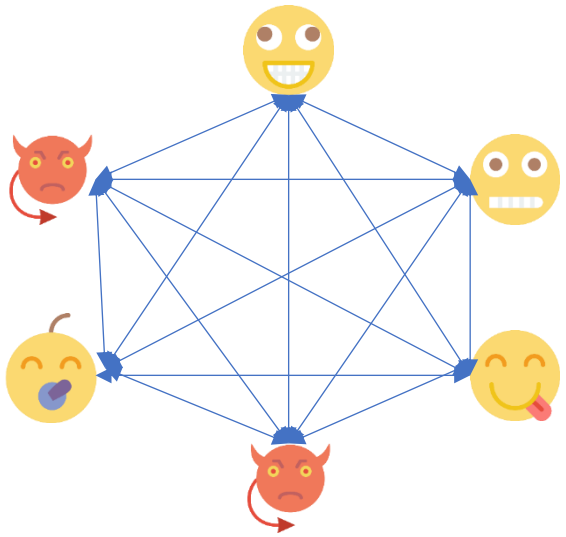
Constant-Round Asynchronous MPC with Optimal Resilience and Linear Communication

Junru Li
Tsinghua University

Yifan Song
Tsinghua University
Shanghai Qi Zhi Institute

Multiparty Computation



Setting

- Asynchronous Network
- Optimal Resilience $t = (n - 1)/3$
- Fully Malicious Adversary (Security with Abort)

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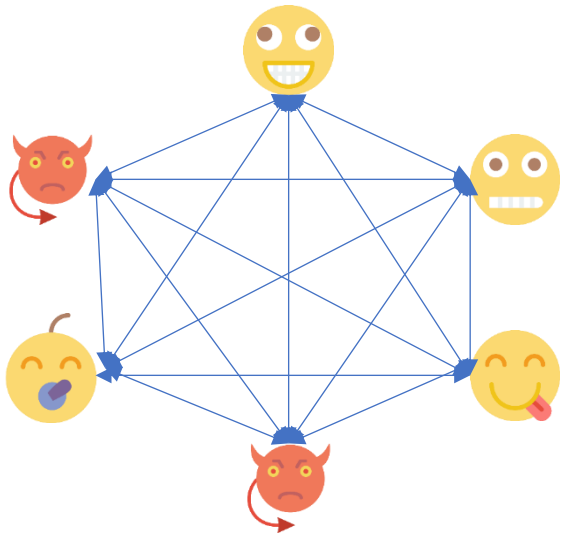

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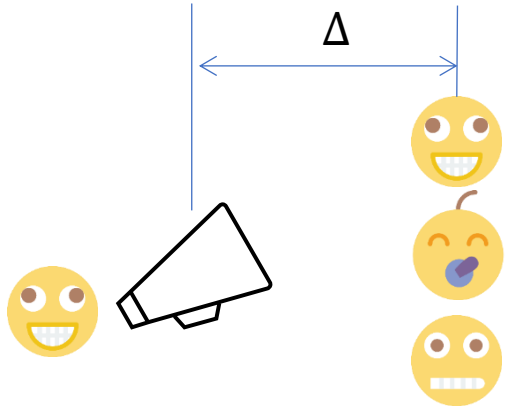
Target

- Lightweight Cryptographic Primitives (no FHE)
- Constant Round Complexity
- Communication Complexity Linear to n

Differences between Sync. and Async.

Sync:



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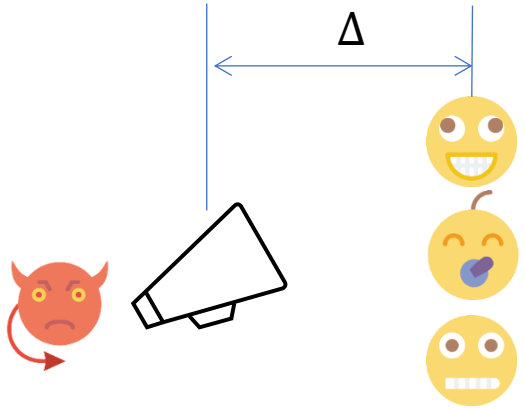

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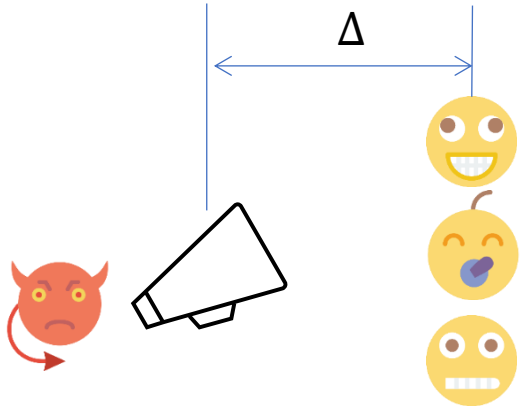

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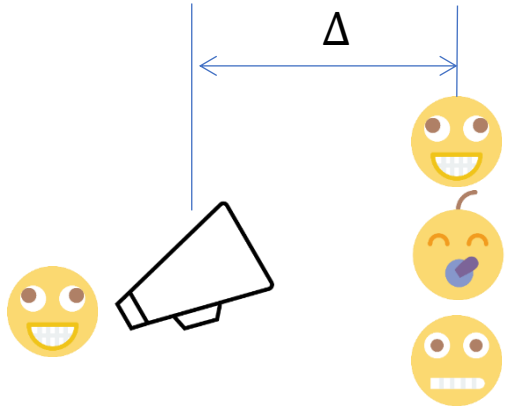

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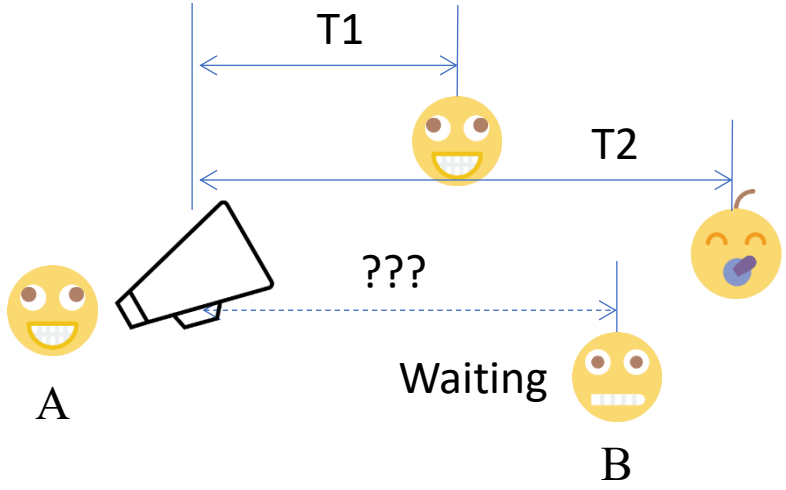

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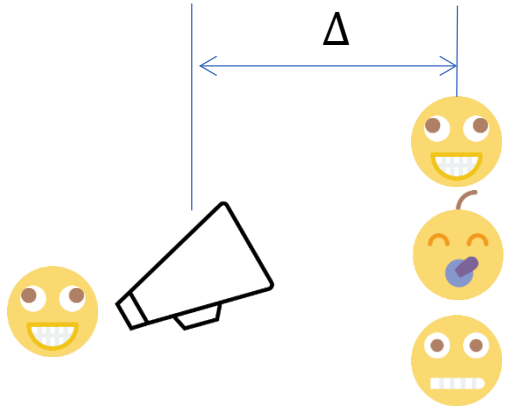

