# Secret sharing-based FHE

## Actively Secure MPC in the Dishonest Majority Setting: Achieving Constant Complexity in Online Communication, Computation Per Gate, Rounds, and Private Input Size

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# Presentation Overview

# Contribution Overview

# Secure Multi-Party Computation (MPC)

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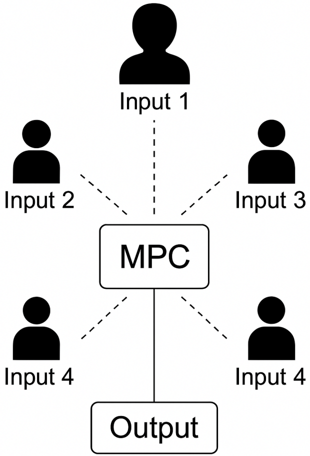
**Security Models:**

- **Honest Majority Setting:**
  - Majority of parties follow the protocol
  - Enables **Guaranteed Output Delivery (GOD)**
- **Dishonest Majority Setting:**
  - Majority may be corrupted
  - At most **Security with Abort** can be guaranteed



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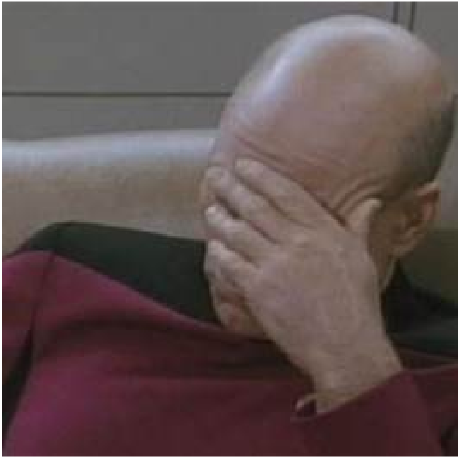
*How about FHE-based MPC? (Threshold FHE)*

# FHE-based MPC ...



# Practical Limitations of FHE-Based MPC

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- High Computational Overhead:
  - FHE operations are significantly slower than plaintext equivalents
- Performance Degrades with Number of Parties:
  - Computation cost increases exponentially with the number of parties
- Security Challenges under Dishonest Majority:
  - Active security typically requires zero-knowledge proofs.
  - ZKPs introduce heavy overhead and complex protocol logic.

# Key Challenges in FHE-based MPC (PPT in NIST MPTS 2023)

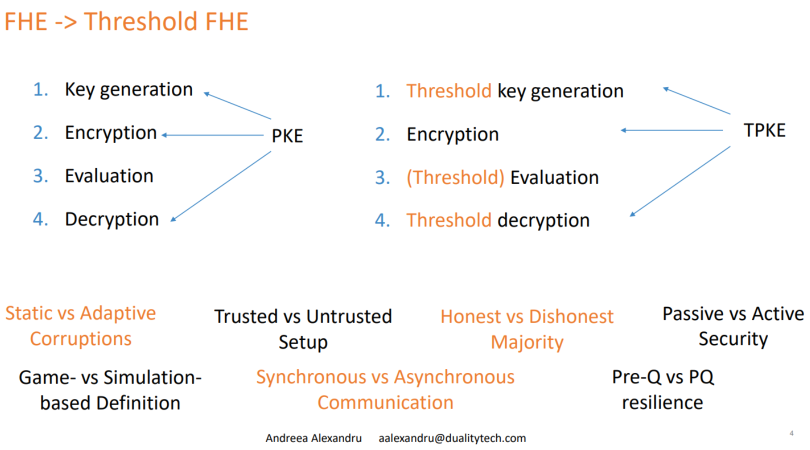


Figure: Content on Page 4 of the NIST MPTS 2023 PPT [13]

