That's AmorE Amortized Efficiency for Pairing Delegation

ia.cr/2025/542




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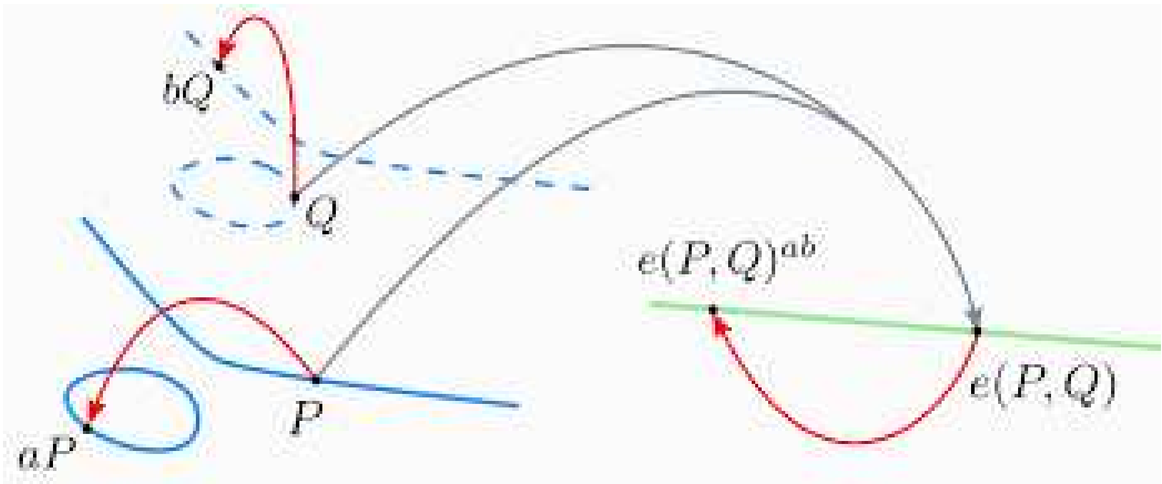
Affiliations: AarhusU (DK), ChalmersU (SE), GöteborgU (SE), TII (UAE)

What are "Pairings"?

bilinear maps on ***groups***

$$e : \mathbb{G}_1 \times \mathbb{G}_2 \longrightarrow \mathbb{G}_T$$



$$e(aP, bQ) = e(P, Q)^{ab}$$

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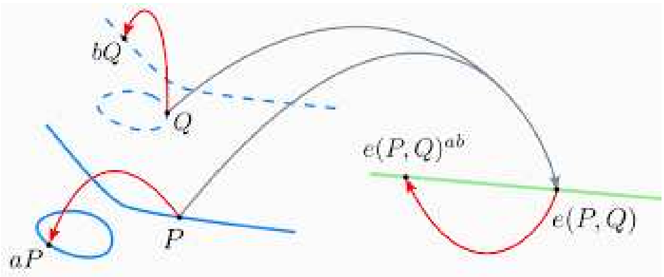

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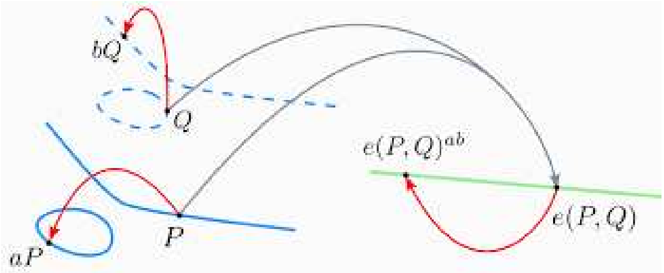

$$e(aP, bQ) = e(P,Q)^{ab}$$

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very abridged reference list

Joux: A **one round** protocol for **tripartite** Diffie-Hellman (2000)
Boneh, Lynn,Shacham: **Short Signatures** from the Weil Pairing (2004)
Sahai, Waters: **Fuzzy identity-based encryption** (2005)
Kate,Zaverucha,Goldberg: **Constant-size commitments to polynomials** [..] (2010)
Groth: **Short** pairing-based **non-interactive** zero-knowledge **arguments** (2010)
Boneh,Drijvers,Neven: **Compact multi-signatures** for smaller blockchains (2018)
Gailly,Maller,Nitulescu, : **SnarkPack**: Practical SNARK aggregation (2022)
Garg,Jain,Mukherjee+: **hints**: Threshold signatures with silent setup (2024)

So, What's the Catch?

costs in 10^3 clock cycles on BLS12-381

So, What's the Catch?

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373

aP in $\mathbb{G}_1$

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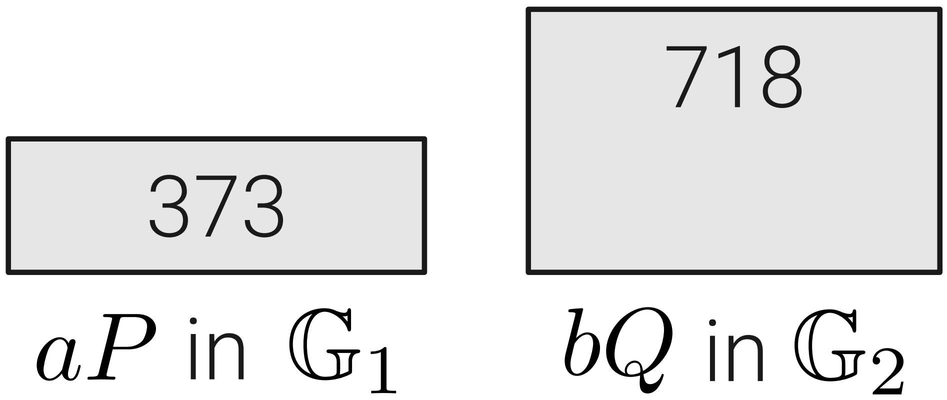

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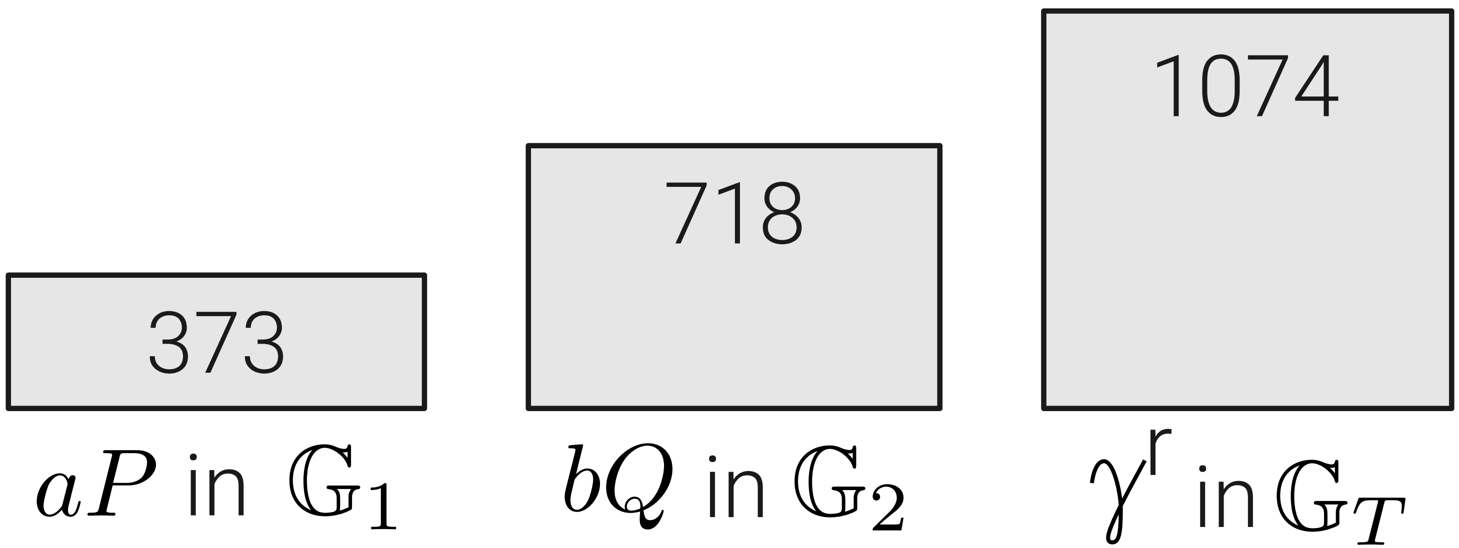