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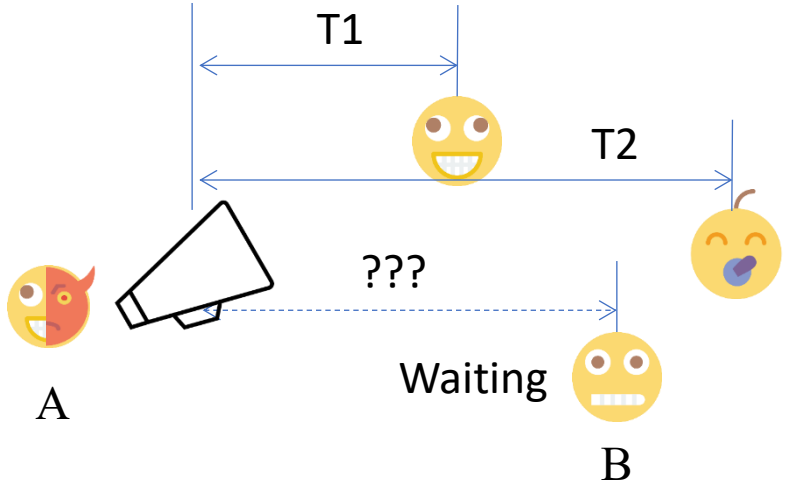

Cannot distinguish *dishonest sender not sending* vs *slow honest sender*

- Protocol runs at speed of actual network delay
- t parties may not be able to provide inputs
- Requiring $t < n/3$

Known Results from Literatures

Communication-Efficient but Non-Constant-Round AMPC (GOD)

- Perfect: $O(|C|n)$ communication is achieved for when $t < \frac{n}{4}$ [AAPP24].
- Statistical: $O(|C|n\kappa)$ communication is achieved for when $t < \frac{n}{3}$ [GLS24], with a large additive overhead $O(n^{14}\kappa^2)$.
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The parties must agree on a common set that provide inputs to the MPC, and this process cannot be constant-round in the plain model

Our Result

Assuming random oracles, there exists a computationally secure (with abort) constant-round AMPC in the $\mathcal{F}_{ACS}$ -hybrid model against a fully malicious adversary controlling up to $t < n/3$ parties with communication $O(|C|n\kappa + D(n+\kappa)^2 n\kappa + \mathrm{poly}(n,\kappa))$ plus 3 invocations of $\mathcal{F}_{ACS}$, where $|C|$ is the circuit size, D is the circuit depth, and κ is the computational security parameter.

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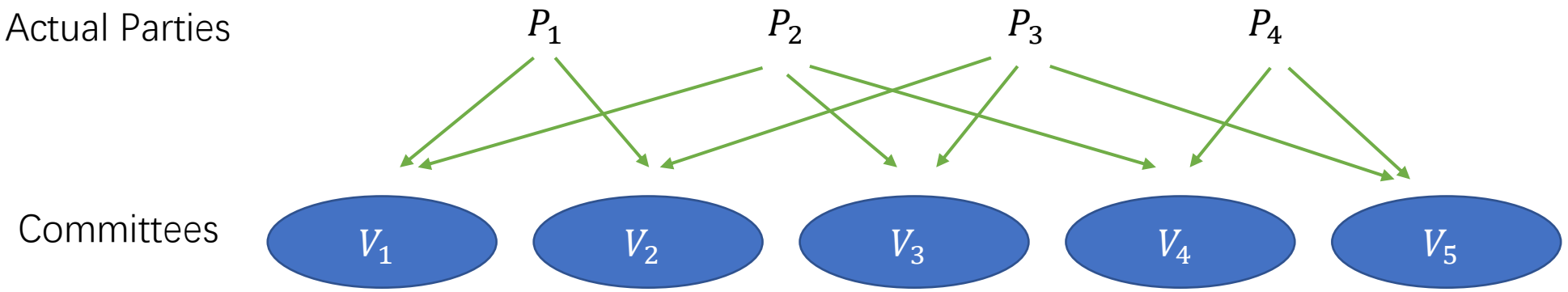
Requiring an $O(|C|\kappa)$-size multiparty garbled circuit (omitting the $D \cdot \text{poly}(n,\kappa)$ term):

- The only known construction in the synchronous case is [GLOS25]

Multiparty Garbling of [GLOS25]



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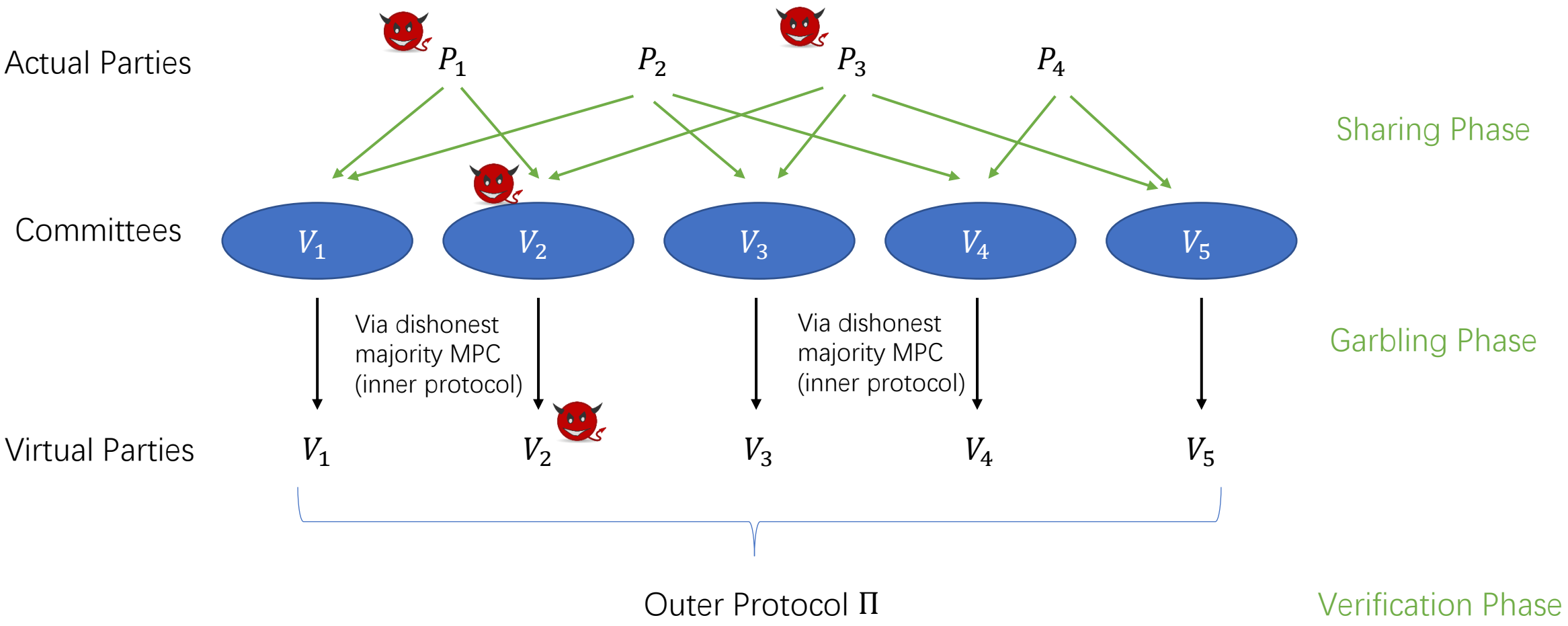

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Protocol Steps

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Difficulties Caused by the Asynchrony

1. Generating preprocessing/input sharings: A corrupted party may send his input sharings to only a part of the parties, and the rest of the parties may wait forever for the shares.
   - Previous solution: ACSS (but only for Shamir sharings)

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Solution: Use Asynchronous Verifiable Information Dispersal (AVID) [CT05, ADD+22]

Can be instantiated from RO

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Solution: Run a synchronous inner protocol

Asynchronous Verifiable Information Dispersal

Goal: A $\xrightarrow{M}$ B

All the parties know M is sent

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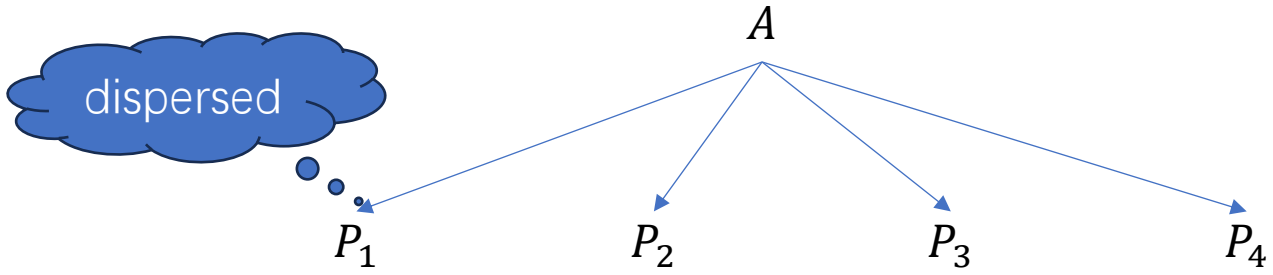
Dispersal



