On One-Shot Signatures, Quantum vs Classical Binding, & Obfuscation Permutations

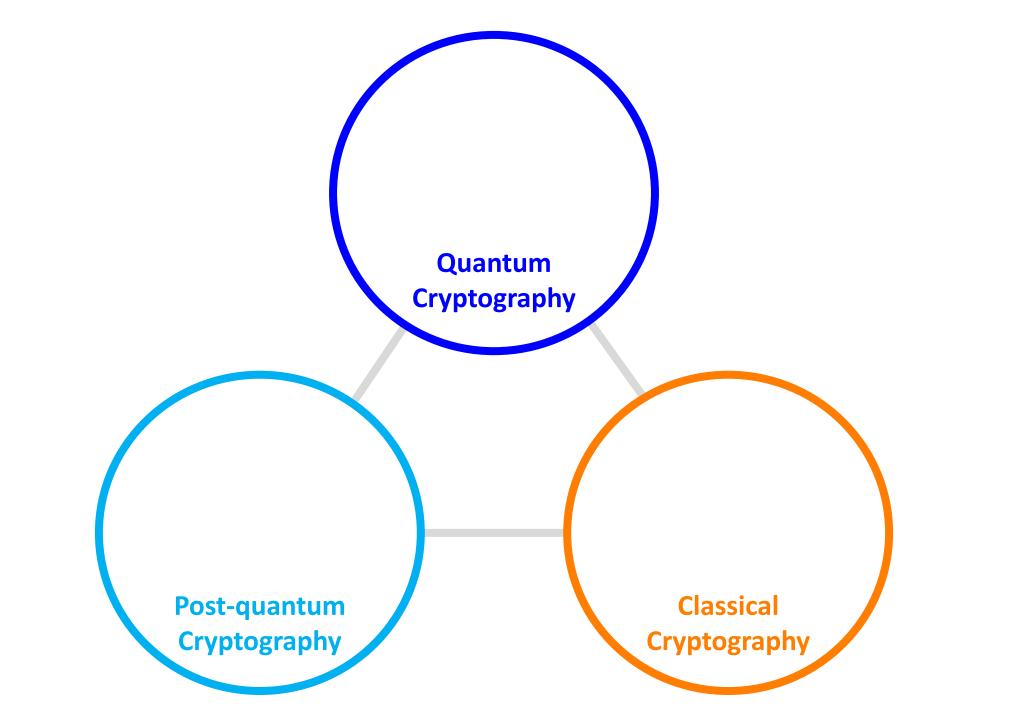
Omri Shmueli

Mark Zhandry









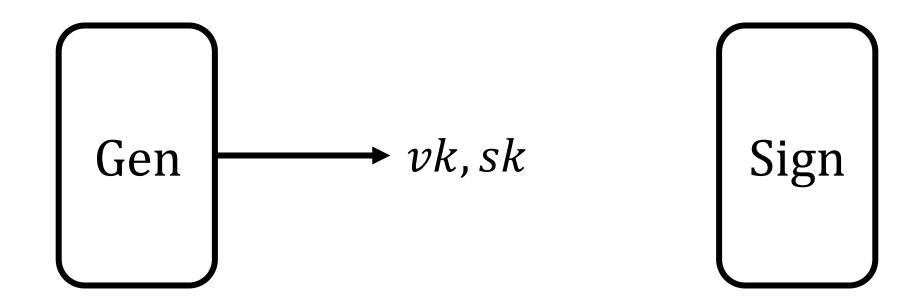
Quantum Cryptography

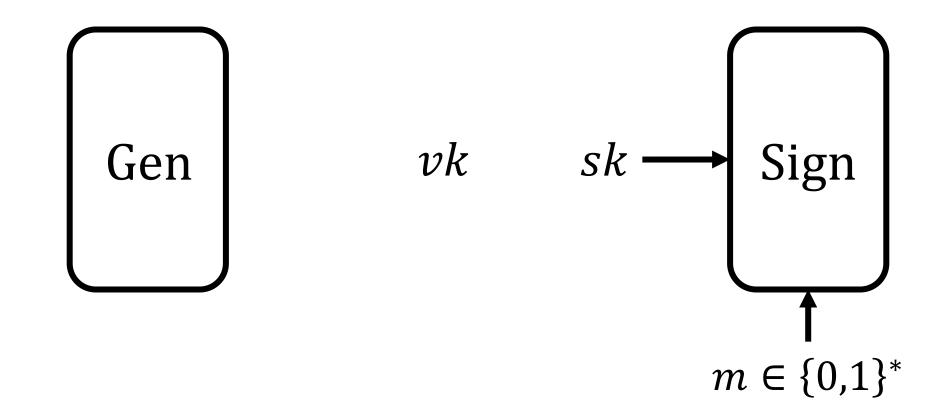
Classical
Commitments with
Quantum Security

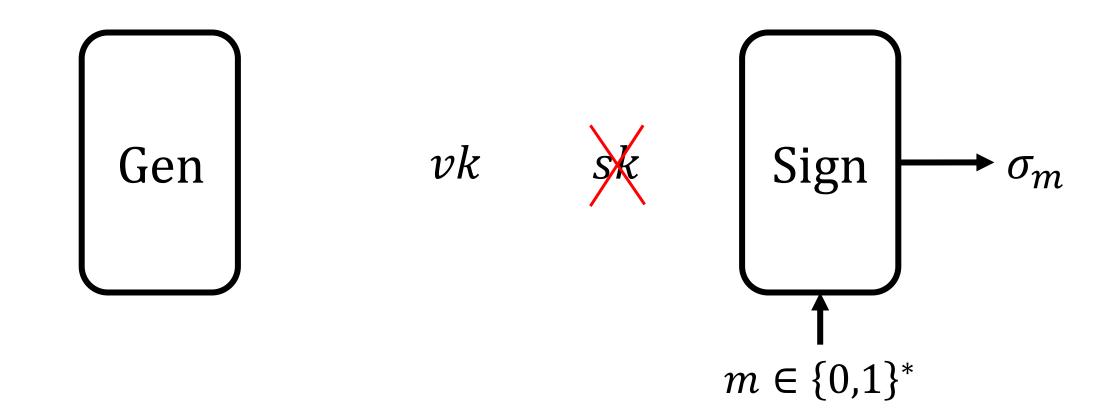
Post-quantum Cryptography

Can we Obfuscate Pseudorandom Permutations?

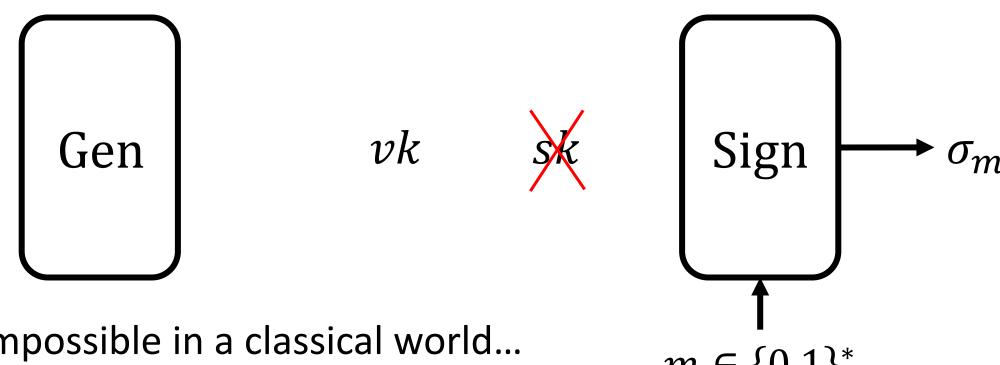
Classical Cryptography





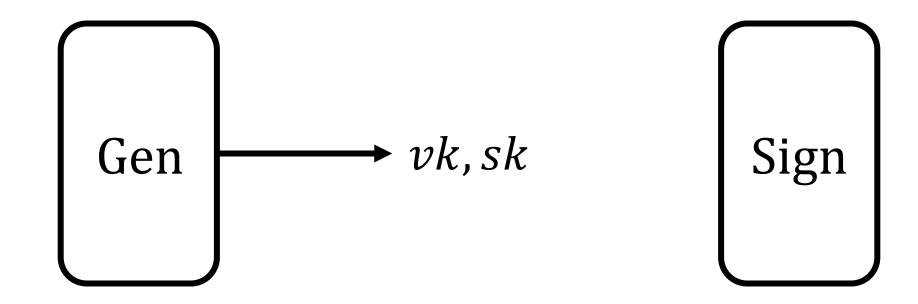


Is it possible to construct a *one-time* signature token?