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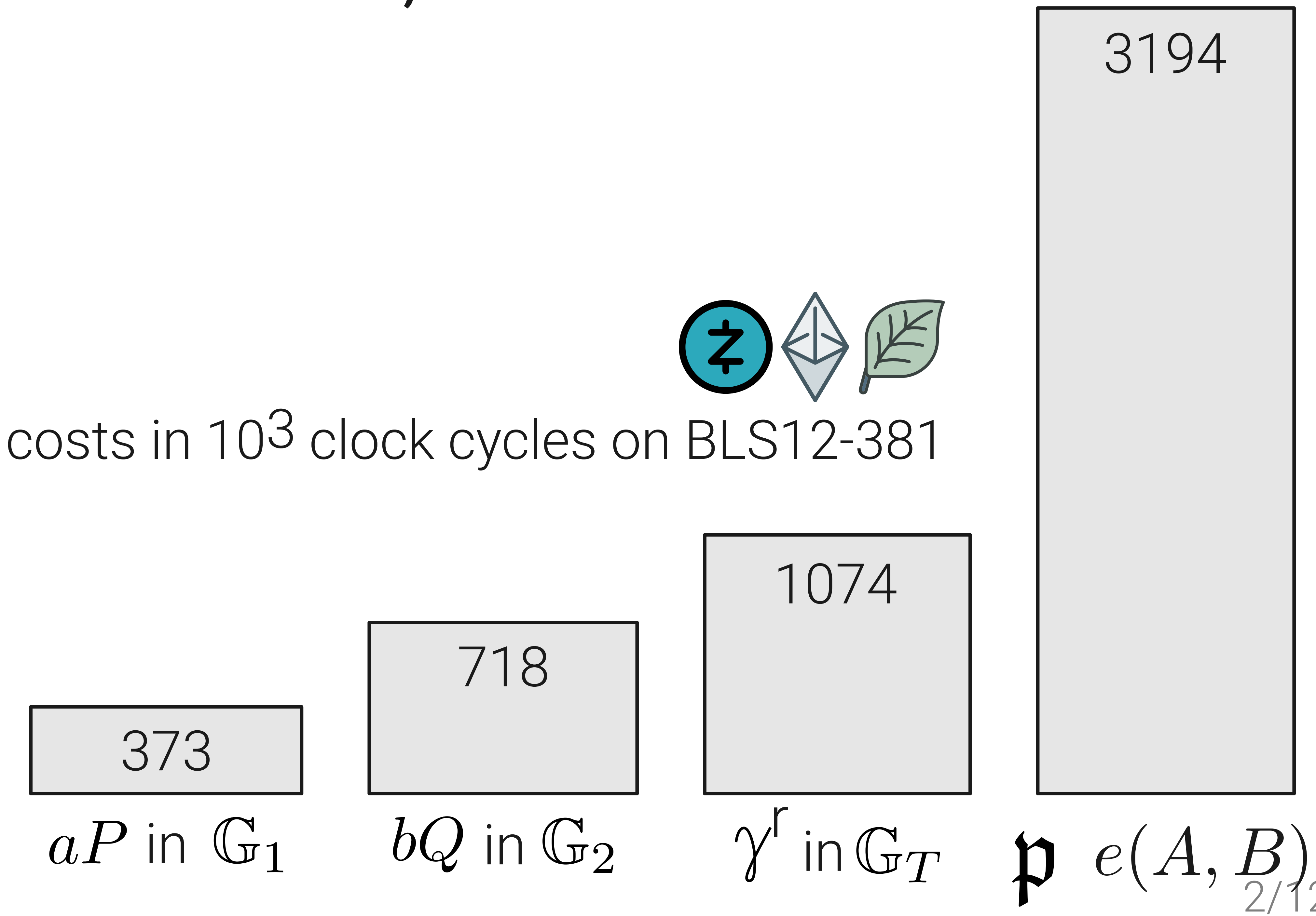

