# Unmasking TRaccoon: A Lattice-Based Threshold Signature with An Efficient Identifiable Abort Protocol

Rafael del Pino PQShield

Shuichi Katsumata PQShield & AIST Guilhem Niot PQShield & Univ Rennes, CNRS, IRISA

Michael Reichle ETH Zurich

Kaoru Takemure PQShield & AIST

### Our Identifiable Abort Protocol

#### Main Contribution: TRaccoon with Identifiable Abort Protocol

- Our interactive IA protocol is a simple add-on to TRaccoon
- Communication cost in IA protocol is  $60 + 6.4 \cdot T$  KB per a signer

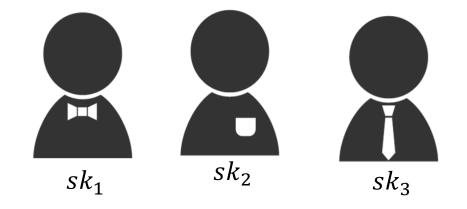
#### **Side Contributions:**

- The first game-based definition of TS with an interactive IA protocol
- The first formal security analysis of a variant of LaBRADOR with ZK

# Background

Verification key vk  $\updownarrow$ Signing key sk

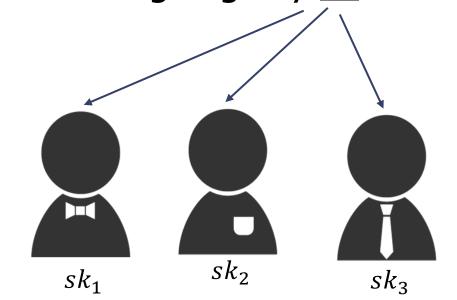
**Key Generation** 



\*2-out-of-3

Verification key vk

\$\Bar{1}\$
Signing key \$sk\$

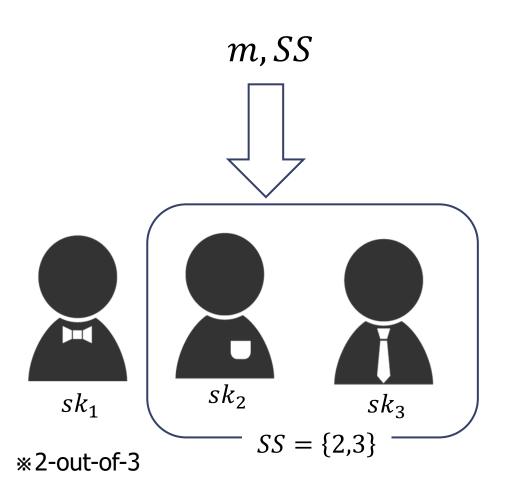


\*2-out-of-3

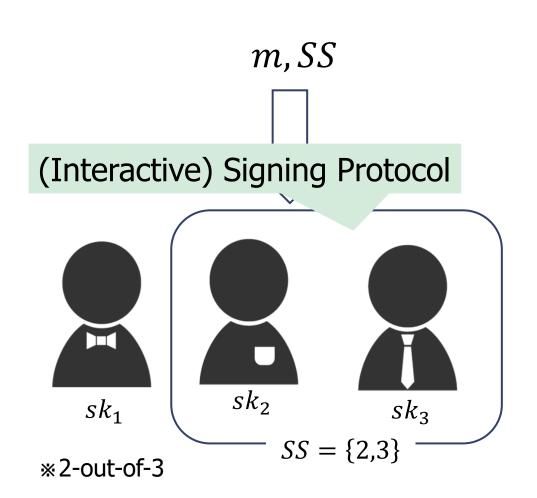
**Key Generation** 

- T or more key shares reconstruct sk
- No signer knows sk
- Less than T key shares leak no information about sk

\*We assume that a trusted party executes distributed key generation as well as [BCK+22,dPKM+24] etc.



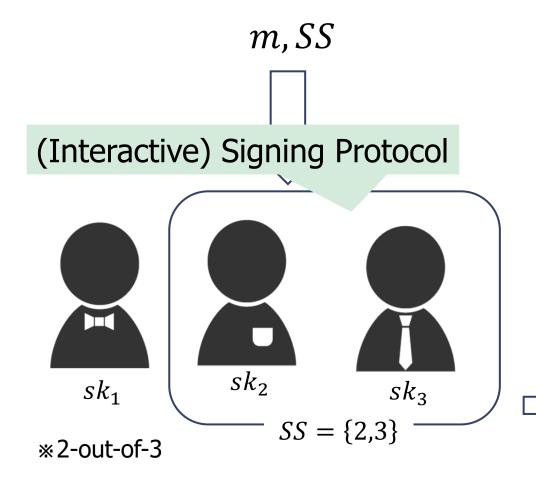
**Signing Protocol** 



**Signing Protocol** 

#### **General Procedure:**

- 1. One decides message m and signer set SS
- 2. Users in SS execute signing protocol