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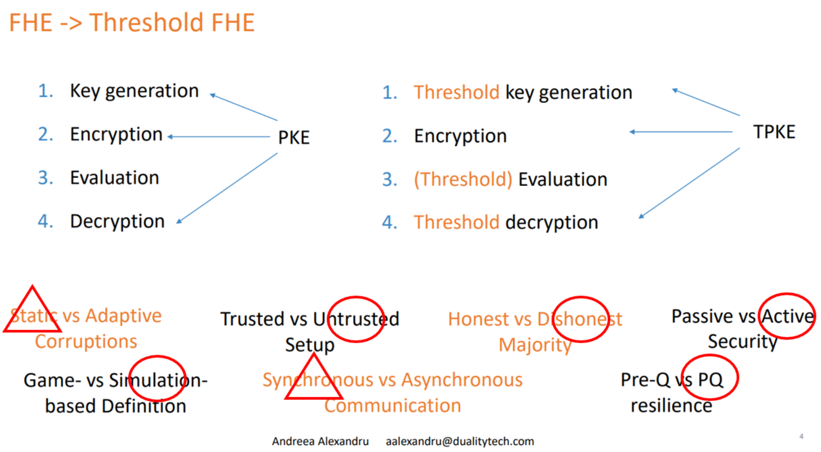


Figure: Today's FHE-based MPC properties

# Comparison of FHE-based MPC Protocols

- Our solution: secret sharing (SPDZ) + FHE + pre-processing model $\rightarrow$ SSFHE

| | Types | $\lvert ct \rvert$ | $\lvert ev \rvert$ | $\lvert evGen \rvert$ |
|---|---|---|---|---|
| GLS15 [5] | ThHE | $O(nd_{\lambda,n}^2 \log d_{\lambda,n})$ | 0 | 0 |
| JS23 [6] | ThHE | $O(d_{\lambda,n})$ | $O(d_{\lambda,n}^2)$ | $O(nd_{\lambda,n}^2)$ |
| DDE+23 [7] | ThHE | $O(d_\lambda)$ | $O(d_\lambda^2)$ | unspecified |
| CCS19 [8] | MkHE | $O(nd_{\lambda,n})$ | $O(n^2 d_{\lambda,n}^2)$ | $O(n^2 d_{\lambda,n}^2)$ |
| TLX+21 [9] | MkHE | $O(d_{\lambda,n})$ | $O(d_{\lambda,n}^2)$ | $O(nd_{\lambda,n}^2)$ |
| Ours | ThHE | $O(d_\lambda)$ | $O(d_\lambda^2)$ | $O(n\kappa d_\lambda^2)$ |

| | Majority | Security | Gate speed | NTT-friendly |
|---|---|---|---|---|
| GLS15 [5] | Honest | Active | $O(\text{poly}(\lambda, n))$ | Yes |
| JS23 [6] | Honest | Passive | $O(\text{poly}(\lambda, n))$ | Yes |
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| Ours | Dishonest | Active | $O(\text{poly}(\lambda))$ | Yes |

Table: Comparison of FHE-based MPC protocols. The parameter $\lambda$ is the computational security bit, $\kappa$ is the statistical security bit, and $d_{\lambda,n}$ is the dimension of LWE sample

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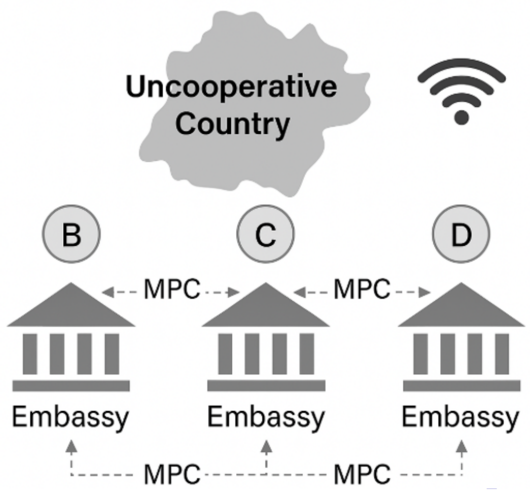
- Another Goal: How to build circuit-private MPC under dishonest majority and eavesdropping environment?