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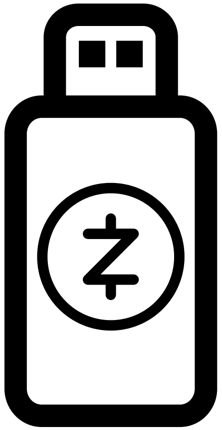
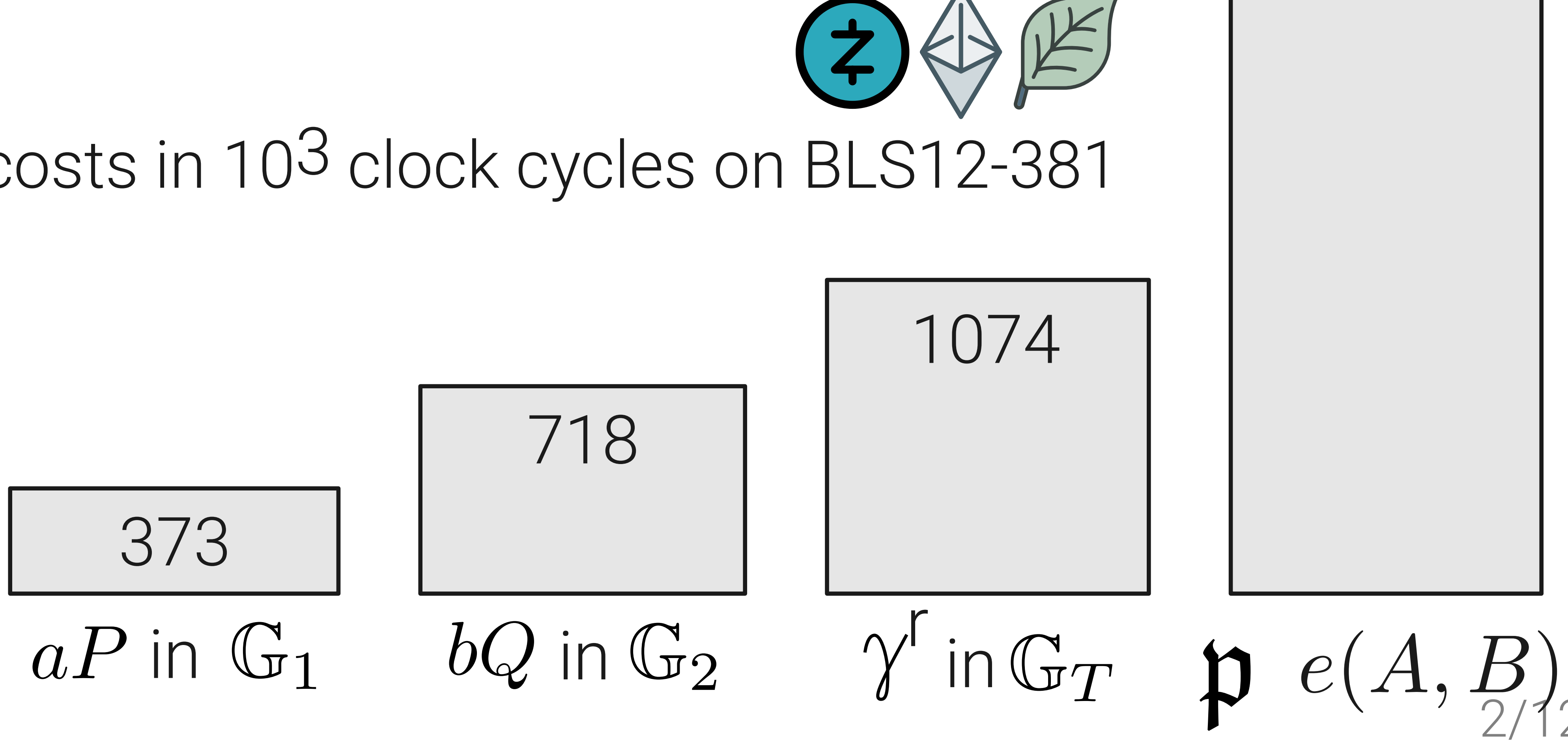


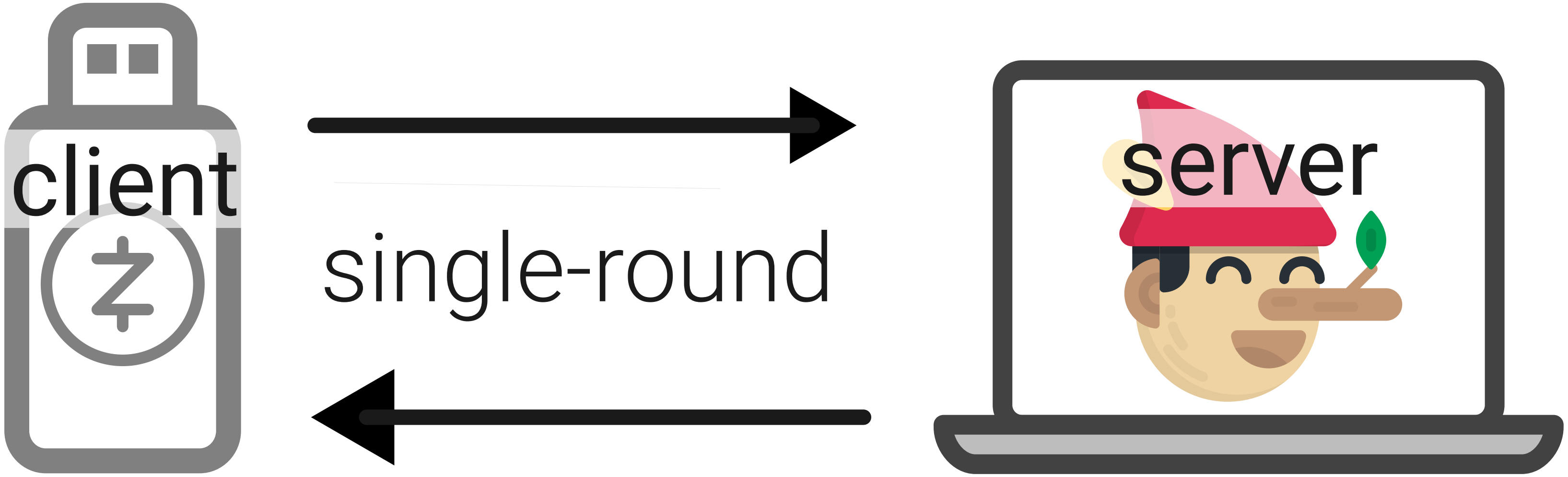
pairings are ***prohibitive***
on weaker IoT devices
(incl. hardware wallets)

costs in 10^3 clock cycles on BLS12-381



Pairing Delegation Protocols

since 2005



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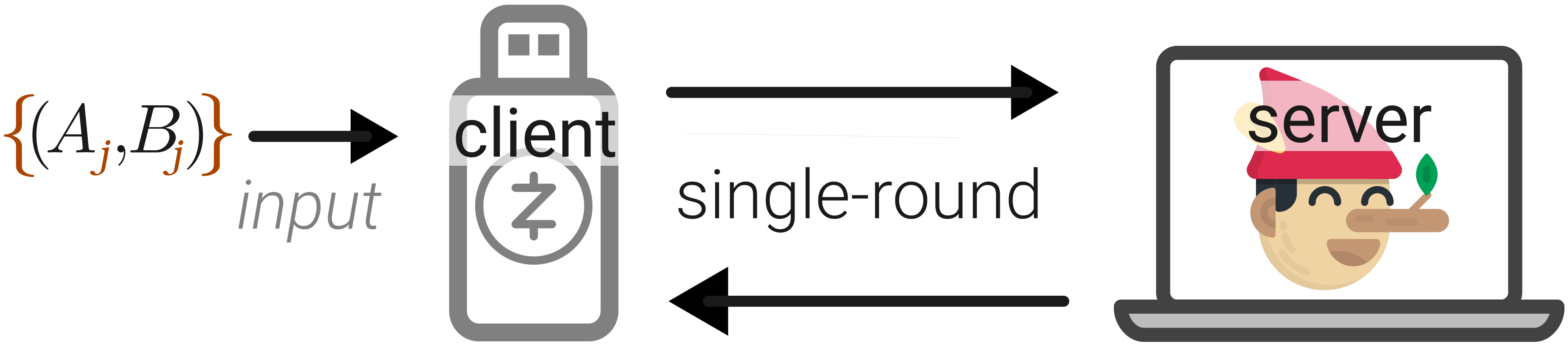