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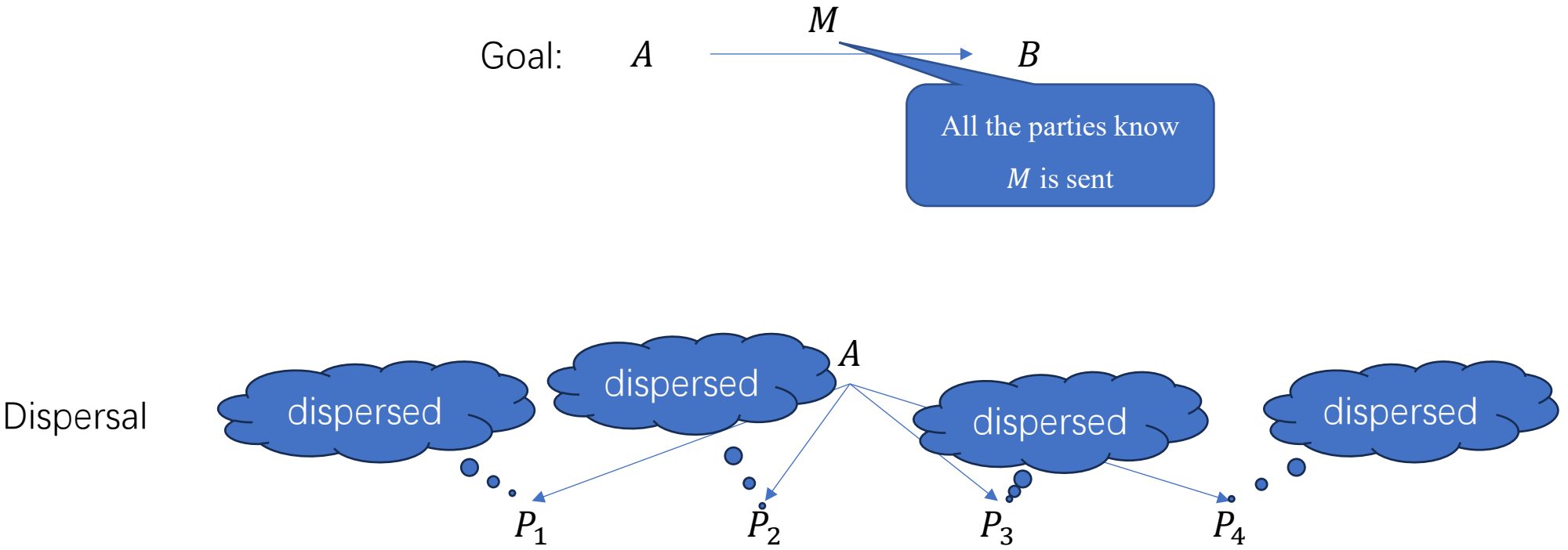

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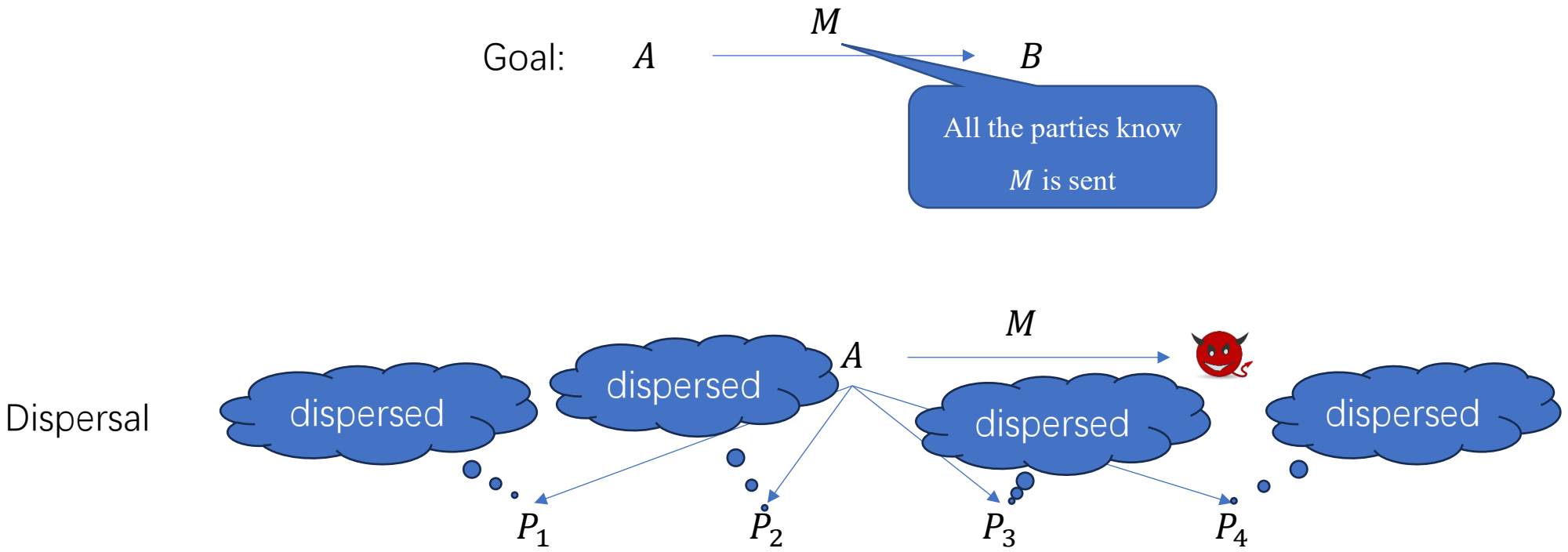

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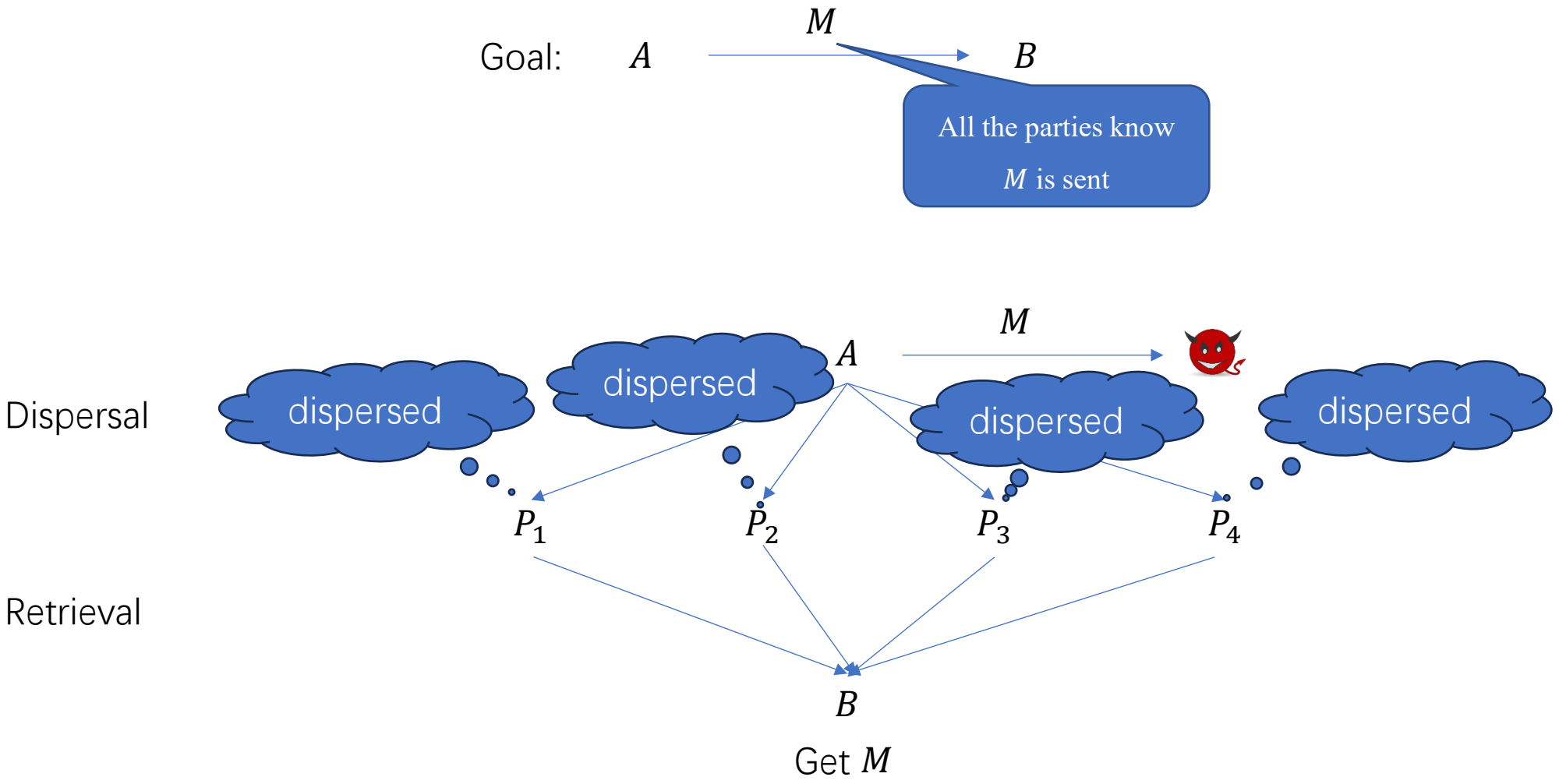

