Pseudorandom Unitaries in the Haar Random Oracle Model

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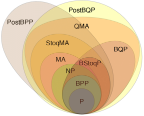
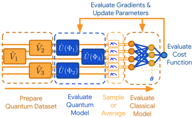
Quantum Pseudorandomness

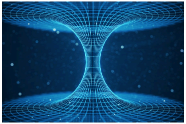
Quantum Complexity



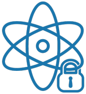
Quantum Learning Theory



Wormholes and AdS/CFT



Quantum Cryptography



Pseudorandom Unitaries

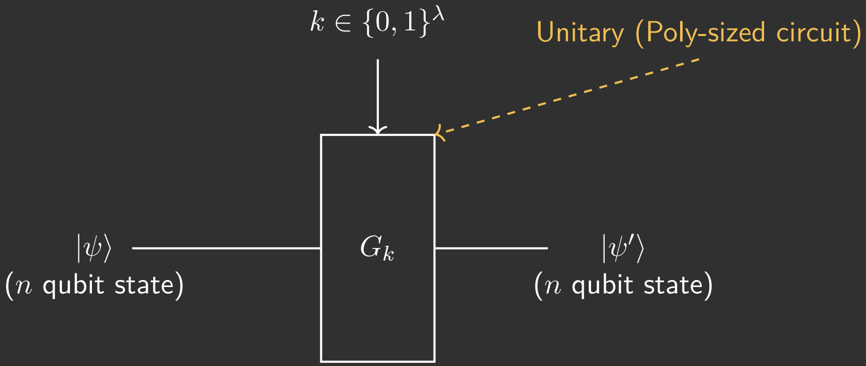
A Central Primitive of Quantum Pseudorandomness

Pseudorandom Unitary (PRU)

Efficiently implementable circuits that “behave like” random unitary.

Pseudorandom Unitary (PRU)

1. Efficient implementation:



Notation: n-PRU

Pseudorandom Unitary (PRU)

2. Pseudorandomness

$$\mathcal{A}^{G_k} \approx \mathcal{A}^{U}$$

U: Haar random unitary

Strong Pseudorandom Unitary (sPRU)

2. Pseudorandomness

$$\mathcal{A}^{G_k, G_k^\dagger} \approx \mathcal{A}^{U, U^\dagger}$$

Previous work

- (JLS18) defined PRU.
- (AGKL22,LQS+23,BM24,MPSY24,CBB+24) gave partial results.
- (MH24) finally gave a construction for adaptive queries.

Defined Path-recording framework

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Why Should We Care About PRUs?

- Succinct Commitments to Quantum States (Chen-Movassagh'24)
- Multi-Copy Secure Encryption Scheme (AGKL24)
- Learning Theory and Complexity Theory
- AdS/CFT correspondence (Bouland-Fefferman-Vazirani'20)
- **Kretschmer (Kre21)** showed evidence that:

  *Quantum pseudorandomness may exist **even if one-way functions do not exist**.*

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Constructing PRUs Without One-Way Functions

- **Many constructions of quantum pseudorandomness:**
  - JLS18, BS19, BS20, AQY22, AGQY22, BBSS23, LQS+23, ABF+24, AGKL24, MPSY24, BM24

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Quantum Haar Random Oracle Model

[Bouland-Fefferman-Vazirani'20, Chen-Movassagh'24]

Quantum Haar Random Oracle Model (QHROM)

All parties P_i as well as the adversary $\mathcal{A}$ get oracle access to a Haar Unitary and its inverse.

$$U \leftarrow \mu_n$$

μ_n: Haar Distribution

$\boxed{P_1}$ $\boxed{P_2}$ $\cdots$ $\boxed{P_t}$ $\boxed{\mathcal{A}}$

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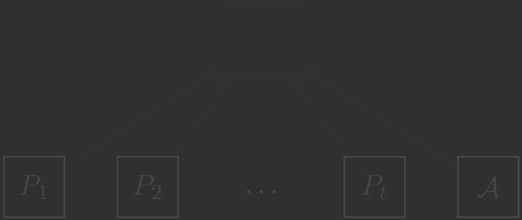