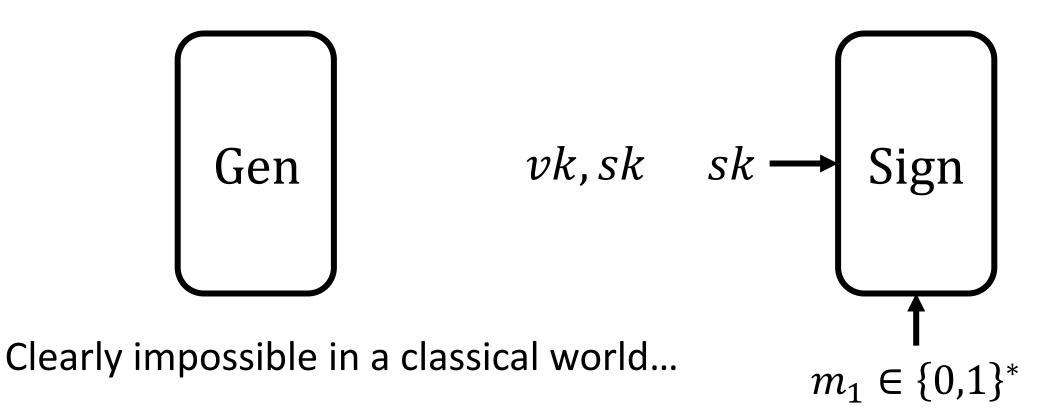


Clearly impossible in a classical world...

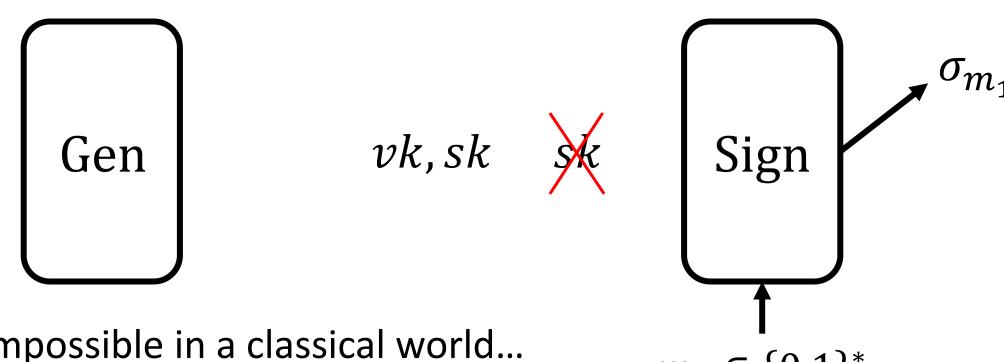
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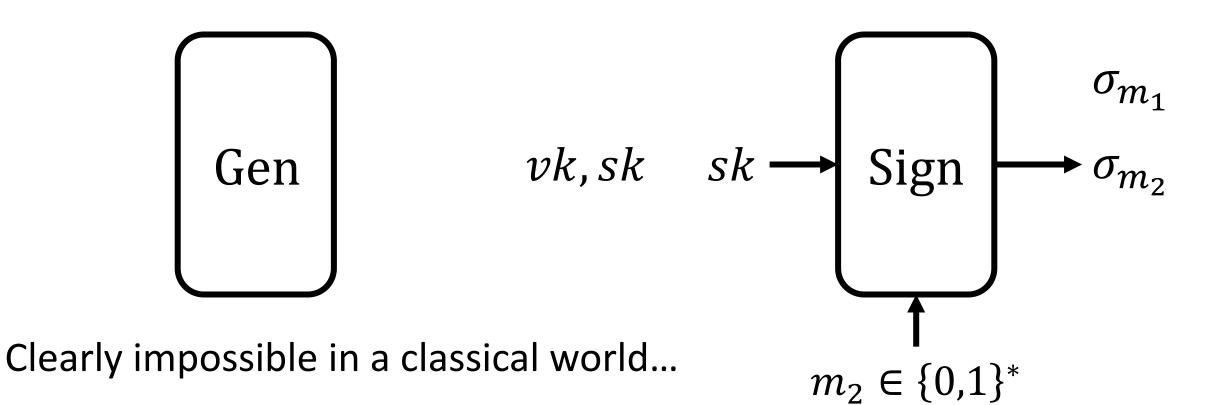
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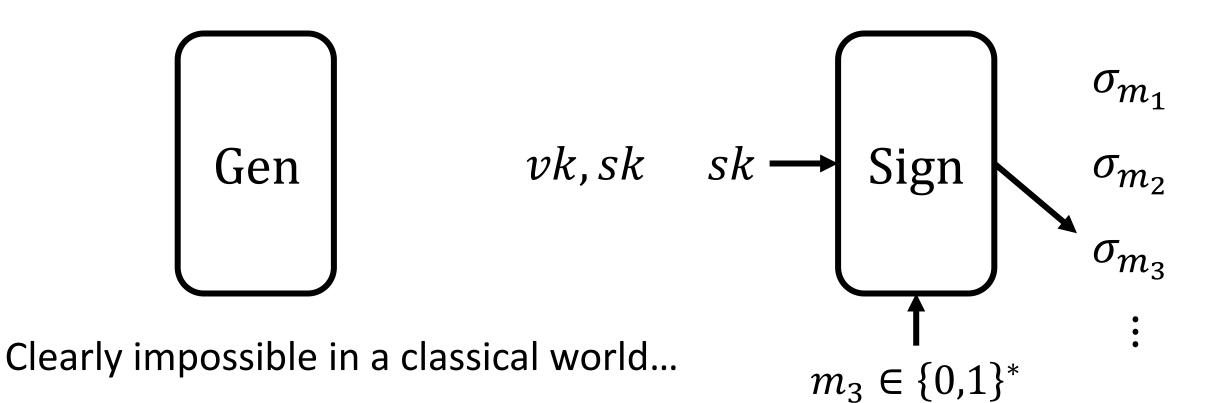


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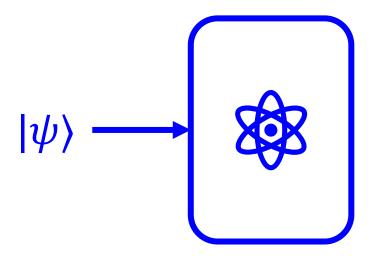
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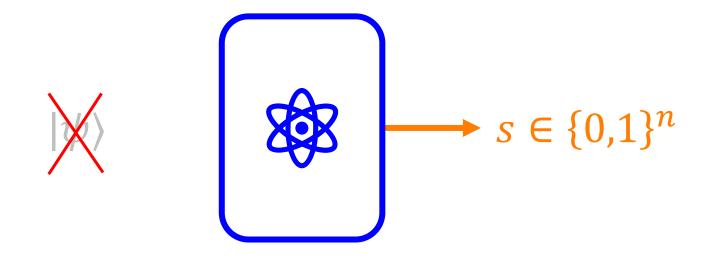
Quantum Information in Cryptography

Extracting classical information from a quantum state can degrade it.