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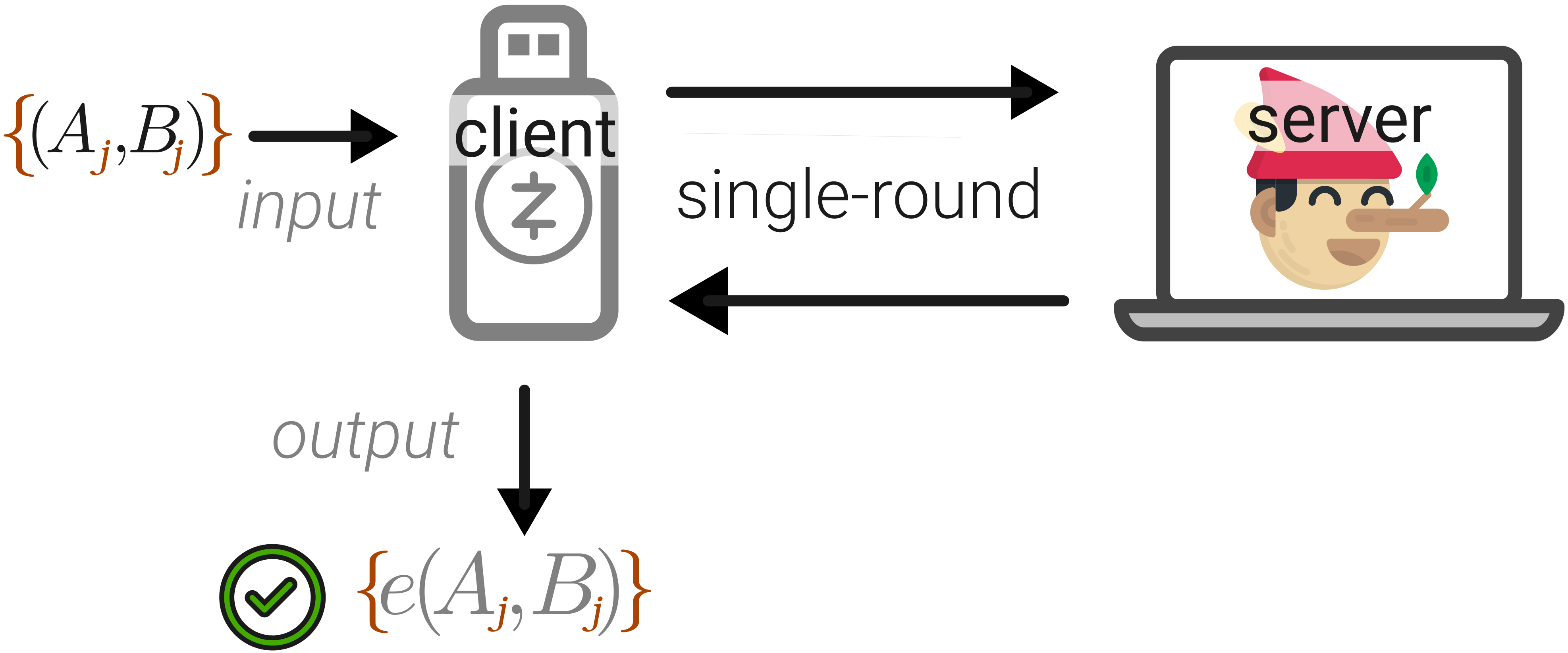

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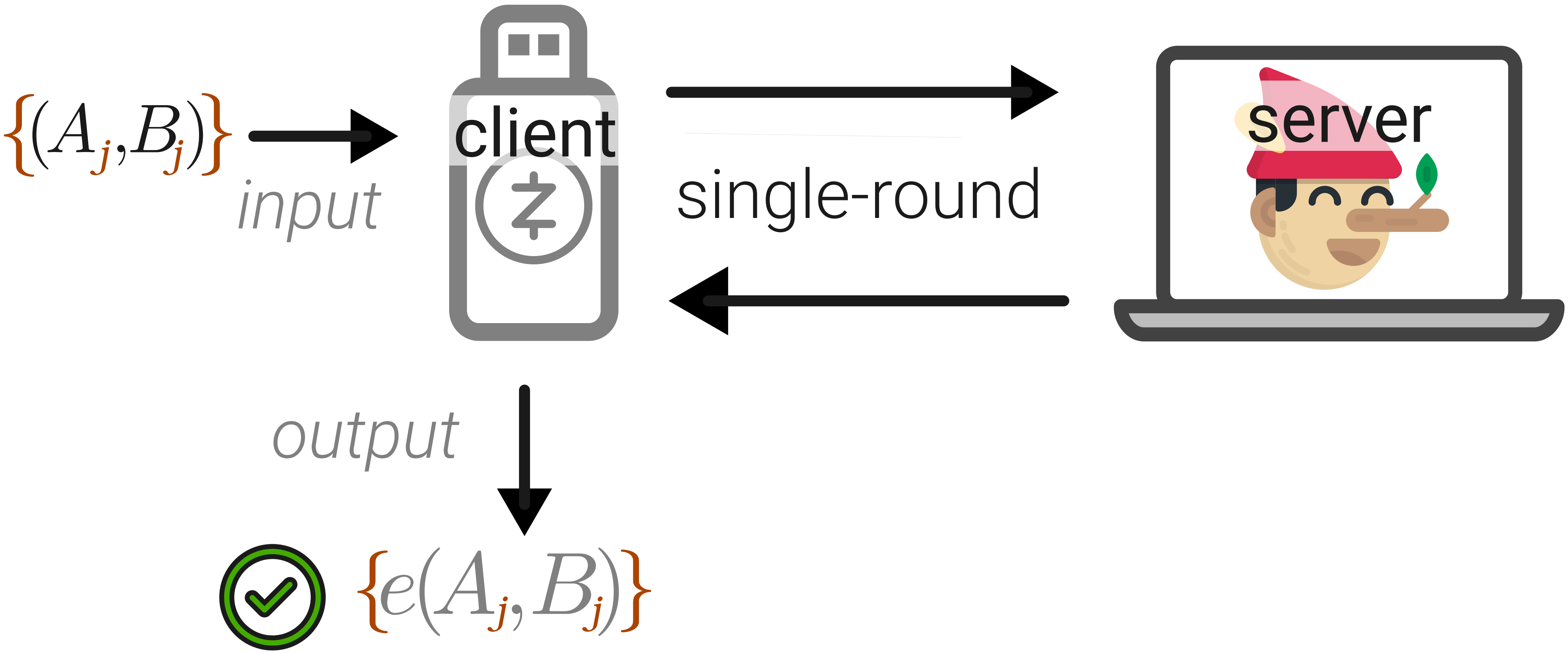
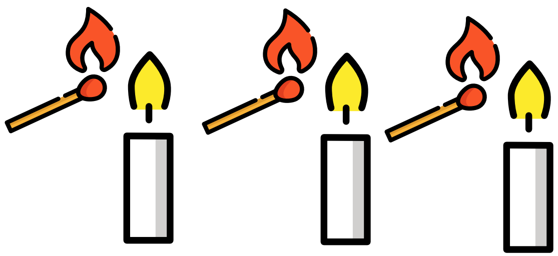


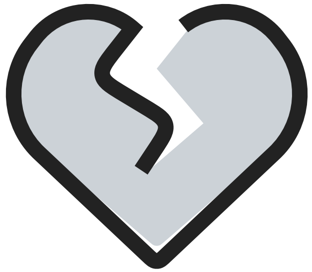
The one-shot framework



Client PreComp + **One Delegation**

State of the Art

State of the Art

Category 1   2 recent protocols broken by *our work*

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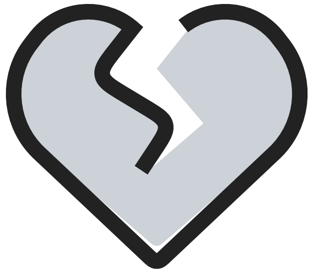
Category 1  