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- Problem in SPDZ, BMR:
  - Communication increases proportional to circuit size
  - Circuit structure is indirectly leaked through total traffic
- **FHE-based MPC is ready to be proven**
  - All evaluation is local
  - No circuit-dependent communication in the online phase
  - Circuit privacy is provably achieved

*Our protocol is the first to achieve circuit-private MPC with active security under dishonest majority.*

# Backgrounds

# LWE Sample and Encryption Review

**LWE Sample:**

- Fix modulus $q$, dimension $n$, and error distribution $\chi$ (e.g., discrete Gaussian)
- Let $s \in \mathbb{Z}_q^n$ be a secret vector
- Sample $A \in \mathbb{Z}_q^{n \times m}$ uniformly at random and noise $e \leftarrow \chi^m$
- Output the LWE sample: $(A, b = sA + e \mod q)$

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## Definition (Regev's Encryption [10])

Let $M = [A^t | b^t] \in \mathbb{Z}_q^{m \times (n+1)}$ be public, and $s \in \mathbb{Z}_q^n$ be a secret key.
To encrypt $x \in \{0, 1\}$:

- Sample $r \in \{0, 1\}^m$, $e_1 \leftarrow \chi^n$ $e_2 \leftarrow \chi$ and compute $c = (c_1, c_2)$:

$$c_1 = rA^t + e_1 \in \mathbb{Z}_q^n, \quad c_2 = rb^t + \left\lfloor \frac{q}{2} \right\rfloor m + e_2 = rA^t s^t + re^t + e_2 + \left\lfloor \frac{q}{2} \right\rfloor$$

# Addition and Multiplication in LWE ciphertext

**Addition (Linear):**

- Ciphertexts can be added.
- Example:
  $\mathrm{Dec}\big(\mathrm{Enc}(m_1) + \mathrm{Enc}(m_2)\big) = \big((A_1, b_2) + (A_2, b_2)\big) = m_1 + m_2$ (noise grows linearly).

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**Multiplication (Nontrivial):**

- Multiplying ciphertexts is not straightforward and limited up to at most $L$ times.

# Bootstrapping: From Leveled HE to FHE

LHE ciphertext with message m after performing $L$ NAND gates

$c_1[m]$ →

$c[sk]$ →

**Bootstrapping ↔ Performing Decryption circuit $\mathcal{C}(c_1[m], sk) = m$ on the ciphertext domain with $|\mathcal{C}| = L' < L$**

Evaluation key
:= encrypting secret key $sk$

↓

LHE ciphertext with message m after performing $L' < L$ NAND gates

$$c_2[m] \leftarrow \mathrm{Eval}(\mathcal{C}, c[sk], c_1[m])$$

* $c_1[m]$ is no longer ciphertext, but plaintext values in terms of the circuit $\mathcal{C}$

# SPDZ Secret Sharing:

**(*n*-*out*-*of*-*n*)Linear Secret Sharing:** We denote the share of $m$ as

$$[m] = ([m]_1, [m]_2, \ldots, [m]_n) \in \mathbb{Z}_q^n$$

such that

$$m = \sum_{i=1}^{n} [m]_i \mod p$$

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## Definition (Ideal functionality $\mathcal{F}_{\text{SPDZ}}$)

- $\mathcal{F}^p_{\text{SPDZ}}$.Input($\cdot$): from input $x$, output $[x]_i$ to the $P_i$.
- $\mathcal{F}^p_{\text{SPDZ}}$.Rand(): output $[u]_i$ for some uniformly random $u \in \mathbb{Z}_p$.
- $\mathcal{F}^p_{\text{SPDZ}}$.RandBit(): output $[r]_i$ for some random bit $r \in \mathbb{Z}_p$.
- $\mathcal{F}^p_{\text{SPDZ}}$.MUL($[x], [y]$): output $[xy]_i$ to the $P_i$.

# Arithmetic on Secret Shares: SPDZ-style MPC

**Linear operations: Local and efficient**

- $[x + y]$ can be computed locally:

$$[x + y]_i = [x]_i + [y]_i$$

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0.5em **Multiplication: Requires communication + preprocessing**

- $[x \cdot y]$ is computed using a **Beaver triple**:

$$\text{Preprocess: } \langle a \rangle,\ \langle b \rangle,\ \langle ab \rangle$$

- Parties use $x - a$ and $y - b$ to compute $xy$ via interactive protocol.
- Hence, multiplicative depth of the circuit $\mathcal{C}$ determines the number of communication round, and the number of multiplication in $\mathcal{C}$ determines the total communication.

# Threshold FHE: Idea and Challenges

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- **KDM security:** Must support

$$\mathrm{Enc}_{\vec{s}_1}(\vec{s}_2), \quad \mathrm{Enc}_{\vec{s}_1}(\vec{s}_1)$$

while $\mathrm{Enc}_{\vec{s}_1}$ and $\mathrm{Enc}_{\vec{s}_2}$ could use difference modulus, respectively.

