Multi-Holder Anonymous Credentials from BBS Signatures

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Anonymous Credentials (ACs)[Cha83, CL01]

Privacy preserving digital credentials whose authorship can be cryptographically verified

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Privacy preserving digital credentials whose authorship can be cryptographically verified

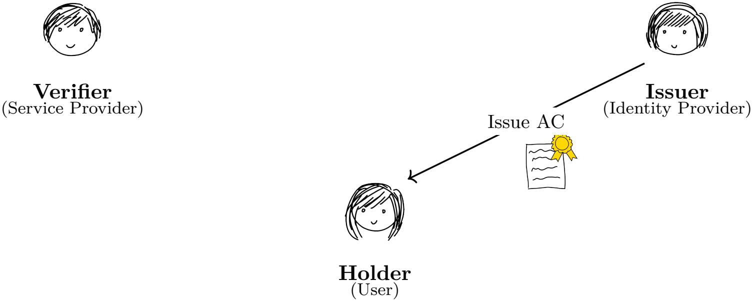
Verifier
(Service Provider)

Issuer
(Identity Provider)

Holder
(User)

Anonymous Credentials (ACs)[Cha83, CL01]

Privacy preserving digital credentials whose authorship can be cryptographically verified



Verifier
(Service Provider)

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Issue AC

Holder
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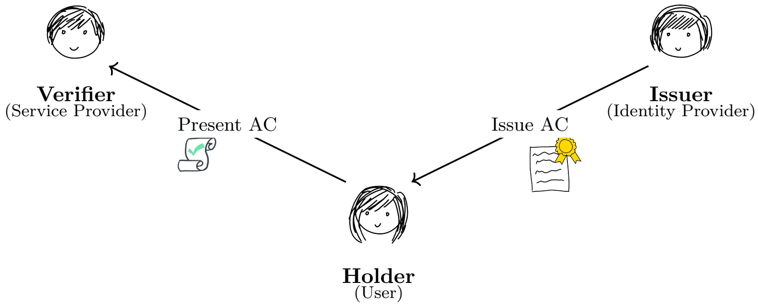


Verifier
(Service Provider)

Issuer
(Identity Provider)

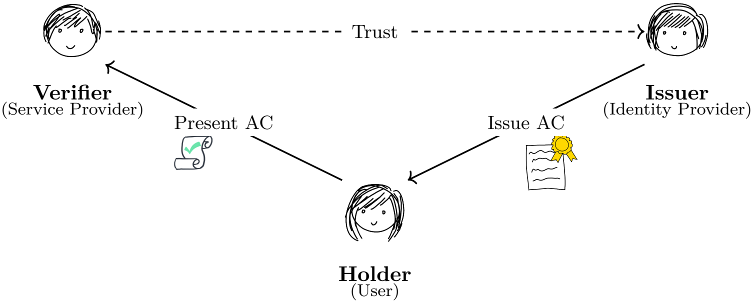
Present AC

Issue AC

Holder
(User)

Anonymous Credentials (ACs)[Cha83, CL01]

Privacy preserving digital credentials whose authorship can be cryptographically verified



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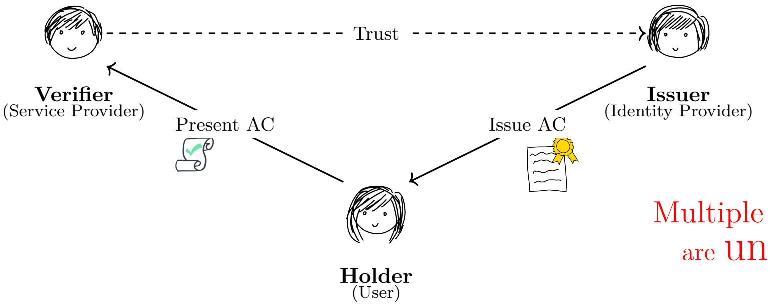

Multiple presentations are unlinkable!

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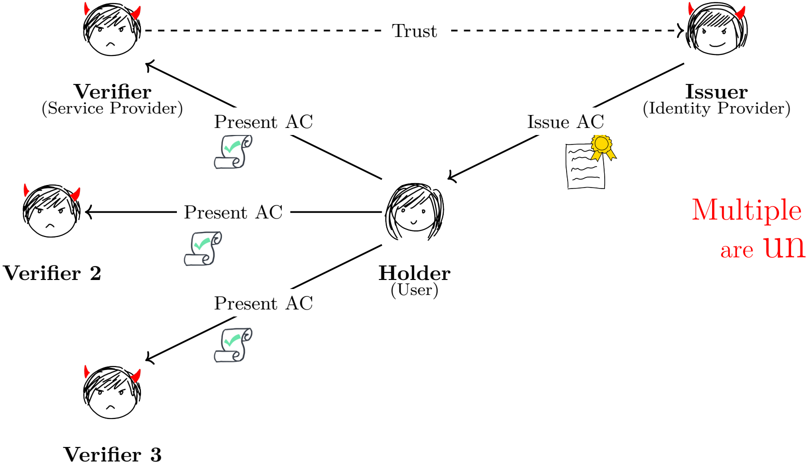

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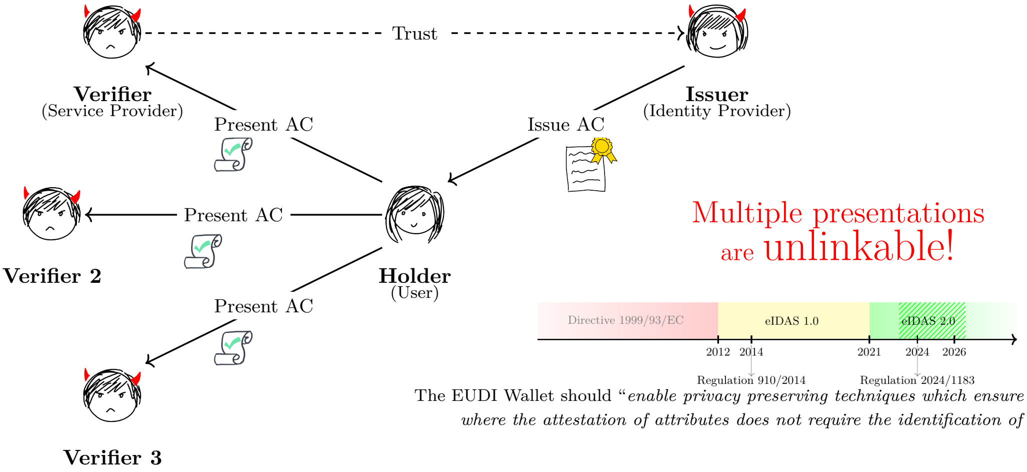

Verifier
(Service Provider)

Trust

Issuer
(Identity Provider)

Issue AC

Present AC

Present AC

Present AC

Holder
(User)

Verifier 2

Verifier 3

Multiple presentations are unlinkable!

The EUDI Wallet should “*enable privacy preserving techniques which ensure **unlinkability**, where the attestation of attributes does not require the identification of the user*”

The CL Framework [CL02]

A signature scheme with efficient NIZKPoK

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Verifier

Issuer $(\mathsf{sk}_{\mathsf{Iss}}, \mathsf{pk}_{\mathsf{Iss}})$

Holder

The CL Framework [CL02]

A signature scheme with efficient NIZKPoK

Verifier

Issuer $(\mathsf{sk}_{\mathsf{Iss}}, \mathsf{pk}_{\mathsf{Iss}})$

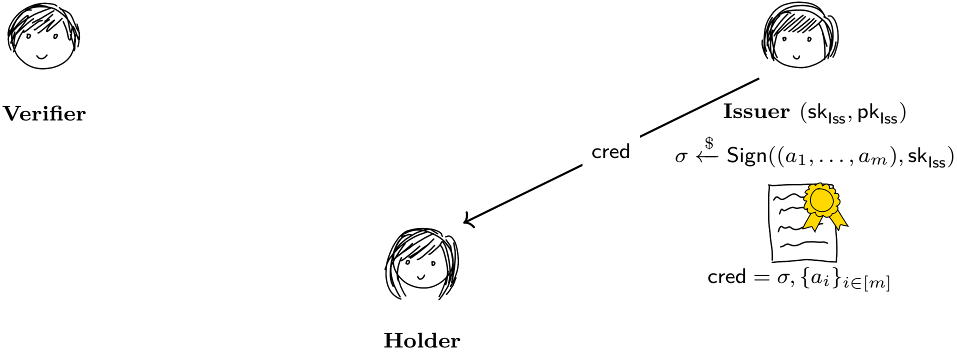
$\sigma \overset{\$}{\leftarrow} \mathsf{Sign}((a_1, \ldots, a_m), \mathsf{sk}_{\mathsf{Iss}})$

$\mathsf{cred} = \sigma, \{a_i\}_{i \in [m]}$

Holder

The CL Framework [CL02]

A signature scheme with efficient NIZKPoK



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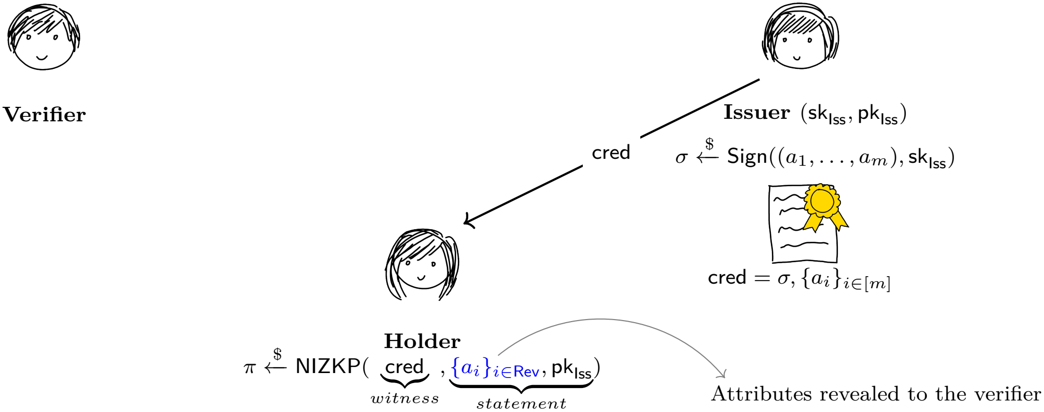

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cred

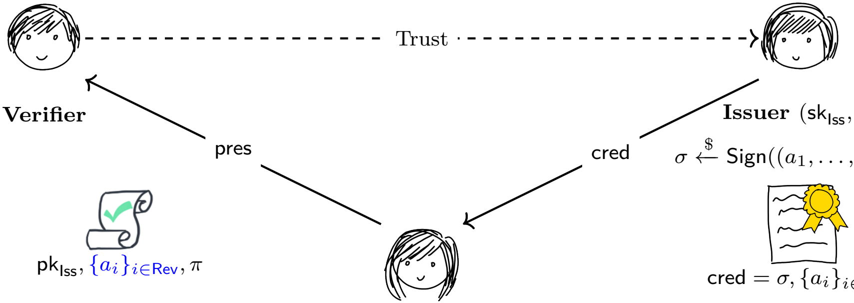
Holder

$\pi \xleftarrow{\$} \mathsf{NIZKP}(\underbrace{\mathsf{cred}}_{witness}, \underbrace{\{a_i\}_{i \in \mathsf{Rev}}, \mathsf{pk}_{\mathsf{Iss}}}_{statement})$

Attributes revealed to the verifier

The CL Framework [CL02]

A signature scheme with efficient NIZKPoK



Verifier

Trust

Issuer $(\mathsf{sk}_{\mathsf{Iss}}, \mathsf{pk}_{\mathsf{Iss}})$

pres

cred

$\sigma \overset{\$}{\leftarrow} \mathsf{Sign}((a_1, \ldots, a_m), \mathsf{sk}_{\mathsf{Iss}})$

$\mathsf{pk}_{\mathsf{Iss}}, \{a_i\}_{i \in \mathsf{Rev}}, \pi$

$\mathsf{cred} = \sigma, \{a_i\}_{i \in [m]}$

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Attributes revealed to the verifier

Multi-Holder Anonymous Credentials (MHAC)

(Our first contribution)

Motivation and Intuition

Increase the security of storage of anonymous credentials to prevent identity theft

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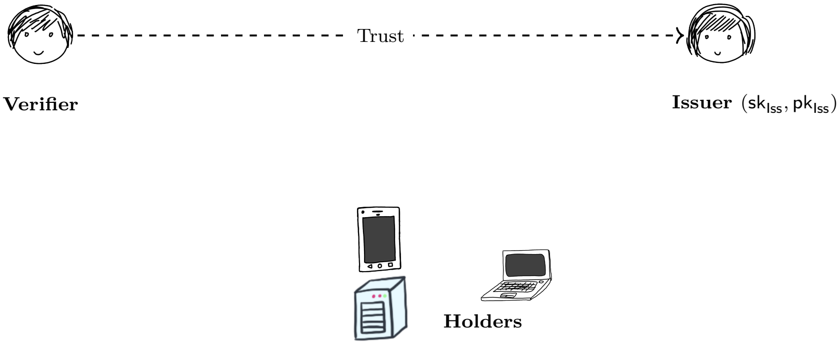

