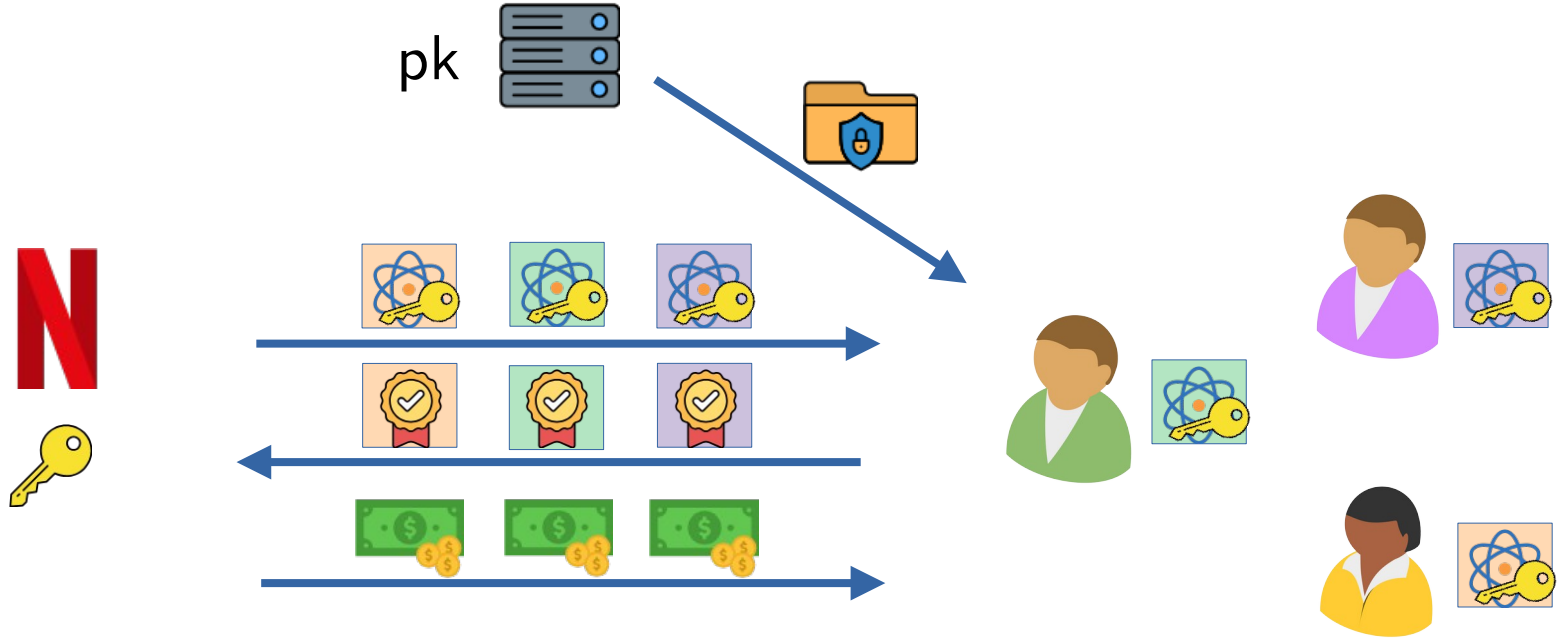
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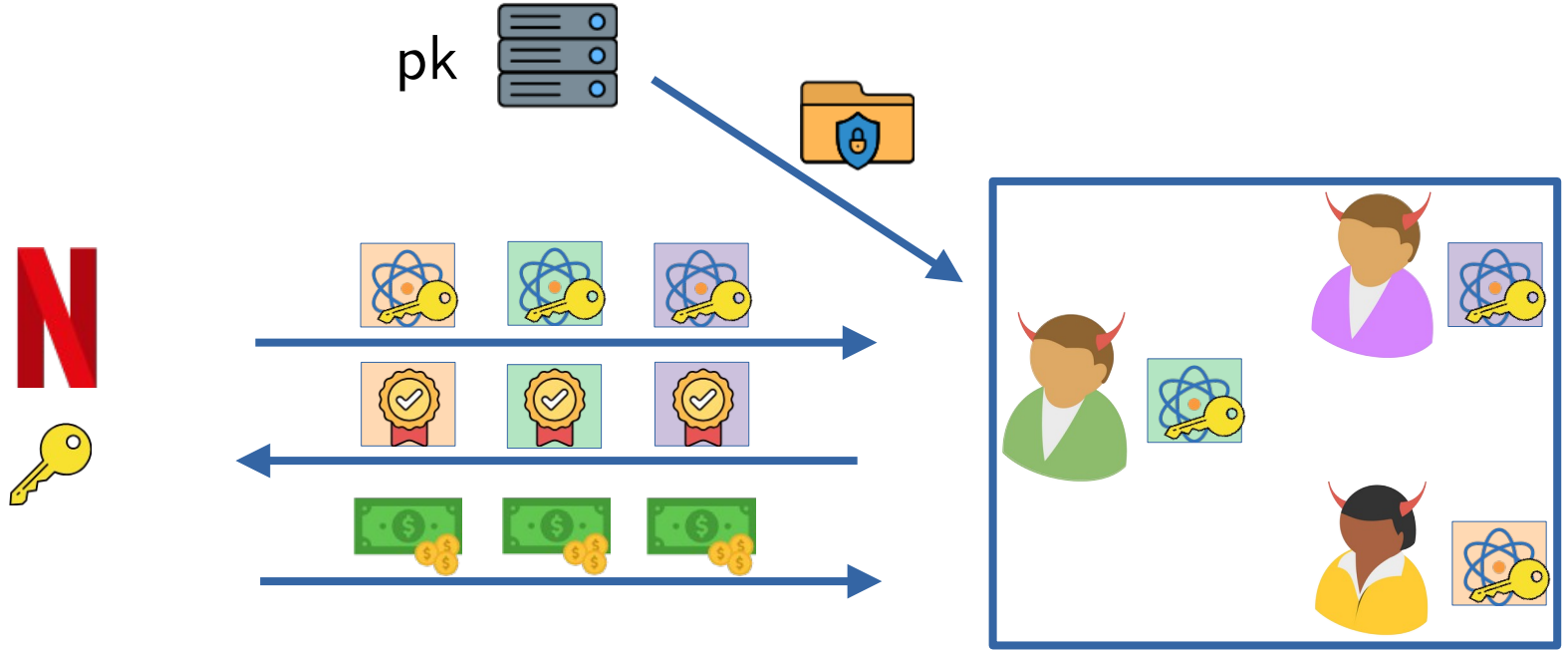
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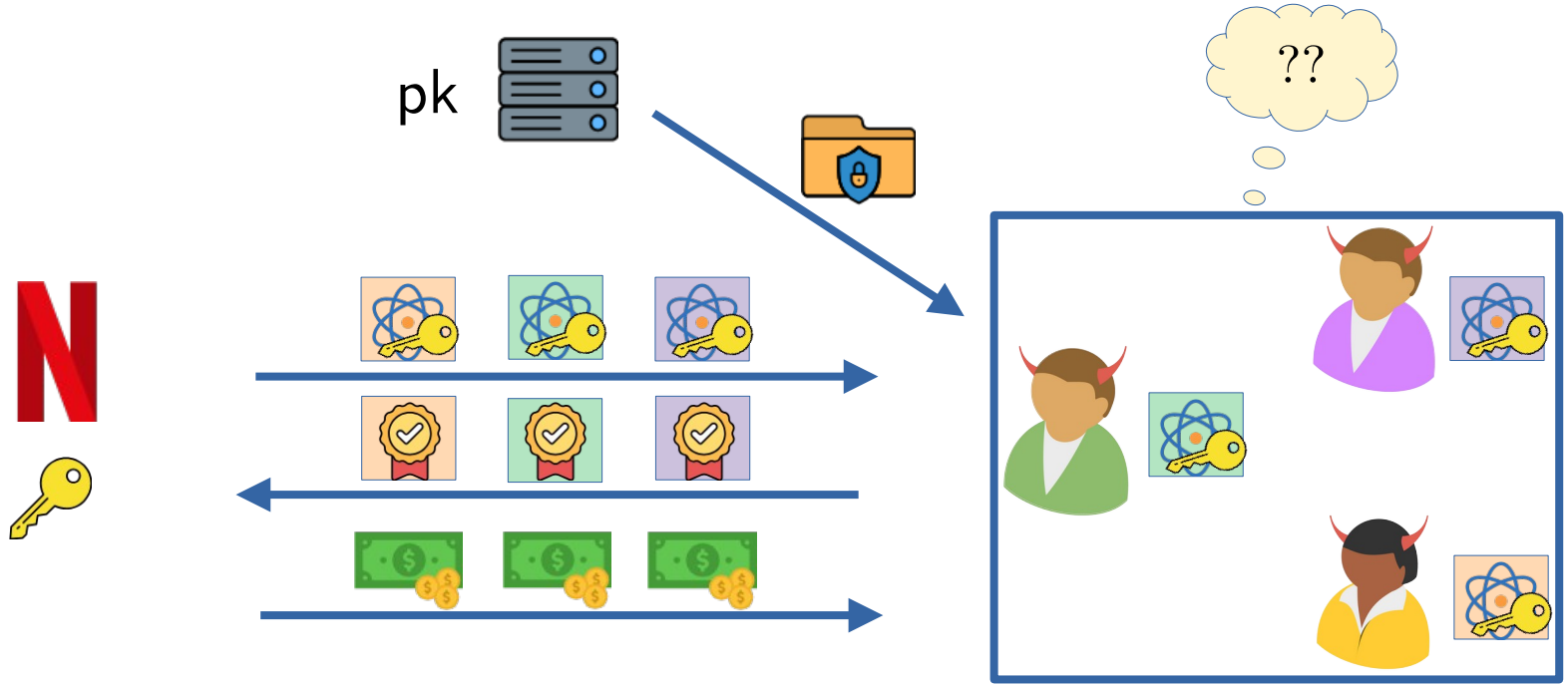
Collusion-Resistance