# Random Bit Sampling over Composite-Modulus Secret Sharing

# Sampling on the secret sharing with Composite Modulus

- **Idea:** Sample secret key and error over a composite modulus

$$Q = p_1 \cdot p_2 \cdots p_\ell$$

where each $p_i$ is pairwise co-prime and $\mathcal{F}_{\mathsf{SPDZ}}^{p_i}$ are available.

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- **Goal:** Construct arbitrary discrete distributions (e.g., Discrete Gaussian) from uniform bits over $Q$ **with active security**

# Sampling on the secret sharing with Composite Modulus

## Lemma (Inverse transform sampling)

*Let $\mathcal{D}$ be any discrete distribution with support size* $\mathrm{poly}(n)$ *and* $u_1, \ldots, u_\kappa$ *be* $\kappa$ *uniformly random samples. Then there exists a Boolean/arithmetic circuit such that:*

- ***Depth:*** *at most* $3\lceil \log \kappa \rceil + 2$
- ***Multiplications:*** *at most* $6 \cdot \mathrm{poly}(n) \cdot \kappa$
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- Actively secure **one-bit sampler** over $Q \Rightarrow$
- Actively secure sampler for **any** $\mathrm{poly}(n)$**-bounded distribution**, including discrete Gaussians with polynomial variance

# Core idea of samping one bit on the composite modulus [12]

- First, perform one-bit sampling of $[m]^p$ over an **auxiliary prime modulus** $p$ and obtaining $k$ such that $m = \sum_{i=1}^{n} [m]_i^p - kp$.

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- Then, **replicate the sampled share** into corresponding shares

$$[m]^{p_1},\ [m]^{p_2},\ \ldots,\ [m]^{p_\ell}$$

over the other prime moduli via $m = (\sum_{i=1}^{n} [m]_i^p - kp) \bmod p_i$.

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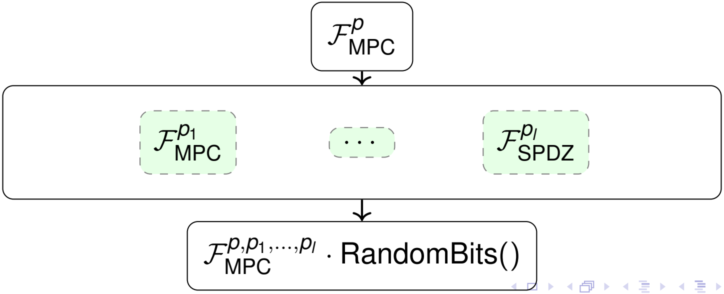

# Open Only High Bits to obtaining $k$ [12]

**$\Delta$-split (high bit only).** Let $p$ be prime and each party hold a share $x_i \in \{0, \ldots, p-1\}$. Fix $\Delta = \lceil p/n \rceil$ and write

$$[m]_i = \ell_i + \Delta h_i, \qquad 0 \leq \ell_i < \Delta.$$

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**Lemma [Correctness].** Let $[m]_1, \ldots, [m]_n$ be uniformly chosen subject to

$$[m]_1 + \cdots + [m]_n \in \{k \cdot p,\ k \cdot p + 1\}$$

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**Lemma [Zero knowledge, Informal] The protocol has perfectly hiding property (no informational leakage).**

# Verification protocol [12]

**Verifying $t$ uniformly random bits consuming $\mathsf{sec}$ random bits**

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5. Otherwise output $[m_i]^{p_v}$.

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1. Sample public $t + \text{sec}$ values $r_{i,j}$ uniformly at random from $\{0, 2^{\text{sec}} - 1\}$.
2. For each $v = 0, \ldots, t-1$, compute linear shares
$$[S_v]^{p_v} = \sum_{i=1}^{t+\text{sec}} r_{i,j} \cdot [m_i]^{p_v}.$$
3. Open $[S_0]^{p_0}$ (auxiliary prime $p_0$) and each $S_v$.
4. Abort if $S_0 \neq S_v \mod p_v$ for some $v$.
5. Otherwise output $[m_i]^{p_v}$.

**Note.** The $r_{i,j}$ randomizers (size $2^{\text{sec}}$) guarantee hiding , while cross-checking ensures consistency across all moduli.