**Signing Protocol** 

#### **General Procedure:**

- 1. One decides message m and signer set SS
- 2. Users in SS execute signing protocol

Signature  $\sigma$ 

### PQ Threshold Signature Schemes

```
Early Schemes: [BKP13], [BGG+18], [ASY22], [GKS23] ⇒The use of heavy tools, e.g., FHE and HTDC
```

Recent Schemes: [dPKM+24], [EKT24], [KRT24], [CATZ24], [BKL+25], etc ⇒No use of such heavy tools

### PQ Threshold Signature Schemes

```
Early Schemes: [BKP13], [BGG+18], [ASY22], [GKS23] ⇒The use of heavy tools, e.g., FHE and HTDC
```

Recent Schemes: [dPKM+24], [EKT24], [KRT24], [CATZ24], [BKL+25], etc ⇒No use of such heavy tools

#### TRaccoon [dPKM+24]:

- Three-round signing protocol
- Efficient sig size compared with early schemes

### PQ Threshold Signature Schemes

Early Schemes: [BKP13], [BGG+18], [ASY22], [GKS23]

⇒The use of heavy tools, e.g., FHE and HTDC

Recent Schemes: [dPKM+24], [EKT24], [KRT24], [CATZ24], [BKL+25], etc

⇒No use of such heavy tools

One drawback: No Availability

Malicious signer can arbitrarily cause the signing protocol to fail

#### TRaccoon [dPKM+24]:

- Three-round signing protocol
- Efficient sig size compared with early schemes

### Availability for TS: Identifiable Abort

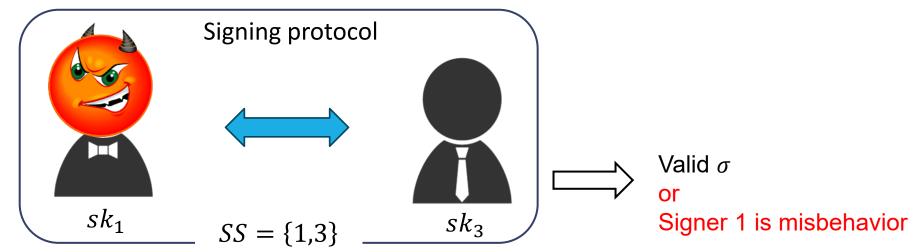


Identifiable Abort:

When the signing protocol fails, honest signers identify misbehaving signers.

Communication Channel:

Synchronous authenticated Broadcast



# Availability for TS: Identifiable Abort



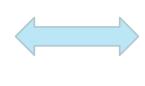
Identifiable Abort:

When the signing protocol fails, honest signers identify misbehaving signers.

Sigi

Can we construct an efficient IA protocol for TRaccoon?





$$SS = \{1,3\}$$



Signer 1 is misbehavior

# TRaccoon

 $vk: A \in \mathcal{R}_q^{k \times \ell}, t = A \cdot s + e$  where short vectors  $(s, e) \in \mathcal{R}_q^{\ell} \times \mathcal{R}_q^k$   $sk_i: s_i$  is a secret share of s,  $\left(seed_{i,j}, seed_{j,i}\right)_{j \in [N]}$  are pair-wise seeds.

Lattice variant of Sparkle[CKM23]

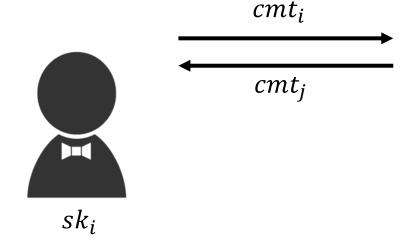


 $vk: A \in \mathcal{R}_q^{k \times \ell}, t = A \cdot s + e$  where short vectors  $(s, e) \in \mathcal{R}_q^{\ell} \times \mathcal{R}_q^k$   $sk_i: s_i$  is a secret share of s,  $\left(seed_{i,j}, seed_{j,i}\right)_{j \in [N]}$  are pair-wise seeds.

Lattice variant of Sparkle[CKM23]

Round 1:

- 1. Sample short vectors  $(r_i, e'_i) \in \mathcal{R}_q^{\ell} \times \mathcal{R}_q^k$
- 2.  $\mathbf{w}_i \leftarrow \mathbf{A} \cdot \mathbf{r}_i + \mathbf{e}'_i$
- 3. Broadcast  $cmt_i \leftarrow H(\mathbf{w}_i)$



 $vk: A \in \mathcal{R}_q^{k \times \ell}, t = A \cdot s + e$  where short vectors  $(s, e) \in \mathcal{R}_q^{\ell} \times \mathcal{R}_q^k$   $sk_i: s_i$  is a secret share of s,  $\left(seed_{i,j}, seed_{j,i}\right)_{j \in [N]}$  are pair-wise seeds.