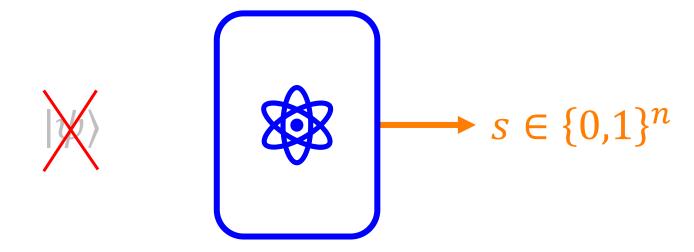
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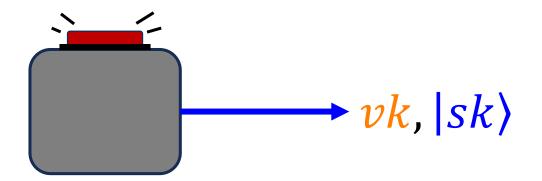


Is it possible to make this degradation *inherent*, for the *benefit* of quantum cryptography?

[Amos-Georgiou-Kiayias-Zhandry-20]

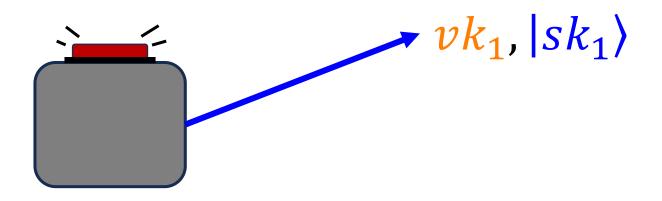
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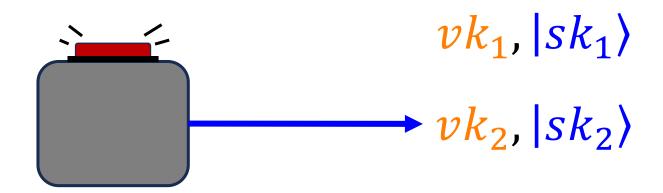
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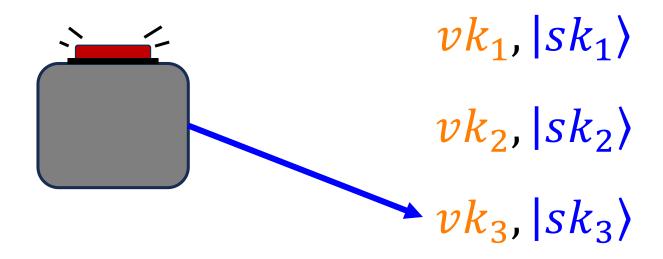
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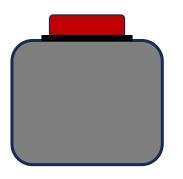
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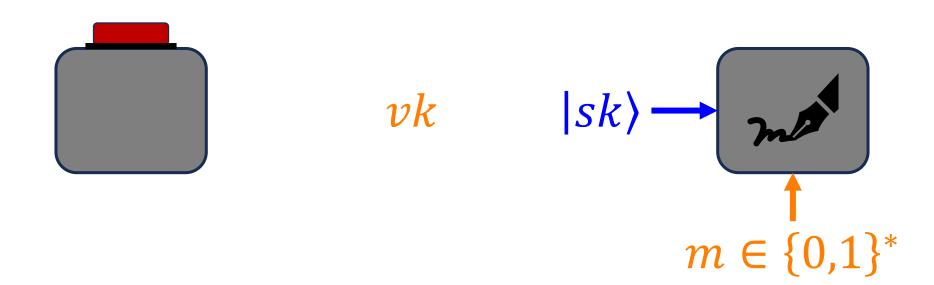


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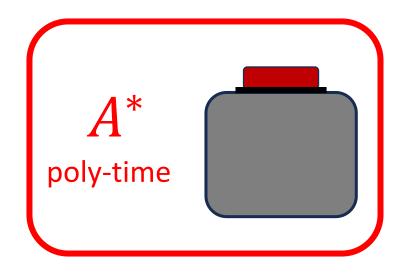
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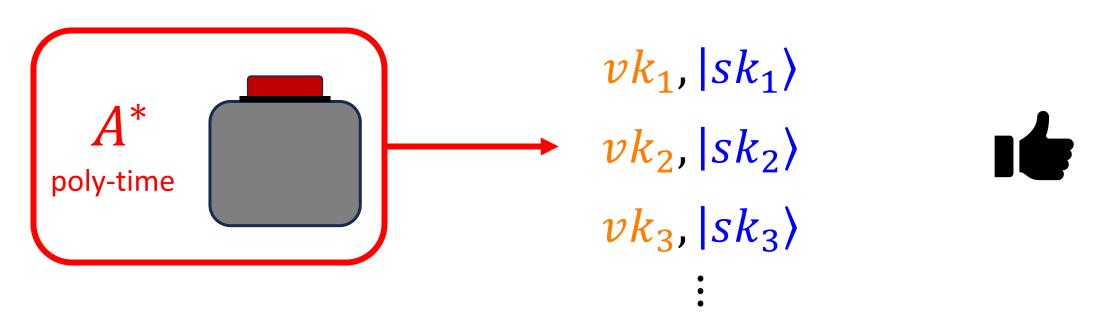
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Security: Intractable to sign twice using the <u>same</u> key