Trust

Verifier

Issuer $(\mathsf{sk}_{\mathsf{Iss}}, \mathsf{pk}_{\mathsf{Iss}})$

Holders

Motivation and Intuition

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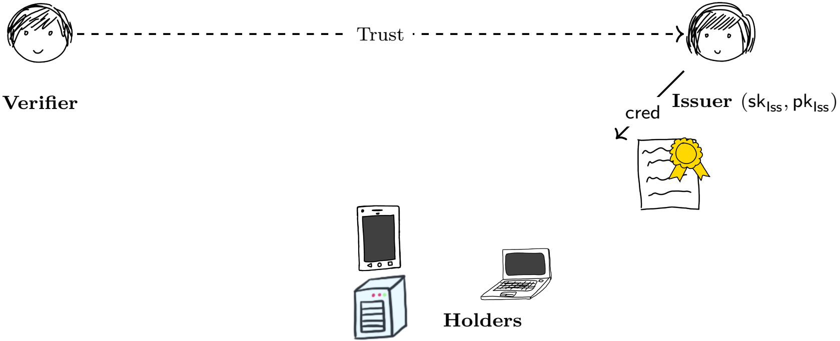

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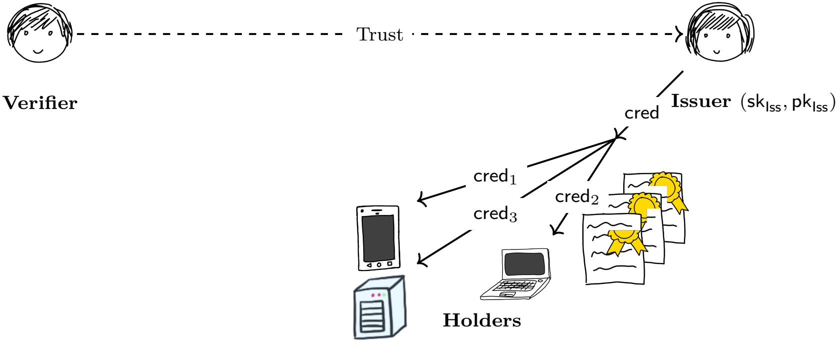

Verifier

Trust

Issuer $(\mathsf{sk}_{\mathsf{Iss}}, \mathsf{pk}_{\mathsf{Iss}})$

cred

cred_1

cred_2

cred_3

Holders

Motivation and Intuition

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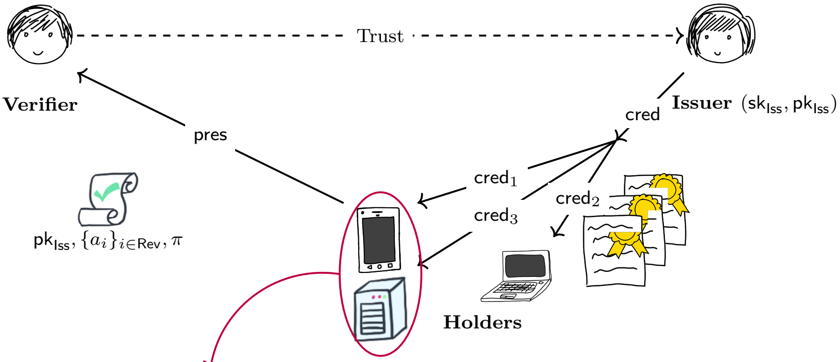

Trust

Verifier

Issuer $(\mathsf{sk}_{\mathsf{Iss}}, \mathsf{pk}_{\mathsf{Iss}})$

cred

pres

cred_1

cred_2

cred_3

Holders

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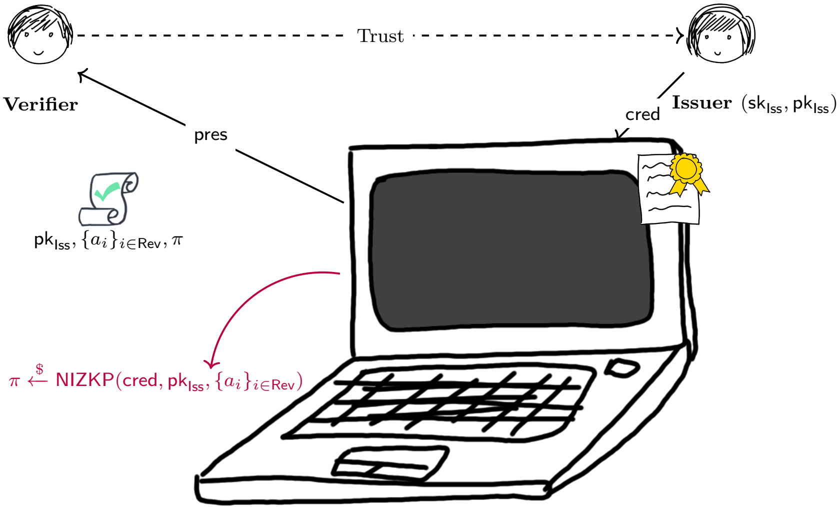

Verifier

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cred

pres

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Security and Privacy Properties

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Correctness

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Unlinkability

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Unforgeability of presentations

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Standard properties for AC
adapted for MHAC

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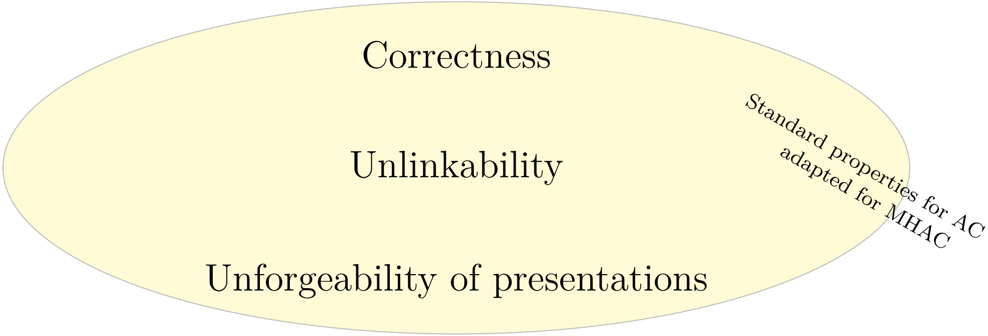


Correctness

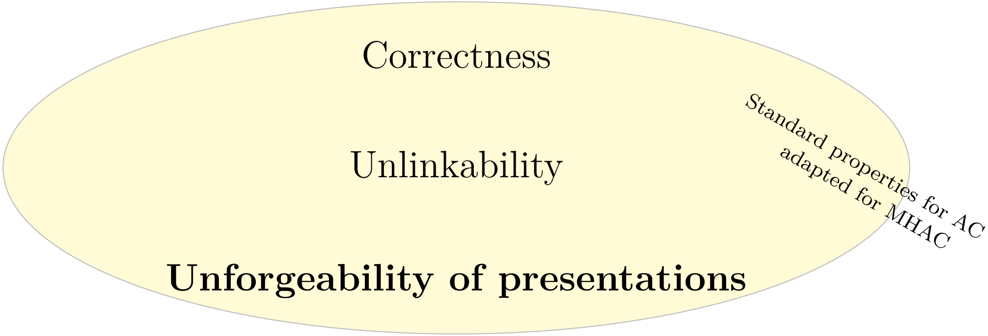
Unlinkability

Unforgeability of presentations

Standard properties for AC adapted for MHAC

Identifiable abort

Security and Privacy Properties



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Unforgeability Experiment

Adversary

Challenger

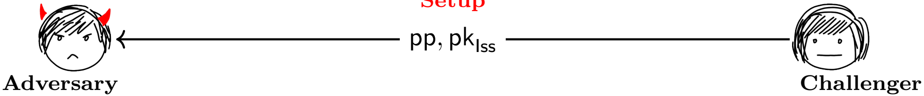
Unforgeability Experiment

Setup

Adversary

Challenger

Unforgeability Experiment



Setup

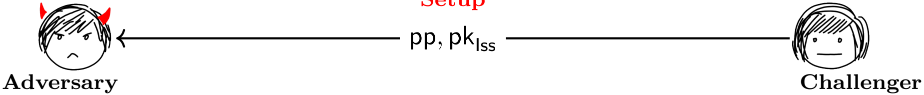
Adversary

$\mathsf{pp}, \mathsf{pk}_{\mathsf{lss}}$

Challenger

Unforgeability Experiment

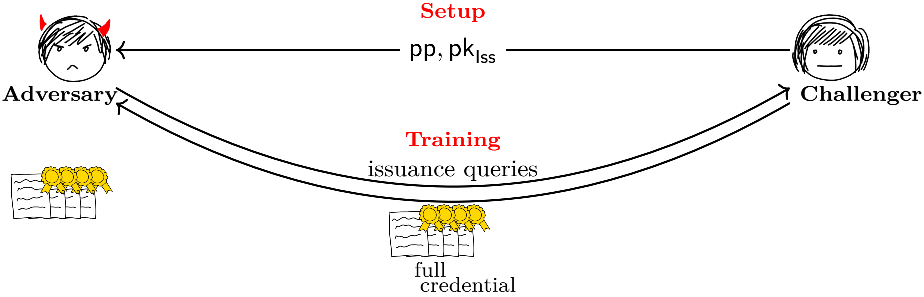
Setup



Adversary ← $\mathsf{pp}, \mathsf{pk}_{\mathsf{lss}}$ — **Challenger**

Training

Unforgeability Experiment



Setup

$\mathsf{pp}, \mathsf{pk}_{\mathsf{Iss}}$

Adversary

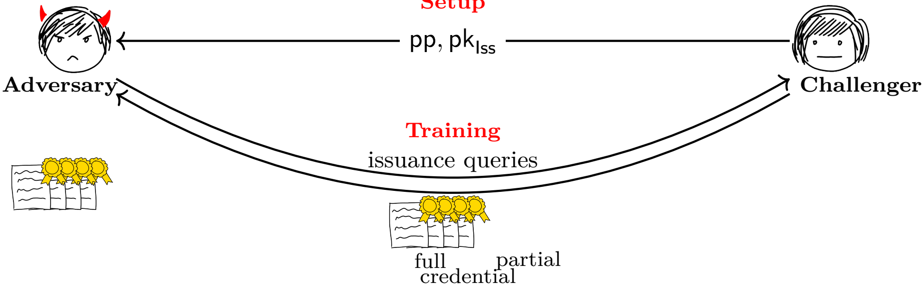
Challenger

Training

issuance queries

full credential

Unforgeability Experiment



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full partial
credential

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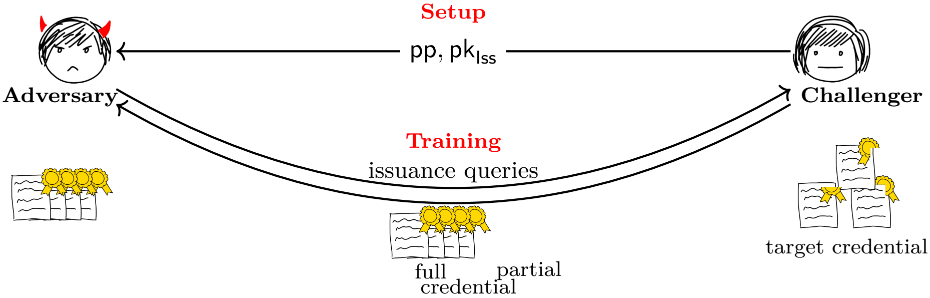

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full partial
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target credential

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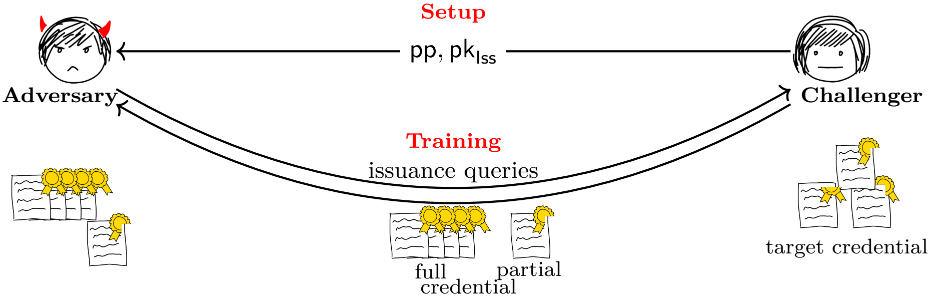

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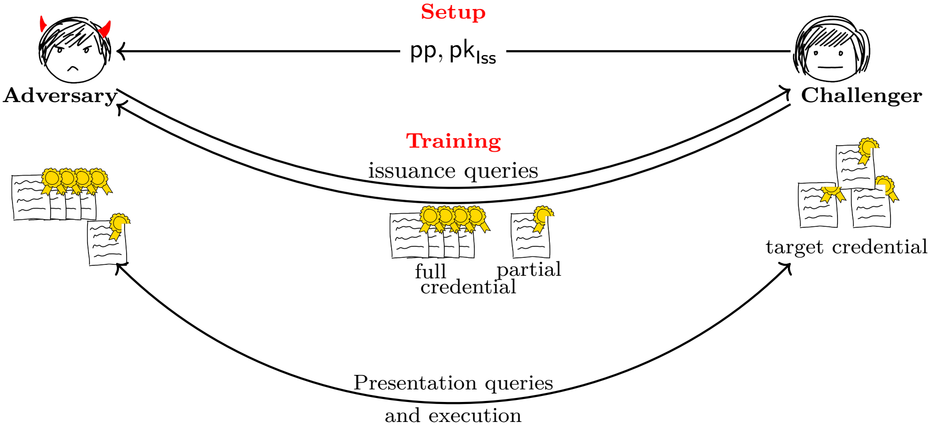