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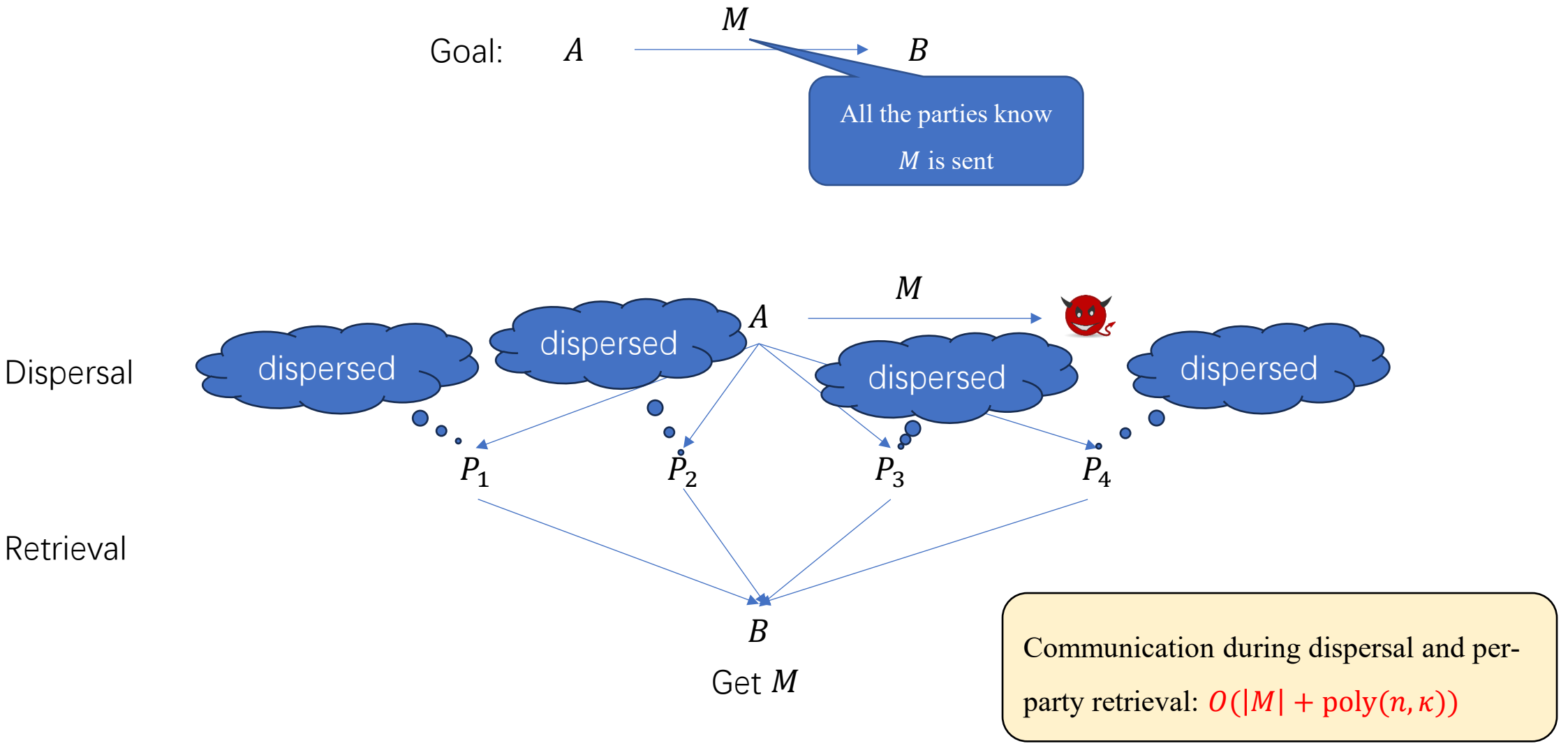

Communication during dispersal and per-party retrieval: $O(|M| + \text{poly}(n, \kappa))$

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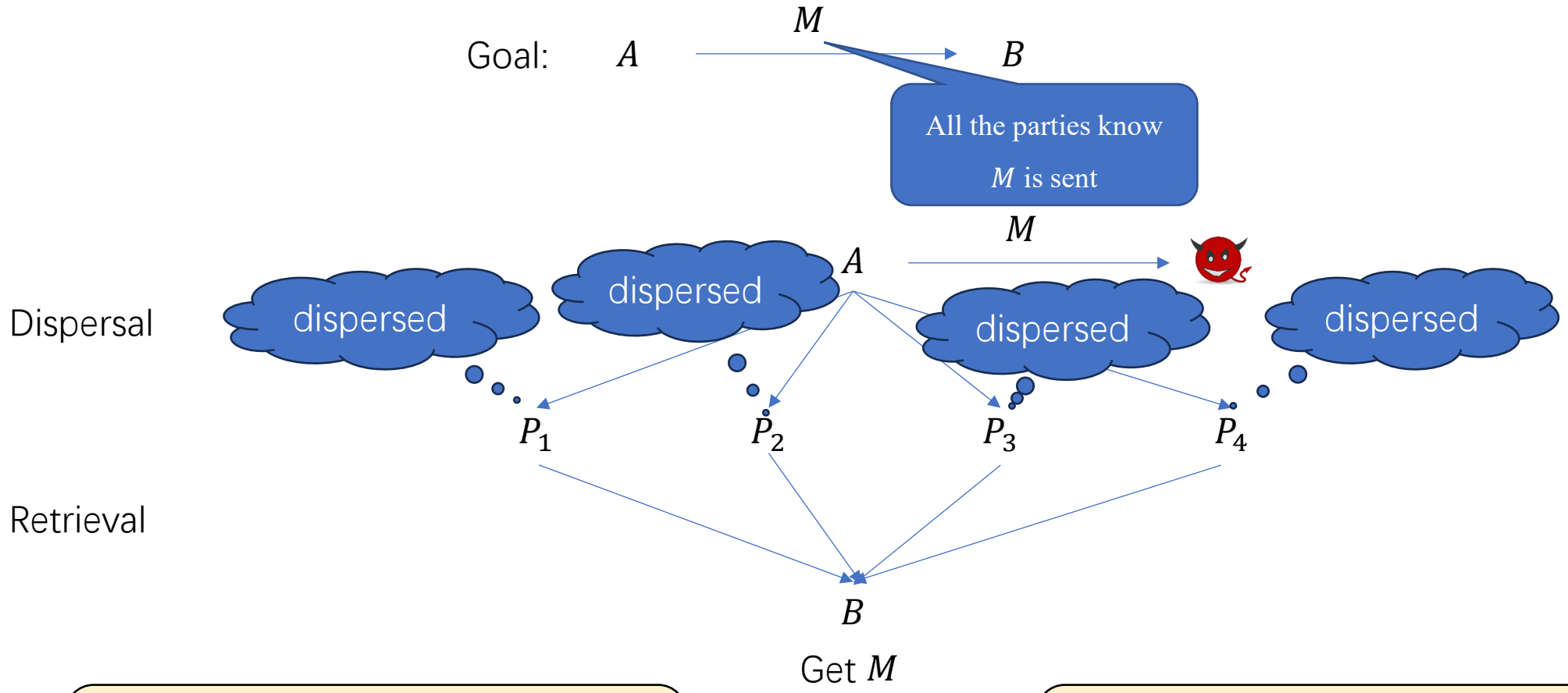

