CISPA
HELMHOLTZ CENTER FOR
INFORMATION SECURITY

Non-Interactive Blind Signatures from RSA Assumption and More

L. Hanzlik, E. Paracucchi, R. Zanotto

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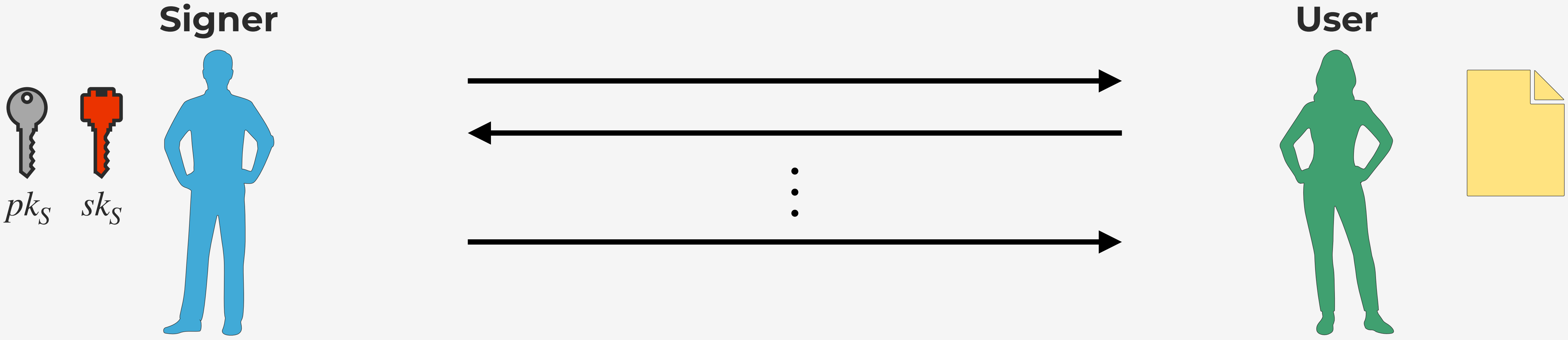
Blind Signatures



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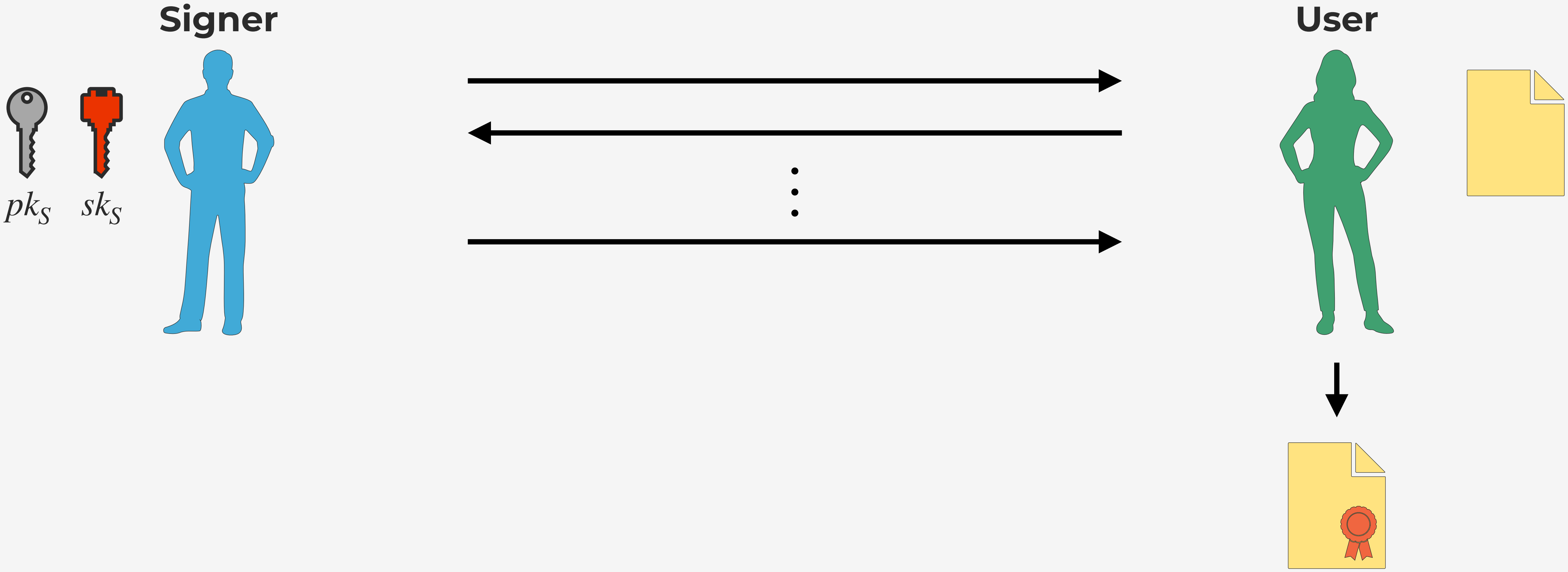

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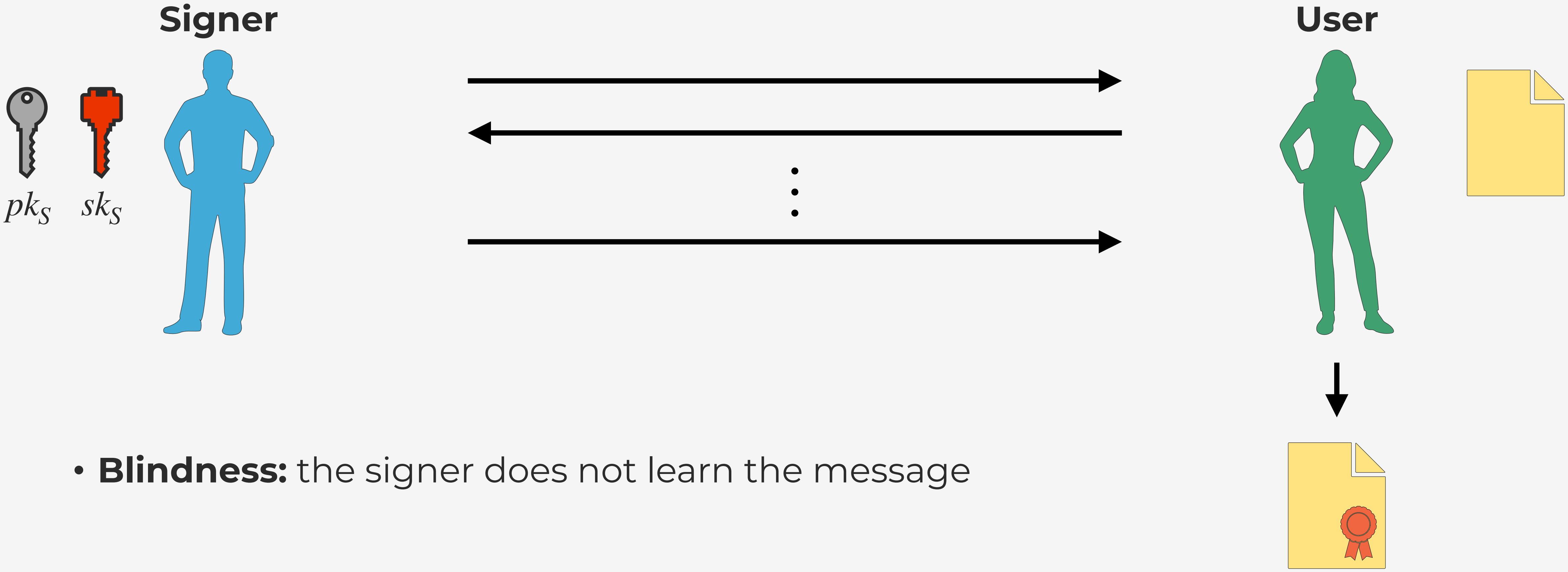