Setup

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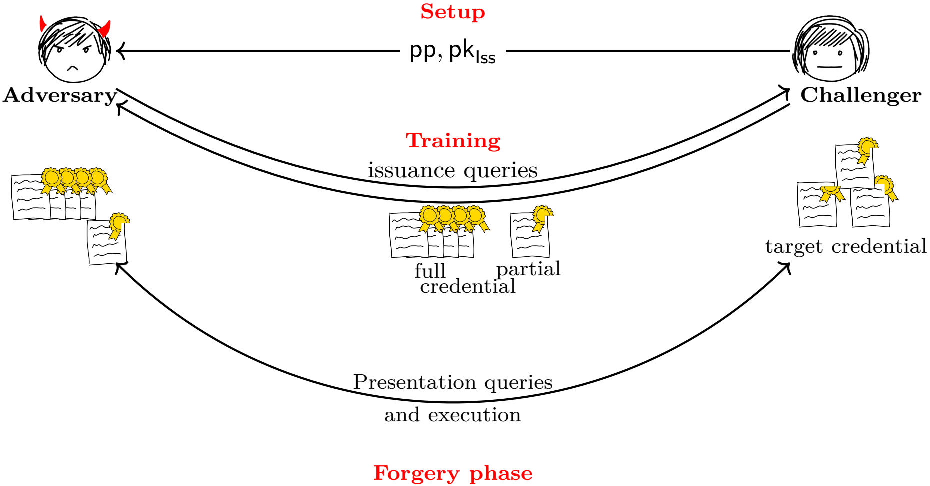
target credential

Presentation queries and execution

Unforgeability Experiment



Setup

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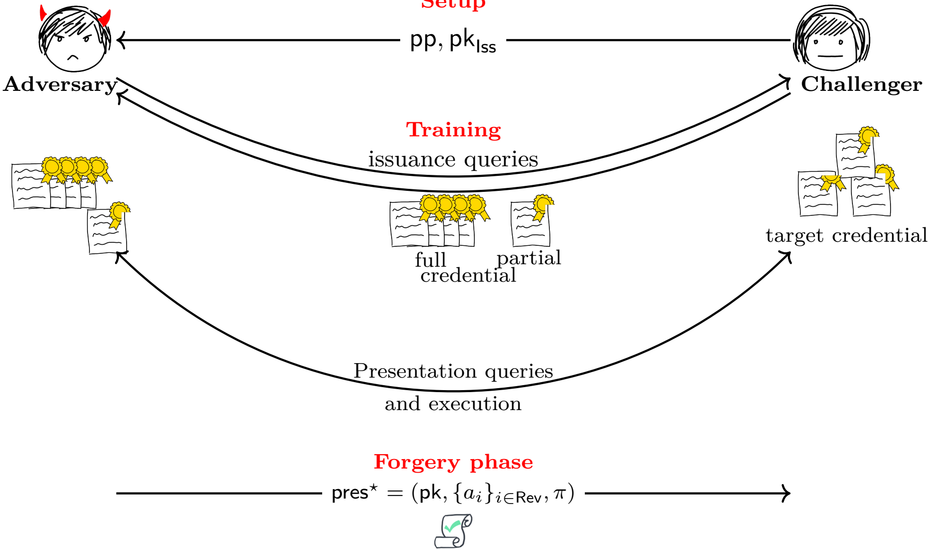
Forgery phase

Unforgeability Experiment



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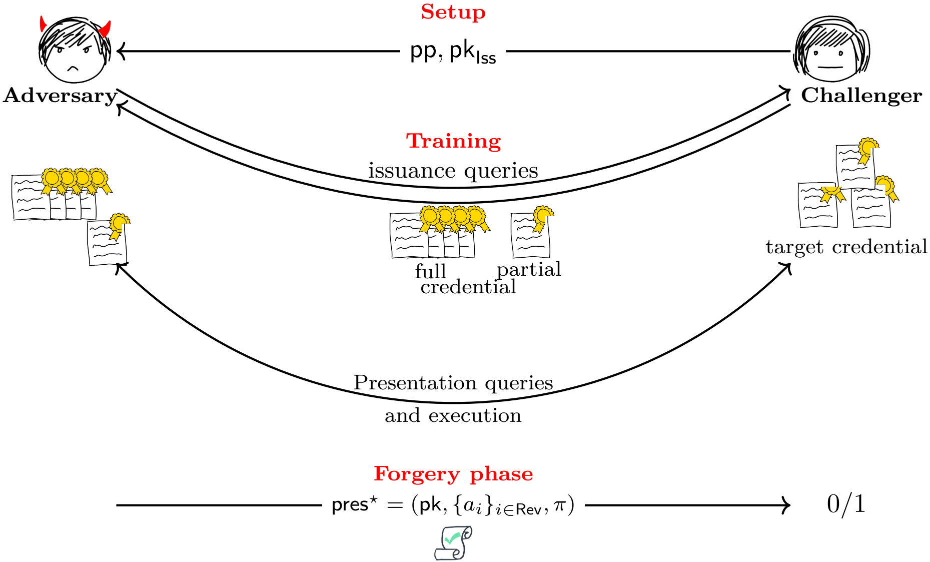
Forgery phase

$\mathsf{pres}^{\star} = (\mathsf{pk}, \{a_i\}_{i \in \mathsf{Rev}}, \pi)$

Unforgeability Experiment



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$0/1$

BBS Anonymous Credentials

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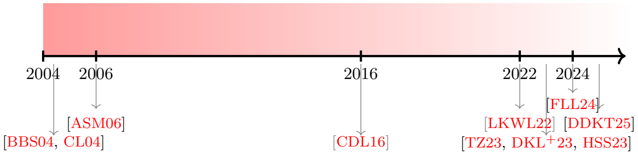
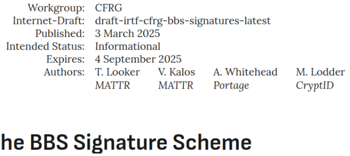

2004
2006
2016
2022
2024
[BBS04, CL04]
[ASM06]
[CDL16]
[LKWL22]
[TZ23, DKL$^+$23, HSS23]
[FLL24]
[DDKT25]

Why BBS?

- multi-message signature
- compact public keys
- efficient signature and NIZKP

Standardization effort by DIF and IRTF


Workgroup: CFRG
Internet-Draft: draft-irtf-cfrg-bbs-signatures-latest
Published: 3 March 2025
Intended Status: Informational
Expires: 4 September 2025
Authors: T. Looker *MATTR*; V. Kalos *MATTR*; A. Whitehead *Portage*; M. Lodder *CryptID*

The BBS Signature Scheme

BBS Issuance

(For a single attribute a_1)

BBS Issuance

(For a single attribute a_1)

Setup

p-order groups $\mathbb{G}_1 = \langle g_1 \rangle, \mathbb{G}_2 = \langle g_2 \rangle$, $\mathbb{G}_T$, and pairing $\mathbf{e} : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$

$$\mathsf{pp} = h_1 \xleftarrow{\$} \mathbb{G}_1 \qquad x \xleftarrow{\$} \mathbb{Z}_p \qquad (\mathsf{sk}_{\mathsf{Iss}}, \mathsf{pk}_{\mathsf{Iss}}) \leftarrow (x, g_2^x)$$

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(For a single attribute a_1)

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Issuance

$$C(a_1) \leftarrow g_1 h_1^{a_1} \qquad e \xleftarrow{\$} \mathbb{Z}_p \qquad A \leftarrow (C(a_1))^{\frac{1}{x+e}}$$

BBS Issuance

(For a single attribute a_1)

Setup

p-order groups $\mathbb{G}_1 = \langle g_1 \rangle, \mathbb{G}_2 = \langle g_2 \rangle$, $\mathbb{G}_T$, and pairing $\mathbf{e} : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$

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$$C(a_1) \leftarrow g_1 h_1^{a_1} \qquad e \overset{\$}{\leftarrow} \mathbb{Z}_p \qquad A \leftarrow (C(a_1))^{\frac{1}{x+e}}$$

$$\mathsf{cred} \leftarrow (\quad \underbrace{(A, e)}_{\text{BBS signature}} \quad , a_1)$$

BBS Presentation[TZ23]

(full disclosure)

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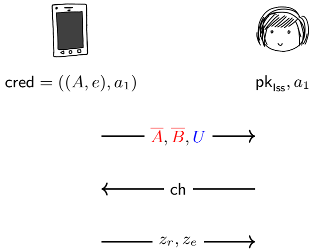
(full disclosure)

Presentation for $(\mathsf{pk}_{\mathsf{Iss}}, a_1)$ **of** $\mathsf{cred} = ((A, e), a_1)$



$\mathsf{cred} = ((A, e), a_1)$

$\mathsf{pk}_{\mathsf{Iss}}, a_1$

$\xrightarrow{\overline{A}, \overline{B}, U}$

$\xleftarrow{\mathsf{ch}}$

$\xrightarrow{z_r, z_e}$

BBS Presentation[TZ23]

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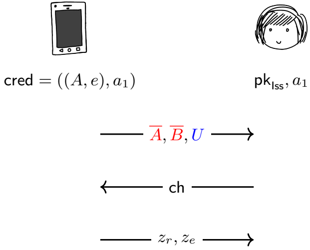
Presentation for $(\mathsf{pk}_{\mathsf{Iss}}, a_1)$ **of** $\mathsf{cred} = ((A, e), a_1)$

- signature randomization: $r \xleftarrow{\$} \mathbb{Z}_p$
  $\overline{A} \leftarrow A^r \qquad \overline{B} \leftarrow C(a_1)^r \overline{A}^{-e}$
- $(U, \mathsf{ch}, z_r, z_e) \xleftarrow{\$} \mathrm{NIZKPoK}\{(\alpha, \beta) : \overline{B} = C(a_1)^\alpha \overline{A}^\beta\}$
- $\pi \leftarrow (\overline{A}, \overline{B}, U, \mathsf{ch}, z_r, z_e)$

$$C(a_1) = g_1 h_1^{a_1}$$



$\mathsf{cred} = ((A, e), a_1)$

$\mathsf{pk}_{\mathsf{Iss}}, a_1$

$\overline{A}, \overline{B}, U \longrightarrow$

$\longleftarrow \mathsf{ch}$

$z_r, z_e \longrightarrow$

BBS Presentation[TZ23]

(full disclosure)

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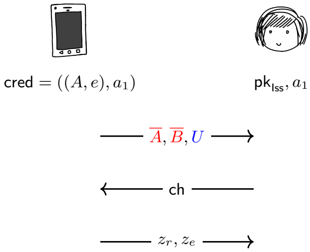

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$\xrightarrow{z_r, z_e}$

$$\mathsf{pres} \leftarrow (\pi, \underbrace{a_1, \mathsf{pk}_{\mathsf{Iss}}}_{\text{statement}})$$

A MHAC compatible with BBS

(Our second contribution)

BBS MHAC Issuance

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Generate a BBS credential $\mathsf{cred} = (A, e), a_1$

BBS MHAC Issuance

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Divide it in shares:

BBS MHAC Issuance

Generate a BBS credential $\mathsf{cred} = (A, e), a_1$

Divide it in shares:

e is never revealed!

$$\{e_i\}_{i\in[n]} \xleftarrow{\$} \mathsf{SS}(t, n, e)$$

BBS MHAC Issuance

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BBS MHAC Issuance

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Crucial observation: giving to each holder A^e is just fine!

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enables the identifiable abort property

BBS MHAC Issuance

Generate a BBS credential $\mathsf{cred} = (A, e), a_1$

e is never revealed!