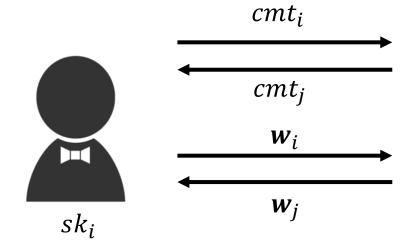
Lattice variant of Sparkle[CKM23]

Round 1:

- 1. Sample short vectors  $(\boldsymbol{r}_i, \boldsymbol{e}_i') \in \mathcal{R}_q^{\ell} \times \mathcal{R}_q^k$
- 2.  $w_i \leftarrow A \cdot r_i + e'_i$
- 3. Broadcast  $cmt_i \leftarrow H(\mathbf{w}_i)$

Round 2:

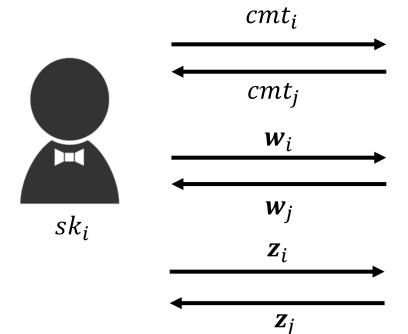
1. Broadcast  $w_i$ 



 $vk: A \in \mathcal{R}_q^{k \times \ell}, t = A \cdot s + e$  where short vectors  $(s, e) \in \mathcal{R}_q^{\ell} \times \mathcal{R}_q^k$   $sk_i: s_i$  is a secret share of s,  $\left(seed_{i,j}, seed_{j,i}\right)_{j \in [N]}$  are pair-wise seeds.

Lattice variant of Sparkle[CKM23]

- Round 1:
- 1. Sample short vectors  $(\mathbf{r}_i, \mathbf{e}'_i) \in \mathcal{R}_q^{\ell} \times \mathcal{R}_q^{k}$
- 2.  $w_i \leftarrow A \cdot r_i + e'_i$
- 3. Broadcast  $cmt_i \leftarrow H(\mathbf{w}_i)$
- Round 2:
- 1. Broadcast  $w_i$
- Round 3:
- 1. Check  $cmt_i = H(\mathbf{w}_i)$
- 2.  $w \leftarrow \sum_{j} w_{j}$
- 3.  $c \leftarrow H_c(vk, m, \mathbf{w})$
- 4. Broadcast  $z_i \leftarrow c \cdot L_{SS,i} \cdot s_i + r_i + \Delta_i$



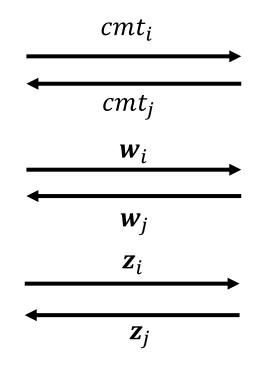
 $vk: A \in \mathcal{R}_q^{k \times \ell}, t = A \cdot s + e$  where short vectors  $(s, e) \in \mathcal{R}_q^{\ell} \times \mathcal{R}_q^k$   $sk_i: s_i$  is a secret share of s,  $\left(seed_{i,j}, seed_{j,i}\right)_{j \in [N]}$  are pair-wise seeds.

Lattice variant of Sparkle[CKM23]

- Round 1:
- 1. Sample short vectors  $(\mathbf{r}_i, \mathbf{e}'_i) \in \mathcal{R}_q^{\ell} \times \mathcal{R}_q^{k}$
- 2.  $w_i \leftarrow A \cdot r_i + e'_i$
- 3. Broadcast  $cmt_i \leftarrow H(\mathbf{w}_i)$
- Round 2:
- 1. Broadcast  $w_i$
- Round 3:
- 1. Check  $cmt_i = H(\mathbf{w}_i)$
- 2.  $\mathbf{w} \leftarrow \sum_{j} \mathbf{w}_{j}$
- 3.  $c \leftarrow H_c(vk, m, \mathbf{w})$
- 4. Broadcast  $z_i \leftarrow c \cdot L_{SS,i} \cdot s_i + r_i + \Delta_i$

Resulting signature:  $(c, \mathbf{z}, \mathbf{h})$  where  $\mathbf{z} = \sum_i \mathbf{z}_i$ ,  $\mathbf{h} = \mathbf{w} - \mathbf{A} \cdot \mathbf{z} + c \cdot \mathbf{t}$ Verification:  $c = H_c(vk, m, \mathbf{A} \cdot \mathbf{z} - c \cdot \mathbf{t} + \mathbf{h})$ 



