A Framework for WE from Linearly Verifiable SNARKs and Applications

Sanjam Garg, Mohammad Hajiabadi, Dimitris Kolonelos, Abhiram Kothapalli, and
Guru-Vamsi Policharla

Witness Encryption [GGSW13, GKPVZ13]

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Today: Focus on efficient WE for special relations and applications.

Not going to build WE for NP

Many Examples!

Over the last 25 years:

**not an exhaustive list*

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Identity-Based Encryption
[BF01]

Hash-Proof Systems
[CS02], [BC16]

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Identity-Based Encryption

At first glance, constructions seem "arbitrary" and unrelated 😕

Can we <u>systematically</u> study special purpose WE?

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Taxonomy of WE

Taxonomy of WE schemes

$(x, w) \in R$

Gen 1

$\text{Enc}(x, m) \rightarrow \text{ct}$

$\text{Dec}(w, \text{ct}) \rightarrow m$

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$T_E = T_D = O(|R|)$

$|\mathsf{ct}| = O(|R|)$

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Gen 2

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Can build "Laconic" primitives!

- **Laconic OT**: Hash the receiver's choice bits
- **Laconic PSI**: Hash the receiver's database

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(SNARK the relation!)

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Today: A framework to build Gen 3 WE and applications

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- … and more!

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Improve best known result [GLWW24]

Registered ABE with a Linear CRS

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New feasibility results

Registered Threshold Encryption

What class of relations support efficient WE?

Relations with “Linear” verifiers

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Express the verification circuit for $R_L(x, w) = 1$ as a set of PPEs.

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Compiler [BC16, BL20, GKPW24]: Linear PPE → WE

The Missing Piece

Linear Relation

PPE Constraint System: $\prod e(x_i, x_j) \cdot \prod e(x_i, w_j) \cdot \prod e(w_i, x_j) = c_T$

The Missing Piece

Natural Relation	$\mathcal{R} = \{(x, w) \mid f(x, w) = 1\}$
Linear Relation	PPE Constraint System: $\prod e(x_i, x_j) \cdot \prod e(x_i, w_j) \cdot \prod e(w_i, x_j) = c_T$

The Missing Piece

Natural Relation

$$\mathscr{R} = \{(x, w) \mid f(x, w) = 1\}$$

???

How do we *linearize* natural relations?
How do we leverage SNARK machinery for *succinctness*?

Linear Relation

PPE Constraint System: $\prod e(x_i, x_j) \cdot \prod e(x_i, w_j) \cdot \prod e(w_i, x_j) = c_T$

Closing the gap:
Our Framework to build WE

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Goal: Simplify the process of translating:

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Natural Relations → Linear Relations

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1. Signatures
2. Algebraic PRF
3. Inner Product
4. Zero Check
5. Degree Check

... and more!

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- Gadgets can be composed to build WE for larger relations!
- Gadgets fully capture succinctness!
- Easy to use and extend with new gadgets!

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This Work

Inner Product × Signature Check × Degree Check × ...

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Inner Product × Signature Check × Degree Check × ...

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Remainder of the Talk

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Improve best known result [GLWW24]

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Distributed Broadcast Encryption