Divide it in shares:

$$\{e_i\}_{i \in [n]} \xleftarrow{\$} \mathsf{SS}(t, n, e) \quad \mathsf{cred}_i \leftarrow (e_i, \underbrace{A, A^e, \{A^{e_i}\}_{i \in [n]}, a_1}_{\text{same for every holder}})$$

Crucial observation: giving to each holder A^e is just fine!

enables the identifiable abort property

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can be made constant size

BBS MHAC Presentation protocol

$\mathsf{cred}_i \leftarrow (A, e_i, A^e, \{A^{e_j}\}_{j \in [n]}, a_1)$

Multi-holder case

Centralised case

$\mathsf{cred} \leftarrow ((A, e), a_1)$

$\mathsf{cred}_i \leftarrow (A, e_i, A^e, \{A^{e_j}\}_{j\in[n]}, a_1)$

$r \xleftarrow{\$} \mathbb{Z}_p$

Multi-holder case

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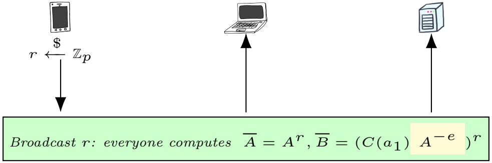

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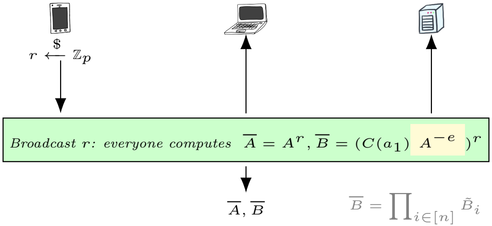

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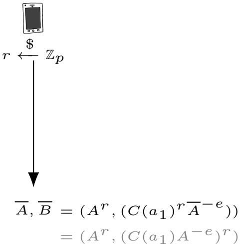

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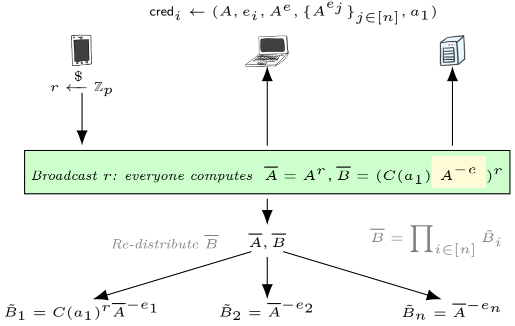

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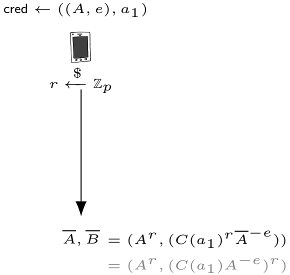

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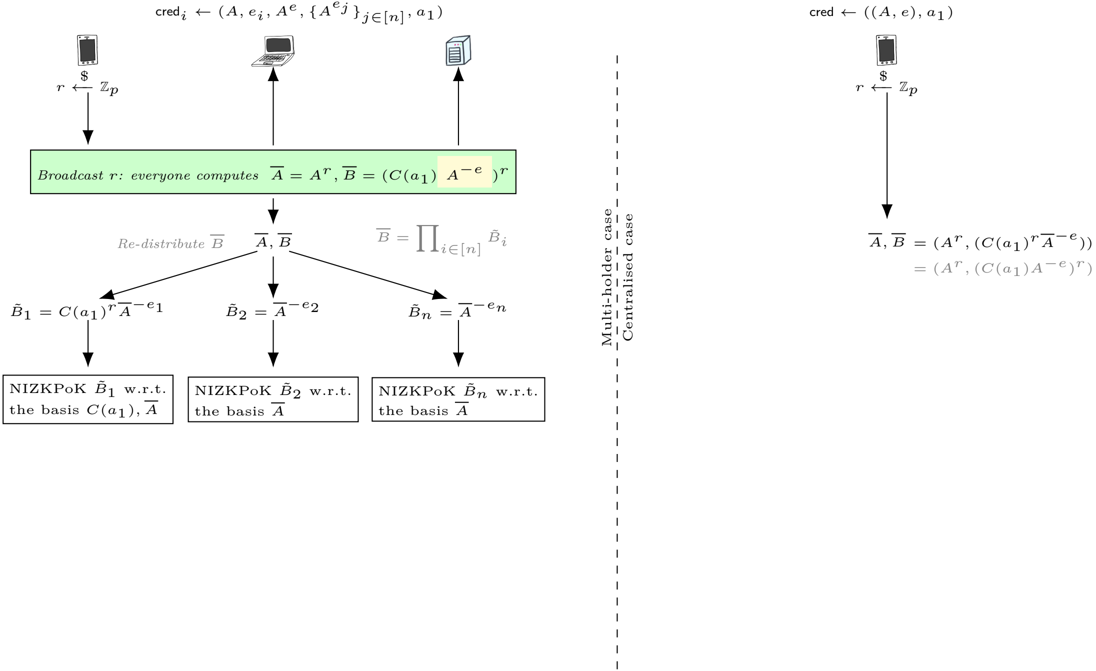

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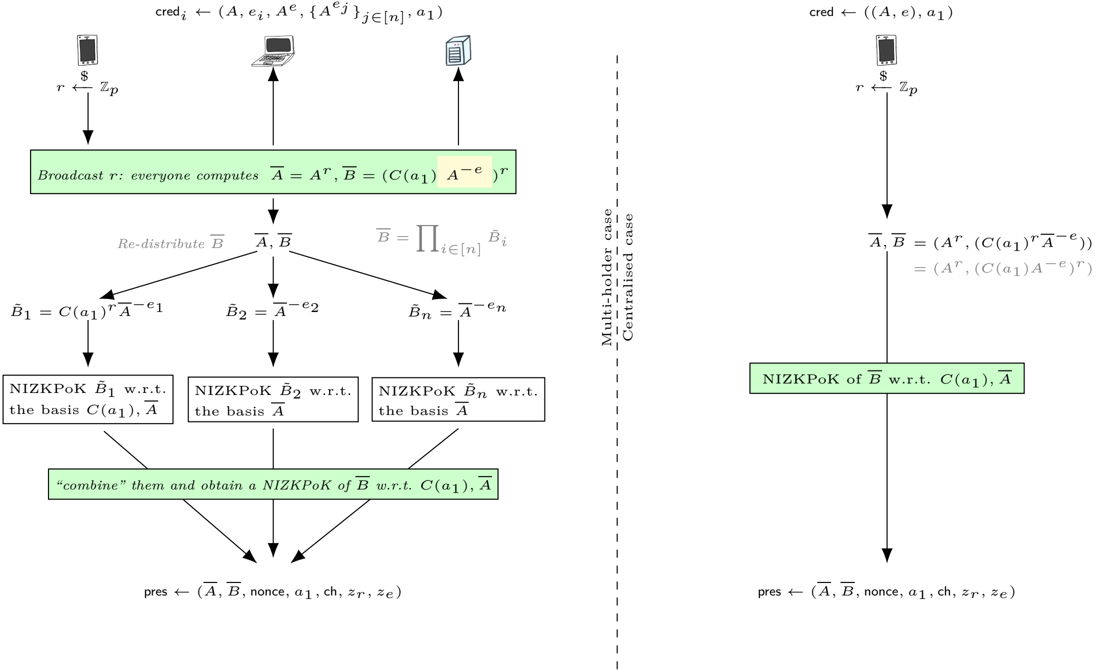

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Multi-holder case

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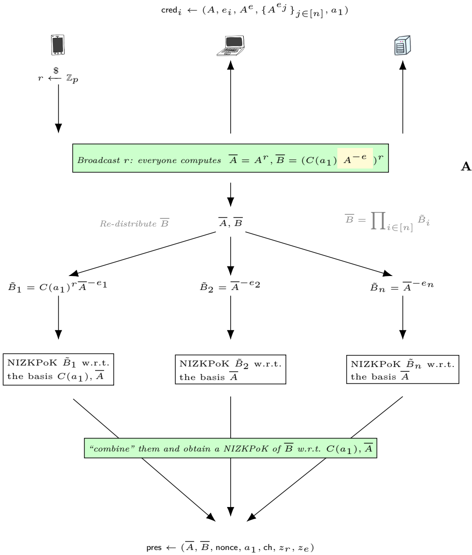
Unforgeability reduces to DL assumption

(When the adversary forges the target credential)



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Adversary

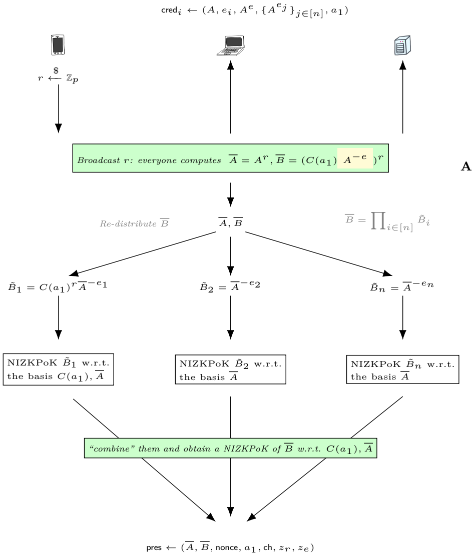
Reduction

DL **challenger**

Unforgeability reduces to DL assumption

(When the adversary forges the target credential)

Setup



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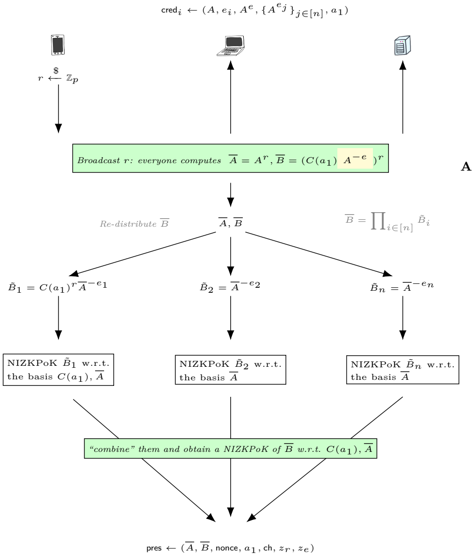
Unforgeability reduces to DL assumption

(When the adversary forges the target credential)



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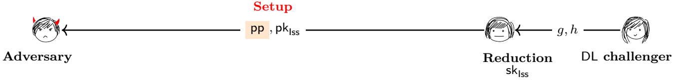


Setup

pp, $\mathsf{pk}_{\mathsf{Iss}}$

g, h

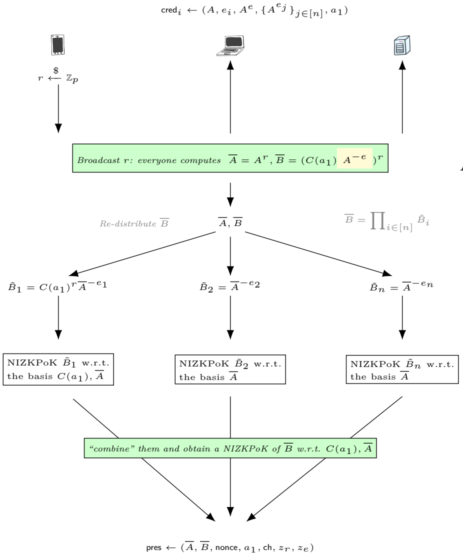
Adversary

Reduction
$\mathsf{sk}_{\mathsf{Iss}}$

DL **challenger**

Unforgeability reduces to DL assumption

(When the adversary forges the target credential)



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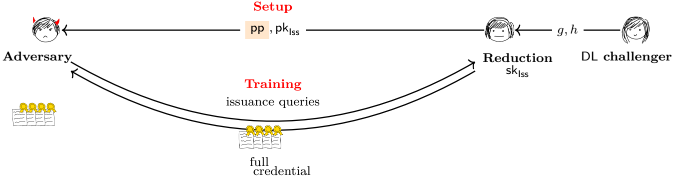


Setup

pp, $\mathsf{pk}_{\mathsf{Iss}}$

g, h

Adversary

Reduction

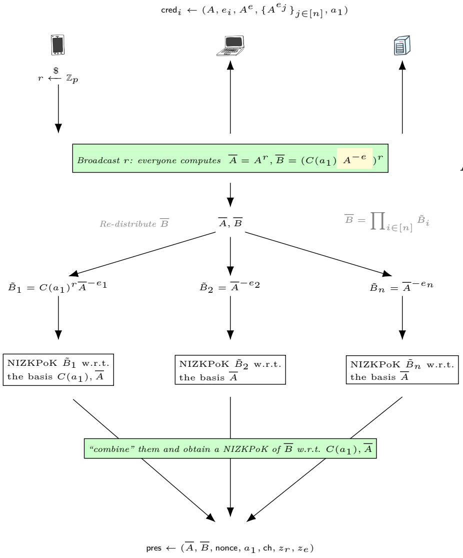
$\mathsf{sk}_{\mathsf{Iss}}$

DL **challenger**

Training

issuance queries

full credential



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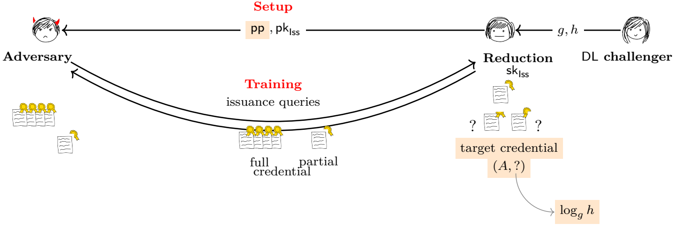
Unforgeability reduces to DL assumption

(When the adversary forges the target credential)



Setup

pp , $\mathsf{pk}_{\mathsf{Iss}}$

g, h

Adversary

Reduction

$\mathsf{sk}_{\mathsf{Iss}}$

DL **challenger**

Training

issuance queries

full credential

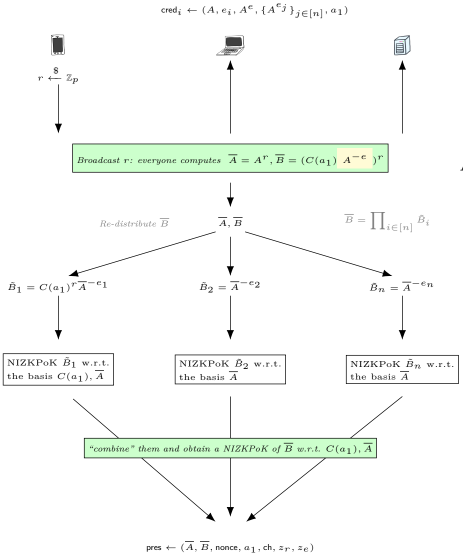
partial

? ?

target credential

$(A, ?)$

$\log_g h$



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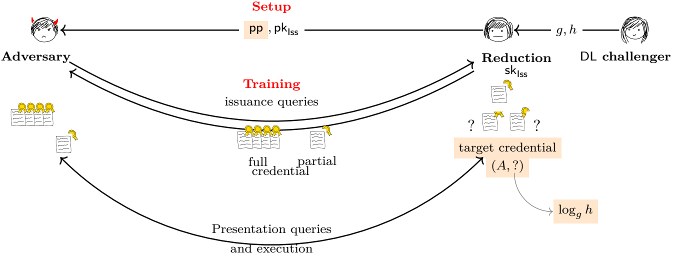

Setup

pp , $\mathsf{pk}_{\mathsf{Iss}}$

g, h

Adversary

Reduction

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DL challenger

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full credential

partial

? ?