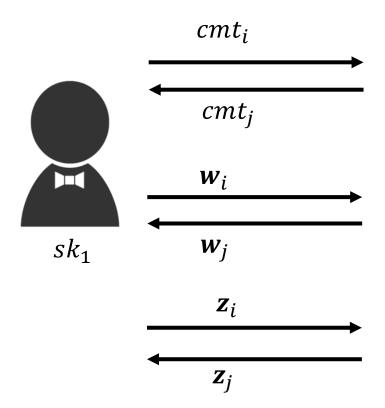


```
vk:
        Important difference from Sparkle:
 Rour Masking Term: \Delta_i = \sum_j ({m m}_{i,j} - {m m}_{j,i}) such that \sum_i \Delta_i = 0
        where \mathbf{m}_{i,i} = H_{msk}(seed_{i,i}, ctnt_z), ctnt_z = SS||m||(cmt_i, w_i)_{i \in SS}.
        This is a crucial component to prevent lattice-specific attacks.
 Rou
                                                                                              Sk_1
                           c \leftarrow H_c(vk, m, \mathbf{w})
                     4. Broadcast z_i \leftarrow c \cdot L_{SS,i} \cdot s_i + r_i + \Delta_i
                                                                                                                         \boldsymbol{z}_i
Resulting signature: (c, \mathbf{z}, \mathbf{h}) where \mathbf{z} = \sum_{i} \mathbf{z}_{i}, \mathbf{h} = \mathbf{w} - \mathbf{A} \cdot \mathbf{z} + c \cdot \mathbf{t}
Verification: c = H_c(vk, m, \mathbf{A} \cdot \mathbf{z} - c \cdot \mathbf{t} + \mathbf{h})
```

# Our Approach

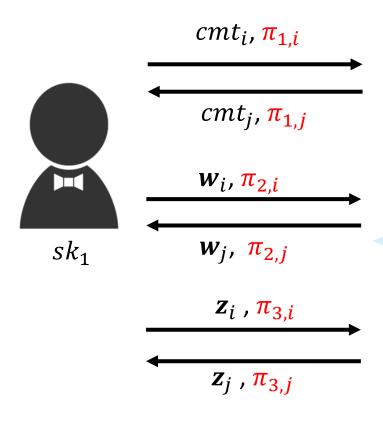
# Straightforward Approach

All signers prove that they honestly executed the signing protocol for each round.

