WHIR 

Proximity testing for Reed–Solomon+

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Motivation

SNARKs

Succinct Non-interactive Arguments of Knowledge

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Want to show “knowledge” of w s.t. $(x, w) \in R$

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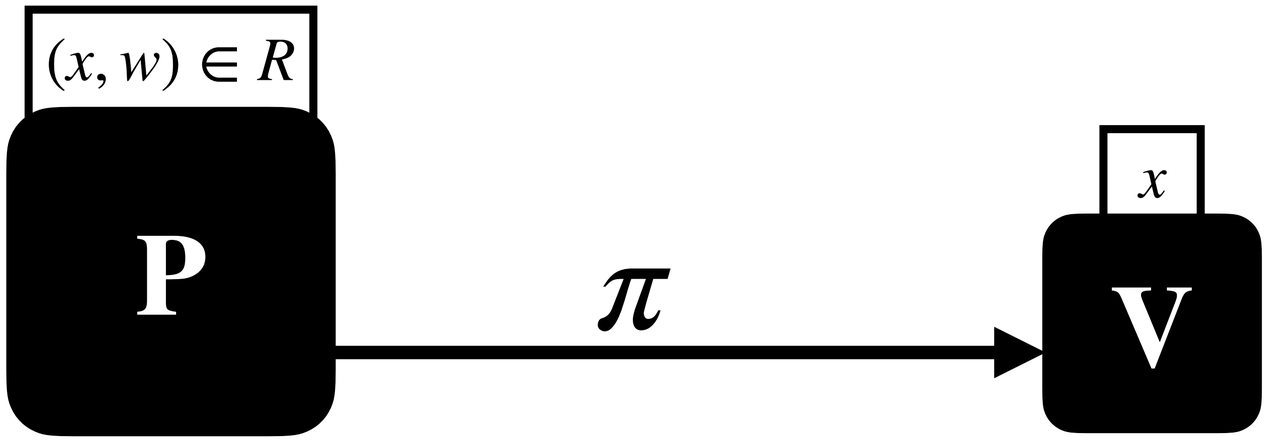

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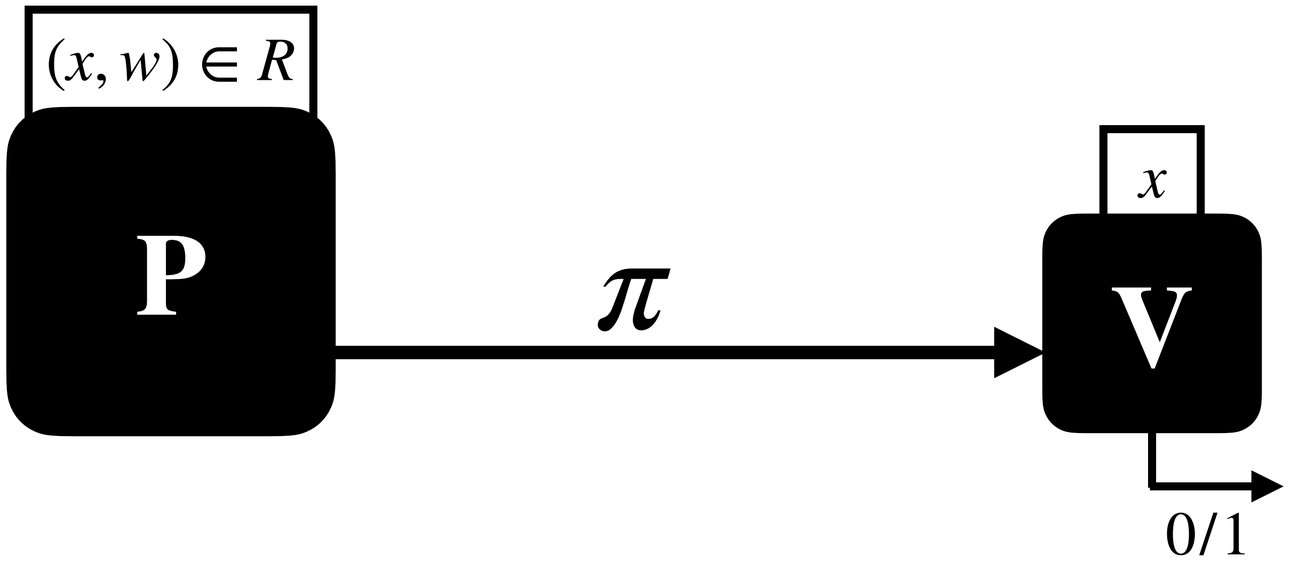

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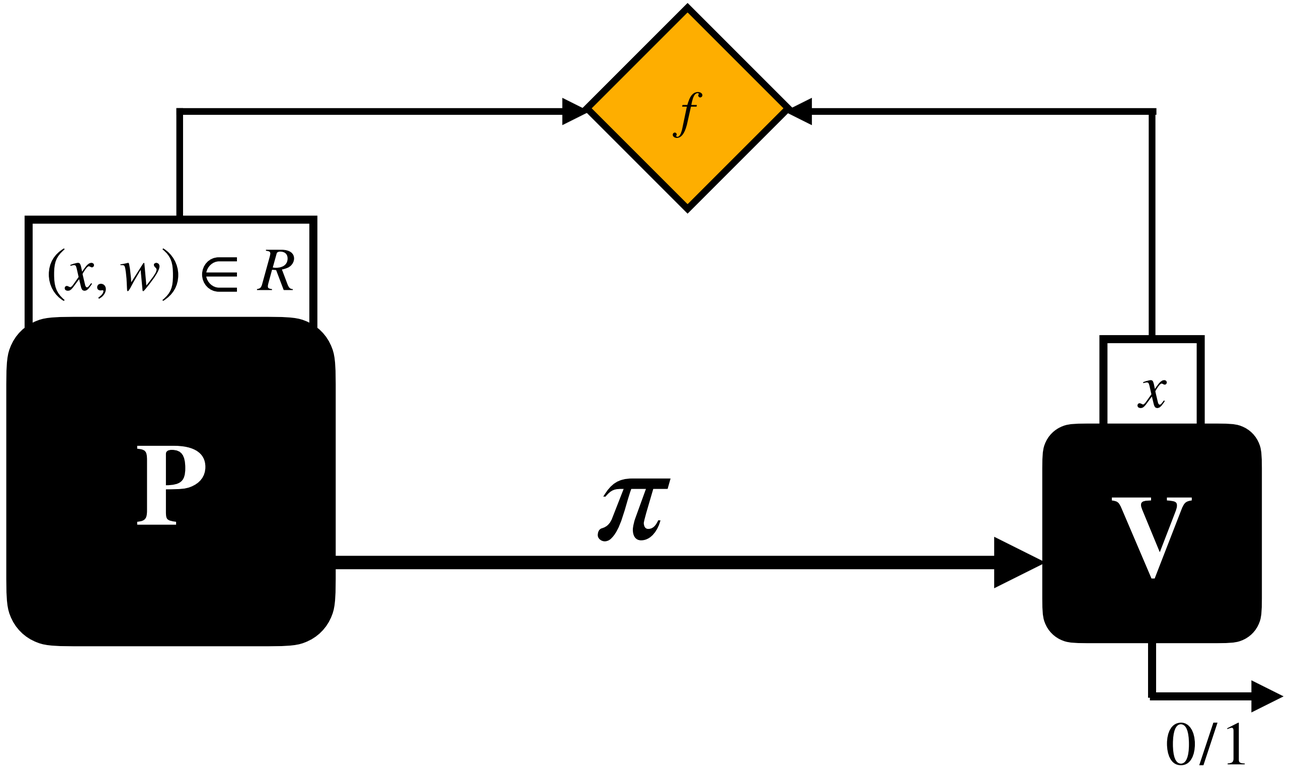

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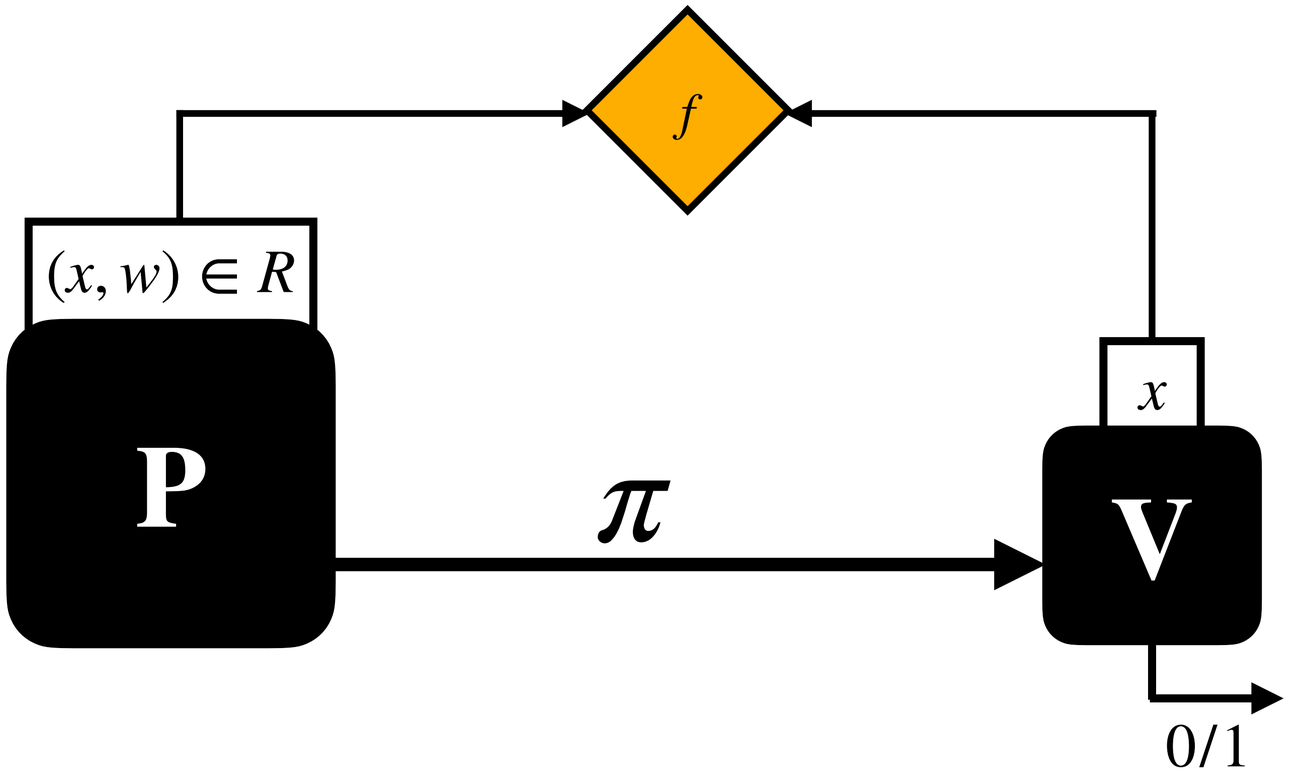

Today: we focus on

hash-based SNARKs

i.e. sound in the **pure** random oracle model.

Hash-based SNARKs

In practice

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Used to secure billions of dollars in real-world blockchains:

STARKWARE

polygon

Irreducible

LAMBDA CLASS

zkSync

dYdX

Matter Labs

RISC ZERO

Succinct

And many more…

Constructing SNARKs

[BCS16] Construction

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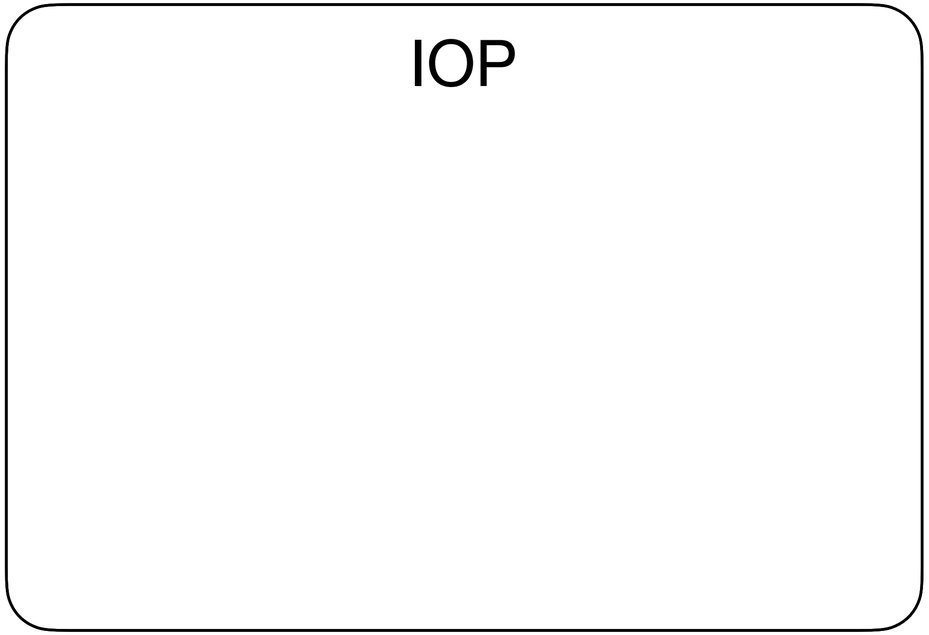

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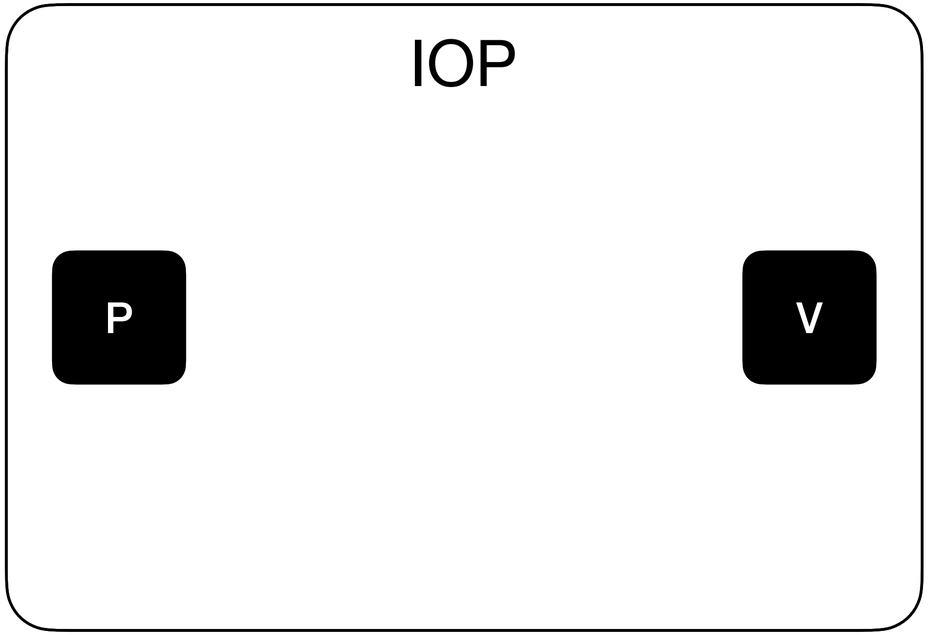

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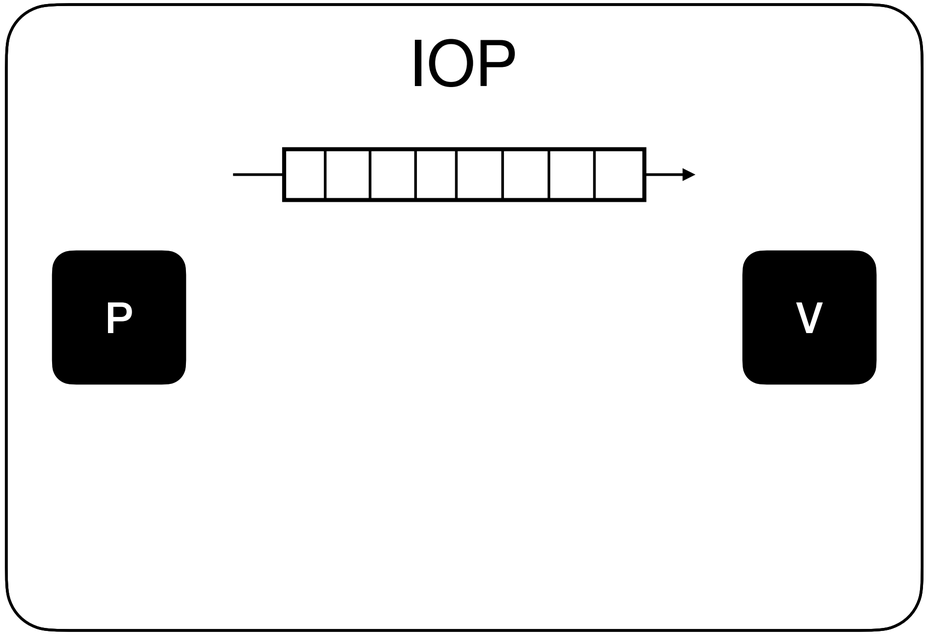

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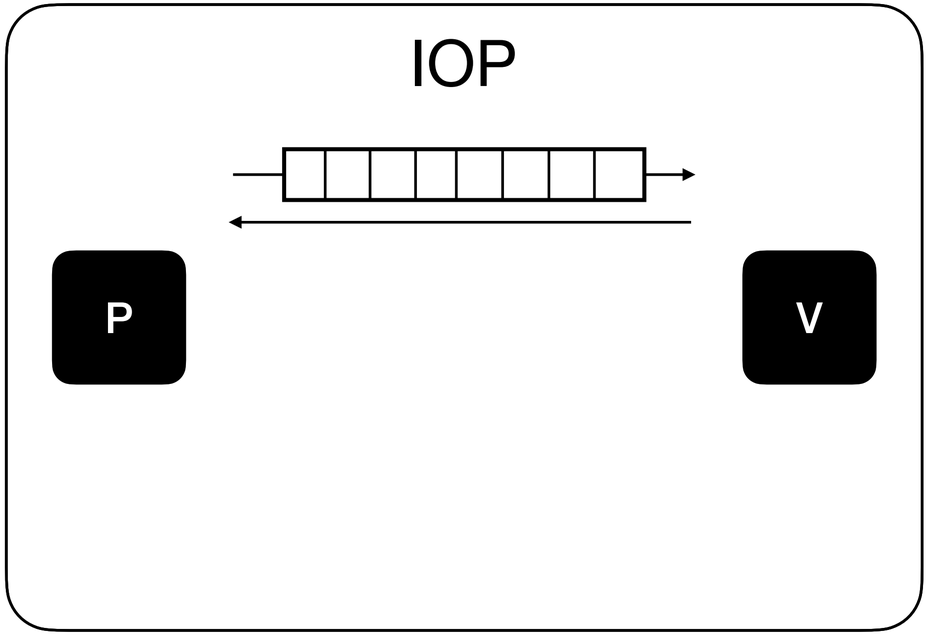

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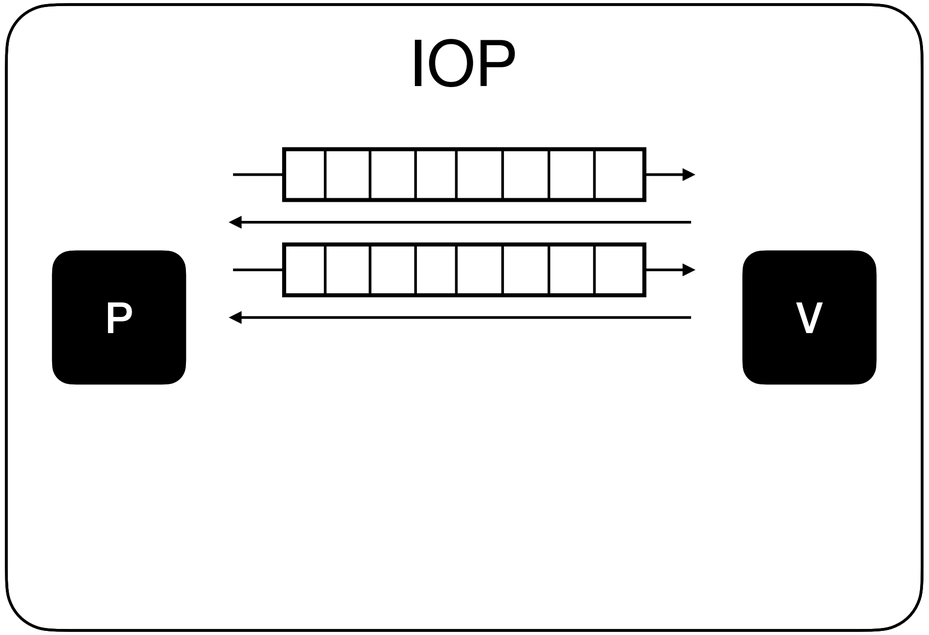

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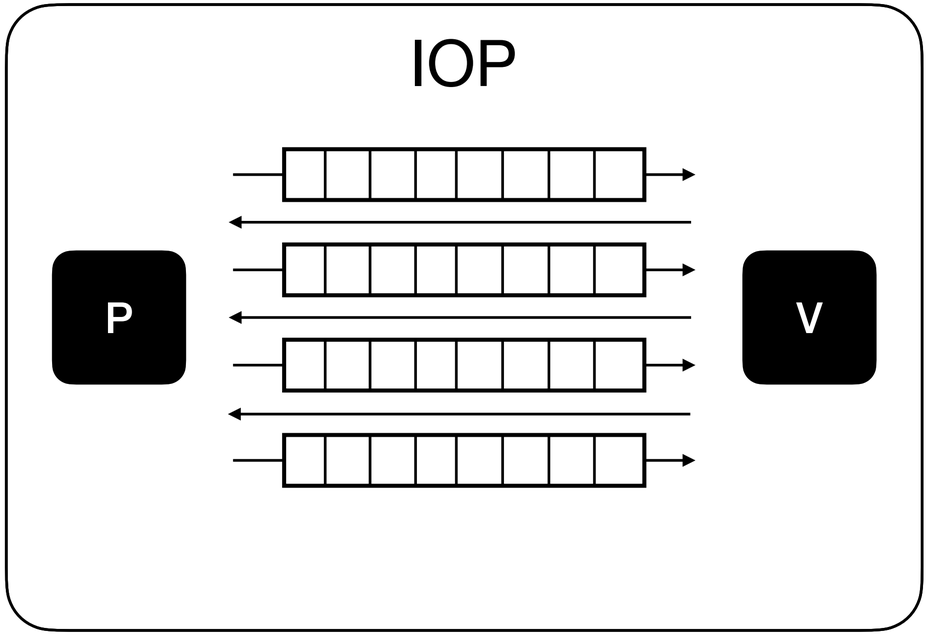

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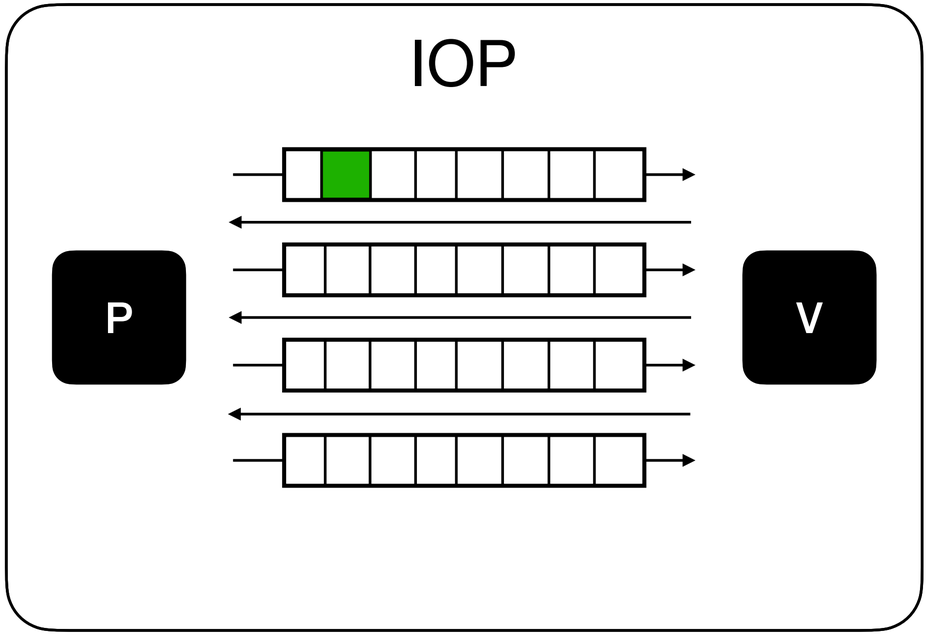

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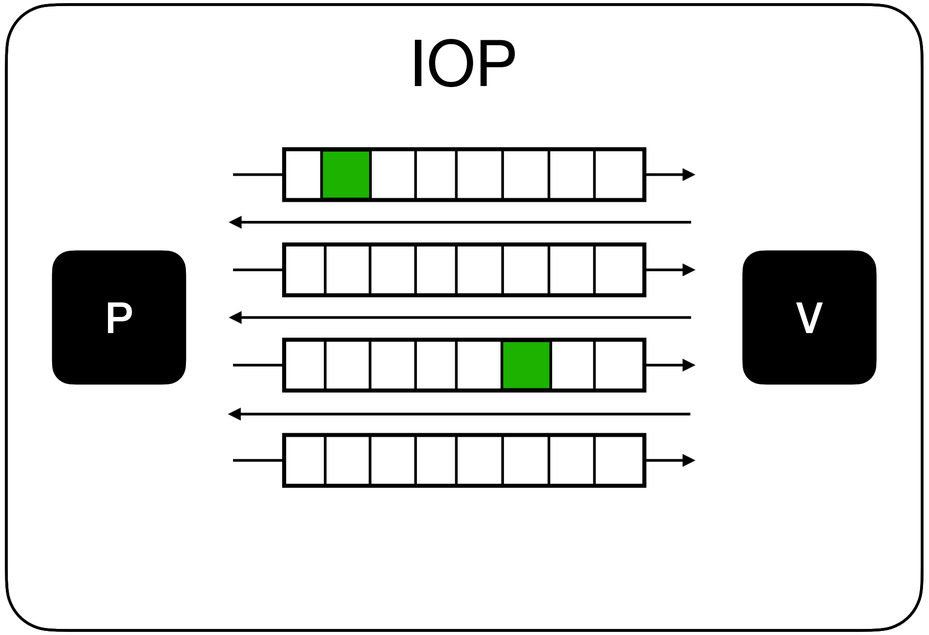

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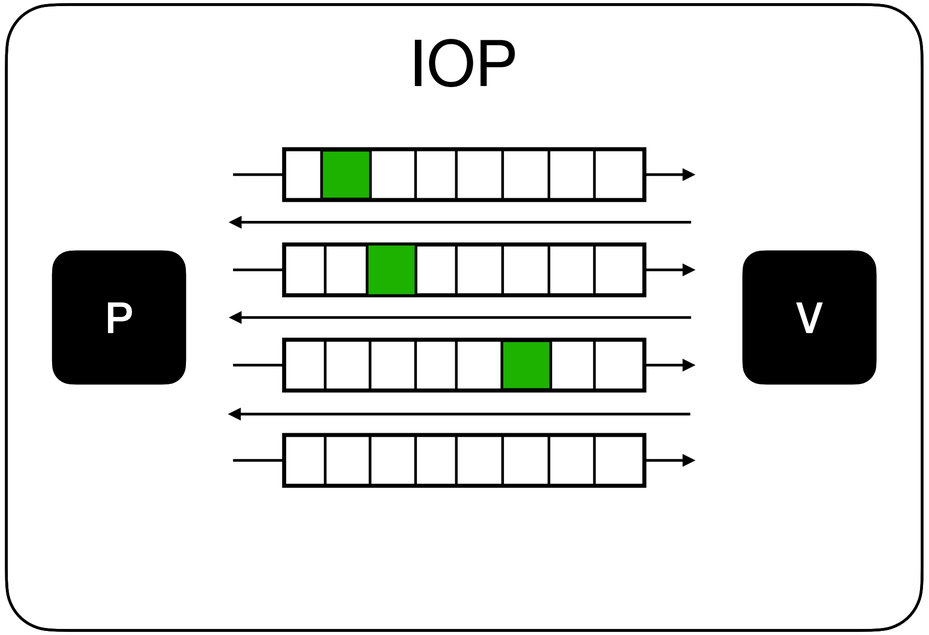

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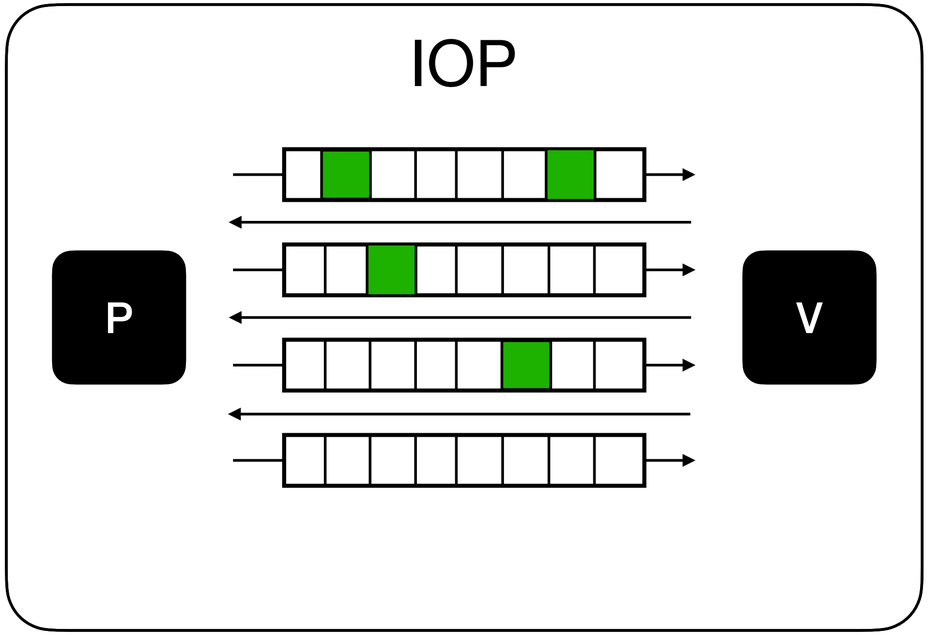

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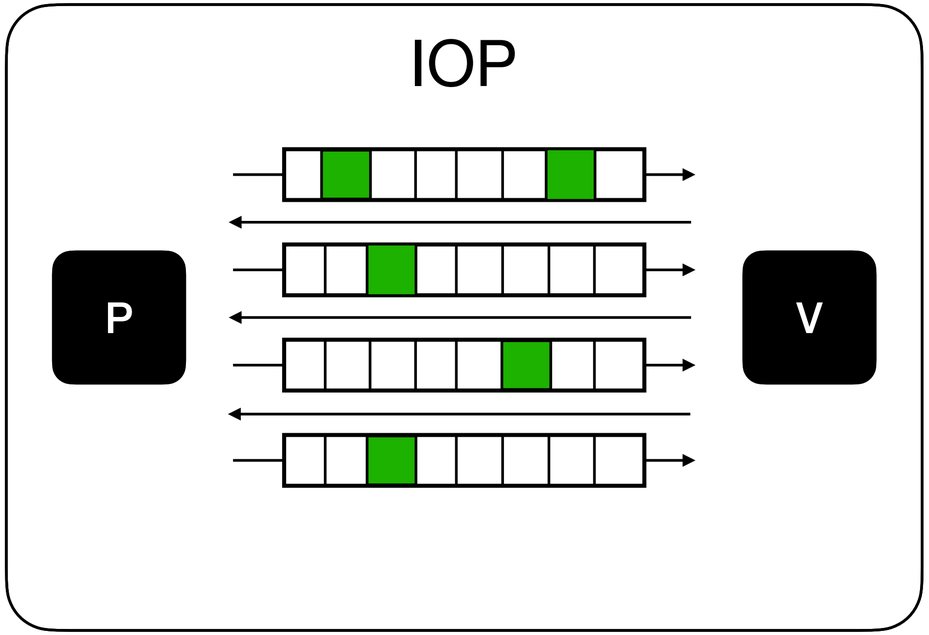

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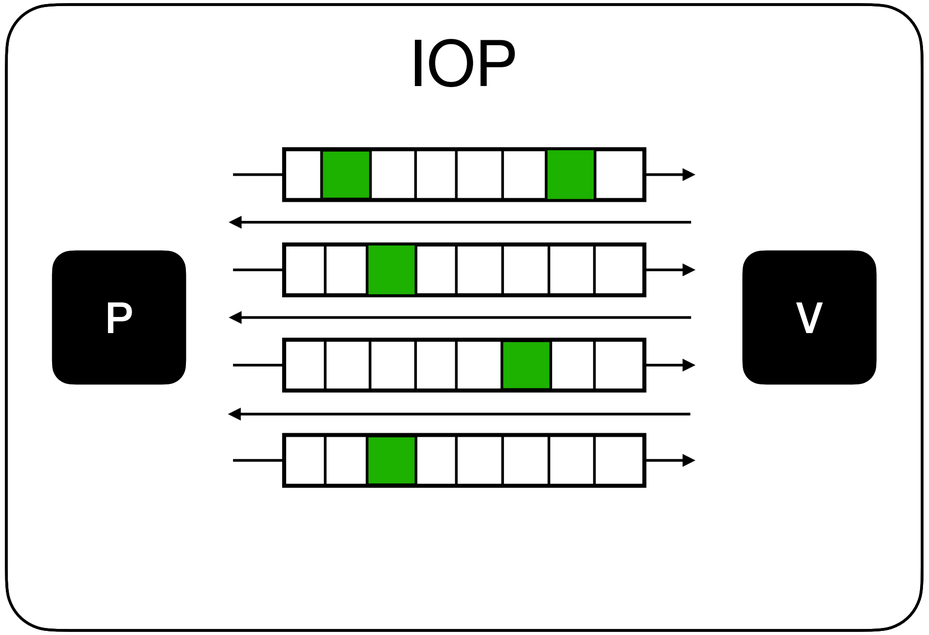

BCS construction:
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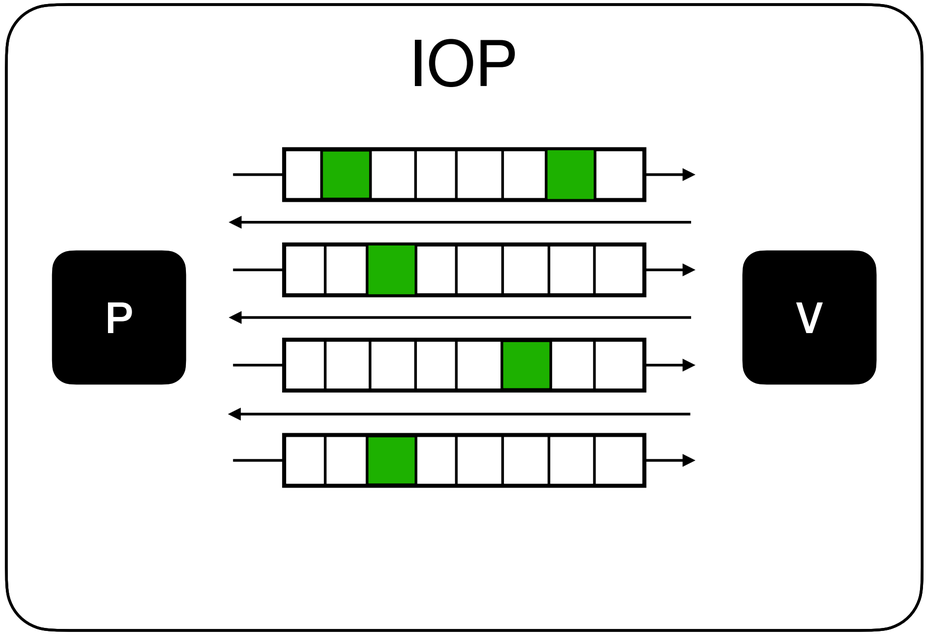

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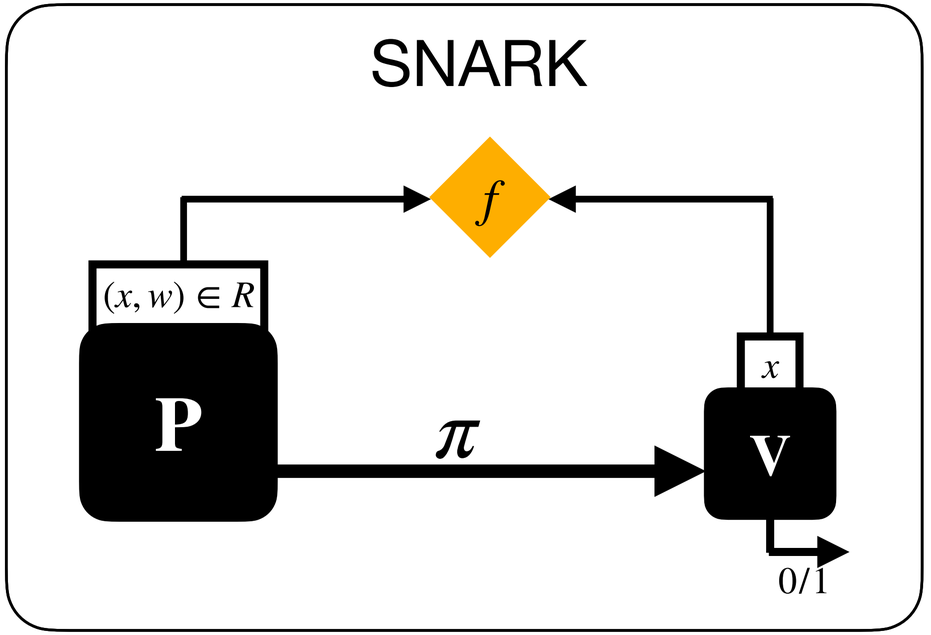

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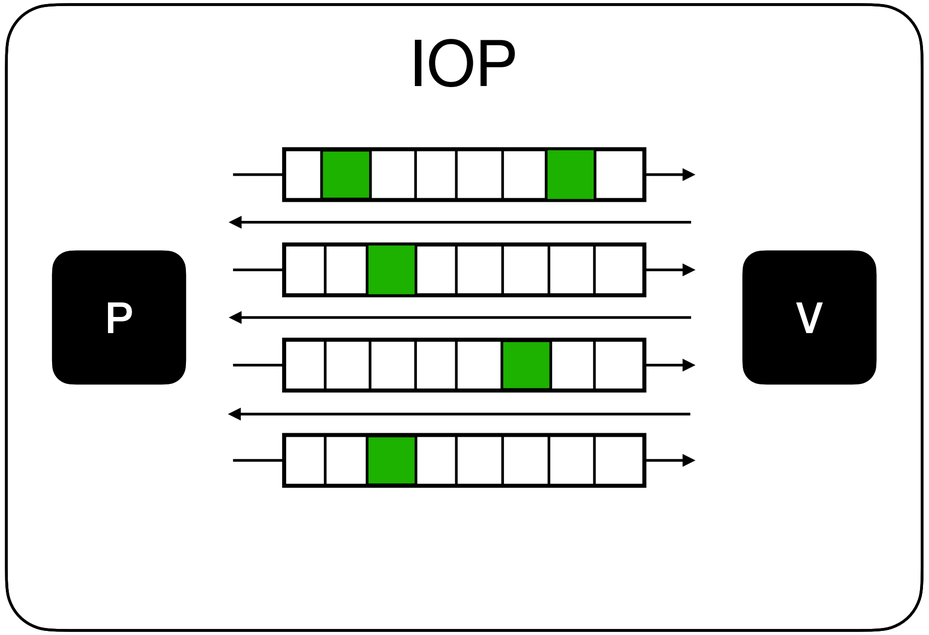