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$(A, ?)$

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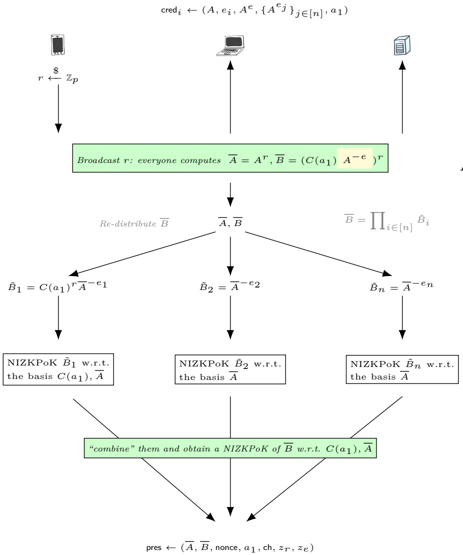
Presentation queries and execution



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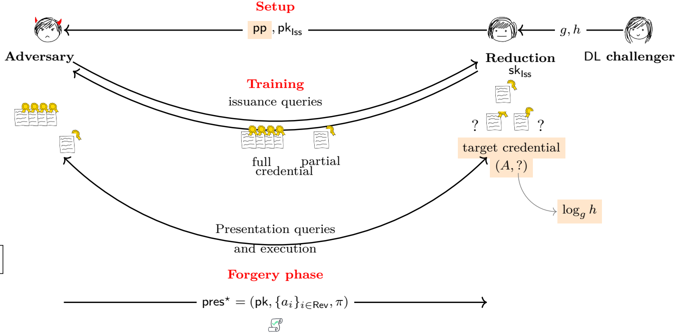
Unforgeability reduces to DL assumption

(When the adversary forges the target credential)



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pp, $\mathsf{pk}_{\mathsf{Iss}}$

g, h

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target credential

$(A, ?)$

$\log_g h$

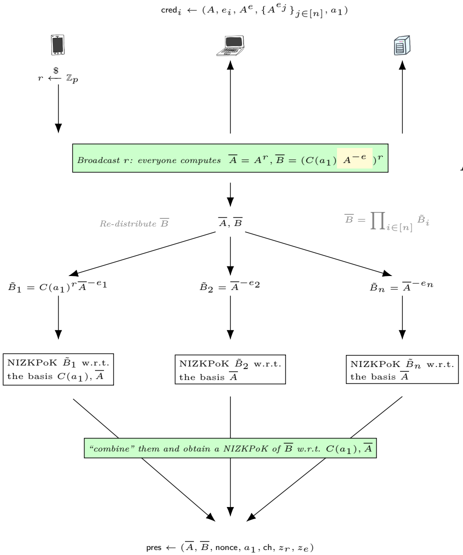
Presentation queries and execution

Forgery phase

$\mathsf{pres}^\star = (\mathsf{pk}, \{a_i\}_{i\in\mathsf{Rev}}, \pi)$

Unforgeability reduces to DL assumption

(When the adversary forges the target credential)



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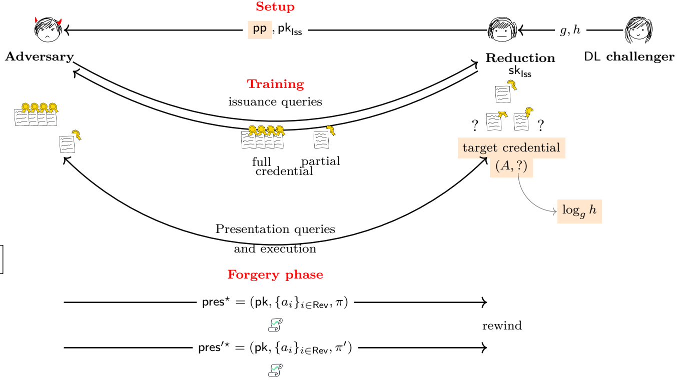

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Training

issuance queries

full credential

partial

? ?

target credential

$(A, ?)$

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Presentation queries and execution

Forgery phase

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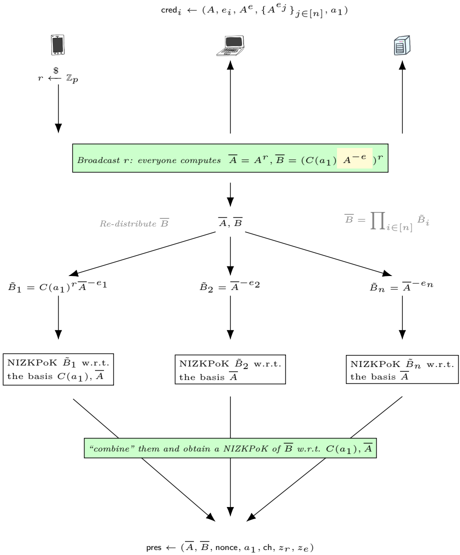
rewind

$\mathsf{pres}'^\star = (\mathsf{pk}, \{a_i\}_{i\in\mathsf{Rev}}, \pi')$

Unforgeability reduces to DL assumption

(When the adversary forges the target credential)



$\mathsf{cred}_i \leftarrow (A, e_i, A^e, \{A^{e_j}\}_{j\in[n]}, a_1)$

$r \xleftarrow{\$} \mathbb{Z}_p$

Broadcast r: everyone computes $\overline{A} = A^r, \overline{B} = (C(a_1)\ A^{-e}\ )^r$

Re-distribute $\overline{B}$

$\overline{A}, \overline{B}$

$\overline{B} = \prod_{i\in[n]} \tilde{B}_i$

$\tilde{B}_1 = C(a_1)^r\overline{A}^{-e_1}$

$\tilde{B}_2 = \overline{A}^{-e_2}$

$\tilde{B}_n = \overline{A}^{-e_n}$

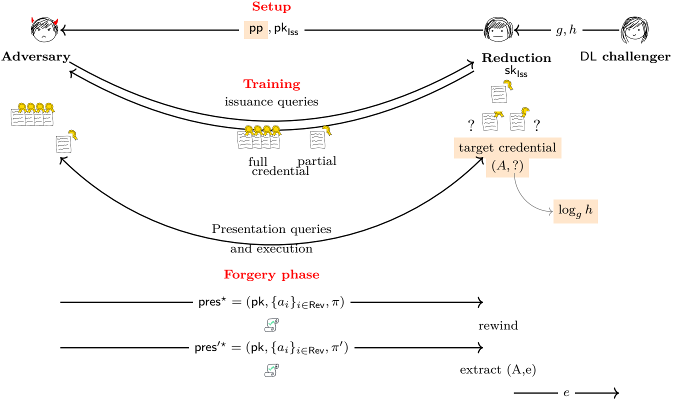
NIZKPoK $\tilde{B}_1$ w.r.t. the basis $C(a_1), \overline{A}$

NIZKPoK $\tilde{B}_2$ w.r.t. the basis $\overline{A}$

NIZKPoK $\tilde{B}_n$ w.r.t. the basis $\overline{A}$

"combine" them and obtain a NIZKPoK of $\overline{B}$ *w.r.t.* $C(a_1), \overline{A}$

$\mathsf{pres} \leftarrow (\overline{A}, \overline{B}, \mathsf{nonce}, a_1, \mathsf{ch}, z_r, z_e)$



Setup

pp, $\mathsf{pk}_{\mathsf{Iss}}$

g, h

Adversary

Reduction

$\mathsf{sk}_{\mathsf{Iss}}$

DL **challenger**

Training

issuance queries

full credential

partial

? ?

target credential

$(A, ?)$

$\log_g h$

Presentation queries and execution

Forgery phase

$\mathsf{pres}^\star = (\mathsf{pk}, \{a_i\}_{i\in\mathsf{Rev}}, \pi)$

rewind

$\mathsf{pres}'^\star = (\mathsf{pk}, \{a_i\}_{i\in\mathsf{Rev}}, \pi')$

extract (A,e)

e

Thank you for your attention!

to Eysa Lee for the Alice-and-Bobs illustrations
https://github.com/eysalee/alice-and-bobs/tree/main

and to the QUBIP European project for funding my trip here

QUBIP

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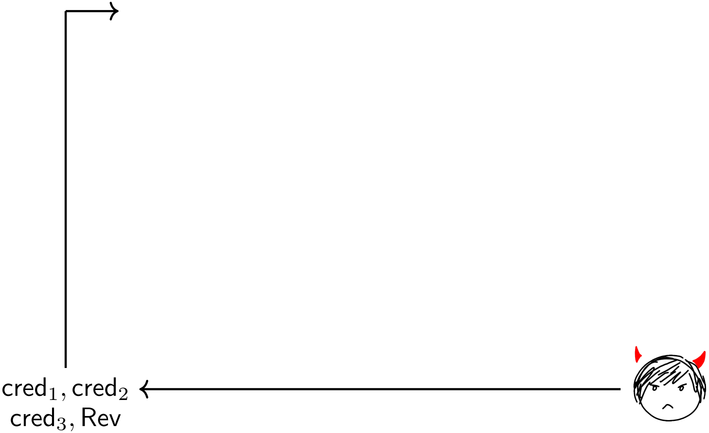
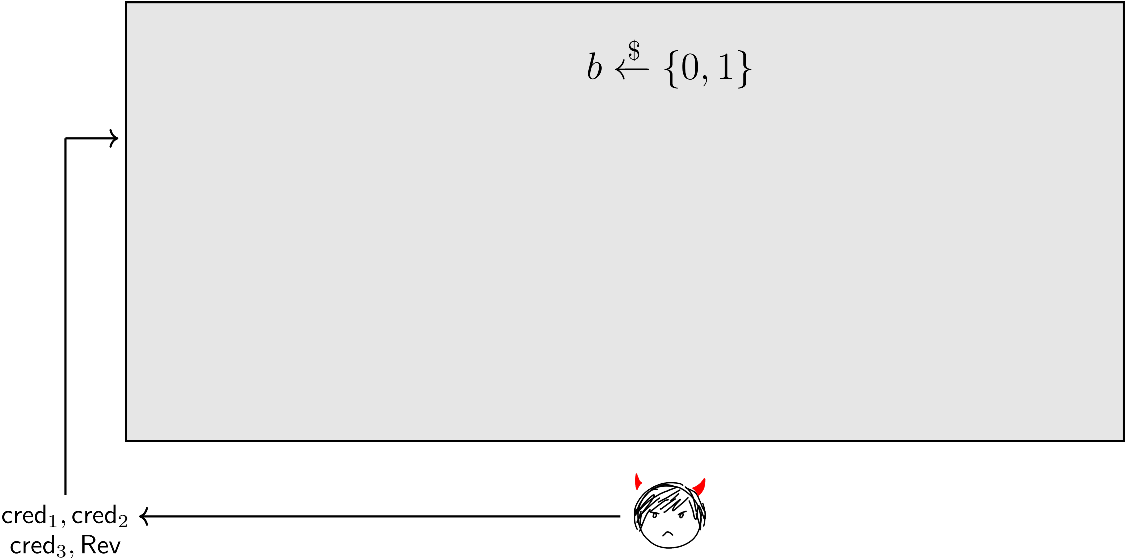
Unlinkability Experiment



$\mathsf{cred}_1, \mathsf{cred}_2$
$\mathsf{cred}_3, \mathsf{Rev}$

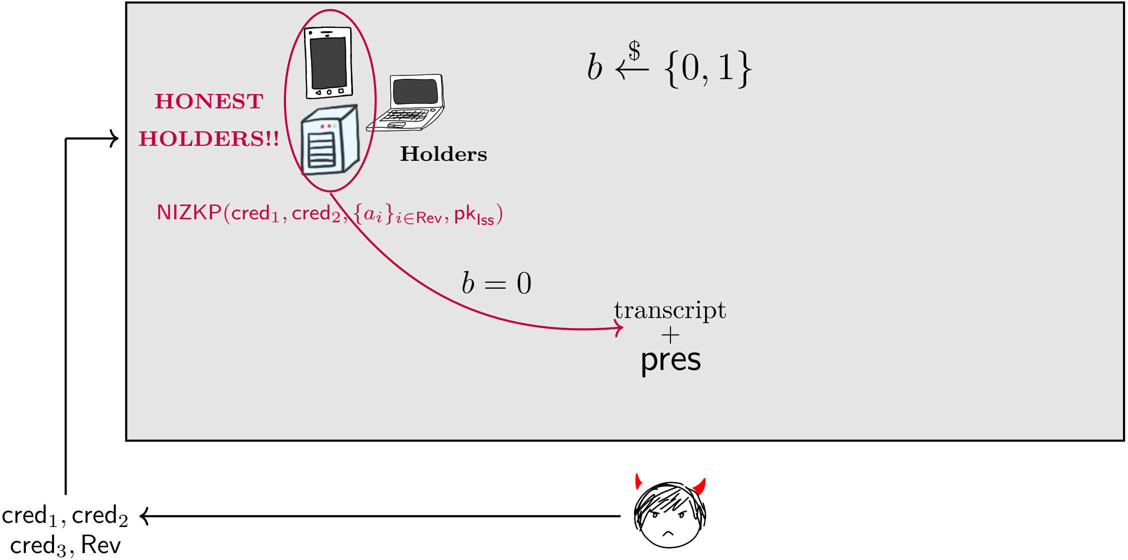
Unlinkability Experiment



$b \xleftarrow{\$} \{0,1\}$

$\mathsf{cred}_1, \mathsf{cred}_2$
$\mathsf{cred}_3, \mathsf{Rev}$

Unlinkability Experiment



$b \xleftarrow{\$} \{0, 1\}$

HONEST HOLDERS!!