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**Note.** The $r_{i,j}$ randomizers (size $2^{\text{sec}}$) guarantee hiding , while cross-checking ensures consistency across all moduli.

**Drawback.** The opened value $[S_0]^{p_0}$ must satisfy $0 < S_0 < p_0$, which requires the auxiliary prime $p_0$ to be much larger than $2^{\text{sec}}$. Hence, a very large modulus $p_0$ is needed.

# [Our works] Opening High Bits: Counterexample & Lemma (in One Slide)

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**Lemma (no leakage except boundary cases).** Write an honest share as $[m]_i = \ell_i + \Delta h_i$ with $0 \le \ell_i < \Delta$, $h_i \in \{0, 1\}$. If $[m]_i$ is *not* in any of the following boundary cases:

$$\text{(i) } \ell_i = 0, \qquad \text{(ii) } \ell_i = \Delta - 1, \qquad \text{(iii) } [m]_i = p - 1,$$

then revealing $h_i$ leaks no information about $m$ (i.e., $h_i \perp m$).

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**Note (rejection sampling restores perfect hiding property).** Whenever a bad event is detected, discard that sample (rejection) and resample; the resulting message-bit shares $[m]$ are again uniformly random and thus perfectly hiding.

# [Our contribution] Protocol Fix: Boundary-Share Filter Before Opening $h_i$

**Idea.** Honest parties locally screen out boundary cases before any $h_i$ is opened.

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**Note (probability & restart policy).** If all parties' shares are uniformly random, the probability that *some* honest share hits a boundary is at most $3np^{-1}$. Hence one can monitor the number of restarts and *abort*.

# [Our contribution] Verification with One-Bit $r$

- In the previous verification protocol, We reduce $r$ down to a *single bit*, leveraging the *Random Smudging Lemma*:
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## Lemma (Random Smudging Lemma)

*Let $X \xleftarrow{\$} U_{2^\kappa}$ and $Y \xleftarrow{\$} D_B$ be independent. Let $\mathcal{E}$ be the event that $B-1 \leq X+Y < 2^\kappa$, and let $\mathcal{E}^c$ be its complement. Then:*

(i) $X+Y$ *and* $Y$ *are independent conditioned on* $\mathcal{E}$.

(ii) $\Pr[\mathcal{E}^c] \leq (B-1)\,2^{-\kappa}$.

(iii) $\Delta\big((X+Y),\ Y\big) \leq (B-1)\,2^{-\kappa}$.

# Improving Random Bit Sampling Overview

**Rotaru et al. [12]**

- Assumes **no leakage** during the sharing
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**Our Improvements**

- Provides **full leakage analysis** of all possible cases
- Remains secure even when **corrupted parties choose shares arbitrarily**
- Requires using primes $p \approx 32$ bits via rejection procedure

# secret-shared FHE, SSFHE

# Informal Construction of SSFHE

**Step-by-step Construction:**

- Combine all primes $p_1, \ldots, p_l$ used in the given FHE into a single composite modulus $Q = p_1 \cdots p_l$.

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**$\epsilon$-Correctness and $\delta$-IND-CPA Security Guarantee:**

- Let FHE be $(\delta_F, \epsilon_F)$-secure
- Let the LSSS-based MPC have $\kappa$-soundness
- Then the resulting SSFHE satisfies:

$$\delta_S \leftarrow \delta_F, \quad \epsilon_S \leftarrow 2^{-O(\kappa)} + \epsilon_F \cdot \text{poly}(\lambda)$$

# Communication Cost of Gate Bootstrapping

**Key Insight:**

- Recall the structure of FHE (or LWE) encryption:

$$\mathrm{Enc}_{\vec{s}}(m) = (\vec{a},\ b = \sum_{i=1}^{d} a_i s_i + e)$$

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**Communication Complexity:**

- For LWE dimension $d$:
  - random bit sampling: $O(1)$ rounds, $O(d^2)$ multiplications
  - Inverse transform: $O(\log \kappa)$ rounds, $O(d^2 \kappa)$ multiplications

