Row Reduction Techniques for n-Party Garbling

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CRYPTO August 19th, 2025

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joint work with Kelong Cong[1], Emmanuela Orsini[2] and Oliver Zajonc[3]

1 Zama, 2 Bocconi University, 3 COSIC, KU Leuven

Overview

Full Row Reduction:

Circuit size:

- $3n\kappa$ (for [HSS17]-style)
- $3(n-1)\kappa$ (for authenticated garbling [WRK17,YZW20])

Improved Preprocessing:

- previous schemes have $4n\kappa$ and $(4n-6)\kappa$ circuit size
- solves open problem from [WRK17]

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Improved Preprocessing:

Authenticated field triples with $\mathcal{O}(n^3\sqrt{|C|})$ comm.

Mask preparation with $\mathcal{O}(2\rho|C|)$ comm.

- previous schemes have $4n\kappa$ and $(4n-6)\kappa$ circuit size
- solves open problem from [WRK17]

- generalizes approach by [DILO22] to $n \geq 3$ parties
- improves communication for large circuits

Overview

Full Row Reduction:

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- $3n\kappa$ (for
- $3(n-1)\kappa$ garbling

What will follow

- Two-Party Garbled Circuits
- Authenticated Multi-Party GC
- Our Construction
- Preprocessing
- Results

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- solves open problem from [WRK17]

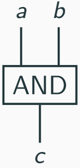
[DILO22] to $n \geq 3$ parties

- improves communication for large circuits

Two-Party Garbled Circuits

Garbler

Evaluator



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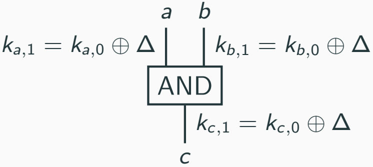

Evaluator

- $\Delta \leftarrow\!\!\$ \{0,1\}^\kappa$ for the whole circuit
- $k_{a,0}, k_{b,0}, k_{c,0} \leftarrow\!\!\$ \{0,1\}^\kappa$ for the gate

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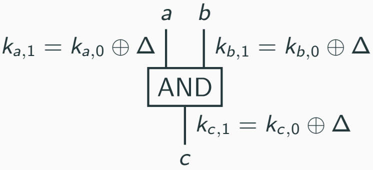

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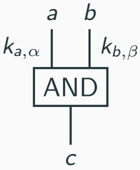
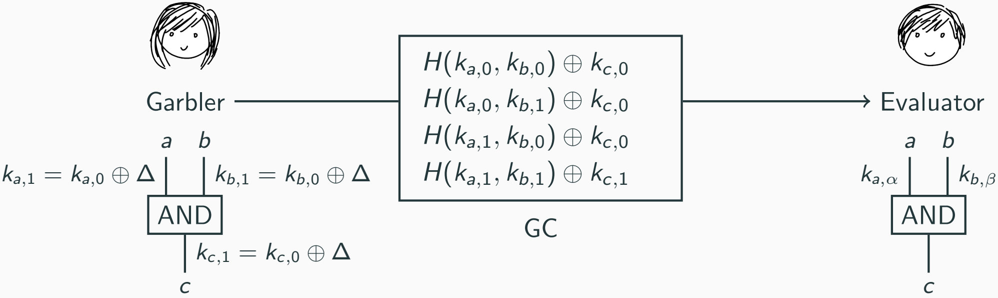


- knows $k_{a,\alpha} = k_{a,0} \oplus \alpha \cdot \Delta$ and $k_{b,\beta} = k_{b,0} \oplus \beta \cdot \Delta$

Two-Party Garbled Circuits



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Two-Party Garbled Circuits

Garbler

a b

$k_{a,1} = k_{a,0} \oplus \Delta$ $k_{b,1} = k_{b,0} \oplus \Delta$

AND

$k_{c,1} = k_{c,0} \oplus \Delta$

c

$$
\boxed{
\begin{array}{l}
H(k_{a,0}, k_{b,0}) \oplus k_{c,0} \\
H(k_{a,0}, k_{b,1}) \oplus k_{c,0} \\
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\end{array}
}
$$