Communication during dispersal and per-party retrieval: $O(|M| + \text{poly}(n, \kappa))$

Only $O(\text{poly}(n, \kappa))$ overhead for sending all messages of a constant-round MPC via AVID

Generating Sharings via AVID

A corrupted party may send his sharings to only a part of the parties, and the rest of the parties may wait forever for the shares.

1. Preparing pair-wise symmetric keys.

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   - Via a general constant-round AMPC, only requiring $O(\text{poly}(n, \kappa))$ communication

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Generating Commitments via AVID

Commit:

1. Preparing a secret-shared seed (using a general constant-round ACSS).
2. Reconstruct the seed to the sender.
3. Mask the message using the seed (via RO).
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Open to a Party:

1. Reconstructing the seed to the party.
2. Let the party retrieve the masked message.
3. Decrypt the message using the seed.

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Observation 1: We don't need all the virtual parties' garbled circuits
(only need enough garbled circuits for reconstructions of the label shares)

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Idea: We can run synchronous inner protocols

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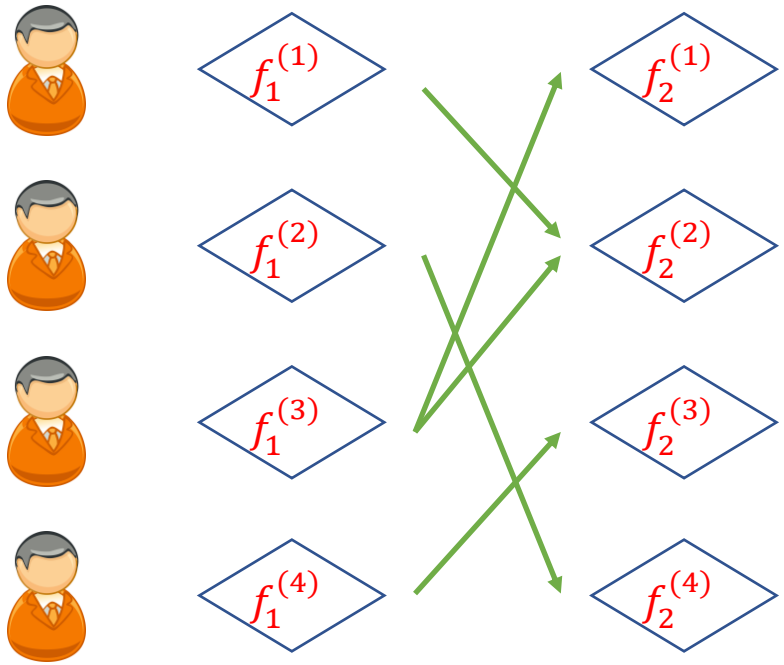
Run a synchronous protocol in the asynchronous setting



A Synchronous Round

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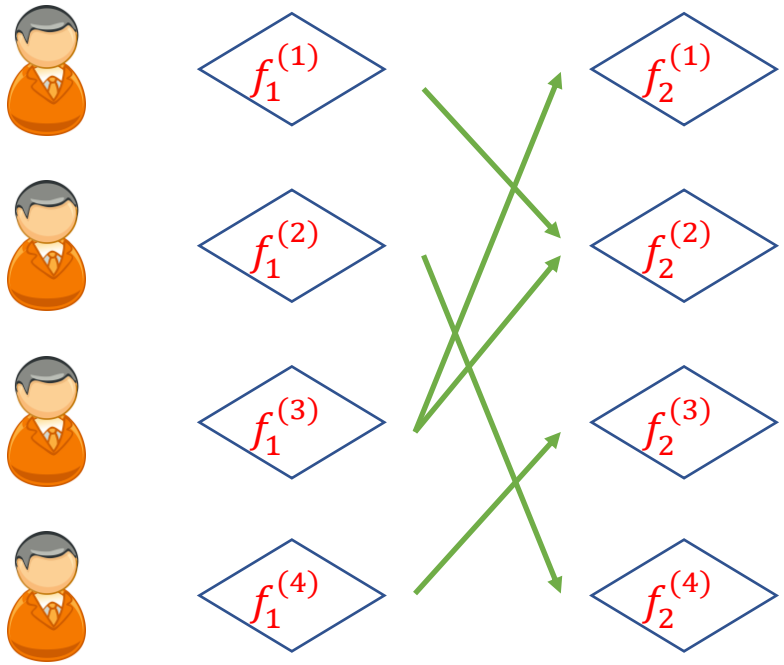

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- When a round finishes, everyone knows.

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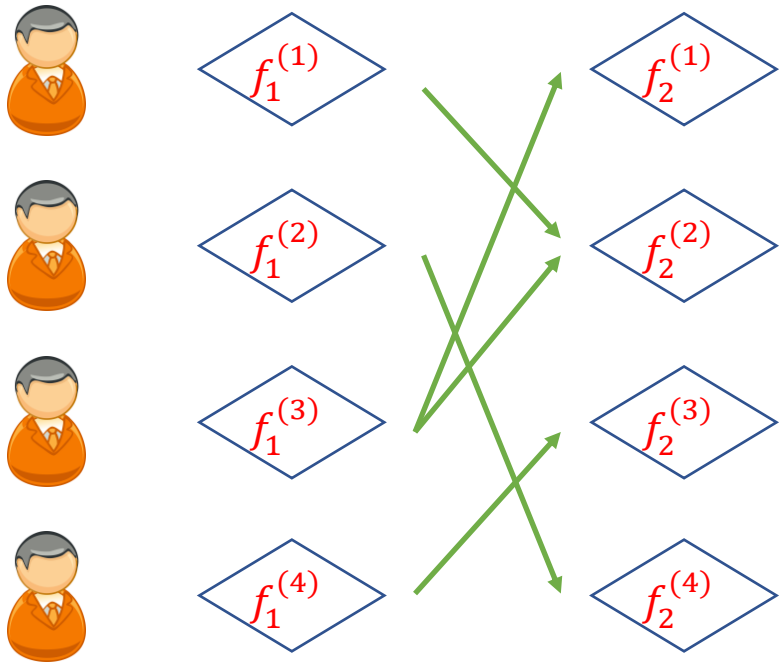

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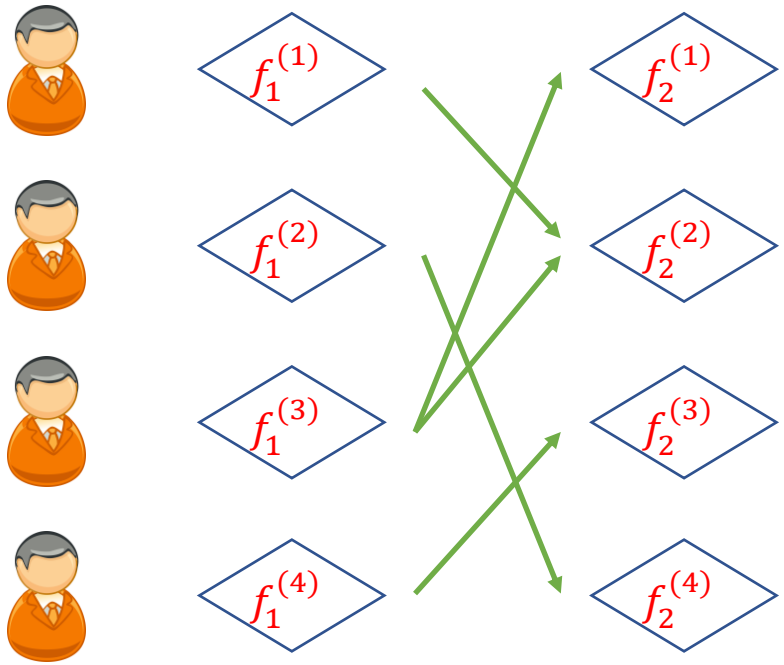