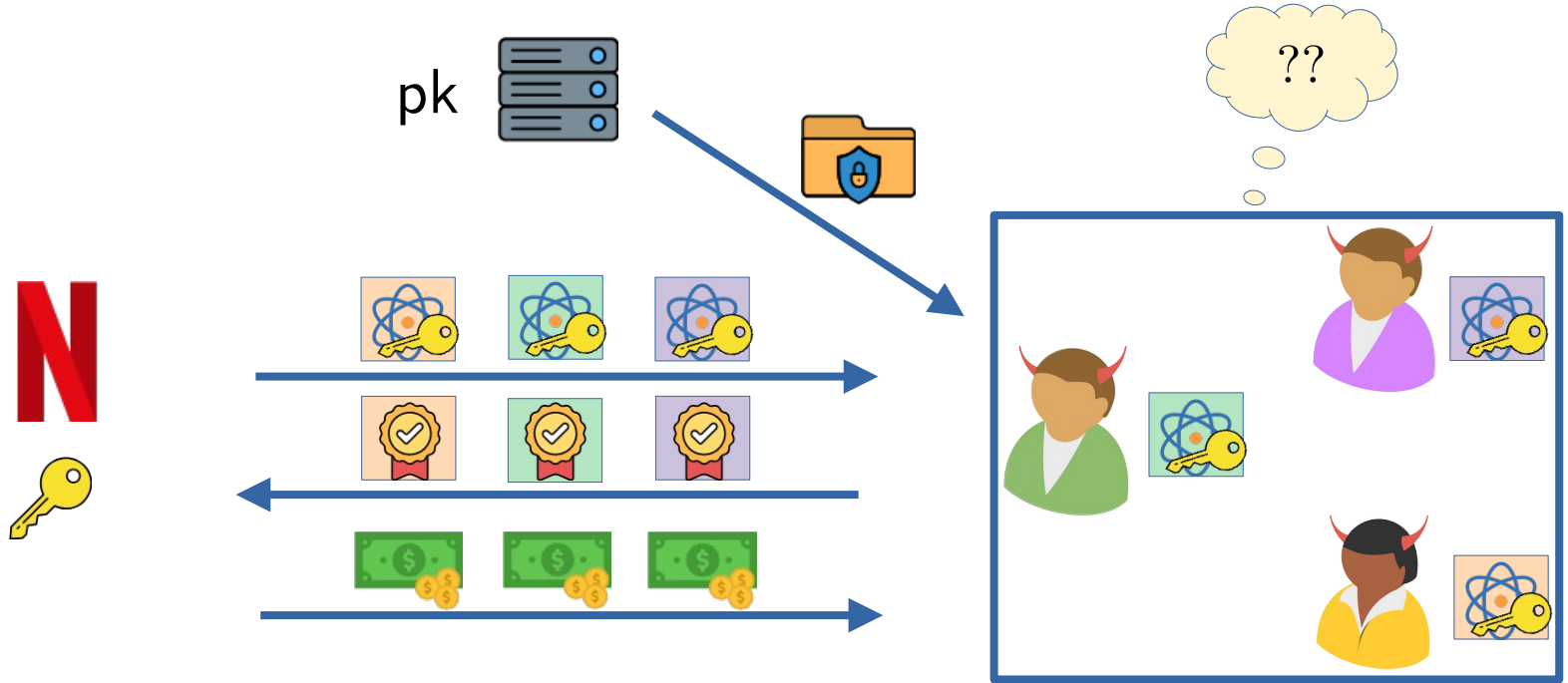
Collusion-Resistance



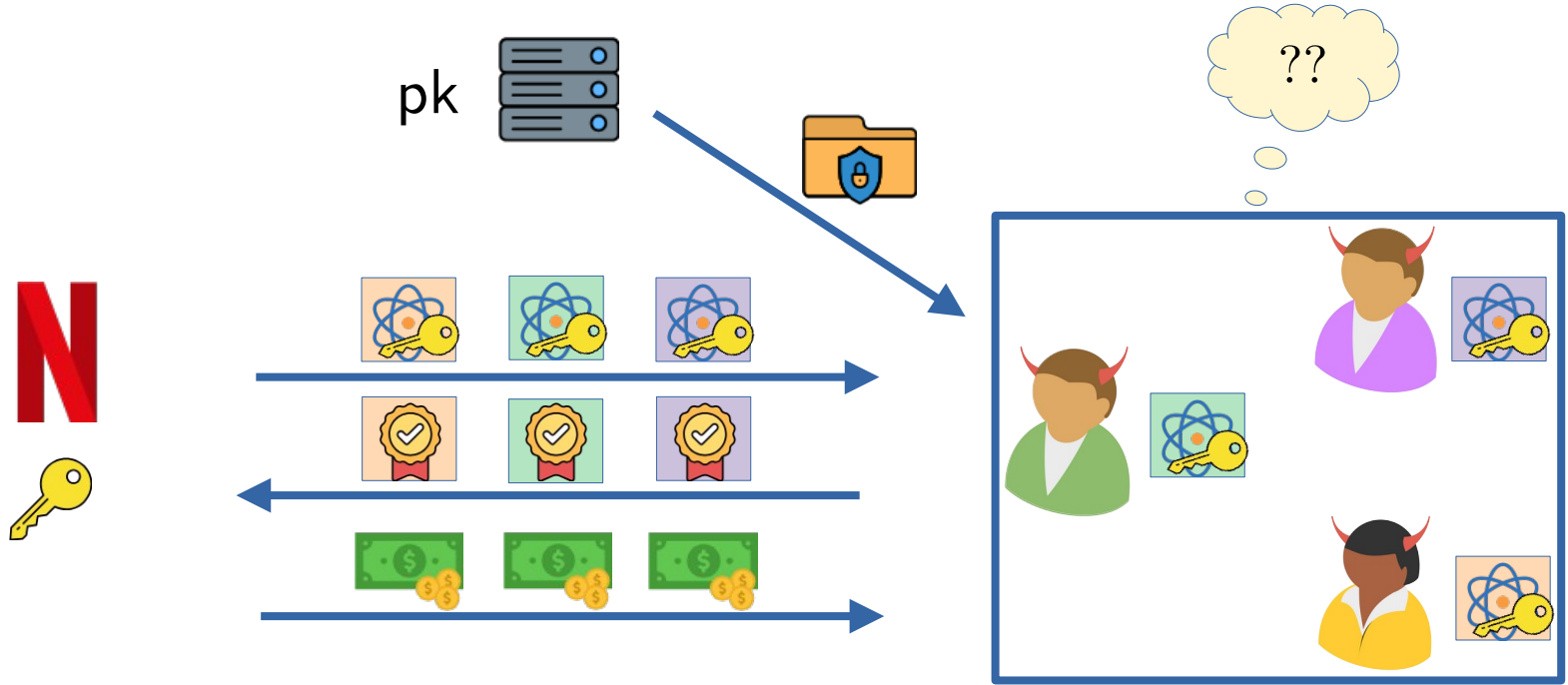
Collusion-Resistance



Collusion-Resistance



Collusion-Resistance



Unbounded Collusion-Resistance: Parameters scale poly-logarithmically with the number of users.

Prior Work

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[AKN+23, APV23, AHH24, CGJL25, KMY25]:

- Various PKE-SKL Constructions
- Standard Assumptions

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1) Correlate keys before deletion



Completely Broken with Collusions

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Completely Broken with Collusions



- 1) Correlate keys before deletion
- 2) Learn crucial classical info

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Completely Broken with Collusions



- 1) Correlate keys before deletion
- 2) Learn crucial classical info
- 3) Leave states undisturbed

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Challenging Setting.

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Challenging Setting.
Provable-Security even more so!

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[AKN+23, BGK+24]:

- (Bounded + Unbounded) Collusion-Resistant Constructions

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Completely Broken with Collusions

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Challenging Setting.
Provable-Security even more so!

[AKN+23, BGK+24]:

- (Bounded + Unbounded) Collusion-Resistant Constructions



Bounded is **Inefficient**. Unbounded rely on **FE/IO (Strong! Post-Quantum?)**

Our Contributions

[KNP₂₅] :



Our Contributions

[KNP25] :

1) Collusion-Resistant Definition:

Our Contributions

[KNP25] :

1) Collusion-Resistant Definition:

PKE-CR-SKL

Our Contributions

[KNP25] :

1) Collusion-Resistant Definition:

PKE-CR-SKL

2)

Our Contributions

[KNP25] :

1) Collusion-Resistant Definition:

PKE-CR-SKL

2)

LWE



PKE-CR-SKL

Our Contributions

[KNP25] :

1) Collusion-Resistant Definition:

PKE-CR-SKL

2)

LWE



PKE-CR-SKL

&

ABE-CR-SKL

Our Contributions

[KNP25] :

1) Collusion-Resistant Definition:

PKE-CR-SKL

2)

LWE



PKE-CR-SKL

&

ABE-CR-SKL

3) New Techniques and Building Blocks:

Our Contributions

[KNP25] :

1) Collusion-Resistant Definition:

PKE-CR-SKL

2)

LWE



PKE-CR-SKL

&

ABE-CR-SKL

3) New Techniques and Building Blocks:

SKE-CR-SKL

Our Contributions

[KNP25] :

1) Collusion-Resistant Definition:

PKE-CR-SKL

2)

LWE



PKE-CR-SKL

&

ABE-CR-SKL

3) New Techniques and Building Blocks:

SKE-CR-SKL

4)

Multi-Input ABE



Variant w/ Classical



Main Result

[KNP25]

Main Result

[KNP₂₅]

PKE-CR-SKL

Main Result

[KNP25]



PKE-CR-SKL

Main Result

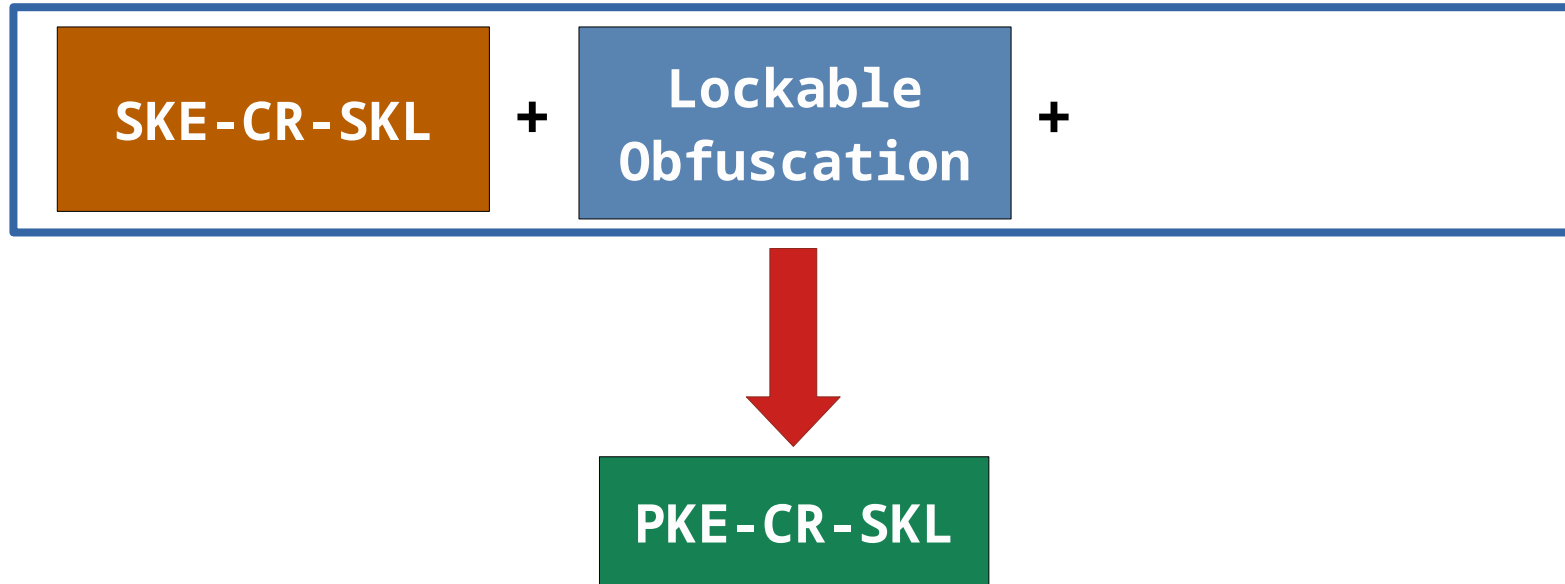
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PKE-CR-SKL

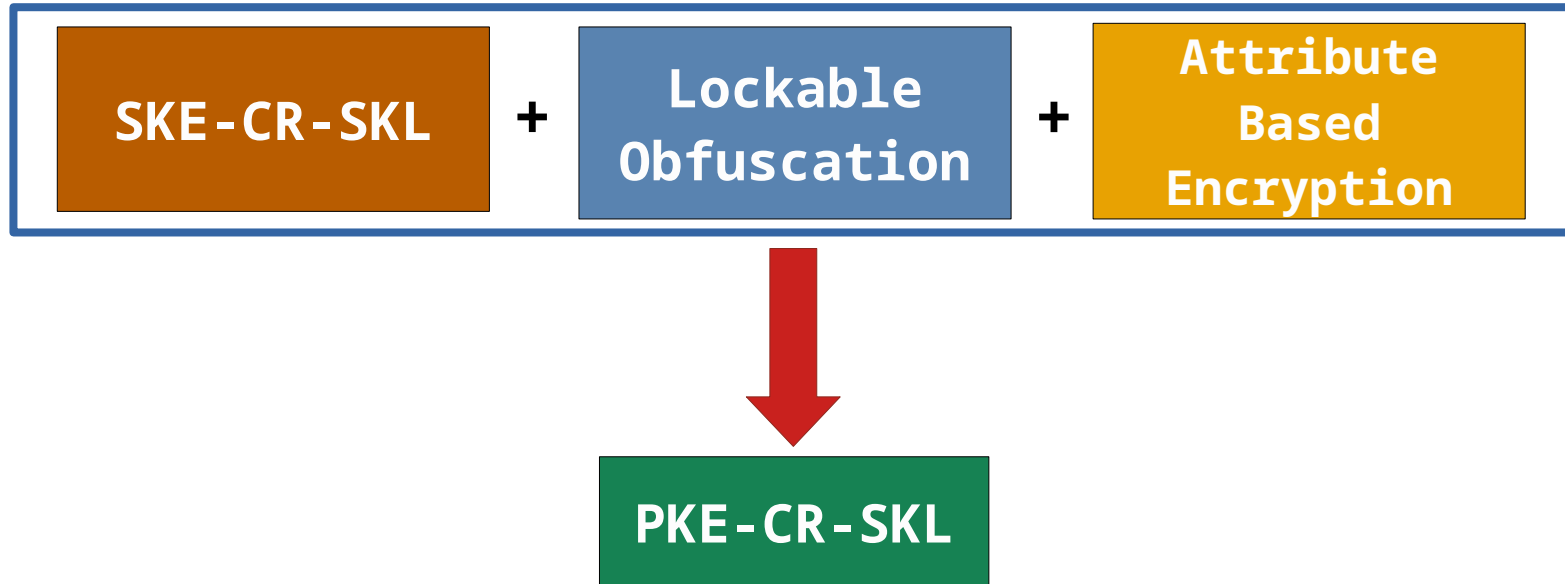
Main Result

[KNP25]



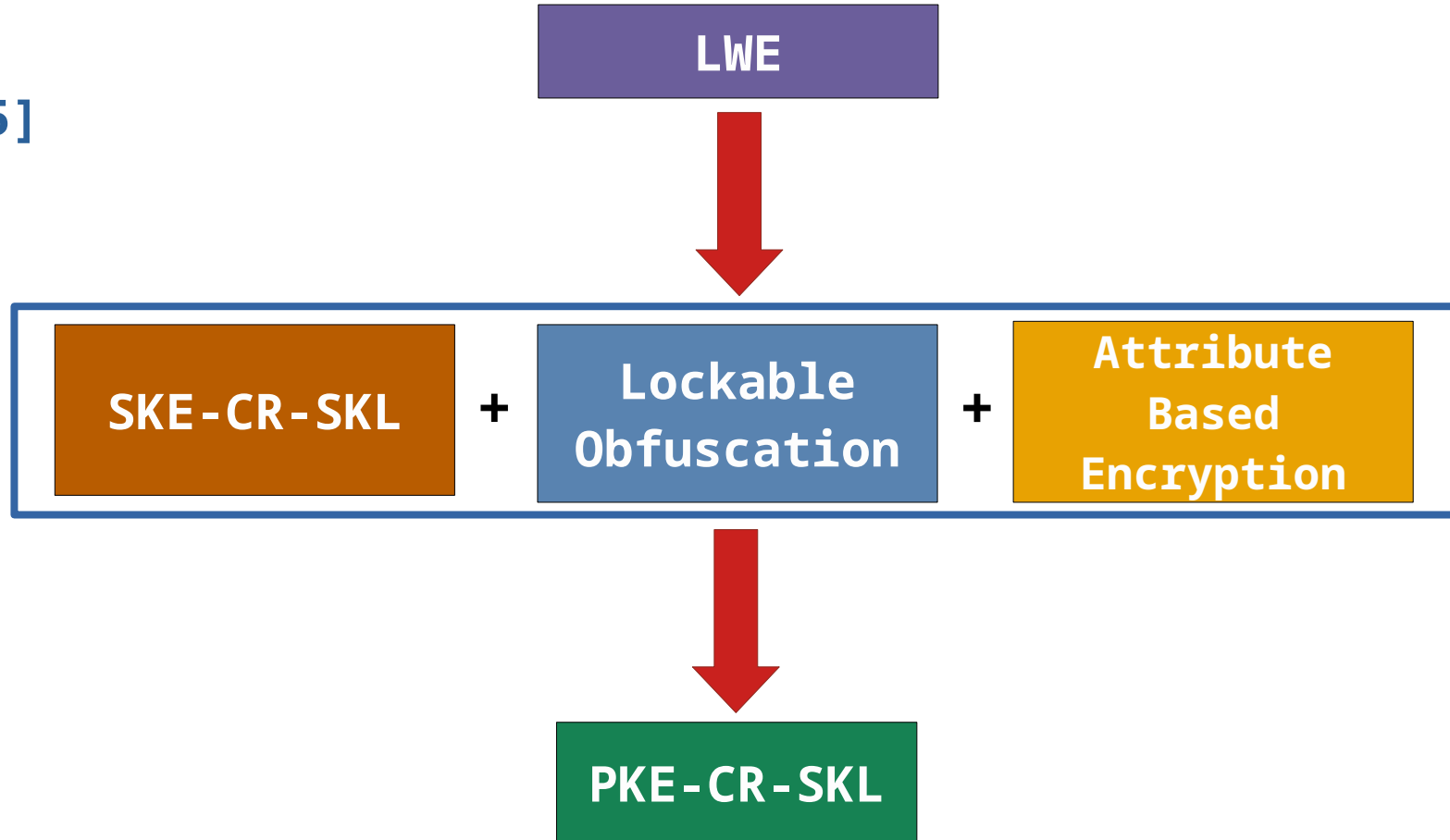
Main Result

[KNP25]