Holders

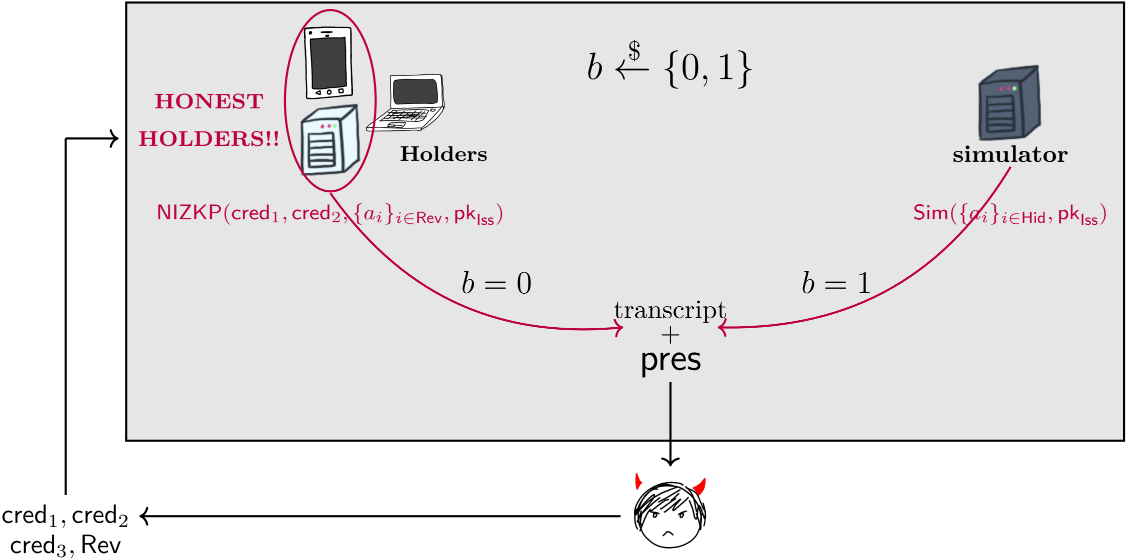
$\mathsf{NIZKP}(\mathsf{cred}_1, \mathsf{cred}_2, \{a_i\}_{i \in \mathsf{Rev}}, \mathsf{pk}_{\mathsf{Iss}})$

$b = 0$

transcript + pres

$\mathsf{cred}_1, \mathsf{cred}_2$
$\mathsf{cred}_3, \mathsf{Rev}$

Unlinkability Experiment



$b \xleftarrow{\$} \{0, 1\}$

HONEST HOLDERS!!

Holders

simulator

$\mathsf{NIZKP}(\mathsf{cred}_1, \mathsf{cred}_2, \{a_i\}_{i \in \mathsf{Rev}}, \mathsf{pk}_{\mathsf{Iss}})$

$\mathsf{Sim}(\{a_i\}_{i \in \mathsf{Hid}}, \mathsf{pk}_{\mathsf{Iss}})$

$b = 0$

$b = 1$

transcript
+
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$\mathsf{cred}_1, \mathsf{cred}_2$
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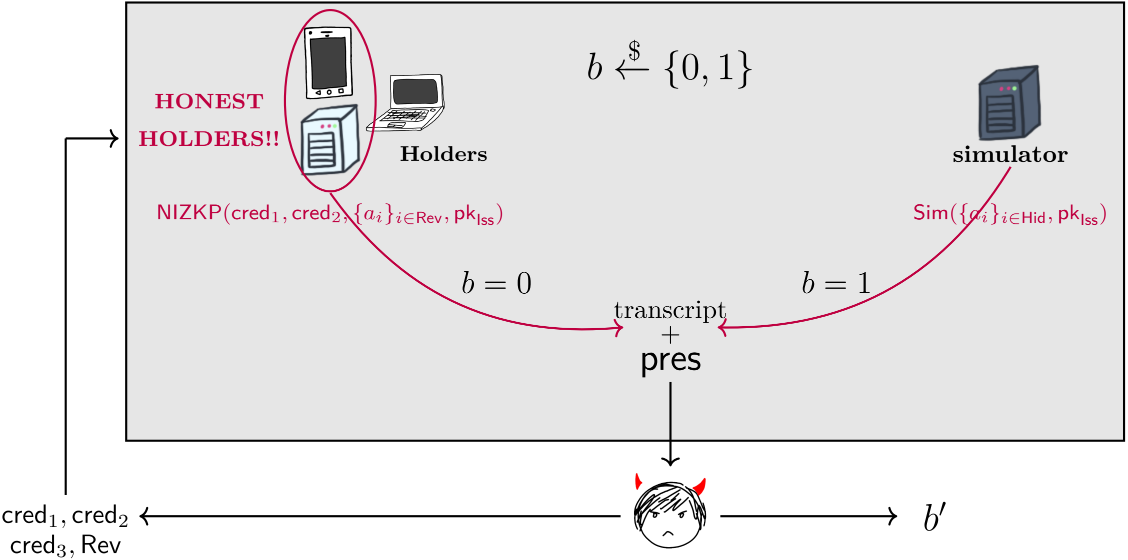
Unlinkability Experiment



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HONEST HOLDERS!!

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$\mathsf{NIZKP}(\mathsf{cred}_1, \mathsf{cred}_2, \{a_i\}_{i \in \mathsf{Rev}}, \mathsf{pk}_{\mathsf{Iss}})$

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$b = 0$

$b = 1$

transcript
+
pres

$\mathsf{cred}_1, \mathsf{cred}_2$
$\mathsf{cred}_3, \mathsf{Rev}$

b'

How does the combined NIZKP works?

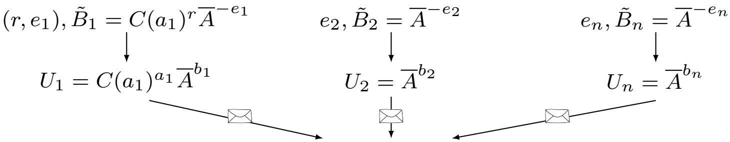
$(r, e_1), \tilde{B}_1 = C(a_1)^r \overline{A}^{-e_1}$ $e_2, \tilde{B}_2 = \overline{A}^{-e_2}$ $e_n, \tilde{B}_n = \overline{A}^{-e_n}$

How does the combined NIZKP works?

$$\begin{tikzcd}
 & (r, e_1), \tilde{B}_1 = C(a_1)^r \overline{A}^{-e_1} \arrow[d] & e_2, \tilde{B}_2 = \overline{A}^{-e_2} \arrow[d] & e_n, \tilde{B}_n = \overline{A}^{-e_n} \arrow[d] \\
a_1, \{b_i\}_{i \in [n]} \xleftarrow{\$} \mathbb{Z}_p & {} & {} & {}
\end{tikzcd}$$

How does the combined NIZKP works?

$a_1, \{b_i\}_{i\in[n]} \xleftarrow{\$} \mathbb{Z}_p$



$(r, e_1), \tilde{B}_1 = C(a_1)^r \overline{A}^{-e_1}$

$U_1 = C(a_1)^{a_1} \overline{A}^{b_1}$

$e_2, \tilde{B}_2 = \overline{A}^{-e_2}$

$U_2 = \overline{A}^{b_2}$

$e_n, \tilde{B}_n = \overline{A}^{-e_n}$

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commit and release

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U_1 = C(a_1)^{a_1} \overline{A}^{b_1} \arrow[dr, "✉"] & U_2 = \overline{A}^{b_2} \arrow[d, "✉"] & U_n = \overline{A}^{b_n} \arrow[dl, "✉"'] \\
& U = \prod_{i=1}^{n} U_i \arrow[d] & \\
& &
\end{tikzcd}$$

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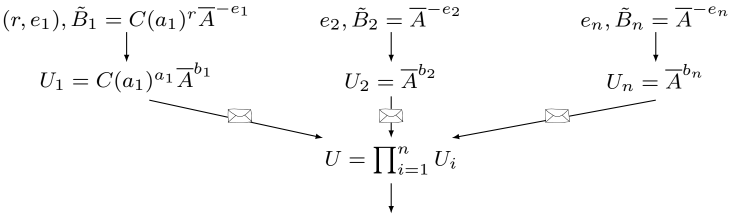

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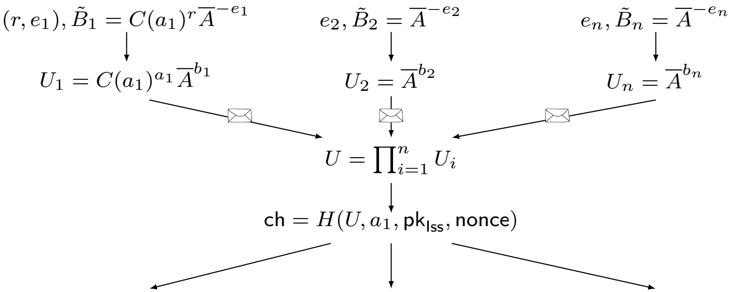

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$\mathsf{ch} = H(U, a_1, \mathsf{pk}_{\mathsf{Iss}}, \mathsf{nonce})$

How does the combined NIZKP works?

$a_1, \{b_i\}_{i \in [n]} \xleftarrow{\$} \mathbb{Z}_p$

commit and release

$$\begin{tikzcd}
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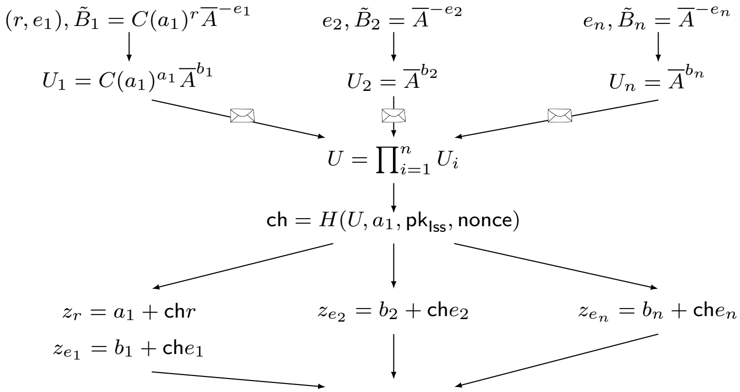
 & U = \prod_{i=1}^{n} U_i \arrow[d] & \\
 & \mathsf{ch} = H(U, a_1, \mathsf{pk}_{\mathsf{Iss}}, \mathsf{nonce}) \arrow[dl] \arrow[d] \arrow[dr] & \\
z_r = a_1 + \mathsf{ch}r & z_{e_2} = b_2 + \mathsf{ch}e_2 & z_{e_n} = b_n + \mathsf{ch}e_n \\
z_{e_1} = b_1 + \mathsf{ch}e_1 & &
\end{tikzcd}$$

How does the combined NIZKP works?

$a_1, \{b_i\}_{i \in [n]} \xleftarrow{\$} \mathbb{Z}_p$

commit and release



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$z_r = a_1 + \mathsf{ch} r$

$z_{e_1} = b_1 + \mathsf{ch} e_1$

$z_{e_2} = b_2 + \mathsf{ch} e_2$

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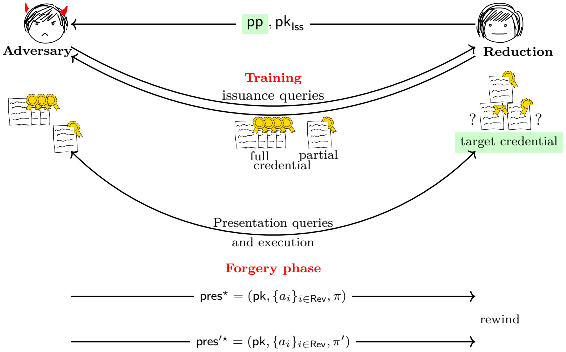
z_r = a_1 + \mathsf{ch} r & & z_{e_2} = b_2 + \mathsf{ch} e_2 \arrow[dd] & & z_{e_n} = b_n + \mathsf{ch} e_n \arrow[ddll] \\
z_{e_1} = b_1 + \mathsf{ch} e_1 \arrow[drr] & & & & \\
& & z_r, z_e \leftarrow \sum_{i=1}^n z_{e_i} & &
\end{tikzcd}$$

Unforgeability of BBS MHAC

How to prove Unforgeability of Presentations?