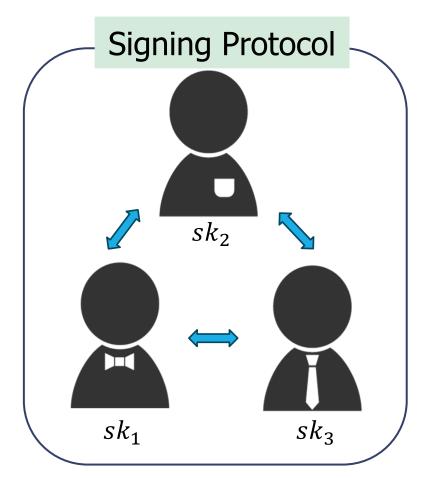


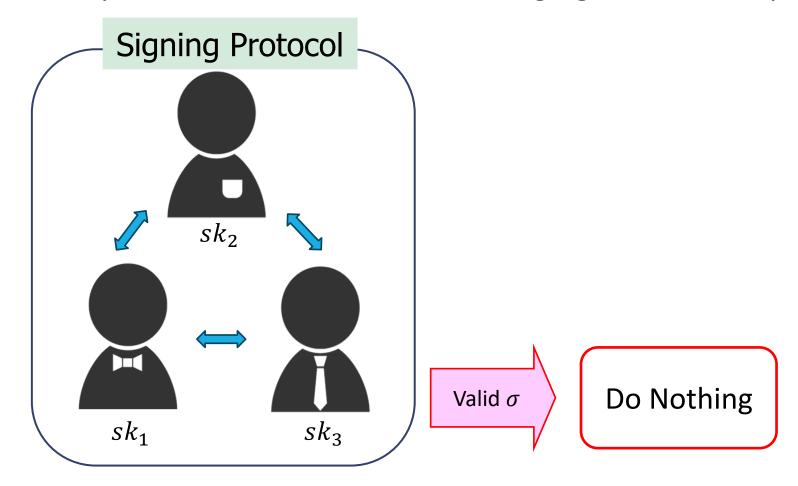
# Straightforward Approach

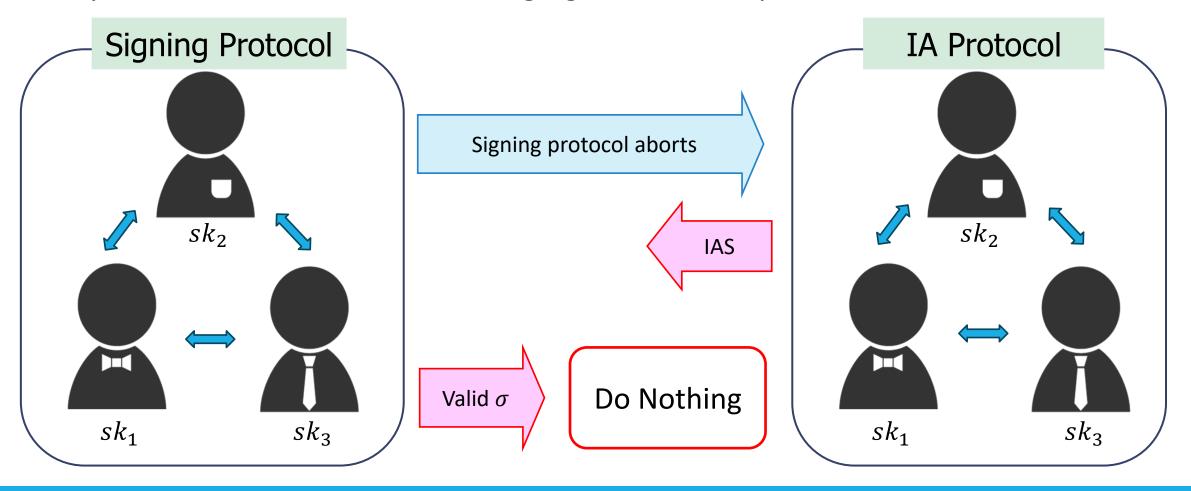
All signers prove that they honestly executed the signing protocol for each round.

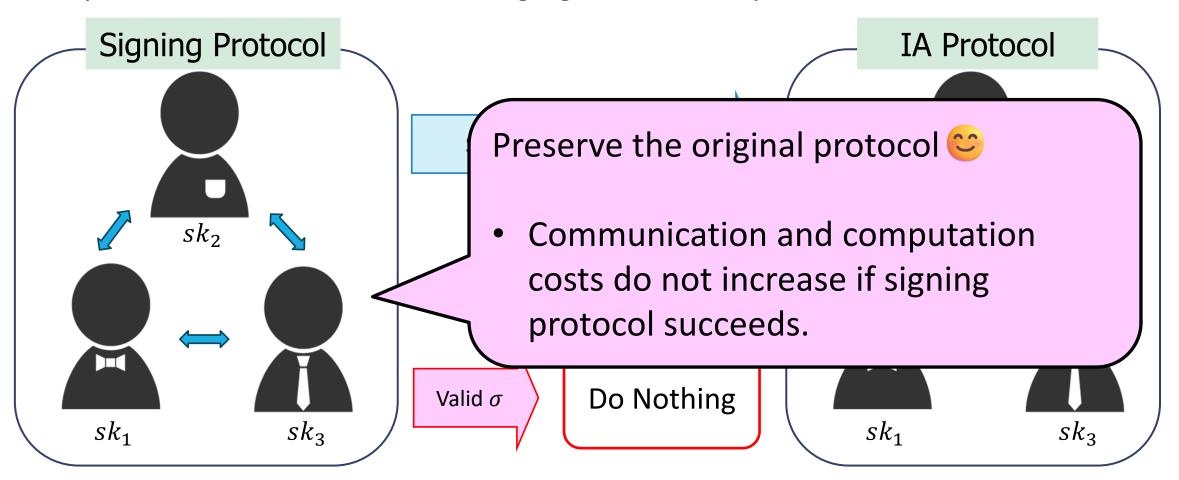


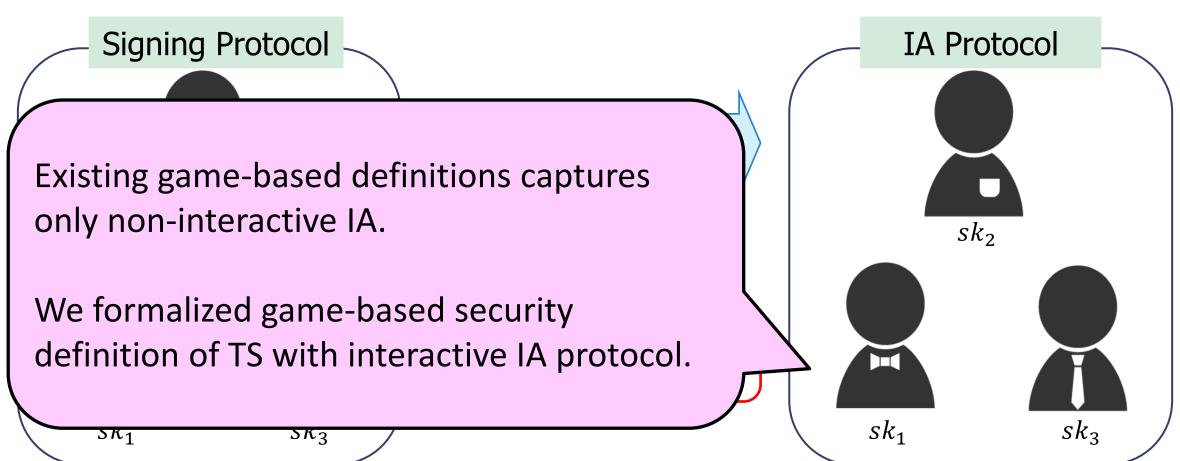
Increase communication cost during signing protocol