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Also providing commitments to the view of virtual parties

Multiparty Garbling Outline

Run a Setup Phase for pair-wise keys and secret-shared seeds

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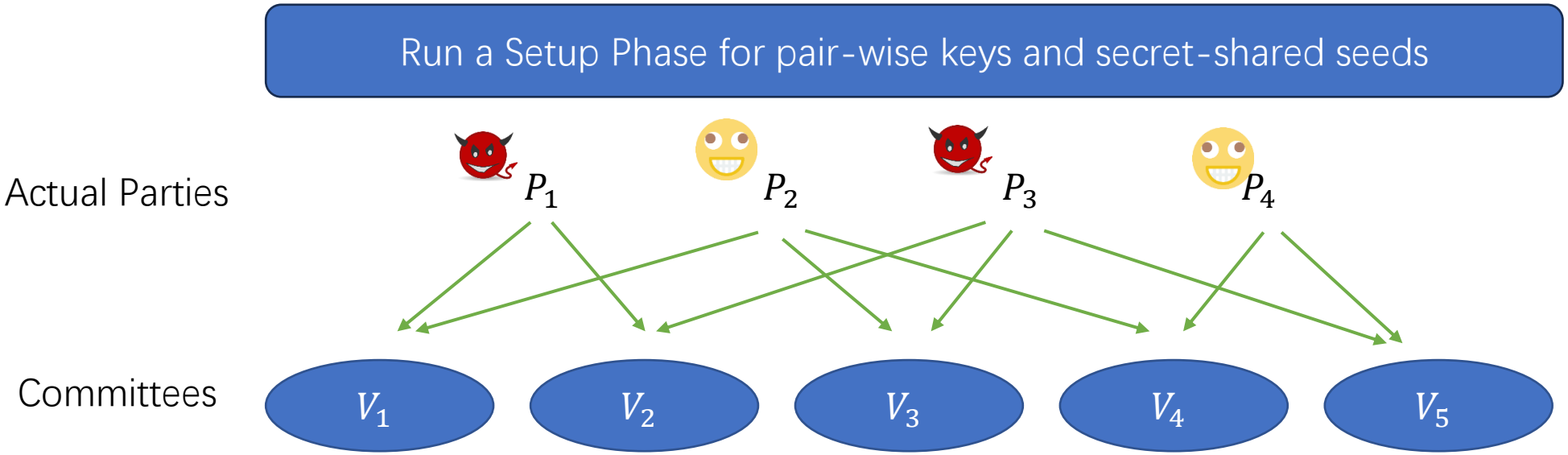

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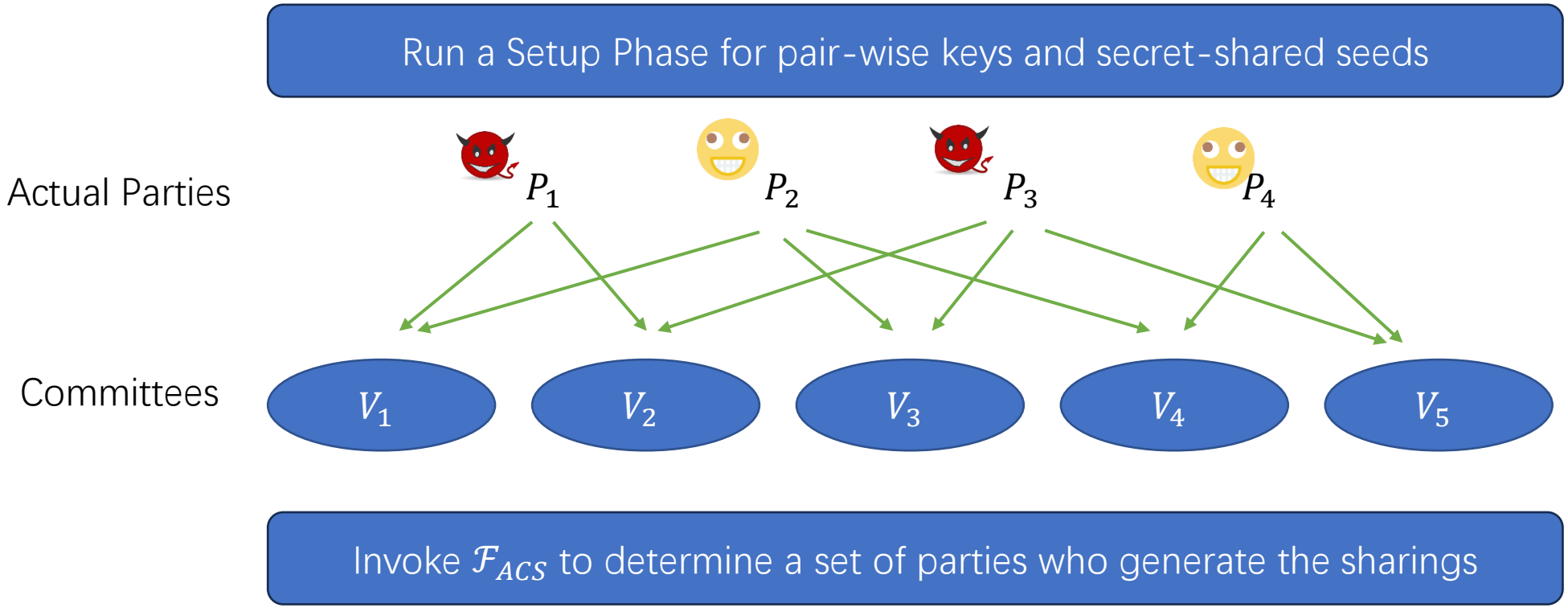