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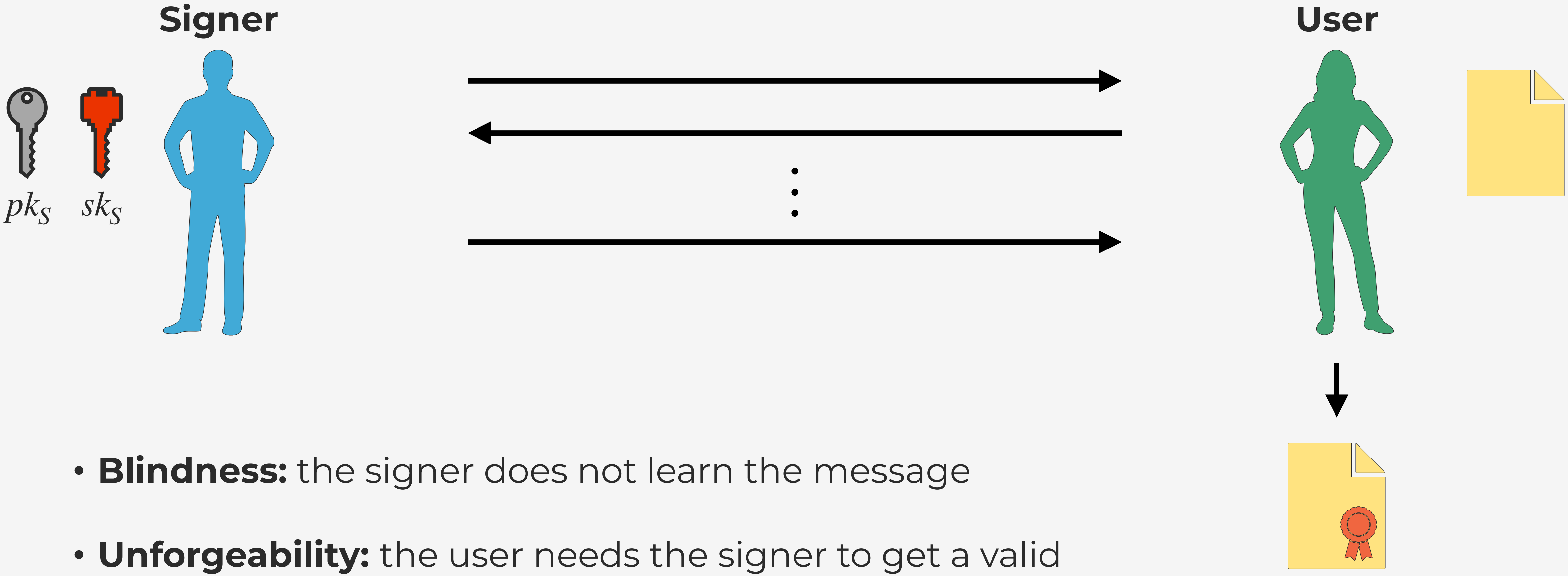

- **Blindness:** the signer does not learn the message
- **Unforgeability:** the user needs the signer to get a valid signature

Applications of BS

Introduced by David Chaum in 1980s. Used as **one-time anonymous tokens**

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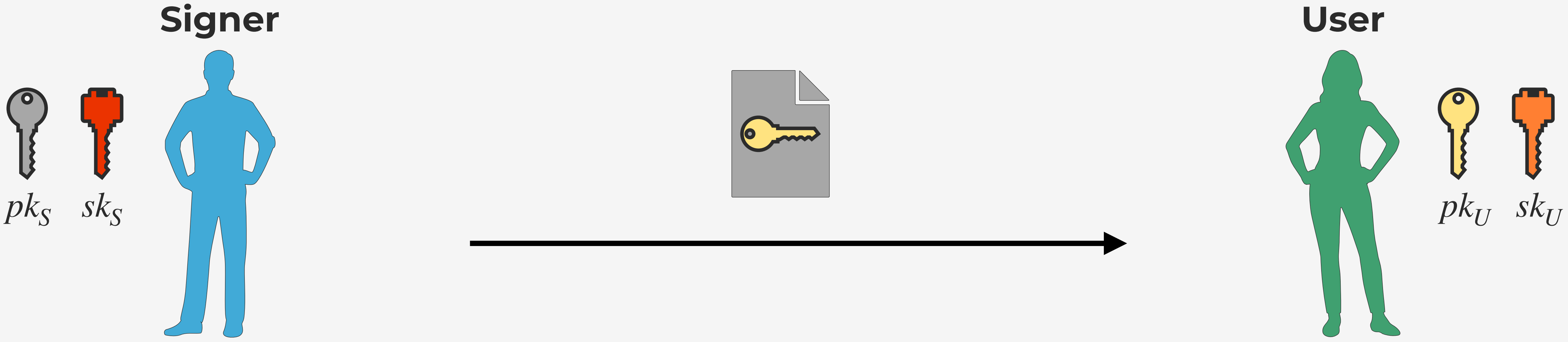
No need of interaction

Non-Interactive Blind Signatures [Han23]