### Relations to be Proven via NIZK

Our IA protocol follows the approach using NIZK.

#### Relations to be proven:

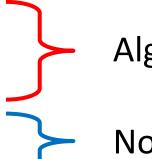
- (1)  $r_i$  is short
- (2)  $\mathbf{z}_i = c \cdot L_{SS,i} \cdot \mathbf{s}_i + \mathbf{r}_i + \Delta_i$
- (3)  $\Delta_i = \sum_j (\boldsymbol{m}_{i,j} \boldsymbol{m}_{j,i})$
- (4)  $\mathbf{m}_{i,j} = H_{msk}(seed_{i,j}, ctnt_z)$

### Relations to be Proven via NIZK

Our IA protocol follows the approach using NIZK.

#### Relations to be proven:

- (1)  $r_i$  is short
- (2)  $\mathbf{z}_i = c \cdot L_{SS,i} \cdot \mathbf{s}_i + \mathbf{r}_i + \Delta_i$
- (3)  $\Delta_i = \sum_j (\boldsymbol{m}_{i,j} \boldsymbol{m}_{j,i})$
- (4)  $m_{i,j} = H_{msk}(seed_{i,j}, ctnt_z)$



Algebraic

Non-Algebraic

### Relations to be Proven via NIZK

Our IA protocol follows the approach using NIZK.

#### Relations to be proven:

- (1)  $r_i$  is short
- (2)  $\mathbf{z}_i = c \cdot L_{SS,i} \cdot \mathbf{s}_i + \mathbf{r}_i + \Delta_i$
- (3)  $\Delta_i = \sum_j (\boldsymbol{m}_{i,j} \boldsymbol{m}_{j,i})$
- (4)  $m_{i,j} = H_{msk}(seed_{i,j}, ctnt_z)$



Non-Algebraic

Proving "mixed" relations is impractical >> How can we avoid this?

Why is (4)  $m_{i,j} = H_{msk}(seed_{i,j}, ctnt_z)$  required?

```
Why is (4) m_{i,j} = H_{msk}(seed_{i,j}, ctnt_z) required?

\Rightarrow Ensure \sum_i \Delta_i = 0
```

Why is (4)  $m_{i,j} = H_{msk}(seed_{i,j}, ctnt_z)$  required?

- $\Rightarrow$  Ensure  $\sum_i \Delta_i = 0$
- ⇒ (★) Each pair of signers uses the same masks

Why is (4)  $m_{i,j} = H_{msk}(seed_{i,j}, ctnt_z)$  required?

- $\Rightarrow$  Ensure  $\sum_i \Delta_i = 0$
- ⇒ (★) Each pair of signers uses the same masks

#### Our observation:

As long as each pair uses the same  $m_{i,j}$  even though it is not honestly generated,  $\sum_i \Delta_i = 0$  holds.

Why is (4)  $m_{i,j} = H_{msk}(seed_{i,j}, ctnt_z)$  required?

- $\Rightarrow$  Ensure  $\sum_i \Delta_i = 0$
- ⇒ (★) Each pair of signers uses the same masks

#### Our observation:

As long as each pair uses the same  $m_{i,j}$  even though it is not honestly generated,  $\sum_i \Delta_i = 0$  holds.