Main Result

[KNP25]



Main Result

[KNP25]

LWE

Standard Assumption



SKE-CR-SKL

+

Lockable
Obfuscation

+

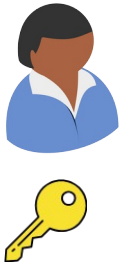
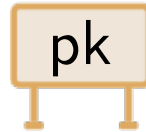
Attribute
Based
Encryption



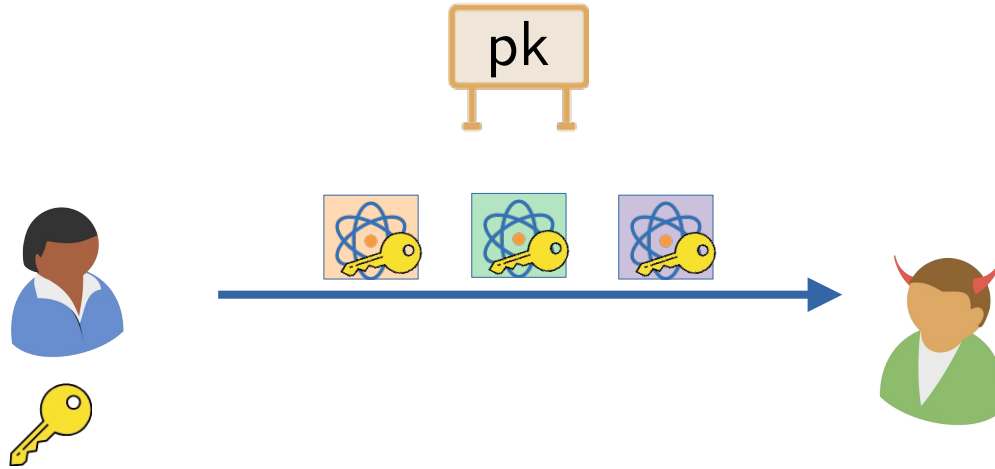
PKE-CR-SKL

PKE-CR-SKL and SKE-CR-SKL

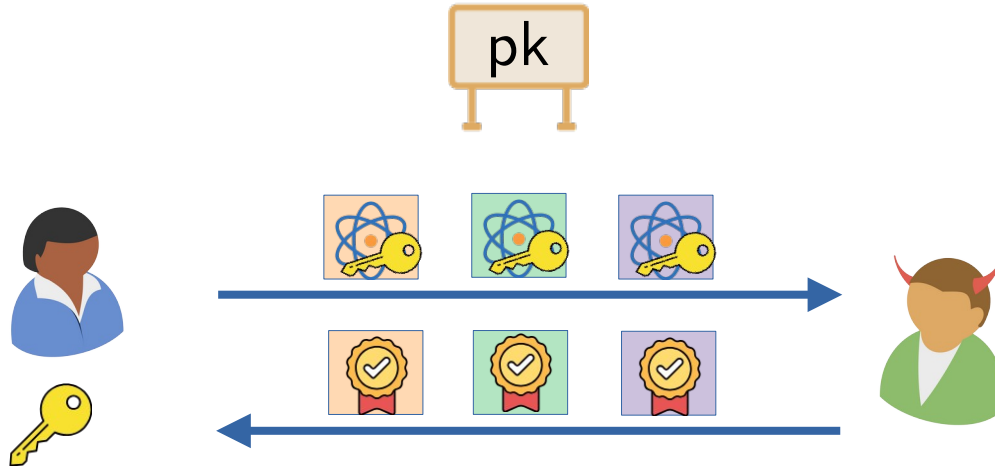
PKE-CR-SKL and SKE-CR-SKL



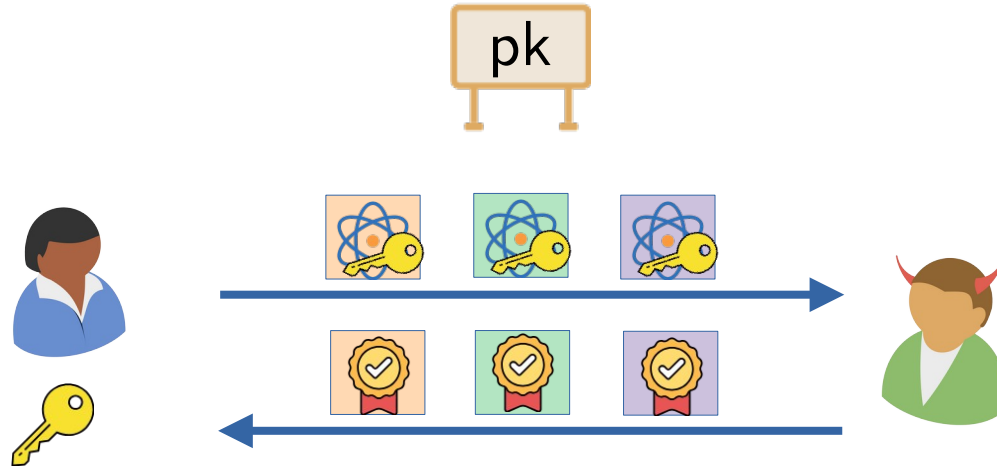
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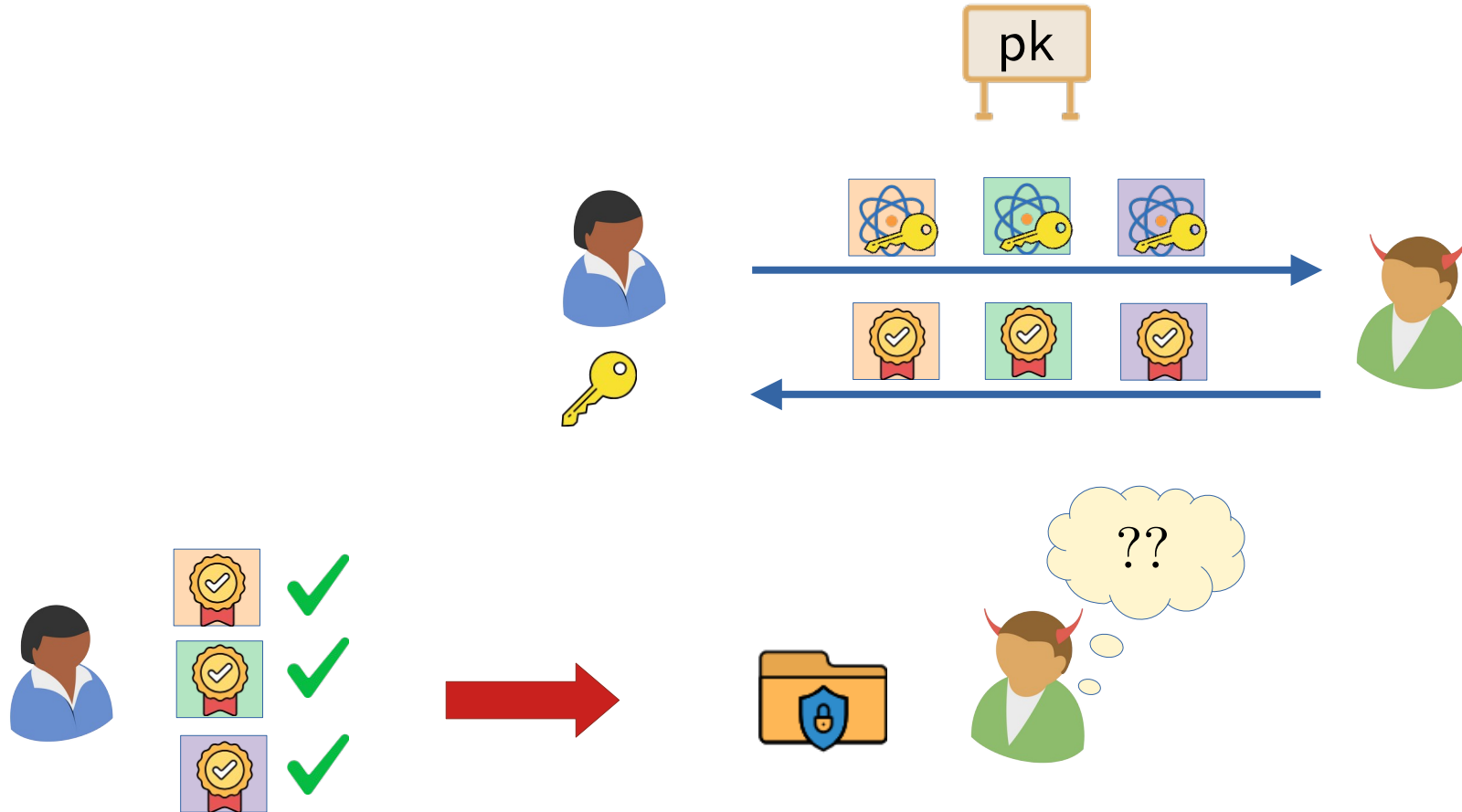
PKE-CR-SKL and SKE-CR-SKL



PKE-CR-SKL and SKE-CR-SKL

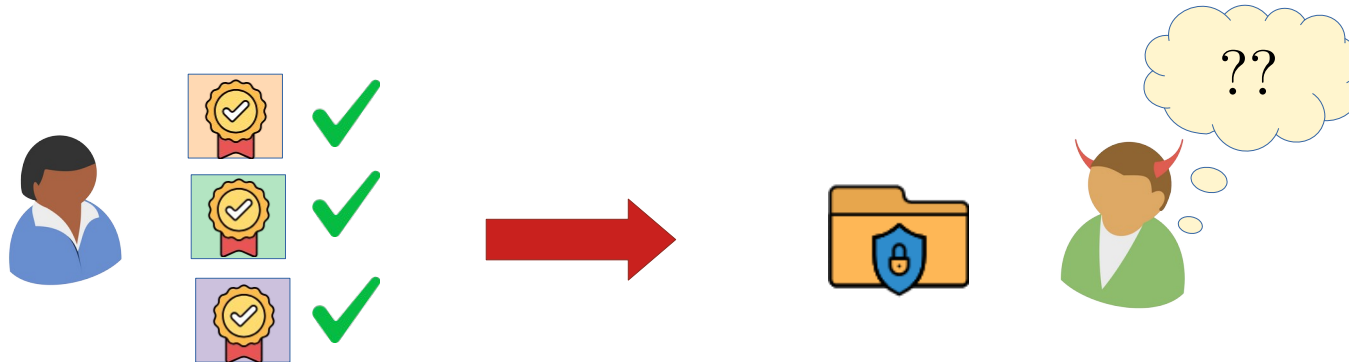
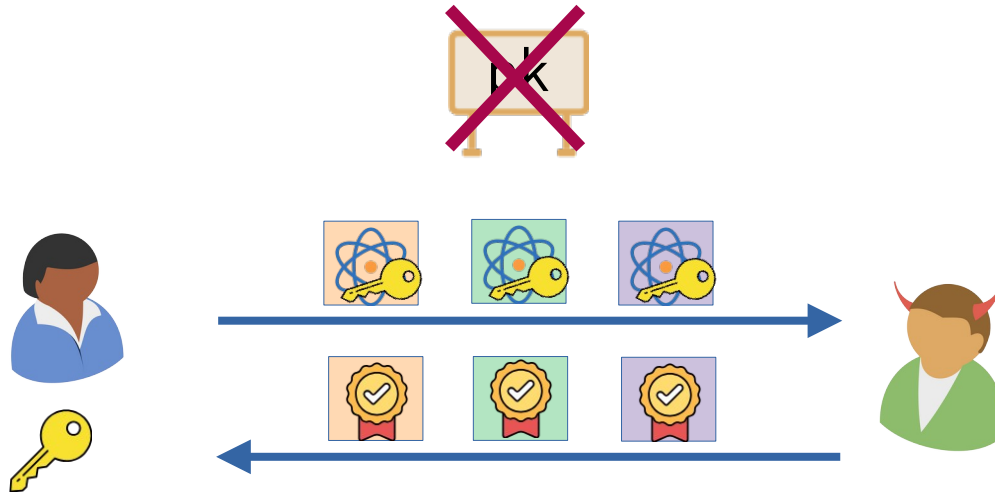