Signer

User

pk_S sk_S

pk_U sk_U

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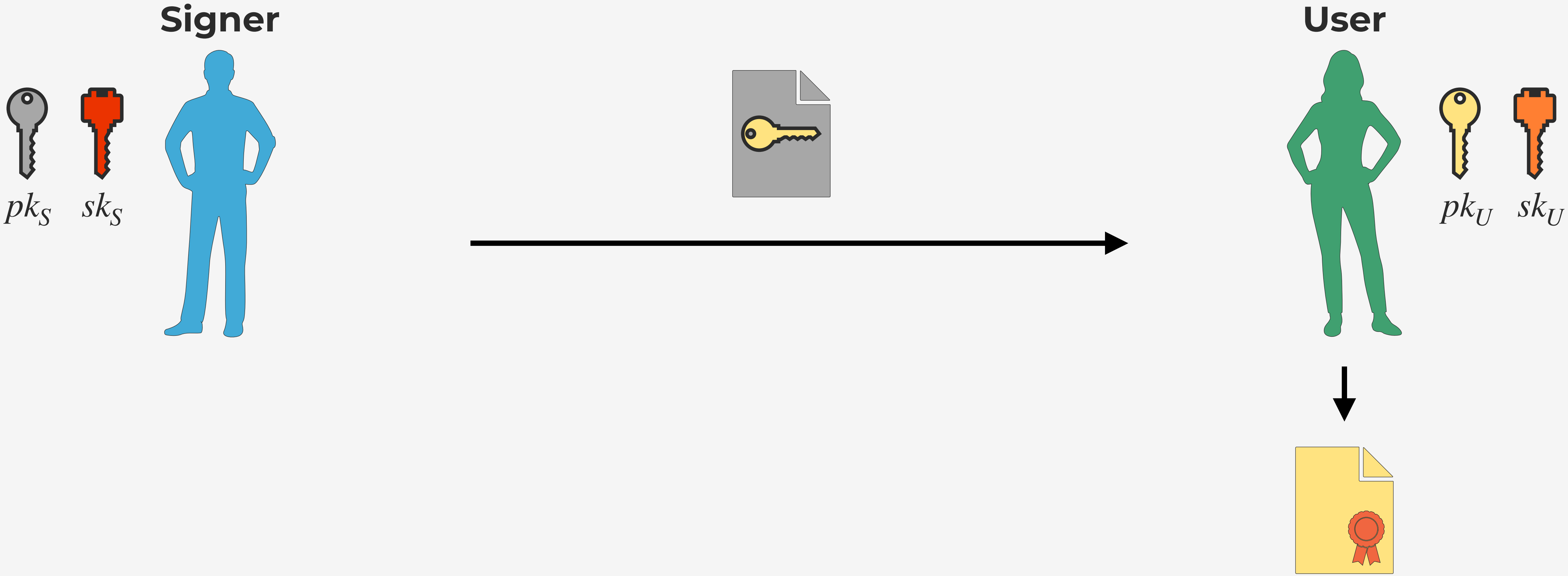

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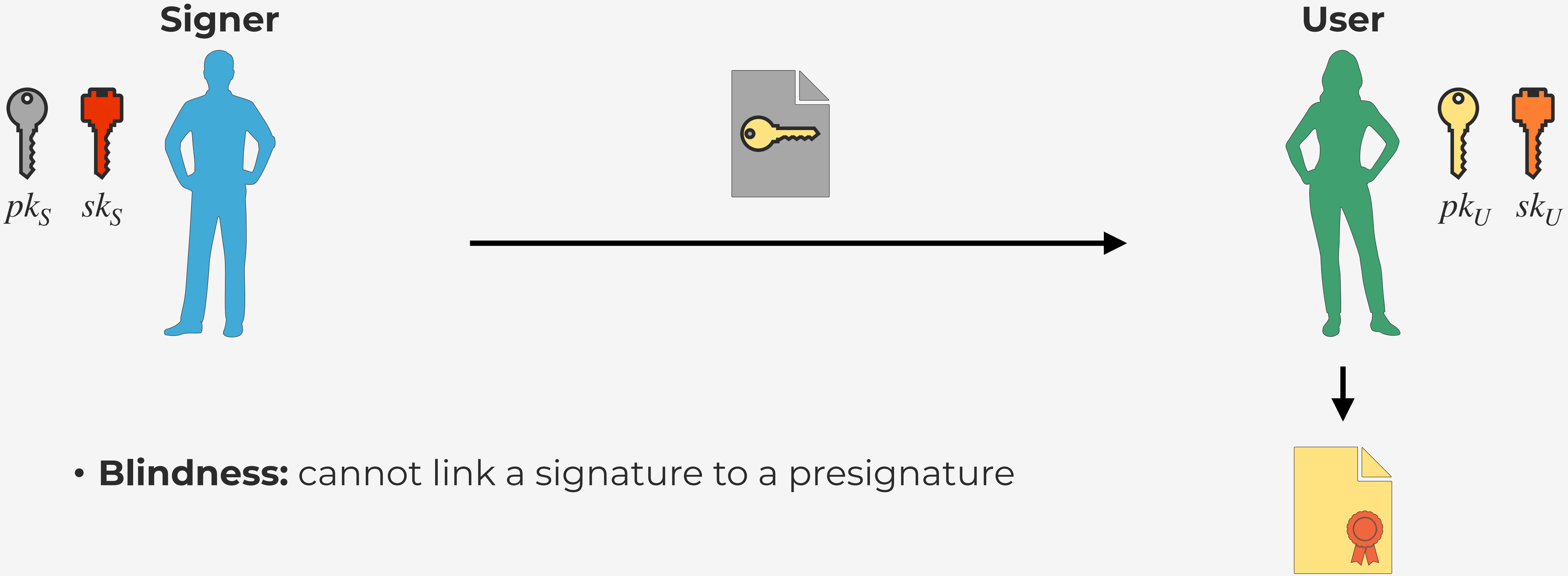

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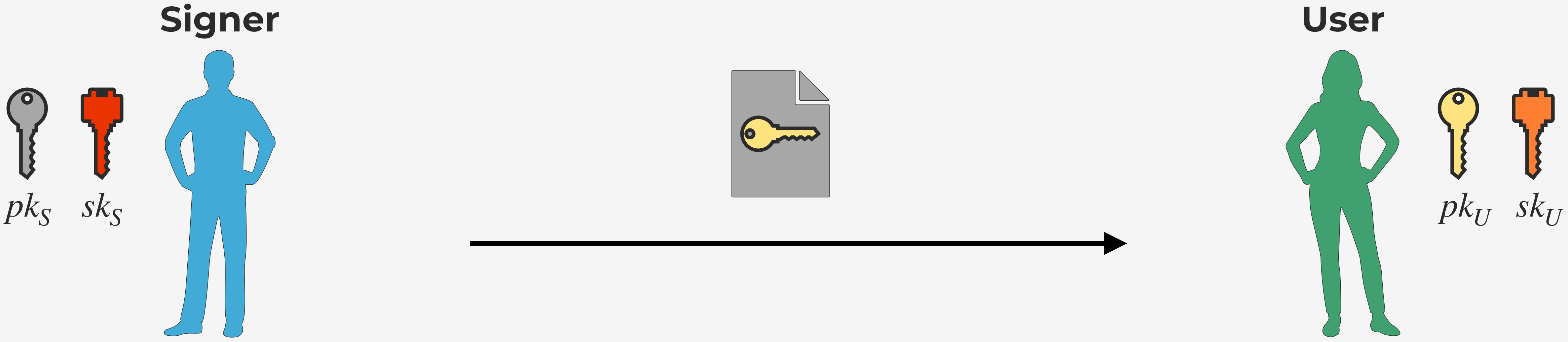

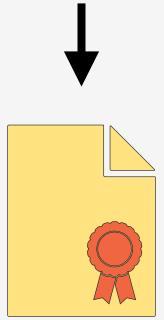

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- **Unforgeability:** cannot create $\ell + 1$ signatures from ℓ presignatures