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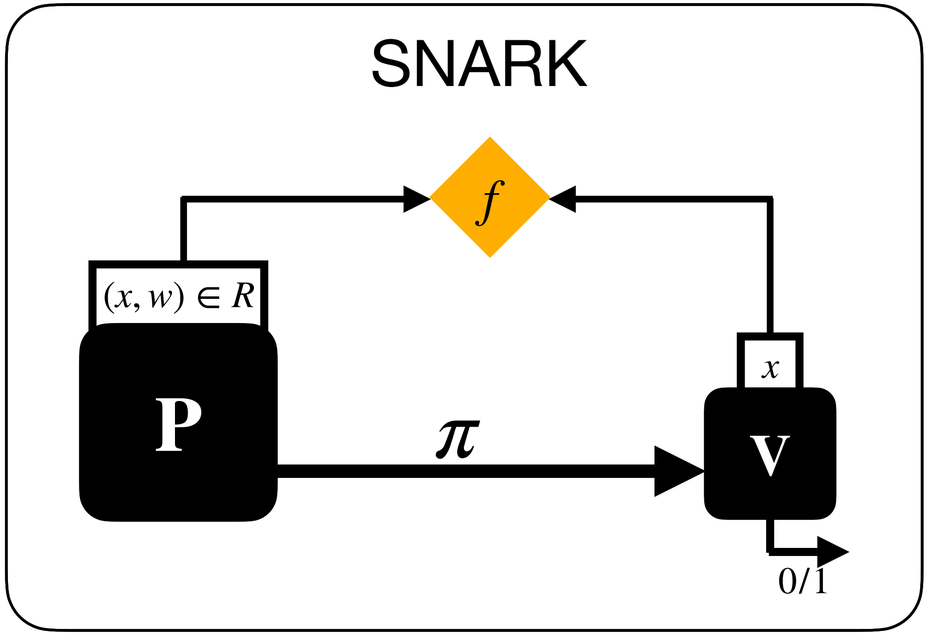

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Queries q $\approx O(\log n)$

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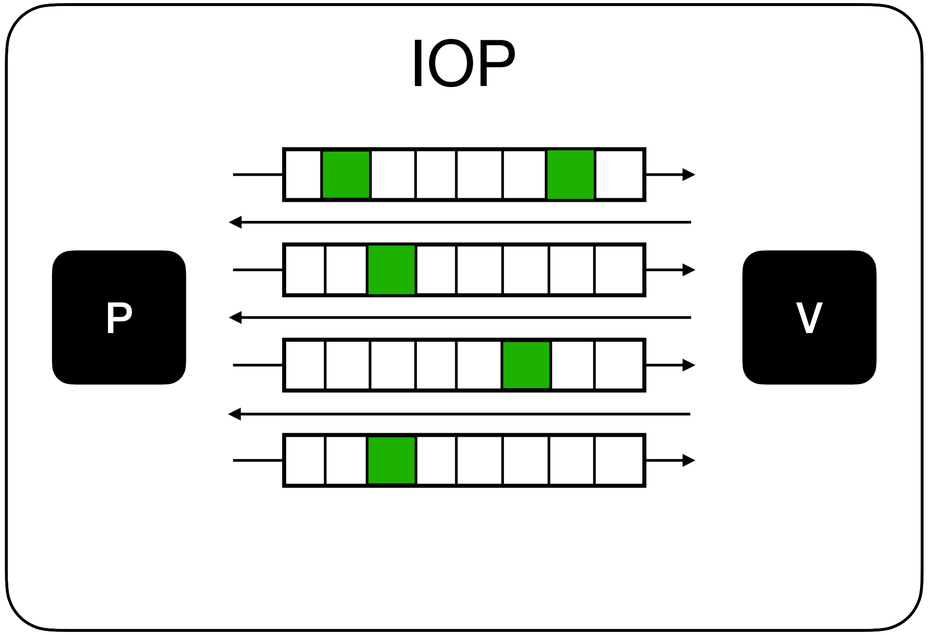

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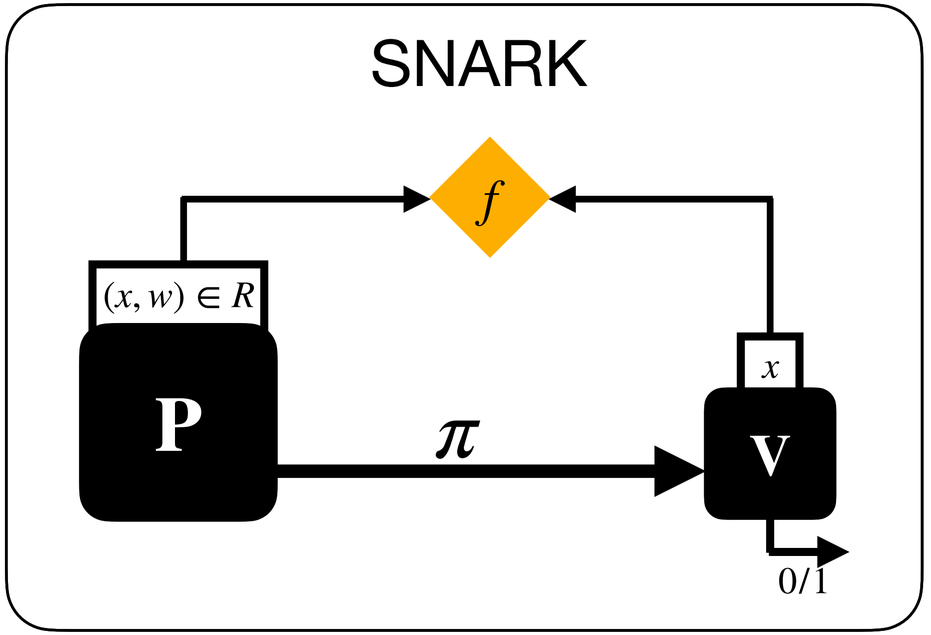

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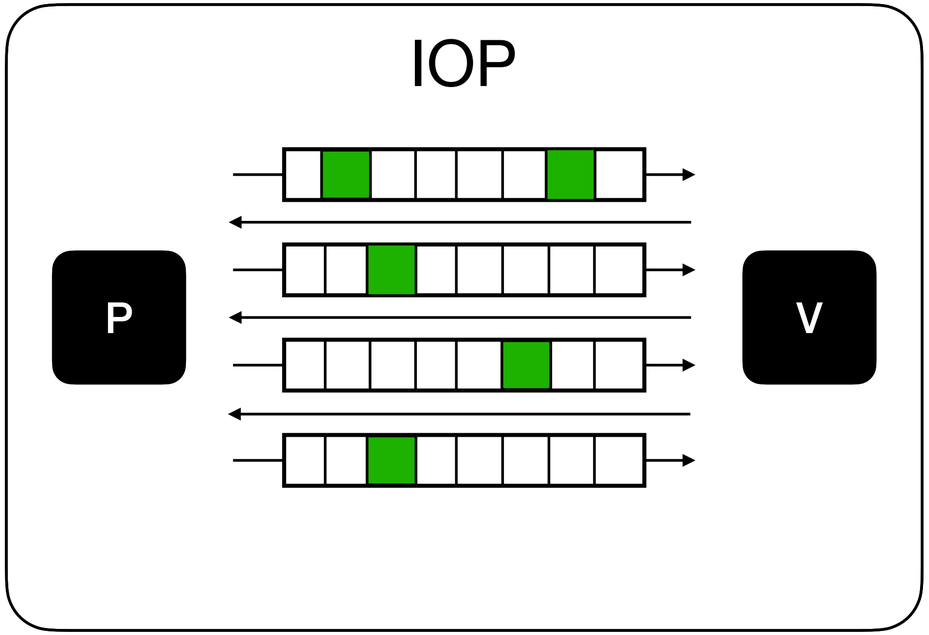

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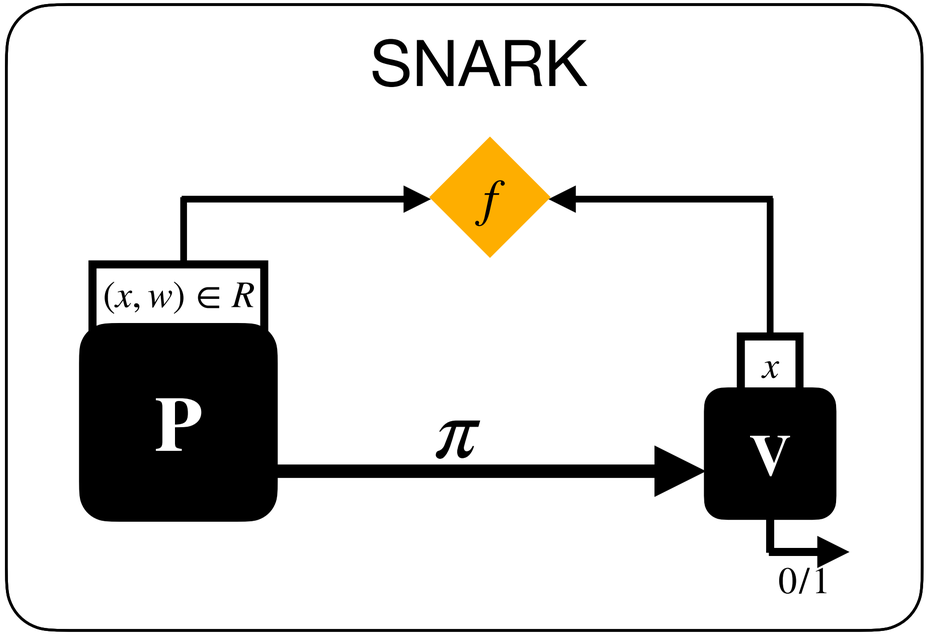

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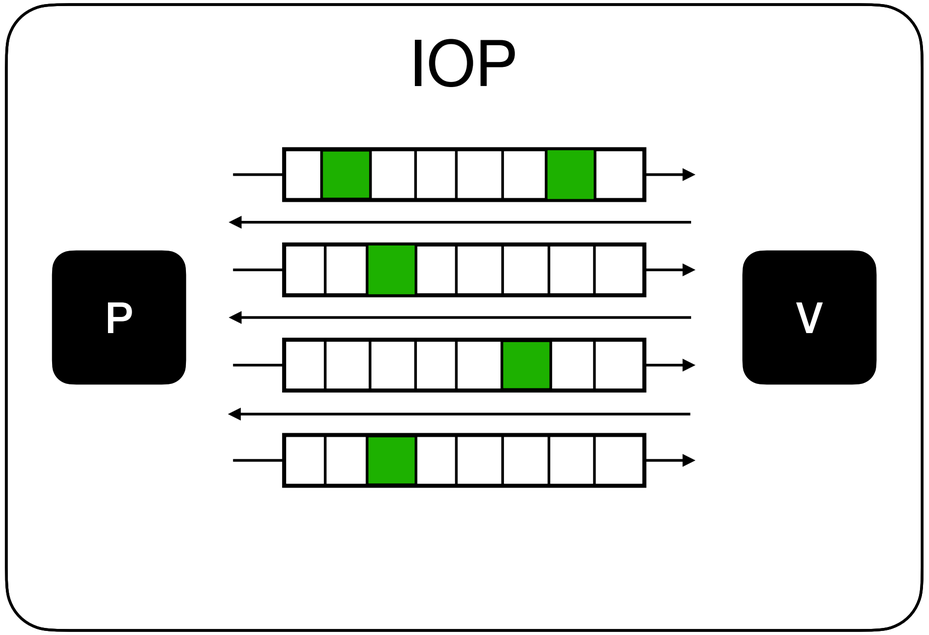

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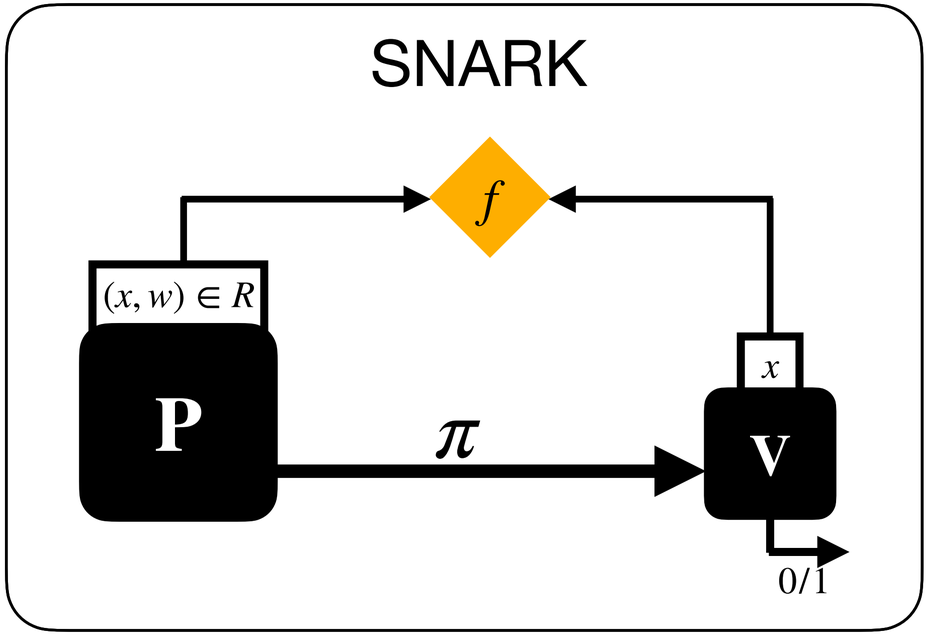

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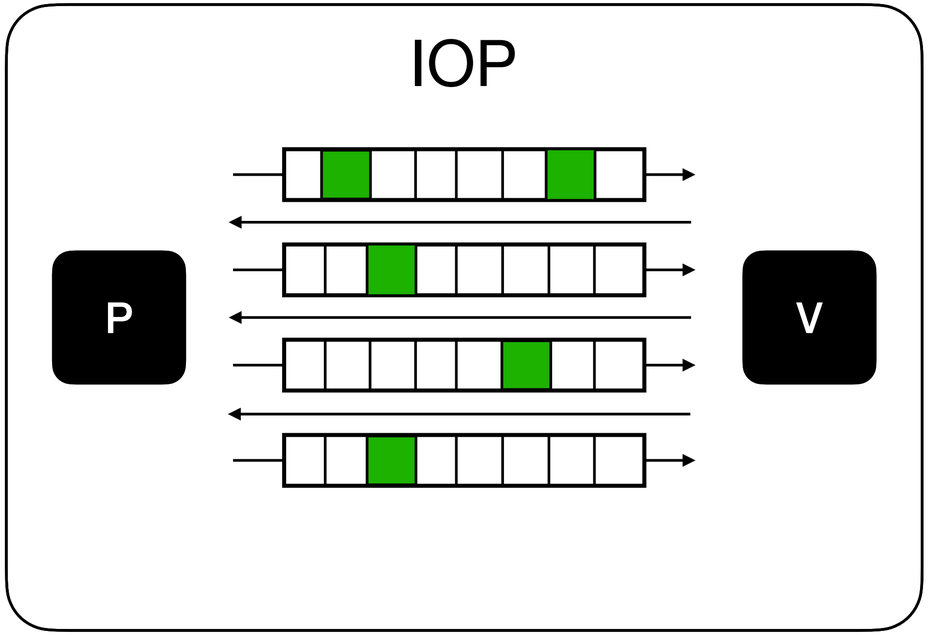
Constructing SNARKs

[BCS16] Construction

In this talk, we focus on the IOP!



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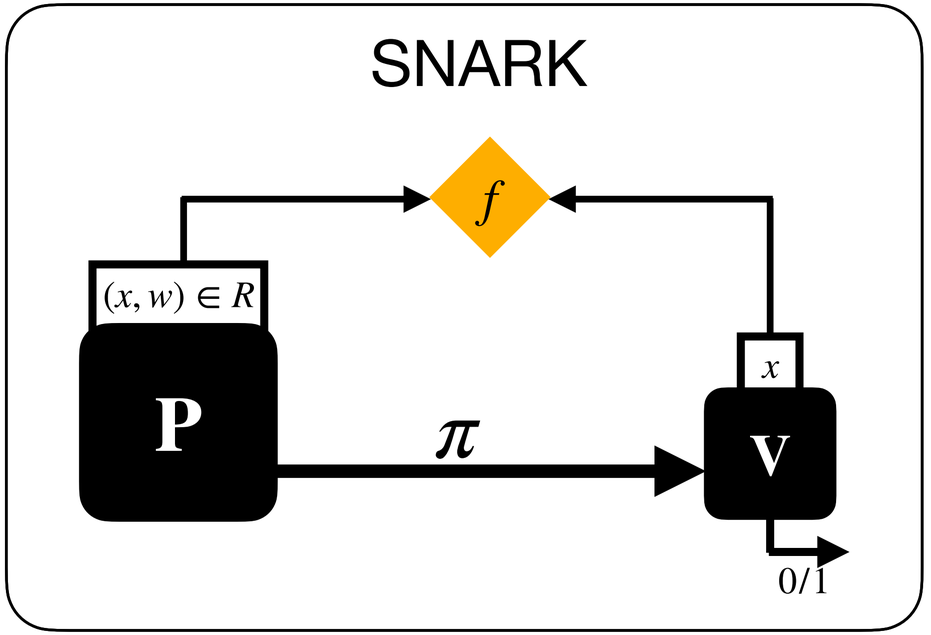

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Traditionally

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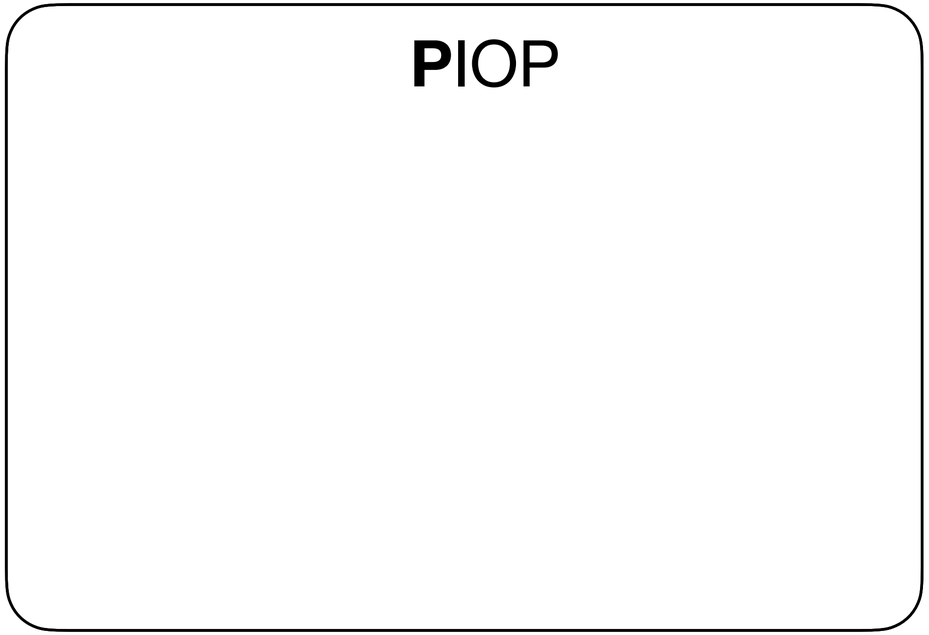

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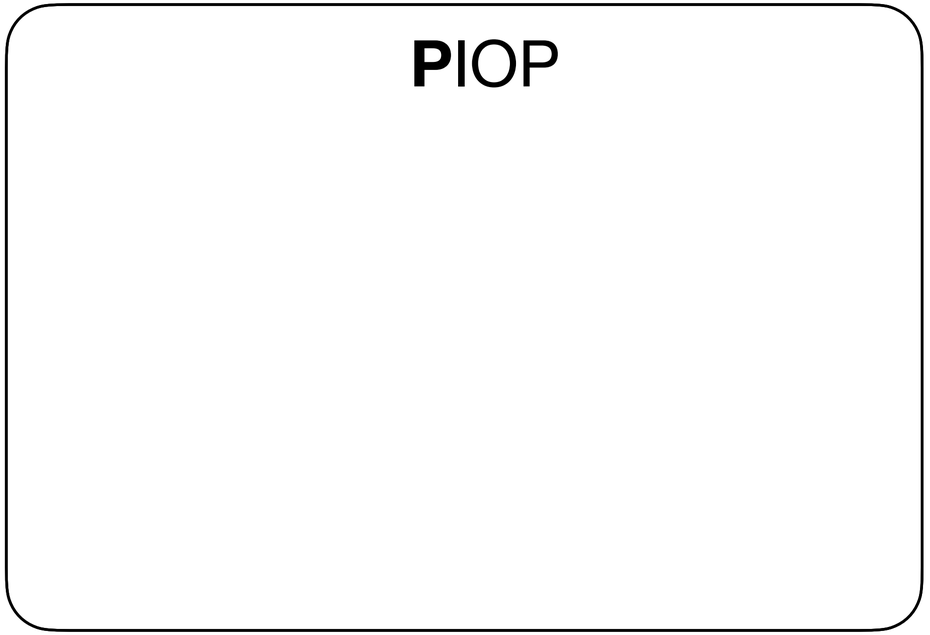

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E.g. Aurora, STARK PIOP etc.

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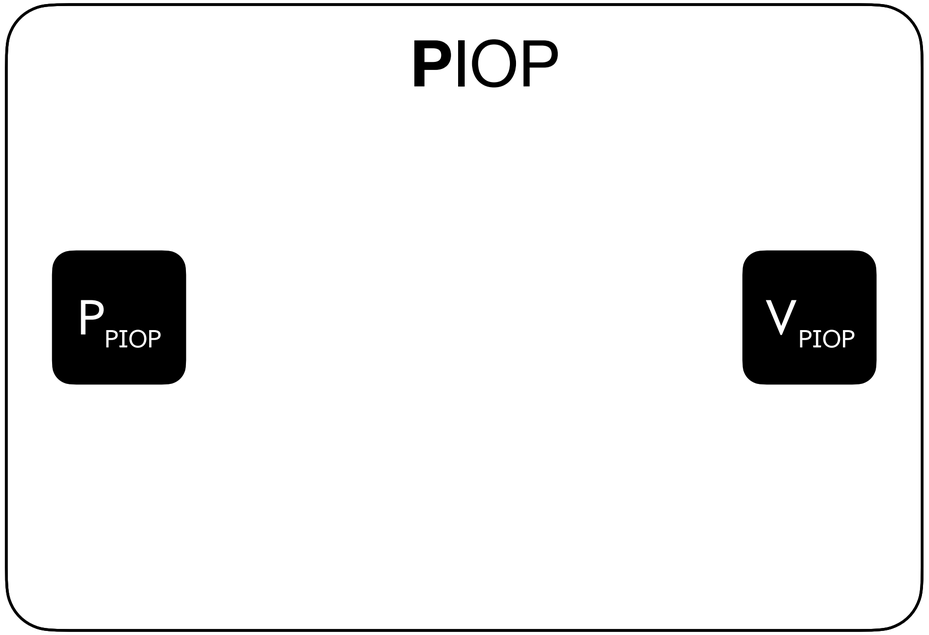

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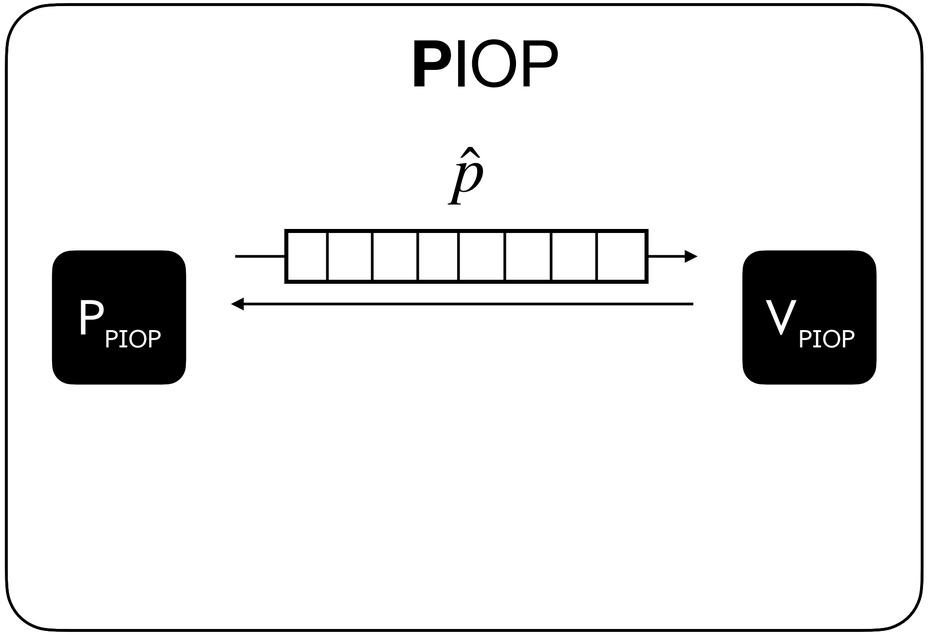

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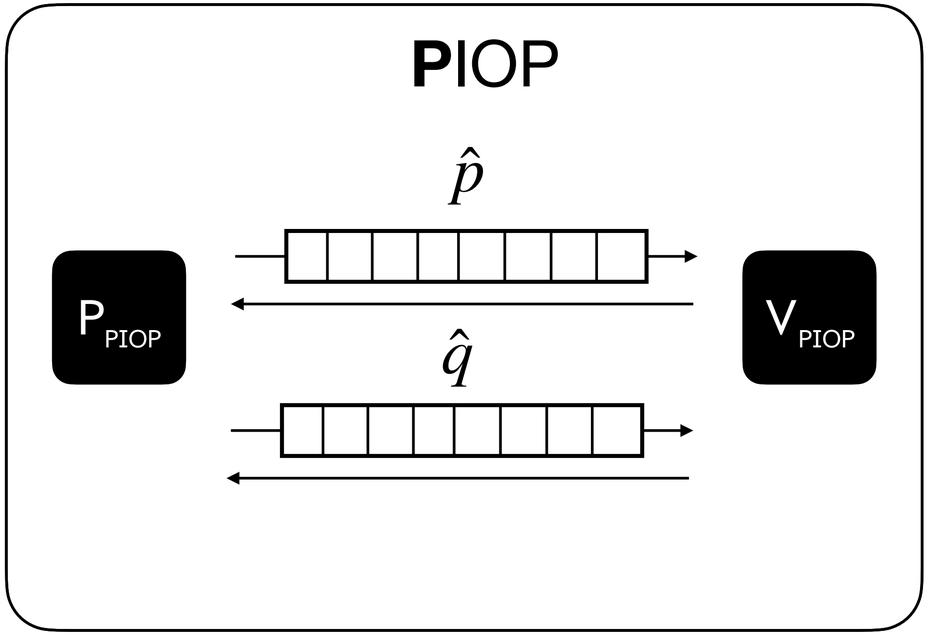

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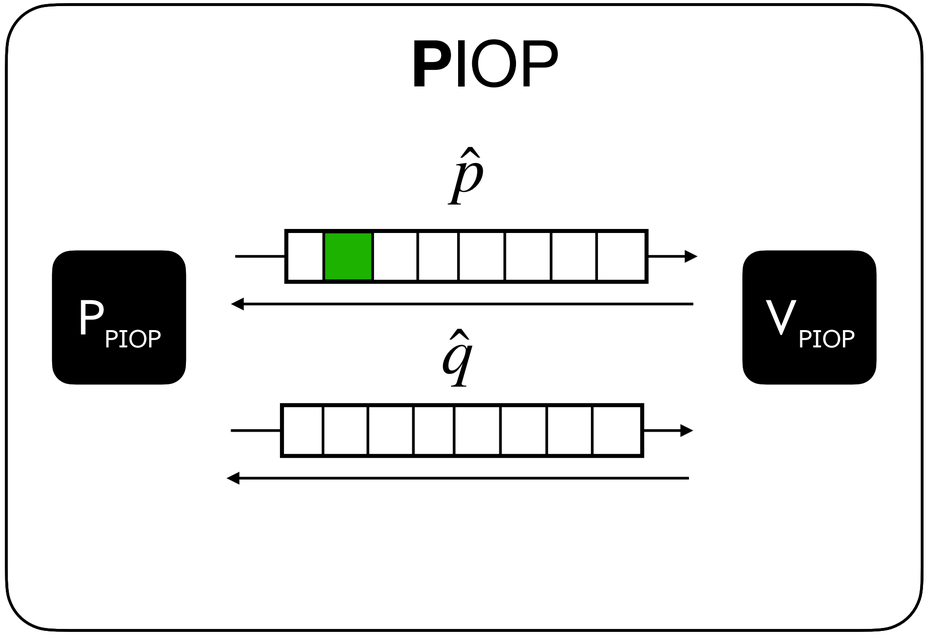

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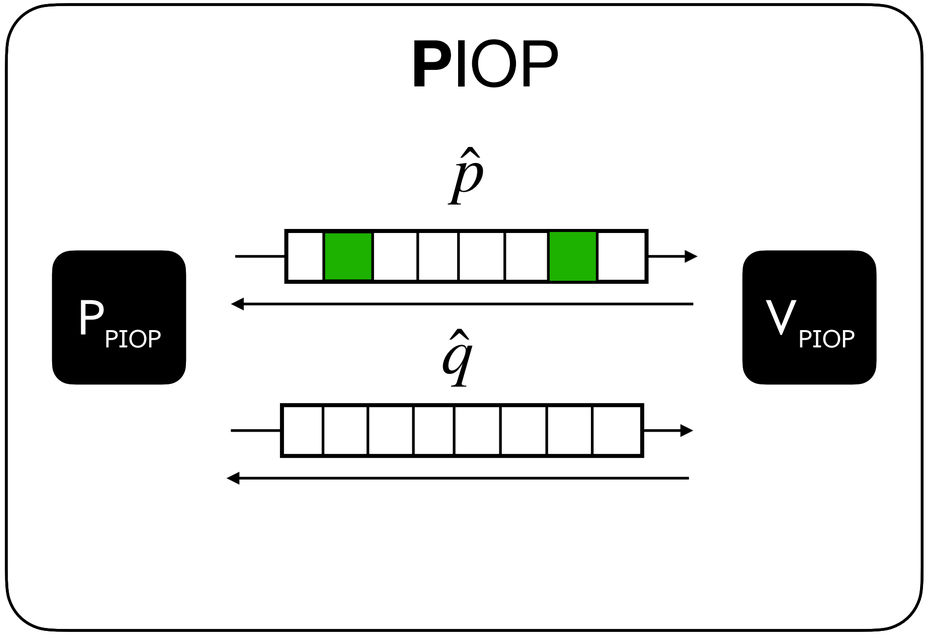

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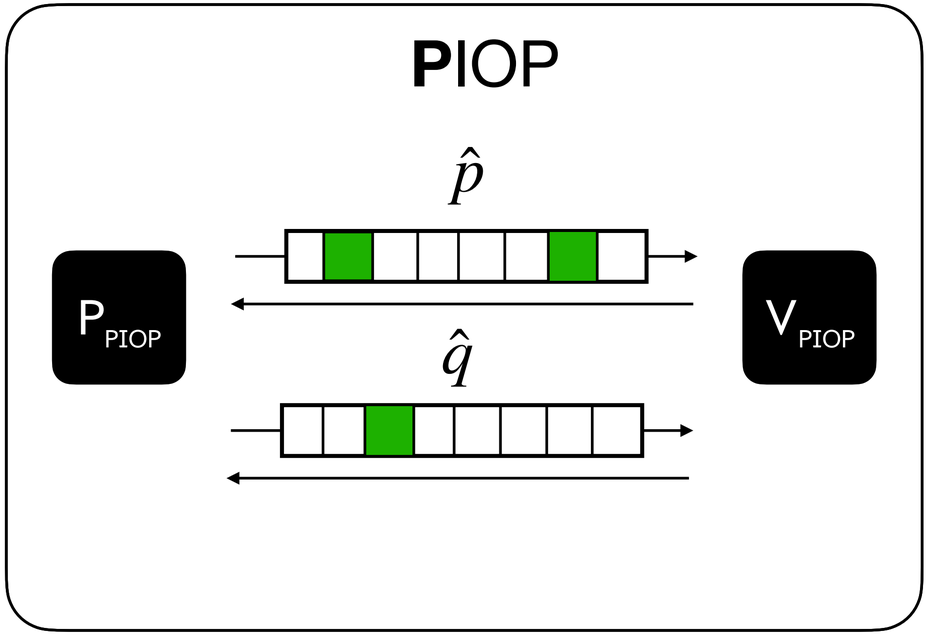

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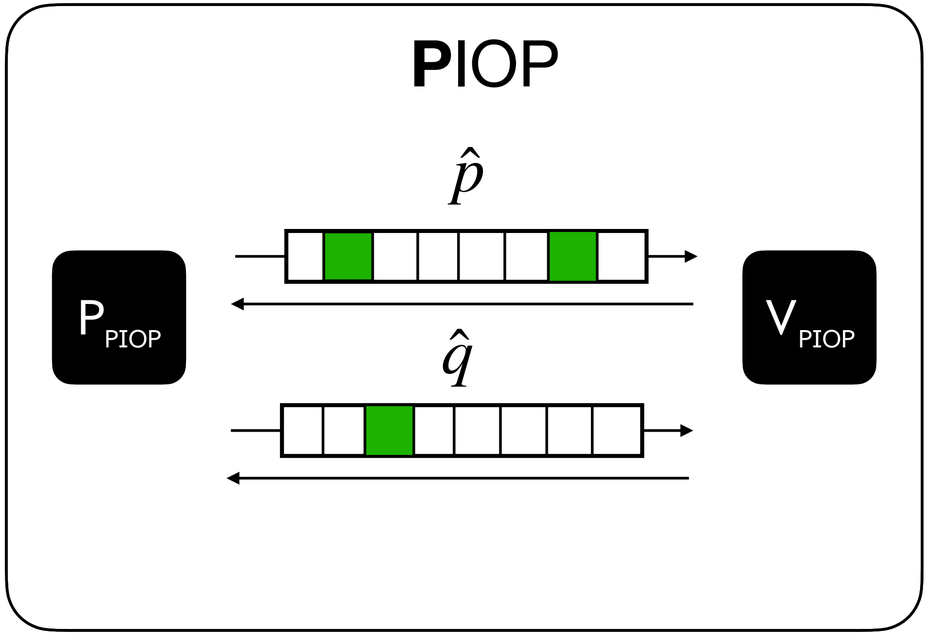

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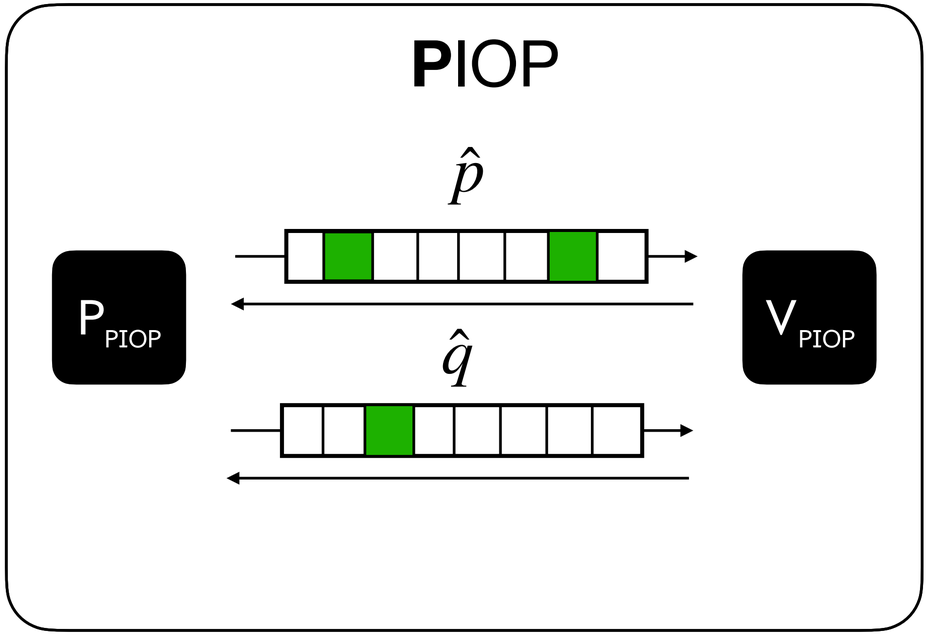

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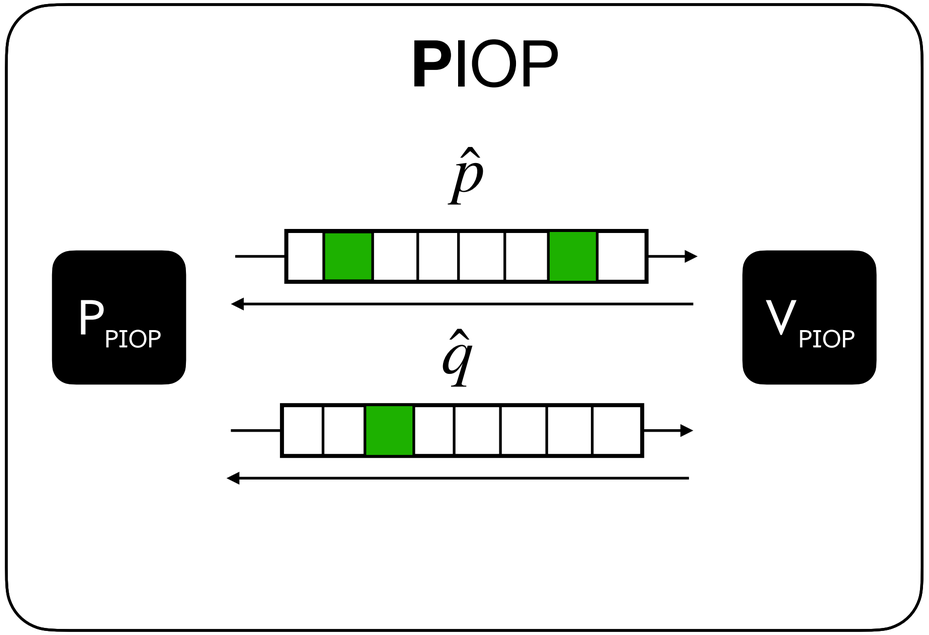

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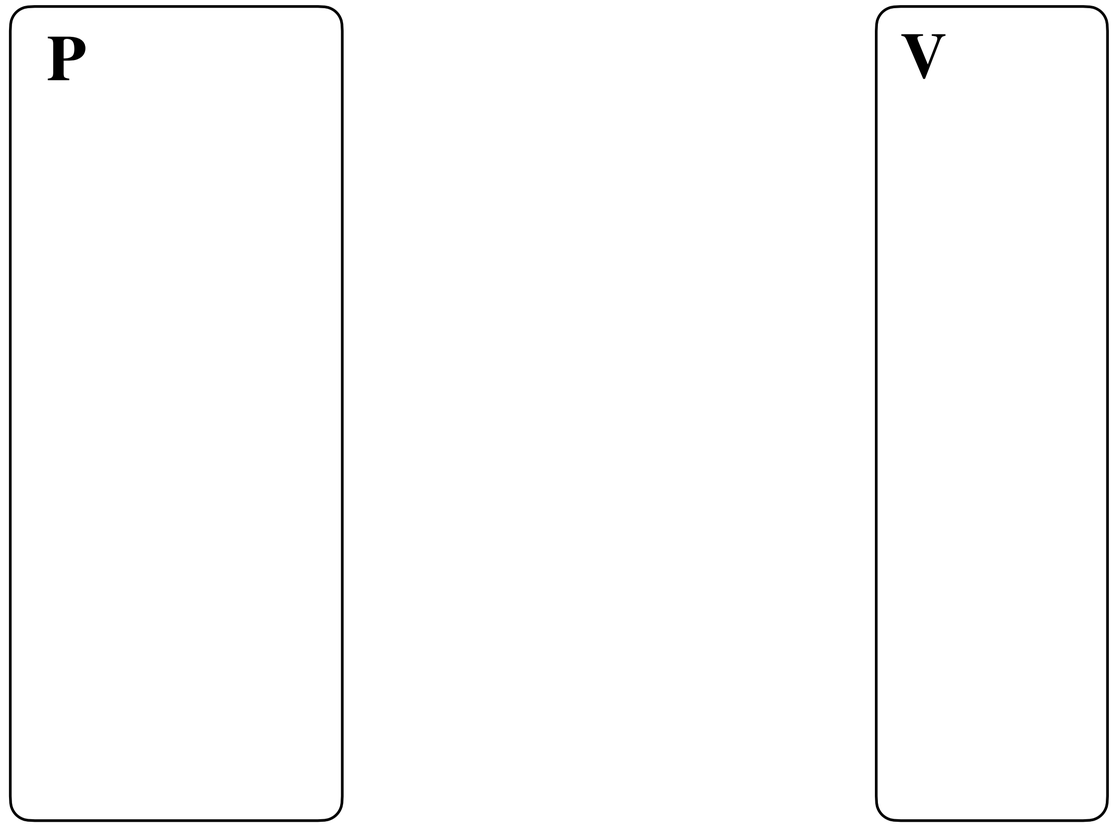