PKE-CR-SKL and SKE-CR-SKL



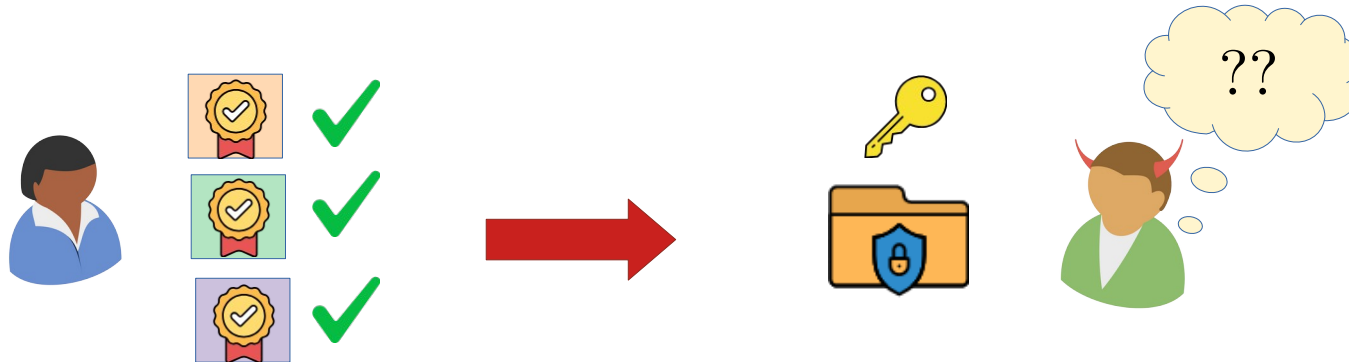
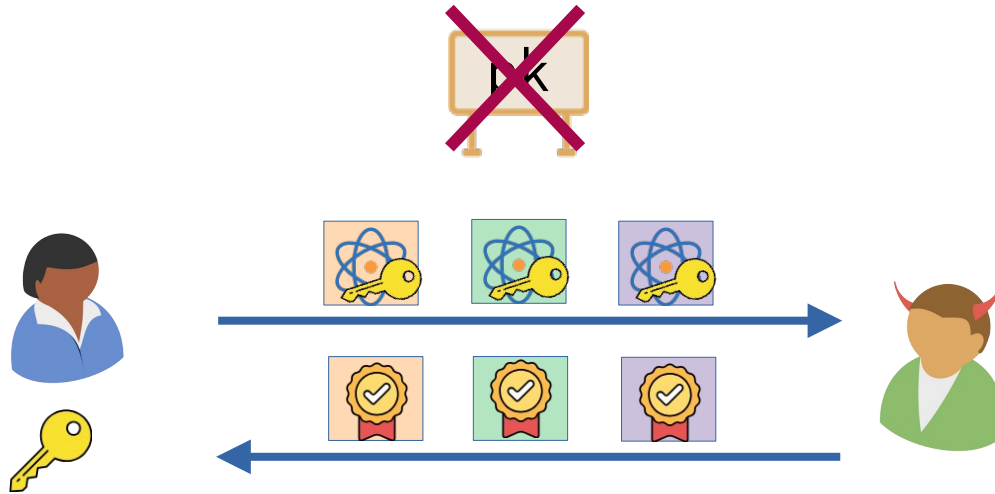
PKE-CR-SKL and SKE-CR-SKL

SKE-CR-SKL



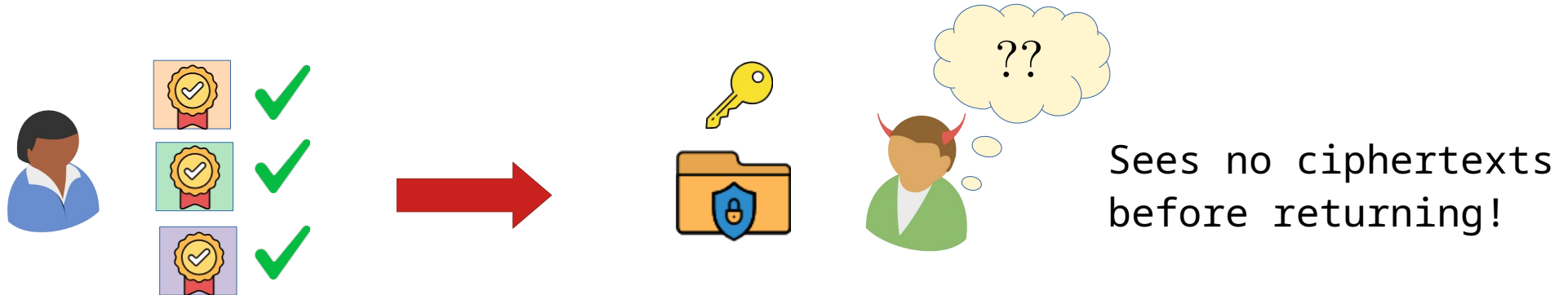
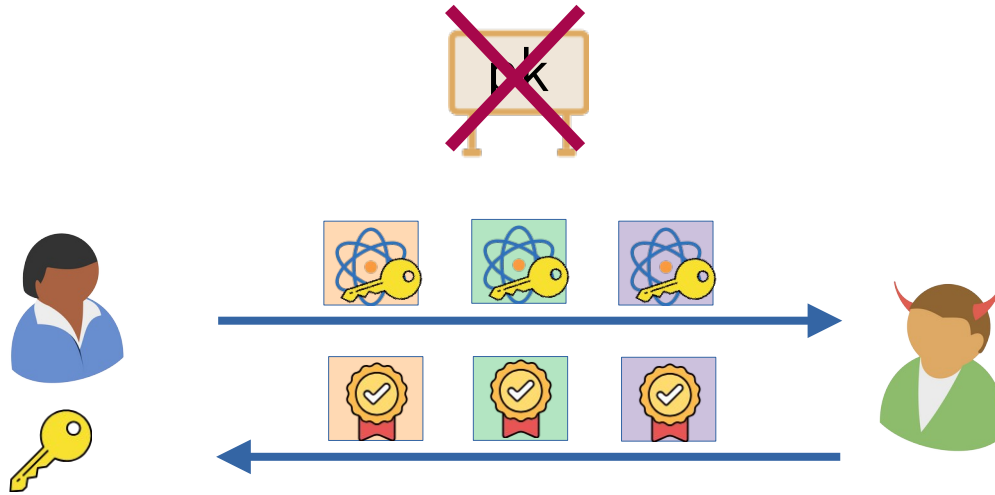
PKE-CR-SKL and SKE-CR-SKL

SKE-CR-SKL

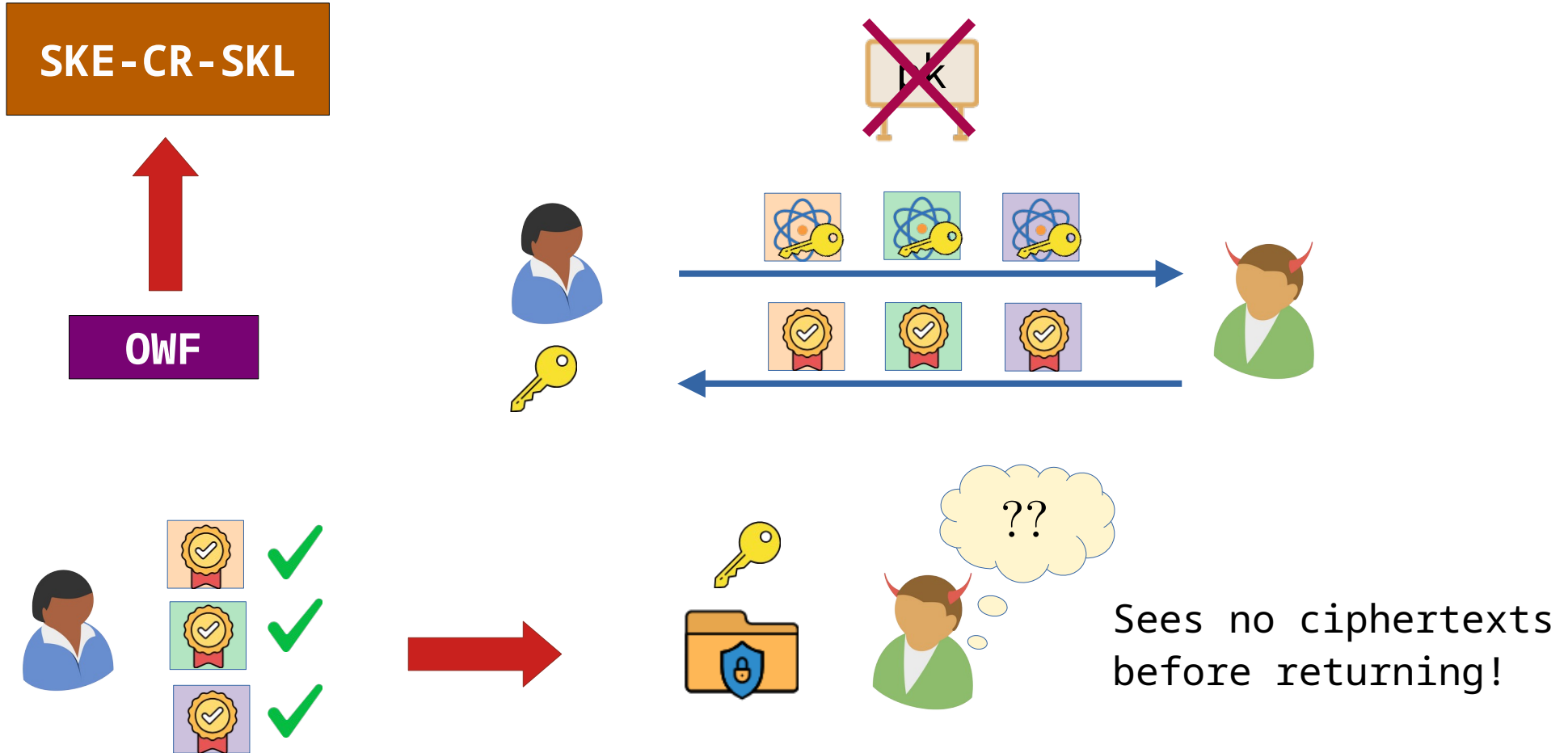


PKE-CR-SKL and SKE-CR-SKL

SKE-CR-SKL



PKE-CR-SKL and SKE-CR-SKL



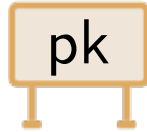
Building Block: **ABE**

**Attribute Based
Encryption**

[GPS+06]

Building Block: ABE

Attribute Based
Encryption



[GPS+06]

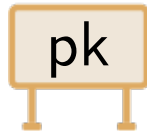


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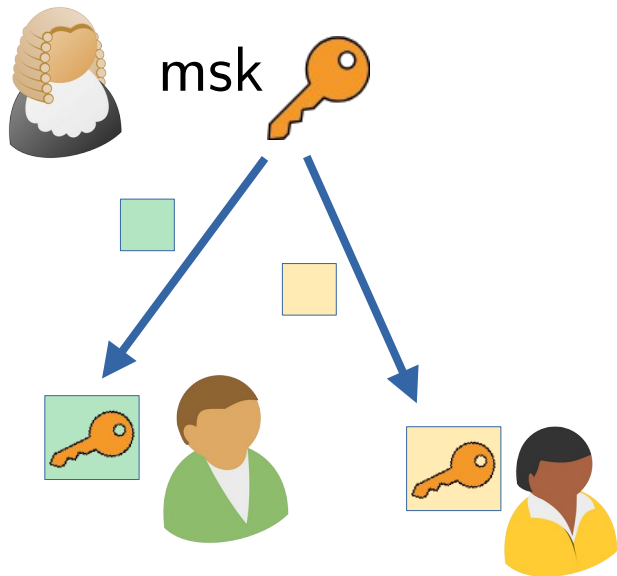


Building Block: ABE

Attribute Based
Encryption



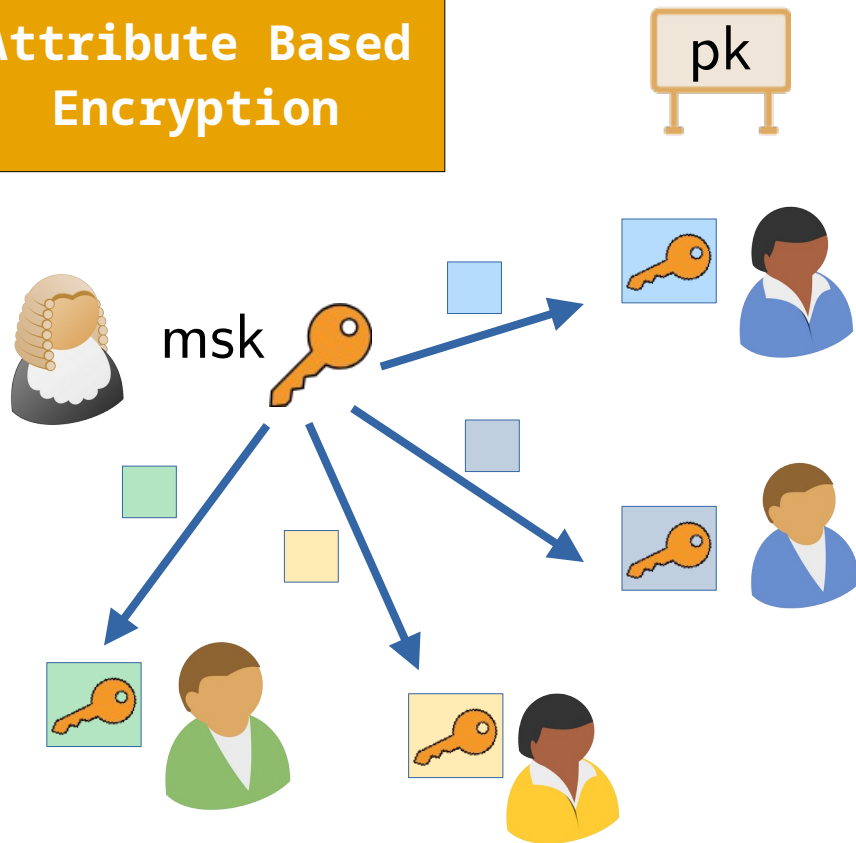
[GPS+06]