Via a reduction to DL assumption (and the unforgeability of BBS)



pp, $\mathsf{pk}_{\mathsf{Iss}}$

Adversary

Reduction

Training

issuance queries

full credential

partial credential

? ?

target credential

Presentation queries and execution

Forgery phase

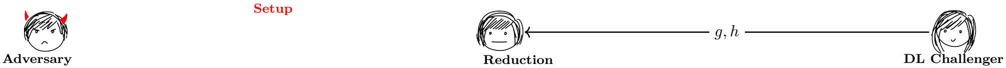
$\mathsf{pres}^\star = (\mathsf{pk}, \{a_i\}_{i\in\mathsf{Rev}}, \pi)$

rewind

$\mathsf{pres}'^\star = (\mathsf{pk}, \{a_i\}_{i\in\mathsf{Rev}}, \pi')$

How to prove Unforgeability of Presentations?

Via a reduction to DL assumption (and the unforgeability of BBS)



Setup

Adversary

Reduction

g, h

DL Challenger

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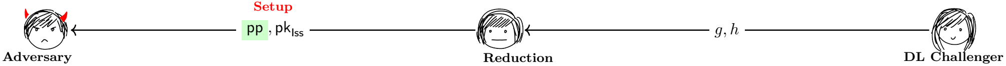

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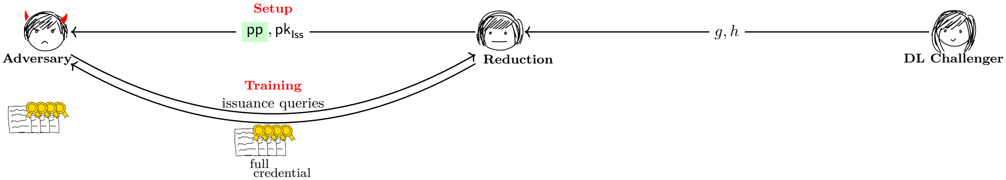

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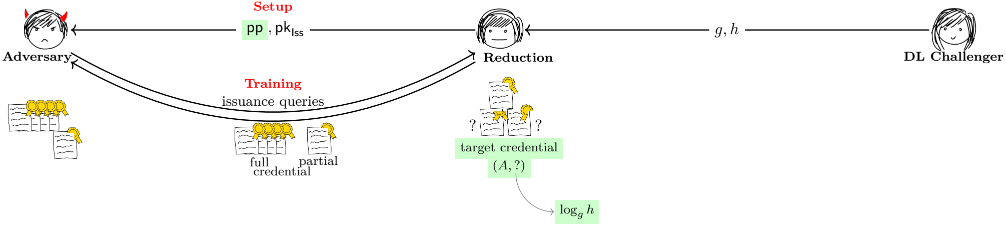

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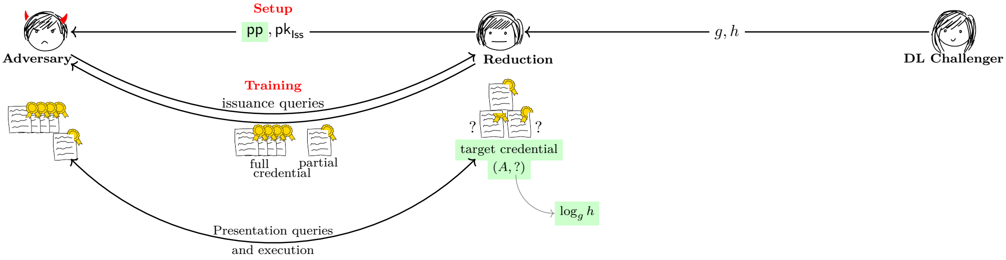

Setup

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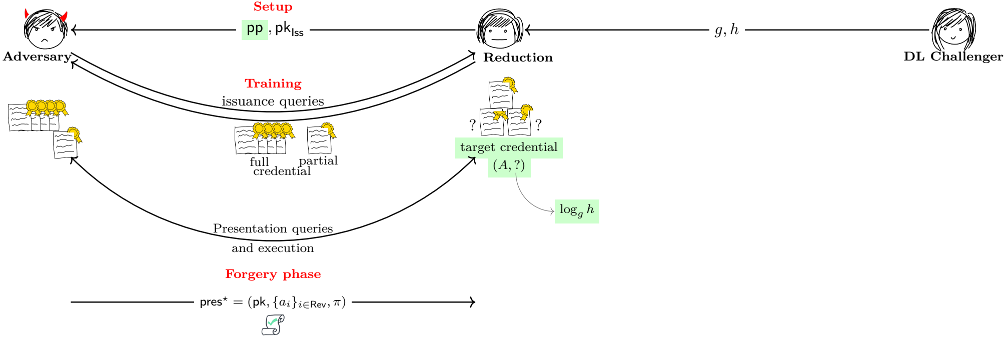

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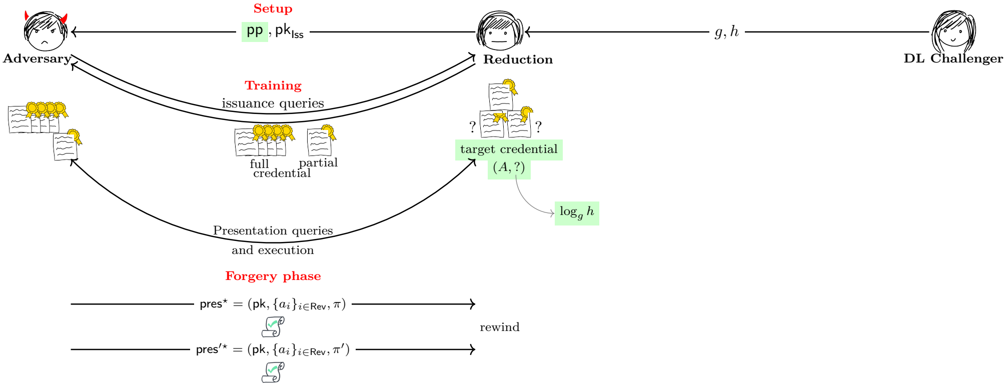

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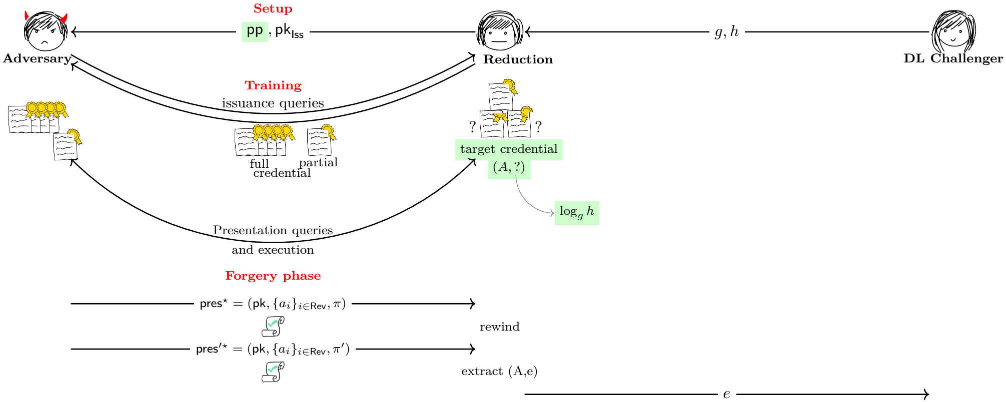

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extract (A,e)

e

A technical challenge

target credential issuance

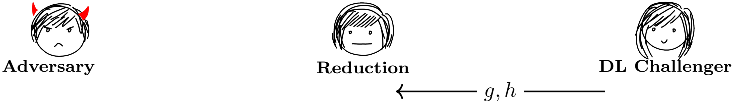
Adversary

Reduction

DL Challenger

A technical challenge

target credential issuance



Adversary

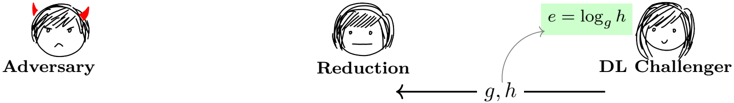
Reduction

DL Challenger

$\xleftarrow{\quad g, h \quad}$

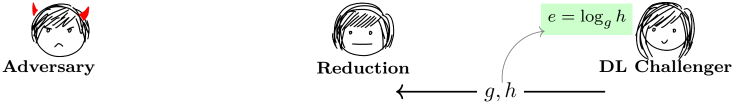
A technical challenge

target credential issuance



A technical challenge

target credential issuance


Adversary

Reduction

DL Challenger

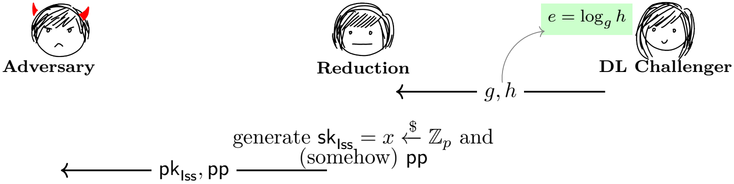
$e = \log_g h$

$\xleftarrow{\quad g, h \quad}$

generate $\mathsf{sk}_{\mathsf{Iss}} = x \overset{\$}{\leftarrow} \mathbb{Z}_p$ and
(somehow) pp

A technical challenge

target credential issuance



Adversary

Reduction

$e = \log_g h$

DL Challenger

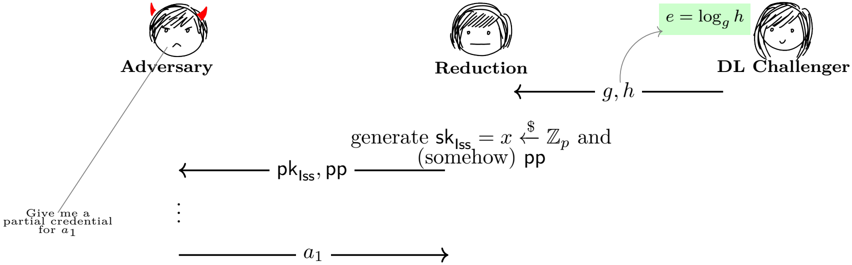
$\longleftarrow g, h \longrule$

generate $\mathsf{sk}_{\mathsf{Iss}} = x \xleftarrow{\$} \mathbb{Z}_p$ and (somehow) pp

$\longleftarrow \mathsf{pk}_{\mathsf{Iss}}, \mathsf{pp}$

A technical challenge

target credential issuance



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$e = \log_g h$

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generate $\mathsf{sk}_{\mathsf{Iss}} = x \xleftarrow{\$} \mathbb{Z}_p$ and
(somehow) pp

$\xleftarrow{\quad \mathsf{pk}_{\mathsf{Iss}}, \mathsf{pp} \quad}$

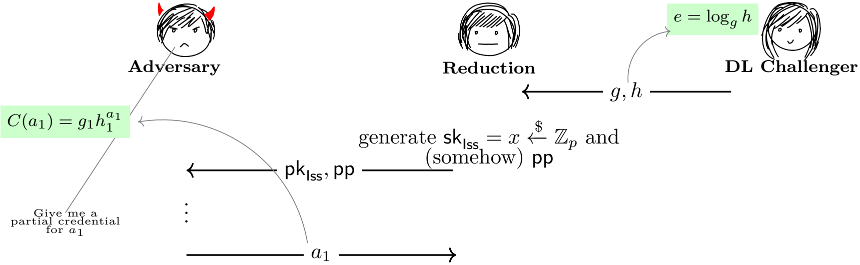
$\vdots$

Give me a
partial credential
for a_1

$\xrightarrow{\quad a_1 \quad}$

A technical challenge

target credential issuance



Adversary

Reduction

DL Challenger

$e = \log_g h$

$\xleftarrow{\quad g, h \quad}$

generate $\mathsf{sk}_{\mathsf{Iss}} = x \xleftarrow{\$} \mathbb{Z}_p$ and (somehow) pp

$\xleftarrow{\quad \mathsf{pk}_{\mathsf{Iss}}, \mathsf{pp} \quad}$

$\vdots$

$\xrightarrow{\quad a_1 \quad}$

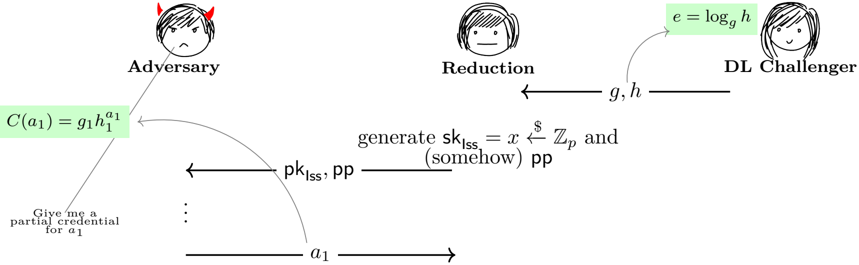
$C(a_1) = g_1 h_1^{a_1}$

Give me a partial credential for a_1

$\mathsf{cred} = ((A, e), a_1)$ is univocally determined! $(A = C(a_1)^{\frac{1}{x+e}})$