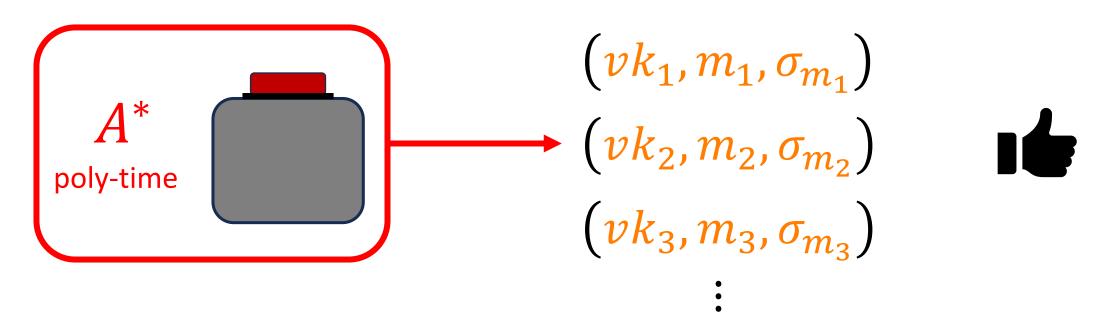
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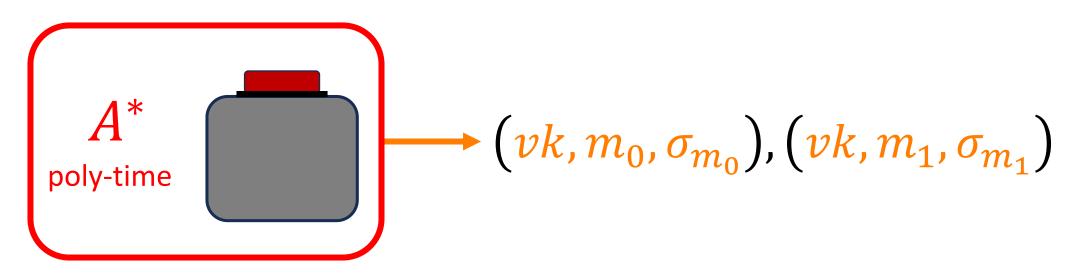
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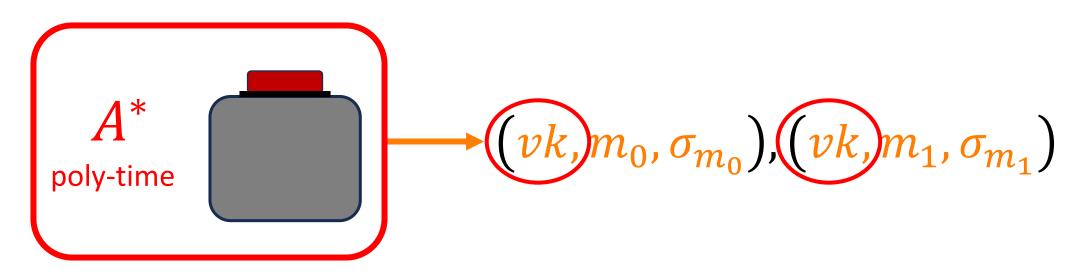
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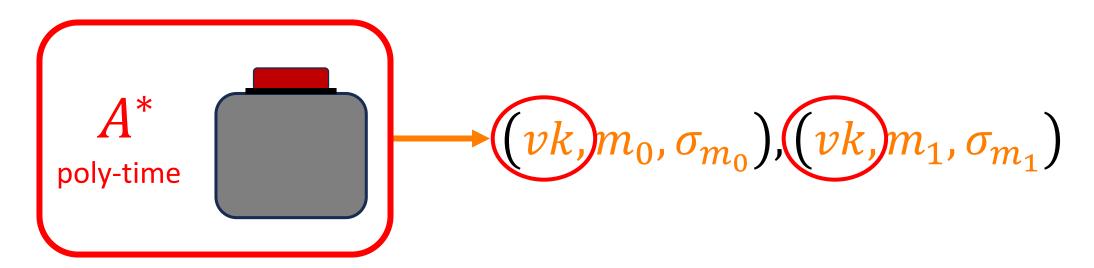
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Computationally intractable for $m_0 \neq m_1$

A master primitive in decentralization

- ➤ Cryptocurrency (based on PoW) without a blockchain [Zha-17].
- ➤ Blockchain-free smart contracts [Sat-22].
- ➤ Solves the Blockchain Scalability Problem [Col-Sat-20].
- >A perfect-finality solution to the double spending problem.
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We do not know of any other primitive in (quantum) cryptography that solves any of these problems

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- ➤ The proof was found to contain a fatal bug [Bar-23].
- To date, the security of that construction remains unknown.

A Paradigm for Constructing One-Shot Signatures: Detour into Post-quantum Cryptography

Detour into Post-quantum Cryptography

[Unruh-15]:

Classical **commitments** that are <u>post-quantum computationally</u> <u>binding</u>, may nonetheless be "insecure" against quantum computers.

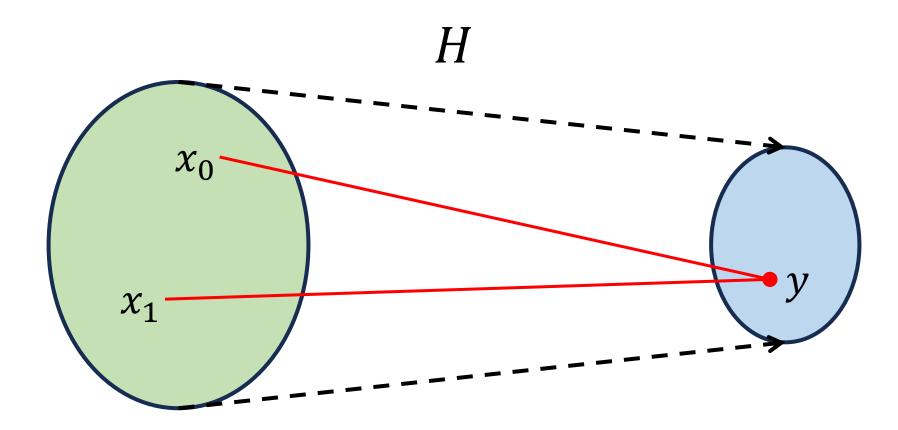
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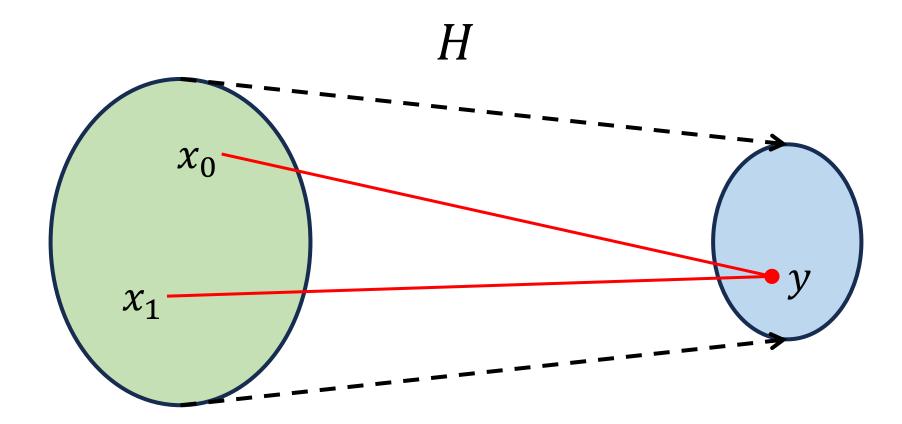
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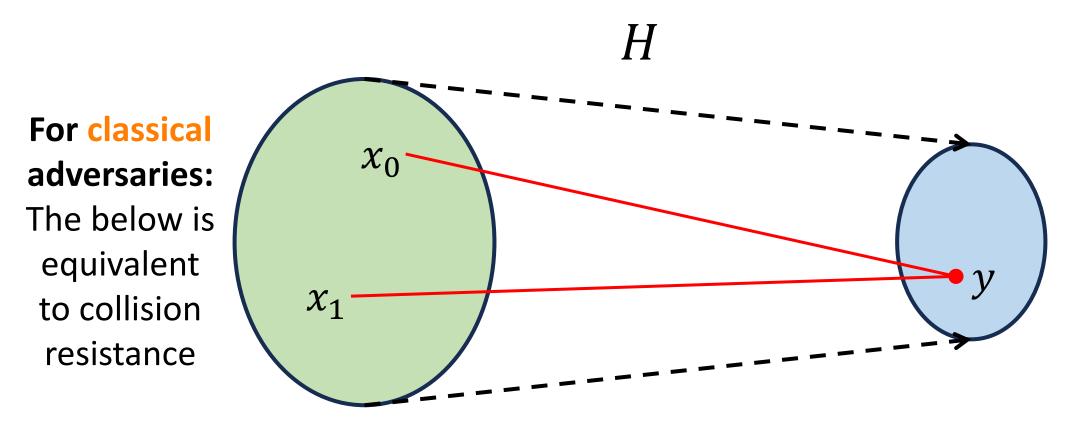
Classical **hash functions** that are <u>post-quantum collision-resistant</u>, may nonetheless be "insecure" against quantum computers.