Building Block: ABE

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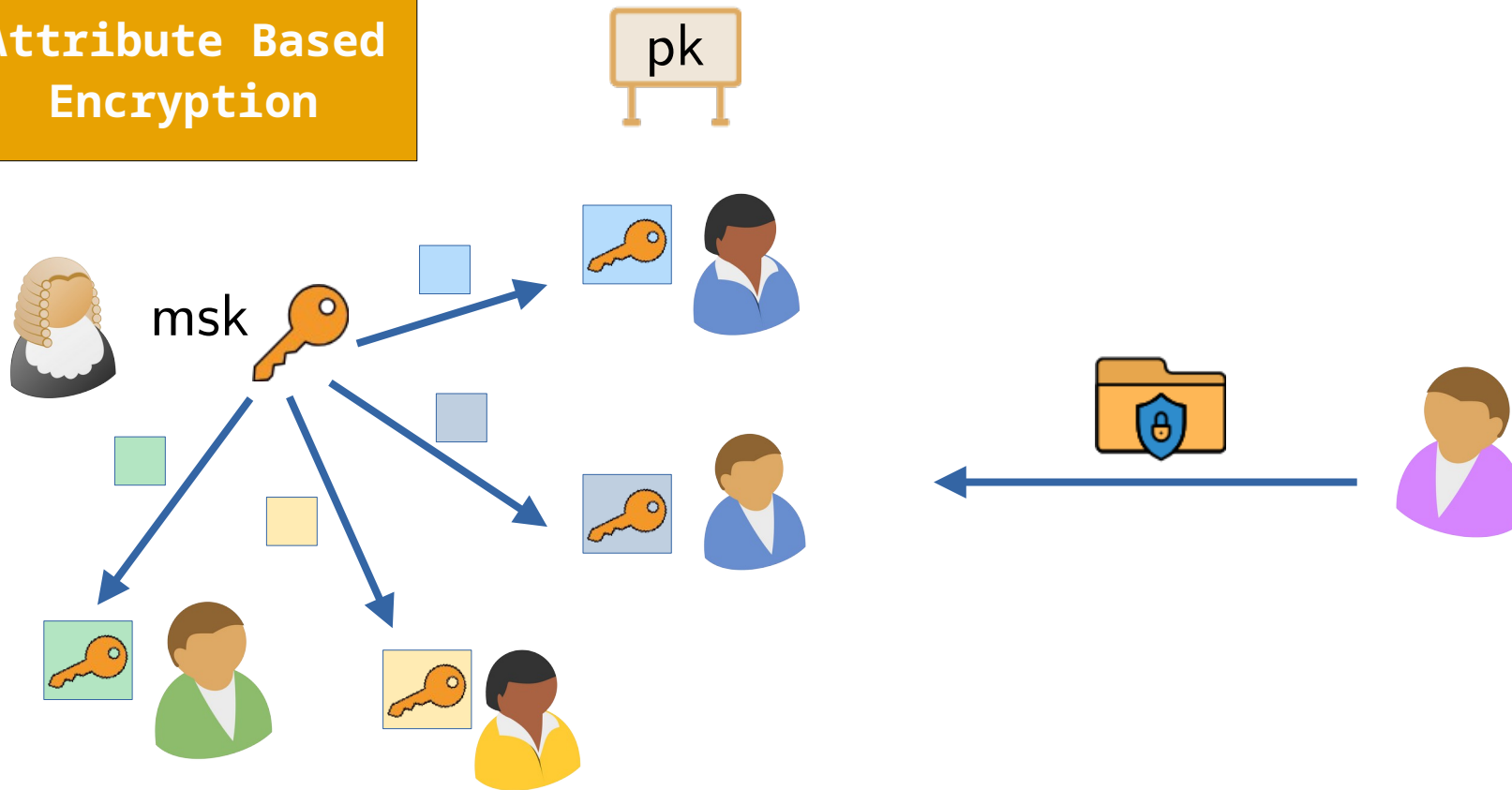
[GPS+06]



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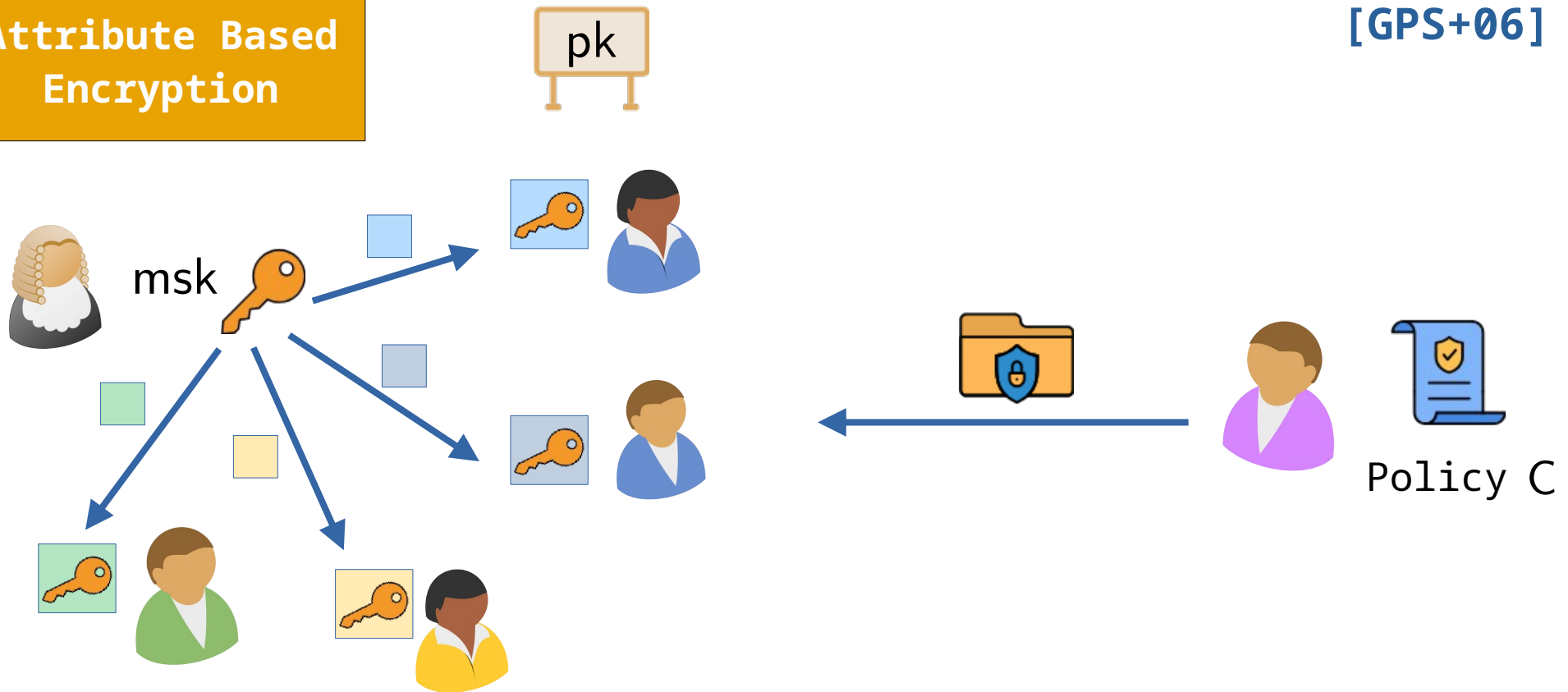
[GPS+06]



Building Block: ABE

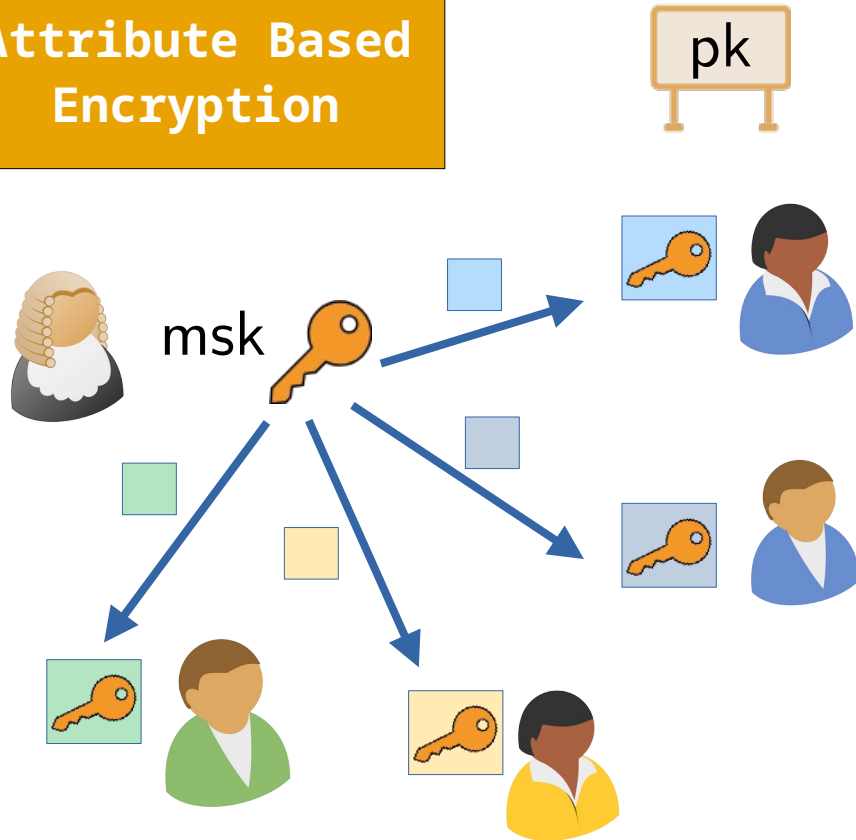
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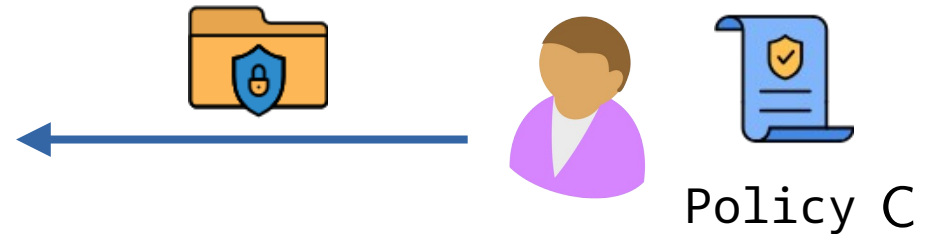
Building Block: ABE

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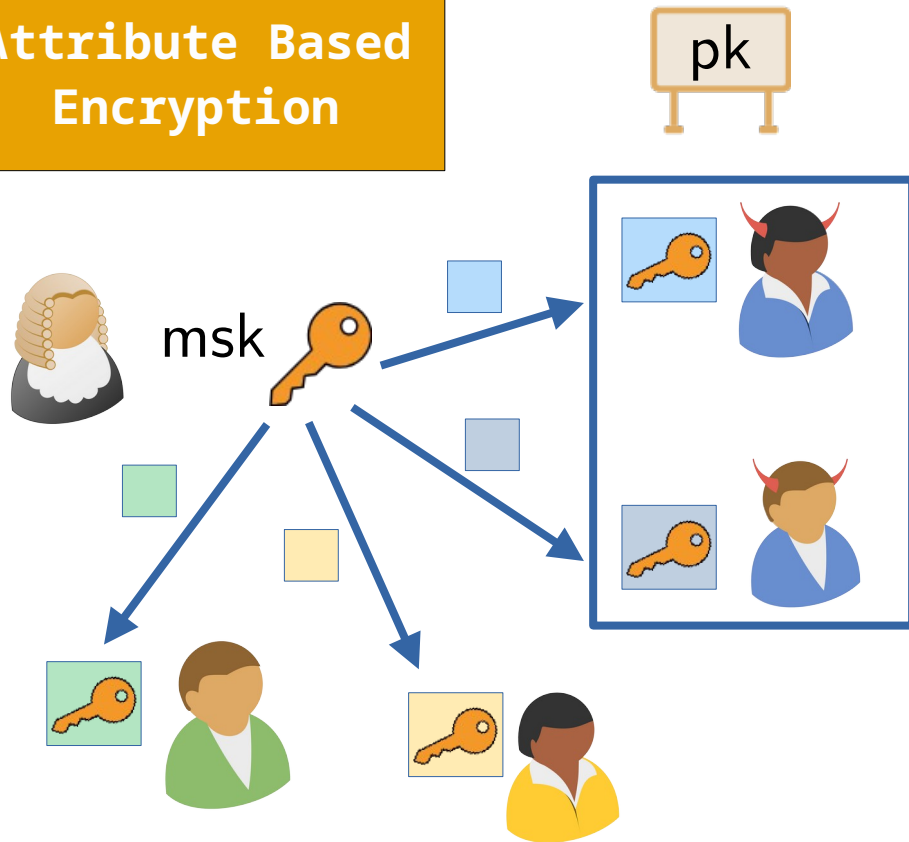
[GPS+06]

Only **non-blue** keys can decrypt.