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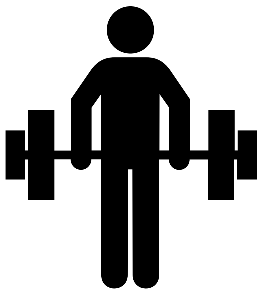
Category 2  

unconditional security one-shot framework

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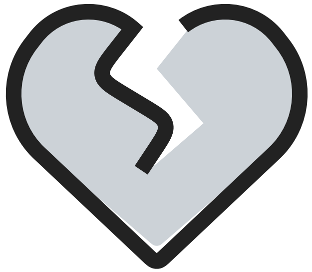
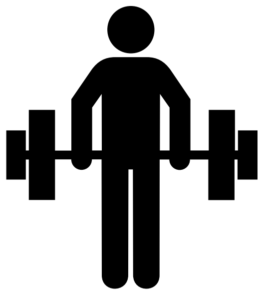
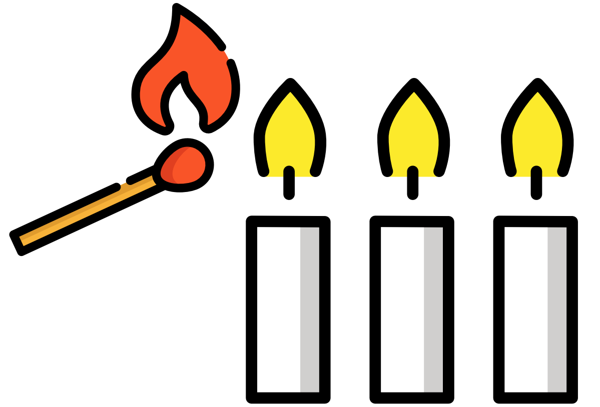
Category 1   2 recent protocols broken by *our work*

Category 2   unconditional security one-shot framework

WANTED: a pairing delegation protocol that is

reasonably *secure* **and** *efficient*

Our Approach



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Client PreComp + *Delegation 1* + + *Delegation N*

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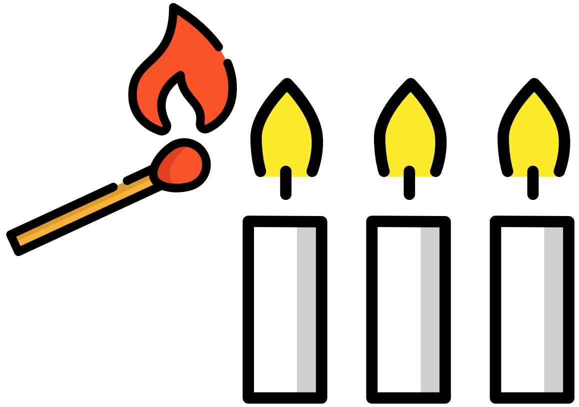

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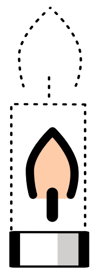

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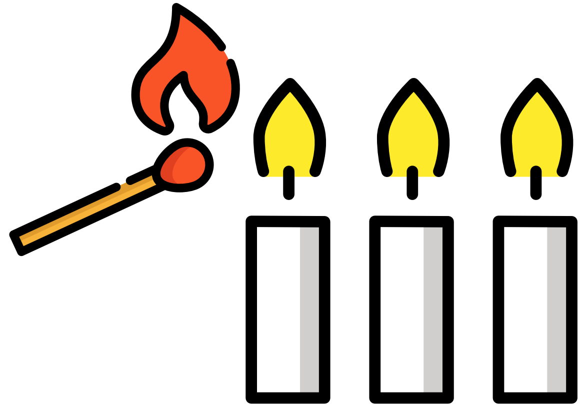

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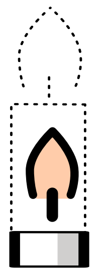

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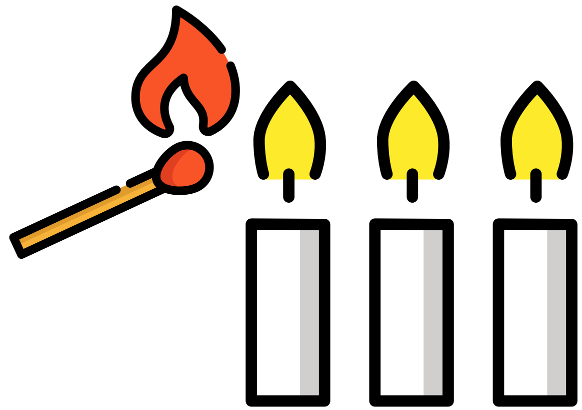

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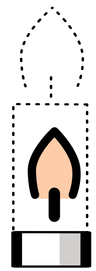

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4. **New Security Assumption**

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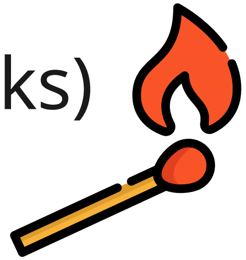
1. **Impossibility result** (inspired by our new attacks)



- client PreComp needs to be expensive

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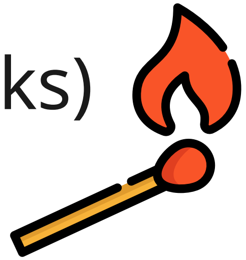

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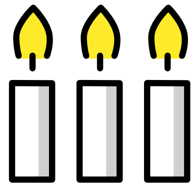

- cost(PreCom) is amortized over several delegations

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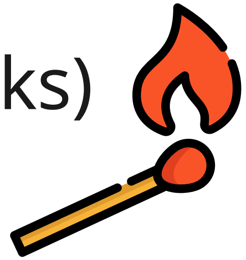

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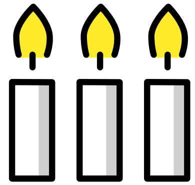

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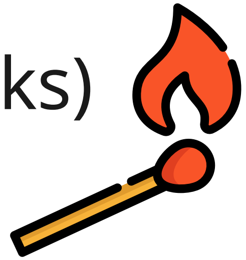

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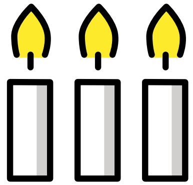

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   and Efficient Short Scalar Sampling

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AmorE
in a Nutshell

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$$\mathsf{t}_{\mathsf{start}} \leftarrow \texttt{time.now}()$$

AmorE in a Nutshell

exponentiation in $\mathbb{G}_T$

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$q = |\mathbb{G}_1| = |\mathbb{G}_2|$