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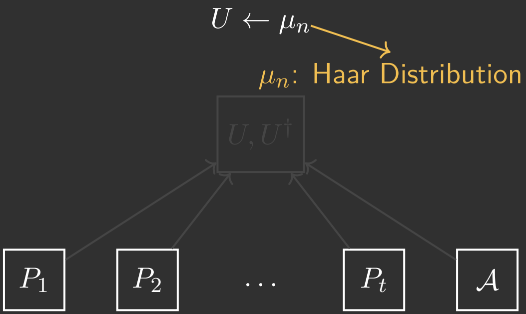

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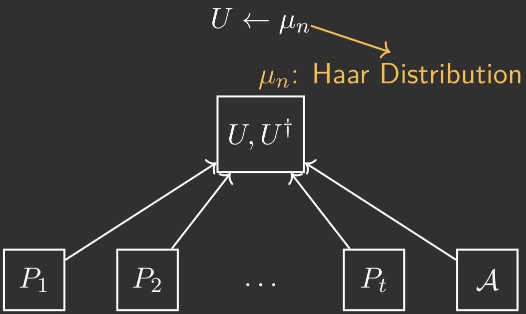
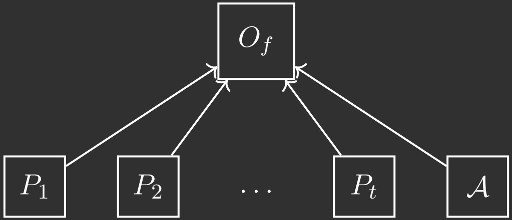


Similarity to QROM

This model is similar to the Quantum Random Oracle Model (QROM) where all parties and the adversary get access to a random function oracle.

$$f \leftarrow \mathcal{F}_n$$



Why use QHROM

- Can Model Behaviour of Random Quantum Circuits.
- Abstraction to study Blackhole Dynamics.
- Gives a pathway to get results from PRU in the plain model.
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Strong Pseudorandom Unitary (PRU)

Psuedorandomness

$$\mathcal{A}^{U,U^\dagger,G_k^{U,U^\dagger}} \approx \mathcal{A}^{U,U^\dagger,V}$$

What can we do in QHROM

- [Bouland-Fefferman-Vazirani'21] Give a candidate construction of PRUs in the QHROM without proof.
- [ABGY25,HY25] Made partial progress on showing PRUs exist in QHROM.

Results

In QHROM

- **Unbounded-poly-query secure strong PRUs in QHROM:** Achieved with **two queries** to the Haar random oracle.
-

In QHROM

- **Unbounded-poly-query secure strong PRUs in QHROM:**
  Achieved with **two queries** to the Haar random oracle.
- **Strong Gluing Theorem for Haar Unitaries:**
  How to construct “Large” Haar Unitaries from smaller ones.
  [SHH24] shows how to do this in the inverseless case.

Consequences

Shrinking strong PRU Keys for Free, in Plain Model:

Unbounded query secure strong PRUs exist with keys of size $O(\lambda^{1/c})$ for any constant c, if strong PRU exists

Previously, GJMZ22 showed 1 query PRU with short keys exists if PRU exists.

Construction of other primitives in QHROM:

sPRU $\rightarrow$ PRU $\rightarrow$ PRFS $\rightarrow$ PRS $\rightarrow$ OWSG $\rightarrow$ EFI.