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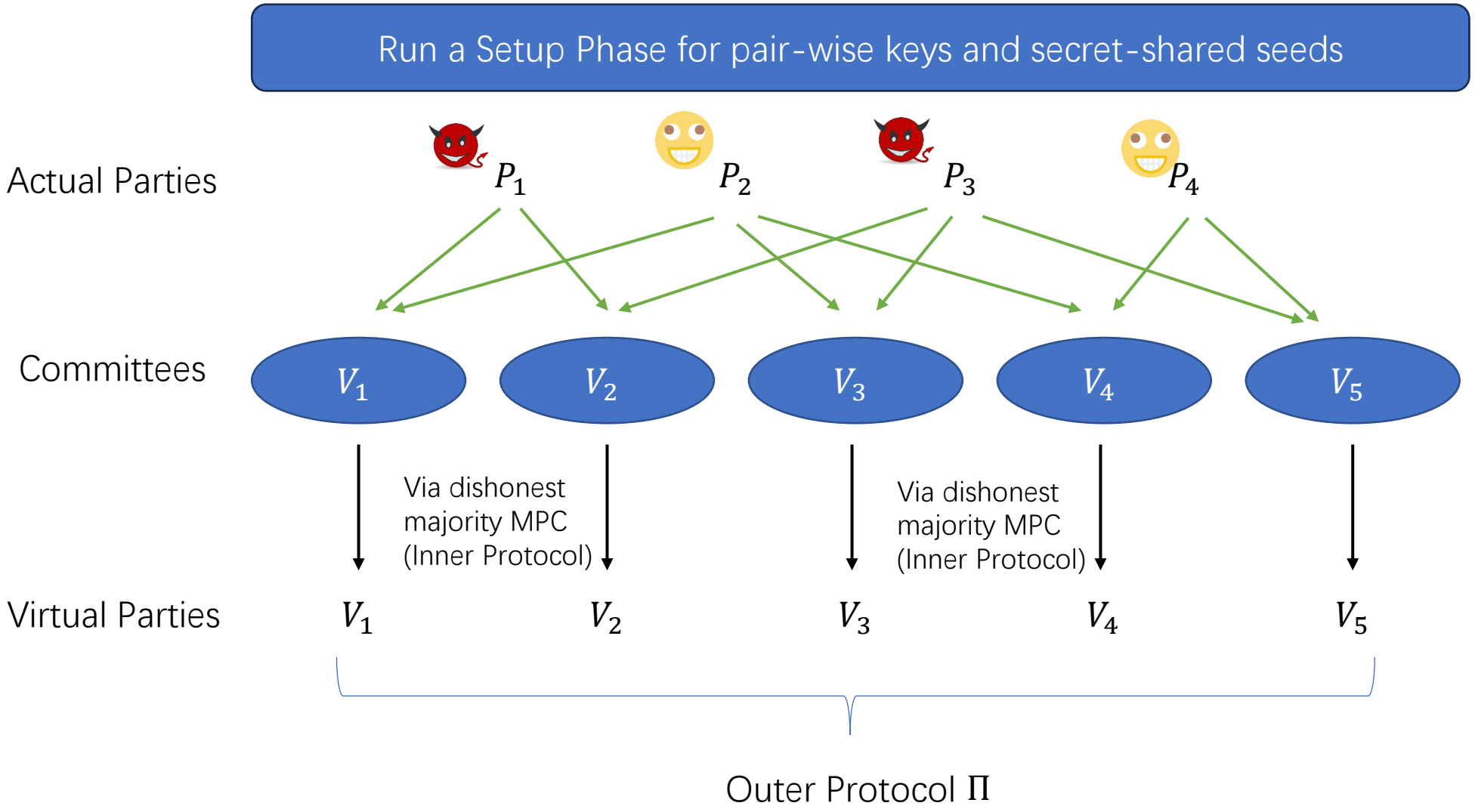

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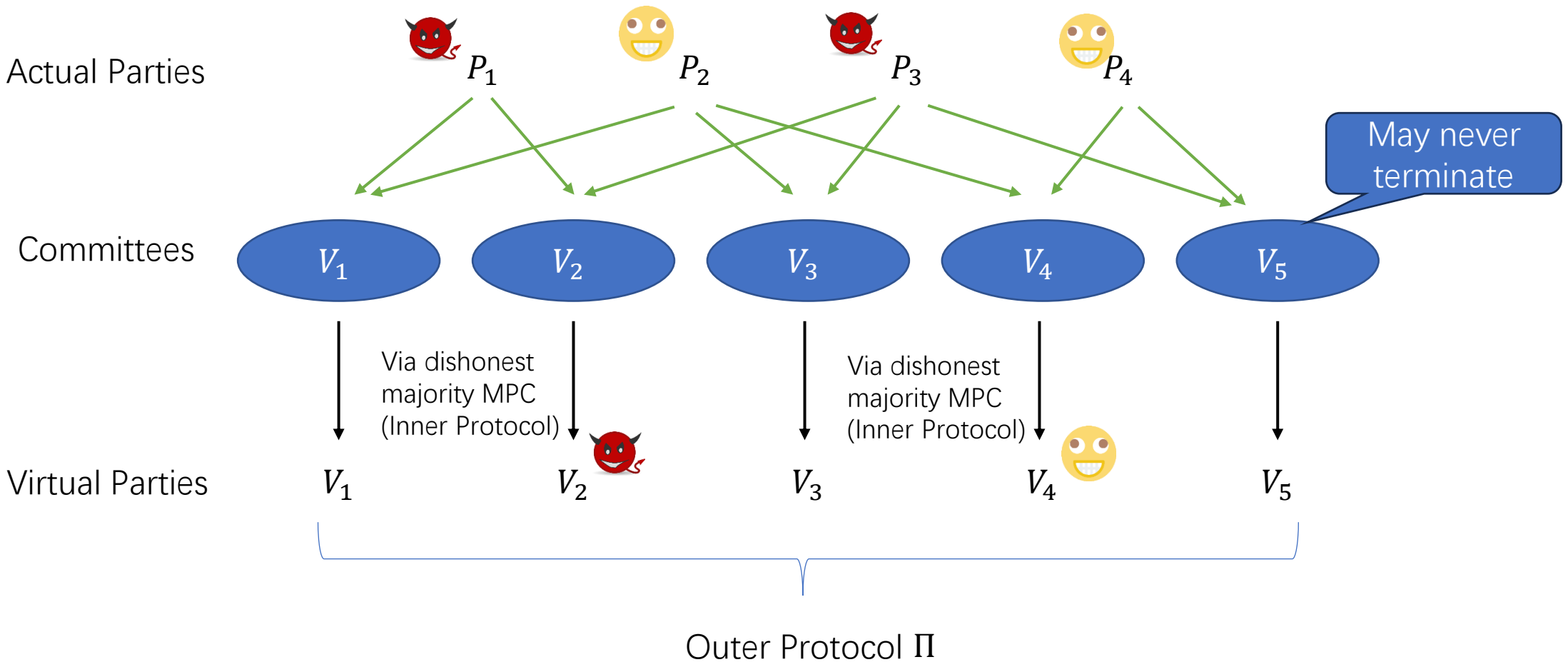

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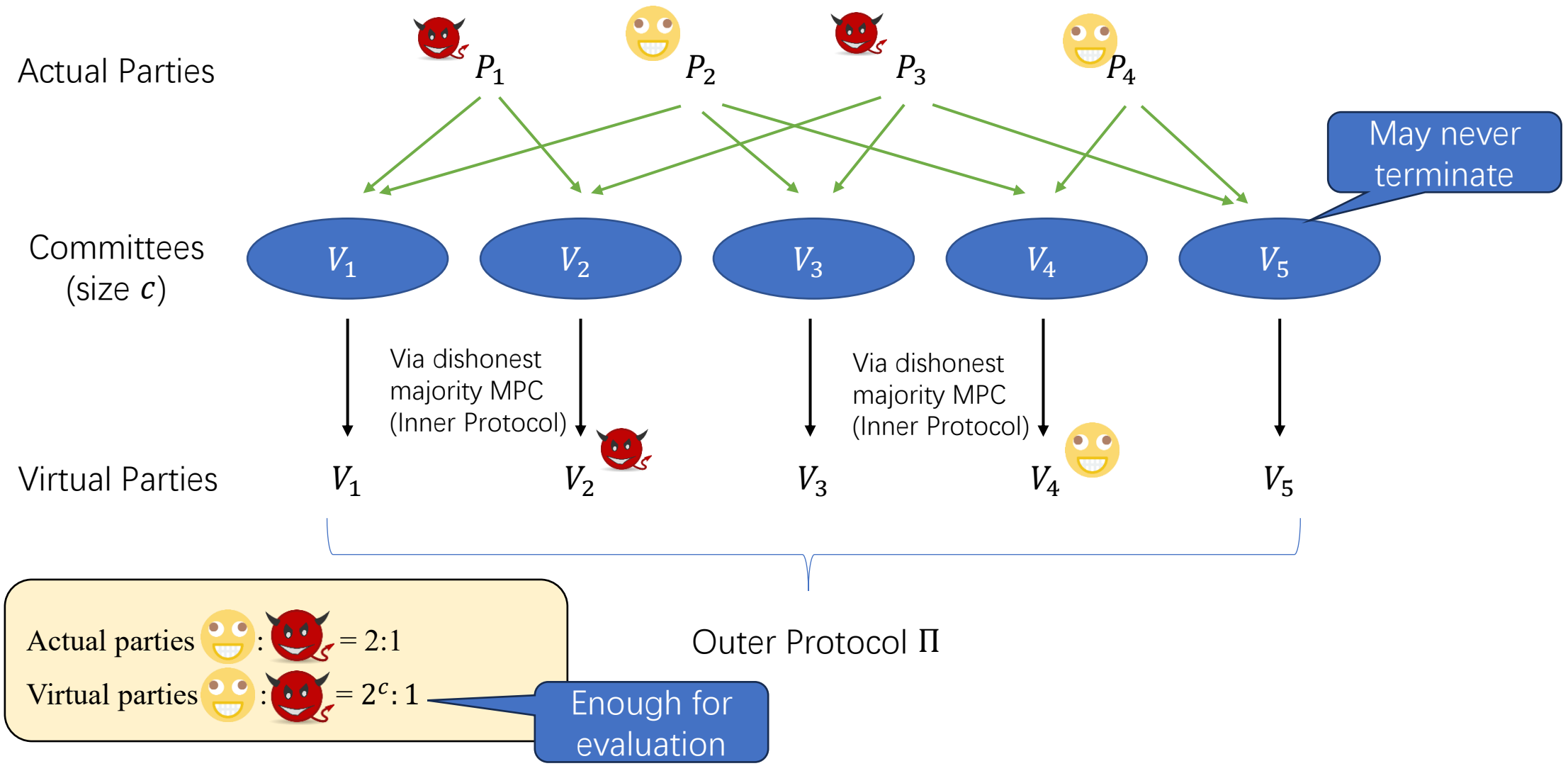