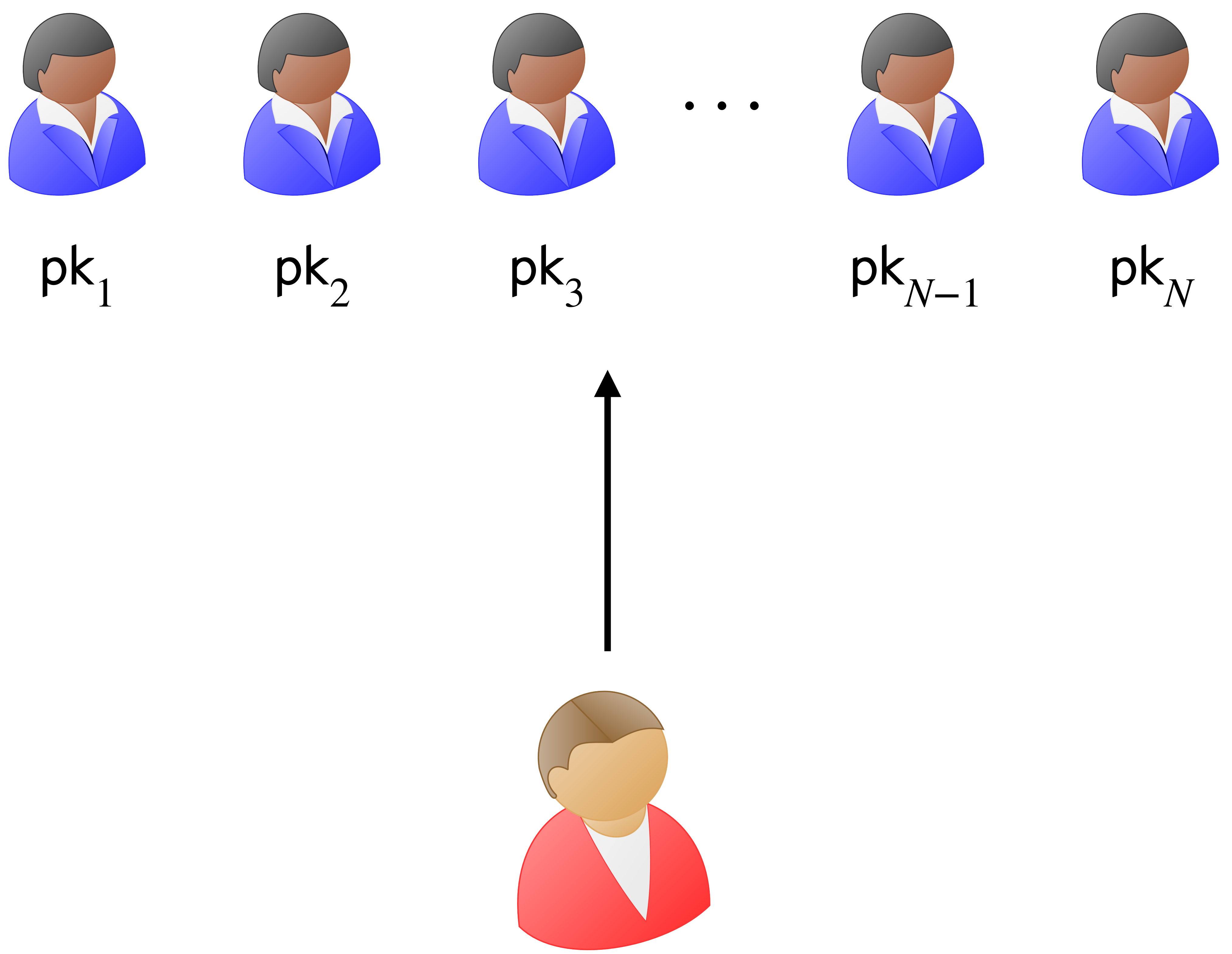
Distributed Broadcast Encryption [WQZD10,BZ14]

Goal: Send a message to n parties with $|\mathsf{ct}| = O(1)$

No interaction during setup except for a PKI.

(Note: Can achieve $|\mathsf{ct}| = O(n)$ using public key encryption)

Distributed Broadcast Encryption [WQZD10,BZ14]