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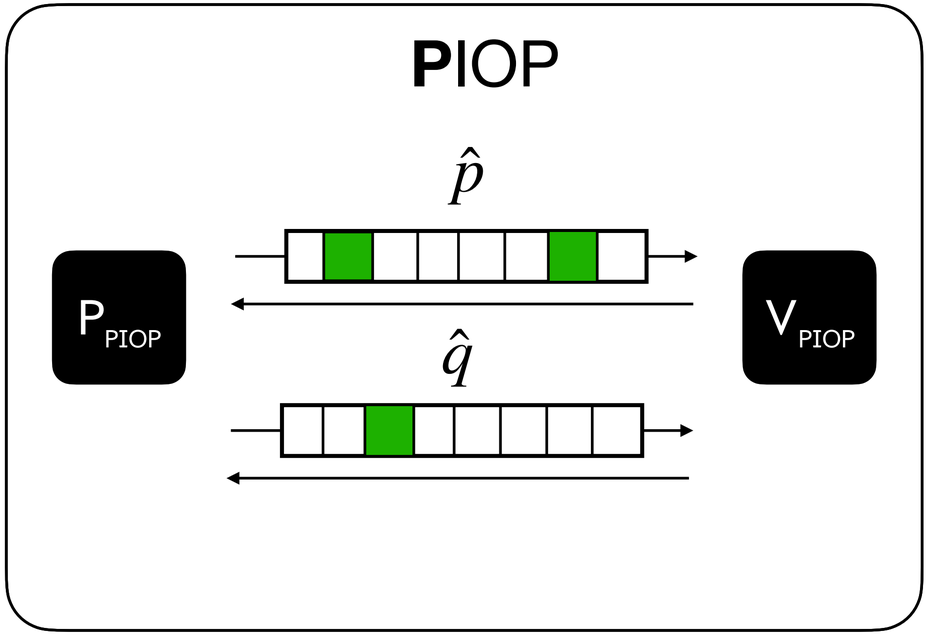

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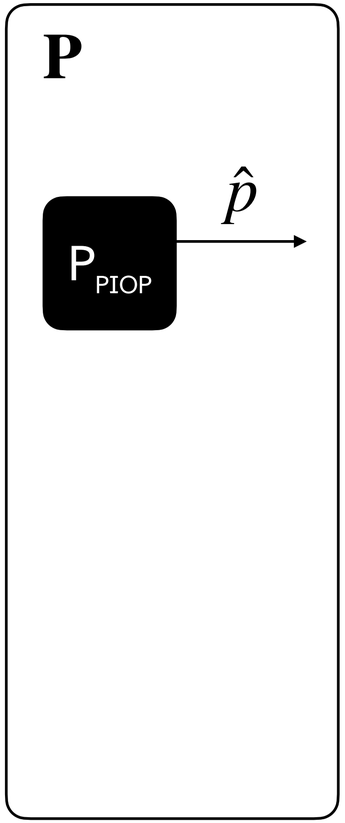

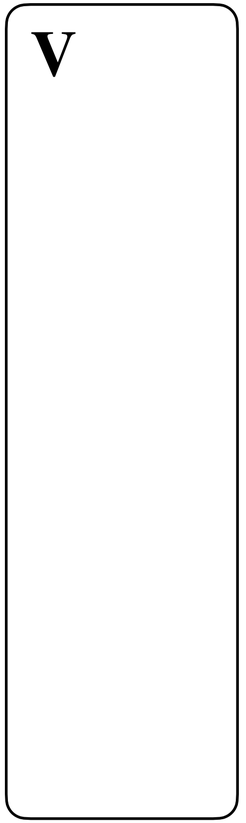

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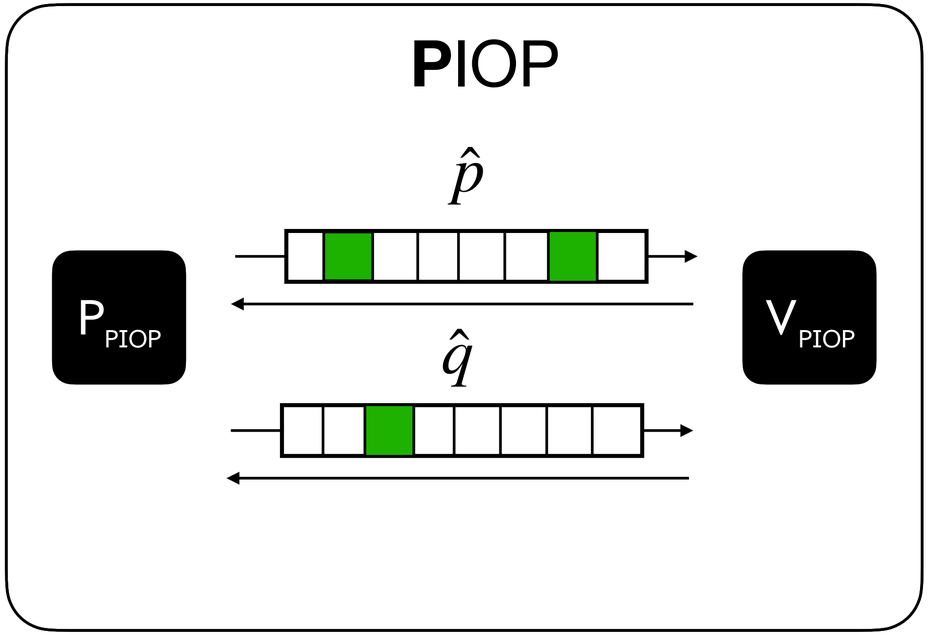

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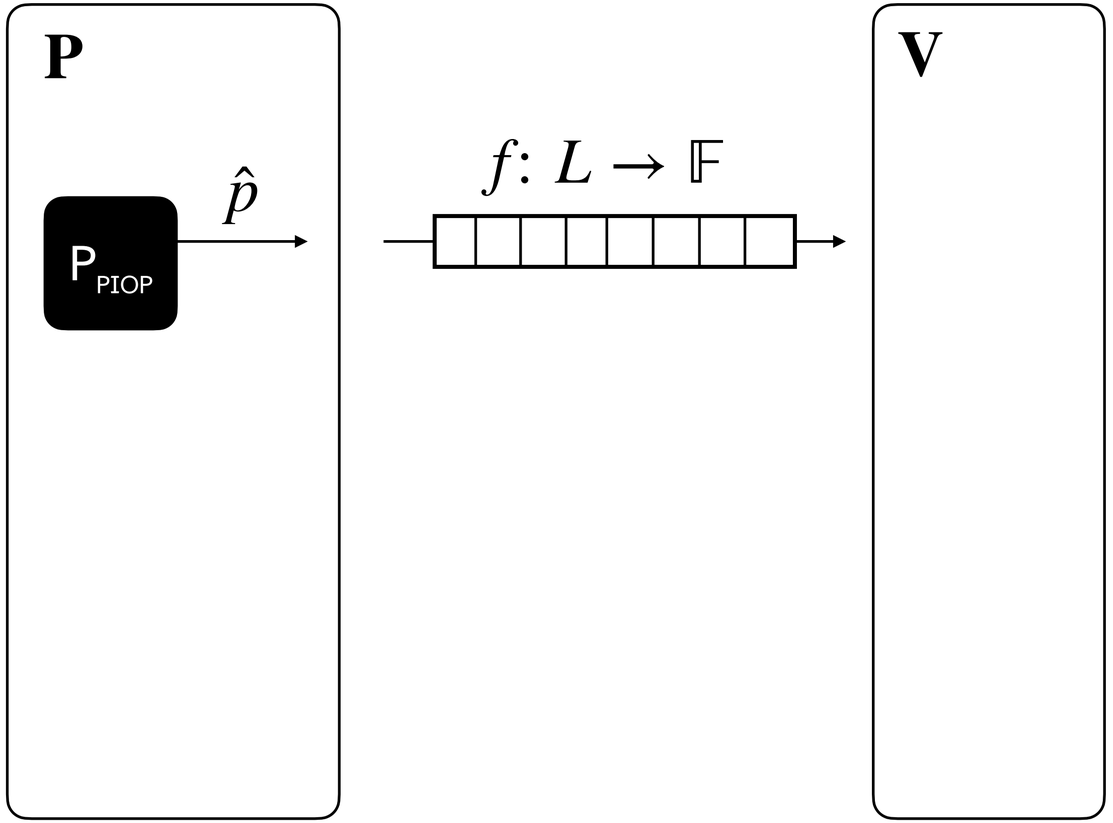

Constructing IOPs

Traditionally

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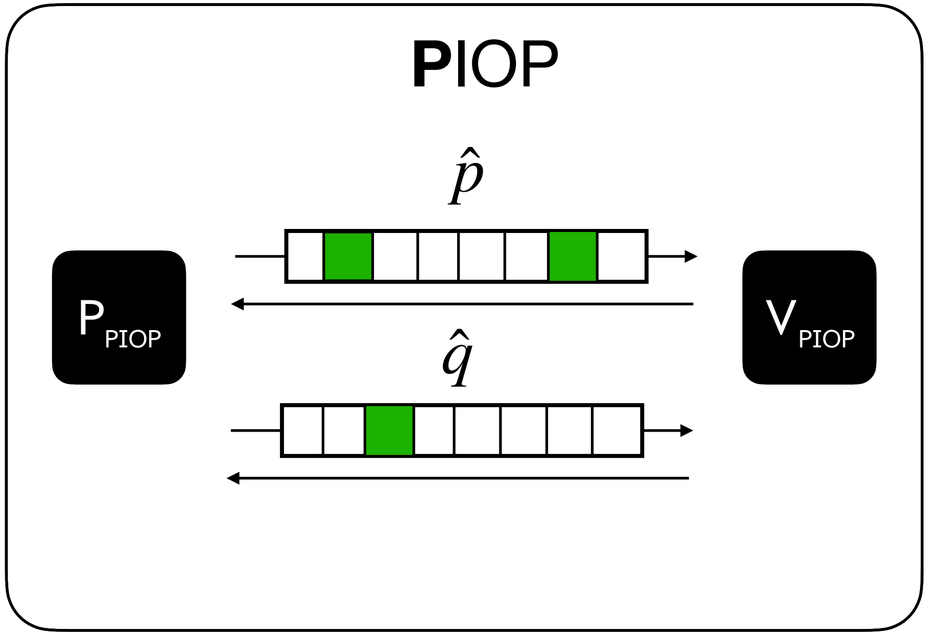

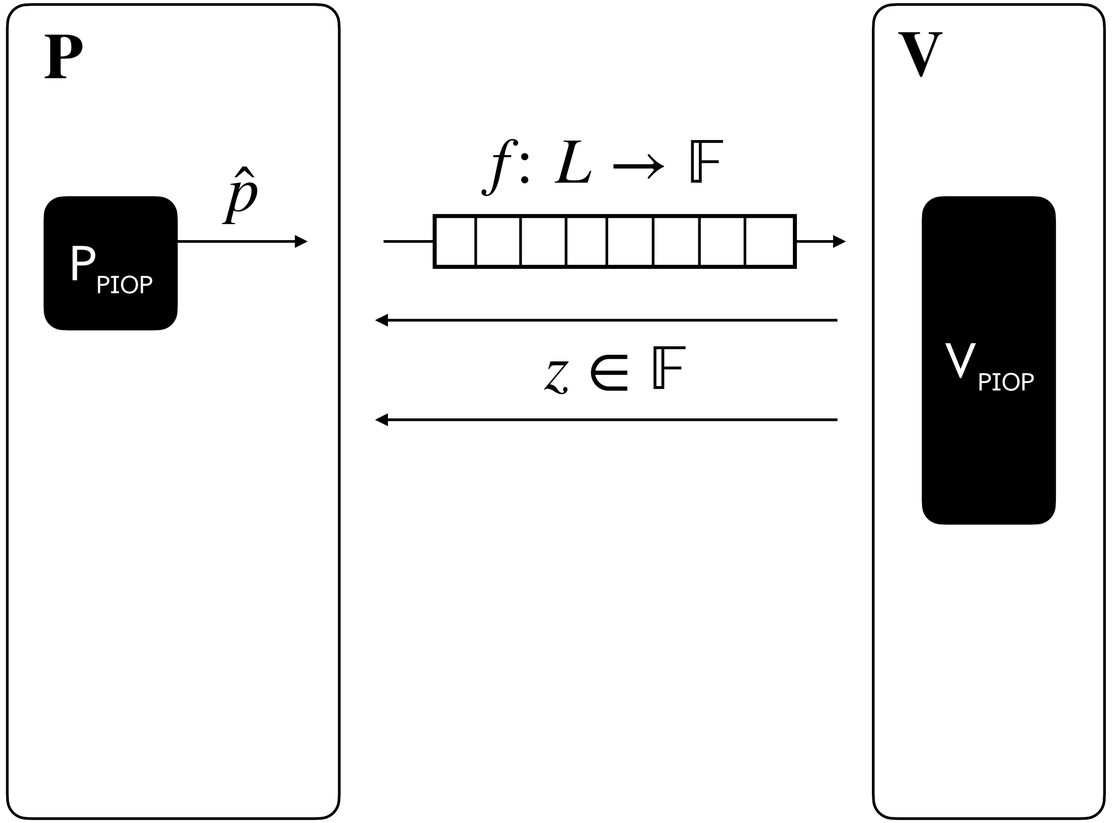

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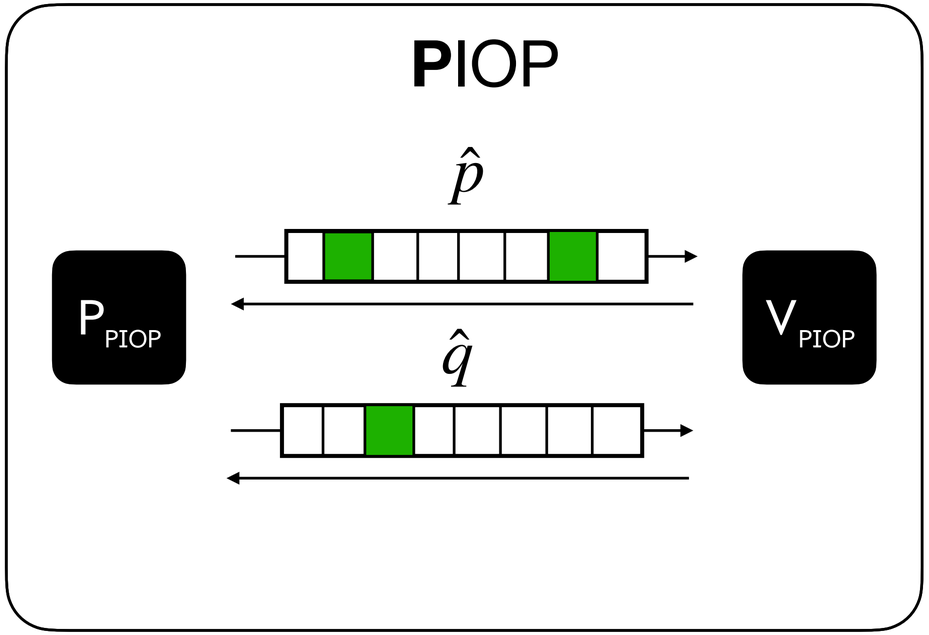

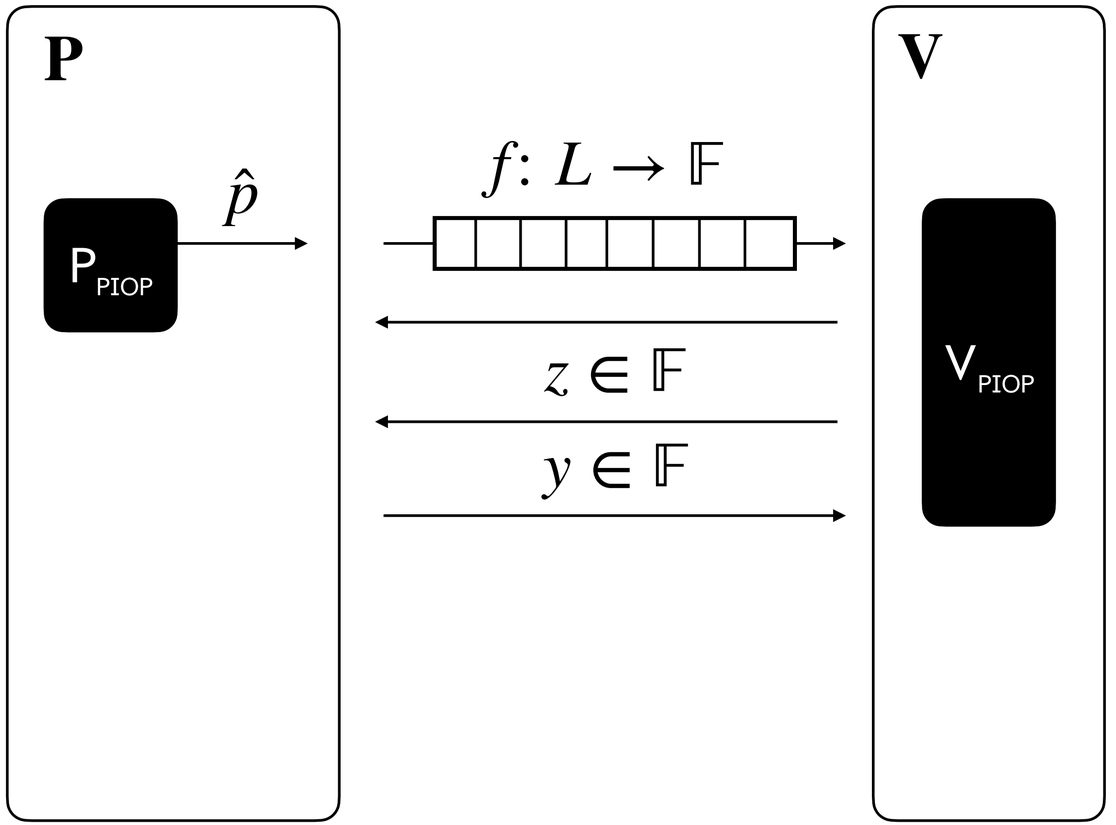

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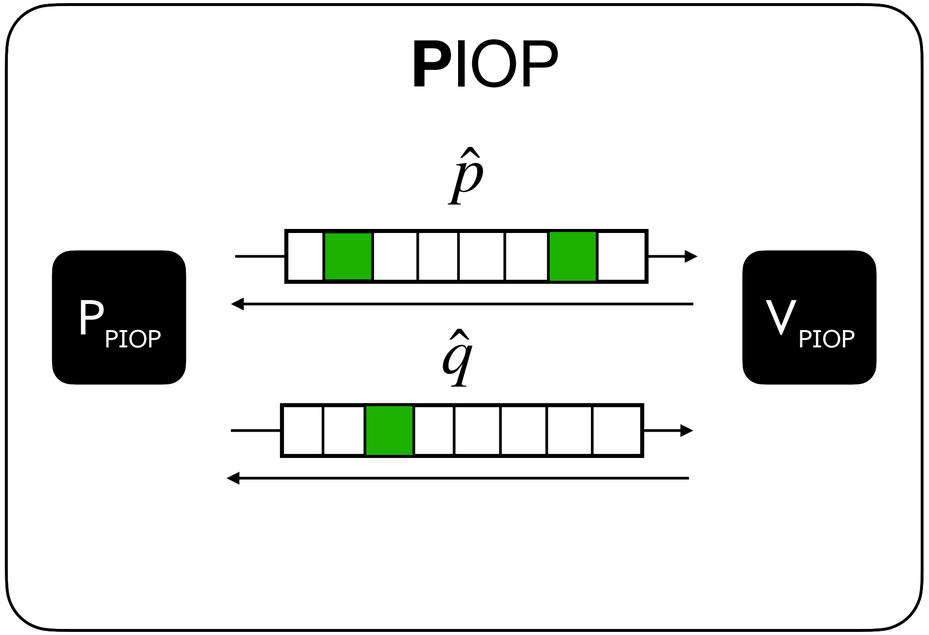

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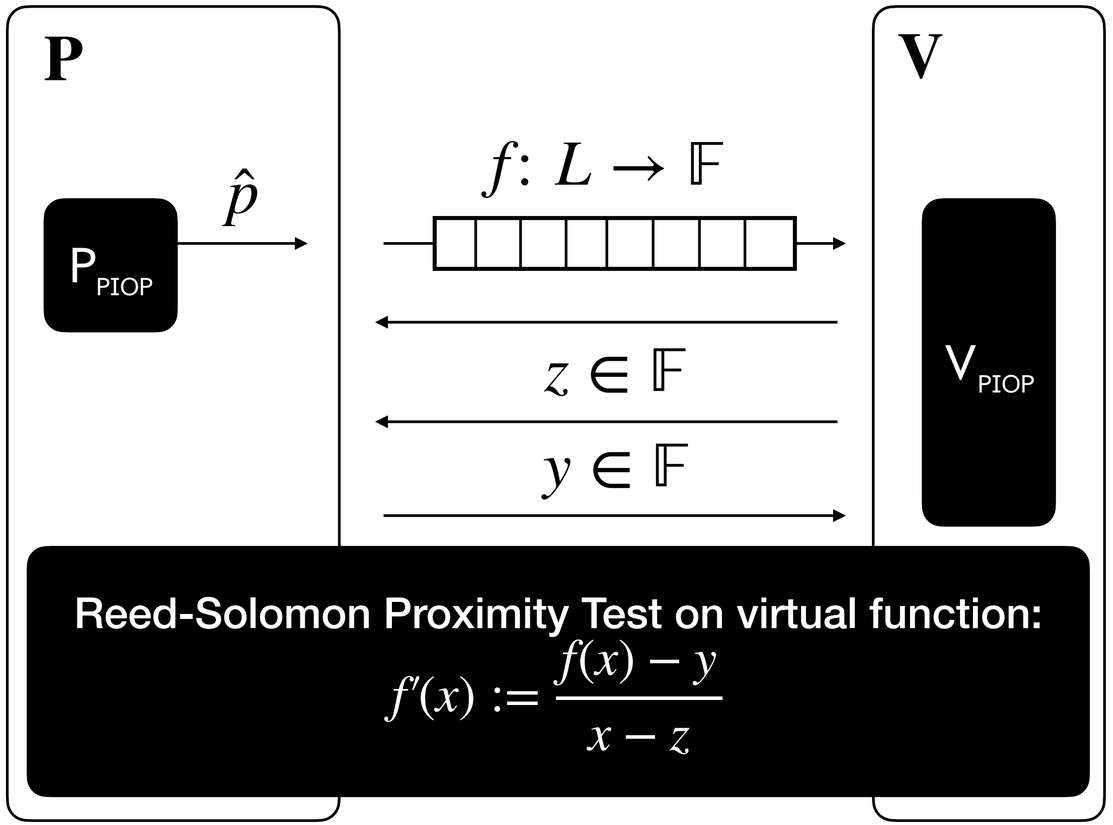

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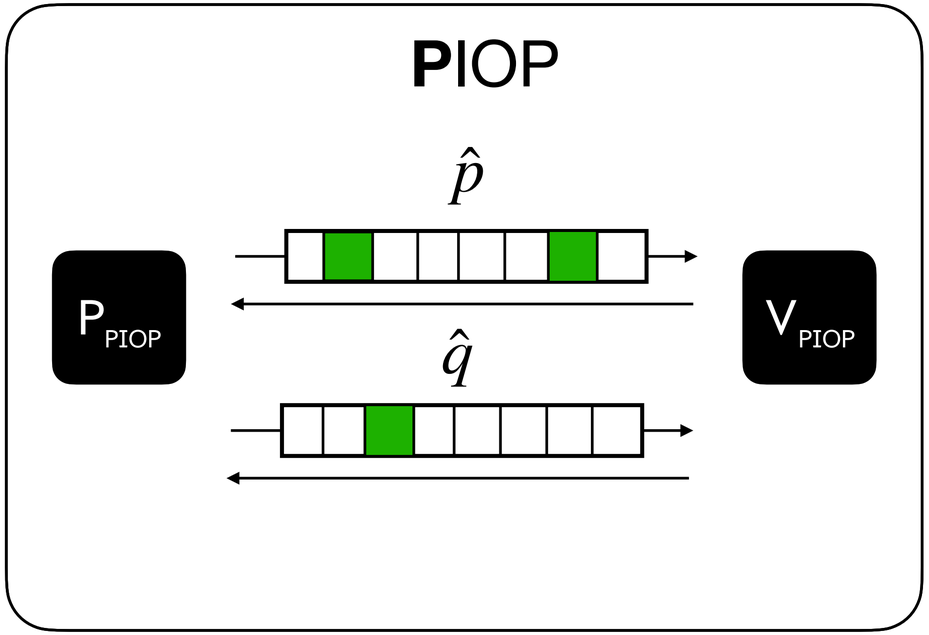

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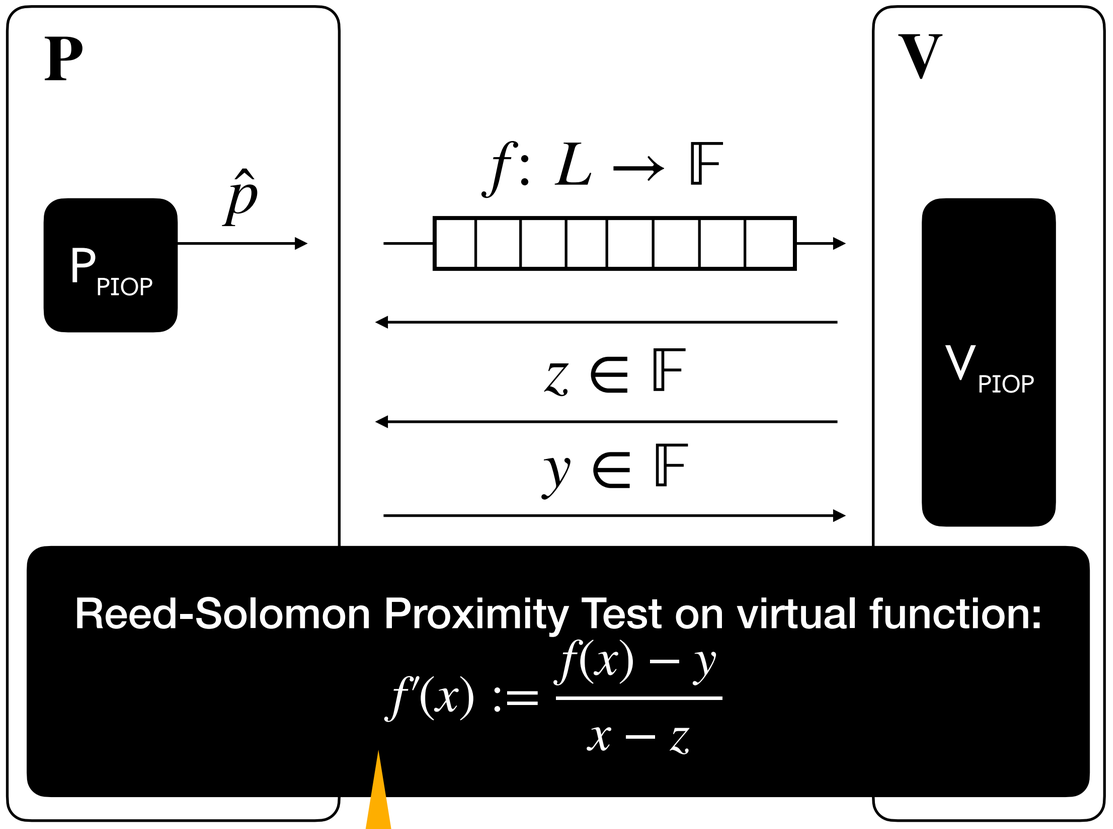

The IOP inherits asymptotics almost entirely from the proximity test

IOP of Proximity to RS codes

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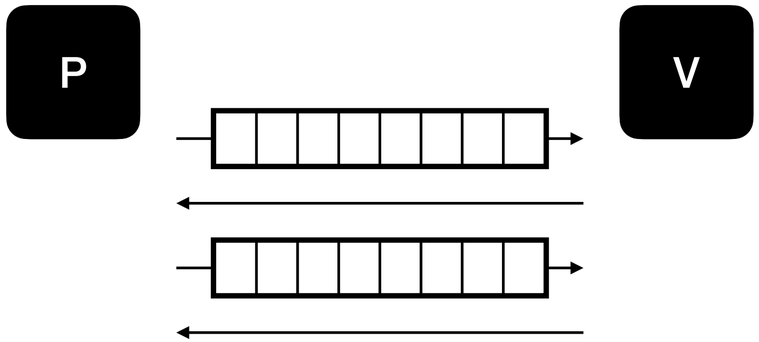

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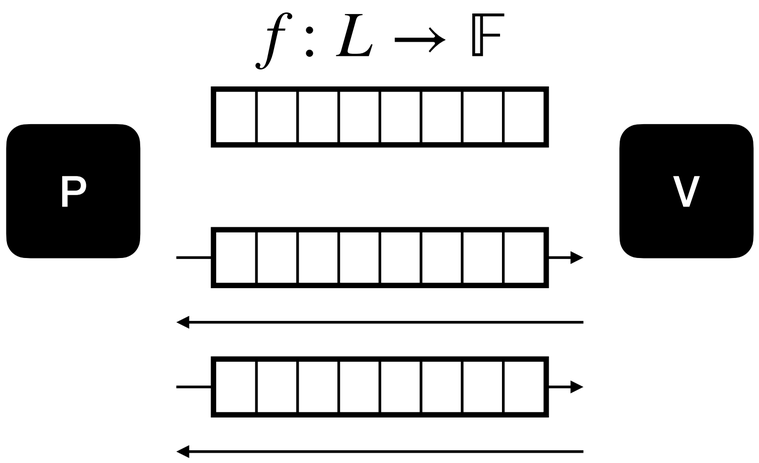

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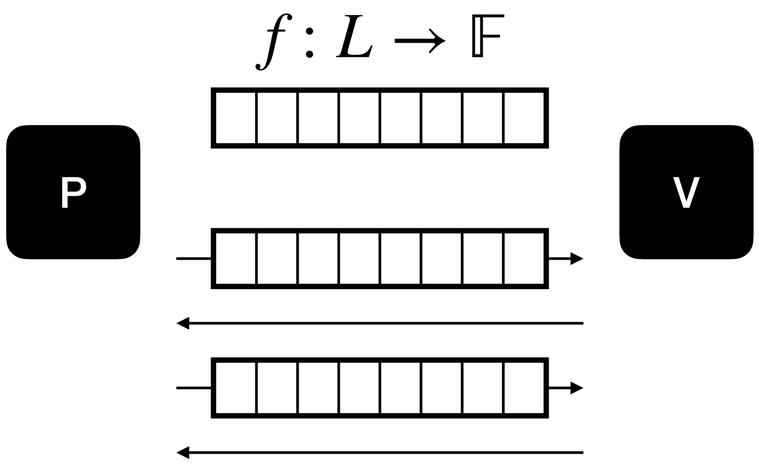

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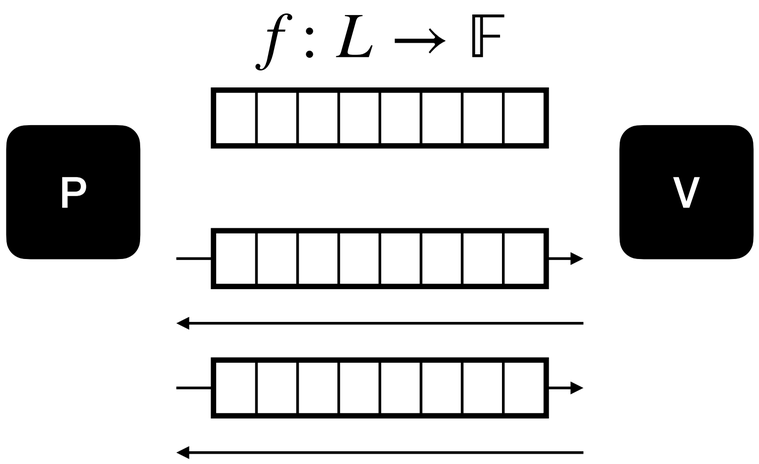

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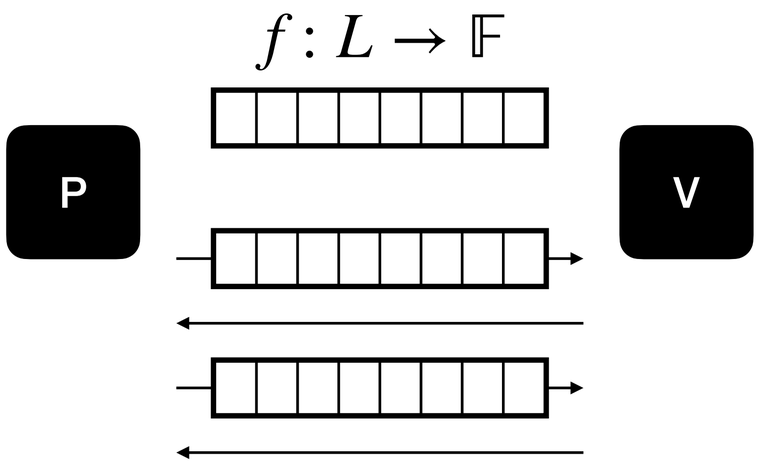

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Round by round, required by BCS transform.

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What we are running:

Reed-Solomon Proximity Test on virtual function:

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Yes! IOPP for **constrained codes**

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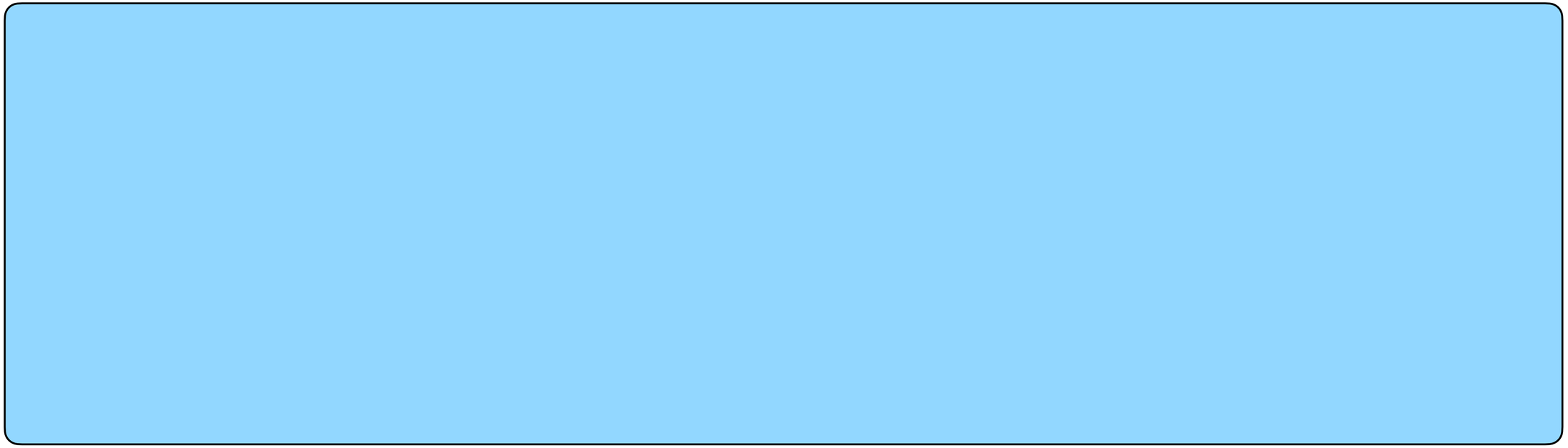
We test **proximity** to CRS!

Our results

WHIR

$k > 1$ is a folding parameter

A constrained Reed-Solomon proximity test



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Proof length: $O(n/2^k)$

Verifier time: $O\left(q_{\text{WHIR}} \cdot (2^k + m)\right)$

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WHIR 🌪️

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Σ-IOP + CRS IOPP (WHIR 🌪️) = IOP

WHIR

$k > 1$ is a folding parameter

A constrained Reed-Solomon proximity test

Rounds: $O(m)$

Alphabet: $\mathbb{F}^{2^k}$

Proof length: $O(n/2^k)$

Verifier time: $O\left(q_{\text{WHIR}} \cdot (2^k + m)\right)$

Query complexity:

$$q_{\text{WHIR}} = O\left(\frac{\lambda}{k} \cdot \log m\right) = O(\lambda)$$

$k \approx \log m$

$\lambda \gg m$

Σ-IOP + CRS IOPP (WHIR) = IOP

High soundness analogue of RS PIOP compiler (w/o quotients)

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Implementation available at WizardOfMenlo/whir

Comparison with prior work

	Queries	Verifier Time	Alphabet
BaseFold	$q_{\text{BF}} = O(\lambda \cdot m)$	$O(q_{\text{BF}})$	$\mathbb{F}^2$
FRI	$q_{\text{FRI}} = O\left(\frac{\lambda}{k} \cdot m\right)$	$O(q_{\text{FRI}} \cdot 2^k)$	$\mathbb{F}^{2^k}$
STIR	$q_{\text{STIR}} = O\left(\frac{\lambda}{k} \cdot \log m\right)$	$O(q_{\text{STIR}} \cdot 2^k + \lambda^2 \cdot 2^k)$	$\mathbb{F}^{2^k}$
WHIR	$q_{\text{WHIR}} = O\left(\frac{\lambda}{k} \cdot \log m\right)$	$O(q_{\text{WHIR}} \cdot (2^k + m))$	$\mathbb{F}^{2^k}$

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When $k \approx \log m$
$\Sigma = \mathbb{F}^m$
Improving is an open question

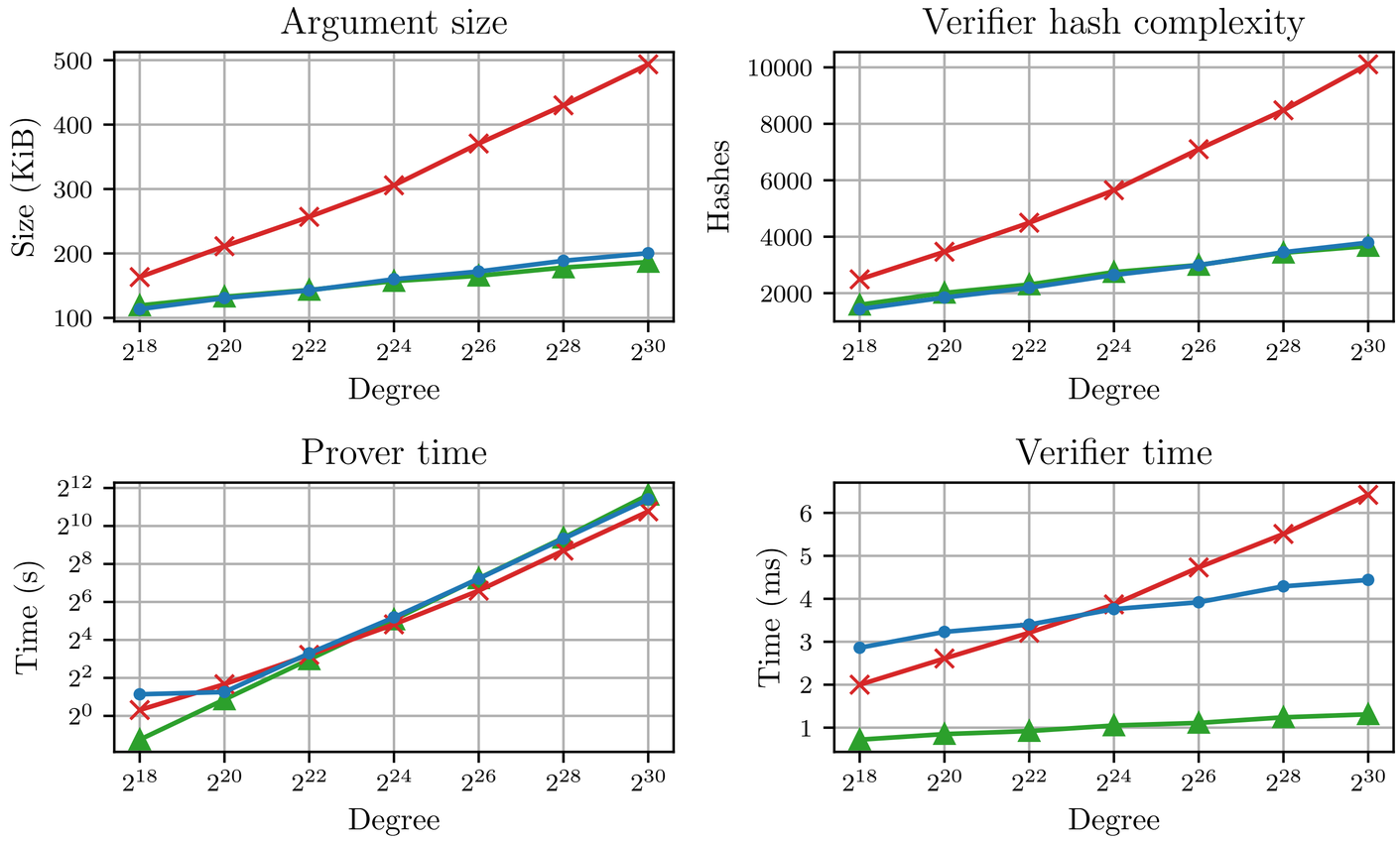
Comparison with FRI and STIR

Note: prover time graph is now outdated due to new optimizations discovered

128-bits security level.