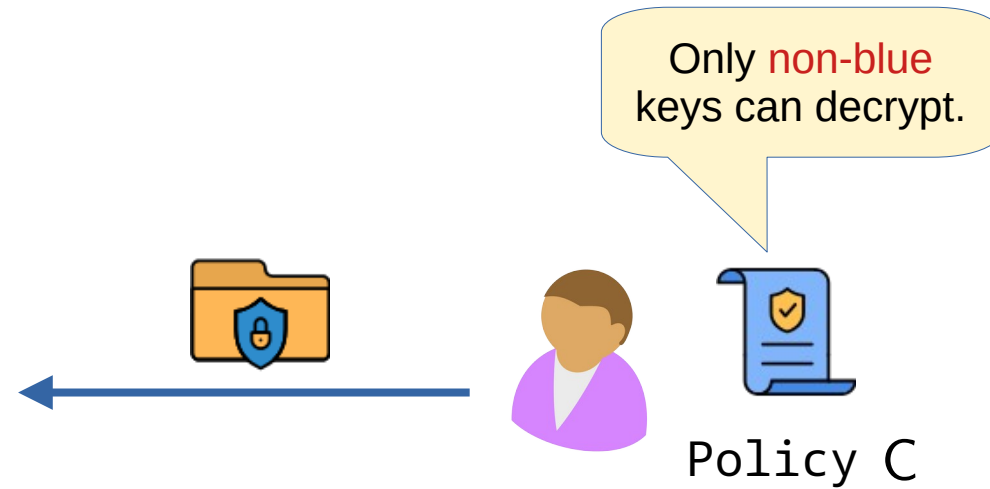


Building Block: ABE

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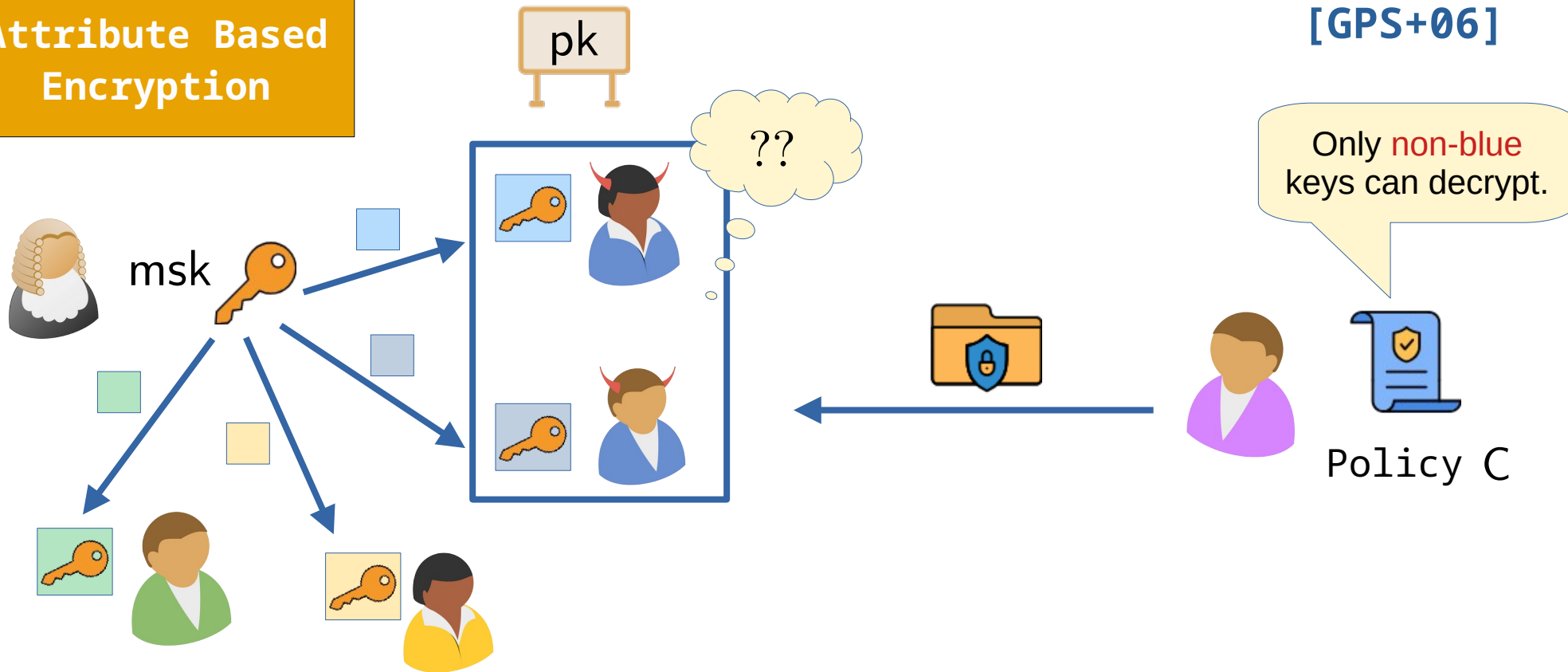


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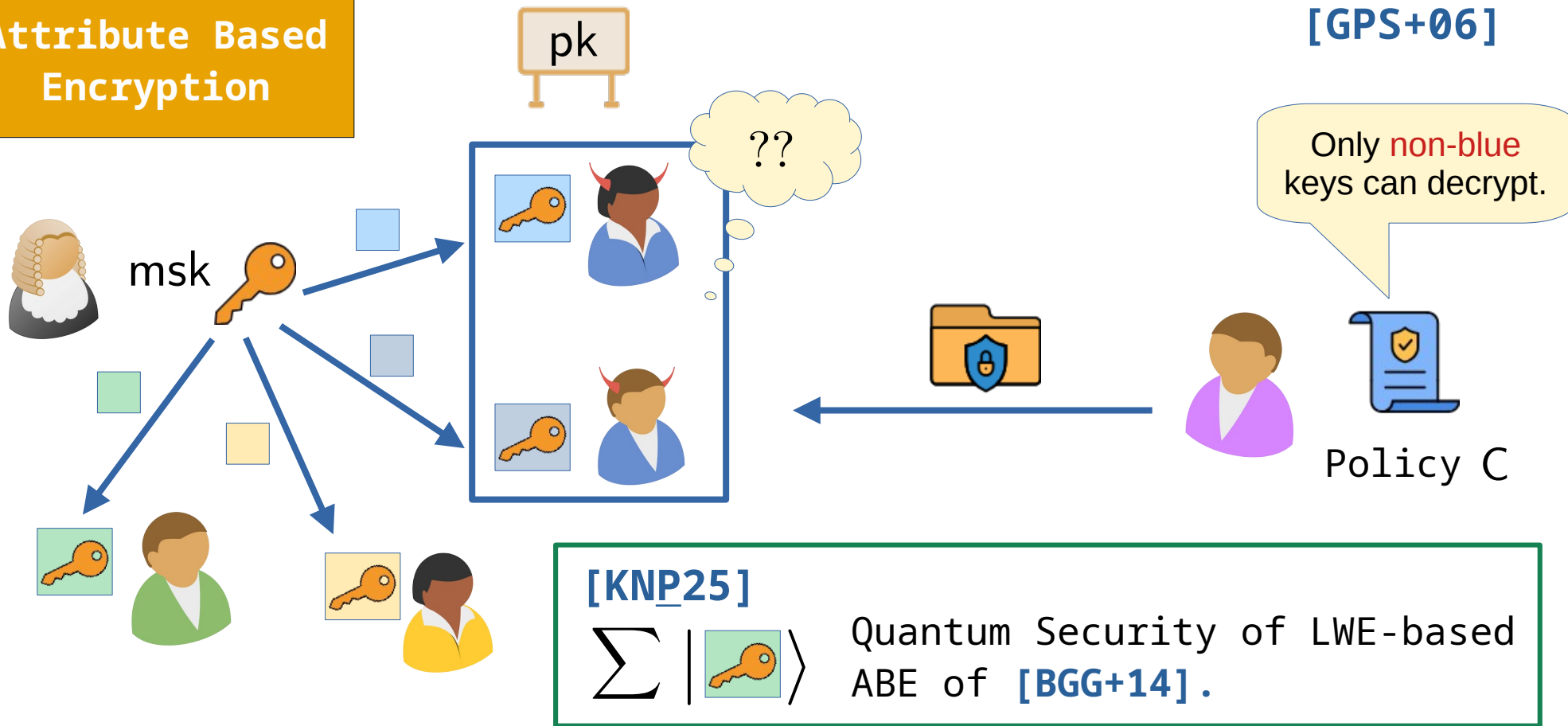
Building Block: ABE

Attribute Based Encryption



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Attribute Based Encryption



Main Idea: **Part I**

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$$\text{ske.dk} = \alpha_1 |\text{green}\rangle + \alpha_2 |\text{yellow}\rangle + \dots + \alpha_N |\text{blue}\rangle$$

SKE-CR-SKL

Main Idea: Part I

$$\text{ske.dk} = \alpha_1 |\text{green}\rangle + \alpha_2 |\text{yellow}\rangle + \dots + \alpha_N |\text{blue}\rangle$$

SKE-CR-SKL

Classical Decryption Property

$$\text{CDec}(\text{ske.ct}, \text{green}) = \text{CDec}(\text{ske.ct}, \text{yellow}) = \dots = \text{CDec}(\text{ske.ct}, \text{blue}) = m$$

Main Idea: Part I

$$\text{ske.dk} = \alpha_1 |\text{green}\rangle + \alpha_2 |\text{yellow}\rangle + \dots + \alpha_N |\text{blue}\rangle$$

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$$|\text{atom key}\rangle = \alpha_1 |\text{green}\rangle |\text{green key}\rangle + \alpha_2 |\text{yellow}\rangle |\text{yellow key}\rangle + \dots + \alpha_N |\text{blue}\rangle |\text{blue key}\rangle$$

PKE-CR-SKL

Main Idea: Part I

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$$\text{atom-key} = \alpha_1 |\text{green}\rangle |\text{key-green}\rangle + \alpha_2 |\text{yellow}\rangle |\text{key-yellow}\rangle + \dots + \alpha_N |\text{blue}\rangle |\text{key-blue}\rangle$$

PKE-CR-SKL

ABE Secret-Key

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SKE-CR-SKL

Classical Decryption Property

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$$\text{atom-key} = \alpha_1 |\text{green}\rangle |\text{green-key}\rangle + \alpha_2 |\text{yellow}\rangle |\text{yellow-key}\rangle + \dots + \alpha_N |\text{blue}\rangle |\text{blue-key}\rangle$$

PKE-CR-SKL

ABE Secret-Key



Main Idea: Part I

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SKE-CR-SKL

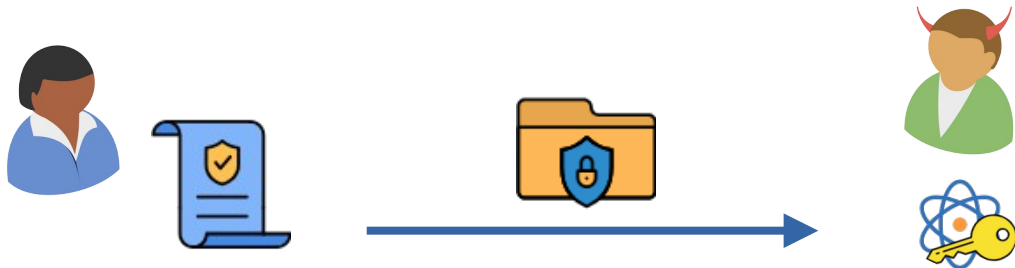
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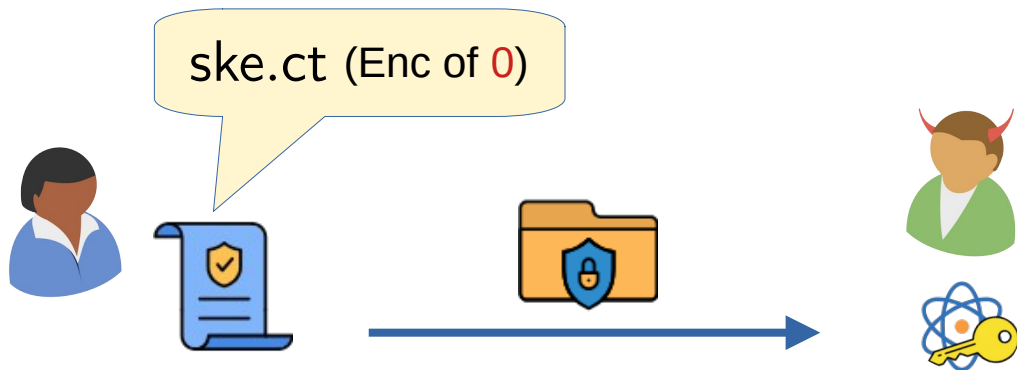
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$$\text{key} = \alpha_1 |\text{green}\rangle |\text{key-green}\rangle + \alpha_2 |\text{yellow}\rangle |\text{key-yellow}\rangle + \dots + \alpha_N |\text{blue}\rangle |\text{key-blue}\rangle$$

PKE-CR-SKL

ABE Secret-Key



Main Idea: Part I

$$\text{ske.dk} = \alpha_1 |\text{■}\rangle + \alpha_2 |\text{■}\rangle + \dots + \alpha_N |\text{■}\rangle$$

SKE-CR-SKL

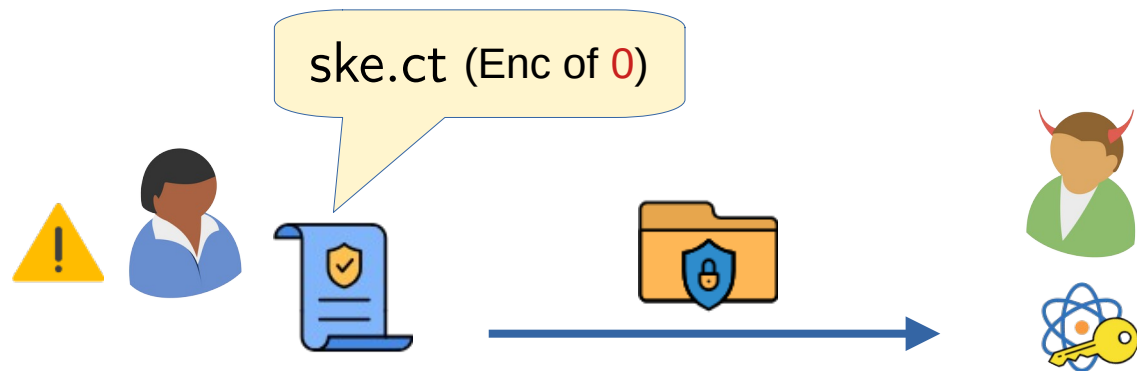
Classical Decryption Property

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$$\text{⚛️} = \alpha_1 |\text{■}\rangle |\text{■}\rangle + \alpha_2 |\text{■}\rangle |\text{■}\rangle + \dots + \alpha_N |\text{■}\rangle |\text{■}\rangle$$

PKE-CR-SKL

ABE Secret-Key



Main Idea: Part I

$$\text{ske.dk} = \alpha_1 |\text{green}\rangle + \alpha_2 |\text{yellow}\rangle + \dots + \alpha_N |\text{blue}\rangle$$

SKE-CR-SKL

Classical Decryption Property

$$\text{CDec}(\text{ske.ct}, \text{green}) = \text{CDec}(\text{ske.ct}, \text{yellow}) = \dots = \text{CDec}(\text{ske.ct}, \text{blue}) = m$$

$$\text{atom-key} = \alpha_1 |\text{green}\rangle |\text{green-key}\rangle + \alpha_2 |\text{yellow}\rangle |\text{yellow-key}\rangle + \dots + \alpha_N |\text{blue}\rangle |\text{blue-key}\rangle$$