GC

Evaluator

a b

$k_{a,\alpha}$ $k_{b,\beta}$

AND

$k_{c,\alpha\wedge\beta}$

c

- $\Delta \leftarrow\$ \{0,1\}^\kappa$ for the whole circuit
- $k_{a,0}, k_{b,0}, k_{c,0} \leftarrow\$ \{0,1\}^\kappa$ for the gate

- knows $k_{a,\alpha} = k_{a,0} \oplus \alpha \cdot \Delta$ and $k_{b,\beta} = k_{b,0} \oplus \beta \cdot \Delta$
- can decrypt one row correctly with $H(k_{a,\alpha}, k_{b,\beta})$
- obtains $k_{c,\alpha\wedge\beta}$

Row Reduction in Two-Party Setting [NPR99]

- Garbler sets $k_{c,0} = H(k_{a,0}, k_{b,0})$
- only 3 rows left

Garbler

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\boxed{\begin{array}{l}
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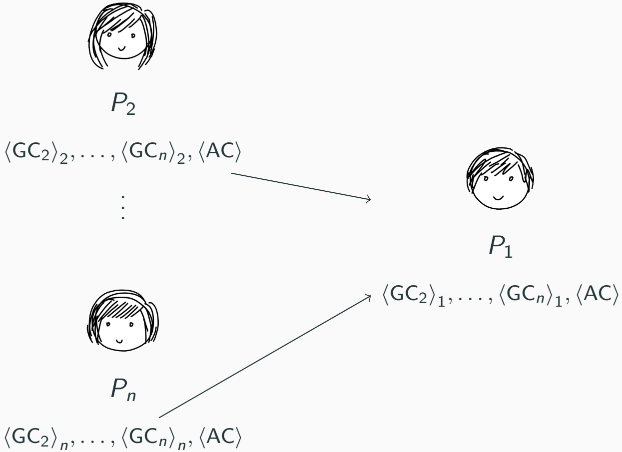
GC

Evaluator

⚠ garbler can no longer choose $k_{c,0}$ freely $\implies$ cannot choose all $k_{c,0}$ at the same time!

Authenticated Garbling for Multi-Party Garbled Circuits [WRK17,YWZ20]

- all parties jointly create *shares* of garbled circuit(s)



- Setting: active security, dishonest majority
- in [WRK17]: $n-1$ garblers, 1 evaluator
- $(n-1)$ GC's to evaluate + information for evaluator to check correctness

AND Gate

- every wire $k^i_{a,1} = k^i_{a,0} \oplus (\underbrace{\lambda_a \oplus \alpha}_{\hat{a}})\Delta^i$

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sharing between P_i and P_j

- use *authenticated* bit shares $[\![\lambda]\!] = \left(\lambda^i, \{\ \langle \lambda^i \Delta^j \rangle, \langle \lambda^j \Delta^i \rangle\ \}_{j \neq i}\right)$
- same Δ as MAC key and as FreeXOR offset

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- parties hold $[\![r_{\hat{a}\hat{b}}]\!] = [\![\overbrace{(\lambda_a \oplus \hat{a})}^{\alpha}\overbrace{(\lambda_b \oplus \hat{b})}^{\beta} \oplus \lambda_c]\!] \qquad \forall(\hat{a}, \hat{b}) \in \{0,1\}^2$

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$$\mathrm{GC}_i[\hat{a}, \hat{b}] = H(k^i_{a,\hat{a}}, k^i_{b,\hat{b}}) \oplus \left(\underset{\text{masked output bit}}{r^i_{\hat{a}\hat{b}}}, \{\ \underset{\langle \mathrm{GC}_j \rangle}{\langle k^j_{w,0} \oplus r_{\hat{a}\hat{b}}\Delta^j\rangle_i}\ \}_{j>1},\ \underset{\langle AC \rangle}{\langle r_{\hat{a}\hat{b}}\Delta^1\rangle} \right) \qquad \forall(\hat{a}, \hat{b}) \in \{0,1\}^2$$

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masked output bit

Evaluator P_1:

- active key $\boldsymbol{k}_{a,\hat{a}} = \big(k^2_{a,0} \oplus (\lambda_a \oplus \alpha)\Delta^2, \ldots, k^n_{a,0} \oplus (\lambda_a \oplus \alpha)\Delta^n\big)$ decrypts $\mathrm{GC}_2[\hat{a},\hat{b}], \ldots, \mathrm{GC}_n[\hat{a},\hat{b}]$

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- obtains $r_{\hat{a}\hat{b}} = \hat{c}$ and active key $\boldsymbol{k}_{c,\hat{c}}$ and checks correctness of $r_{\hat{a}\hat{b}} = \alpha\beta \oplus \lambda_c$

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masked output bit $\qquad \langle \mathrm{GC}_j\rangle \qquad \langle AC\rangle$

Evalu

Sending $r_{\hat{a}\hat{b}}$ and $r_{\hat{a}\hat{b}}\Delta^1$ can be removed at cost of additional online rounds [YWZ20]

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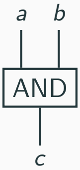
Our Row Reduction

Split AND equation into three parts: $\alpha\beta \oplus \lambda_c$



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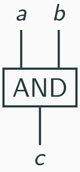
Split AND equation into three parts: $\alpha\beta \oplus \lambda_c = \underbrace{(\alpha \oplus \lambda_a)\beta}_{\text{prod}_1} \oplus \underbrace{(\beta \oplus \lambda_b)\lambda_a}_{\text{prod}_2} \oplus \underbrace{\lambda_a\lambda_b \oplus \lambda_c}_{\text{masks}}$



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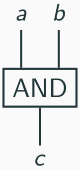

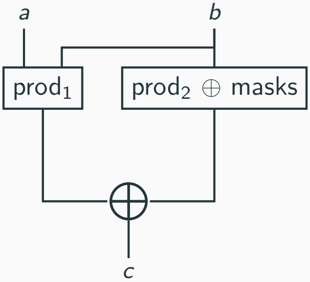

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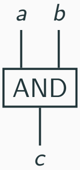

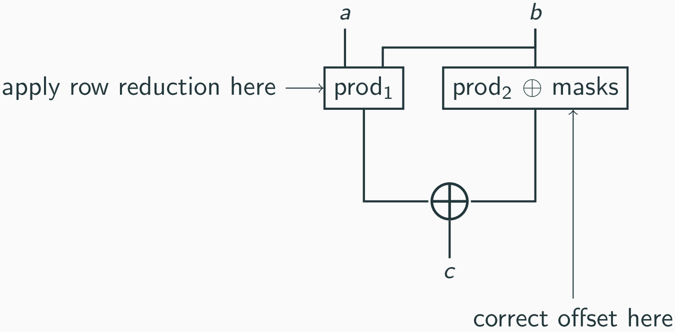

The prod_1 gadget $(\alpha \oplus \lambda_a)\beta$

(adapted multi-party version of the length-2 one-hot garbling construction [HK21])

- the parties hold $[\![r_1]\!] = [\![\lambda_a]\!]$ and compute

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$$C_1^i = H(k_{a,0}^i) \oplus H(k_{a,1}^i) \oplus \left(r_1^i,\ \{\langle k_{b,0}^j \oplus r_1 \Delta^j \rangle\}_{j>1},\ \langle r_1 \Delta^1 \rangle_i\right)$$

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- evaluator P_1 computes $H(k_{a,\hat{a}}^i) \oplus \hat{a} C_1^i \oplus \hat{a} k_{b,\hat{b}}^i = \langle \mathsf{prod}_1 \rangle_i, \{\langle k^j \rangle_i\}_{j>1}, \langle \mathsf{prod}_1 \Delta^1 \rangle_i$

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- but: $k^j = \mathsf{prod}_1 \Delta^j \oplus \sum_i H(k_{a,0}^i)$ with undesired offset
- the offset will be corrected in the prod_2 gadget