$\lambda = 106 + 22$ bits of PoW + “list-decoding” assumptions.

Rate of the code $\rho = 1/2$



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2^24 coeffs rate = 1/4	FRI	WHIR
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Verifier time	2.4ms	700µs

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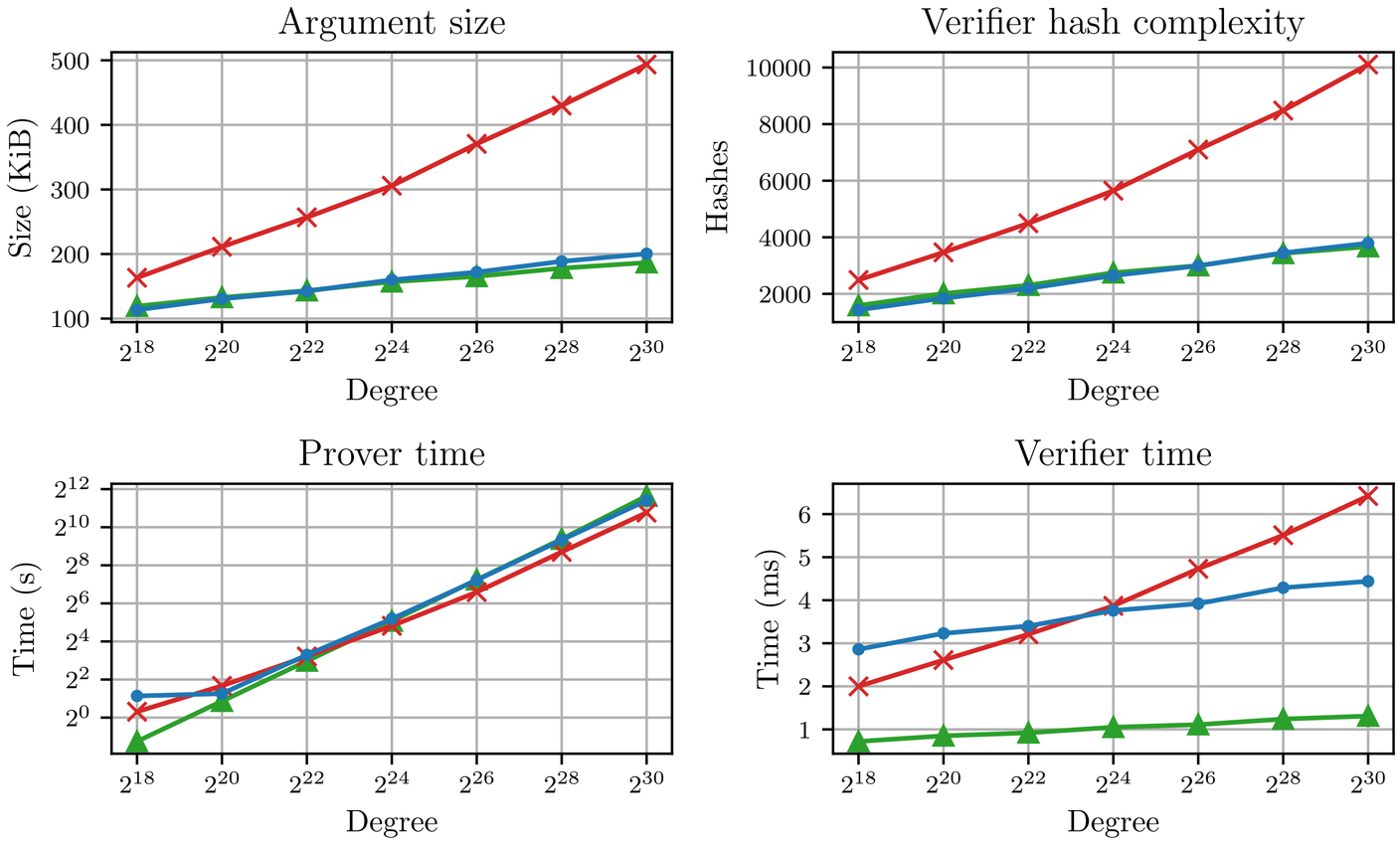

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2^30 coeffs rate = 1/2	FRI	WHIR
Size (KiB)	494	187
Verifier time	4.4ms	1.3ms

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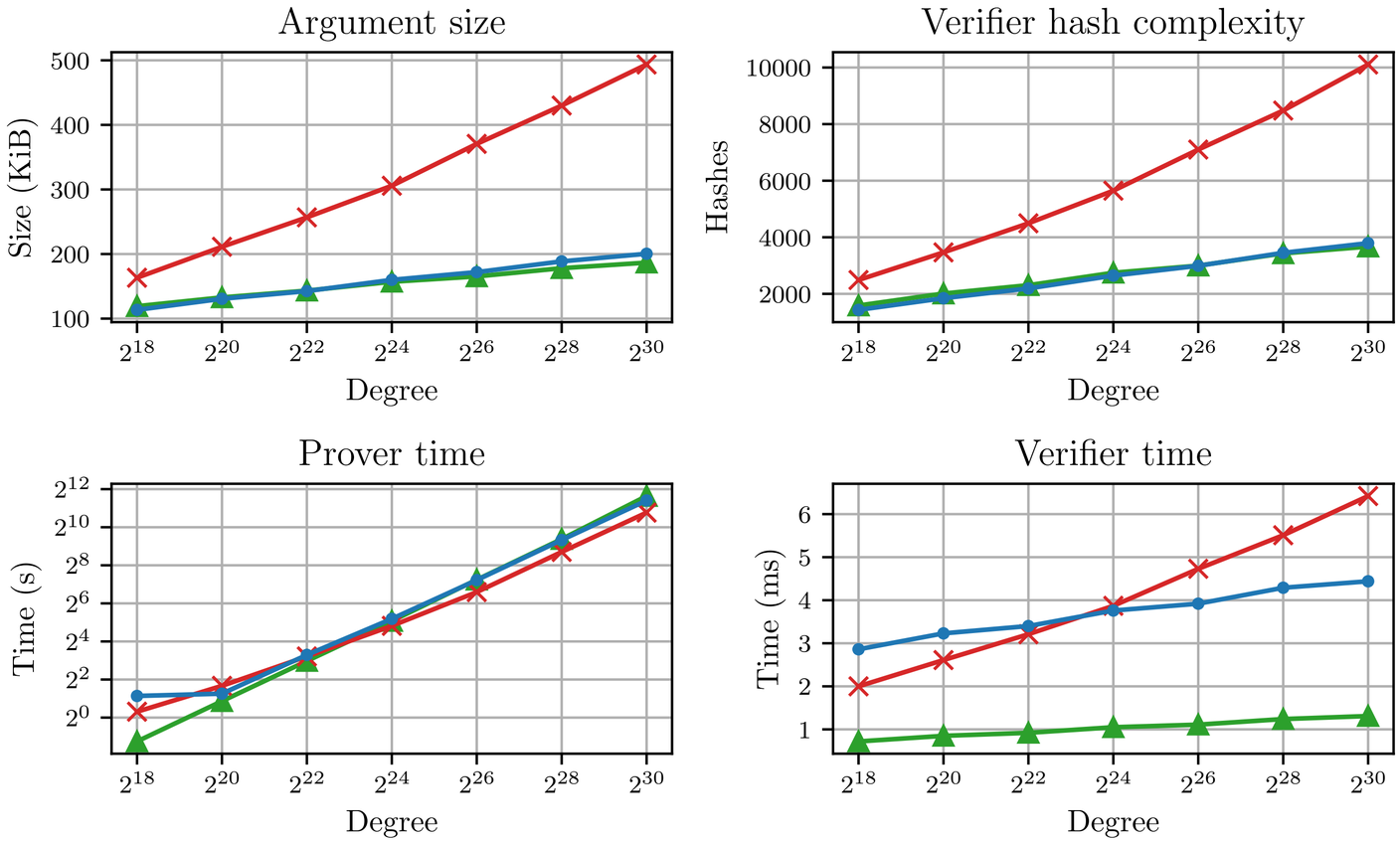

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Super fast verifier

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$\lambda = 100$	3500	733	-	100	7.81	2.42	0.61	0.29
$\lambda = 128$	3680	750	130	151	9.92	3.66	1.4	0.6

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Schemes with **trusted** setup using pairings!

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Practical adoption

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Implementation available
@ WizardOfMenlo/whir

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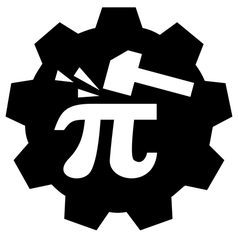

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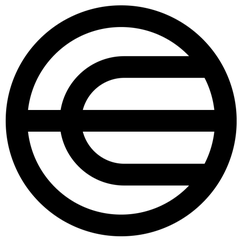

World client-side prover
@ worldfnd/ProveKit

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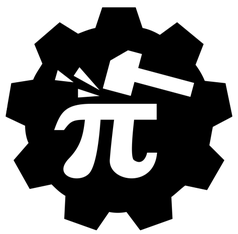

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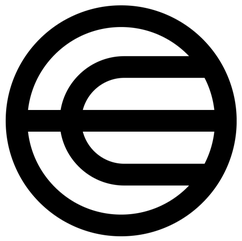

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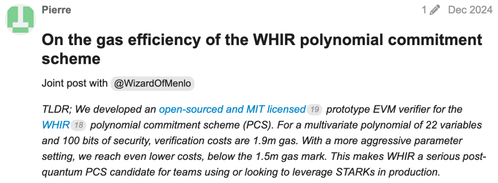

Solidity verifier implementation
@ privacy-scaling-explorations/sol-whir

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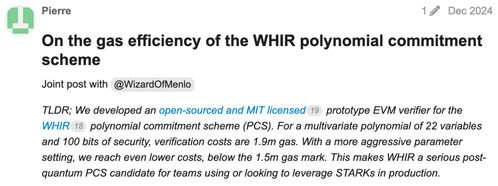

~1M gas for verification, competitive with pre-quantum alternatives

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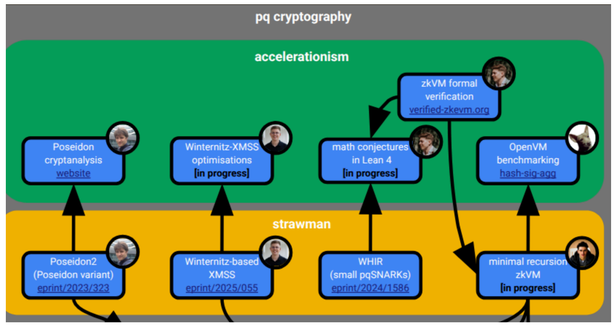
Ethereum pq transition



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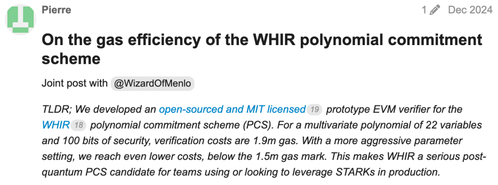

Pierre 1 Dec 2024

On the gas efficiency of the WHIR polynomial commitment scheme

Joint post with @WizardOfMenlo

TLDR; We developed an open-sourced and MIT licensed 19 prototype EVM verifier for the WHIR 18 polynomial commitment scheme (PCS). For a multivariate polynomial of 22 variables and 100 bits of security, verification costs are 1.9m gas. With a more aggressive parameter setting, we reach even lower costs, below the 1.5m gas mark. This makes WHIR a serious post-quantum PCS candidate for teams using or looking to leverage STARKs in production.

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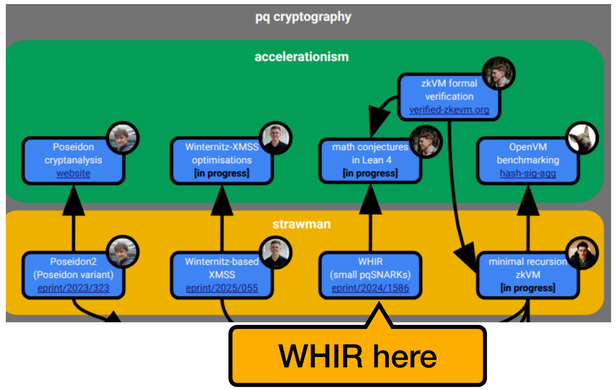

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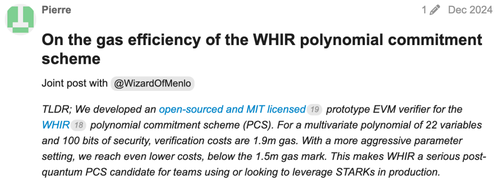

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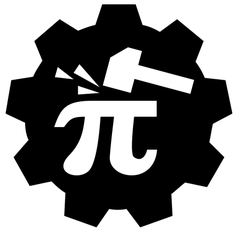

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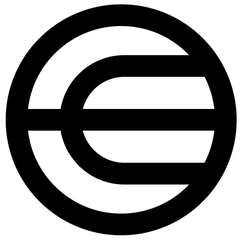

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Plonky3

POWERED BY polygon

Plonky3 implementation
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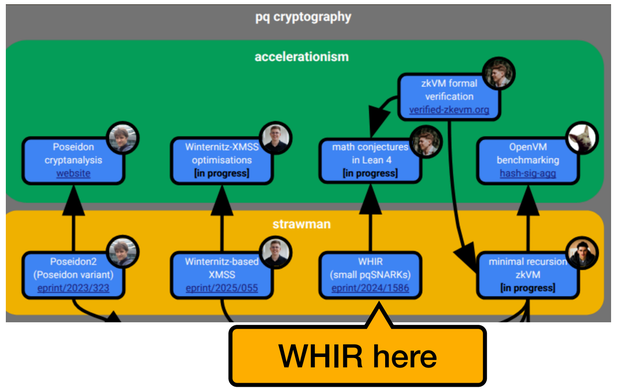

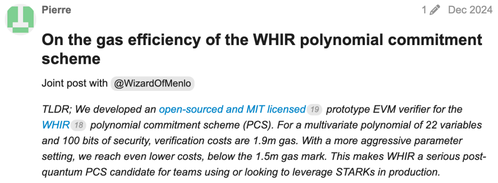

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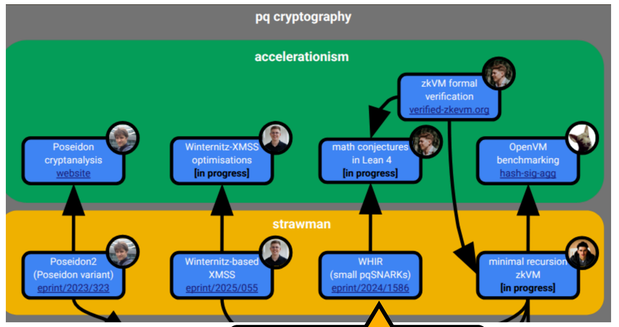

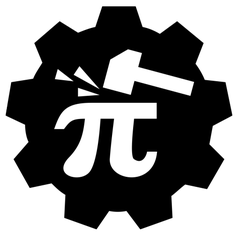

Implementation available
@ WizardOfMenlo/whir

Whirlaway

A hash-based SNARK with lightweight proofs, powered by the Whir Polynomial Commitment Scheme.

Specifications

- **Arithmetization**: AIR (Algebraic Intermediate Representation) with preprocessed columns
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Hash-based SNARK
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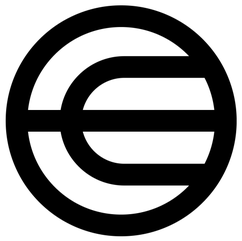
@ TomWambsgans/Whirlaway



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To be deployed to
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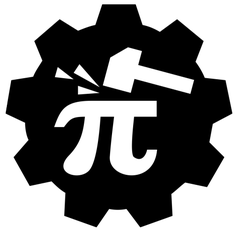

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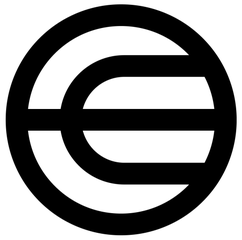

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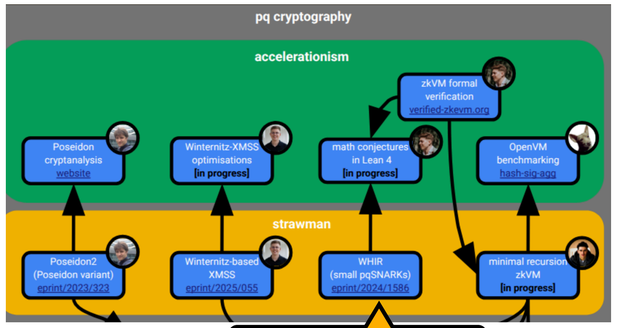

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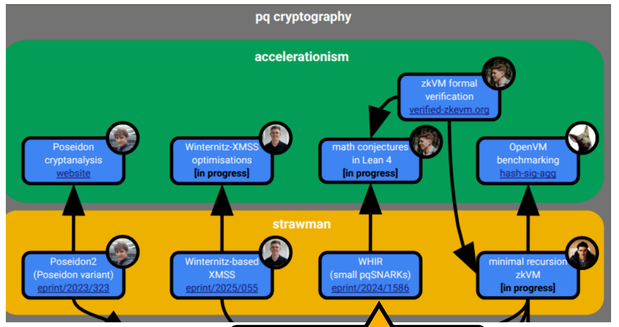
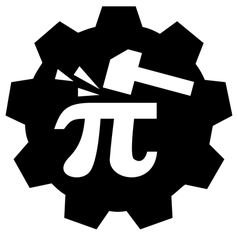


WHIR here



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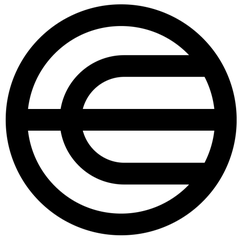

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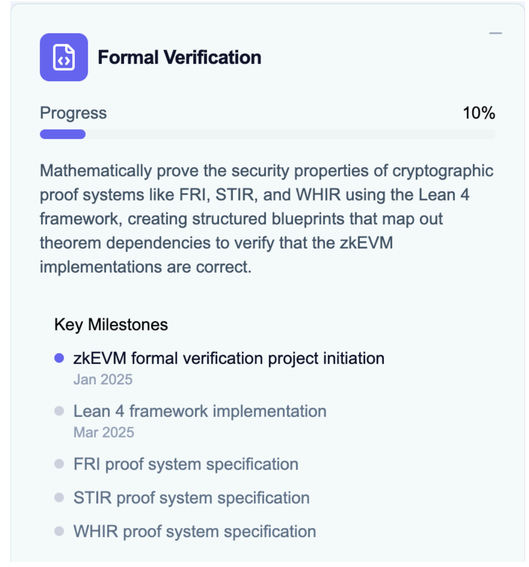

Formal verification effort

Conclusion

Summary

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WHIR 🌪️: a new IOPP for CRS codes.

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Query complexity:

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Verifier complexity:

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Open question:
Can argument size be improved at the **same prover cost?**

Extra slides

Techniques

FRI & STIR Folding

Reduce RS$[n, m, \rho]$ **to** RS$[n/2^k, m - k, \rho]$

(Think $k = 4$)

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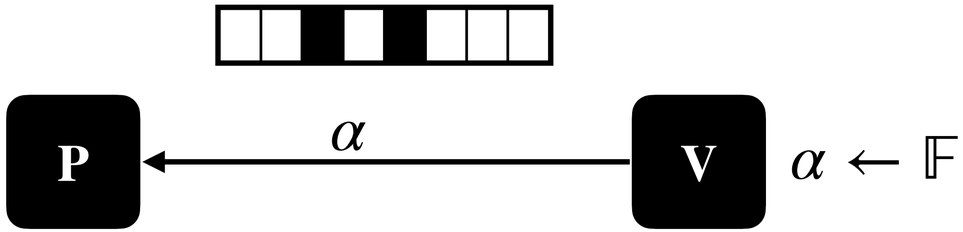

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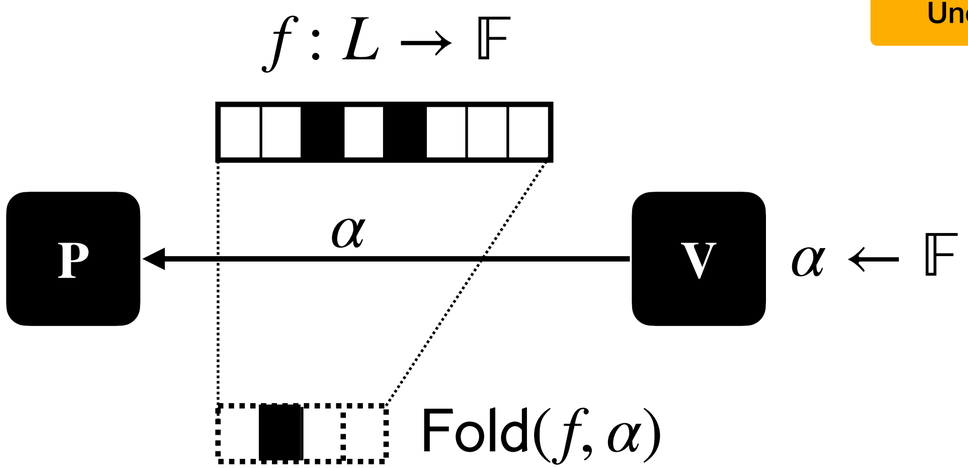

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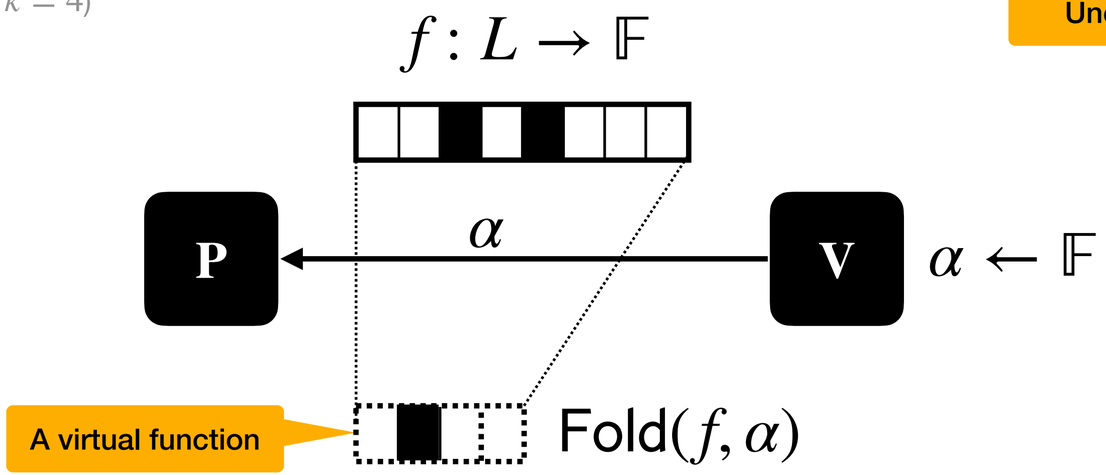

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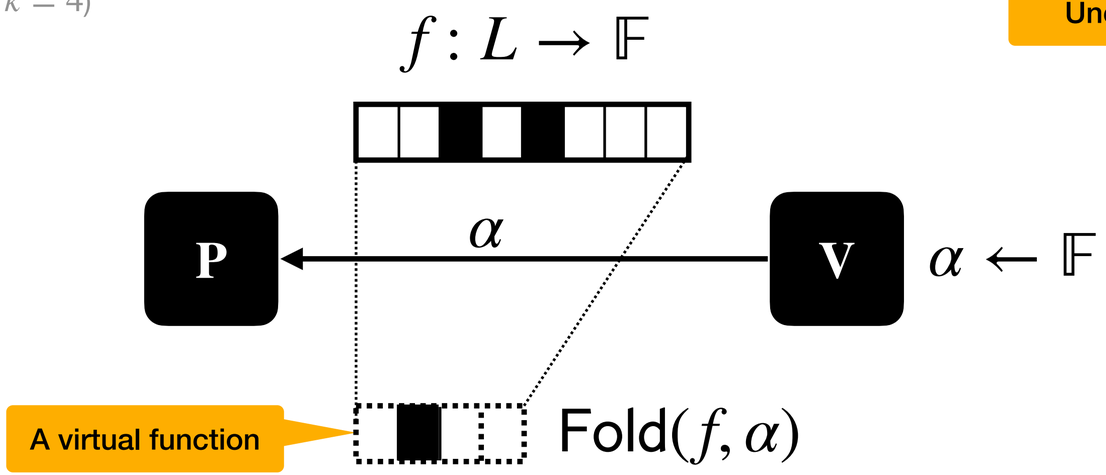

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$$\mathrm{Fold}(f, \alpha) := f_{\text{odd}} + \alpha \cdot f_{\text{even}}$$

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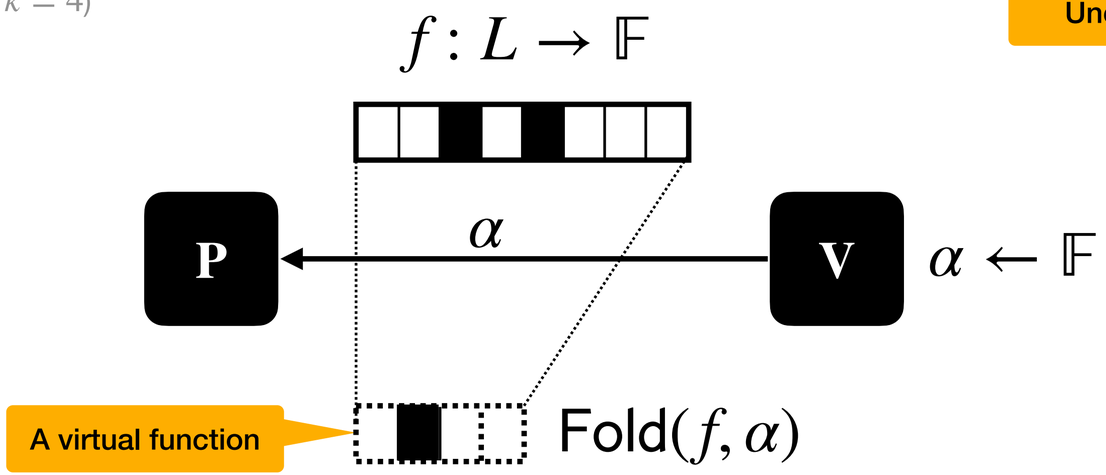

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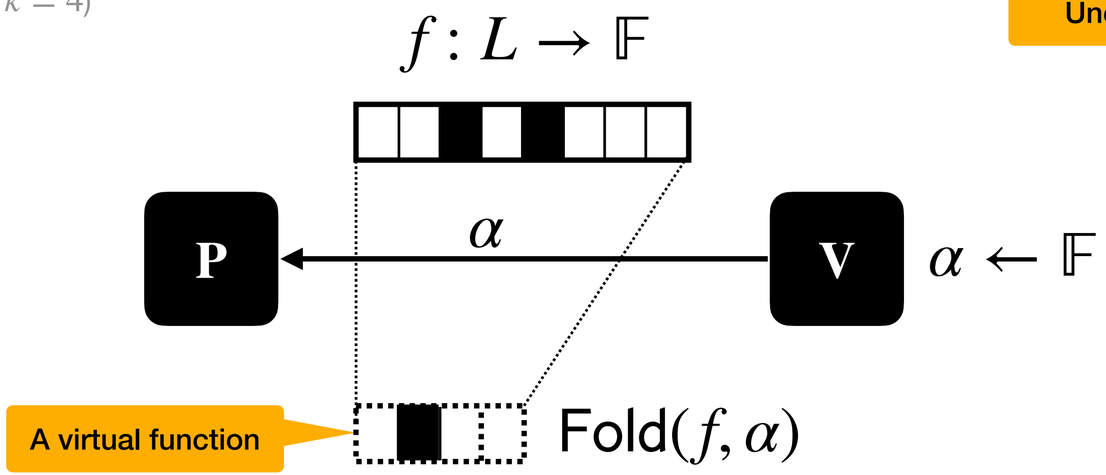

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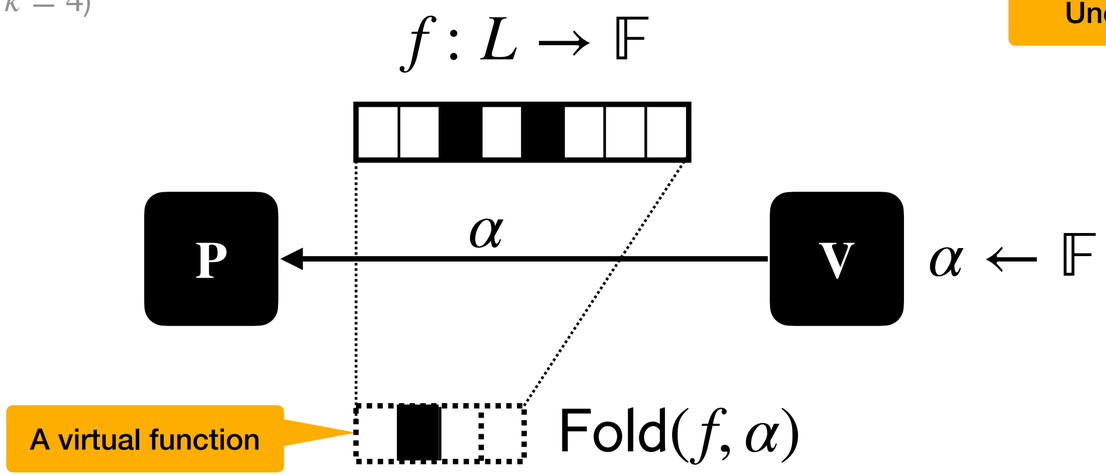

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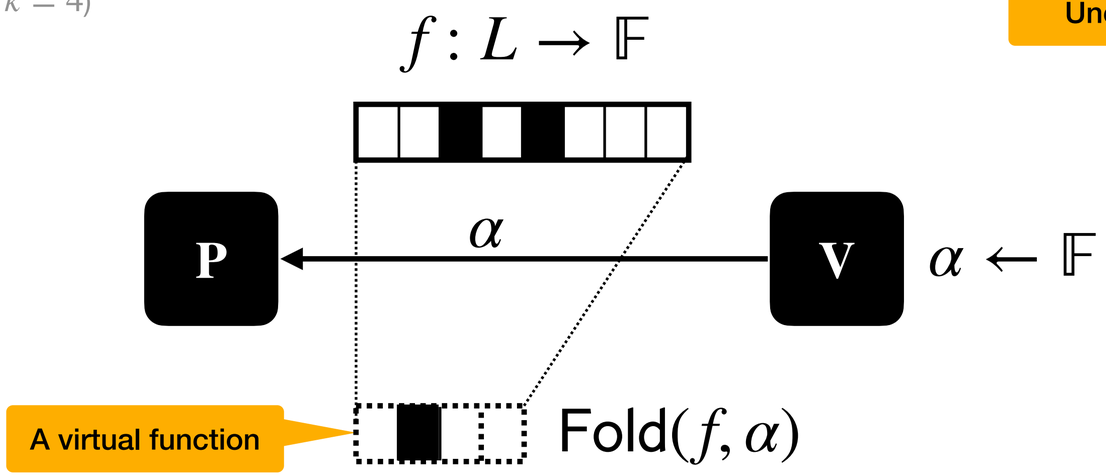

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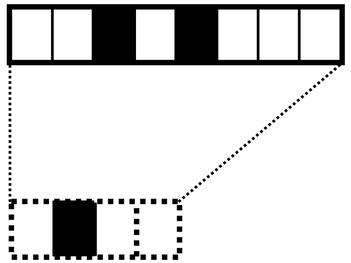

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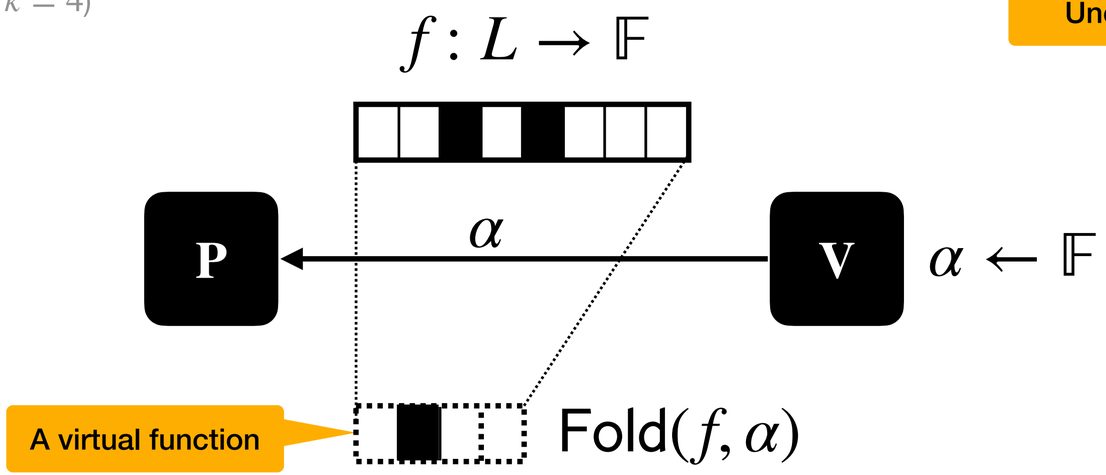

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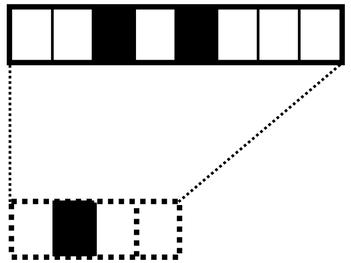

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Proximity Gaps for Reed–Solomon Codes

Eli Ben-Sasson* Dan Carmon* Yuval Ishai† Swastik Kopparty ‡
Shubhangi Saraf§

July 3, 2021

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Test a random linear combination

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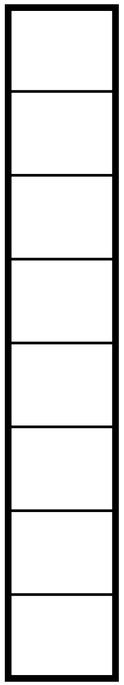

f_1

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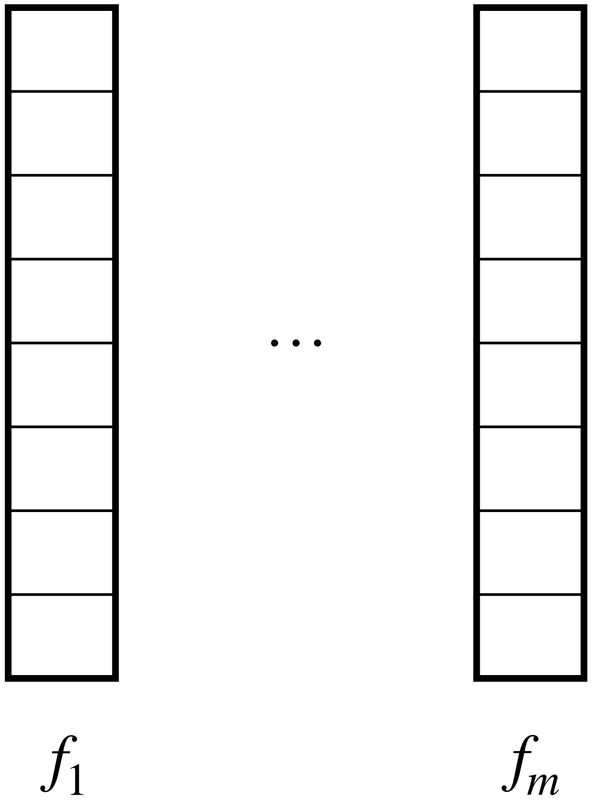

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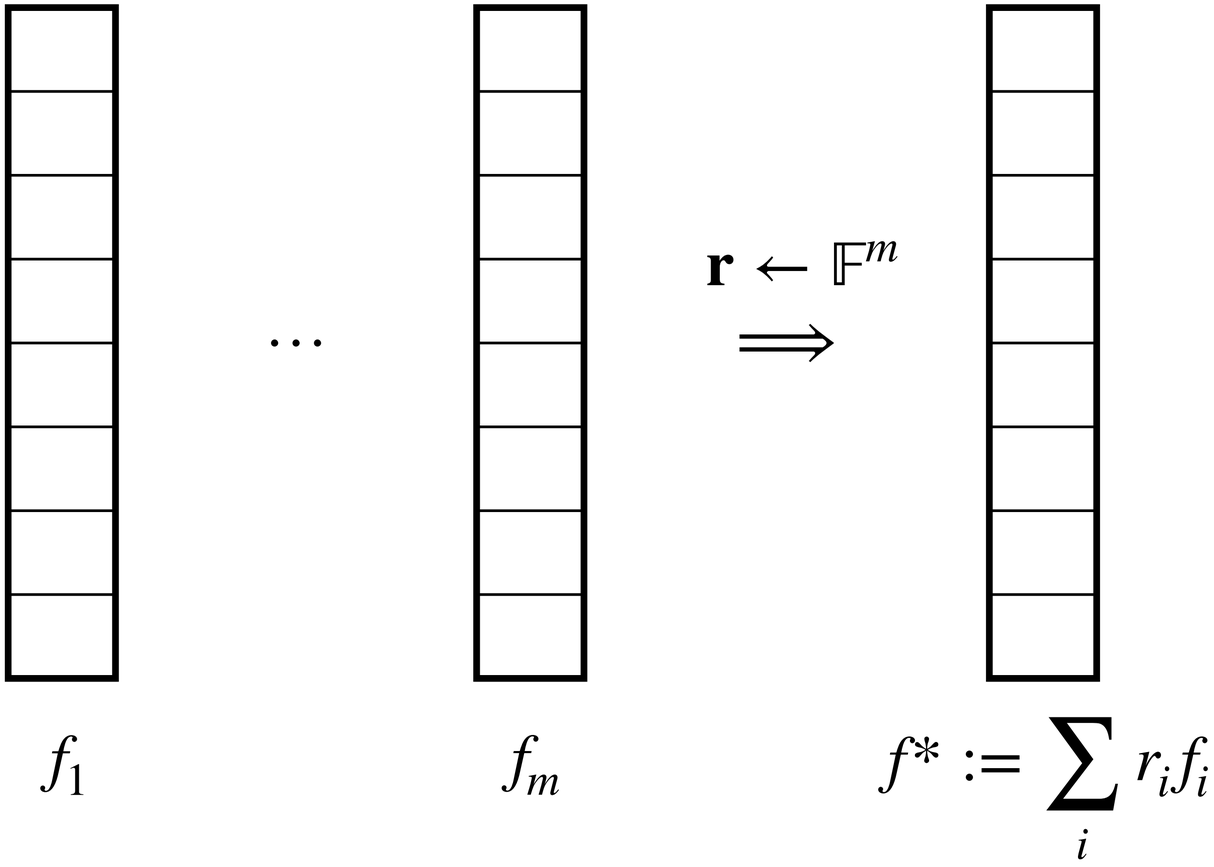