Idea: Ensure (\*) outside of NIZK

### How to Check $(\bigstar)$

Com: Lattice-based commitment scheme

1. For 
$$j \in SS \setminus \{i\}$$
, compute  $D_{i,j}^{(i)} \leftarrow Com(\boldsymbol{m}_{i,j}; \delta_{i,j})$  and  $D_{j,i}^{(i)} \leftarrow Com(\boldsymbol{m}_{j,i}; \delta_{j,i})$  where  $\delta_{i,j} = H_{rnd}(seed_{i,j}, ctnt_z)$ ,  $\delta_{j,i} = H_{rnd}(seed_{j,i}, ctnt_z)$ . Deterministic Broadcast  $\left(D_{i,j}^{(i)}, D_{j,i}^{(i)}\right)_{i \in SS \setminus \{i\}}$ 

### How to Check $(\bigstar)$

#### Com: Lattice-based commitment scheme

- 1. For  $j \in SS \setminus \{i\}$ , compute  $D_{i,j}^{(i)} \leftarrow Com(\boldsymbol{m}_{i,j}; \delta_{i,j})$  and  $D_{j,i}^{(i)} \leftarrow Com(\boldsymbol{m}_{j,i}; \delta_{j,i})$  where  $\delta_{i,j} = H_{rnd}(seed_{i,j}, ctnt_z)$ ,  $\delta_{j,i} = H_{rnd}(seed_{j,i}, ctnt_z)$ . Deterministic Broadcast  $\left(D_{i,j}^{(i)}, D_{j,i}^{(i)}\right)_{i \in SS \setminus \{i\}}$
- 2. Broadcast  $\left(seed_{i,j}^{(i)}, seed_{j,i}^{(i)}\right)$  for j s.t.  $D_{i,j}^{(i)} \neq D_{i,j}^{(j)}$  or  $D_{j,i}^{(i)} \neq D_{j,i}^{(j)}$  Inconsistent Mask

20

### How to Check (★)

#### Com: Lattice-based commitment scheme

- 1. For  $j \in SS \setminus \{i\}$ , compute  $D_{i,j}^{(i)} \leftarrow Com(\boldsymbol{m}_{i,j}; \delta_{i,j})$  and  $D_{j,i}^{(i)} \leftarrow Com(\boldsymbol{m}_{j,i}; \delta_{j,i})$  where  $\delta_{i,j} = H_{rnd}(seed_{i,j}, ctnt_z)$ ,  $\delta_{j,i} = H_{rnd}(seed_{j,i}, ctnt_z)$ . Deterministic Broadcast  $\left(D_{i,j}^{(i)}, D_{j,i}^{(i)}\right)_{i \in SS \setminus \{i\}}$
- 2. Broadcast  $\left(seed_{i,j}^{(i)}, seed_{j,i}^{(i)}\right)$  for j s.t.  $D_{i,j}^{(i)} \neq D_{i,j}^{(j)}$  or  $D_{j,i}^{(i)} \neq D_{j,i}^{(j)}$  Inconsistent Mask

$$\underbrace{H_{msk}\left(seed_{k,\ell}^{(k)},ctnt_{z}\right)} \underbrace{H_{rnd}\left(seed_{k,\ell}^{(k)},ctnt_{z}\right)$$

3. Check  $D_{k,\ell}^{(k)} = Com\left(\boldsymbol{m}_{k,\ell}^{(k)}; \delta_{k,\ell}^{(k)}\right)$  and  $C_{i,j} = H_{seed}(seed_{k,\ell}^{(k)})$  If not, k is misbehavior.

# How to Check $(\bigstar)$

#### Com: Lattice-based commitment scheme

- 1. For  $j \in SS \setminus \{i\}$ , compute  $D_{i,i}^{(i)} \leftarrow Com(\boldsymbol{m}_{i,j}; \delta_{i,j})$  commitment scheme, where  $\delta_{i,j} = H_{rnd}(seed_{i,j}, ctnt_z)$ ,  $\delta_{j,i} = H_{rnd}(seed_{i,j}, ctnt_z)$ Broadcast  $\left(D_{i,j}^{(i)}, D_{j,i}^{(i)}\right)_{i \in SS \setminus \{i\}}$
- Thanks to binding of uses the same masks!
- 2. Broadcast  $\left(seed_{i,i}^{(i)}, seed_{j,i}^{(i)}\right)$  for j s.t.  $D_{i,i}^{(i)} \neq D_{i,i}^{(j)}$  or  $D_{i,i}^{(i)} \neq D_{i,i}^{(j)}$

$$\left(H_{msk}\left(seed_{k,\ell}^{(k)},ctnt_{z}\right)\right) \left(H_{rnd}\left(seed_{k,\ell}^{(k)},ctnt_{z}\right)\right)$$

$$H_{rnd}\left(seed_{k,\ell}^{(k)},ctnt_{z}\right)$$

3. Check  $D_{k,\ell}^{(k)} = Com\left(\boldsymbol{m}_{k,\ell}^{(k)}; \delta_{k,\ell}^{(k)}\right)$  and  $C_{i,j} = H_{seed}(seed_{k,\ell}^{(k)})$ If not, k is misbehavior.