The prod_2 gadget

is a regular unary gate with added masks and offset terms

- the parties hold $[\![r_{2,\hat{b}}]\!] = [\![\hat{b} \cdot \lambda_a \oplus \lambda_a \lambda_b \oplus \lambda_c]\!]$ for $\hat{b} \in \{0, 1\}$

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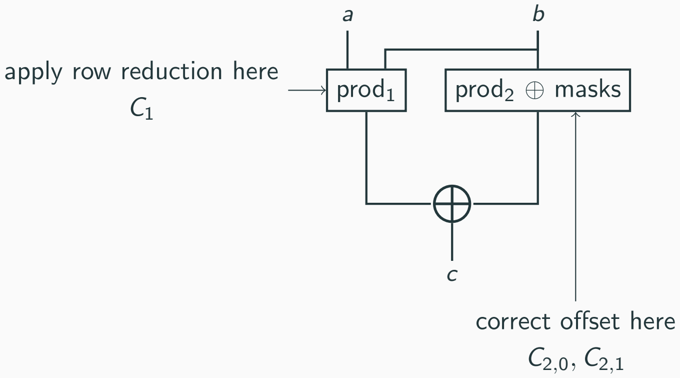
Finally: XORing the prod_1 and prod_2 gadgets yields: $\langle \hat{c} \rangle_i$, $k^i_{c,\hat{c}}$ and $\langle \hat{c}\Delta^1 \rangle_i$

Summary

The general idea:

Split AND equation into three parts: $\alpha\beta \oplus \lambda_c = \underbrace{(\alpha + \lambda_a)\beta}_{\text{prod}_1} \oplus \underbrace{(\beta \oplus \lambda_b)\lambda_a}_{\text{prod}_2} \oplus \underbrace{\lambda_a\lambda_b \oplus \lambda_c}_{\text{masks}}$



Improved Preprocessing

we extend the approach of [DILO22] from 2 to $n \geq 3$ parties

Func. Indep. Prep. authenticated mult. triples in $\mathbb{F}_2$ $\longrightarrow$ **Func. Dep. Prep.** beaver multiplication of $[\![\lambda_a]\!] \times [\![\lambda_b]\!]$ $\longrightarrow$ **Garble/Eval** use $[\![\lambda_a \lambda_b]\!]$ to construct GC

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[YWZ20] expensive $\mathcal{O}(\kappa|C|)$ cheap $|C|$

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[YWZ20]	expensive $\mathcal{O}(\kappa \lvert C \rvert)$		cheap $\lvert C \rvert$		
[DILO22] (and ours)	cheap (sublinear in $\lvert C \rvert$)		ok $\rho \lvert C \rvert$ $\rho < \kappa$		

Improved Preprocessing

we extend the approach of [DILO22] from 2 to $n \geq 3$ parties

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1. design a PCG for correlation $\langle x \rangle, \langle y \rangle, \langle xy \rangle$ with authentication values $\{\langle x\alpha^i \rangle, \langle y\alpha^i \rangle, \langle xy\alpha^i \rangle\}_i$ in $\mathbb{F}_{2^\rho}$

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2. perform a "large field" multiplication for $[\![\lambda_a]\!] \times [\![\lambda_b]\!]$
3. "key switch" to authentication in $\mathbb{F}_{2^\kappa}$ using n-party VOLE

see full paper for details

Results

Full Row Reduction:

Circuit size:

- $3n\kappa$ (for [HSS17]-style)
- $3(n-1)\kappa$ (for authenticated garbling [WRK17,YWZ20])

- 25% to 43% smaller circuit compared to [HSS17], [WRK17] and [YWZ20]

Execution time in ms for AES-128 circuit

Parties	4	8	12
[WRK17]	223	423	629
Ours	168	359	540

Improved Preprocessing:

Authenticated field triples with $\mathcal{O}(n^3\sqrt{|C|})$ comm.

Mask preparation with $\mathcal{O}(2\rho|C|)$ comm.

- $\times 6$ lower comm. cost compared to [HSS17]
- $\times 2.2$ lower comm. cost compared to [YWZ20]

Thank you!

eprint 2025/829

github.com/zama-ai/copz25-code

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