PKE-CR-SKL

ABE Secret-Key

ske.ct (Enc of 0)



satisfies policy if:

$$\text{CDec}(\text{ske.ct}, \text{green}) = 0$$



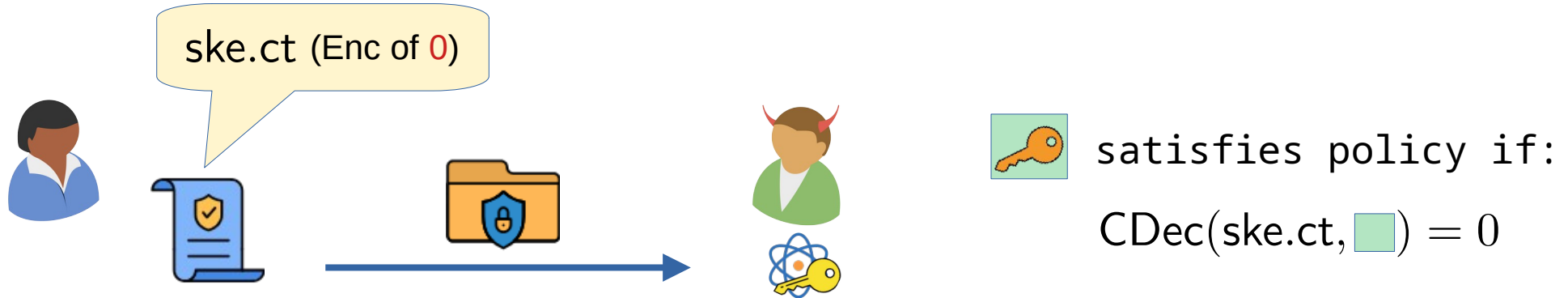
Main Idea: **Part II**

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$$\text{🔑} = \alpha_1 |\text{🟩}\rangle |\text{🔑🟩}\rangle + \alpha_2 |\text{🟨}\rangle |\text{🔑🟨}\rangle + \dots + \alpha_N |\text{🟦}\rangle |\text{🔑🟦}\rangle$$

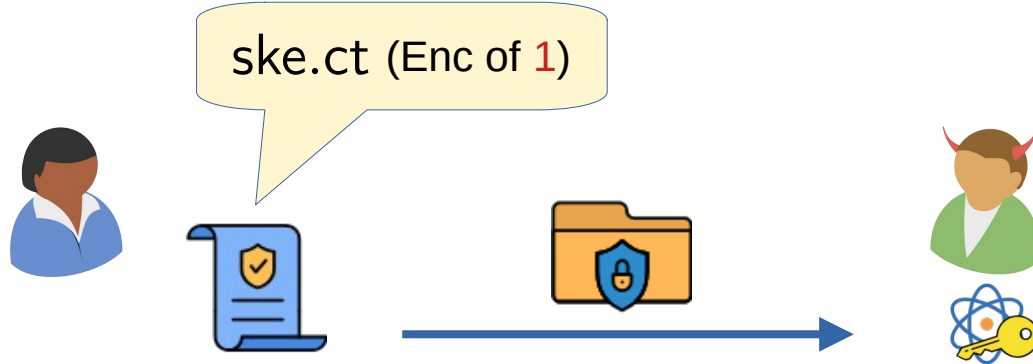
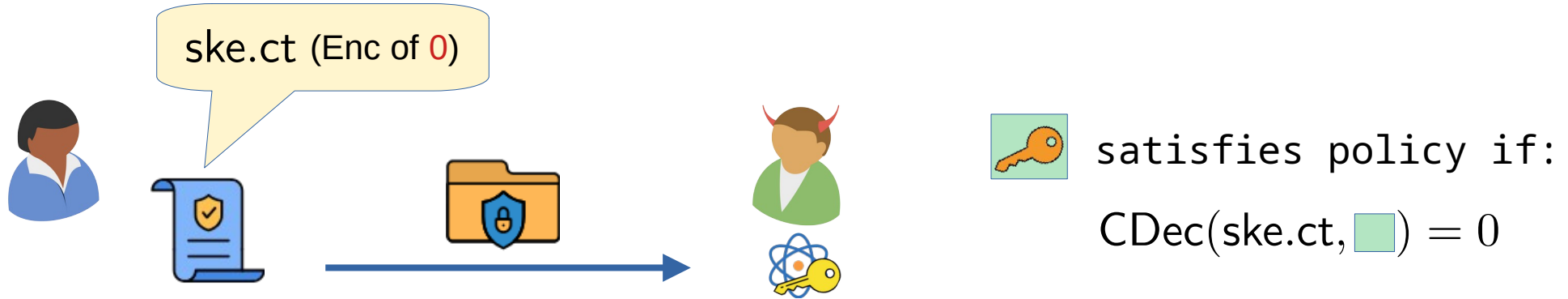
Main Idea: **Part II**

$$\text{key} = \alpha_1 |\text{green box}\rangle |\text{key in green box}\rangle + \alpha_2 |\text{yellow box}\rangle |\text{key in yellow box}\rangle + \dots + \alpha_N |\text{blue box}\rangle |\text{key in blue box}\rangle$$



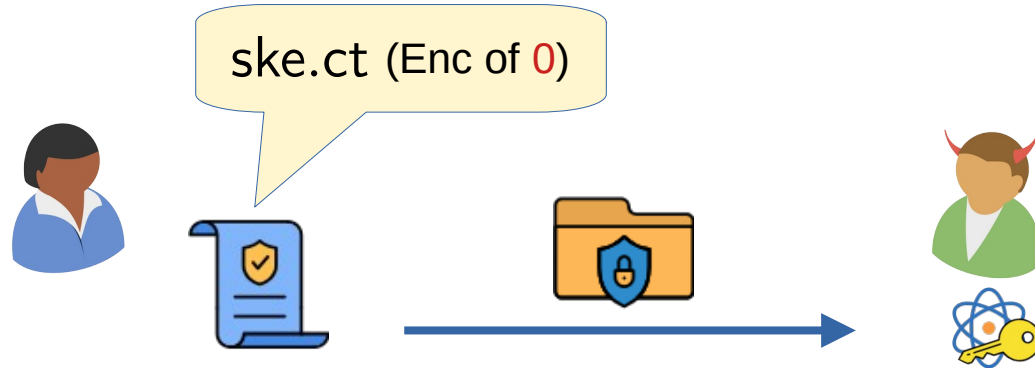
Main Idea: **Part II**


$$\text{atom-key} = \alpha_1 |\text{green box}\rangle |\text{key in green box}\rangle + \alpha_2 |\text{yellow box}\rangle |\text{key in yellow box}\rangle + \dots + \alpha_N |\text{blue box}\rangle |\text{key in blue box}\rangle$$

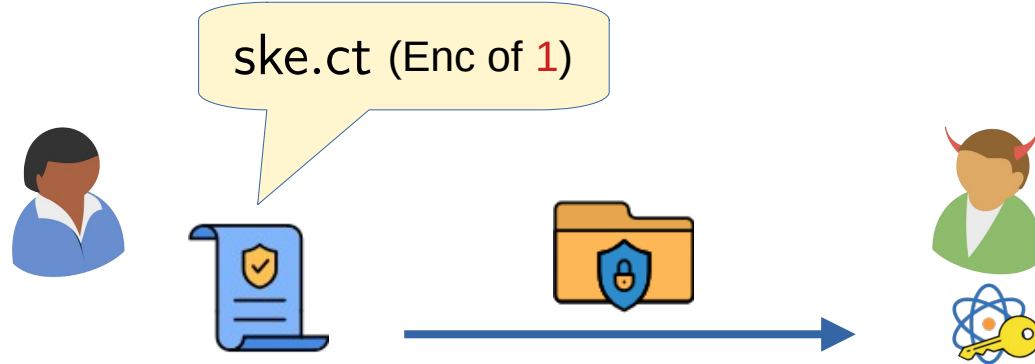


Main Idea: **Part II**

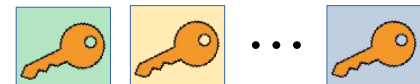
$$\text{key icon} = \alpha_1 |\text{green box}\rangle |\text{key in green box}\rangle + \alpha_2 |\text{yellow box}\rangle |\text{key in yellow box}\rangle + \dots + \alpha_N |\text{blue box}\rangle |\text{key in blue box}\rangle$$



 satisfies policy if:
 $\text{CDec}(\text{ske.ct}, \text{green box}) = 0$

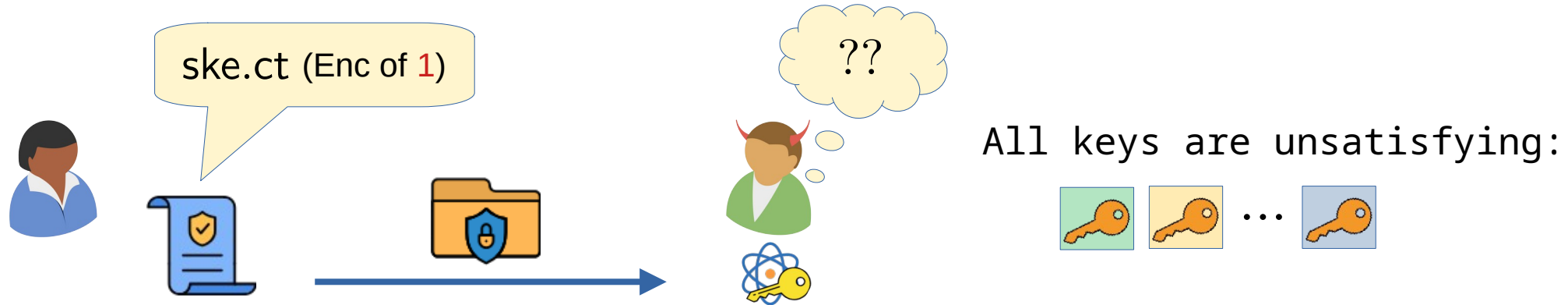
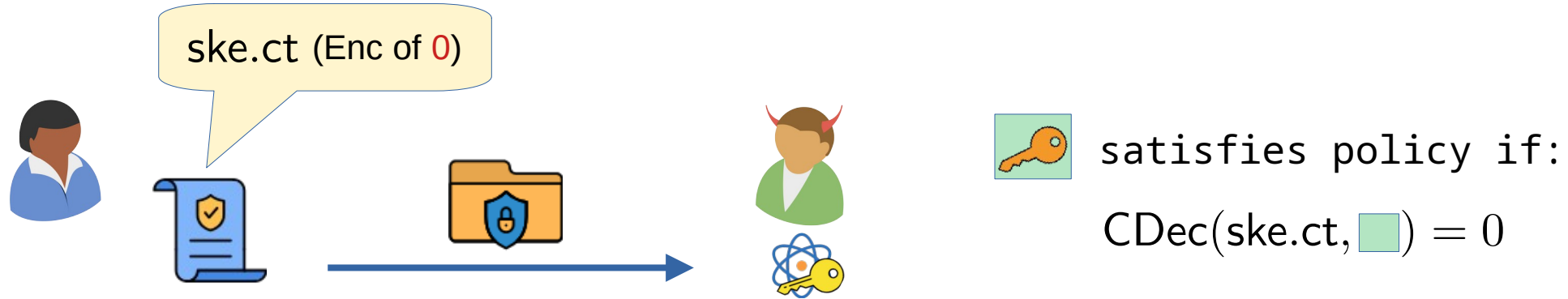


All keys are unsatisfying:



Main Idea: **Part II**

$$\text{key} = \alpha_1 |\text{green box}\rangle |\text{green key}\rangle + \alpha_2 |\text{yellow box}\rangle |\text{yellow key}\rangle + \dots + \alpha_N |\text{blue box}\rangle |\text{blue key}\rangle$$



Main Idea: **Part III**

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$$\text{🔑} = \alpha_1 |\text{🟩}\rangle |\text{🔑🟩}\rangle + \alpha_2 |\text{🟨}\rangle |\text{🔑🟨}\rangle + \dots + \alpha_N |\text{🟦}\rangle |\text{🔑🟦}\rangle$$

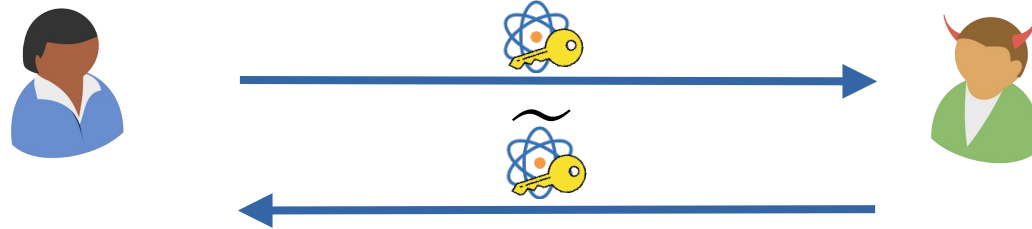
Main Idea: Part III

$$\text{🔑} = \alpha_1 |\text{🟩}\rangle |\text{🔑🟩}\rangle + \alpha_2 |\text{🟨}\rangle |\text{🔑🟨}\rangle + \dots + \alpha_N |\text{🟦}\rangle |\text{🔑🟦}\rangle$$



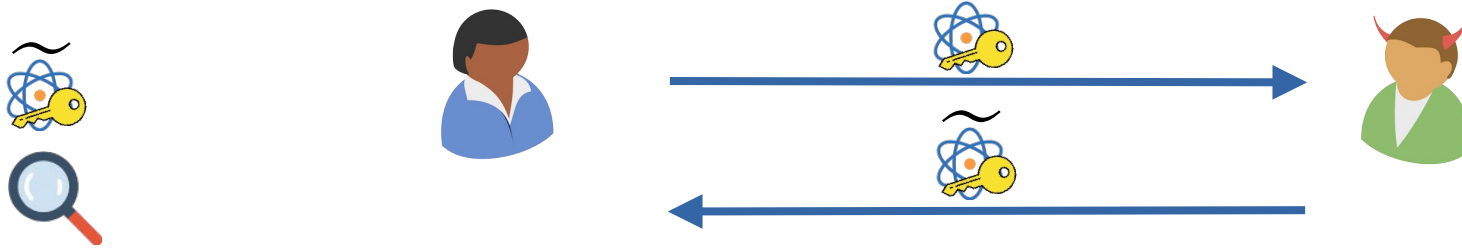
Main Idea: Part III

$$\text{🔑} = \alpha_1 |\text{🟩}\rangle |\text{🔑🟩}\rangle + \alpha_2 |\text{🟨}\rangle |\text{🔑🟨}\rangle + \dots + \alpha_N |\text{🟦}\rangle |\text{🔑🟦}\rangle$$