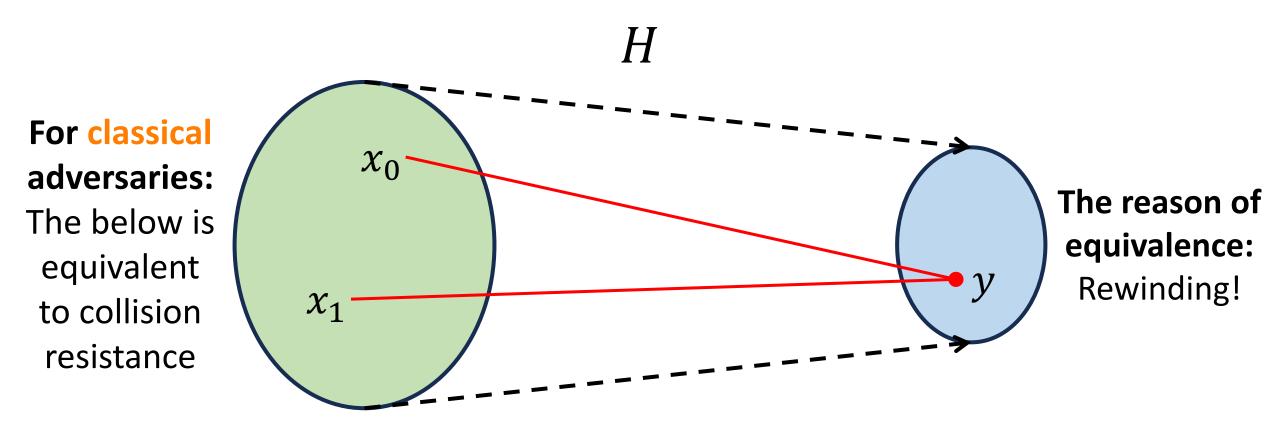
Computationally intractable to find $x_0 \neq x_1$ s.t. $H(x_0) = H(x_1)$, even for a quantum computer.



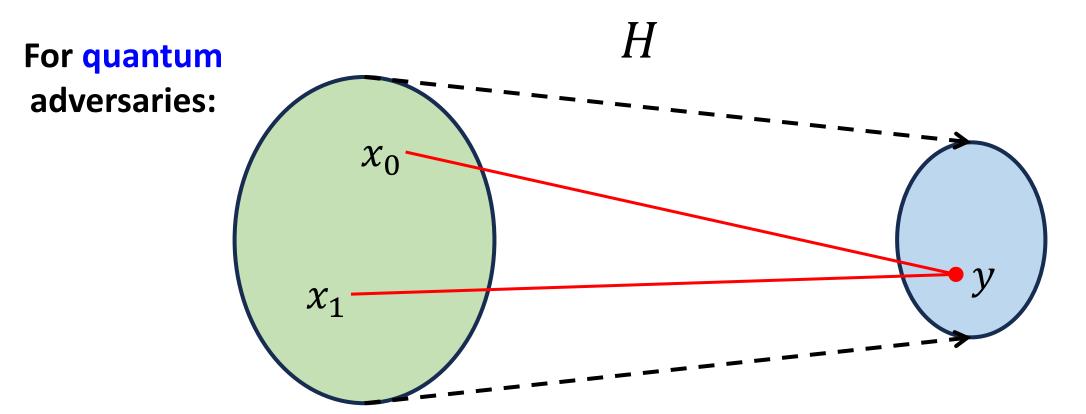
For computationally binding commitments we want: If the adversary sends y, it is intractable for it choose x_b later.



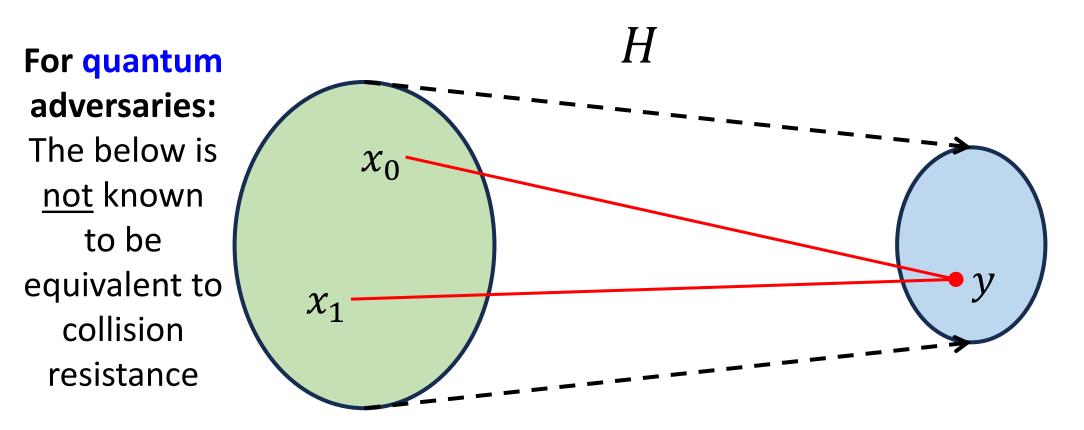
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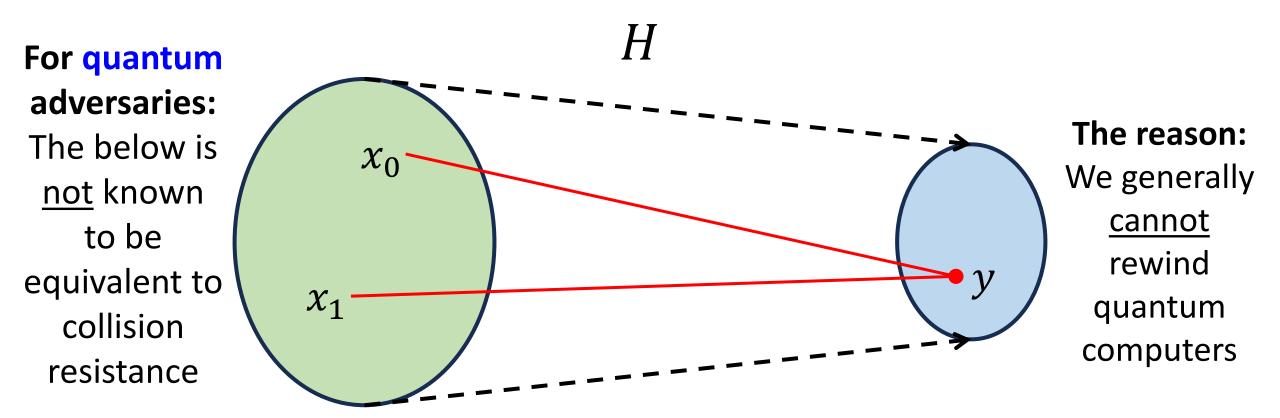
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Quantum Rewinding is Hard

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[VDG-C-97], [Wat-02], [Kob-03], [D-F-S-04], [Wat-09], [Unr-12], [H-S-S-11], [L-N-11], [A-R-U-14], [B-J-S-W-16], [Unr-16a], [Unr-16b], [B-S-20], [C-M-S-Z-21], [L-M-S-22], ...
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- The adversary sends y as the commitment.
- **The issue:** The adversary has $\sum_{x \in \{0,1\}^n : H(x) = y} |x\rangle$. Theoretically, could steer the superposition to a specific preimage x (e.g., that starts with a 0).