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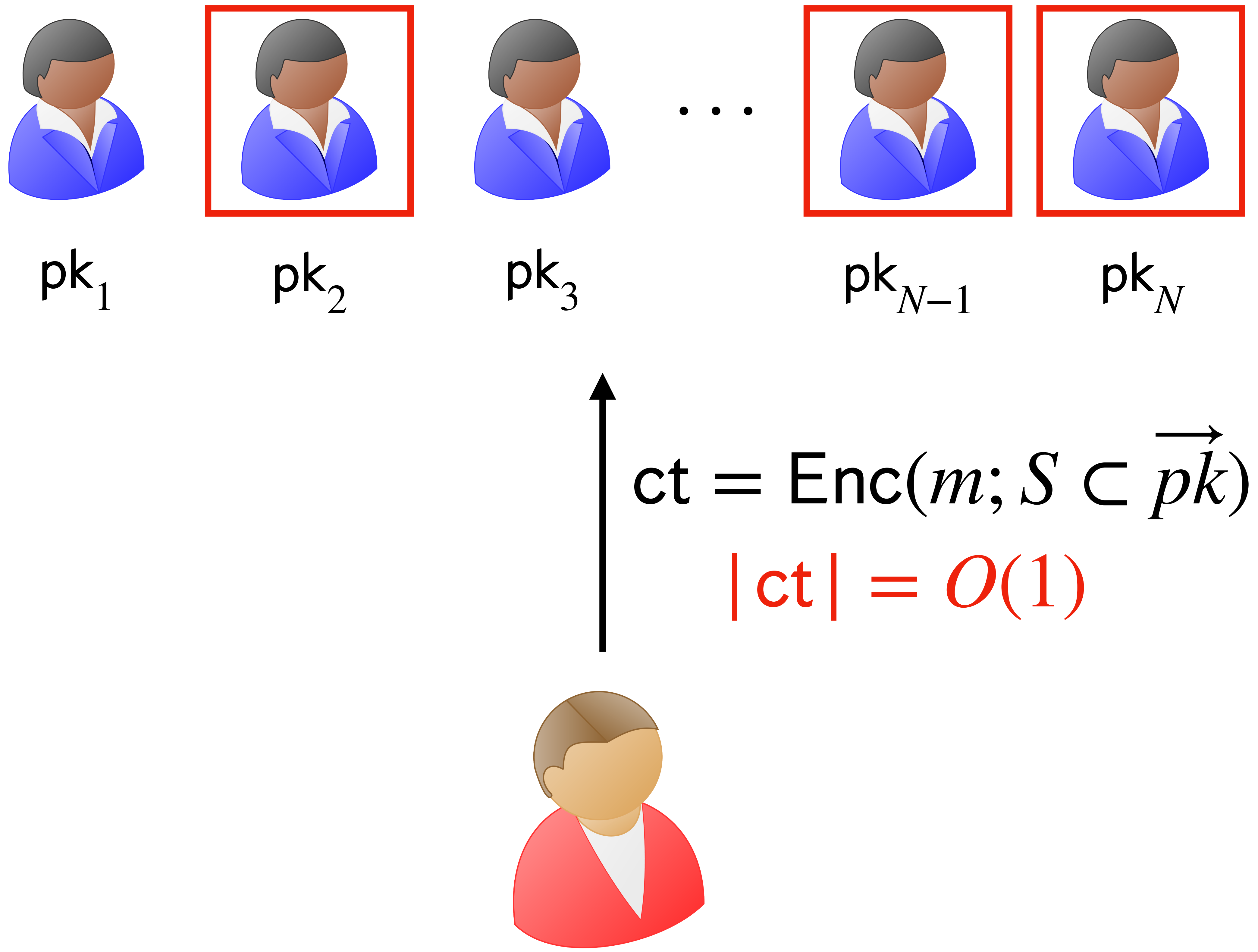

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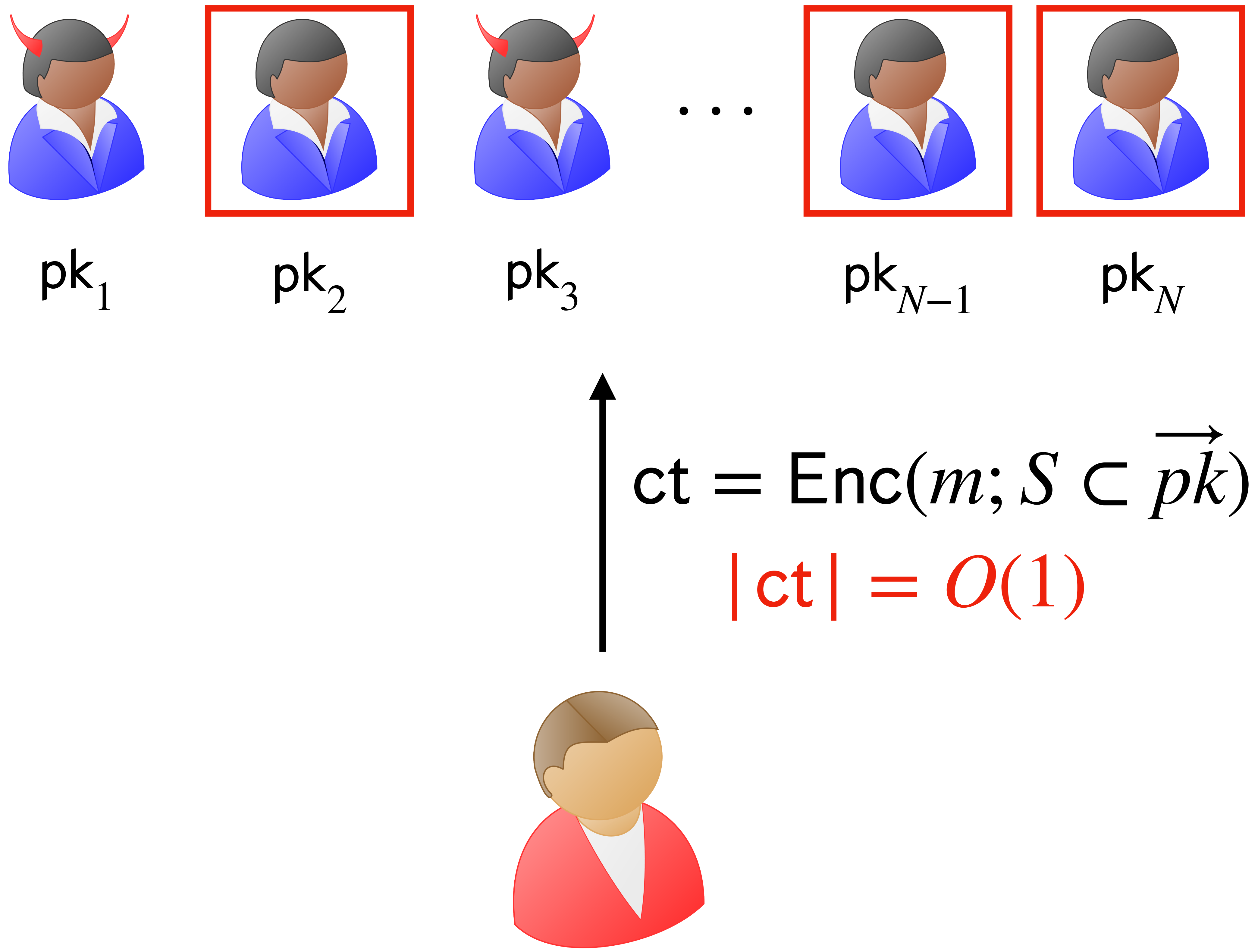