Airdropping

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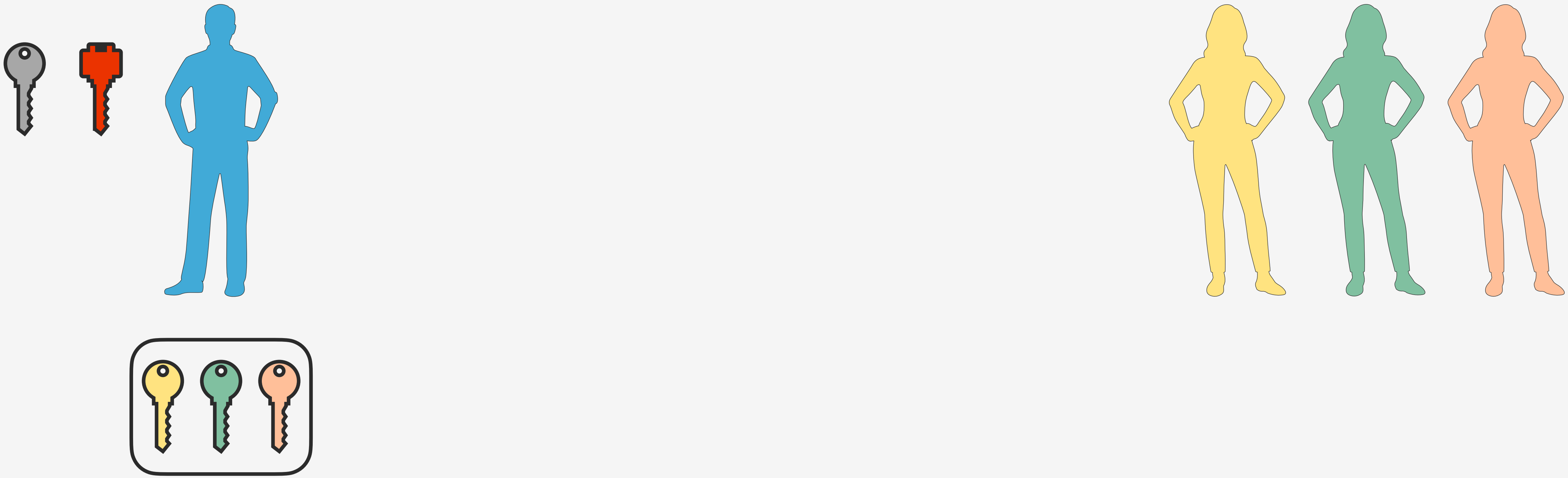
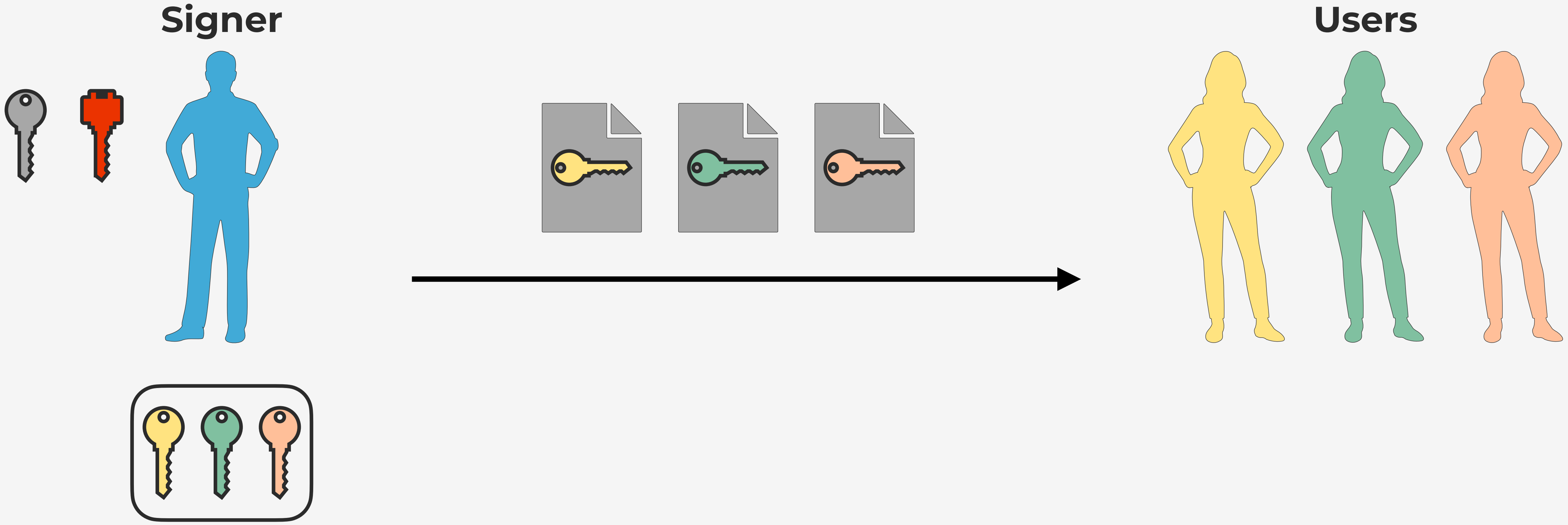


Signer

Users

Selected users public keys

Airdropping



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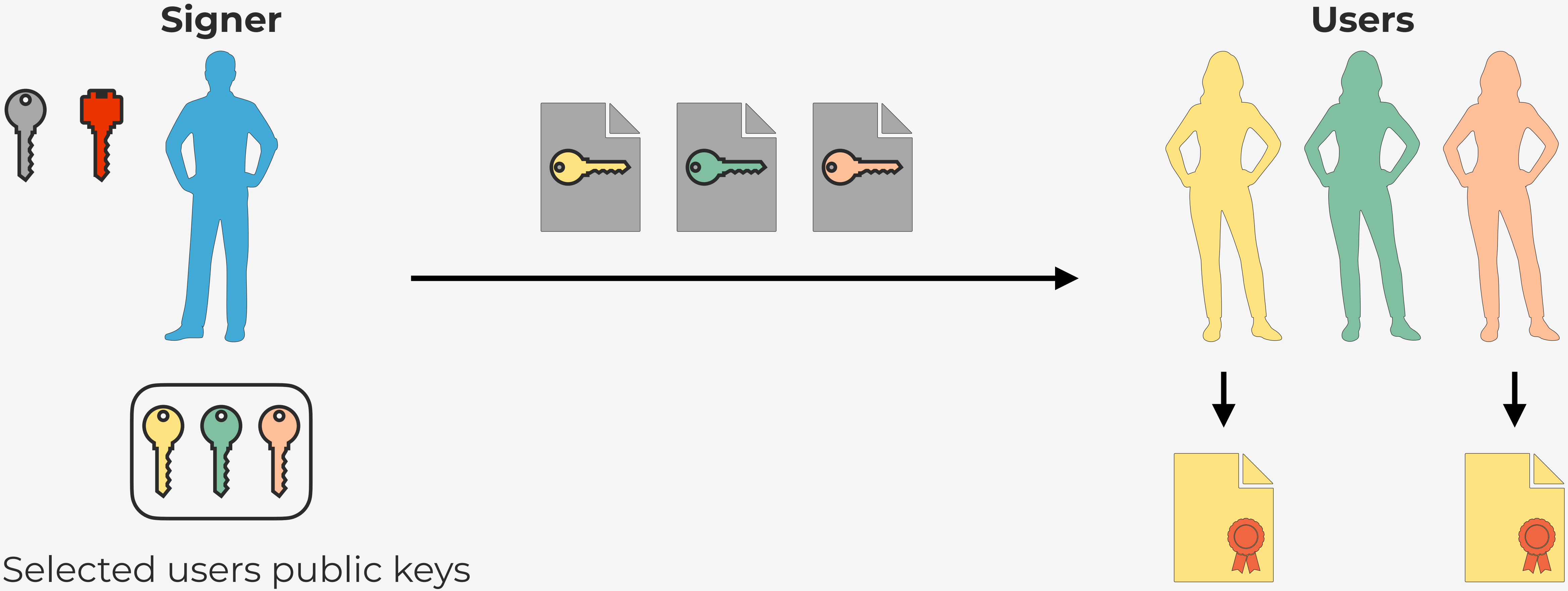

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Challenges

The user’s public key corresponds to a long-term public key for other schemes such as GitHub public keys, PGP keys etc.