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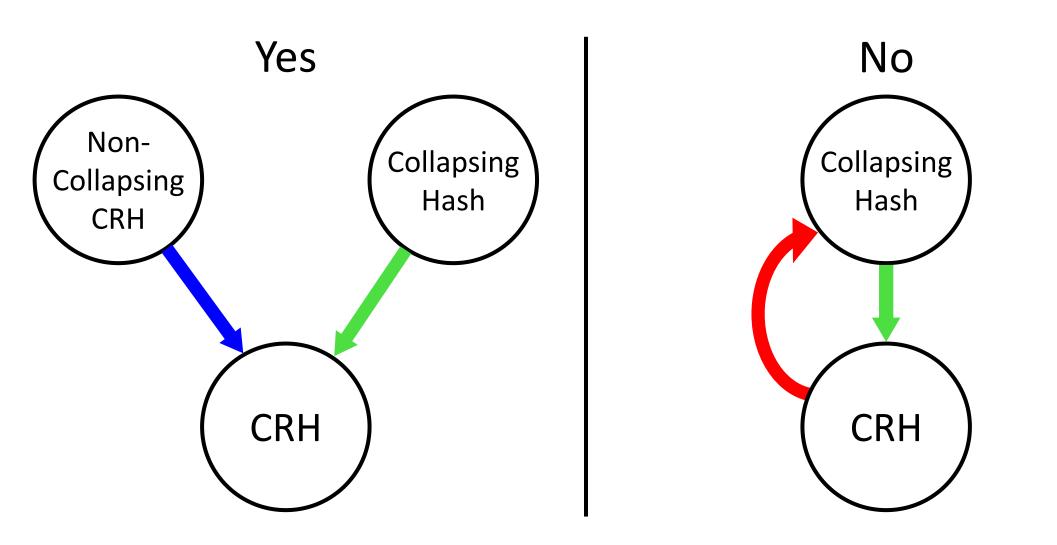
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- For a collapsing $H: \sum_{x \in \{0,1\}^n: H(x)=y} |x\rangle \approx_c \{x: x \leftarrow H^{-1}(y)\}$.
- Plenty of constructions of **collapsing hash functions** in the standard model ([Unr-16], [L-Z-19], [Zha-22], [L-M-Z-23]).



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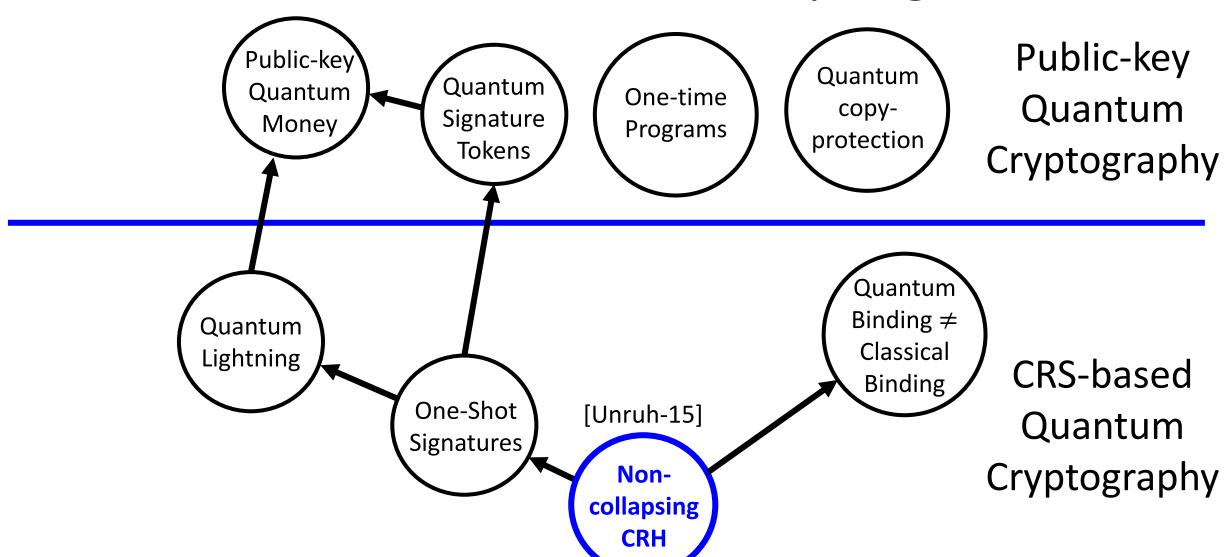
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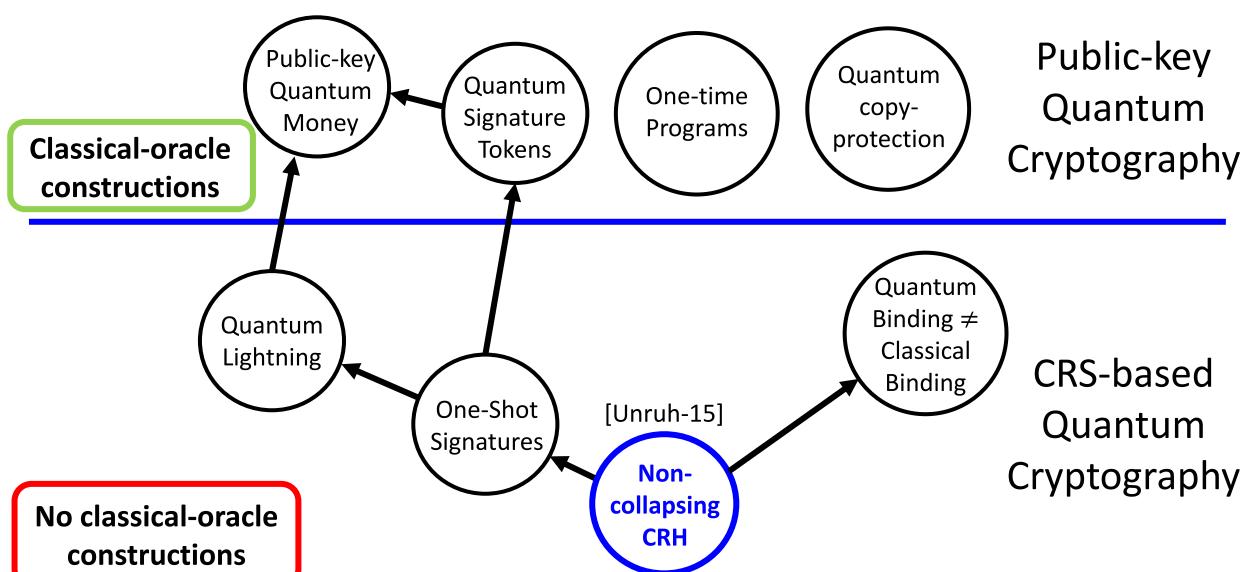
$$\sum_{x \in \{0,1\}^n: H(x) = y} |x\rangle \not\approx \left\{x: x \leftarrow H^{-1}(y)\right\}$$

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- [Zha-17], [A-G-K-Z-20], [D-S-22]:
 Non-collapsing CRH ⇒ One-Shot Signatures .
 - (+ collapsing is **necessary** for post-quantum binding)





Some of our Results

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Relative to a classical oracle there exists a non-collapsing CRH unconditionally.