Semantic security against $S' = \overrightarrow{pk} \backslash S$

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Suppose we had a *succinct* ($|\mathsf{ct}| = O(1)$) WE for the following relation:

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You can decrypt my ciphertext iff you know a secret key

$$\{\mathsf{sk}_i : (\mathsf{pk}_i = g^{\mathsf{sk}_i}) \wedge (\mathsf{pk}_i \in \overrightarrow{\mathsf{pk}})\}$$

How do we build the WE?

Inner-Product Gadget

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1. The above WE has $|\mathsf{ct}| = O(1)$!
2. Enc takes as input a <u>succinct</u> commitment to $\vec{u}$, and runs in $O(1)$ time

Step #2: Reduce DBE $\rightarrow$ Inner-Product

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Honest users decrypt using $\vec{w} = (0, \ldots, \mathsf{sk}_i^{-1}, \ldots, 0)$

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Adversary can be reduced to solving DLOG

Registered Attribute Based Encryption

Registered Attribute Based Encryption [HLWW23]

Goal: Attribute Based Encryption without a Trusted Party

No interaction during setup except for a PKI

+ some notions of efficiency

Registered Attribute Based Encryption [HLWW23]

M users

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M users

Bulletin Board					
Public Key:	pk_1	pk_2	pk_3	pk_4	pk_5
Region:	EU	EU	USA	EU	USA
Area:	Crypto	Crypto	ML	ML	Crypto

Registered Attribute Based Encryption [HLWW23]

M users

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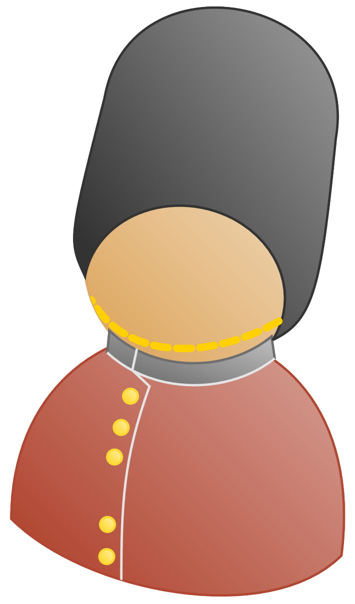

Key Curator

↓

Short aPK

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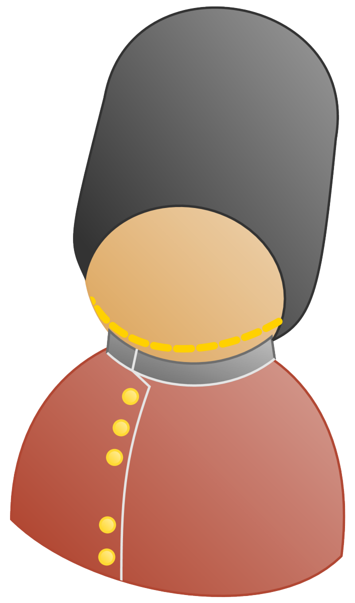