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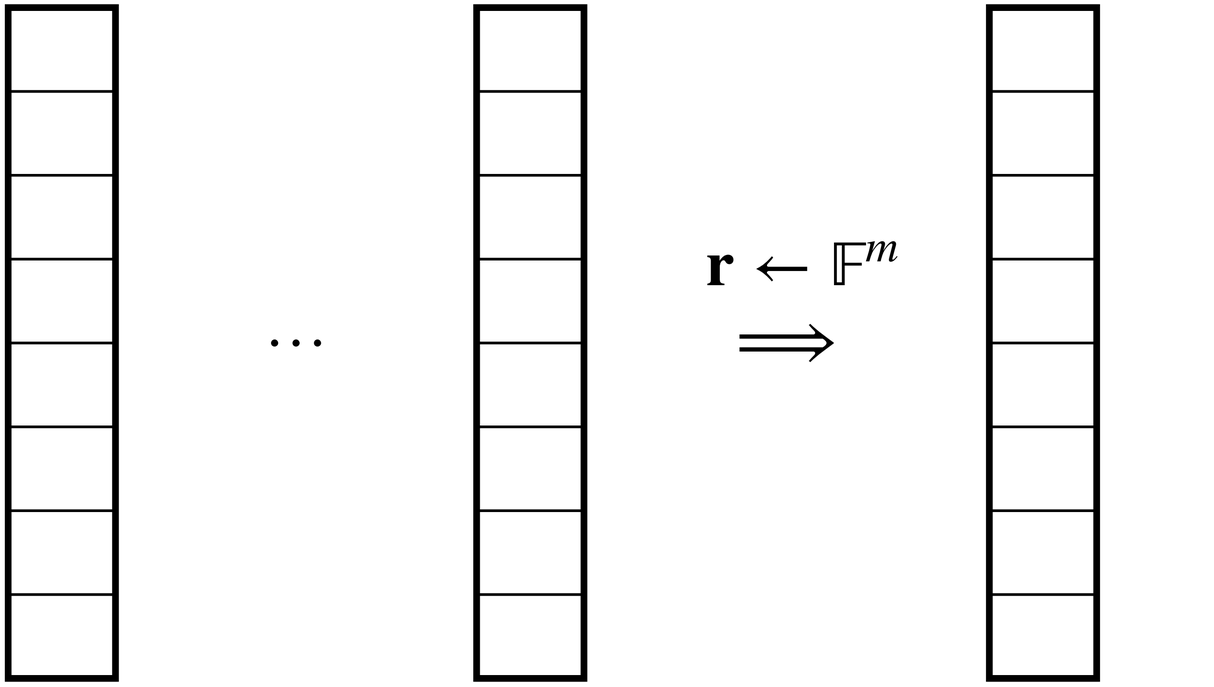

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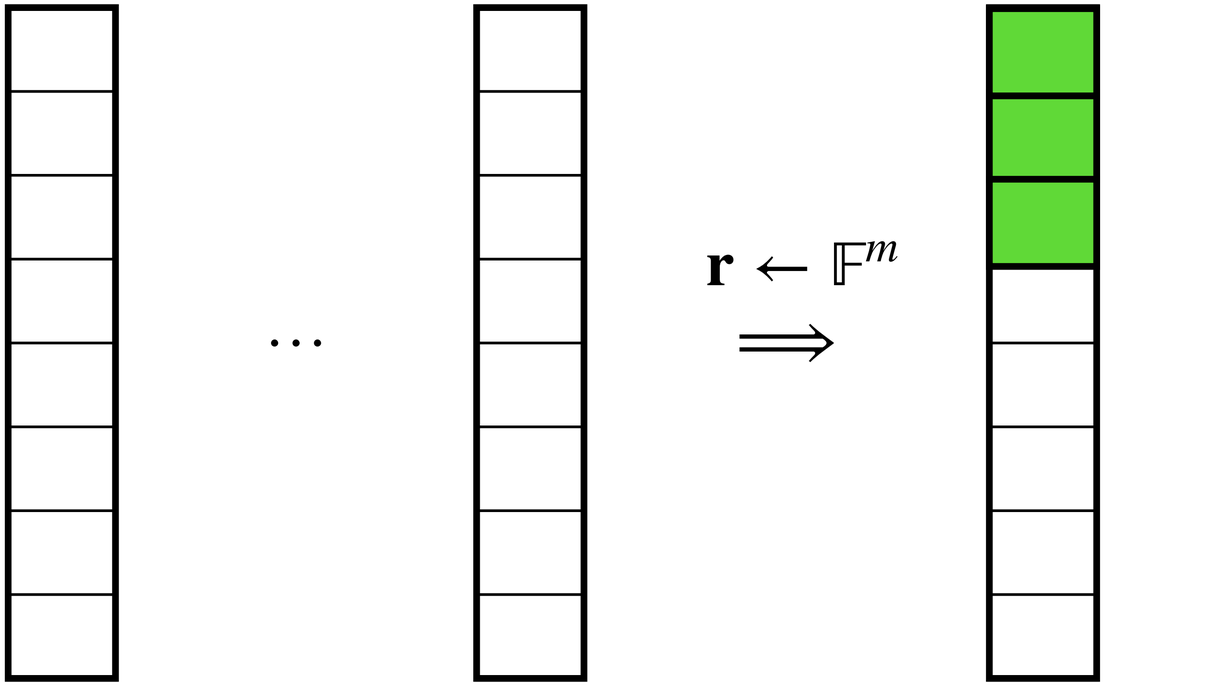

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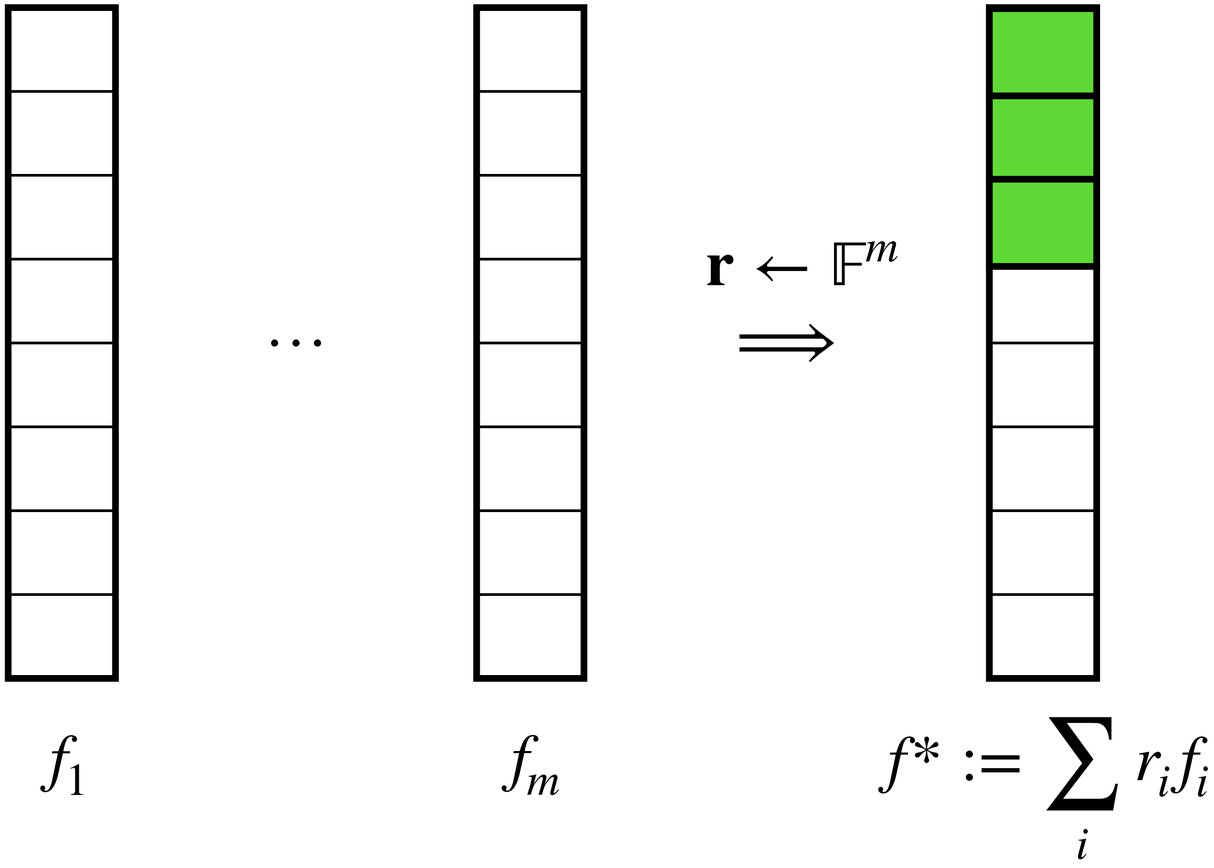

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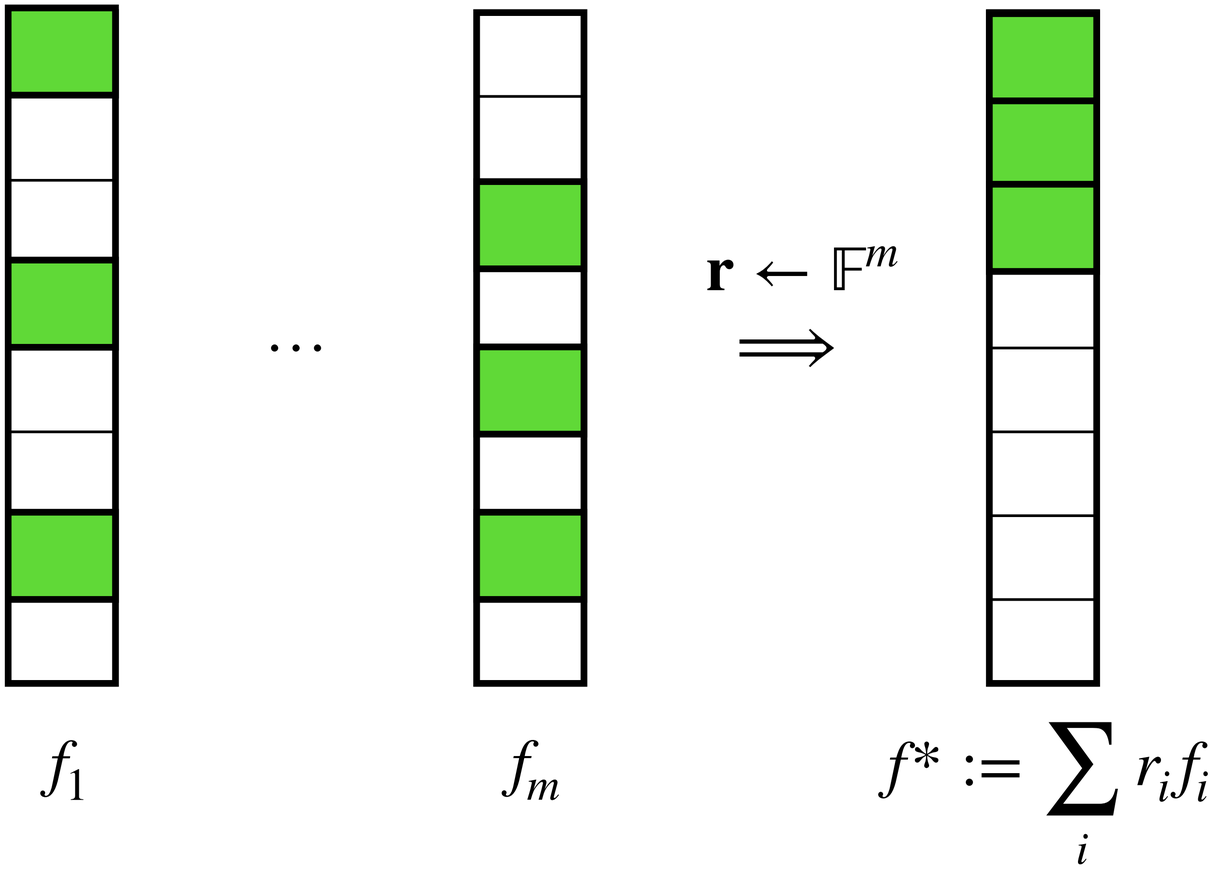

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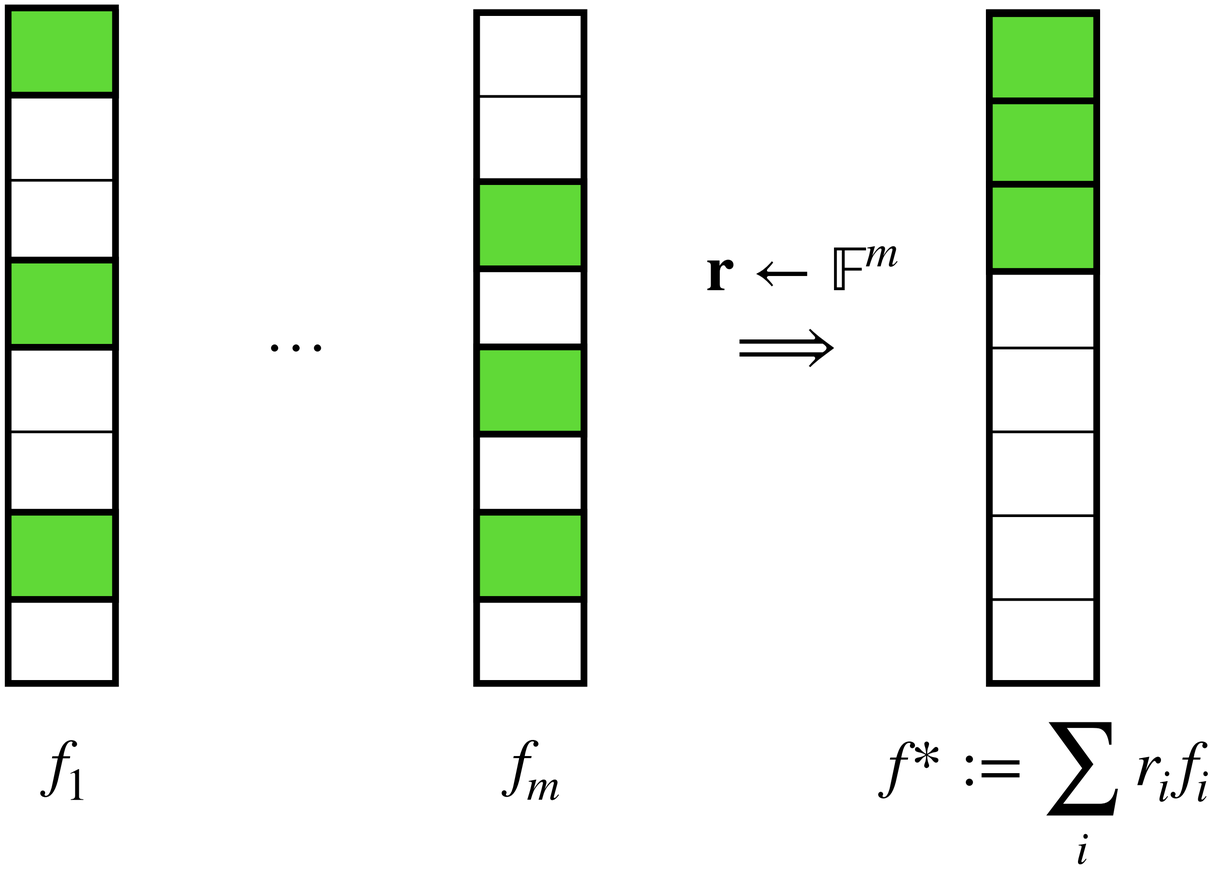

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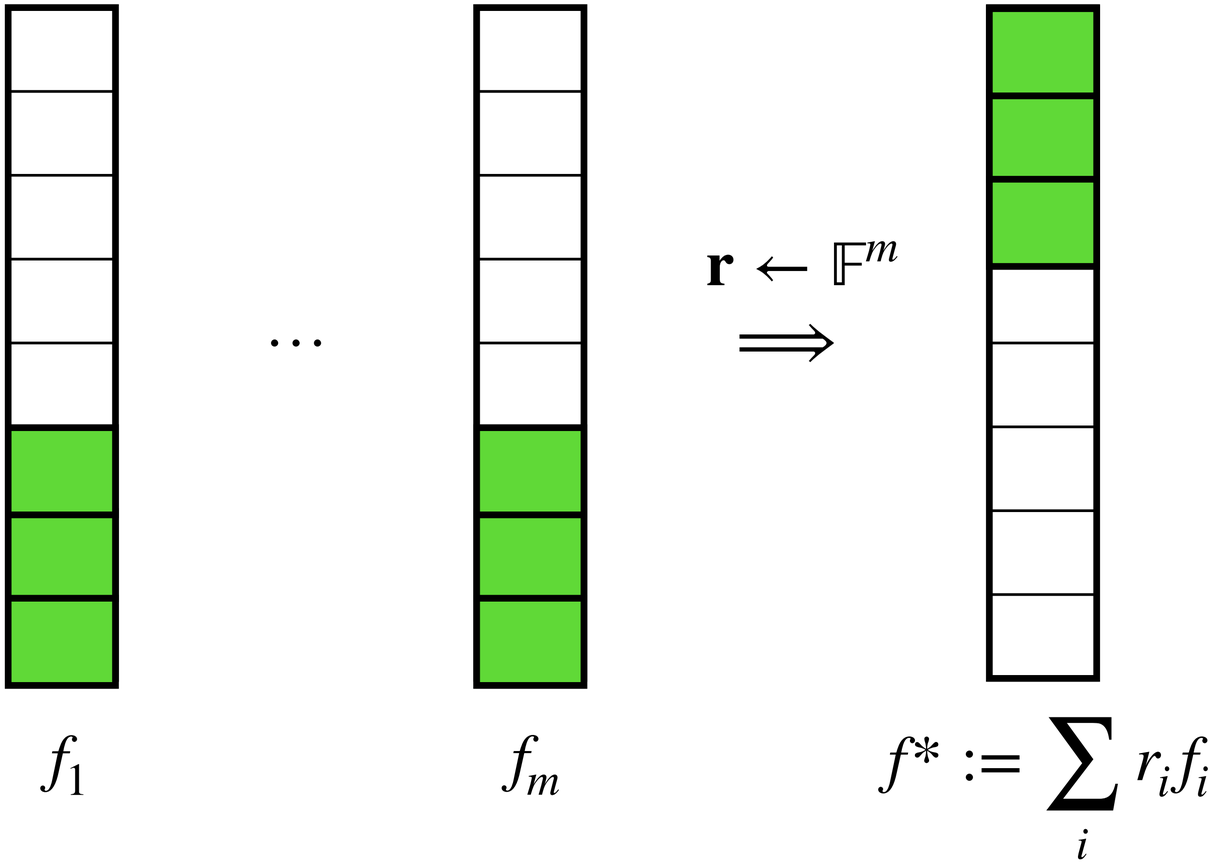

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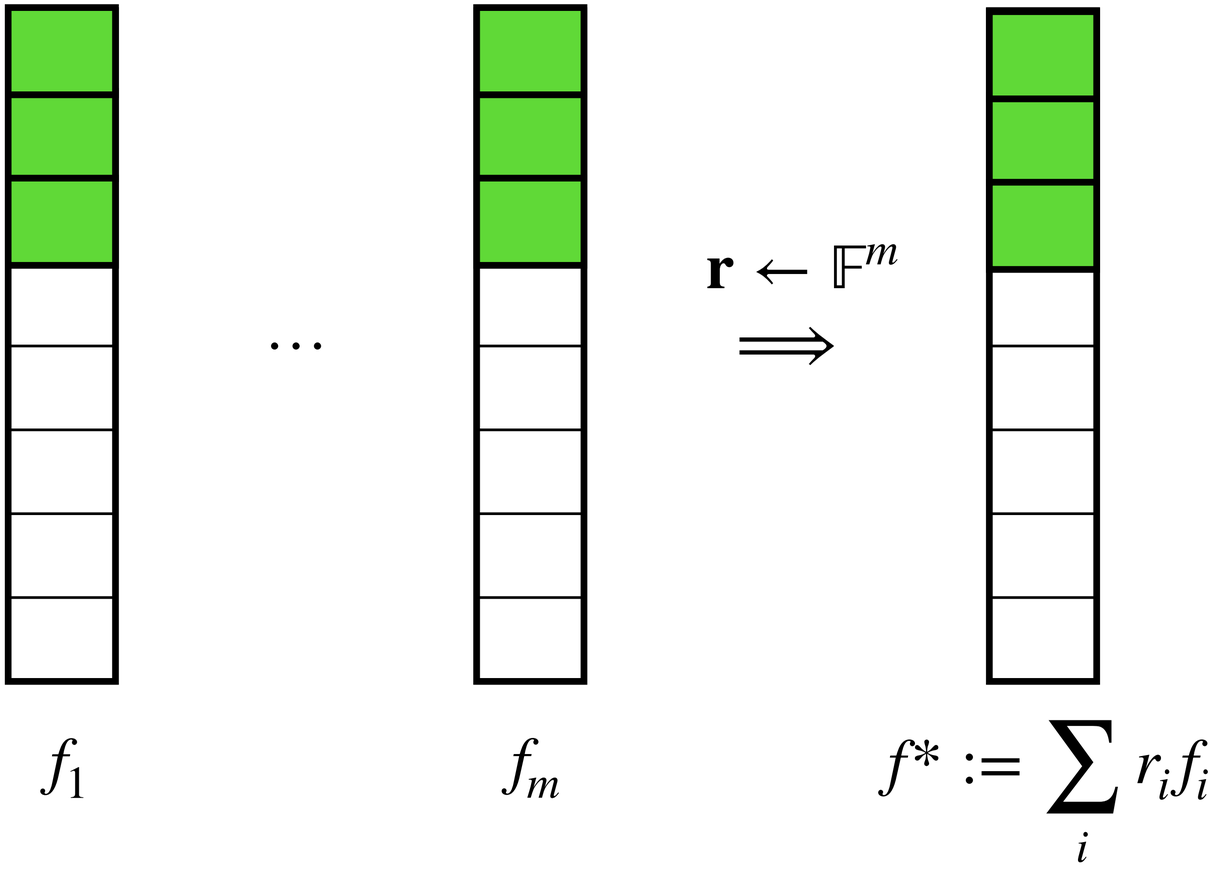

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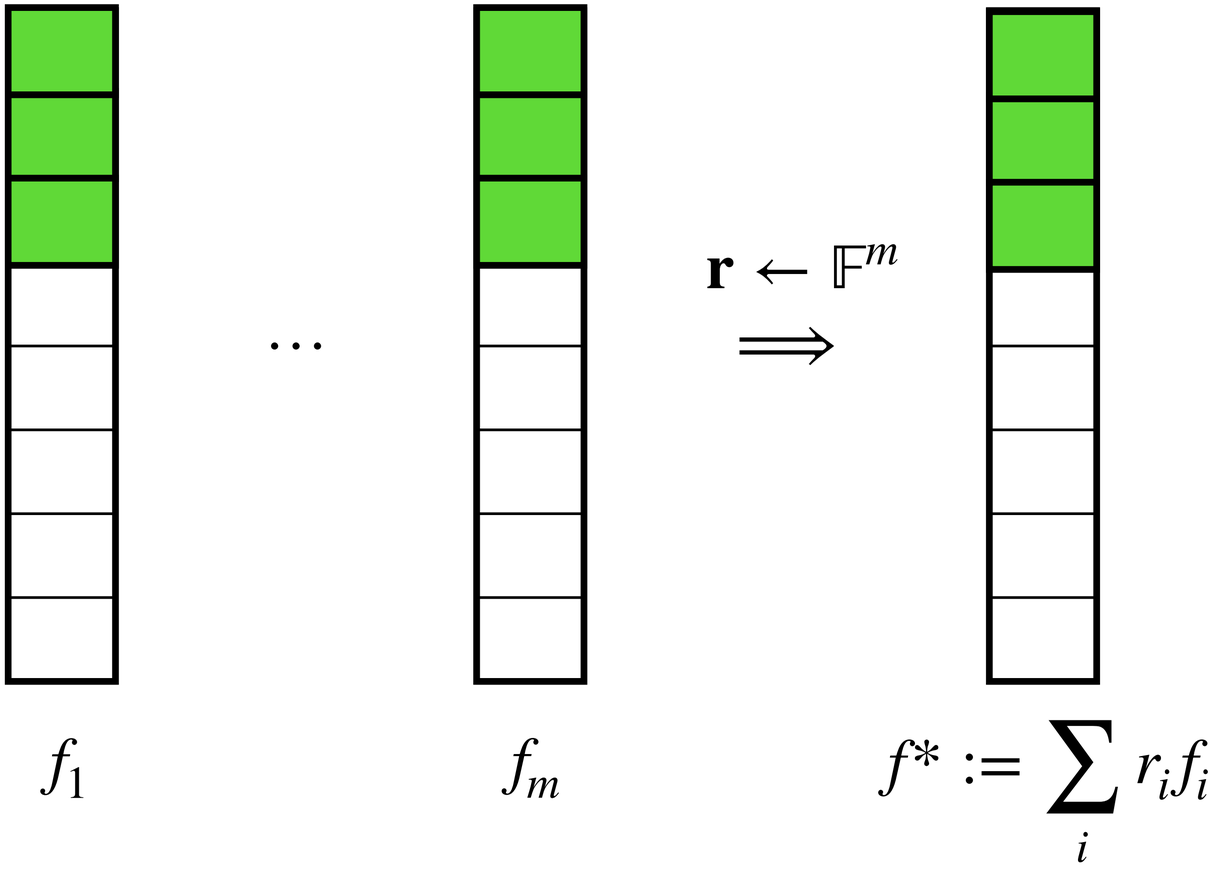

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Mutual correlated agreement: the stripe in which $f_1, \ldots, f_m$ agree with $\mathscr{C}$ is the same on which f^* does:

"*No new correlated domains appear*"

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Implied by mutual correlated agreement

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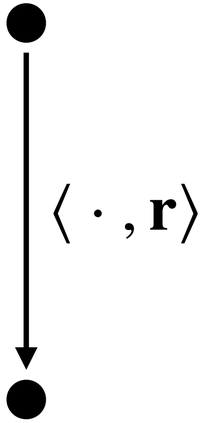

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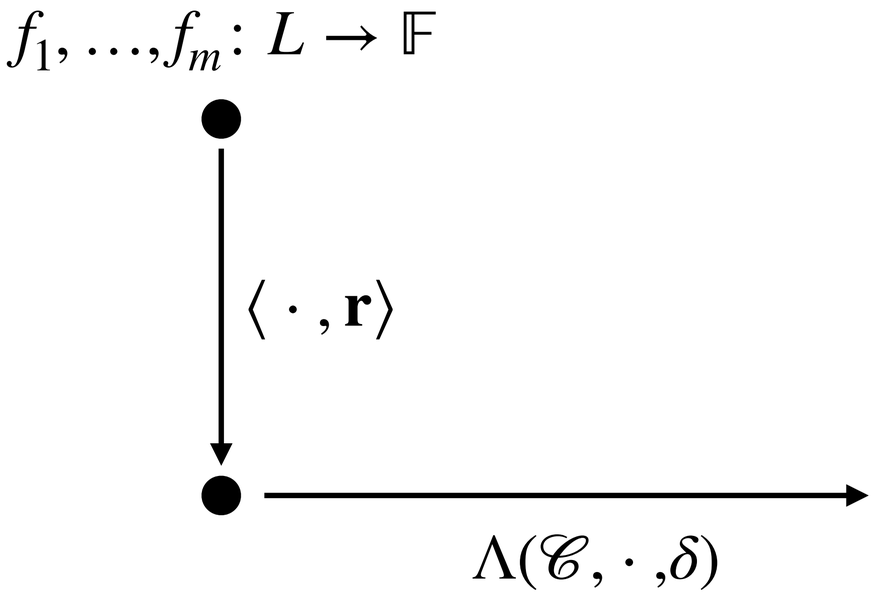

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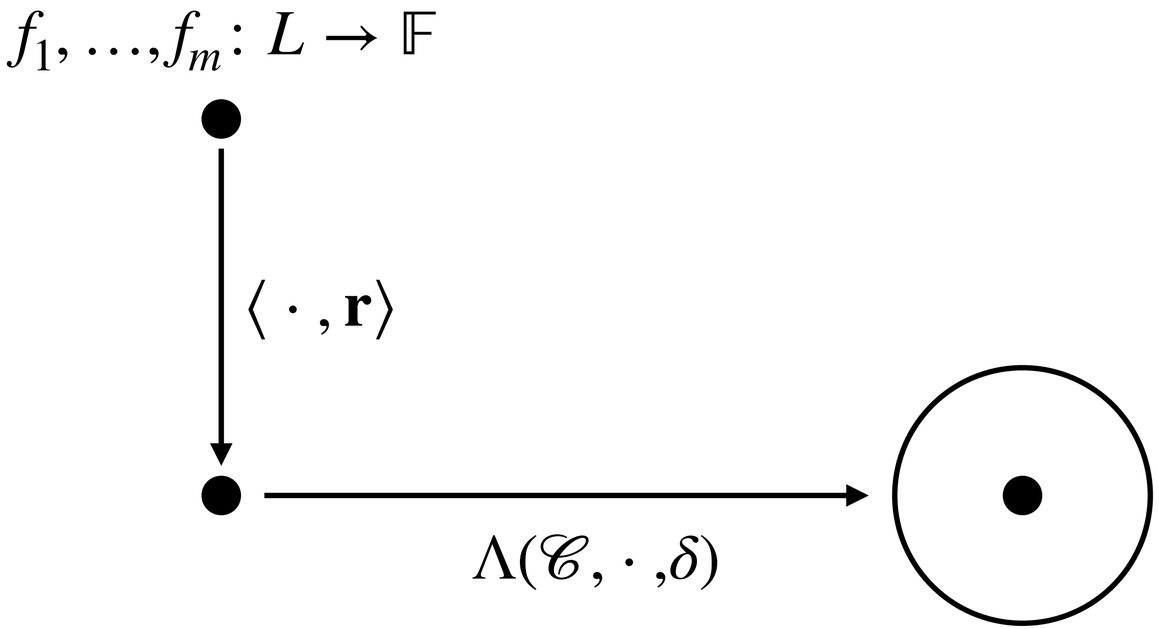

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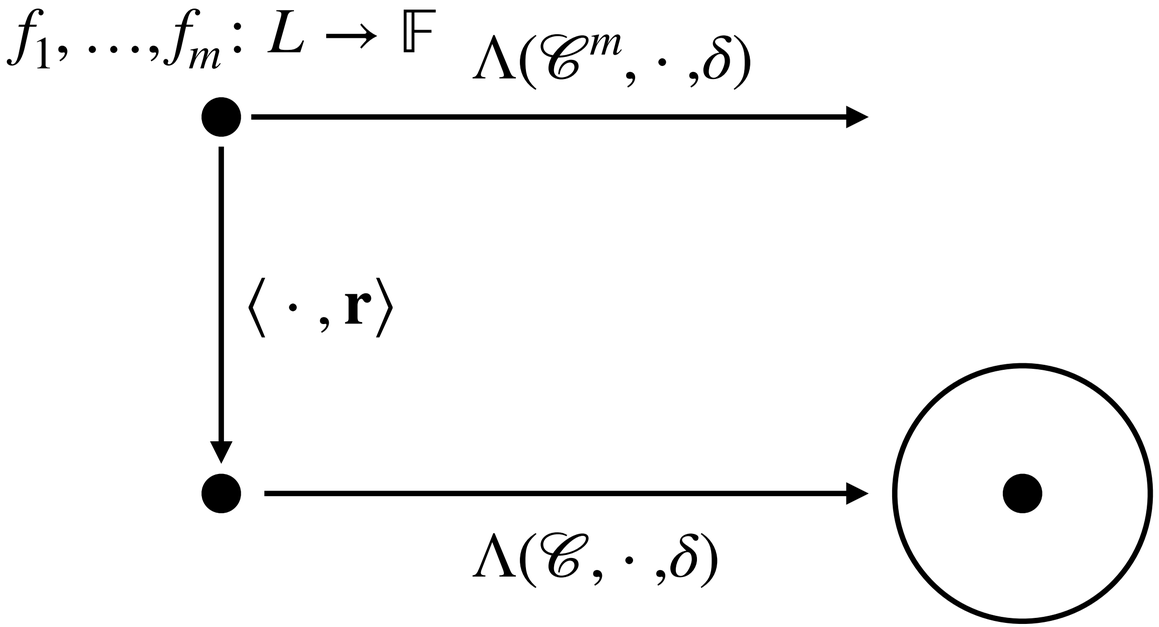

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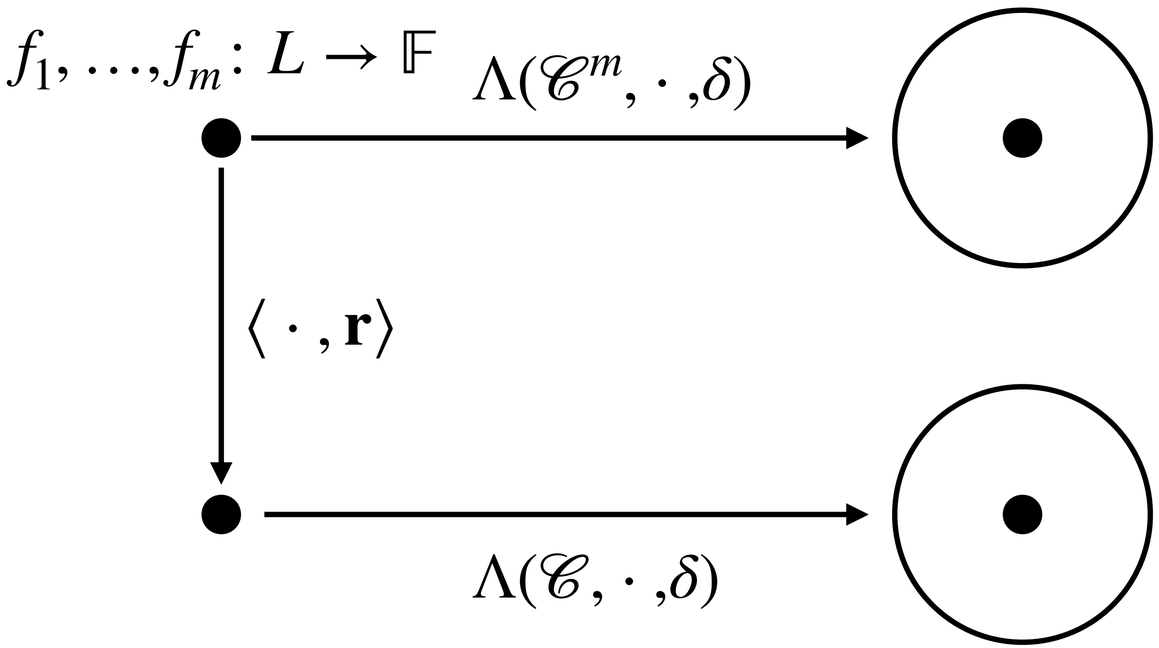

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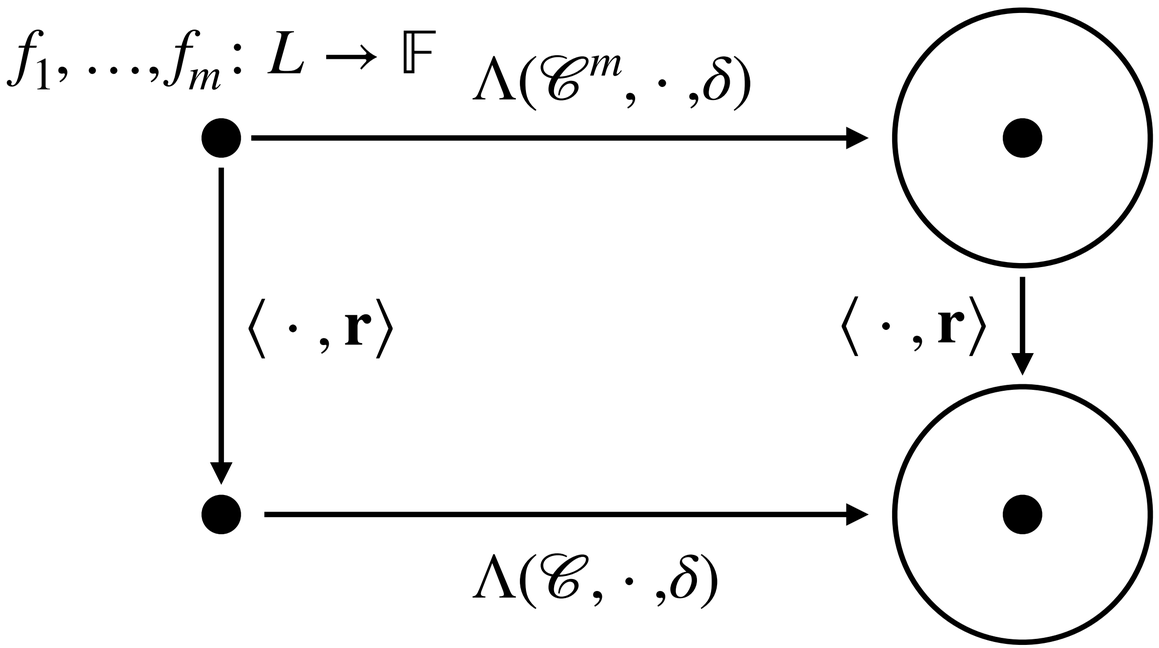

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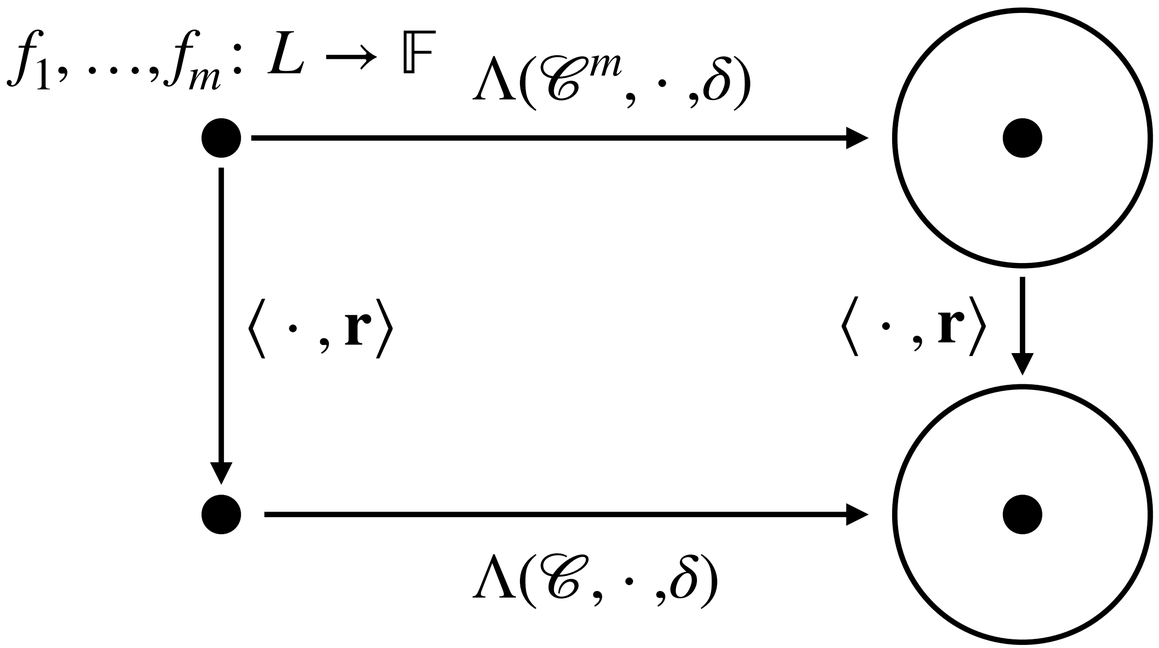