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probabilistic

$\mathbb{G}_1, \mathbb{G}_2$ elements

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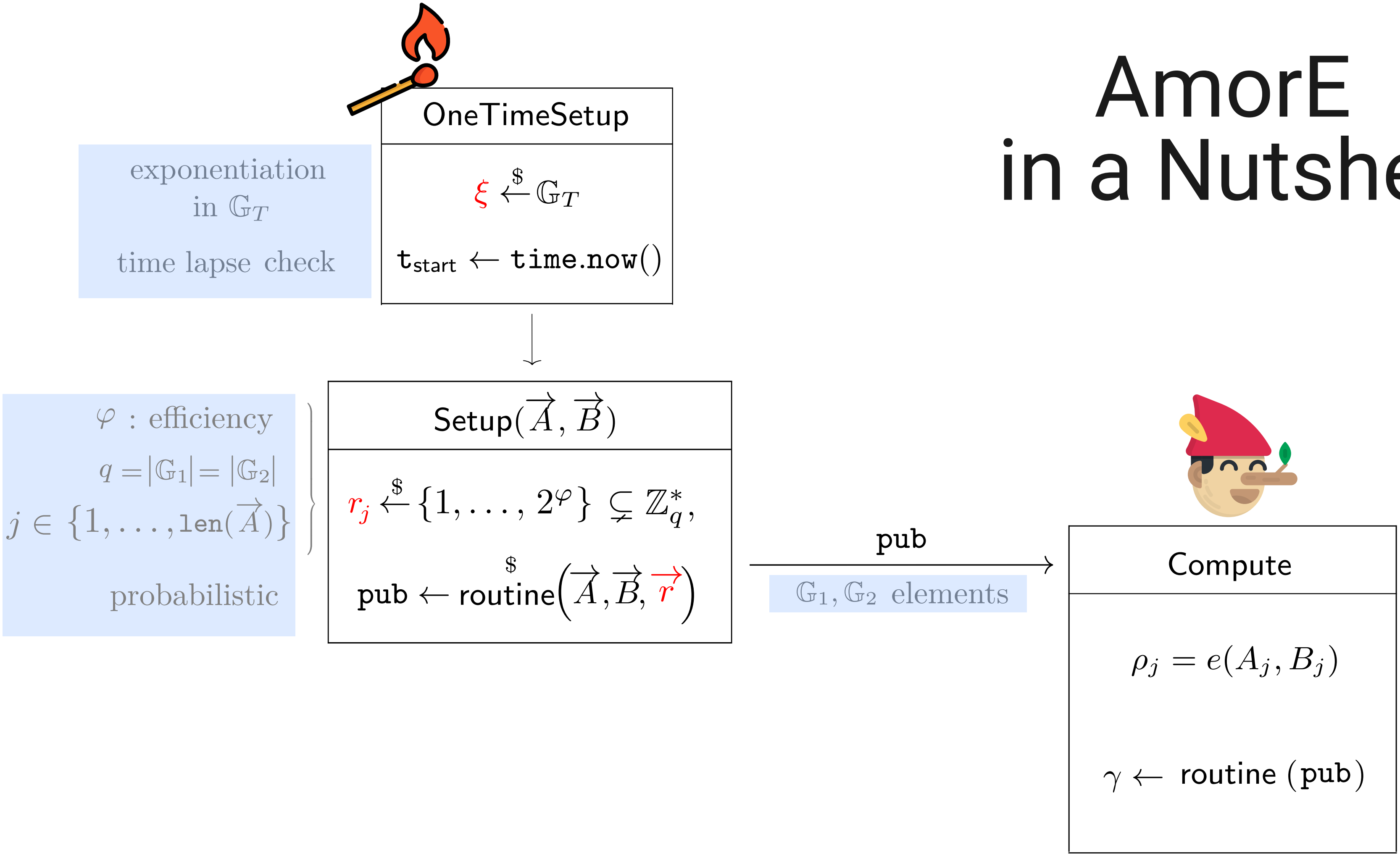

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pairing evaluations

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← $\mathtt{out} = (\gamma, \vec{\rho})$

$\mathbb{G}_T$ elements

Verify

$\mathtt{time.now}() - \mathsf{t}_{\mathsf{start}} \stackrel{?}{>} \tau$

$\rho_j \stackrel{?}{\in} \mathbb{G}_T$

$\xi \stackrel{?}{=} \left(\prod_{j=1}^{\mathtt{len}(\vec{A})} \rho_j^{r_j}\right) \cdot \gamma$

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τ : latency

membership tests

verification equation

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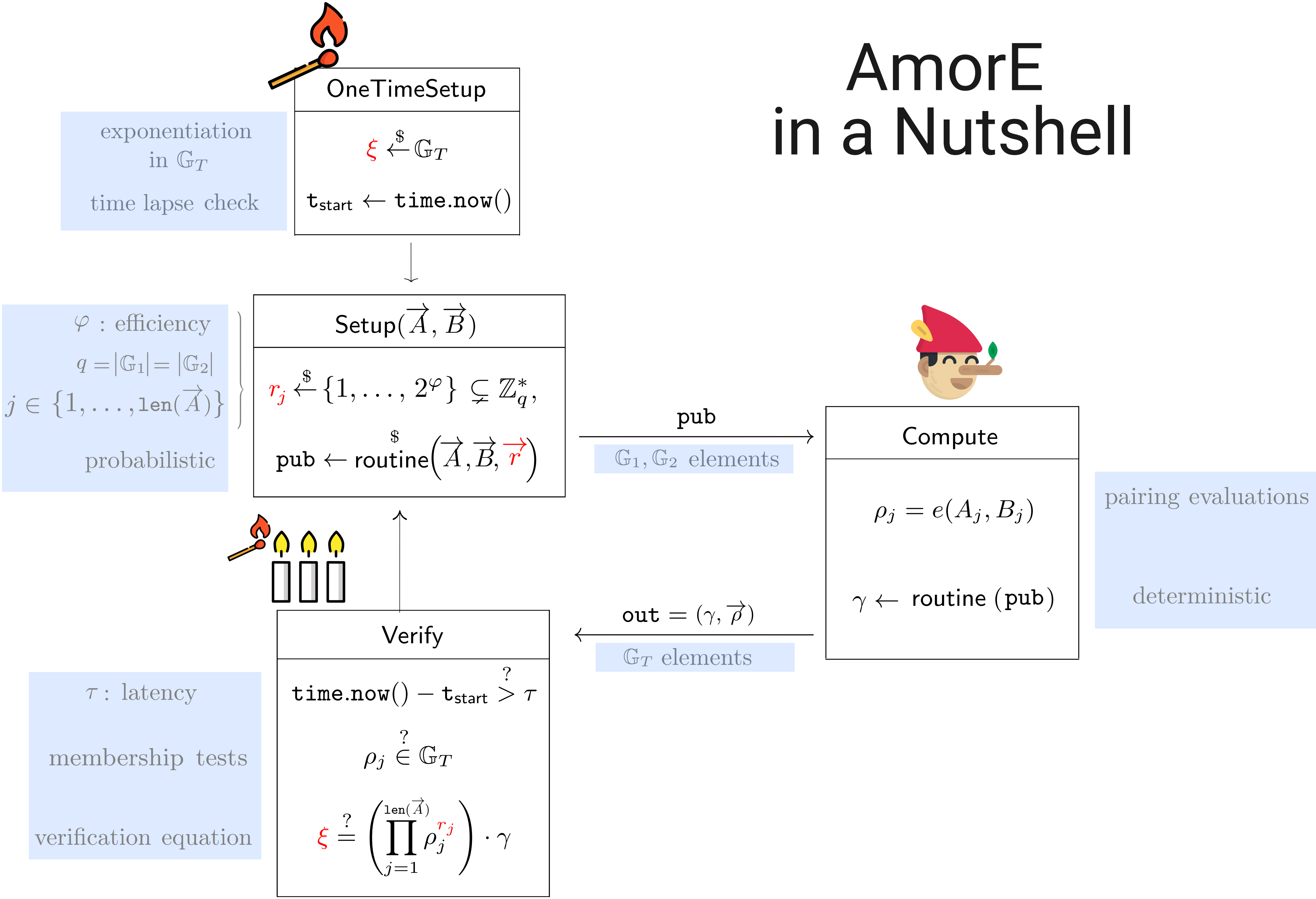