A technical challenge

target credential issuance



Adversary

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Give me a partial credential for a_1

⋮

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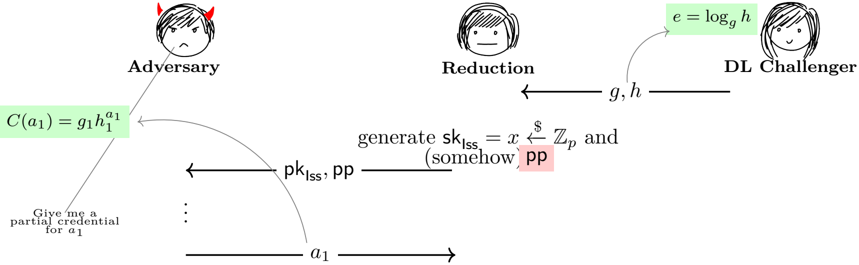
The reduction must produce A, A^e

recall: $\mathsf{cred}_i \leftarrow (A, e_i, \{A^{e_i}\}_{i \in [n]}, a_1)$

A technical challenge

target credential issuance



Adversary

Reduction

$e = \log_g h$ **DL Challenger**

$\xleftarrow{\quad g, h \quad}$

generate $\mathsf{sk}_{\mathsf{Iss}} = x \xleftarrow{\$} \mathbb{Z}_p$ and (somehow) pp

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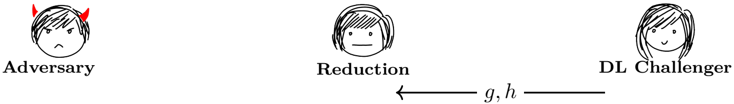
Our Solution

Adversary

Reduction

DL Challenger

Our Solution



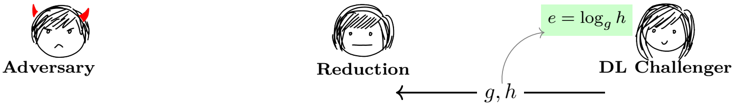
Adversary

Reduction

DL Challenger

$\xleftarrow{\quad g, h \quad}$

Our Solution



Our Solution

Adversary

Reduction

$e = \log_g h$

DL Challenger

$$\xleftarrow{\quad g, h \quad}$$

$$x \xleftarrow{\$} \mathbb{Z}_p, \quad \mathsf{pk}_{\mathsf{lss}} \leftarrow g_2^x$$

Our Solution

Adversary

Reduction

$e = \log_g h$

DL Challenger

$$\xleftarrow{\quad g, h \quad}$$

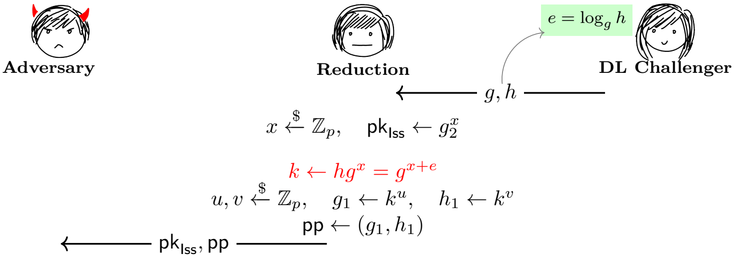
$$x \overset{\$}{\leftarrow} \mathbb{Z}_p, \quad \mathsf{pk}_{\mathsf{lss}} \leftarrow g_2^x$$

$$k \leftarrow hg^x = g^{x+e}$$

$$u, v \overset{\$}{\leftarrow} \mathbb{Z}_p, \quad g_1 \leftarrow k^u, \quad h_1 \leftarrow k^v$$

$$\mathsf{pp} \leftarrow (g_1, h_1)$$

Our Solution



Adversary

Reduction

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g, h

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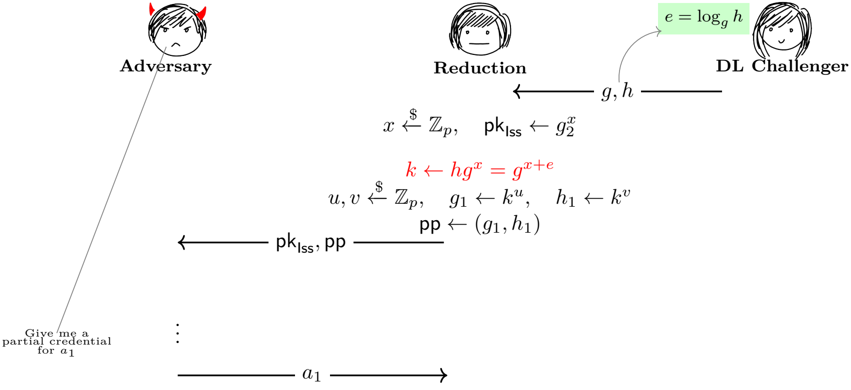
$\mathsf{pk}_{\mathsf{Iss}}, \mathsf{pp}$

Our Solution



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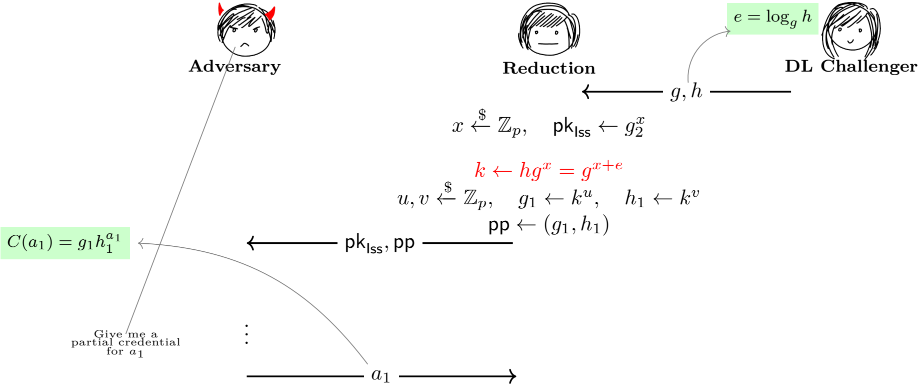
Give me a partial credential for a_1

$\vdots$

$\xrightarrow{\quad a_1 \quad}$

Our Solution



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⋮

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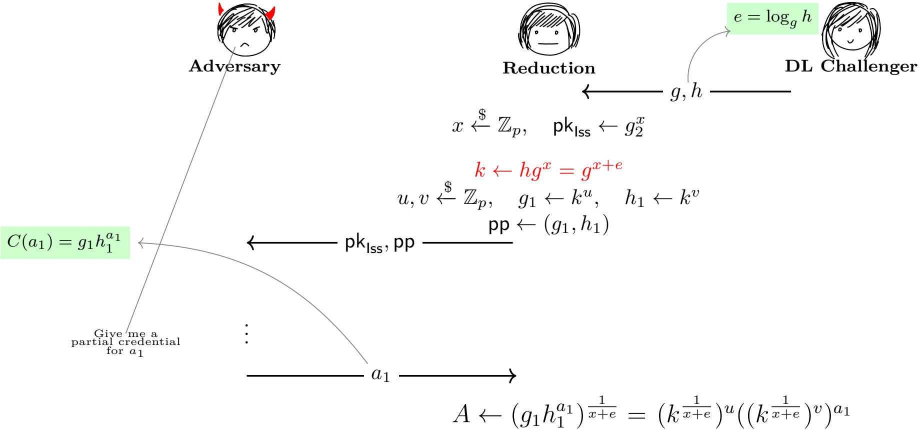
Give me a partial credential for a_1

$\vdots$

$$\xrightarrow{\quad a_1 \quad}$$

$$A \leftarrow (g_1 h_1^{a_1})^{\frac{1}{x+e}} = (k^u (k^v)^{a_1})^{\frac{1}{x+e}}$$

Our Solution



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$\vdots$

Give me a partial credential for a_1

$\xrightarrow{\quad a_1 \quad}$

$$A \leftarrow (g_1 h_1^{a_1})^{\frac{1}{x+e}} = (k^{\frac{1}{x+e}})^u((k^{\frac{1}{x+e}})^v)^{a_1}$$

Our Solution

Adversary

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$k^{\frac{1}{x+e}} = g$

Give me a partial credential for a_1

⋮

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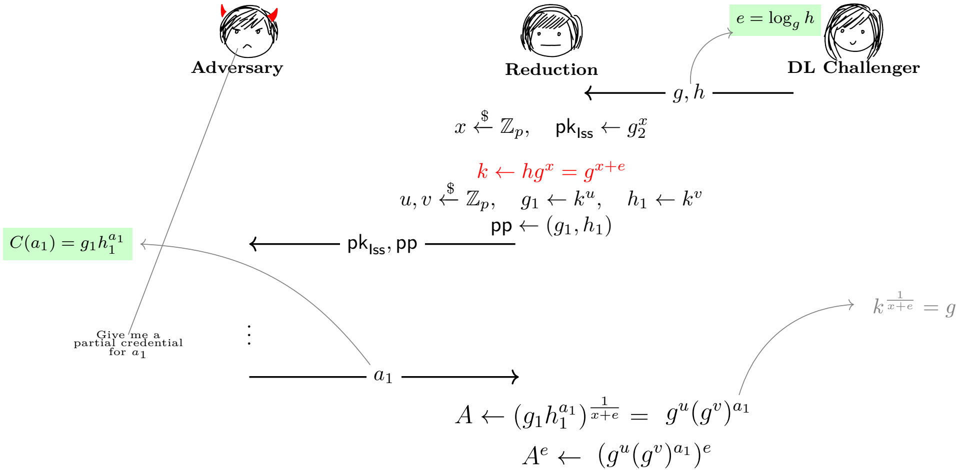
Our Solution



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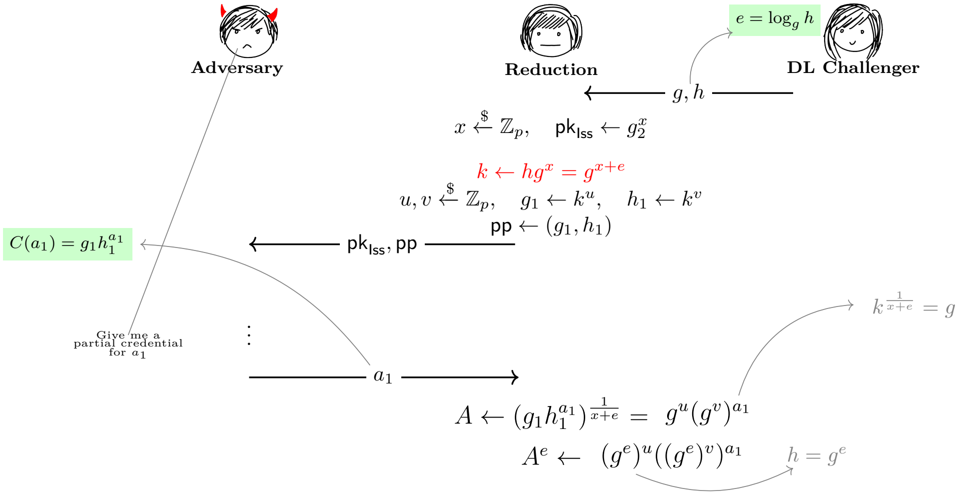
$$A^e \leftarrow (g^u (g^v)^{a_1})^e$$

$k^{\frac{1}{x+e}} = g$

Our Solution



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$\xrightarrow{\quad a_1 \quad}$

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$h = g^e$

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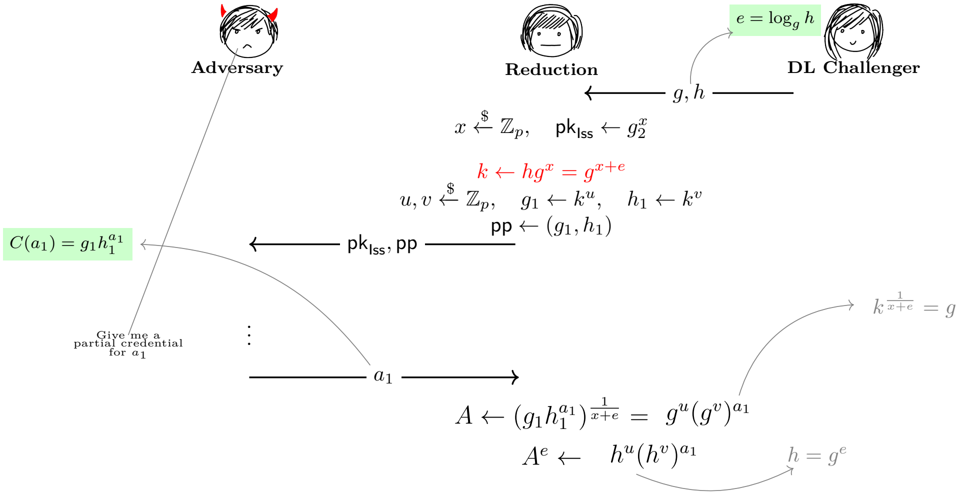

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$A^e \leftarrow h^u (h^v)^{a_1}$

$h = g^e$

The adversary shares are $(A, \{e_i\}_{i \in \mathsf{cor}}, \{A^{e_i}\}_{i \in [n]}, a_1)$