# Circuit-private MPC

# Circuit privacy: Why we use SSFHE

## Previous MPC ideal functionality: $\mathcal{F}_{\text{ABB}}$

**Ideal functionality of $\mathcal{F}_{\text{ABB}}$**

**Initializ:** On input (init, $p^k$) for all parties, the functionality activates and store the modulus $p$.

**Input:** On input (input, $P_i$, varid, $x$) from $P_i$ and (input, $P_i$, varid, ?) from all other parties, with varid a fresh identifier, the functionality stores (varid, $x$)

**Add:** On command (add, varid$_1$, varid$_2$, varid$_3$) from all parties (if varid$_1$ and varid$_2$ are present in memory and varid$_3$ is not), the functionality retrieves (varid$_1$, $x$), (varid$_2$, $y$) and store (varid$_3$, $x + y \mod p$)

**Multiply:** On command (multiply, varid$_1$, varid$_2$, varid$_3$) from all parties (if varid$_1$ and varid$_2$ are present in memory and varid$_3$ is not), the functionality retrieves (varid$_1$, $x$), (varid$_2$, $y$) and store (varid$_3$, $xy \mod p$)

**Output:** On input (output, varid from all honest parties parties (if varid is present in memory), the functionality retrieves (varid, $x$), and outputs it to the environment. If the adversary inputs OK then $x$ is output to all parties. Otherwise $\perp$ is output to all parties.

Figure: The common ideal functionality for arithmetic black-box model.

# Circuit privacy: Why we use SSFHE

## Current circuit-private MPC ideal functionaliy: $\mathcal{F}_{\text{CPMPC}}$

**Ideal functionality of $\mathcal{F}_{\text{CPMPC}}$**

**Init:** On input (**init**, $t$) from all parties, store the threshold $t$. Initialize the register State to 0.

**CircuitInit:** On input (**CircInit**, $i, \mathcal{C}, I$) from all parties $P_i$, if State is 0, store the circuit $\mathcal{C}$ and leak the number of inputs $I$ and the input-to-party mapping to the adversary $\mathcal{A}$. Set State to 1.

**Input:** On input (**input**, $P_i$, varid, $x$) from all parties $P_i$ and (input, $P_i$, varid, ?) from all other parties in $\mathcal{P} \backslash P_i$, store (varid, $x$) if varid is a fresh identifier. Set State to 2.

**Output:** On input (**output**, $i$, varid) from all parties $P_i$, if varid exists in memory, TTP retrieves (varid, $x$) and outputs it to the adversary $\mathcal{A}$. If $\mathcal{A}$ inputs OK, $x$ is output to all parties, and State is reset to 0. If $\mathcal{A}$ inputs $\perp$, then $\perp$ is output to all parties, and the protocol terminates.

Figure: The ideal functionality for circuit-private MPC.

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Figure: The ideal functionality for circuit-private MPC.

### BMR vs SS vs FHE-based MPC

- Without corrupted parties, BMR or secret sharing simulators cannot easily generate views without circuit info.

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Figure: The ideal functionality for circuit-private MPC.

## BMR vs SS vs FHE-based MPC

- Without corrupted parties, BMR or secret sharing simulators cannot easily generate views without circuit info.
- (FHE-based MPC) Simulator need not construct output ciphertexts, thus security proof is feasible.

# Circuit-Private MPC

## Theorem (Informal)

*Let:*

- *An* ($\delta$-IND-CPA, $\epsilon$-correctness)*-secure SSFHE scheme be given,*
- *A* $\kappa$*-soundness secret sharing scheme be used,*
- *And all errors in the decrypted ciphertext are bounded by B..*

*Then, under the UC framework, the adversary's computational advantage is bounded by:*

$$\delta + (B - 1) \cdot 2^{-\kappa+2} + 2\epsilon$$

# Future Work (On Progress...)

- **SPDZ over 16-bit Primes:**
  - Explore implementation of SPDZ using small 16-bit NTT-friendly primes
  - Leverage vector Oblivious Linear Evaluation (vOLE) for efficient multiplication triples
  - Aim to reduce computation and memory cost while preserving active security

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  - Leverage vector Oblivious Linear Evaluation (vOLE) for efficient multiplication triples
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- **SSFHE with GOD in Honest Majority:**
  - Construct SSFHE protocol under honest majority assumption
  - Ensure **Guaranteed Output Delivery (GOD)** despite adversarial behavior
  - Implement and evaluate performance in real-world parameters

# Thank you

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