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Techniques

Strong PRU in QHROM

Potential constructions

Single Query

$$A_k \, U \, B_k$$

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$$(A_k U B_k \otimes I)|EPR\rangle = (A_k U \otimes B_k^T)|EPR\rangle$$

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Apply on EPR and "OR lemma" attack

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Parallel Query

$$A_k \, U^{\otimes s} \, B_k$$

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Unprotected Query

$$U\ A_k\ U$$

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Unprotected Query

$$U \, A_k \, U$$

$$U^\dagger (U \, A_k \, U) U^\dagger \; = \; A_k$$

Potential constructions

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Apply on $U^\dagger$ and Learn A_k

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Two Query

$$A_k U B_k U C_k$$

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No easy way to attack

PRU in QHROM

PRU in QHROM : $G^U(k_1||k_2||k_3) = X^{k_3}UX^{k_2}UX^{k_1}$

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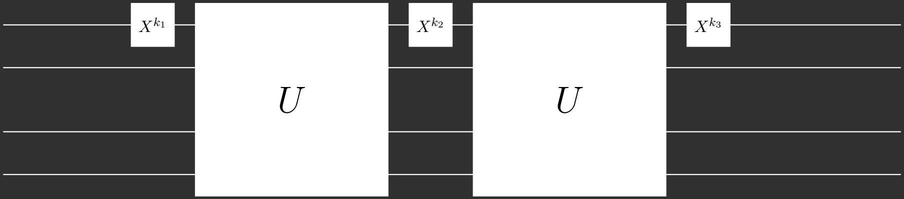

PRU in QHROM

$$U \leftarrow \mu_n$$

$$\rho_{\mathsf{AB}}^{\mathcal{A}} = \mathop{\mathbb{E}}_{\substack{U \leftarrow \mu_n \\ k \leftarrow \{0,1\}^\lambda}} \left[|\mathcal{A}^{U,U^\dagger,G_k^U}\rangle\langle\mathcal{A}^{U,U^\dagger,G_k^U}|_{\mathsf{AB}} \right]$$

Very hard to understand this state.

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Techniques

Path Recording framework

[MH24]

Purification

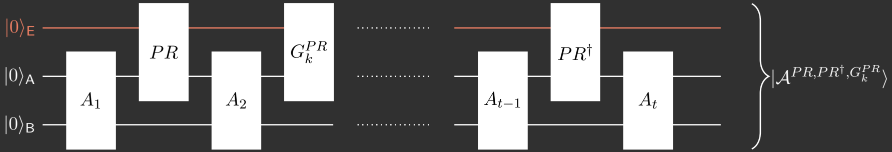
$$U \leftarrow \mu_n$$

$$\rho_{\mathsf{AB}}^{\mathcal{A}} = \mathop{\mathbb{E}}_{\substack{U \leftarrow \mu_n \\ k \leftarrow \{0,1\}^\lambda}} \left[|\mathcal{A}^{U,U^\dagger,G_k^U}\rangle\langle\mathcal{A}^{U,U^\dagger,G_k^U}|_{\mathsf{AB}} \right]$$

By Schmidt decomposition, for some $|\psi_{\mathcal{A}}\rangle$

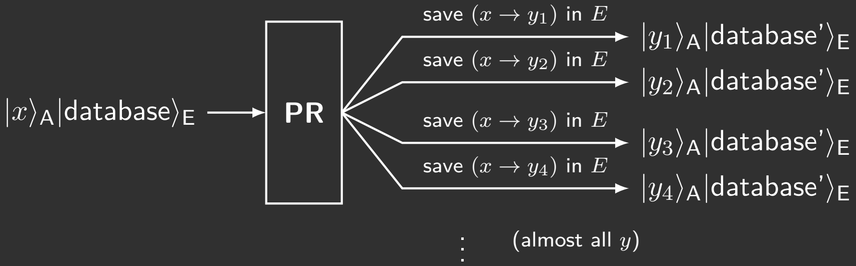
$$\rho_{\mathsf{AB}}^{\mathcal{A}} = Tr_{\mathsf{E}} \left(|\psi_{\mathcal{A}}\rangle\langle\psi_{\mathcal{A}}|_{\mathsf{ABE}} \right)$$