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$$\text{Enc}(\text{aPK}, m, \text{"Crypto"} \wedge \text{"EU"}) \to |\text{ct}|$$

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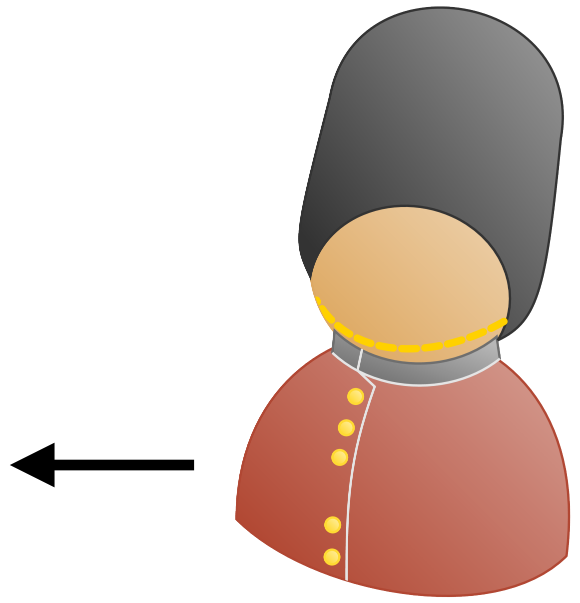

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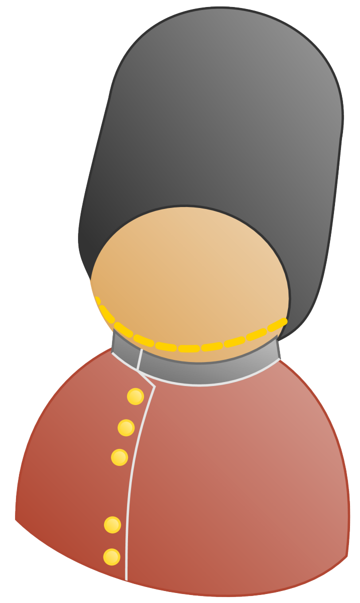

Key Curator

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$$\mathsf{Enc}(\mathsf{aPK}, m, \text{"Crypto"} \wedge \text{"EU"}) \rightarrow |\mathsf{ct}|$$

$$\mathsf{Dec}(\mathsf{aPK}, \mathsf{hk}_2, \mathsf{sk}_2, \mathsf{ct}) \rightarrow m$$

Step #1: Identify a Relation

M users

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USA:	-	-	pk_3	-	pk_5
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You can decrypt my ciphertext iff you know a secret key

$$\{sk : (pk = g^{sk}) \wedge (pk \in \text{Crypto}) \wedge (pk \in \text{EU})\}$$

How do we build it?

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EU:	pk_1	pk_2	0	pk_4	0
Crypto:	pk_1	pk_2	0	0	pk_5

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Almost works… but adversary can use "empty" slots

Zero-Check Gadget

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Statement: $S \subset [n]$

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2. Enc takes as input a succinct commitment to S, and runs in $O(1)$ time

Inner-Product + Zero-Check → rABE

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$\overrightarrow{\text{Crypto}} = (\text{pk}_1, \text{pk}_2, 0, 0, \text{pk}_5)$ $\overrightarrow{\text{EU}} = (\text{pk}_1, \text{pk}_2, 0, \text{pk}_4, 0)$

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Key Curator computes:

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Avoid reading entire bulletin board AND $(\text{Enc}, |\text{ct}|, \text{Dec})$ **are succinct**

Line of Work on Improving CRS

	$\lvert \mathsf{crs} \rvert$	Policy	Setting
[HLWW23, §5]	1	Circuit	iO
[FWW23]	1	Circuit	WE
[HLWW23, §7]	$\lvert\mathbb{U}\rvert M^2$	MSP	Composite, static
[ZZGQ23]	$\lvert\mathbb{U}\rvert M^2$	ABP	Prime, static
[AT24]	M^2	SP	Prime, static
[GLWW24, §4]	$M^{1+o(1)}$	MSP	Prime, q-type
[GLWW24, §5]	$\lvert\mathbb{U}\rvert M^{1+o(1)}$	MSP	Composite, static
Our Scheme	M	DNF	Prime, GGM

M users and $\lvert \mathbb{U} \rvert$ attributes

Matches CRS size of “weaker” primitives like RBE

Thank you!