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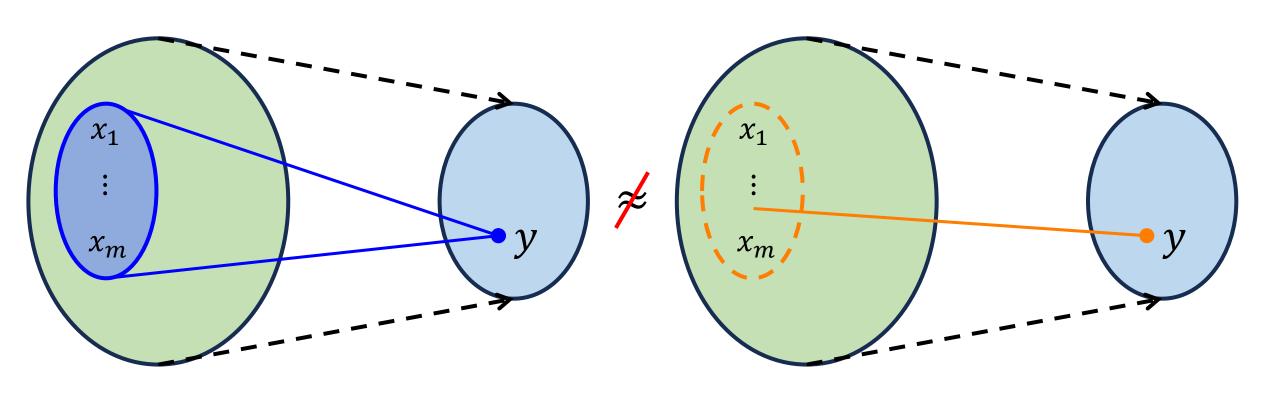
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Then, there exists a non-collapsing CRH in the standard model.



Construct a CRH where the two cases above are distinguishable.

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Collision resistance:

However, what makes a hash function collision resistant is the <u>lack</u> of predictable structure of inputs.

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A random permutation $\Pi: \{0,1\}^n \to \{0,1\}^n$ can be used to mediate between two requirements:

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- 1. Unstructured, collision-resistant sets, and
- 2. Structured sets, detectable in quantum superposition.

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- Define two functions $H, J: \{0,1\}^n \to \{0,1\}^{\frac{n}{2}}$:

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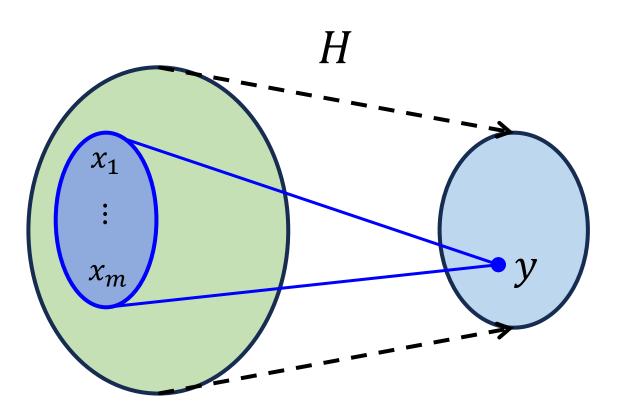
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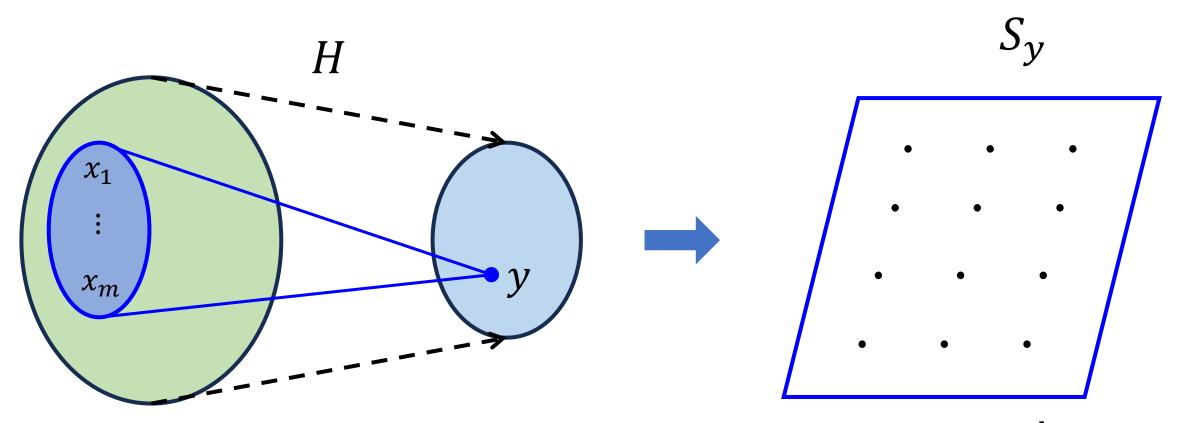
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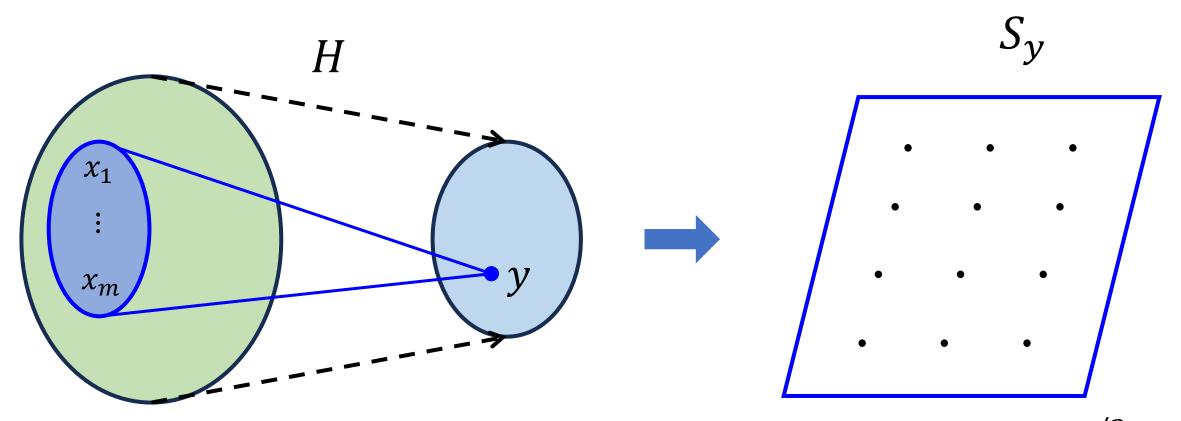
Non-collapsing: How to detect superpositions of preimages of H?



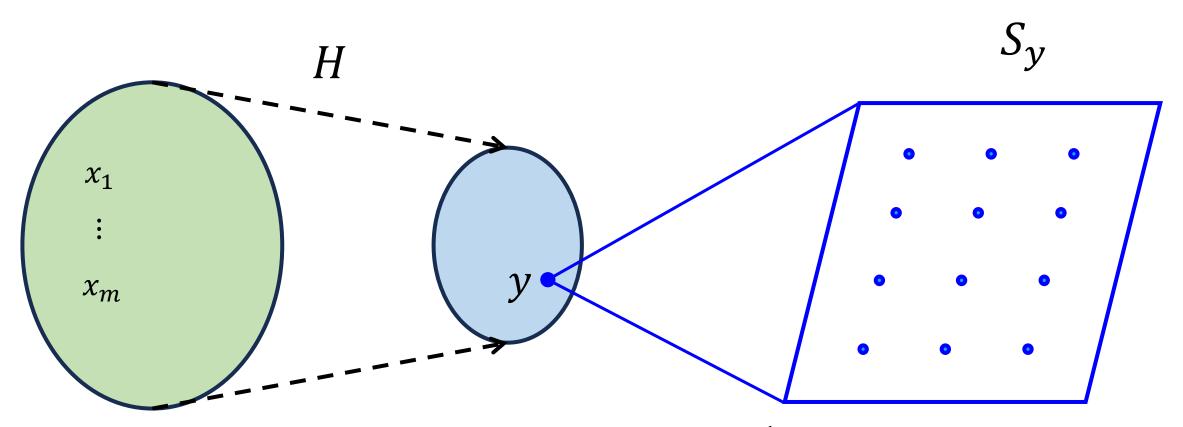
1. Compute H in superposition and measure an output y.