Compressed Purification



$$\mathop{\mathbb{E}}_{U \leftarrow \mu_n}\left[|\mathcal{A}^{U,U^\dagger,G_k^U}\rangle\langle\mathcal{A}^{U,U^\dagger,G_k^U}|_{\mathsf{AB}}\right] \approx Tr_{\mathsf{E}}\left(|\mathcal{A}^{PR,PR^\dagger,G_k^{PR}}\rangle\langle\mathcal{A}^{PR,PR^\dagger,G_k^{PR}}|_{\mathsf{ABE}}\right)$$

Path Recording



$$\mathsf{PR}_{\mathsf{AE}} : |x\rangle_{\mathsf{A}}|R\rangle_{\mathsf{E}} \mapsto \frac{1}{\sqrt{N-|R|}} \sum_{\substack{y\in[N],\\ y\notin \mathrm{Im}(R)}} |y\rangle_{\mathsf{A}}\underbrace{|R \cup \{(x,y)\}\rangle_{\mathsf{E}}}_{\text{Path}}.$$

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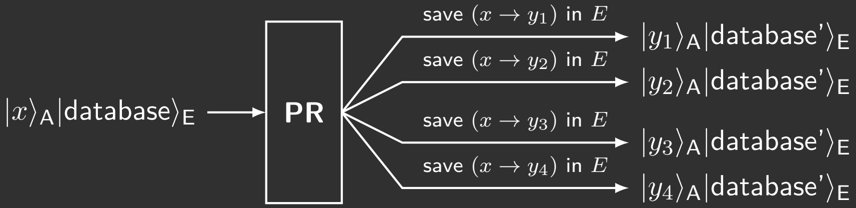

$\vdots$ (almost all y)

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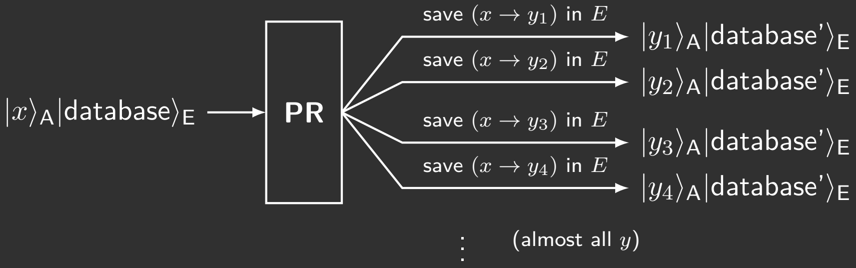

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Techniques

Analysing our construction

Ideal vs Real

Ideal

- First oracle: U_1
- Second oracle: U_2
- Purification is Two "Paths"
- $|\text{Path}_1\rangle \otimes |\text{Path}_2\rangle$

Real

- First oracle: U
- Second oracle: $X^{k_3} U X^{k_2} U X^{k_1}$
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Key Simulator $\mathcal{I}^{\mathsf{Sim}}$

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- $|\text{Path}\rangle \otimes |\text{Keys}\rangle$

Key Simulator $\mathcal{I}^{\mathsf{Sim}}$

Ideal vs Real

Ideal

- First oracle: U_1
- Second oracle: U_2
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Real