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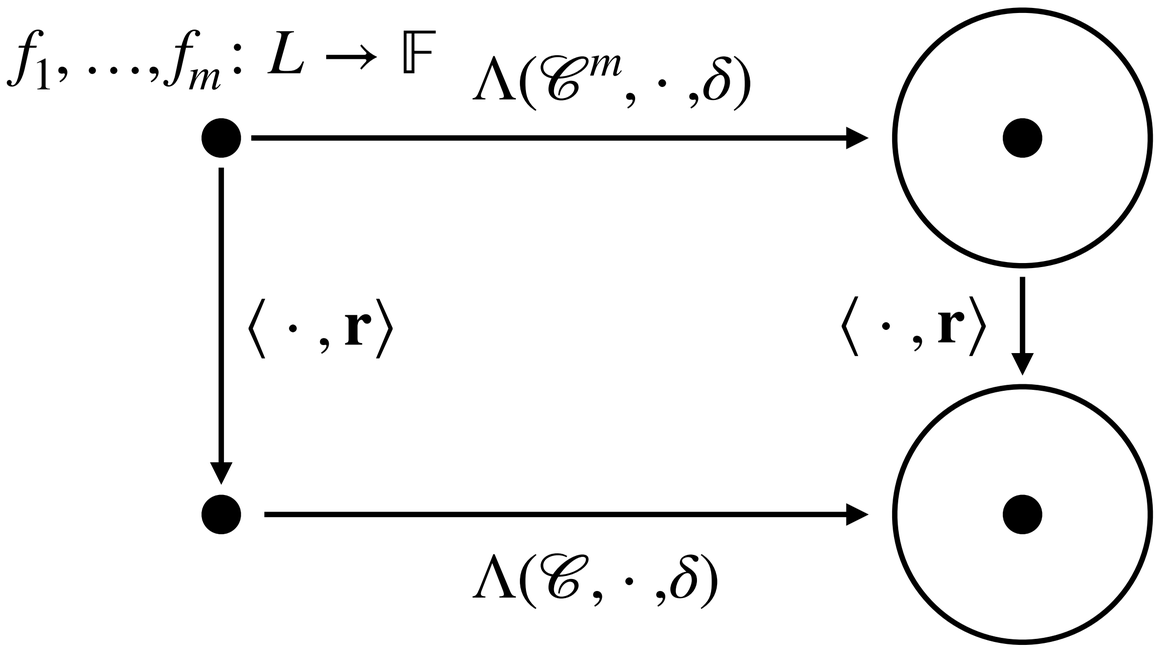

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w.h.p. over $\mathbf{r}$:

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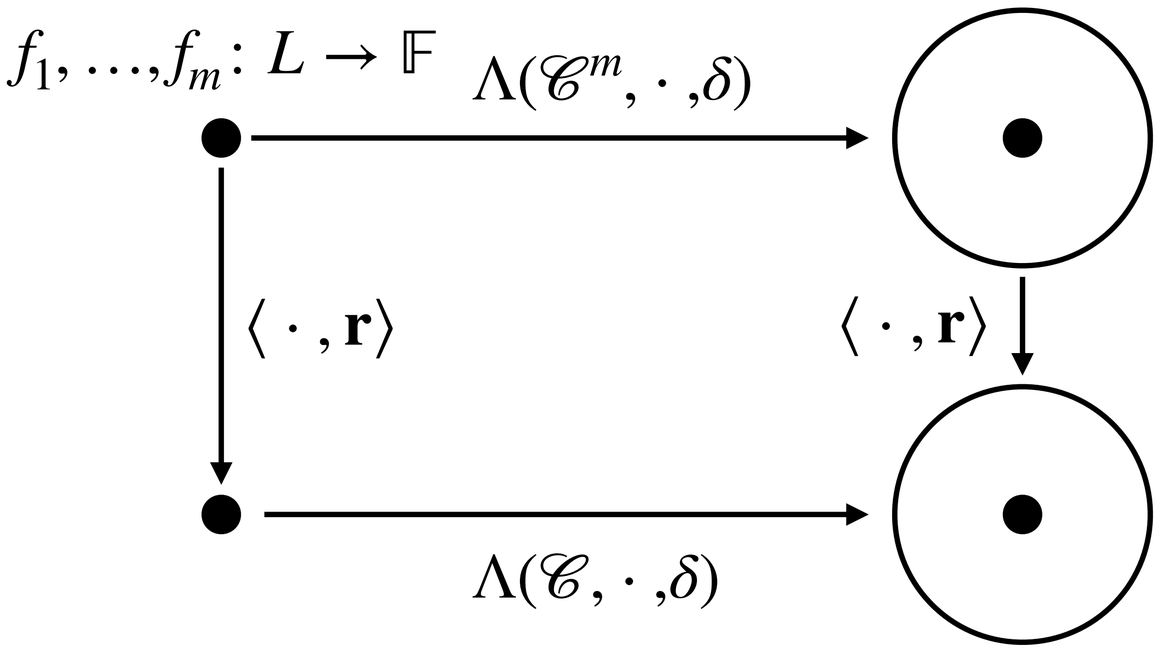

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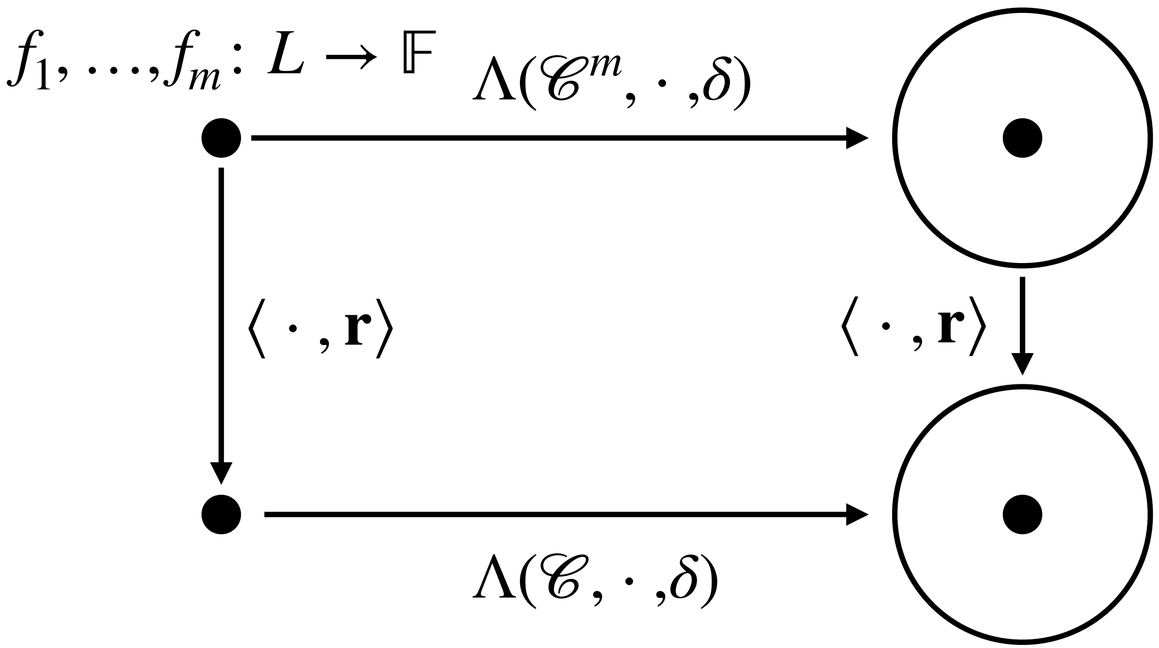

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Folding and lists commute

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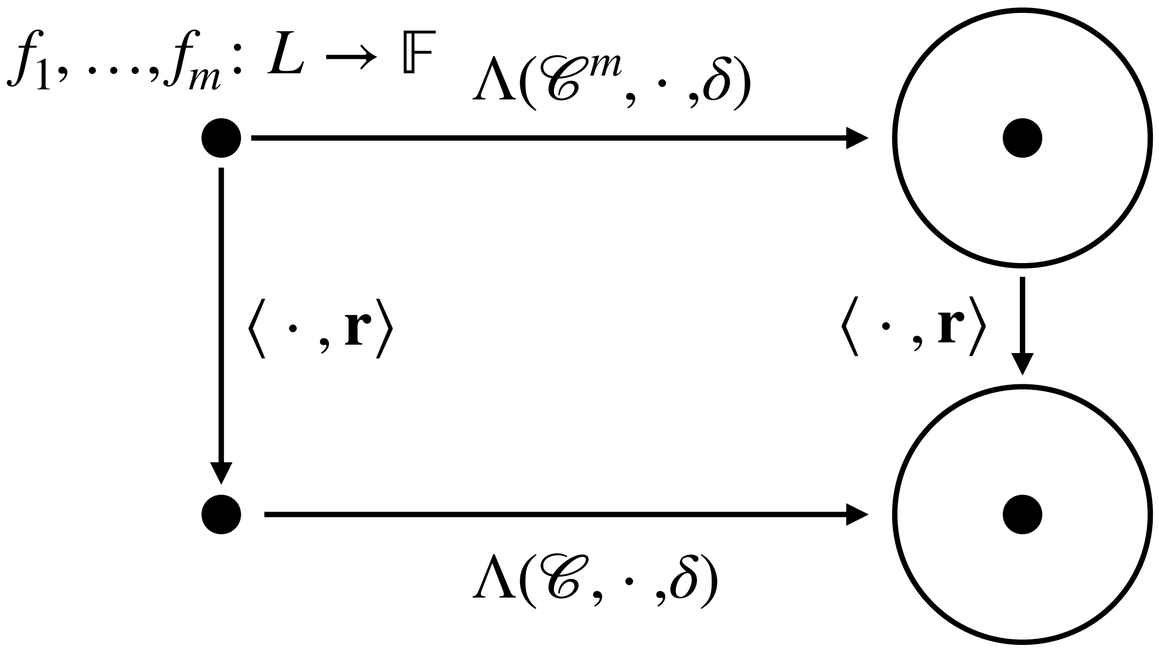
$\Lambda(\mathscr{C}, f, \delta)$ is the list of codewords of $\mathscr{C}$ that are δ-close to f



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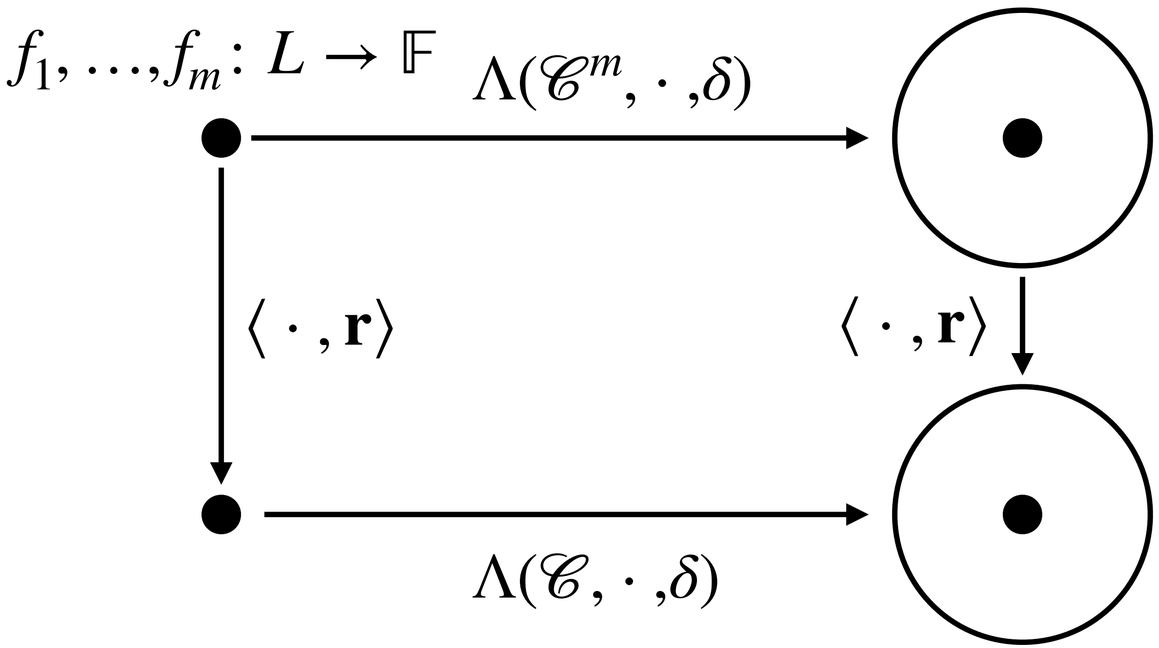

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Recent results show that mutual correlated agreement holds up to 1.5 Johnson for general linear codes!

We show correlated agreement implies mutual correlated agreement in *unique decoding.*

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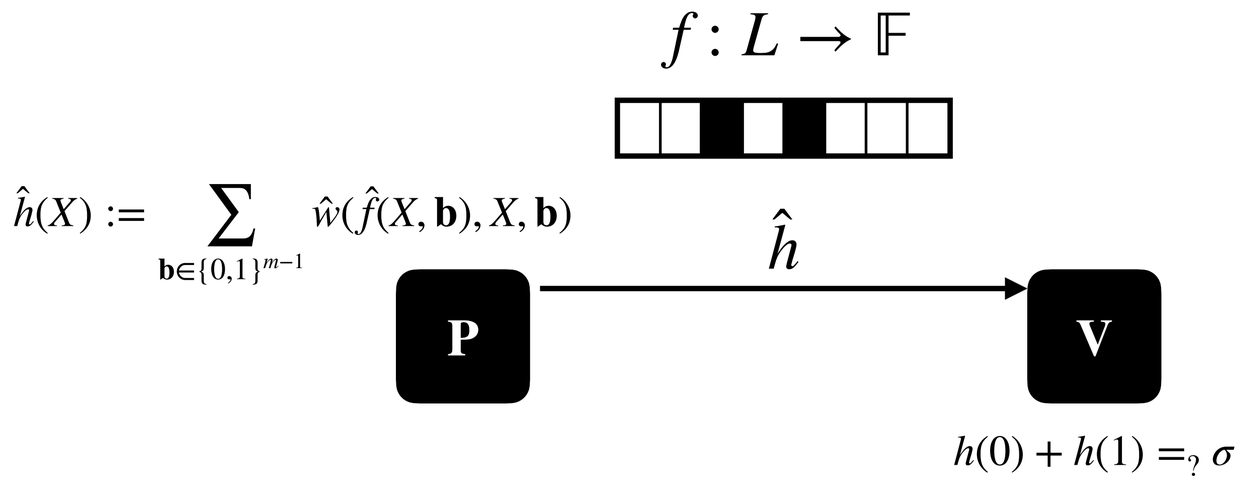

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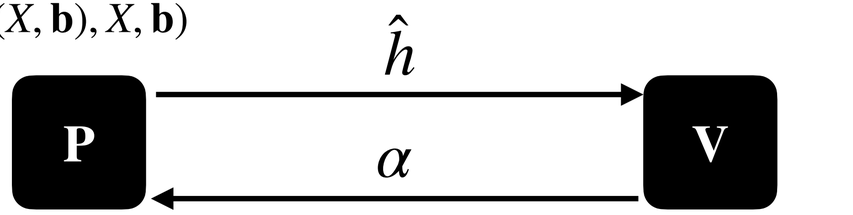

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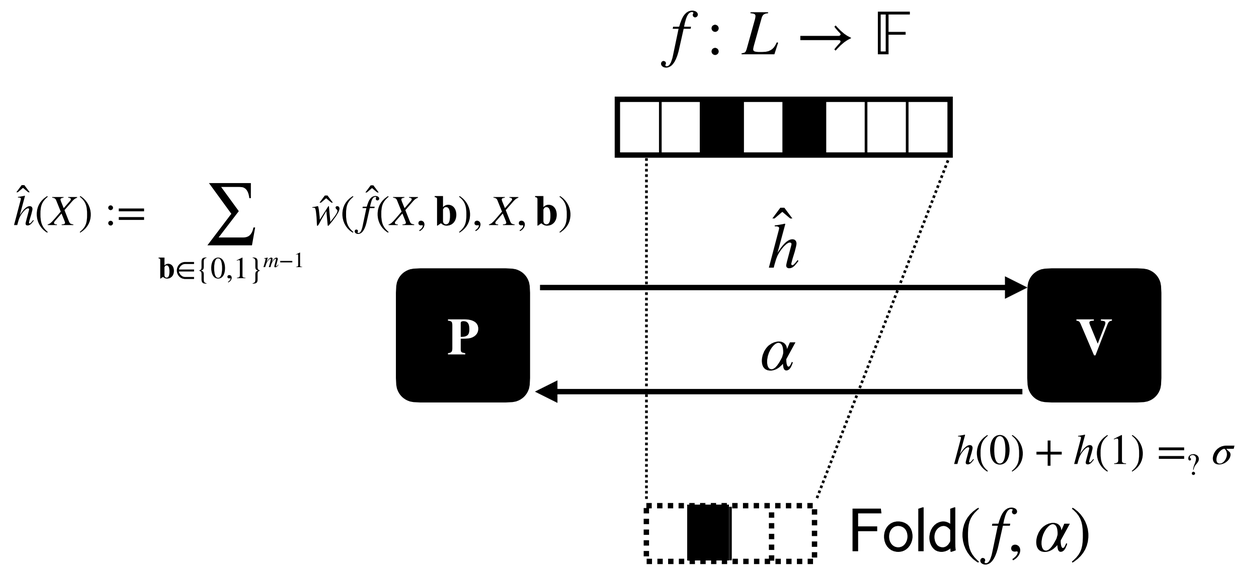

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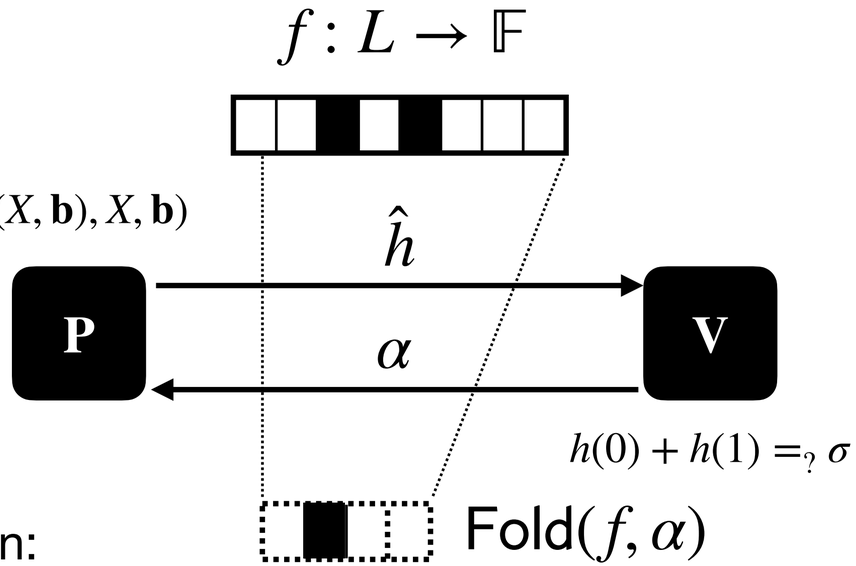

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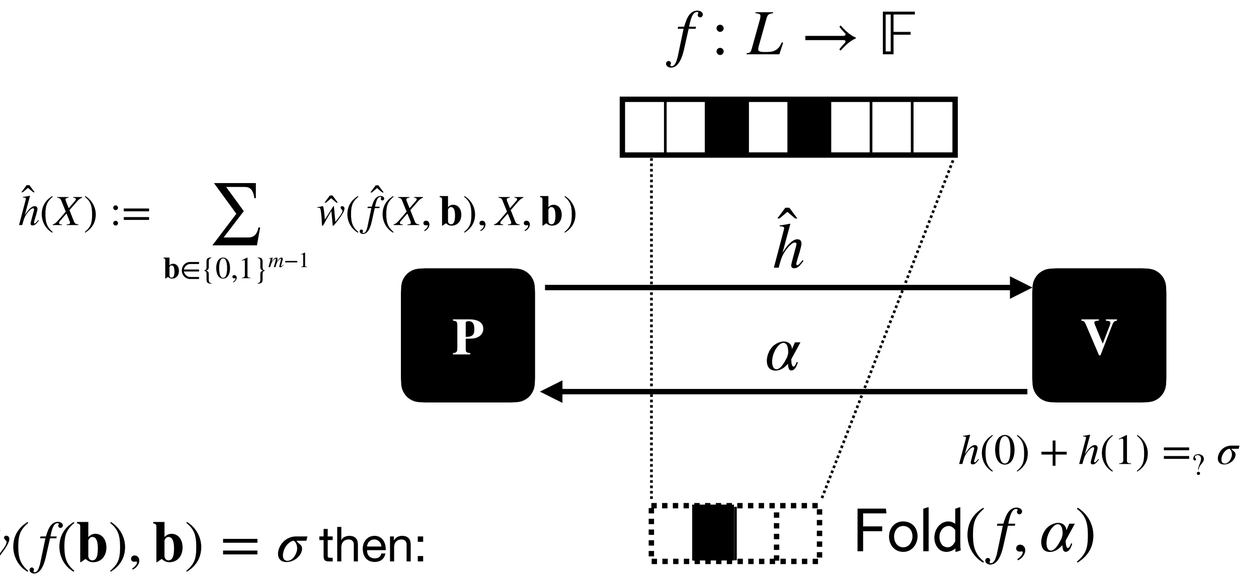

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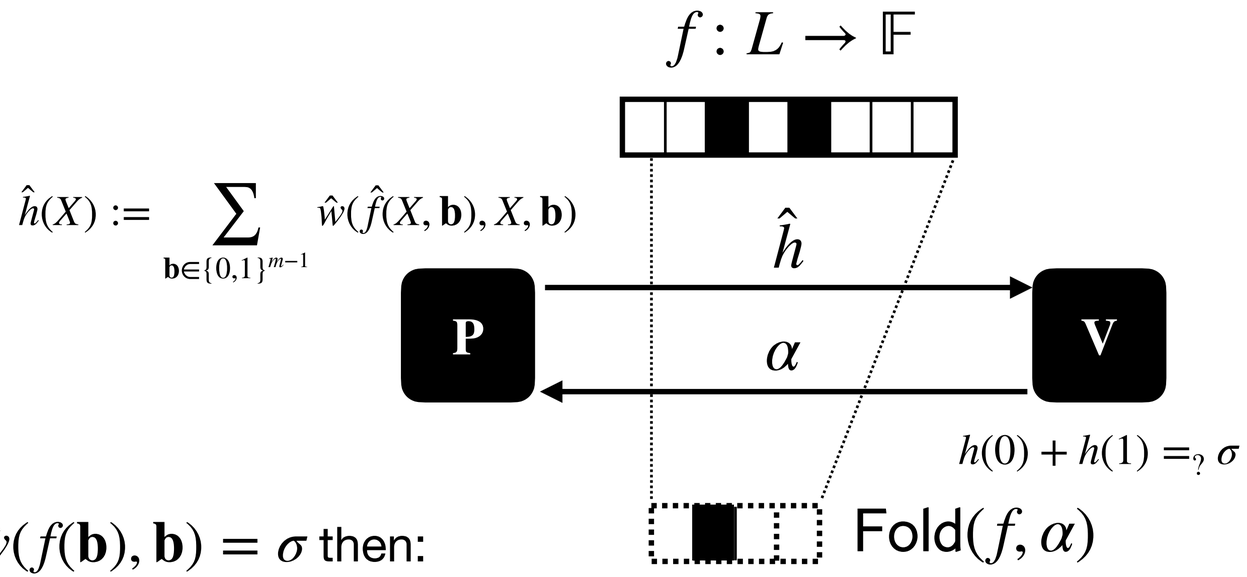

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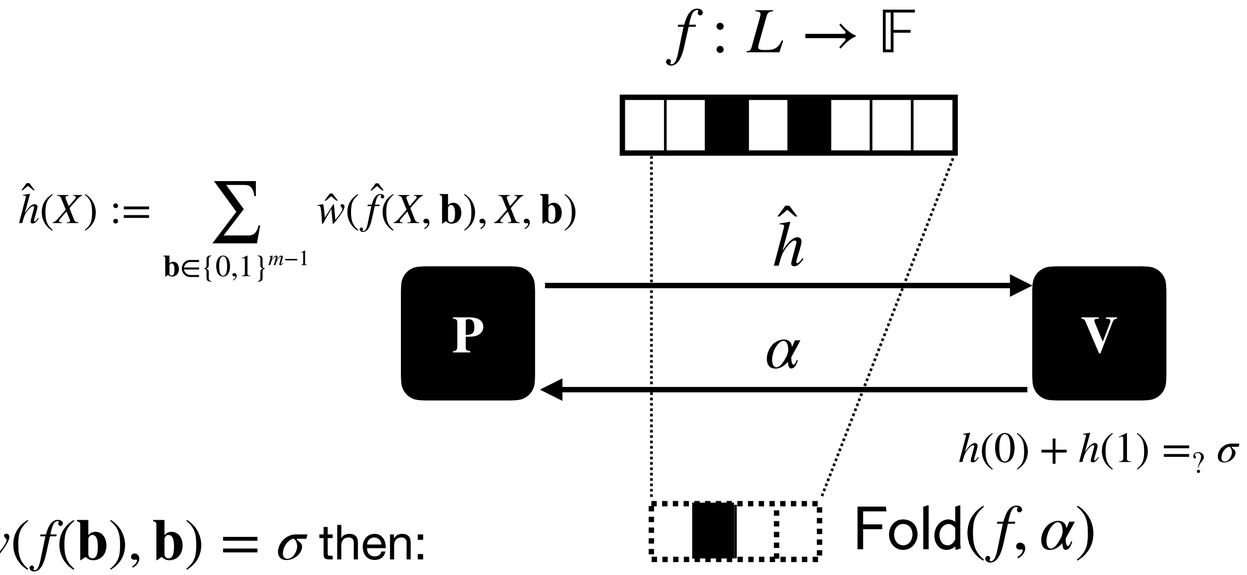

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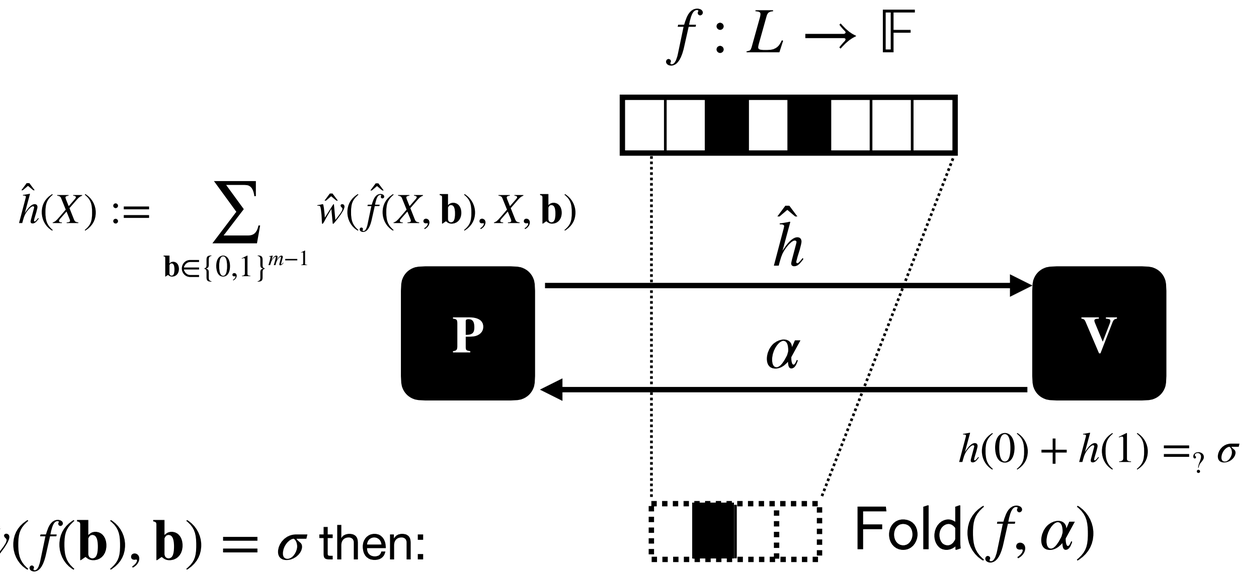

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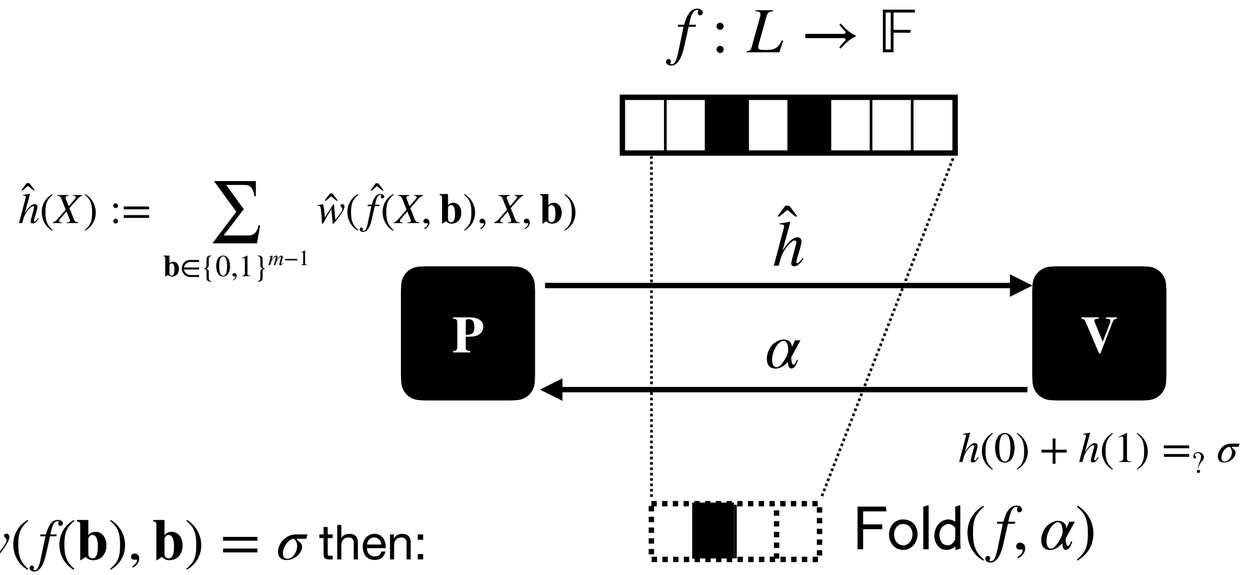

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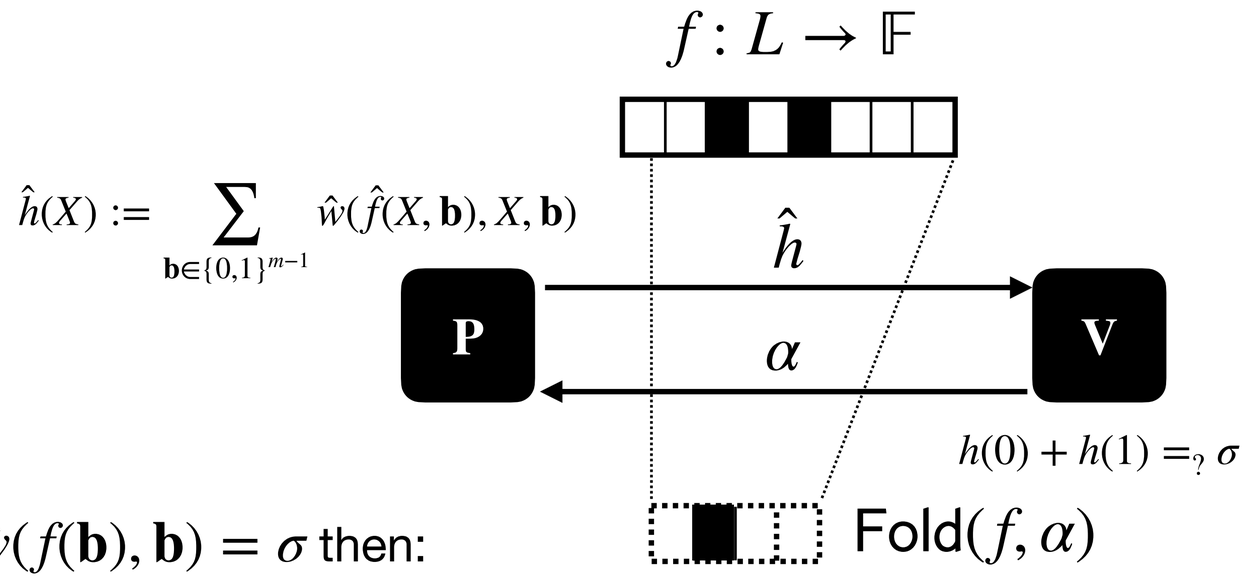

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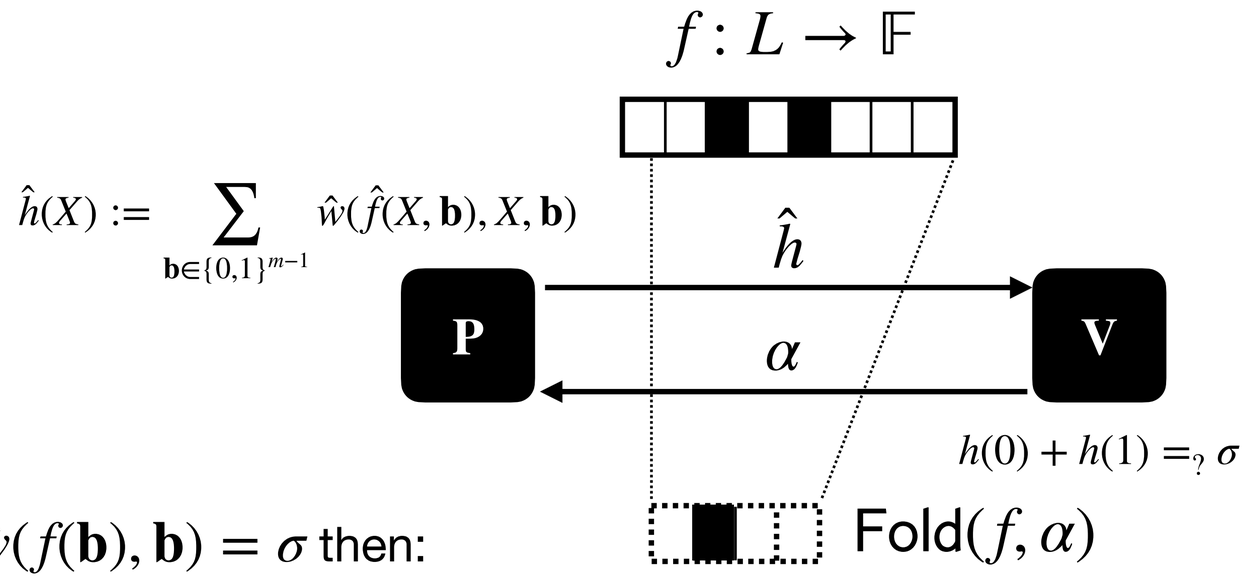

In the full protocol, we fold by 2-by-2 k times. Can also fold by 2^k at a time (nice for first round!)