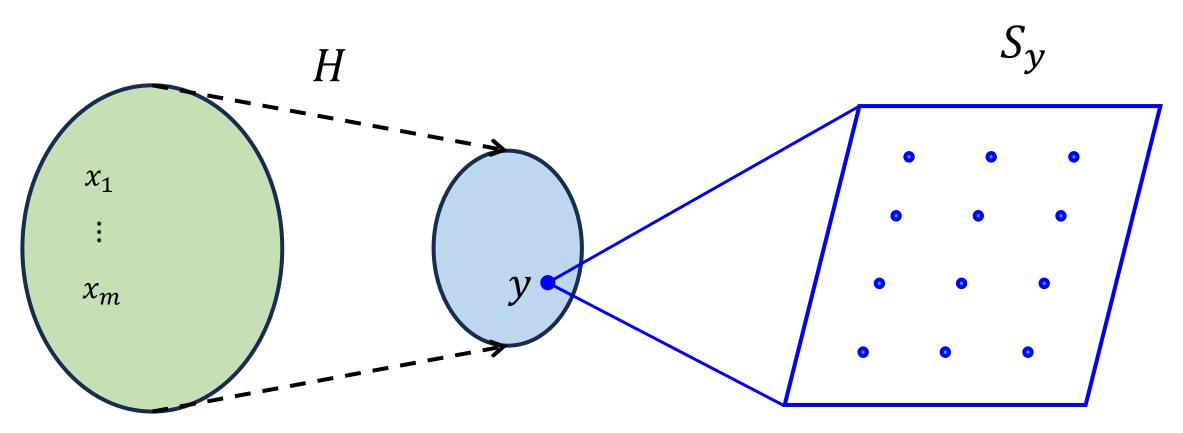
2. Given y, sample a secret sparse subspace $S_y \subseteq \mathbb{Z}_2^k$.



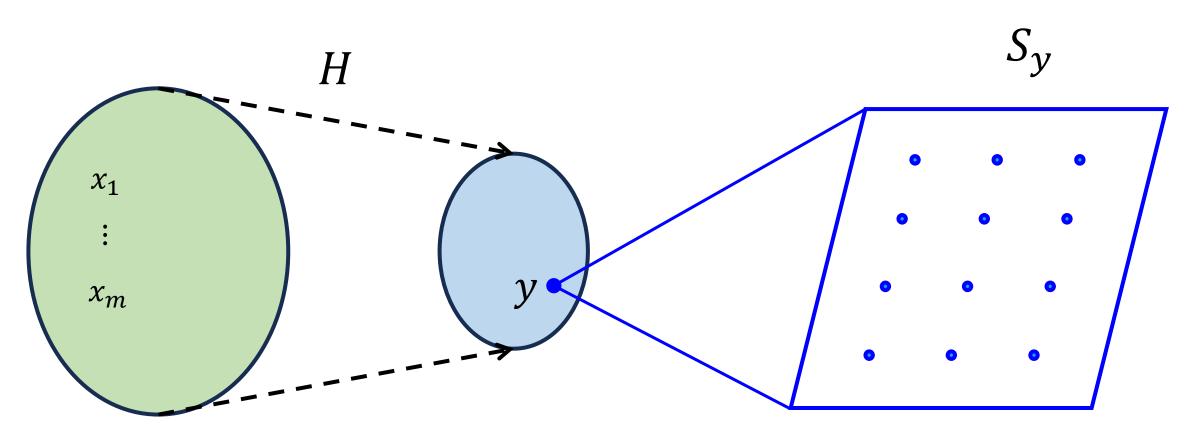
3. Note $\{J(x)\}_{x\in H^{-1}(y)}=\{0,1\}^{n/2}$. We can think of it as $\mathbb{Z}_2^{n/2}$. These can be coordinate vectors for S_y .



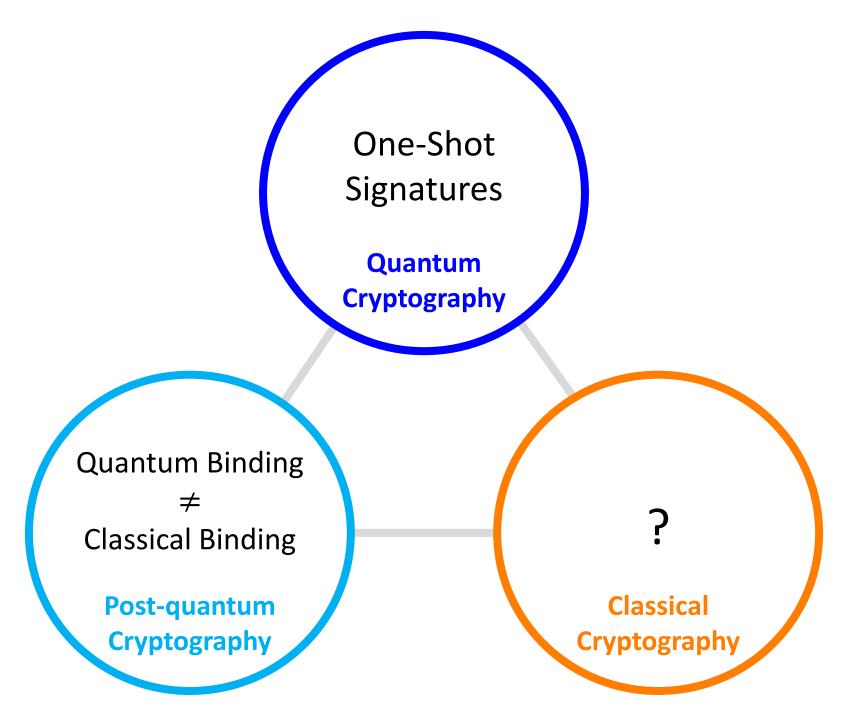
4. We show how to move between $H^{-1}(y)$ and S_y reversibly, while keeping the collision resistance of H.



5. By known techniques [A-C-12]: Superposition over S_y can be detected publicly, without revealing it.



In the paper: We show how to formalize these intuitions to get a Non-collapsing CRH in a classical oracle model.



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- The challenge: We need to obfuscate a PRP (open problem in classical cryptography, for at least a decade).
- We define a new notion: Permutable PRPs.
- Permutable PRPs allow: $iO(\Pi) \approx_c iO(\Gamma \circ \Pi)$, for a known Γ .

We show how to obfuscate a permutable PRP and make the circuit public, without revealing the PRP.

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Theorem 3:

Assume,

- Sub-exponentially-secure One-Way Functions, and
- Sub-exponentially-secure iO for classical circuits.

Then, \exists a trapdoor one-way permutation with domain $\{0,1\}^n$.

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2. Can we construct One-Shot Signatures (or even weaker primitives) without indistinguishability obfuscation?

Questions?