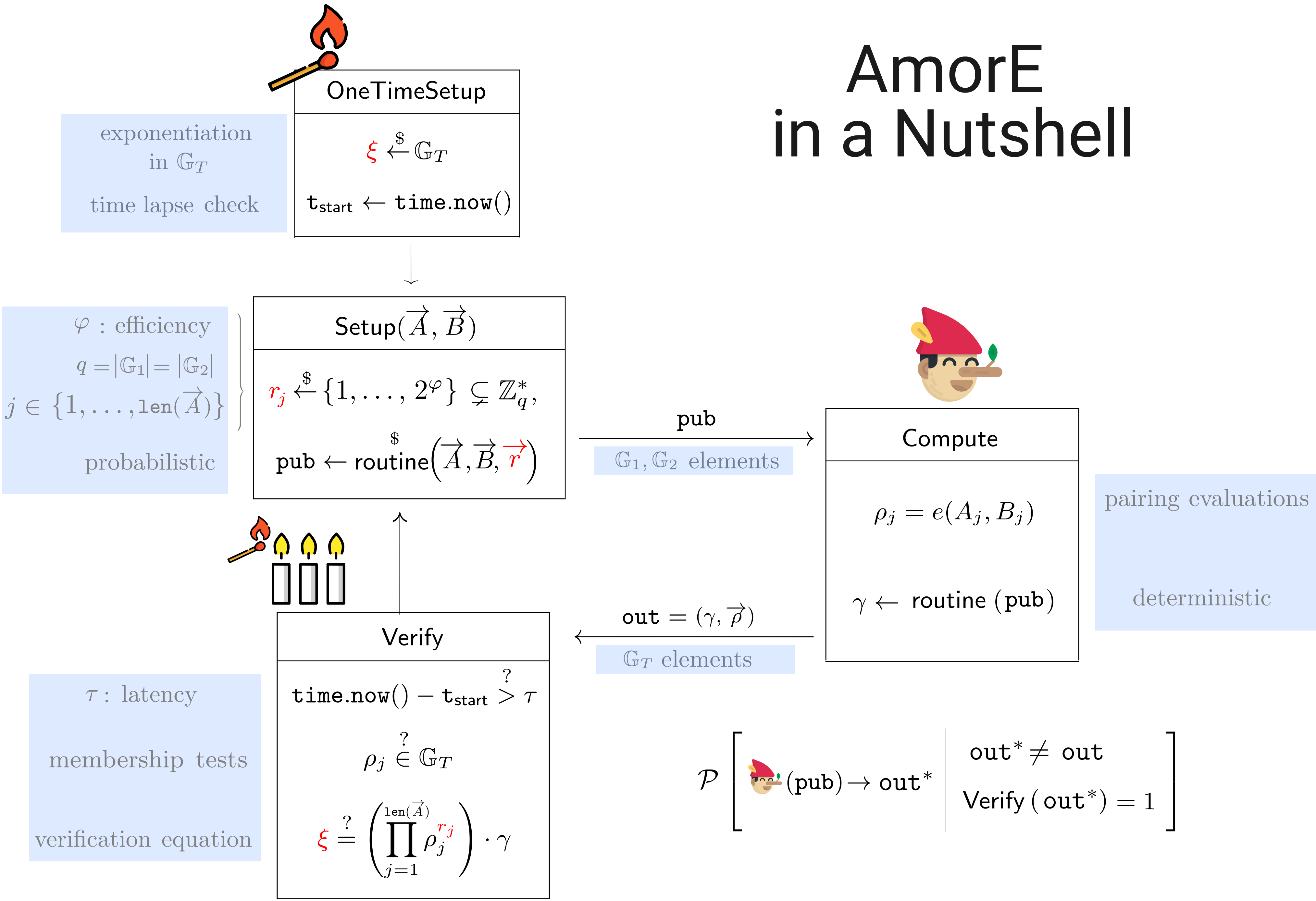
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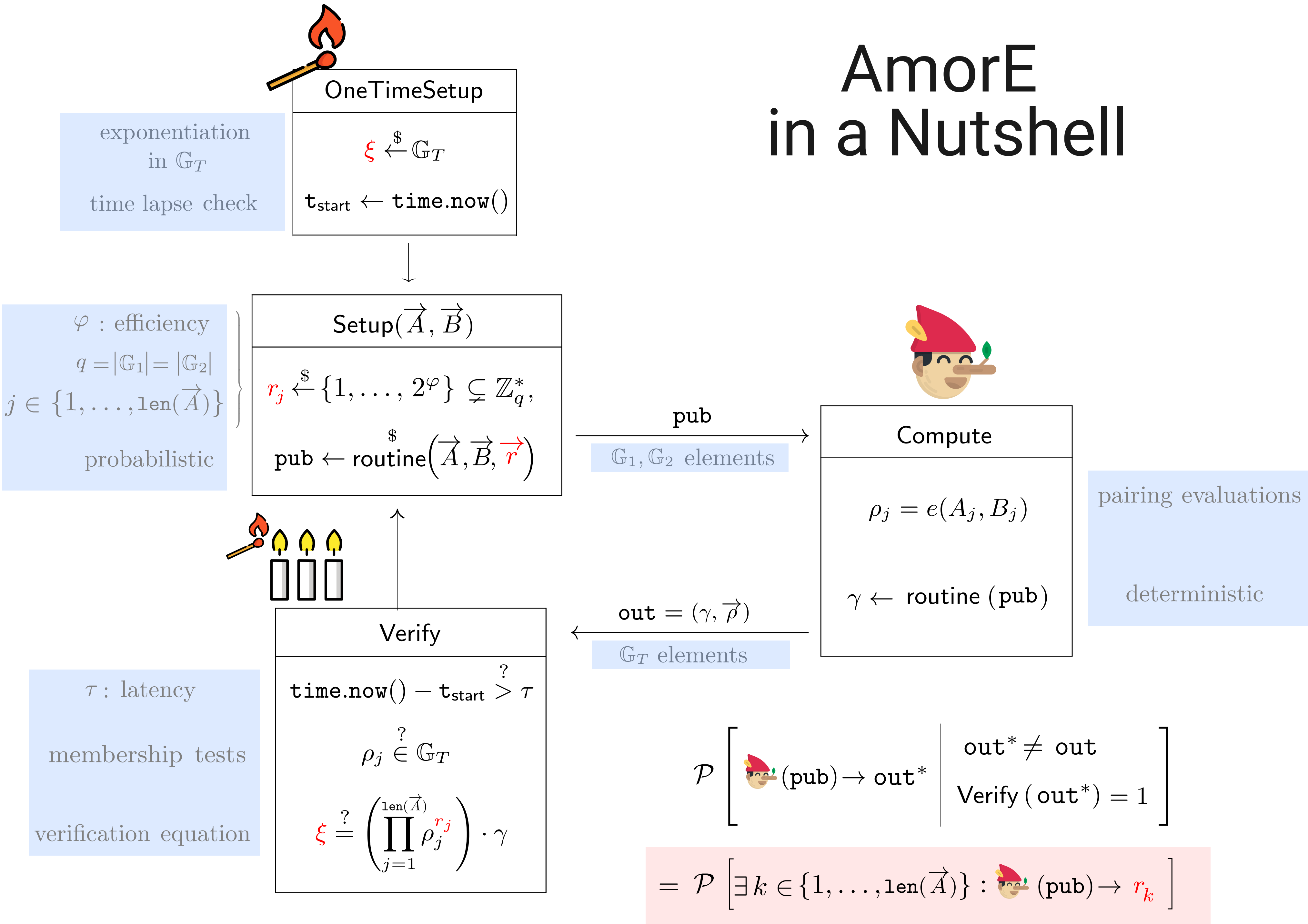
$\mathcal{P}\left[(\mathtt{pub}) \rightarrow \mathtt{out}^* \;\middle|\; \begin{array}{l} \mathtt{out}^* \neq \mathtt{out} \\ \mathsf{Verify}(\mathtt{out}^*) = 1 \end{array} \right]$