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Ideal Experiment

First Oracle

$$|\text{Path}_1\rangle \otimes |\text{Path}_2\rangle$$

$$\downarrow$$

first oracle

U_1

$$\downarrow$$

$$|\text{Path}_1 + \text{Entry}\rangle \otimes |\text{Path}_2\rangle$$

Second Oracle

$$|\text{Path}_1\rangle \otimes |\text{Path}_2\rangle$$

$$\downarrow$$

second oracle

U_2

$$\downarrow$$

$$|\text{Path}_1\rangle \otimes |\text{Path}_2 + \text{Entry}\rangle$$

Ideal Experiment

First Oracle

$$|\text{Path}_1\rangle \otimes |\text{Path}_2\rangle$$

$$\downarrow$$

$$\boxed{\begin{array}{c}\text{first oracle}\\ U_1\end{array}}$$

$$\downarrow$$

$$|\text{Path}_1 + \text{Entry}\rangle \otimes |\text{Path}_2\rangle$$

Second Oracle

$$|\text{Path}_1\rangle \otimes |\text{Path}_2\rangle$$

$$\downarrow$$

$$\boxed{\begin{array}{c}\text{second oracle}\\ U_2\end{array}}$$

$$\downarrow$$

$$|\text{Path}_1\rangle \otimes |\text{Path}_2 + \text{Entry}\rangle$$

Real Experiment

First Oracle

$|\text{Path}\rangle \otimes |\text{Keys}\rangle$

$\downarrow$

first oracle

U

$\downarrow$

$|\text{Path} + \text{entry}\rangle \otimes |\text{Keys}\rangle$

Second Oracle

$|\text{Path}\rangle \otimes |\text{Keys}\rangle$

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second oracle

$X^{k_3} U X^{k_2} U X^{k_1}$

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$|\text{Path} + \underbrace{\{\text{entry}_1, \text{entry}_2\}}_{}\rangle \otimes |\text{Keys}\rangle$

Entries Correlated by Keys

Real Experiment

First Oracle

$|\text{Path}\rangle \otimes |\text{Keys}\rangle$

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Entries Correlated by Keys

Measuring Closeness

To prove closeness, we do Query-by-Query Analysis:

- Take any intermediate state
- The following two processes are close:
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Progress Measure

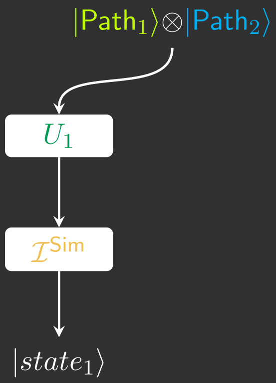
First Oracle

$|\text{Path}_1\rangle \otimes |\text{Path}_2\rangle$

Second Oracle

Progress Measure

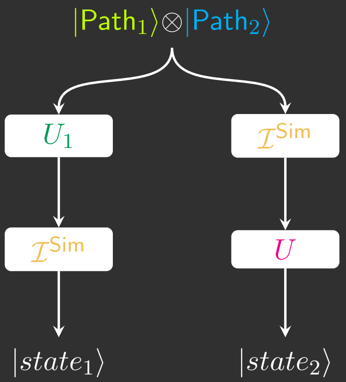
First Oracle



Second Oracle

Progress Measure

First Oracle



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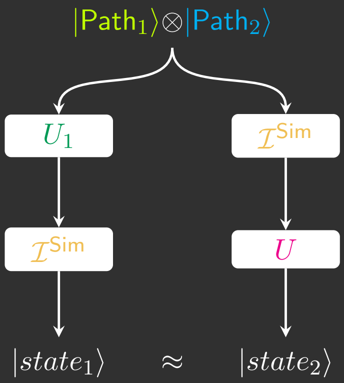

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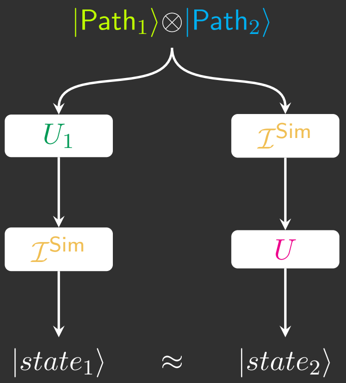

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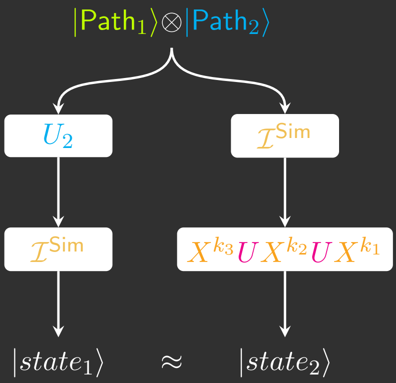

Conclusions and open-problems

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- **Strong Gluing Theorem for Haar Unitaries**
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- **Super-strong PRU in strong QHROM**
  Conjugate and Transpose to Haar oracle and PRU
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Thank You