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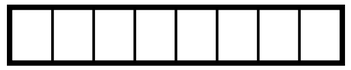

P

V

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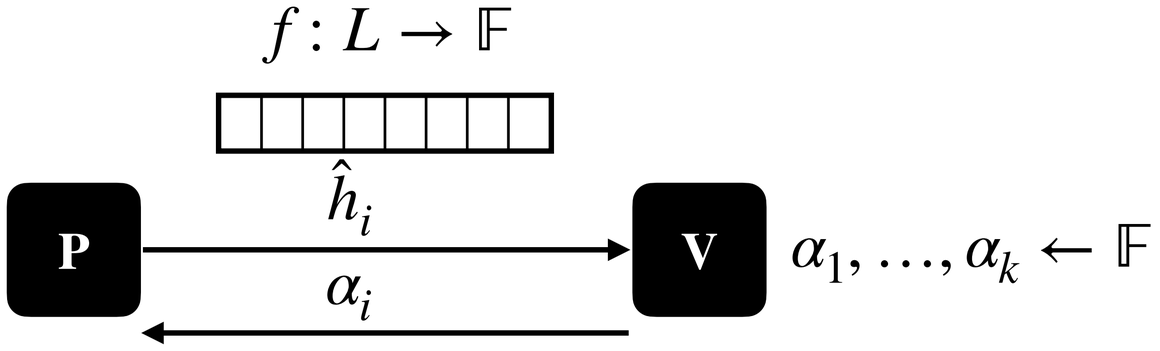

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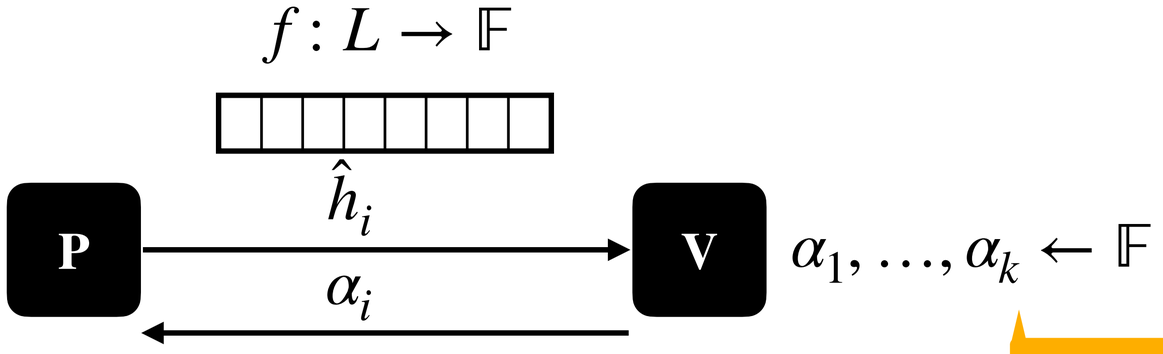

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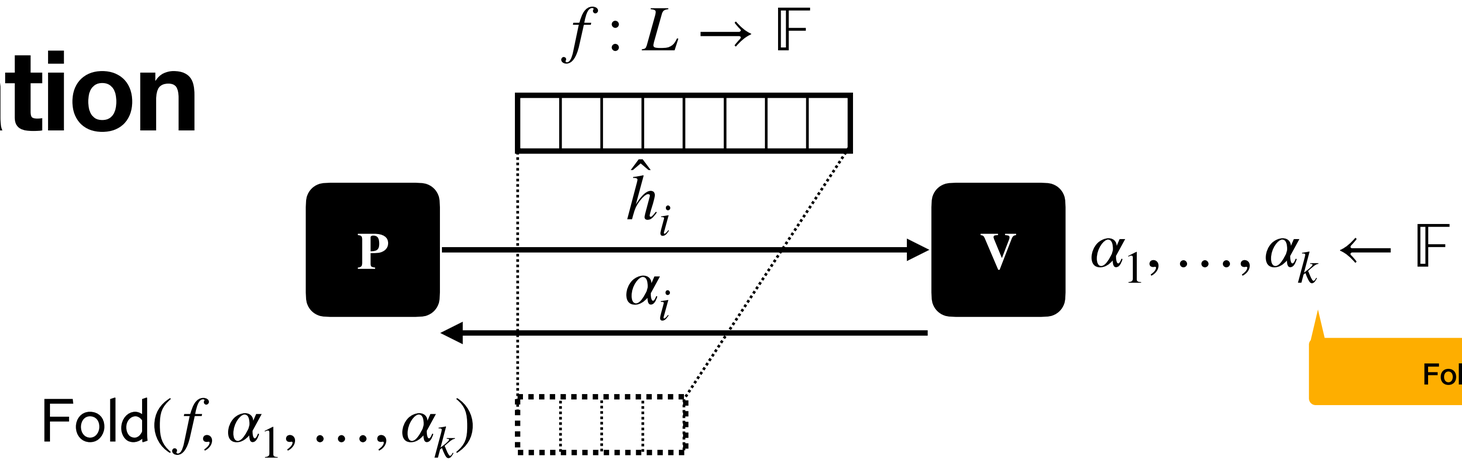

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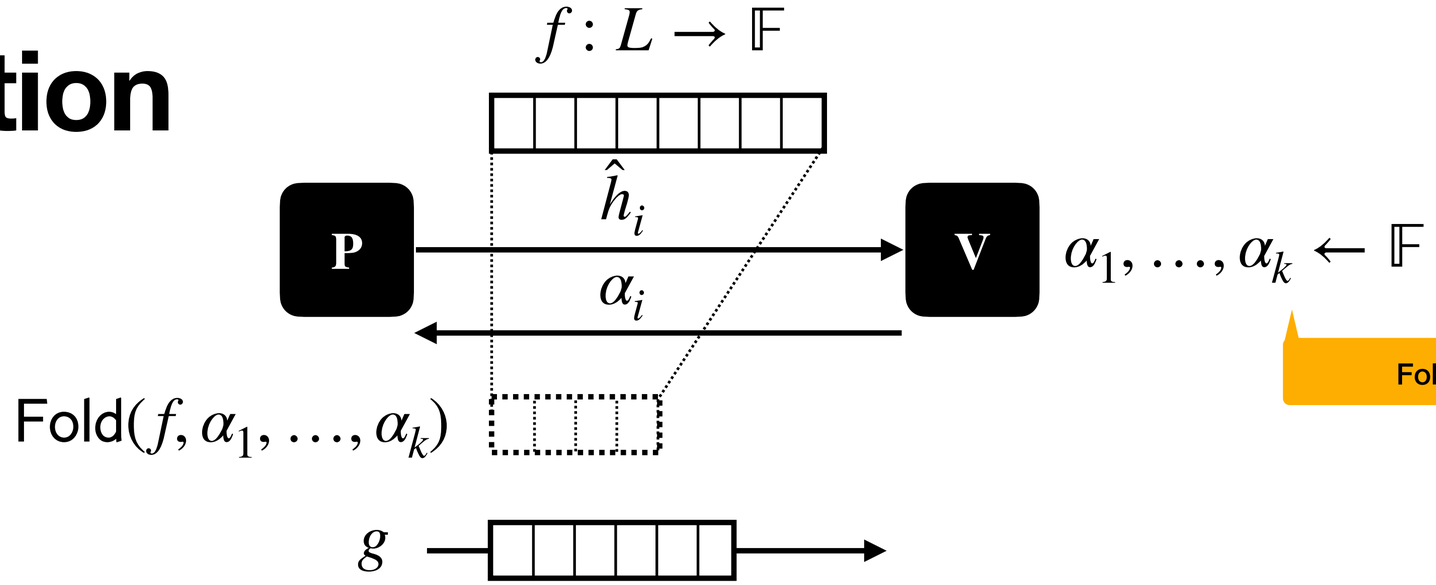

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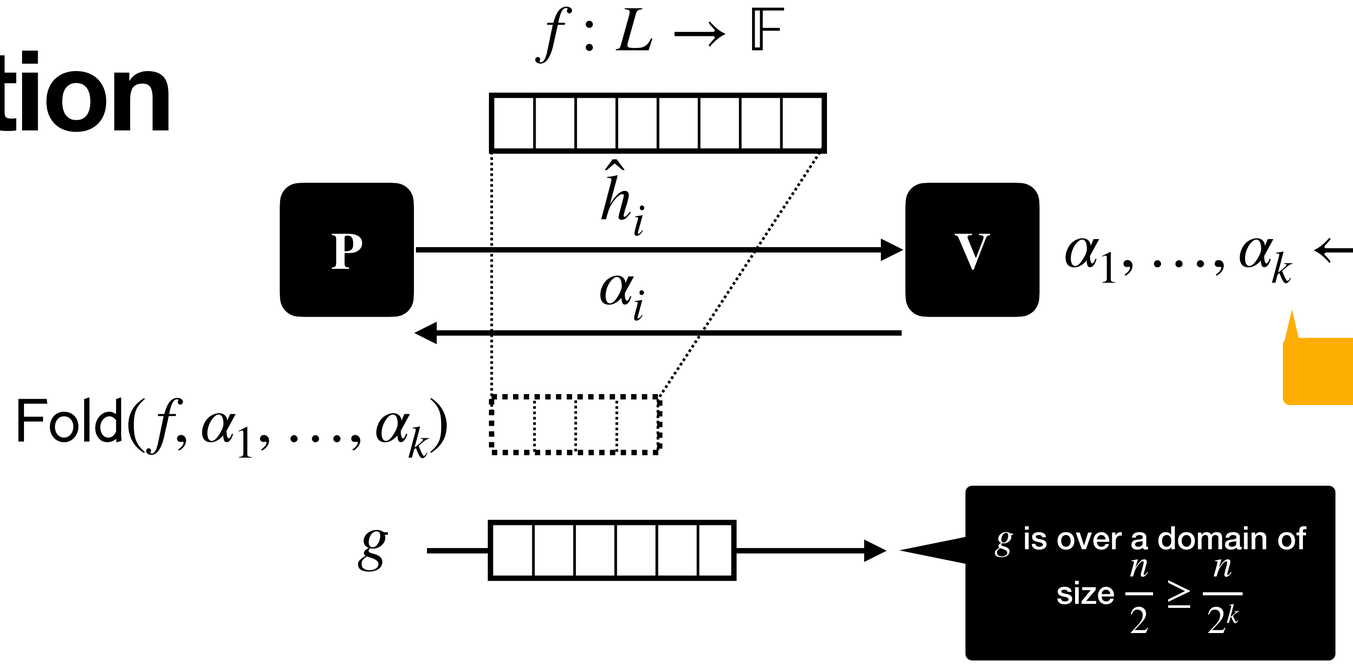

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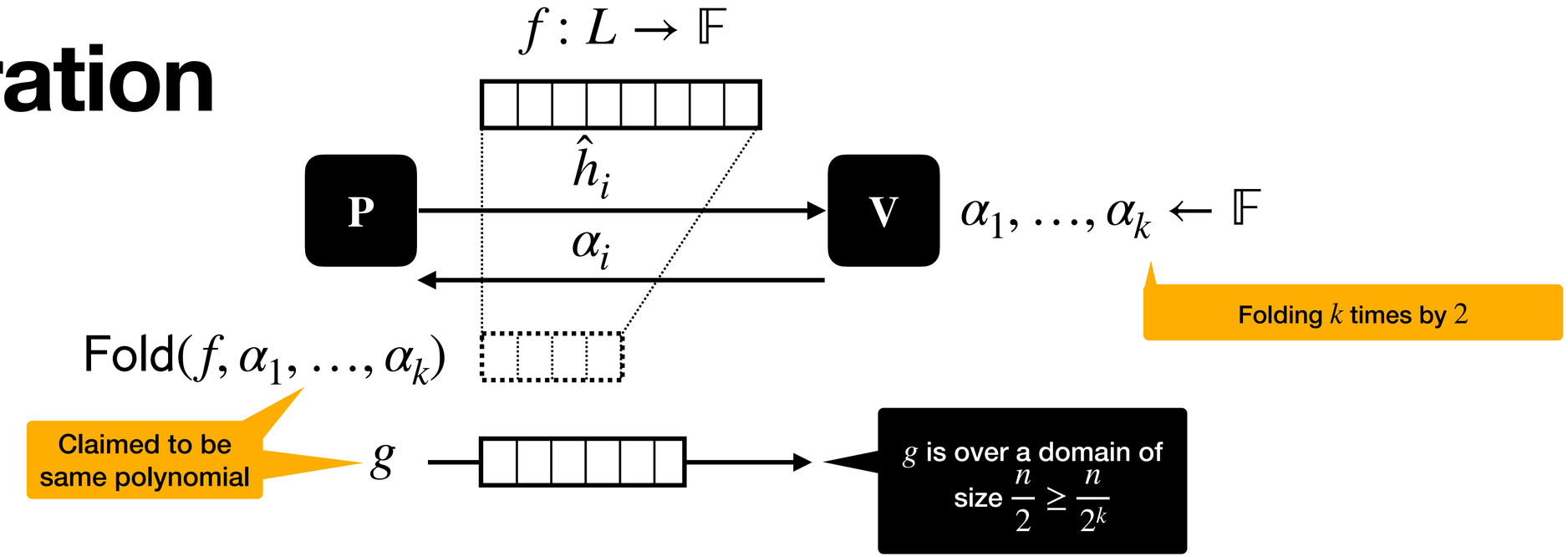

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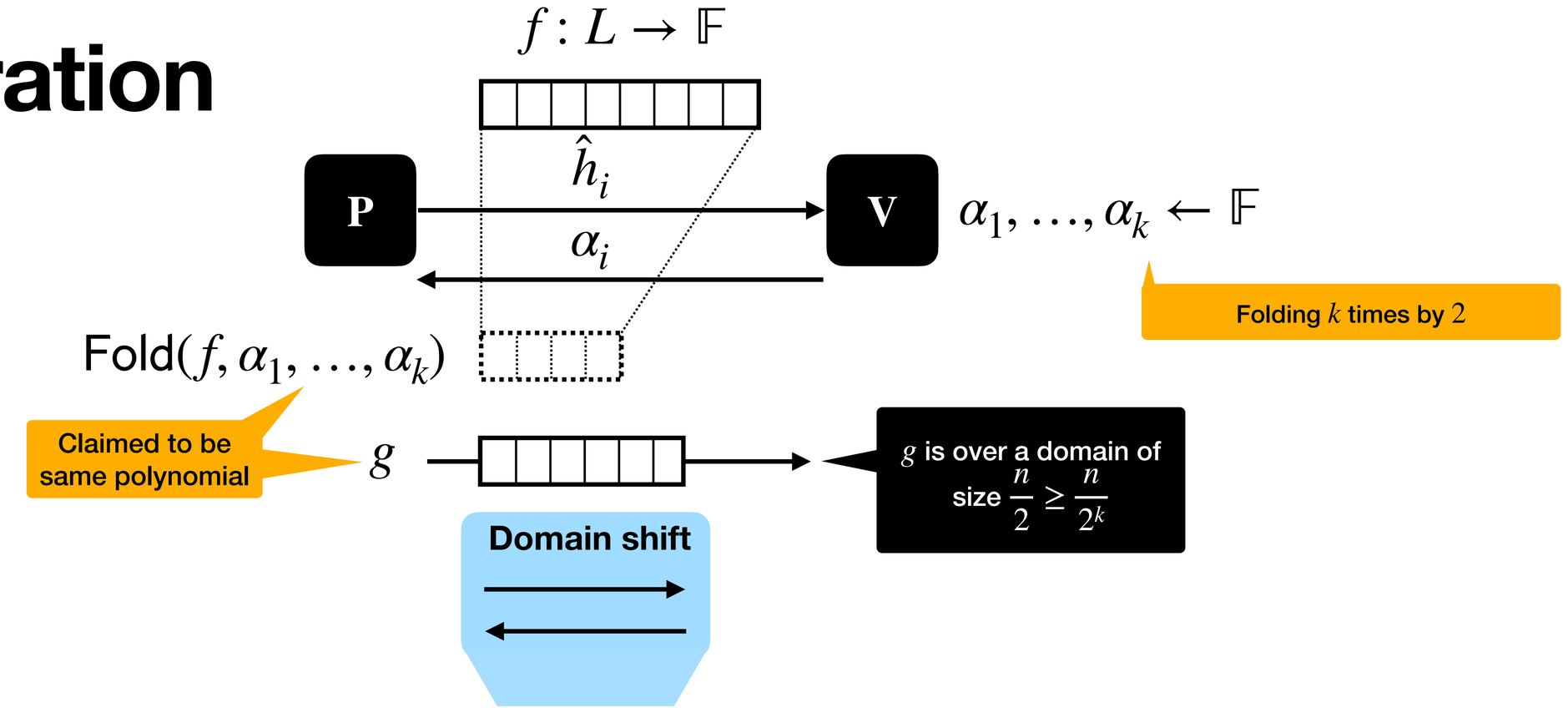

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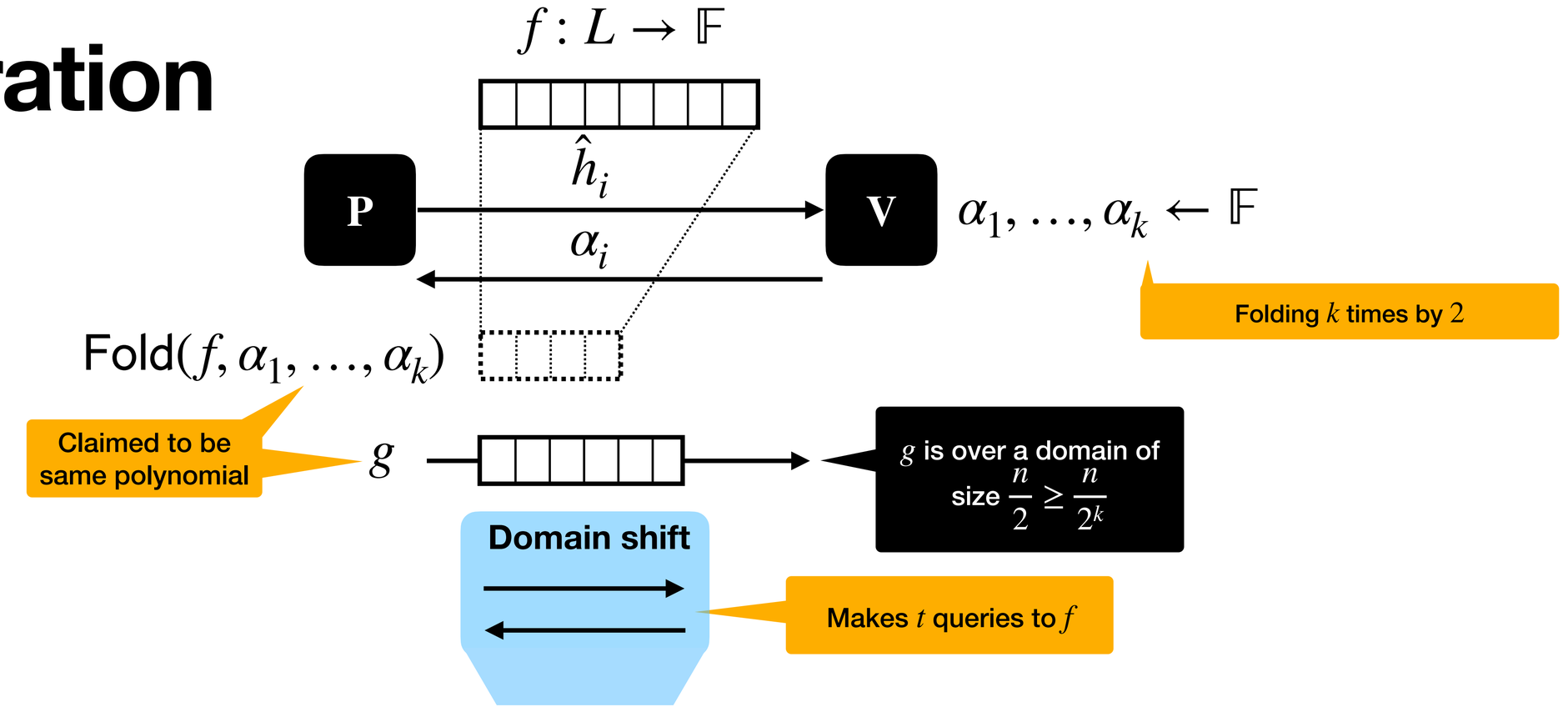

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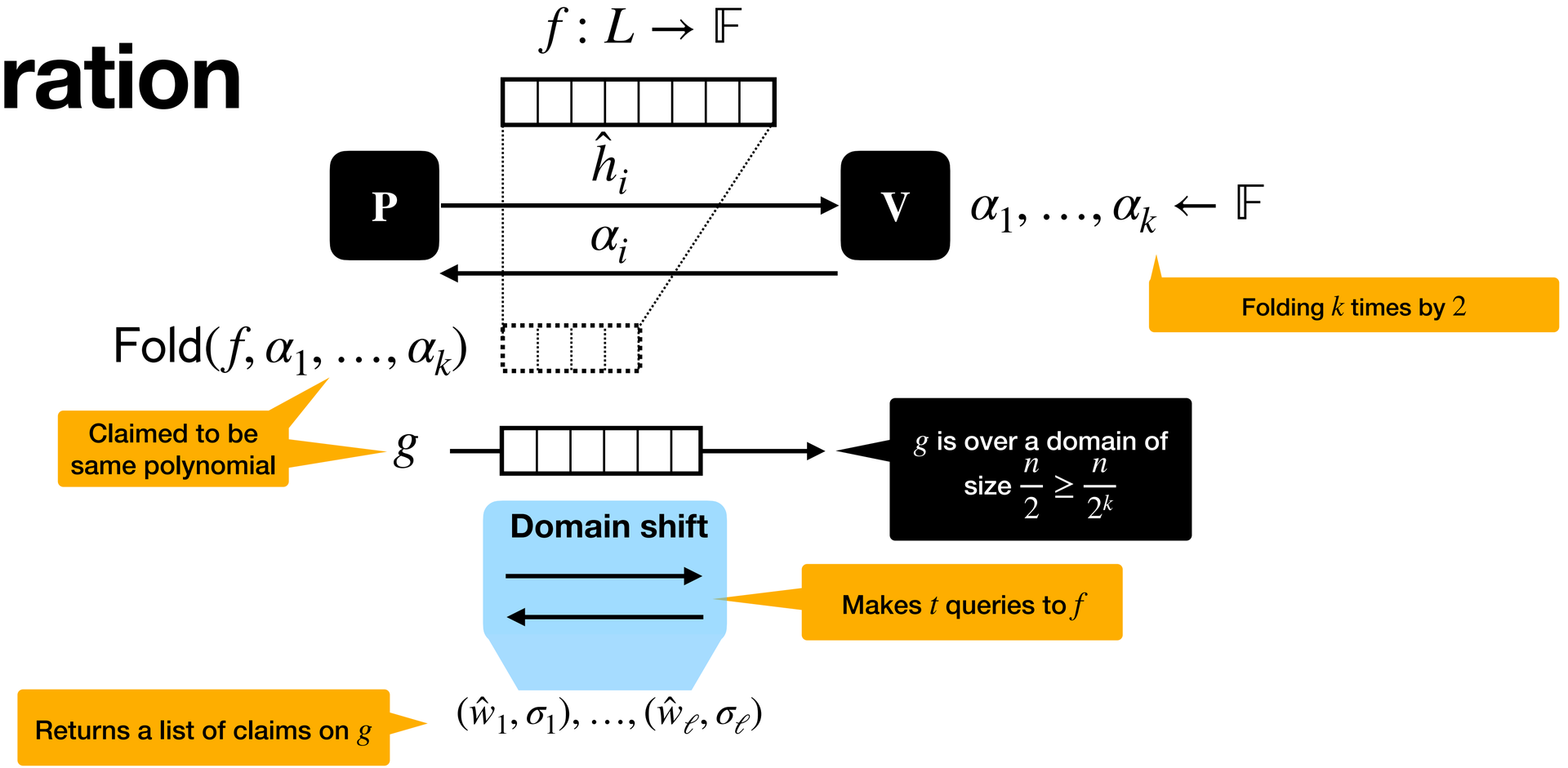

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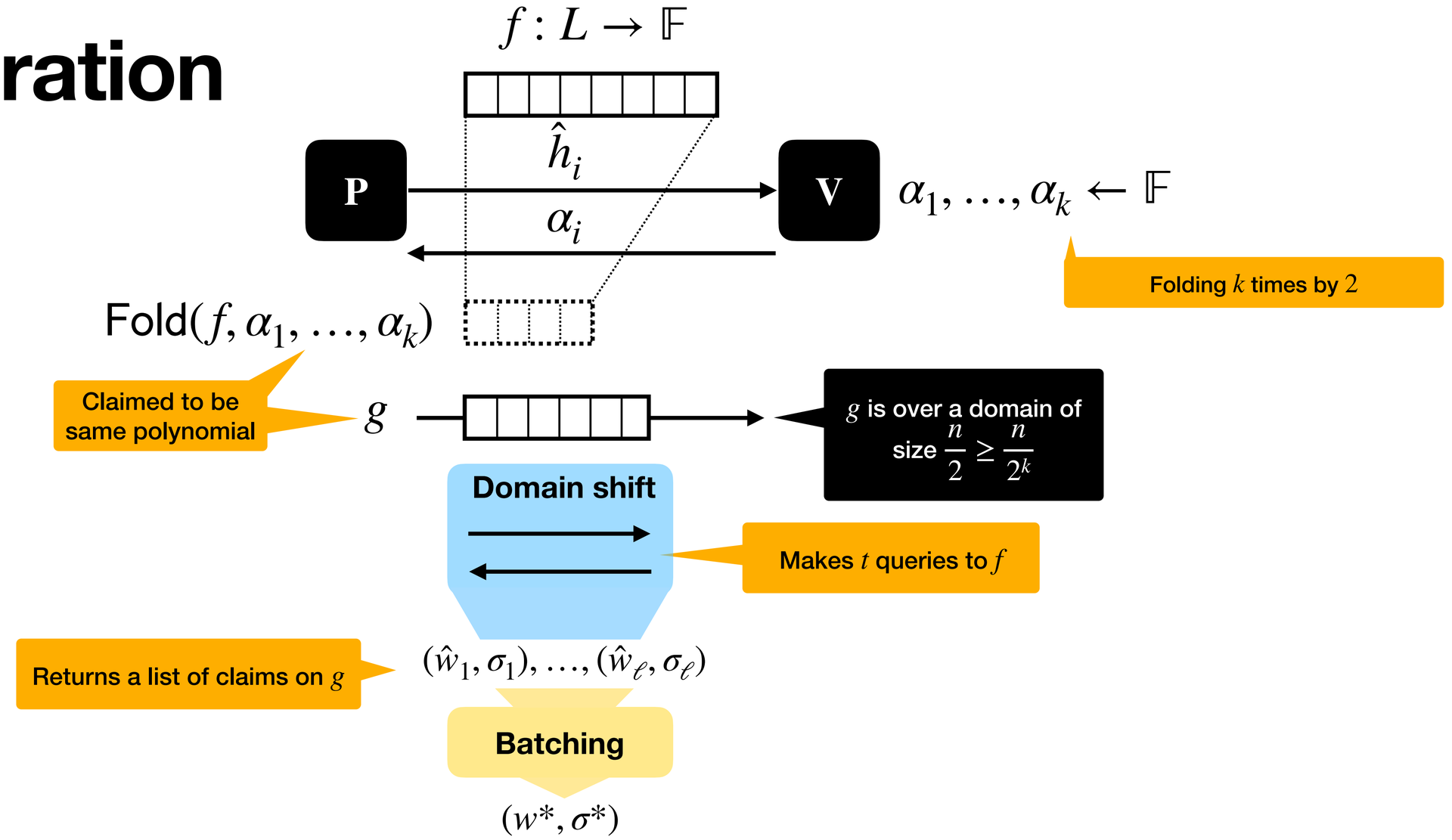

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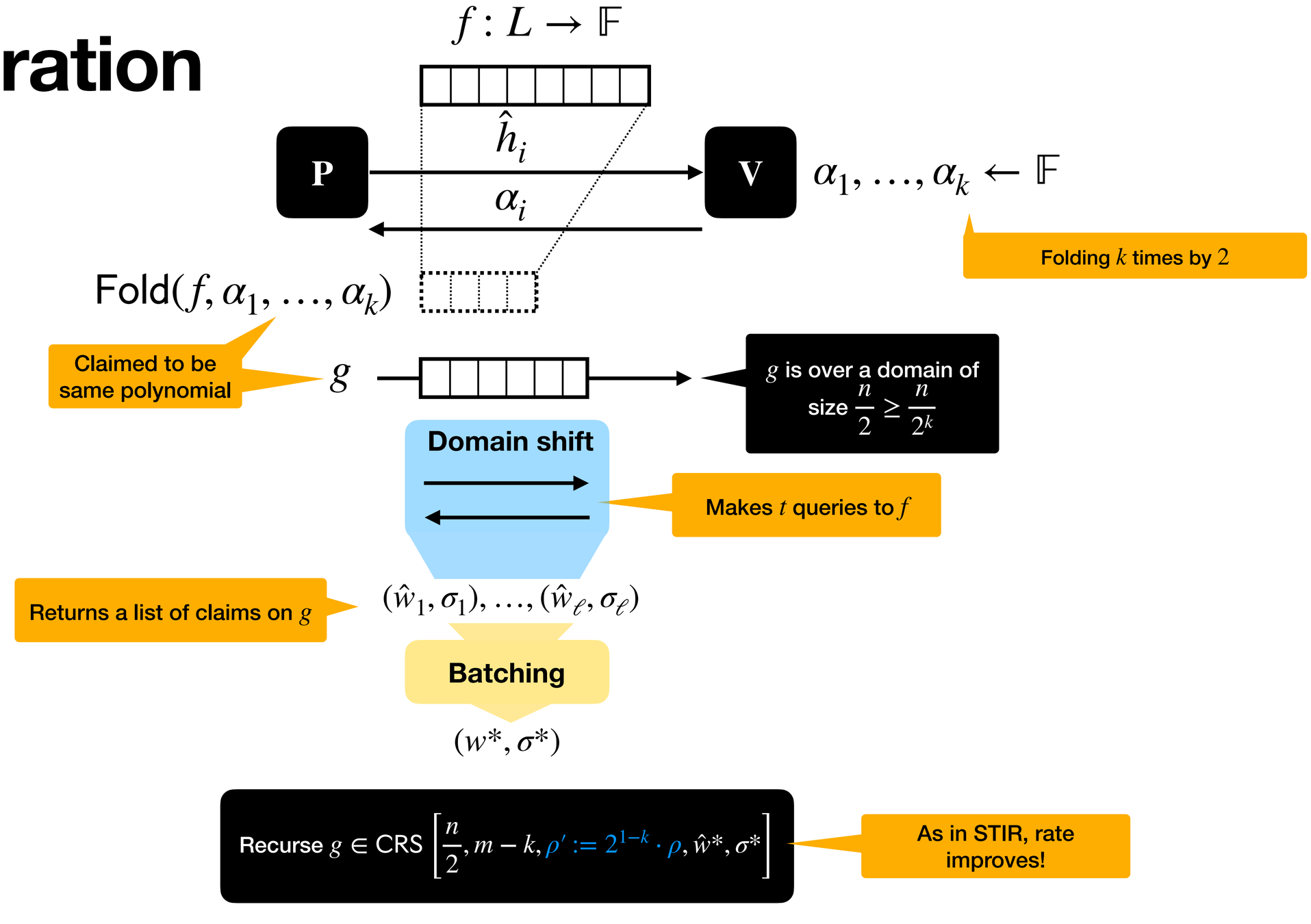

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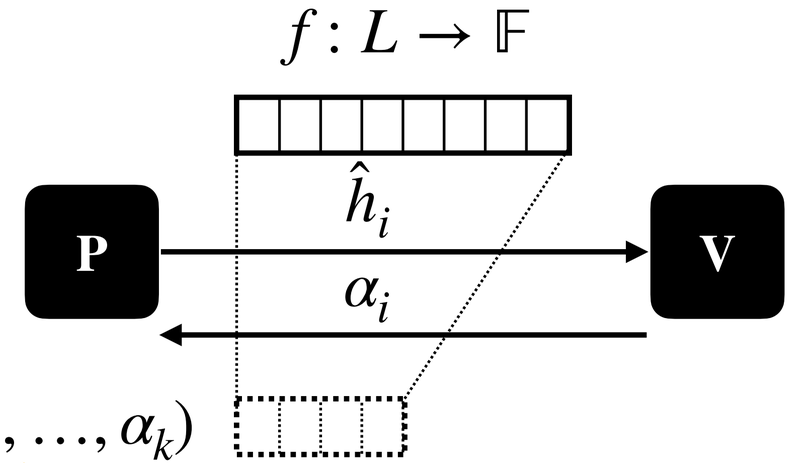

$\alpha_1, \ldots, \alpha_k \leftarrow \mathbb{F}$

Folding k times by 2

$\text{Fold}(f, \alpha_1, \ldots, \alpha_k)$

Claimed to be same polynomial

g

g is over a domain of size $\frac{n}{2} \geq \frac{n}{2^k}$

Domain shift

Makes t queries to f

Returns a list of claims on g

$(\hat{w}_1, \sigma_1), \ldots, (\hat{w}_\ell, \sigma_\ell)$

Batching

(w^*, σ^*)

Recurse $g \in \text{CRS}\left[\frac{n}{2}, m-k, \rho' := 2^{1-k} \cdot \rho, \hat{w}^*, \sigma^*\right]$

As in STIR, rate improves!

Similar structure to STIR! Multilinear structure **forbids** using quotients: we need new ideas to domain shift!

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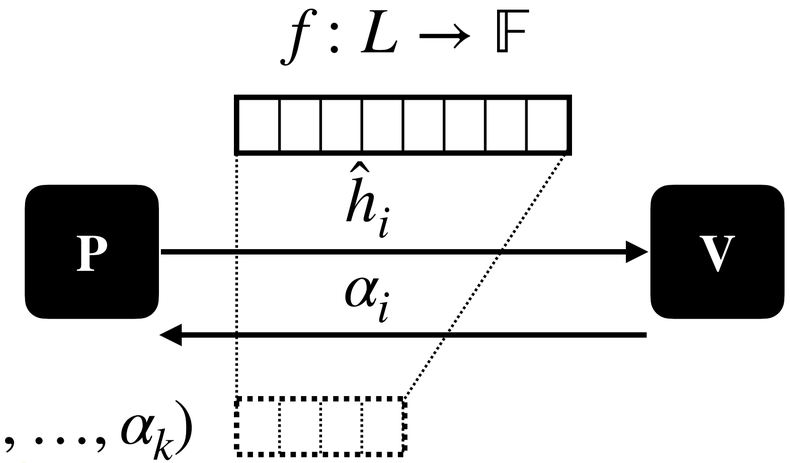

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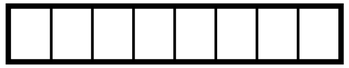

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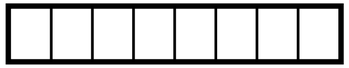

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f and g claimed to be evaluations of same polynomial. Want to output **claims** on g.

Goal: If f is $\left(1 - \sqrt{\rho}\right)$-far from CRS$[|L|, m, \rho, \hat{w}, \sigma]$, w.h.p. g is $\left(1 - \sqrt{\rho'}\right)$-far from CRS$[|L^*|, m, \rho', \hat{w}_i, \sigma_i]$ for at least one $i \in [\ell]$

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Goal: If f is $\left(1 - \sqrt{\rho}\right)$-far from CRS$[|L|, m, \rho, \hat{w}, \sigma]$, w.h.p. g is $\left(1 - \sqrt{\rho'}\right)$-far from CRS$[|L^*|, m, \rho', \hat{w}_i, \sigma_i]$ for at least one $i \in [\ell]$

Assume there is unique polynomial $\hat{p}$ that is $\left(1 - \sqrt{\rho'}\right)$-close to g.

OOD subprotocol (next)

Domain shifting

Claim on f: $(\hat{w}, \sigma)$

$f : L \to \mathbb{F}$

$g : L^* \to \mathbb{F}$

Output claims on g:
$(\hat{w}_1, \sigma_1), \ldots, (\hat{w}_\ell, \sigma_\ell)$

f and g claimed to be evaluations of same polynomial. Want to output **claims** on g.

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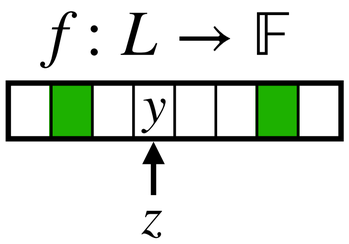
Assume there is unique polynomial $\hat{p}$ that is $\left(1 - \sqrt{\rho'}\right)$-close to g.

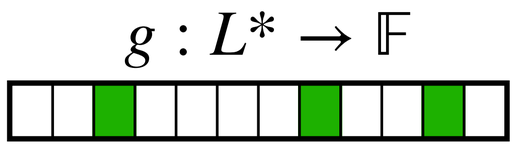
OOD subprotocol (next)

Then, if $\hat{p}$ satisfies the $(\hat{w}, \sigma)$-constraint f must be be $\left(1 - \sqrt{\rho}\right)$-far from it.

Domain shifting

Claim on f: $(\hat{w}, \sigma)$





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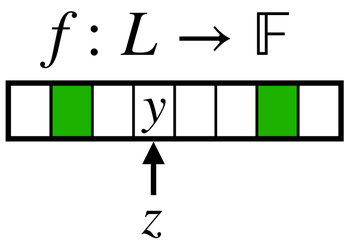

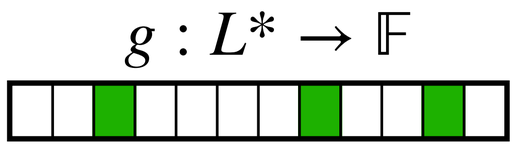

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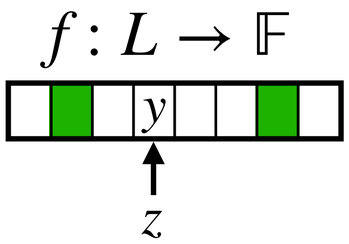
Then, if $\hat{p}$ satisfies the $(\hat{w}, \sigma)$-constraint f must be be $\left(1 - \sqrt{\rho}\right)$-far from it.

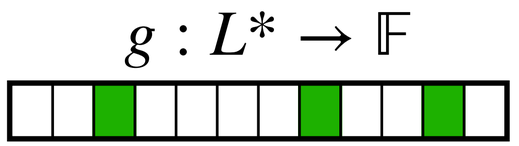
New constraints: (i) original constraint $(\hat{w}, \sigma)$ (ii) $\hat{p}(z) = y$ for some random point z.

Just an evaluation constraint which we know how to handle!

Domain shifting

Claim on f: $(\hat{w}, \sigma)$





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New constraints: (i) original constraint $(\hat{w}, \sigma)$ (ii) $\hat{p}(z) = y$ for some random point z.

Just an evaluation constraint which we know how to handle!

So, except with probability $\sqrt{\rho}$, g is $\left(1 - \sqrt{\rho'}\right)$-far from CRS$[|L^*|, m, \rho', (\hat{w}_1, \sigma_1), \ldots, (\hat{w}_\ell, \sigma_\ell)]$.

Can amplify to $\sqrt{\rho}^t$

Out Of Domain

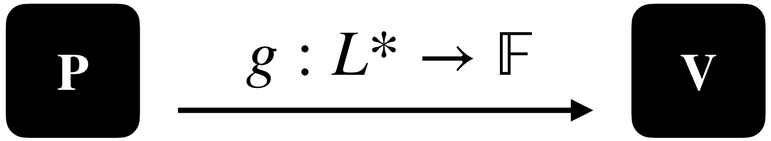
Subprotocol to force unique

Out Of Domain

Subprotocol to force unique

Out Of Domain

Subprotocol to force unique



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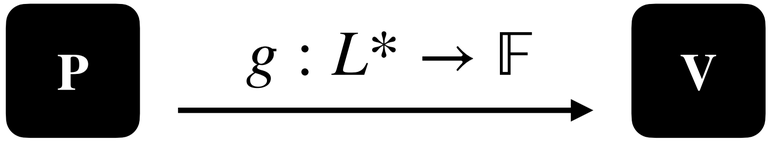

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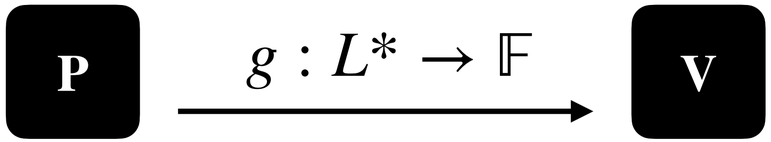

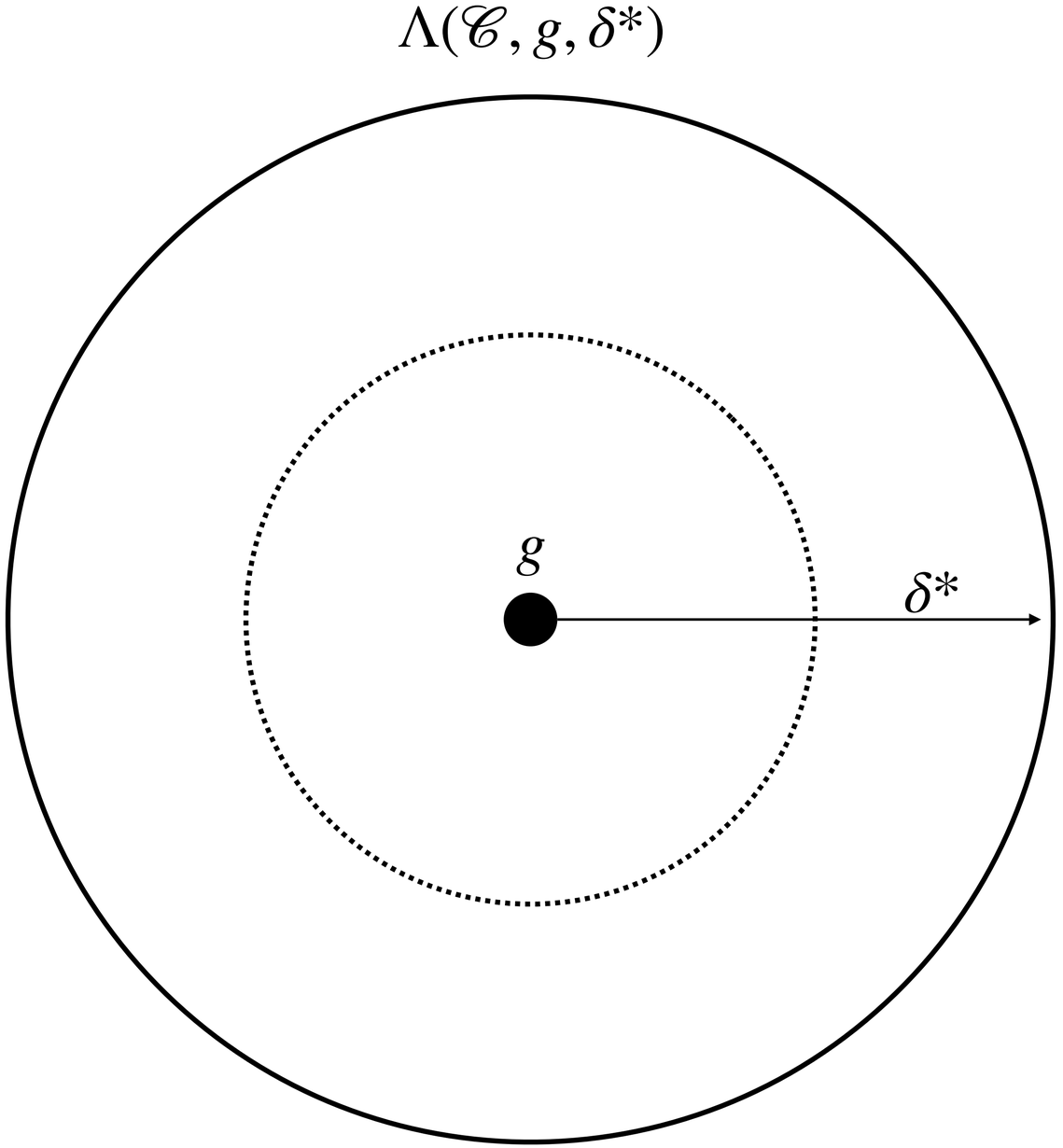

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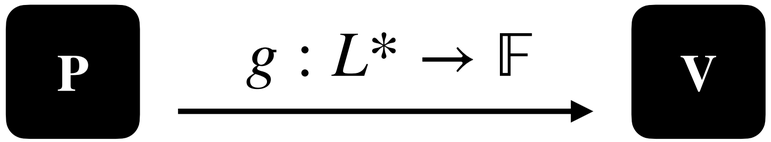