AmorE in a Nutshell



Security and Efficiency Trade-Off

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Unconditional

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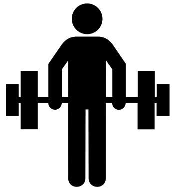
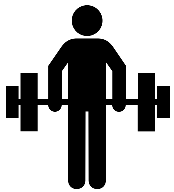


`pub` is expensive

Security and Efficiency Trade-Off

Unconditional $\iff$



`pub` is expensive $\iff$ $\color{red}{\vec{r}}$ masked

Security and Efficiency Trade-Off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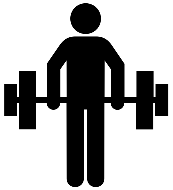
$\Longleftrightarrow$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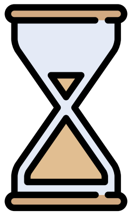
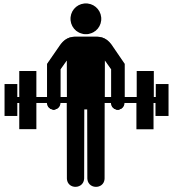
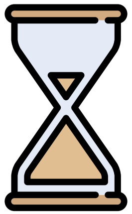
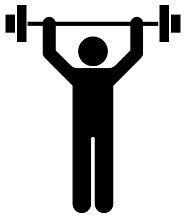
$\Longleftrightarrow$ $\vec{r}$ masked


Time-bound

Security and Efficiency Trade-Off

 $\iff$  $\iff$ $\vec{r}$ masked

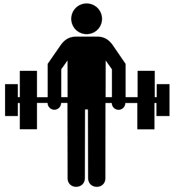
Unconditional — `pub` is expensive

 $\rightsquigarrow$ 

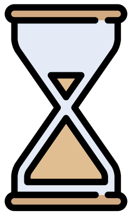
Time-bound — `pub` is cheap

Security and Efficiency Trade-Off

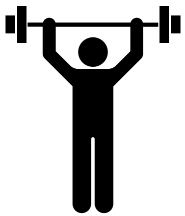
Unconditional $\Longleftrightarrow$



`pub` is expensive $\Longleftrightarrow$ $\vec{r}$ masked



Time-bound $\leadsto$



`pub` is cheap

...but $\vec{r}$ extractable after x computations

Adversary Bound

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$\leq$

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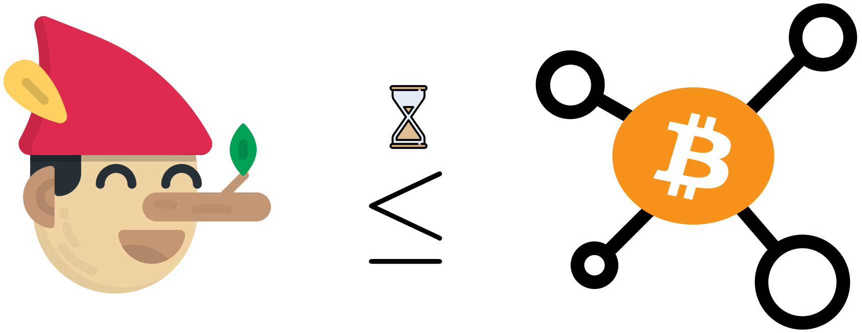
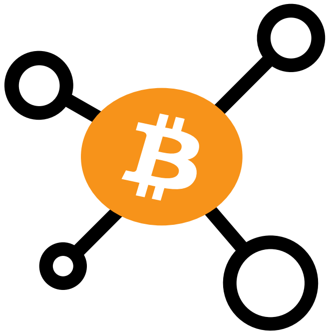


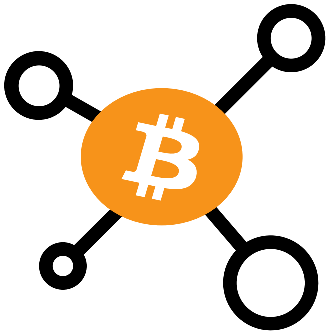
Adversary Bound

$\leq$



$\leq \quad 2^{70}\ \mathrm{H/s}$

Adversary Bound

  $\leq$  $\leq \quad 2^{70}\ \mathrm{H/s} \quad \leq \quad 2^{70}\ \bar{\mathfrak{m}}_{\iota}/\mathrm{s}$

(Fair) assumption:

$$\mathsf{cost}\left(\text{block header hash}\right) < \mathsf{cost}\left(\text{short scalar multiplication in } \mathbb{G}_1\right)$$

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broken after up to $2^{\varphi+1}$ scalar computations in $\mathbb{G}$

$\swarrow$ $\searrow$

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$\hookrightarrow$ if $\varphi = \left\lceil \frac{\sigma - 1}{2} + \kappa \right\rceil$

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↳ if $\varphi = \left\lceil \frac{\sigma - 1}{2} + \kappa \right\rceil$

In this work:

$\tau = 1$	latency
$\kappa = 70$	computational
$\sigma = 40$	statistical
$\varphi = 90$	efficiency

Result Overview

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Curve	Protocol	Client cost	Security
BLS12-381	CDS14	$1.41\ \mathfrak{p}$	
	CKKS20	$2.01\ \mathfrak{p}$	
	LOVE	$1.90\ \mathfrak{p}$	
	AmorE	$0.68\ \mathfrak{p}$	

$\mathtt{len}(\overrightarrow{A}) = 1$

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Table: Amortized Efficiency obtained over $N = 10$ delegations and 40 bits of statistical security (RELIC implementations).

Thank you for your attention :)

Open-source tools used for our presentation:

Inkscape

Sozi

SVG repo