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This work: construction of a NIBS compatible with standard RSA keys (N, e)

2PC and Yao’s Garbled Circuits

We need some way for the signer to send obliviously some data to the user

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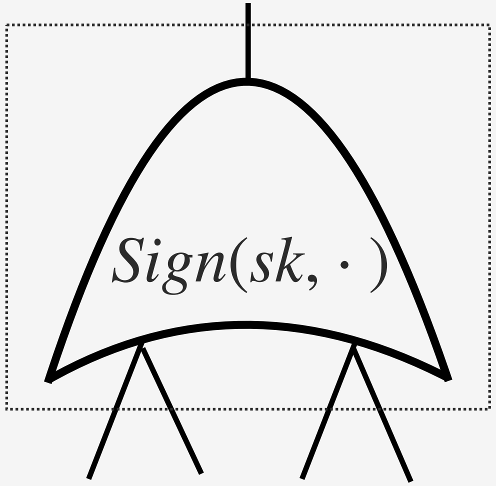

→ **Garbled encoding** of the fuction $Sign(sk, \cdot\,)$

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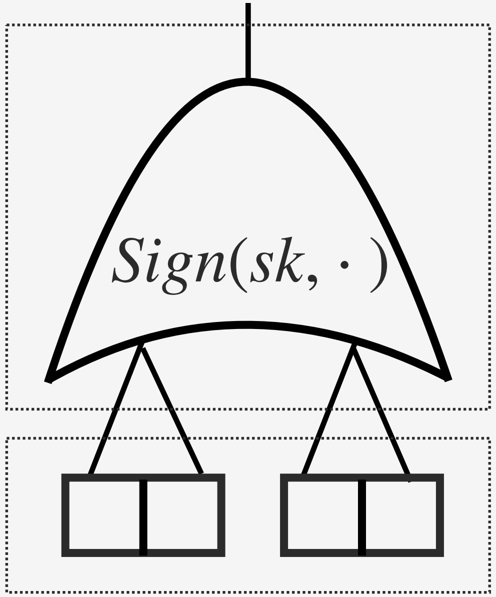

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→ Labels for the circuit sent using **oblivious transfer**

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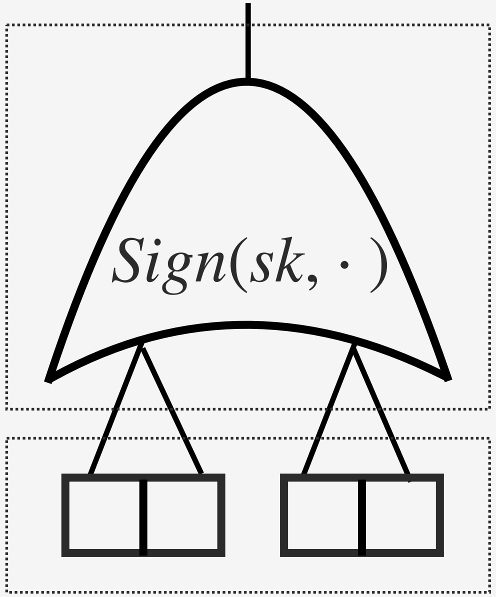

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The construction is interactive!

Non-Interactive Oblivious Transfer (NIOT)

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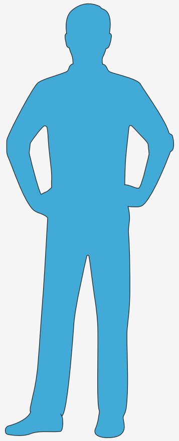
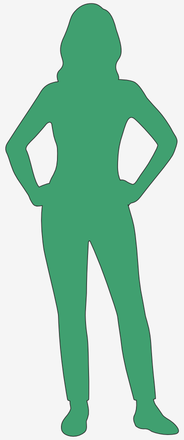
m_0, m_1, pk



Receiver

pk, sk



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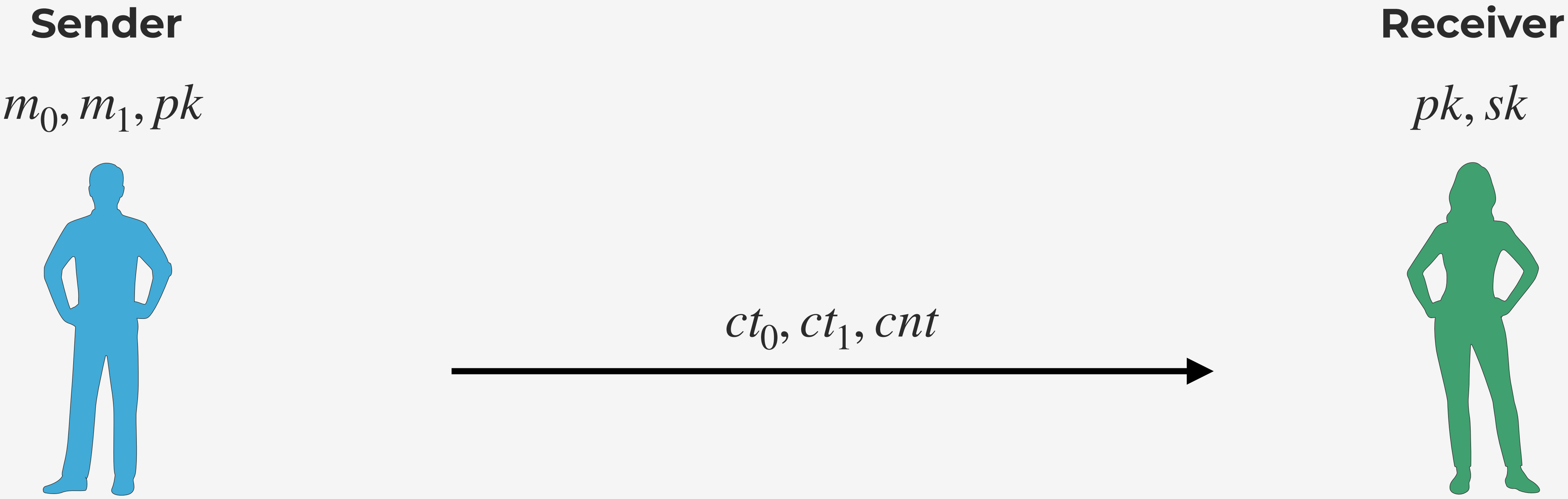