Generated in KeyGen

**Inconsistent Mask** 

# How to Check $(\bigstar)$

Revealing seeds does not harm the security because seeds for honest pairs are not revealed.

lheme

- 
$$Com(\mathbf{m}_{i,j}; \delta_{i,j})$$
  
 $(\mathbf{z}_z), \delta_{j,i} = H_{rnd}(\mathbf{z}_j)$ 

Thanks to binding of  $- Com(\mathbf{m}_{i,j}; \delta_{i,j})$  commitment scheme,  $t_z$ ),  $\delta_{i,i} = H_{rnd}(s)$  we can ensure that each pair uses the same masks!

2. Broadcast  $\left(seed_{i,i}^{(i)}, seed_{j,i}^{(i)}\right)$  for j s.t.  $D_{i,j}^{(i)} \neq D_{i,j}^{(j)}$  or  $D_{j,i}^{(i)} \neq D_{j,i}^{(j)}$ 

$$\left(H_{msk}\left(seed_{k,\ell}^{(k)},ctnt_{z}\right)\right) \left(H_{rnd}\left(seed_{k,\ell}^{(k)},ctnt_{z}\right)\right)$$

$$H_{rnd}\left(seed_{k,\ell}^{(k)},ctnt_{z}\right)$$

3. Check  $D_{k,\ell}^{(k)} = Com\left(\boldsymbol{m}_{k,\ell}^{(k)}; \delta_{k,\ell}^{(k)}\right)$  and  $C_{i,j} = H_{seed}(seed_{k,\ell}^{(k)})$ If not, k is misbehavior.

Generated in KeyGen

**Inconsistent Mask** 

### Eventual Relations to be Proven via NIZK

Our IA protocol follows the approach using NIZK.

#### Relations to be proven:

- (1)  $r_i$  is short
- (2)  $\mathbf{z}_i = c \cdot L_{SS,i} \cdot \mathbf{s}_i + \mathbf{r}_i + \Delta_i$

(3) 
$$\Delta_i = \sum_j (\boldsymbol{m}_{i,j} - \boldsymbol{m}_{j,i})$$

$$\frac{\textbf{(4)} \ \boldsymbol{m}_{i,j} = H_{msk}(seed_{i,j},ctnt_Z)}{}$$

(4)' 
$$D_{i,j}^{(i)} = Com(\boldsymbol{m}_{i,j}; \delta_{i,j})$$

### Eventual Relations to be Proven via NIZK

Our IA protocol follows the approach using NIZK.

#### Relations to be proven:

(1) 
$$\boldsymbol{r}_{i}$$
 is short  
(2)  $\boldsymbol{z}_{i} = c \cdot L_{SS,i} \cdot \boldsymbol{s}_{i} + \boldsymbol{r}_{i} + \Delta_{i}$   
(3)  $\Delta_{i} = \sum_{j} (\boldsymbol{m}_{i,j} - \boldsymbol{m}_{j,i})$   
(4)  $\boldsymbol{m}_{i,j} = H_{msk}(seed_{i,j}, ctnt_{z})$   
(4)  $D_{i,j}^{(i)} = Com(\boldsymbol{m}_{i,j}; \delta_{i,j})$ 

### Eventual Relations to be Proven via NIZK

Our IA protocol follows the approach using NIZK.

#### Relations to be proven:

(1) 
$$\boldsymbol{r}_{i}$$
 is short  
(2)  $\boldsymbol{z}_{i} = c \cdot L_{SS,i} \cdot \boldsymbol{s}_{i} + \boldsymbol{r}_{i} + \Delta_{i}$   
(3)  $\Delta_{i} = \sum_{j} (\boldsymbol{m}_{i,j} - \boldsymbol{m}_{j,i})$   
(4)  $\boldsymbol{m}_{i,j} = H_{msk}(seed_{i,j}, ctnt_{z})$   
(4)  $D_{i,j}^{(i)} = Com(\boldsymbol{m}_{i,j}; \delta_{i,j})$ 

<u>Lattice-based ZK-SNARK combining LNP[LNP22] + LaBRADOR[BS23]</u> which is sketched in prior works [BS23,ADDG24].

We formally analyze security of this approach in a modular manner.

### Performance

	$ \sigma $	Com Cost in Signing	<i>SS</i>	Availability
Traccoon[dPKM+24]	12.7	28.2	T	<del>-</del>
Traccoon-IA	12.7	28.2	T	IA 60+6.4· <i>T</i>

Same cost in signing protocol

Simple add-on

### Thank you for your attention!!

#### **Future Works:**

- ➤ Does our technique work on related lattice-based schemes using masking mechanism [EKT24], [KRT24], [BKL+25].
- Distributed Key Generation for our scheme

#### **Independent and Concurrent Work:**

[dPENP] Del Pino et al. "Simple and Efficient Lattice Threshold Signatures with Identifiable Aborts"

- IA for a variant of TRaccoon based on new short secret sharing technique
- Non-interactive IA
- Efficient when the number of signers or corruption threshold is small