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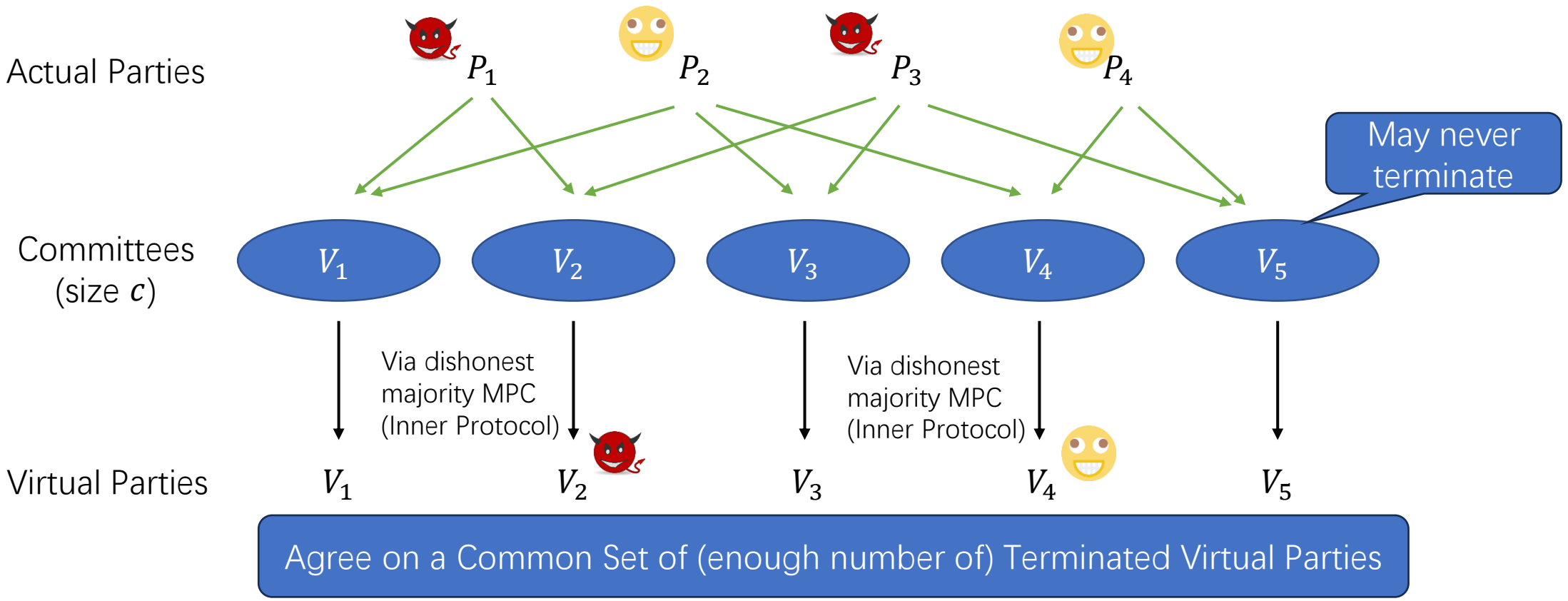

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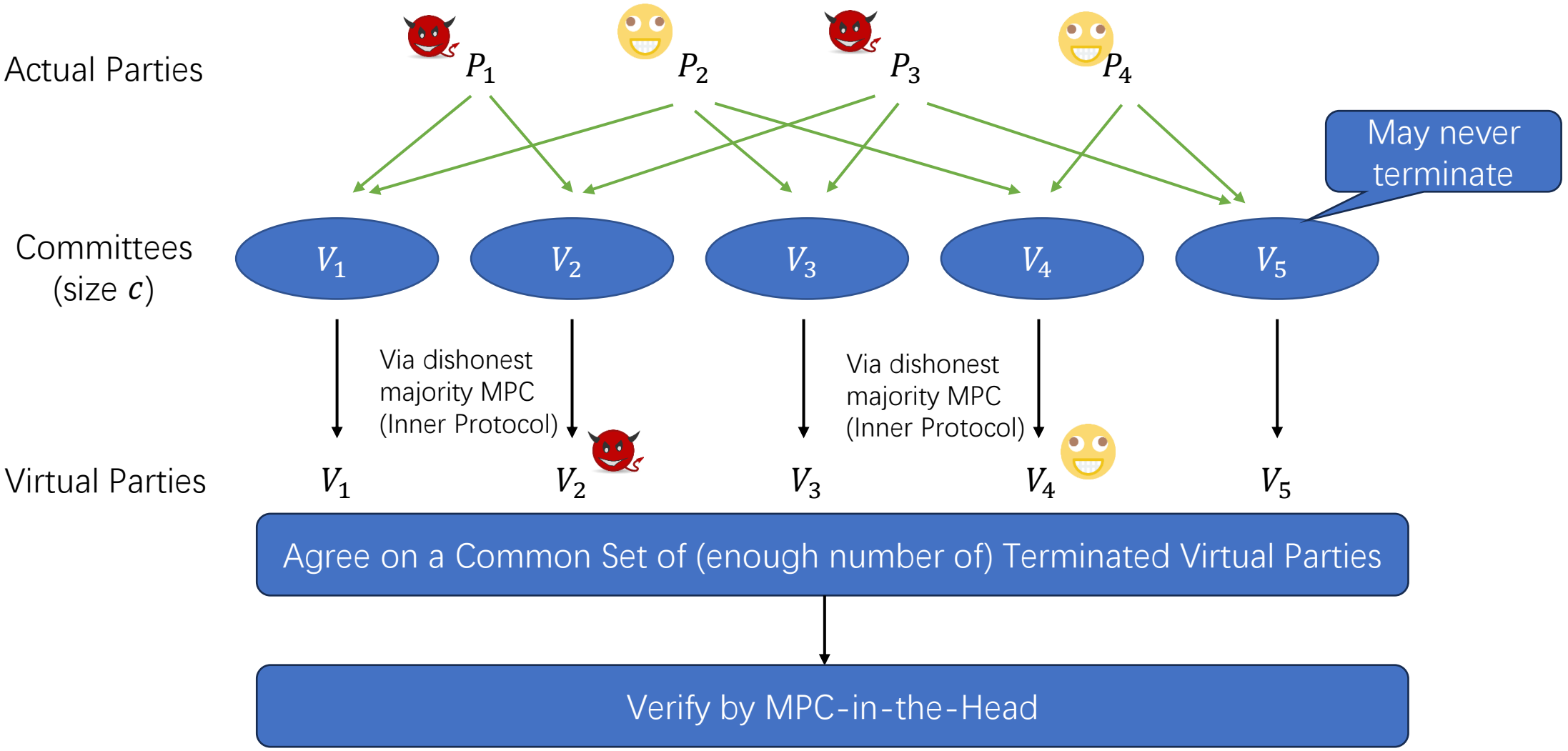

Thanks!

https://eprint.iacr.org/2025/1032