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Receiver

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ct_0, ct_1, cnt

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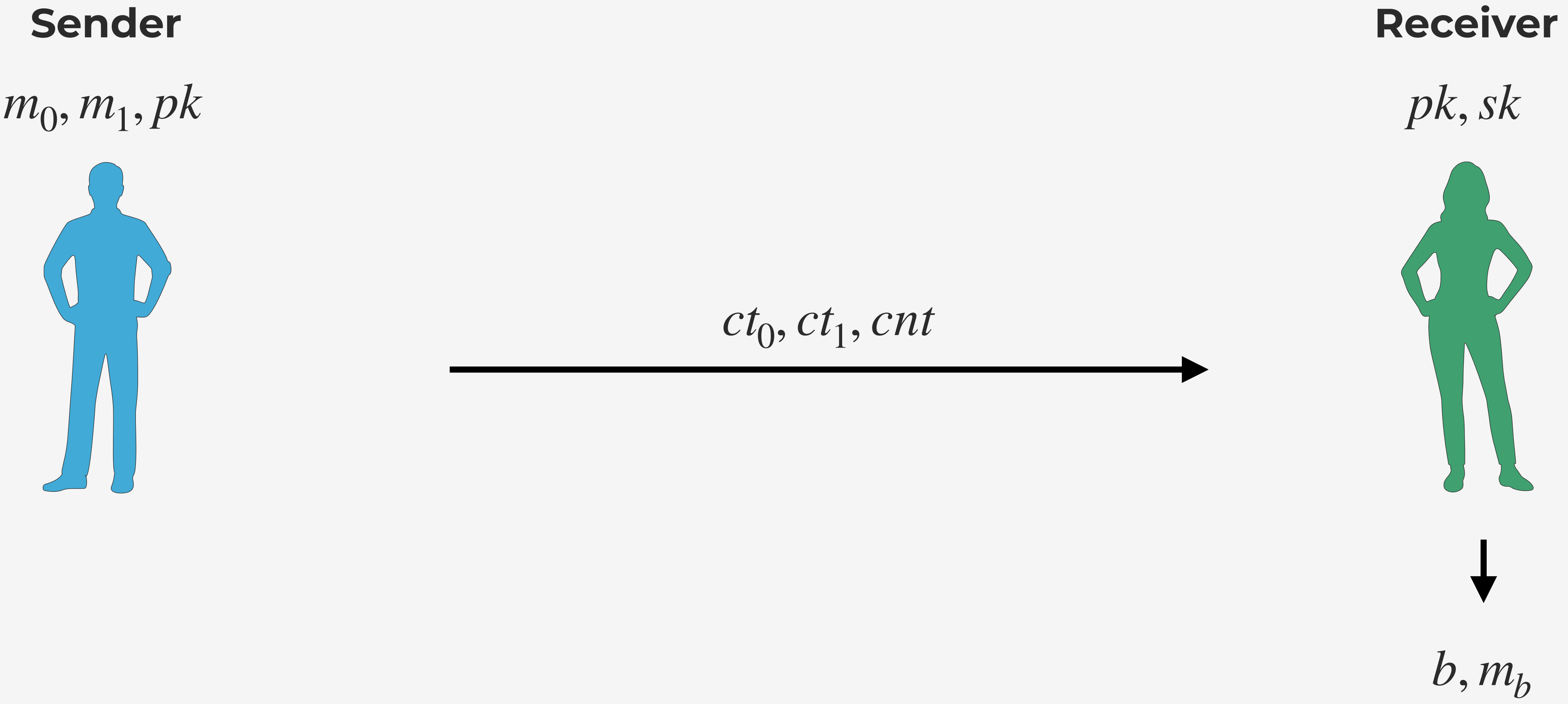

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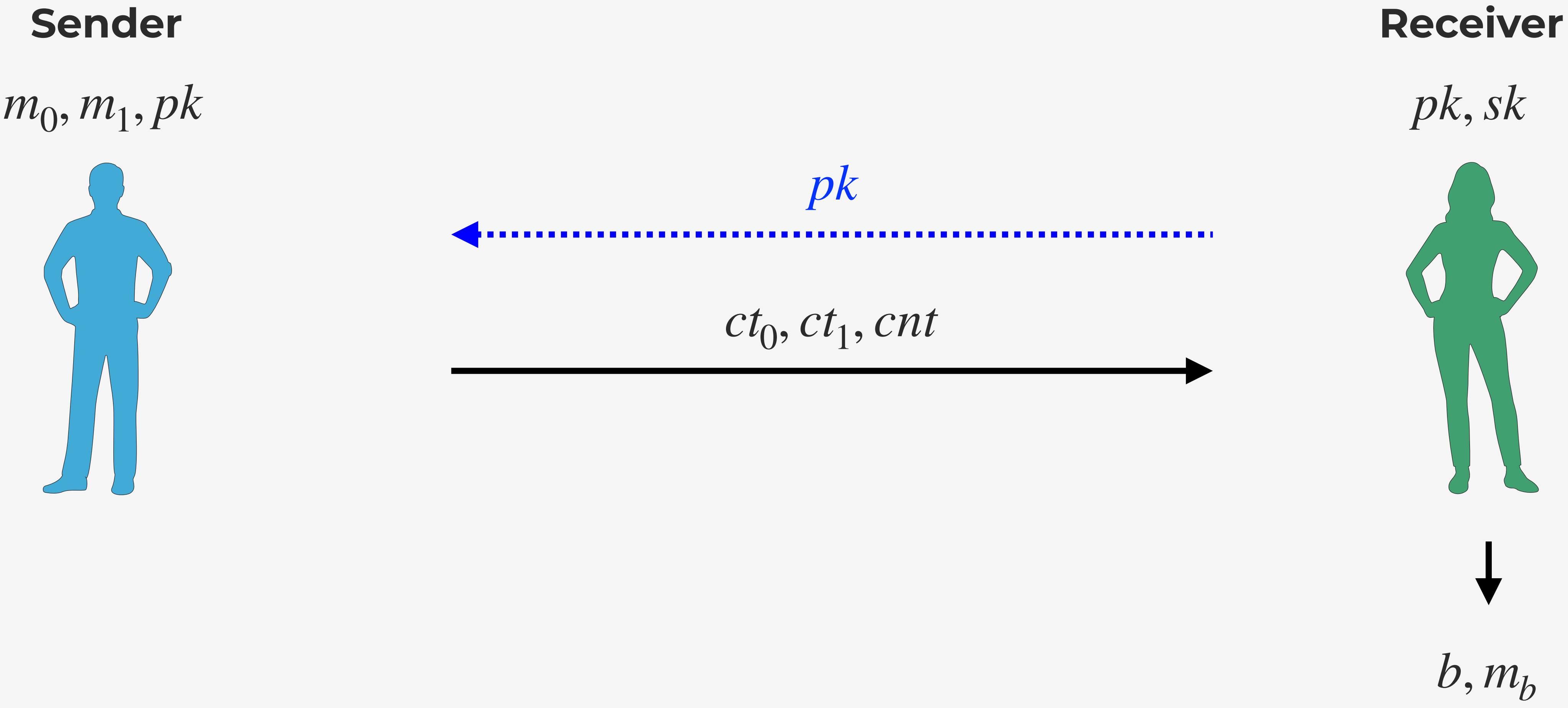