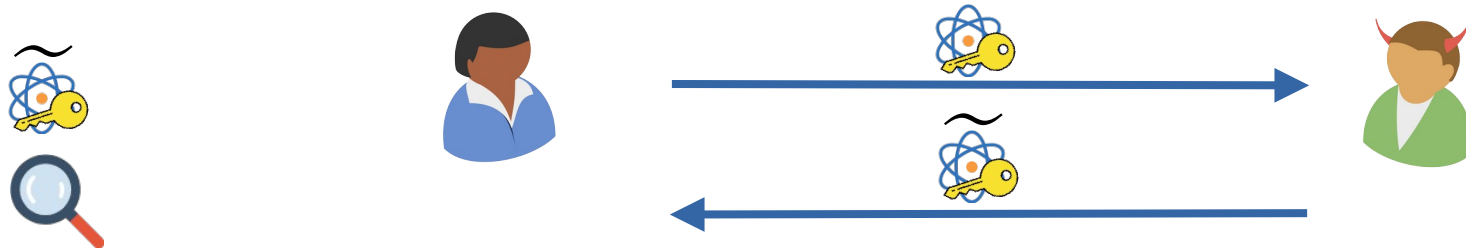
Main Idea: Part III

$$\text{Atom-Key} = \alpha_1 |\text{Green Box}\rangle |\text{Key in Green Box}\rangle + \alpha_2 |\text{Yellow Box}\rangle |\text{Key in Yellow Box}\rangle + \dots + \alpha_N |\text{Blue Box}\rangle |\text{Key in Blue Box}\rangle$$



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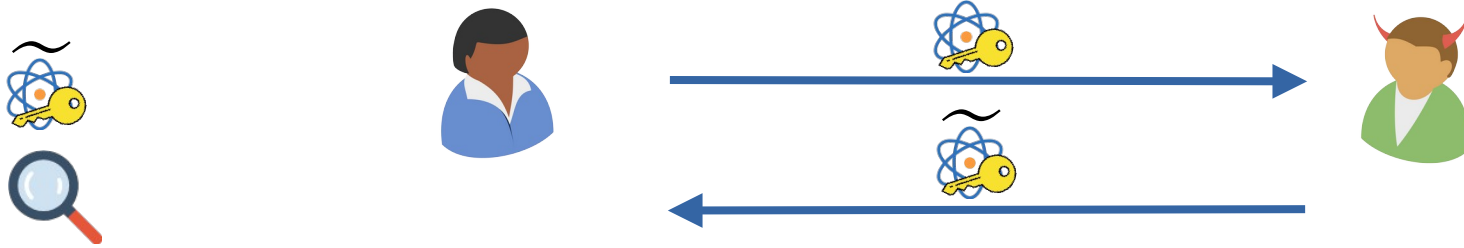


1. Uncompute ABE keys

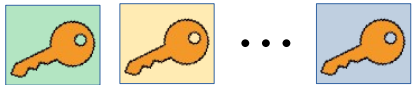


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$$\text{Atom Key} = \alpha_1 |\text{Green Box}\rangle |\text{Key in Green Box}\rangle + \alpha_2 |\text{Yellow Box}\rangle |\text{Key in Yellow Box}\rangle + \dots + \alpha_N |\text{Blue Box}\rangle |\text{Key in Blue Box}\rangle$$



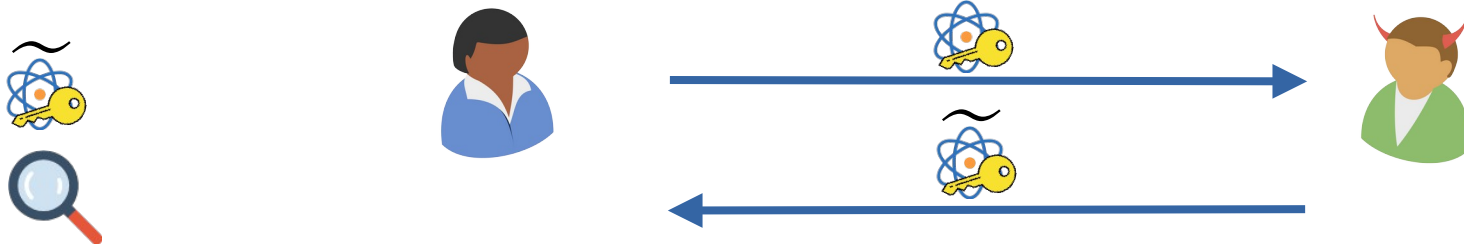
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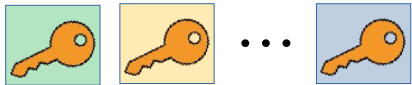
2. Verify $\widetilde{\text{ske.dk}}$.

Main Idea: Part III

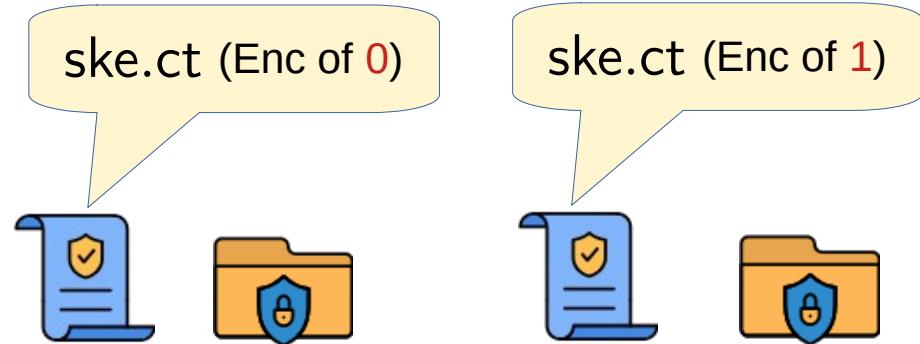
$$\text{atom-key} = \alpha_1 |\text{green box}\rangle |\text{key in green box}\rangle + \alpha_2 |\text{yellow box}\rangle |\text{key in yellow box}\rangle + \dots + \alpha_N |\text{blue box}\rangle |\text{key in blue box}\rangle$$



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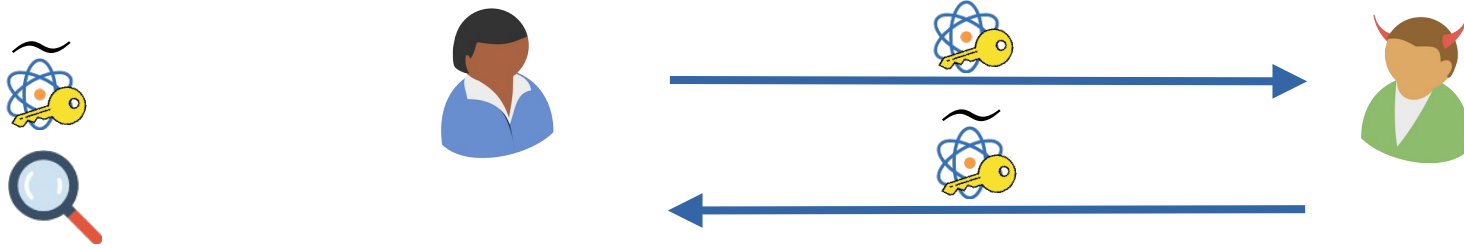


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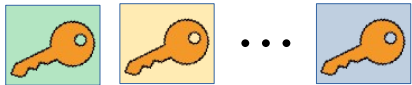


Main Idea: Part III

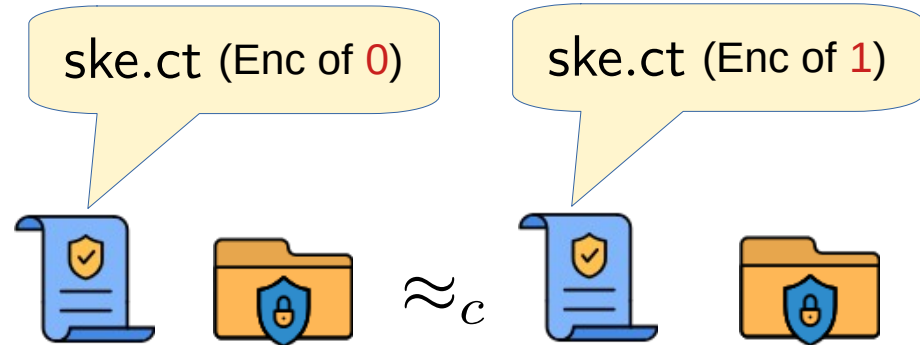
$$\text{atom-key} = \alpha_1 |\text{green box}\rangle |\text{key in green box}\rangle + \alpha_2 |\text{yellow box}\rangle |\text{key in yellow box}\rangle + \dots + \alpha_N |\text{blue box}\rangle |\text{key in blue box}\rangle$$



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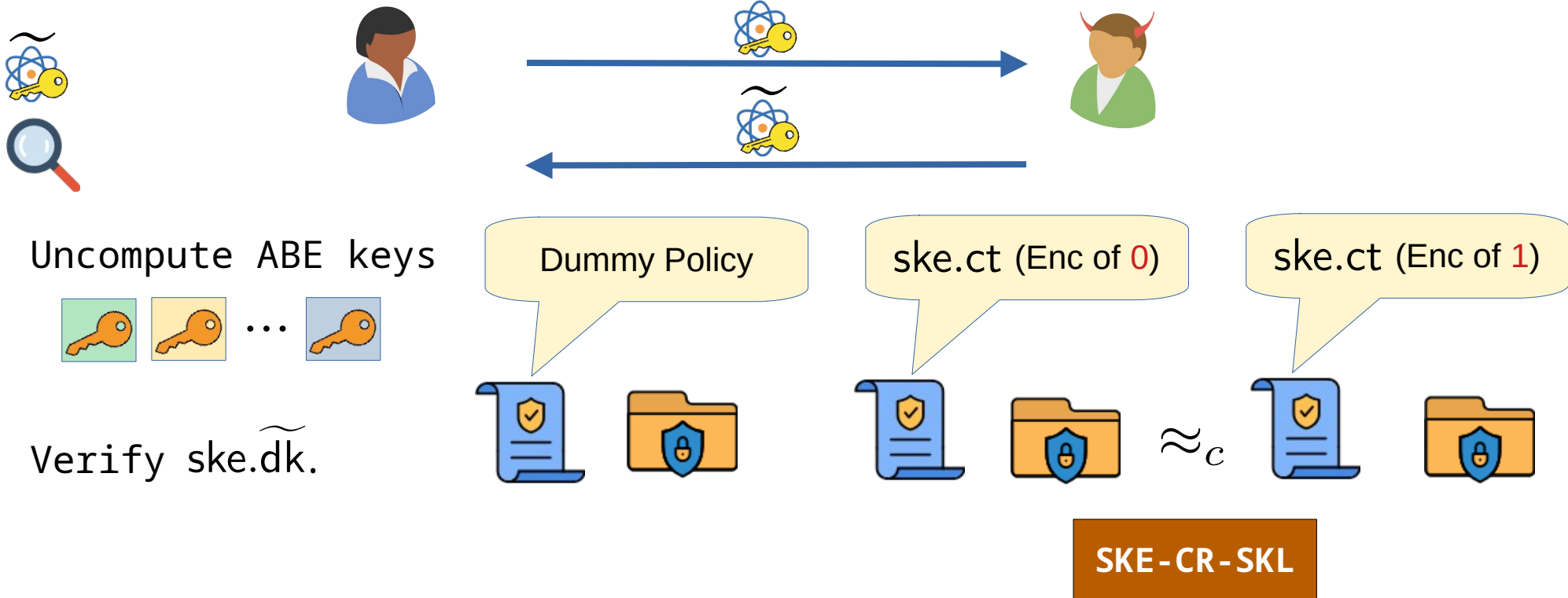
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SKE-CR-SKL

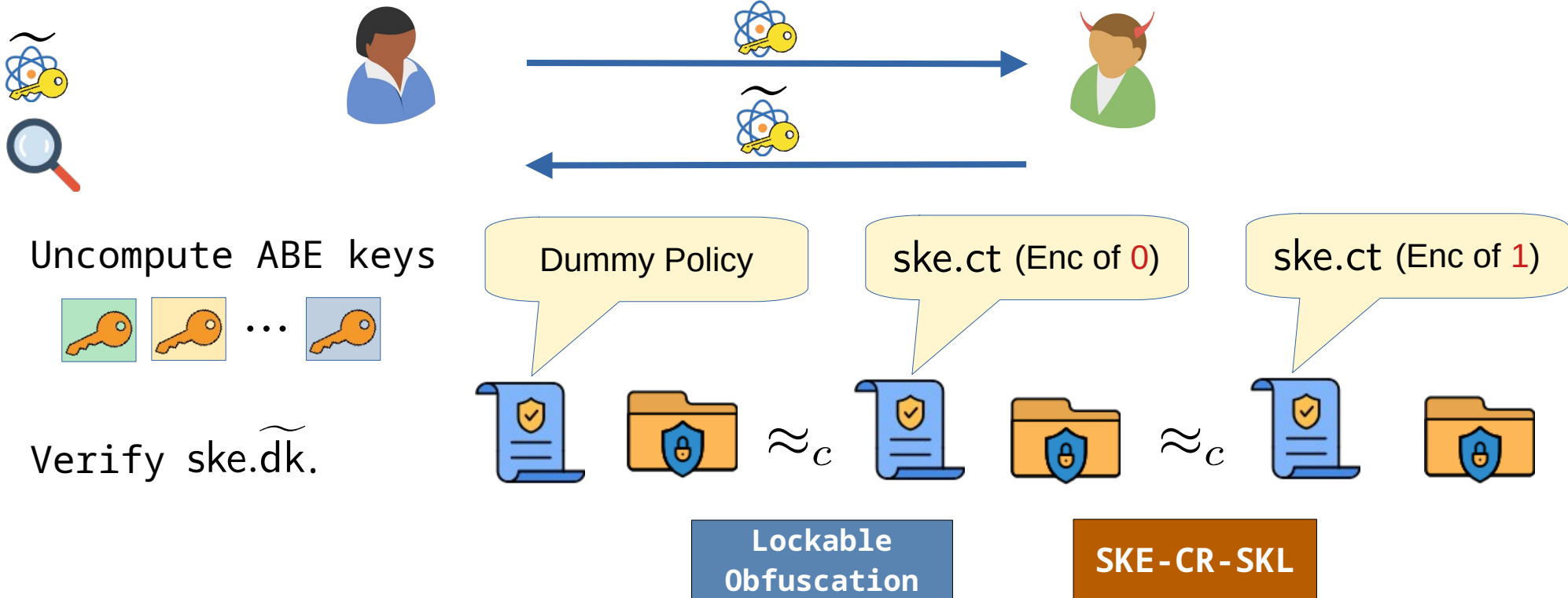
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Thank You!