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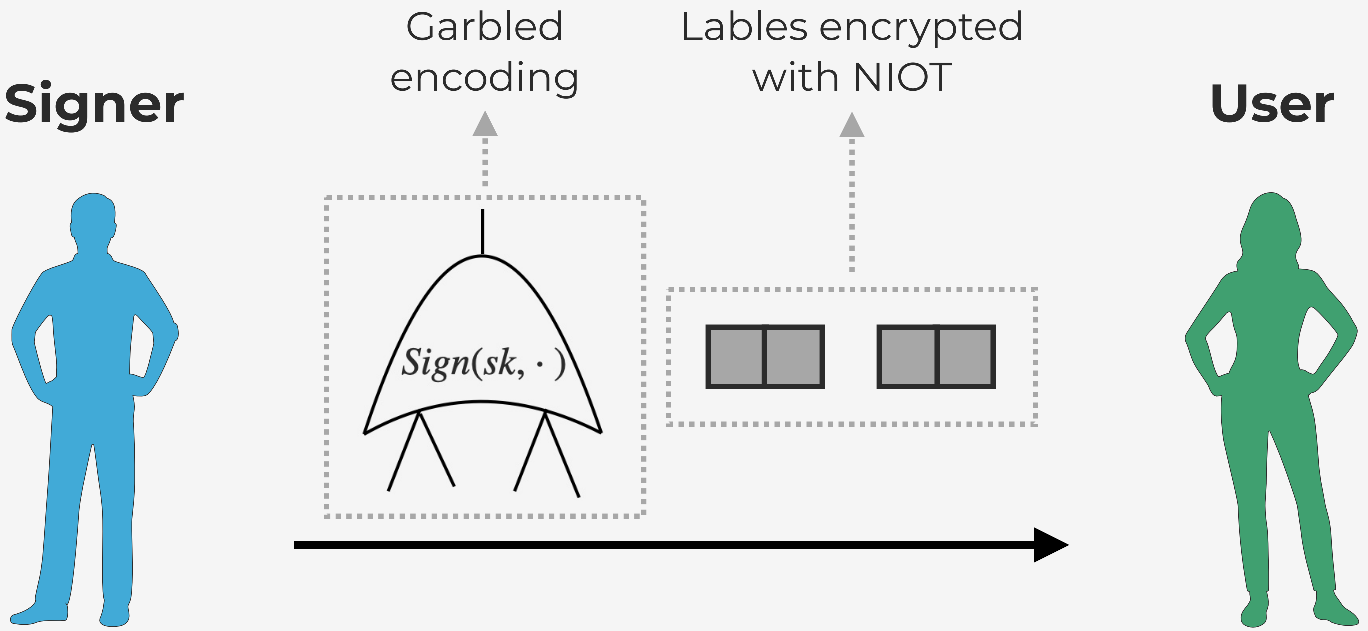

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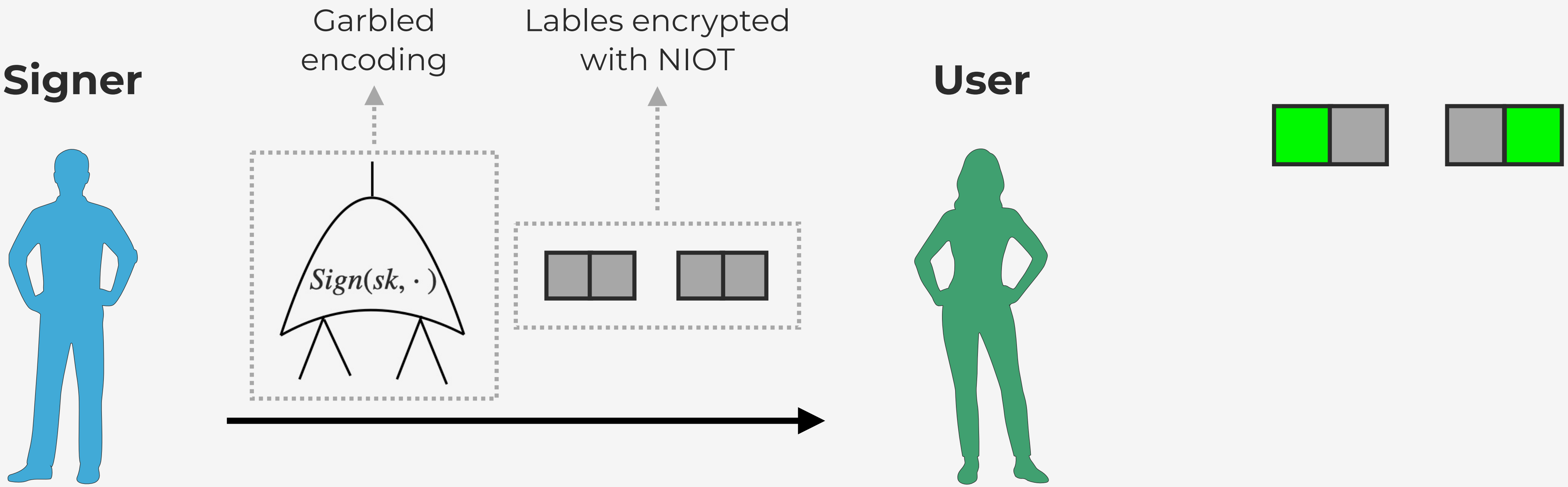

Signer

Garbled encoding

Lables encrypted with NIOT

User

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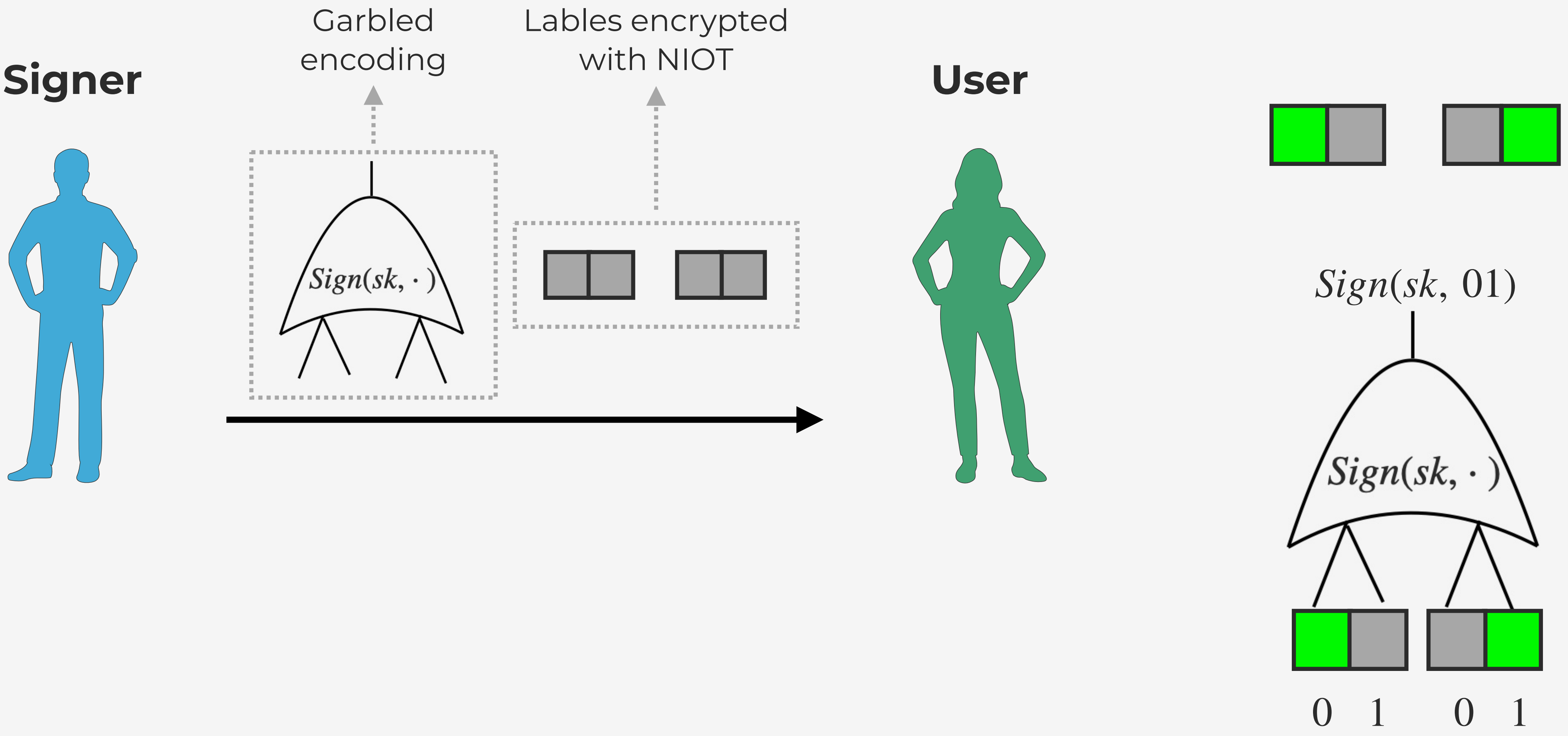

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🔗 Construction of NIOT supporting RSA keys → Rest of this talk

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The public key $N = pq$ must somehow encode the user’s choice in a way that the sender cannot distinguish

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$m_0 \iff$ square $\qquad$ $m_1 \iff$ non-square

Goldwasser-Micali vs Cocks

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pk : $N, x \in \mathbb{Z}_N$ non-square

sk : factorization of N

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Hiding property

If x is a square, $GM.Enc(x, m)$ statistically hides m

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Hiding property

If x is a non-square, and N is squarefree

then $Cocks.Enc(x, m)$ statistically hides m

NIOT Construction, squarefree modulus

Sender

$m_0, m_1, pk = N$

Receiver

$pk = N, sk = fact(N)$

NIOT Construction, squarefree modulus

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$x \leftarrow \mathscr{H}_N(N, cnt)$

$ct_0 \leftarrow Cocks.Enc(x, m_0)$

$ct_1 \leftarrow GM.Enc(x, m_1)$

$$\xrightarrow{\quad ct_0, ct_1, cnt \quad}$$

NIOT Construction, squarefree modulus

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$$\xrightarrow{\quad ct_0, ct_1, cnt \quad}$$

$x \leftarrow \mathscr{H}(N, cnt)$

if x is a square:

$m \leftarrow Cocks.Dec(x, ct_0)$

else:

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Generic Modulus

If N is not squarefree then a malicious receiver might decrypt both ciphertexts

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Idea: encrypt the ciphertext ct_0, ct_1 with a key k that can be recovered only if N is squarefree

Generic Modulus

If N is not sqf $\Rightarrow d = gcd(N, \phi(N)) > 1 \Rightarrow$ the equation $X^N = a$ has (zero or) at least $d > 1$ solutions in $\mathbb{Z}_N^*$

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- If N is sqf then w.h.p. $gcd(N, \phi(N)) = 1$ then the receiver can recover from b_i's the (unique) a_i's and hence k
- Otherwise the equation $X^N = b_i$ has more than d solutions, hence will recover the right a_i with probability less than $1/d$. Therefore a malicious can decrypt with probability less than $(1/d)^\lambda$

NIOT Construction, generic modulus

Sender

$m_0, m_1, pk = N$

$x \leftarrow \mathscr{H}_N(N, cnt)$

$ct_0 \leftarrow Cocks.Enc(x, m_0)$

$ct_1 \leftarrow GM.Enc(x, m_1)$

$a_1, \ldots, a_\lambda \leftarrow \mathbb{Z}_N$

$k \leftarrow \mathscr{H}(a_1, \ldots, a_\lambda)$

$b_i \leftarrow a_i^N$ for $i = 0, \ldots, \lambda$

$$\xrightarrow{\begin{array}{c} Enc_k(ct_0), Enc_k(ct_1), cnt, \\ b_1, \ldots, b_\lambda \end{array}}$$

Receiver

$pk = N, sk = fact(N)$

From b_i recover k

Recover ct_0, ct_1

$x \leftarrow \mathscr{H}(N, cnt)$

if x is a square:

$m \leftarrow Cocks.Dec(x, ct_0)$

else:

$m \leftarrow GM.Dec(x, ct_1)$

Wrapping it up

⭐ We have built a **NIOT supporting RSA user’s public key**

This gives us, combined with efficient garbling of signing functions, the **first NIBS compatible with standard RSA keys**

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🧩 **Generic paradigm** to constuct NIBS

🔮 Future work:

- Post-quantum construction
- More applications

PS Signature

Public parameters: p, G_1, G_2, G_T, e

$KeyGen$: sample $g \leftarrow G_2$ and $(x, y) \leftarrow \mathbb{Z}_p^2$, set $X = g^x$ and $Y = g^y$. Return the pair $pk = (X, Y),\ sk = (x, y)$

$Sign(sk, m)$: sample $h \leftarrow G_1$ and output $\sigma = (h, h^{x+ym})$

$Verify(pk, m, \sigma)$: parse $\sigma = (\sigma_1, \sigma_2)$ and check if $e(\sigma_1, X \cdot Y^m) = e(\sigma_2, g)$

Garbling PS Signature

We garble the second component σ_2

Let $\ell = \lfloor \log p \rfloor$, we consider the binary decomposition of $m = m_1 \ldots m_\ell$

Compute $a_1, \ldots, a_\ell \in G_1$ such that $\prod_{i=0}^{\ell} a_i = 1_{G_1}$, set $d = a_0 \cdot h^x$

For $i = 1, \ldots, \ell$ define $s_i^0 = a_i$ and $s_i^1 = a_i \cdot h^{2^{i-1}y}$

Derive ciphertexts $ct_i^0 = Enc(k_i^0, s_i^0)$ and $ct_i^1 = Enc(k_i^1, s_i^1)$ for some keys k_i^0, k_i^1

Garbled function: $\{ct_i^0, ct_i^1\}_i, h, d$

Labels: $\{k_i^0, k_i^1\}_i$