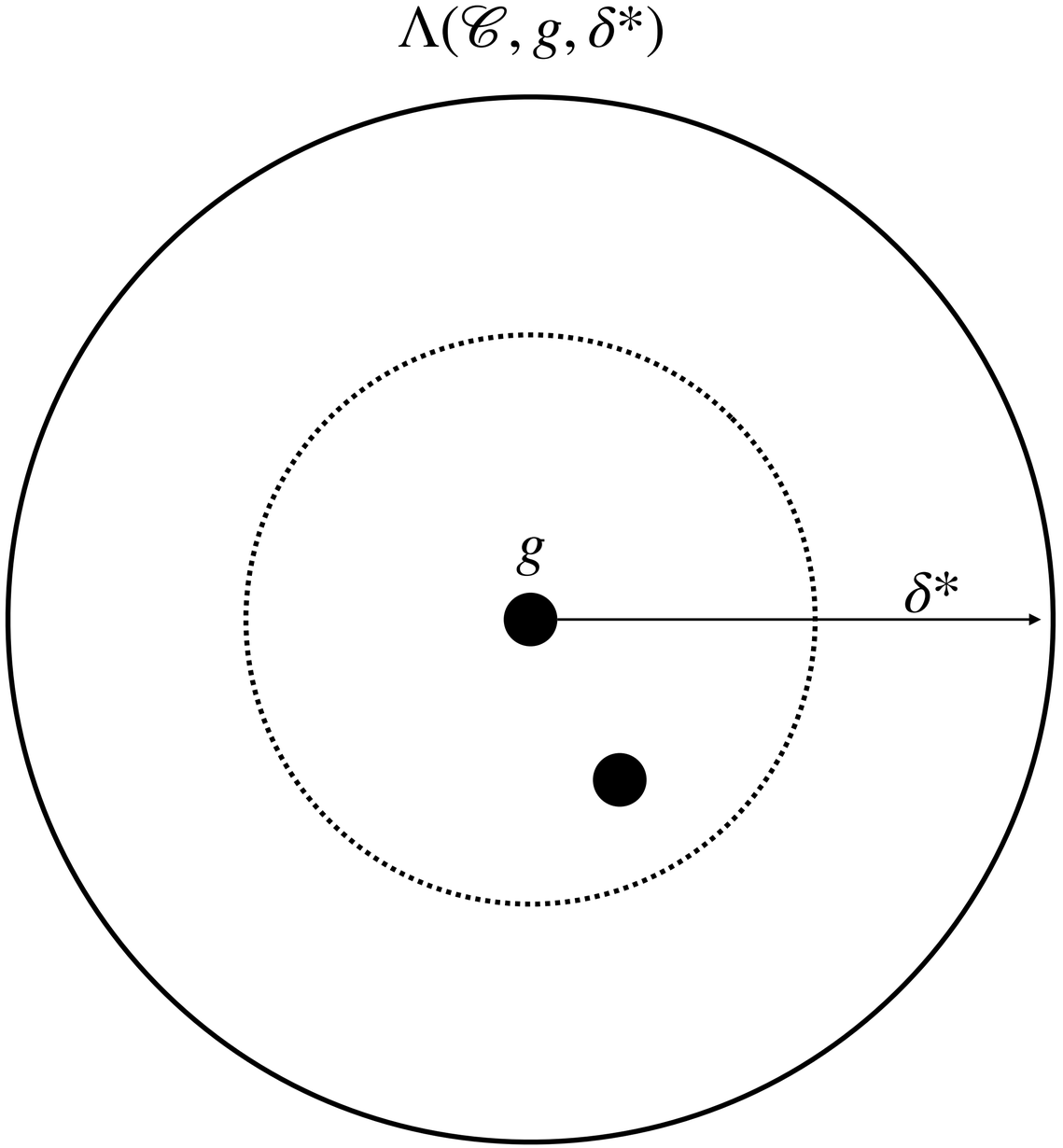

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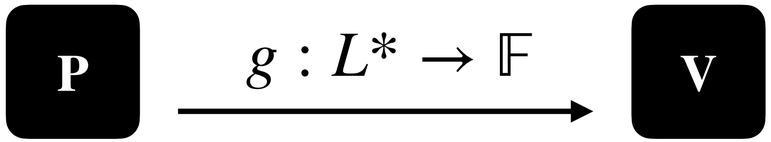

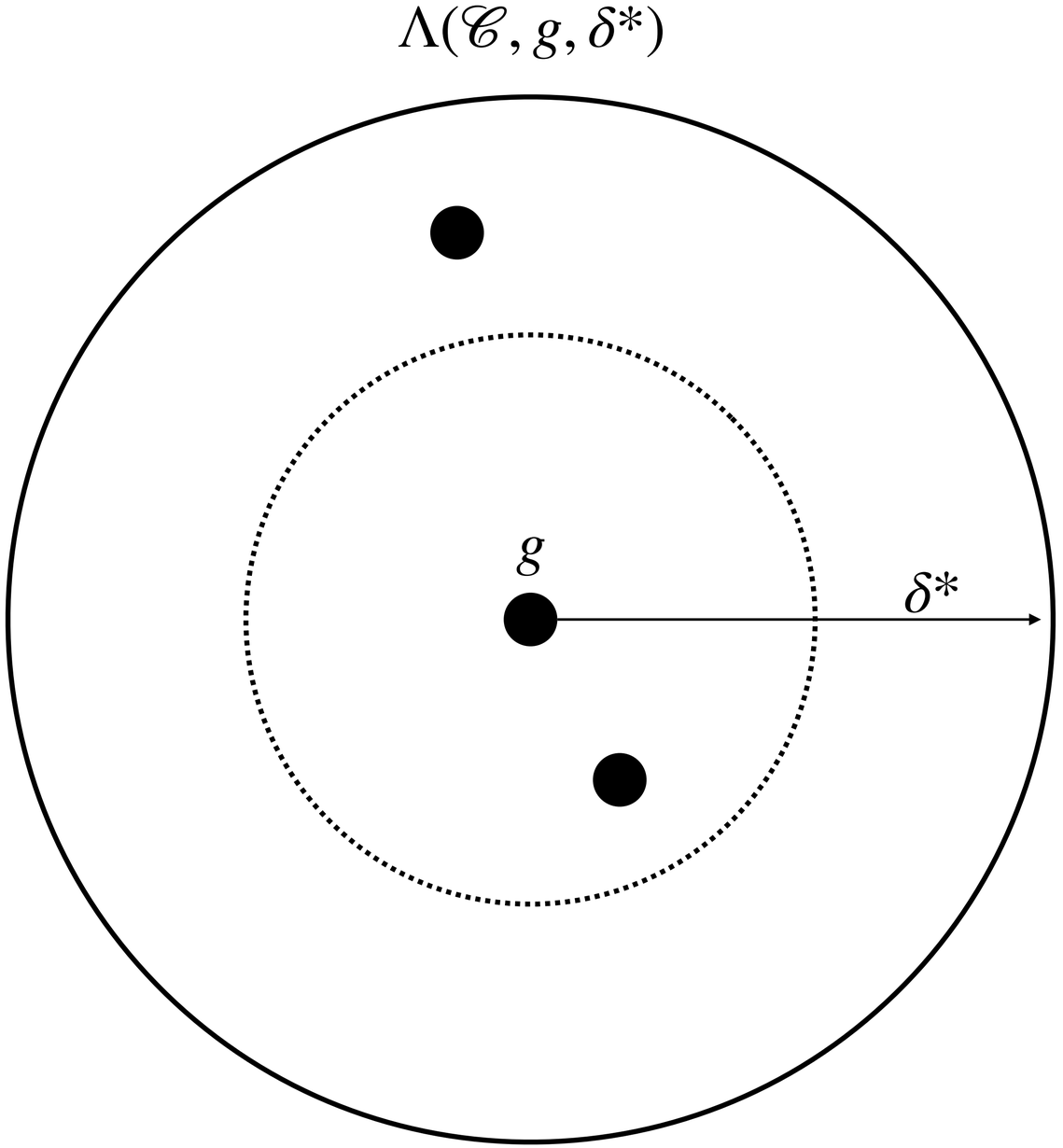

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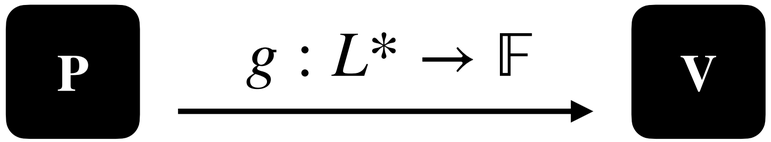

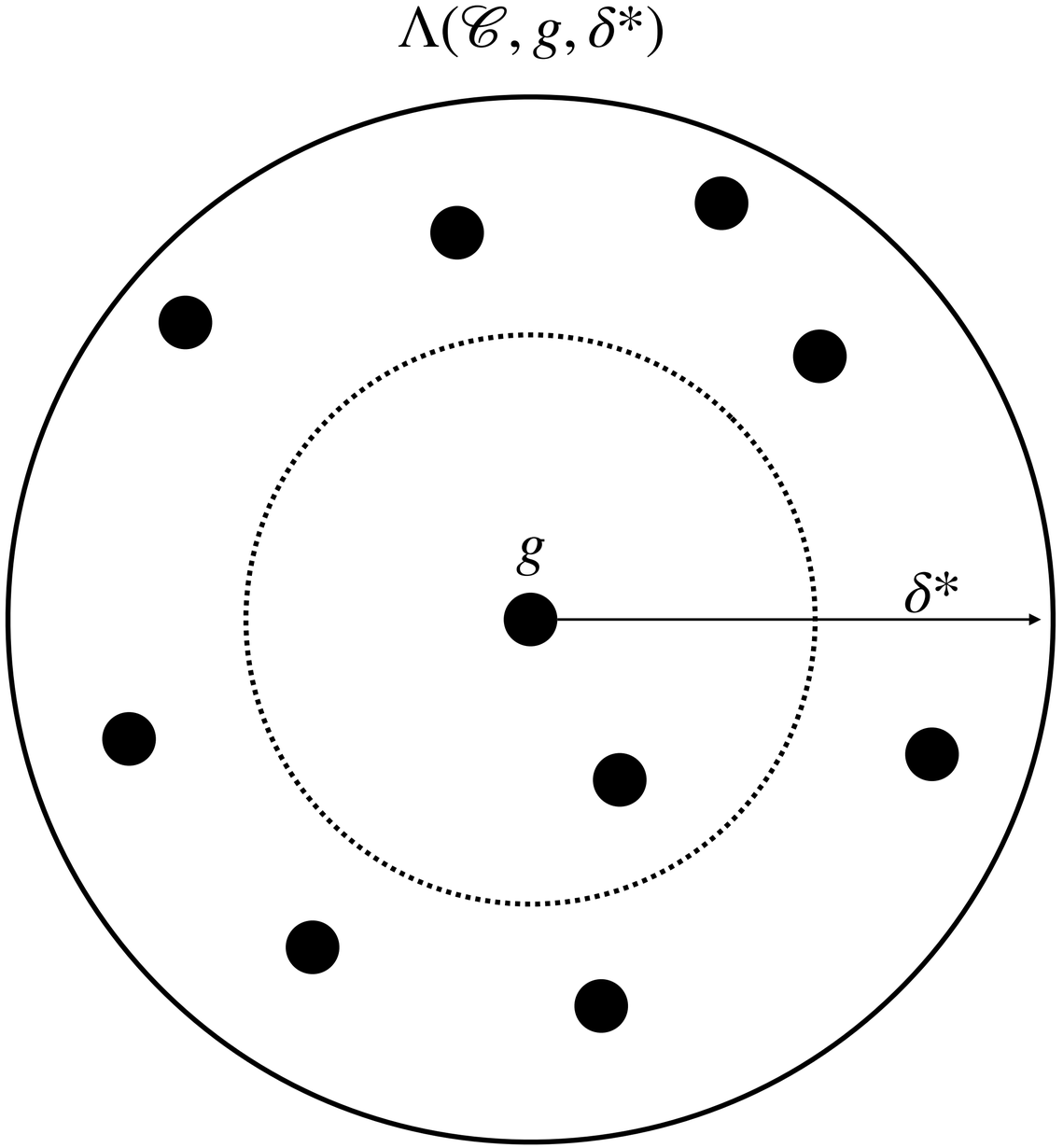

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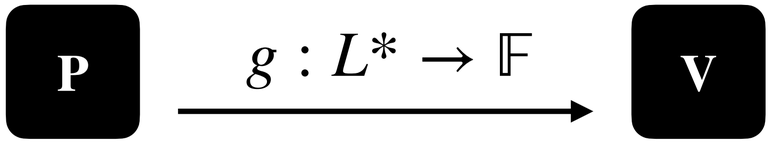

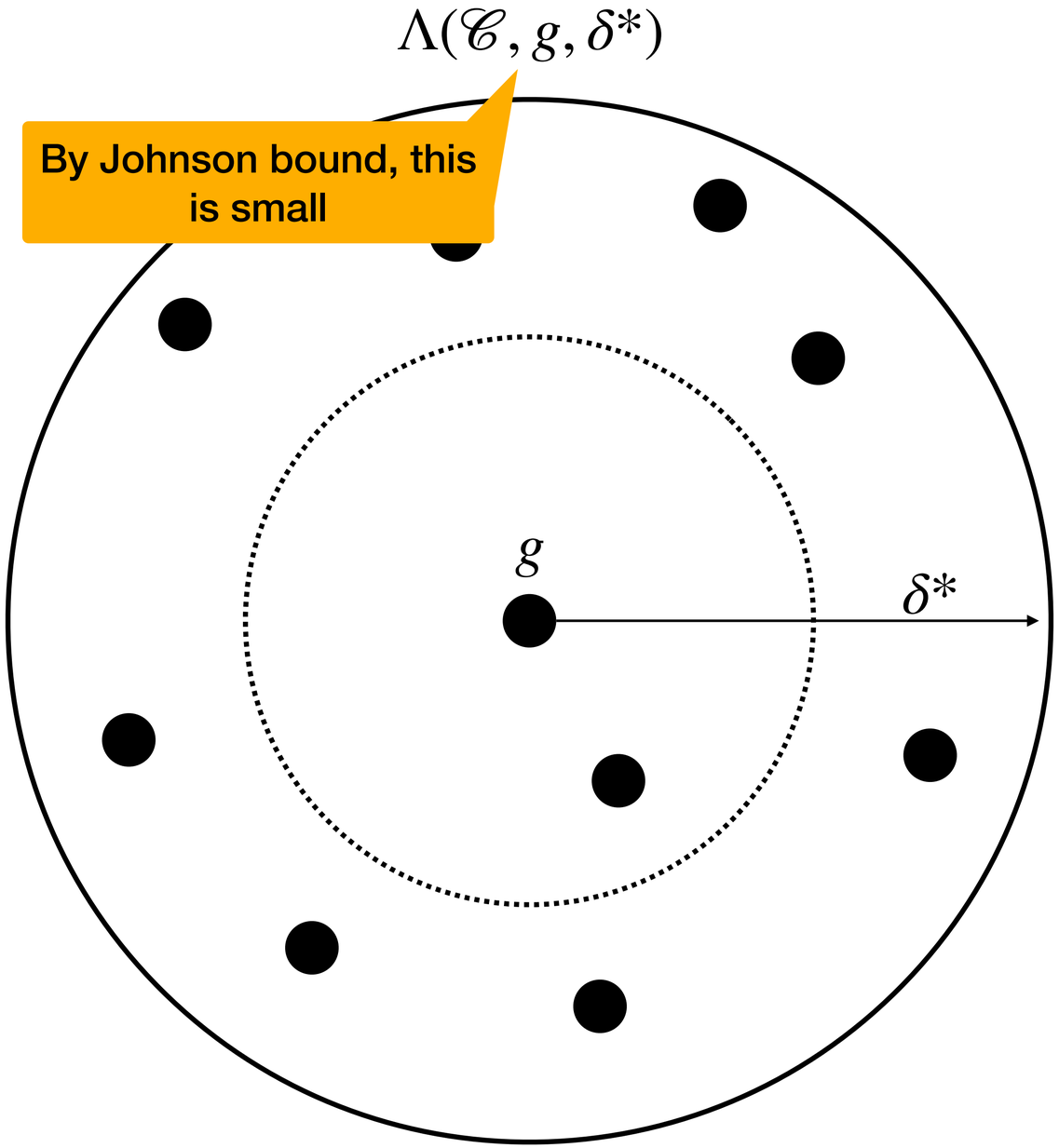

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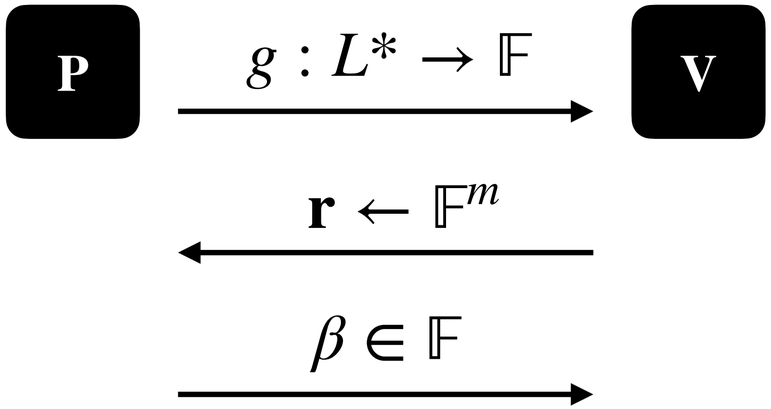

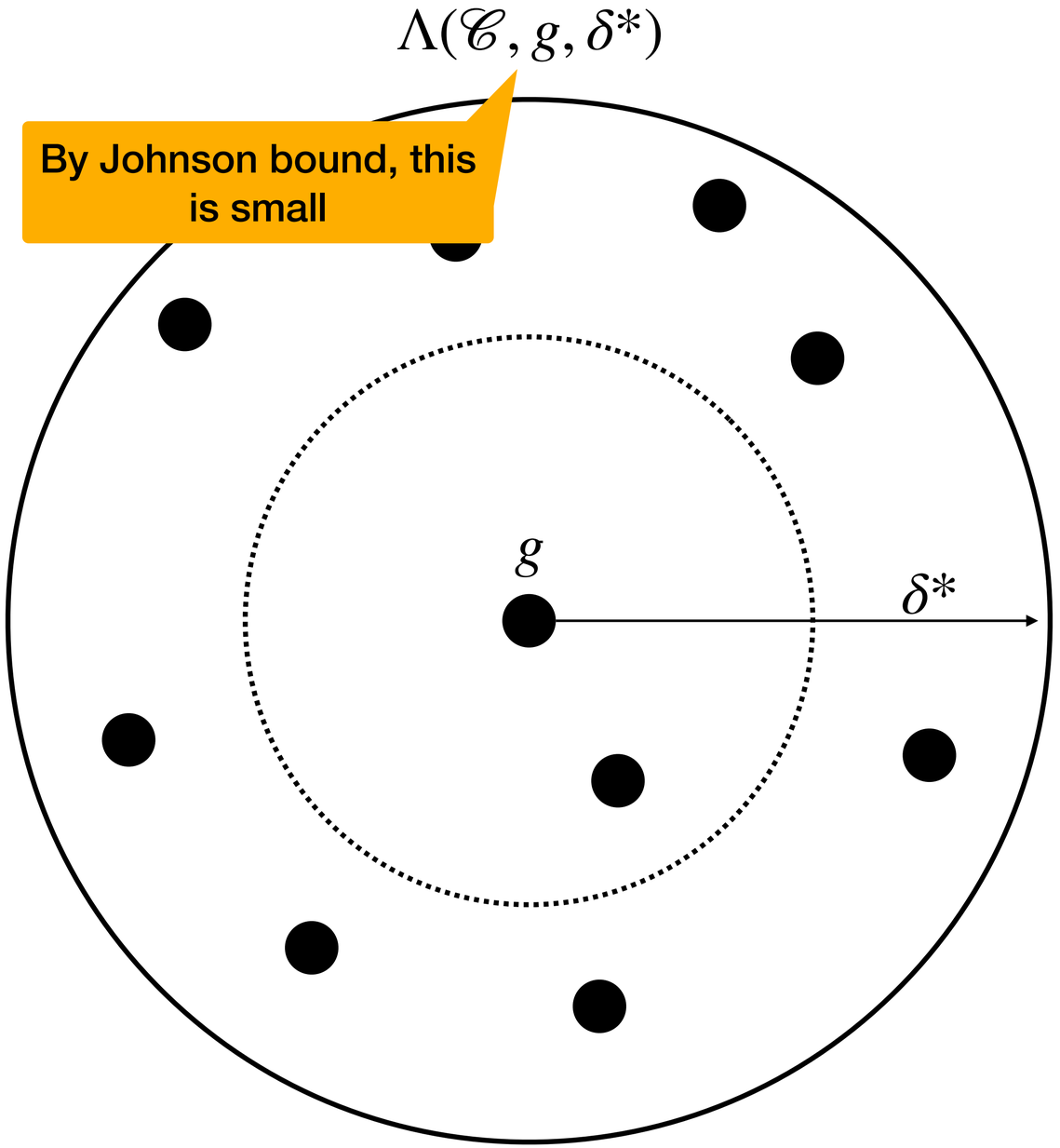

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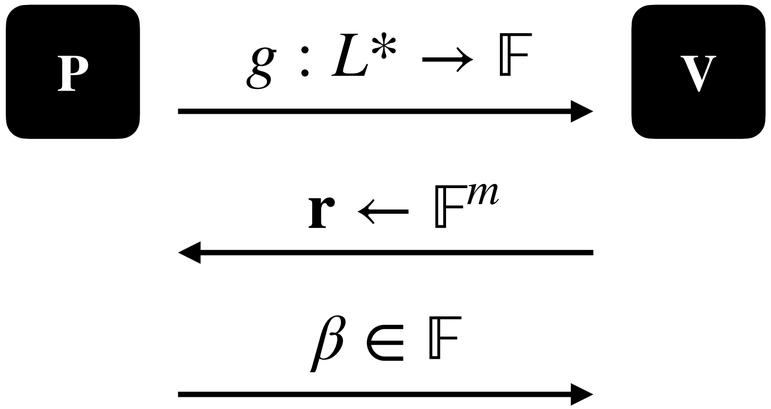
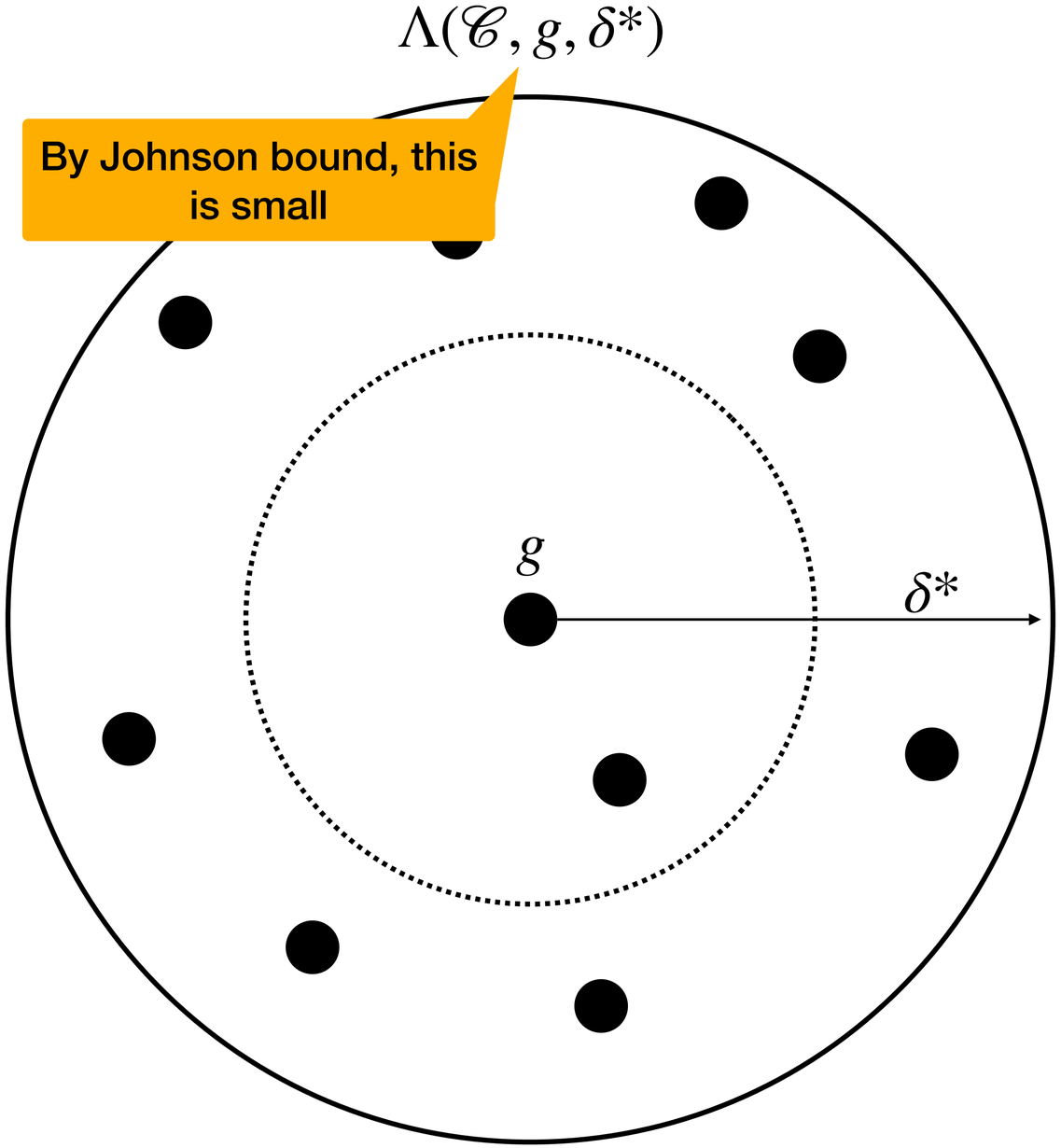


- By SZ lemma w.h.p.
  no pair $\hat{u}, \hat{v}$ with $\hat{u}(\mathbf{r}) = \hat{v}(\mathbf{r})$
- Prover "chooses" which codeword $\hat{u}$ it "commits" to



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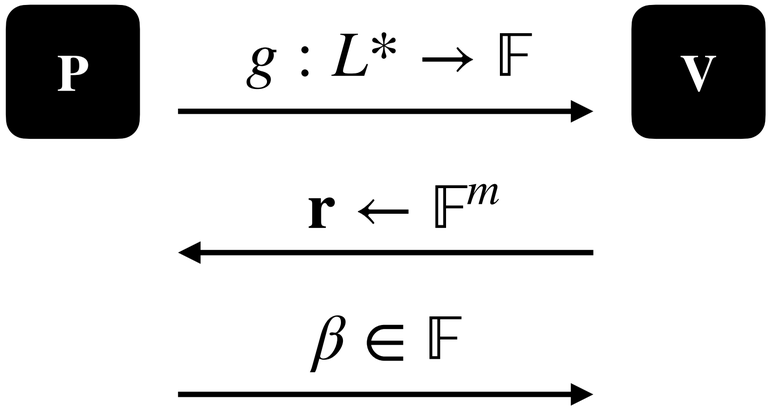

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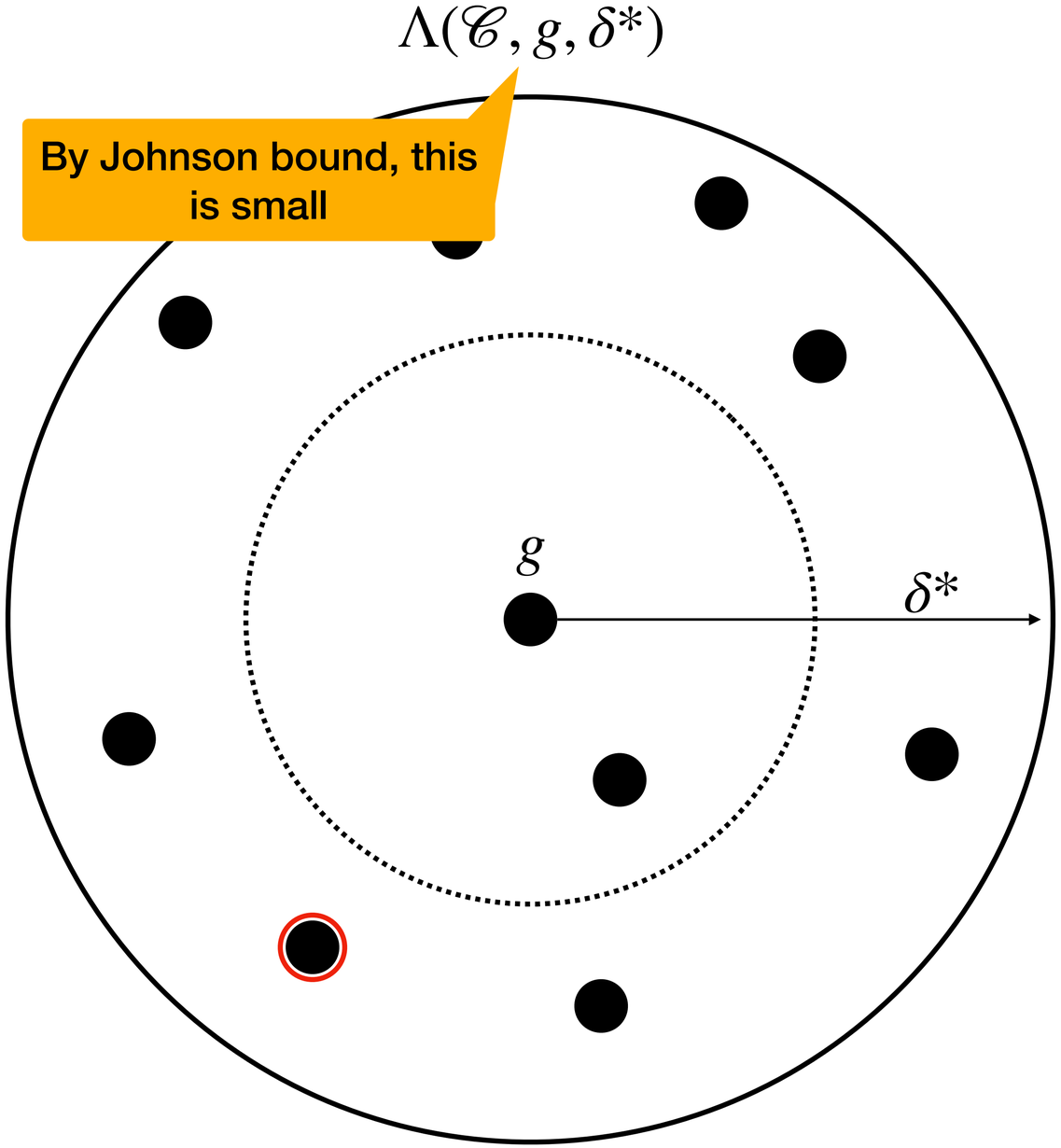

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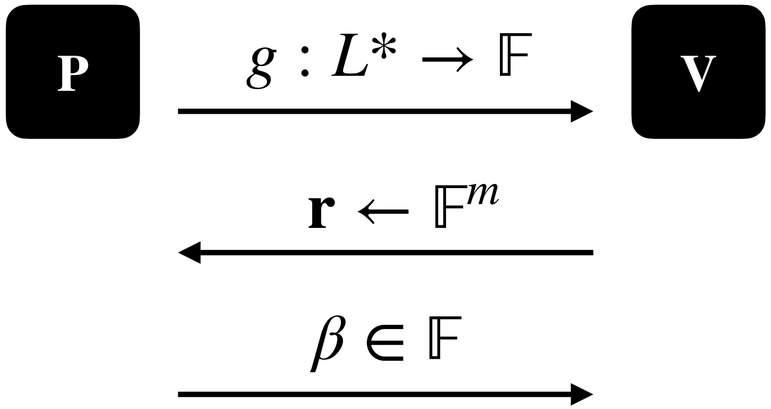

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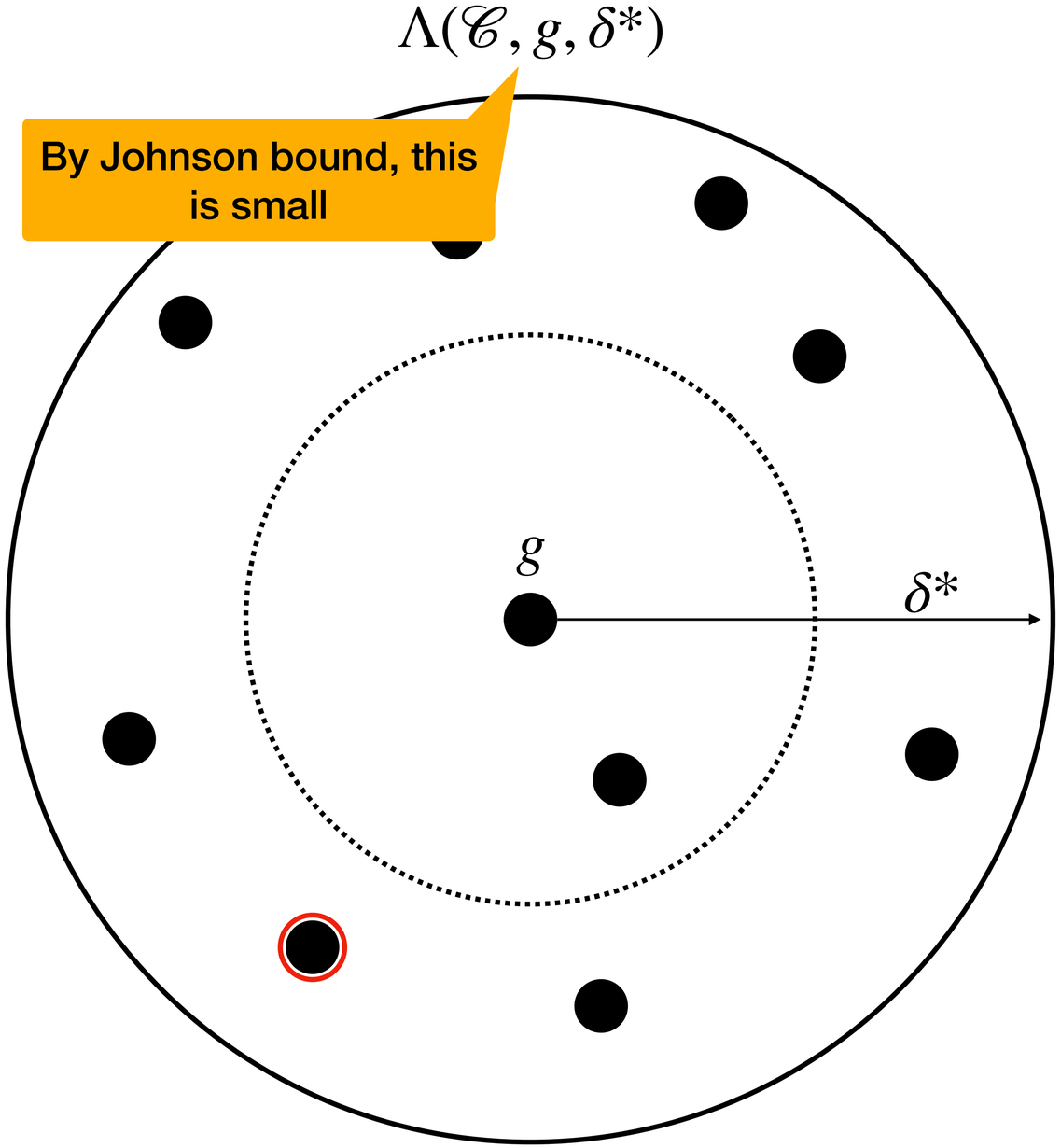
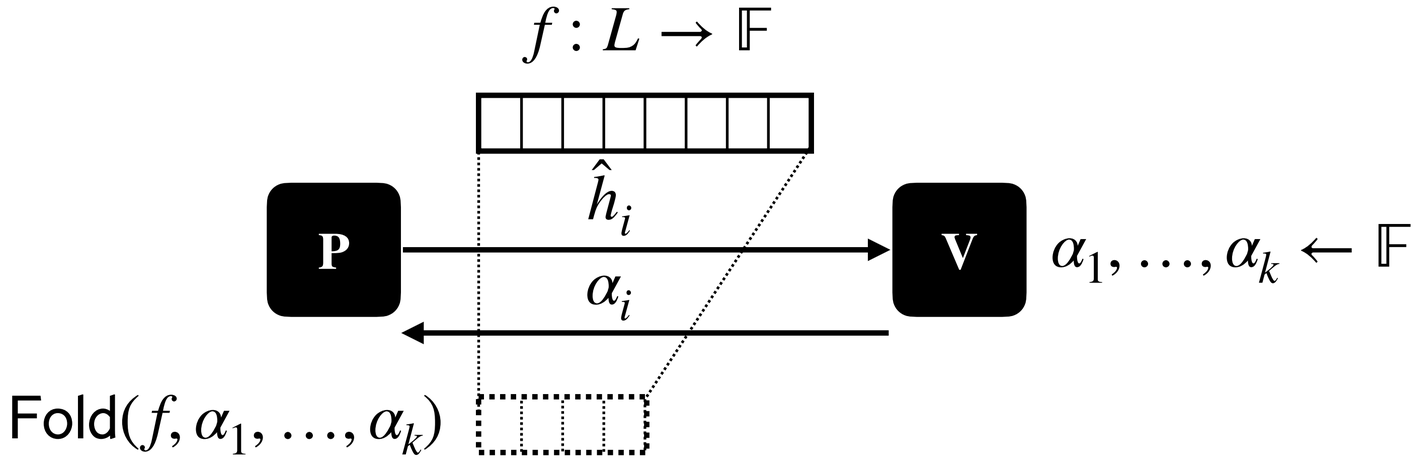


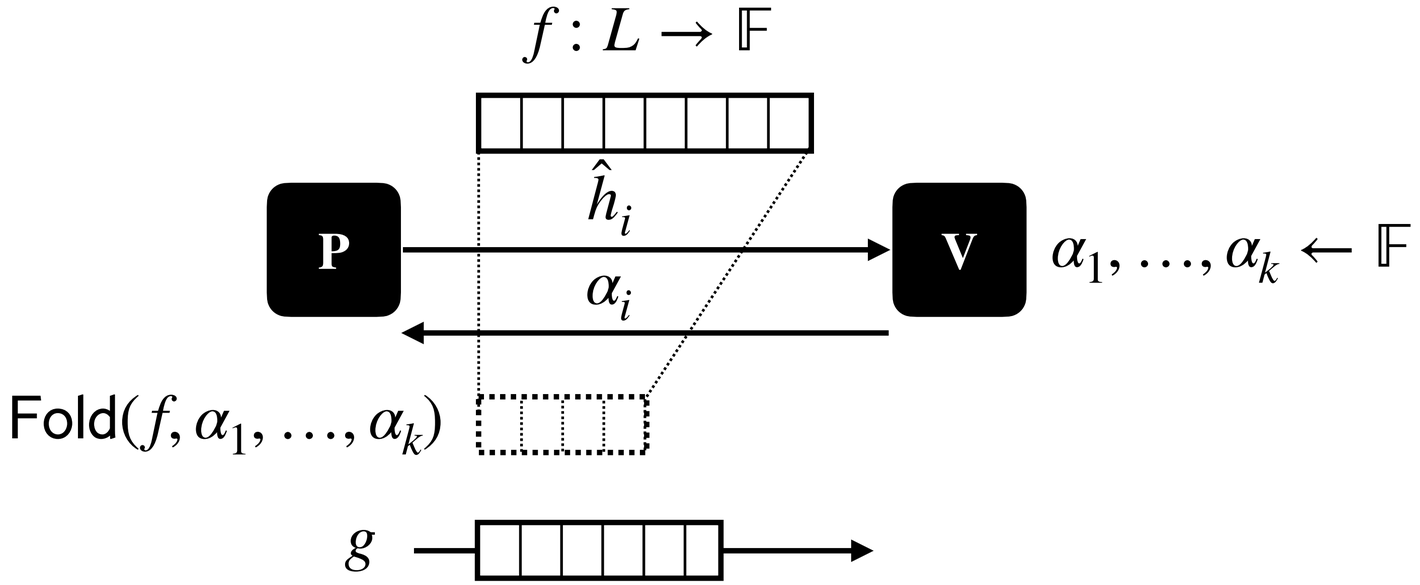
Add to list of constraints to **enforce**!

WHIR 

WHIR


$f : L \to \mathbb{F}$
$\hat{h}_i$
P
V
α_i
$\alpha_1, \ldots, \alpha_k \leftarrow \mathbb{F}$
Fold$(f, \alpha_1, \ldots, \alpha_k)$

WHIR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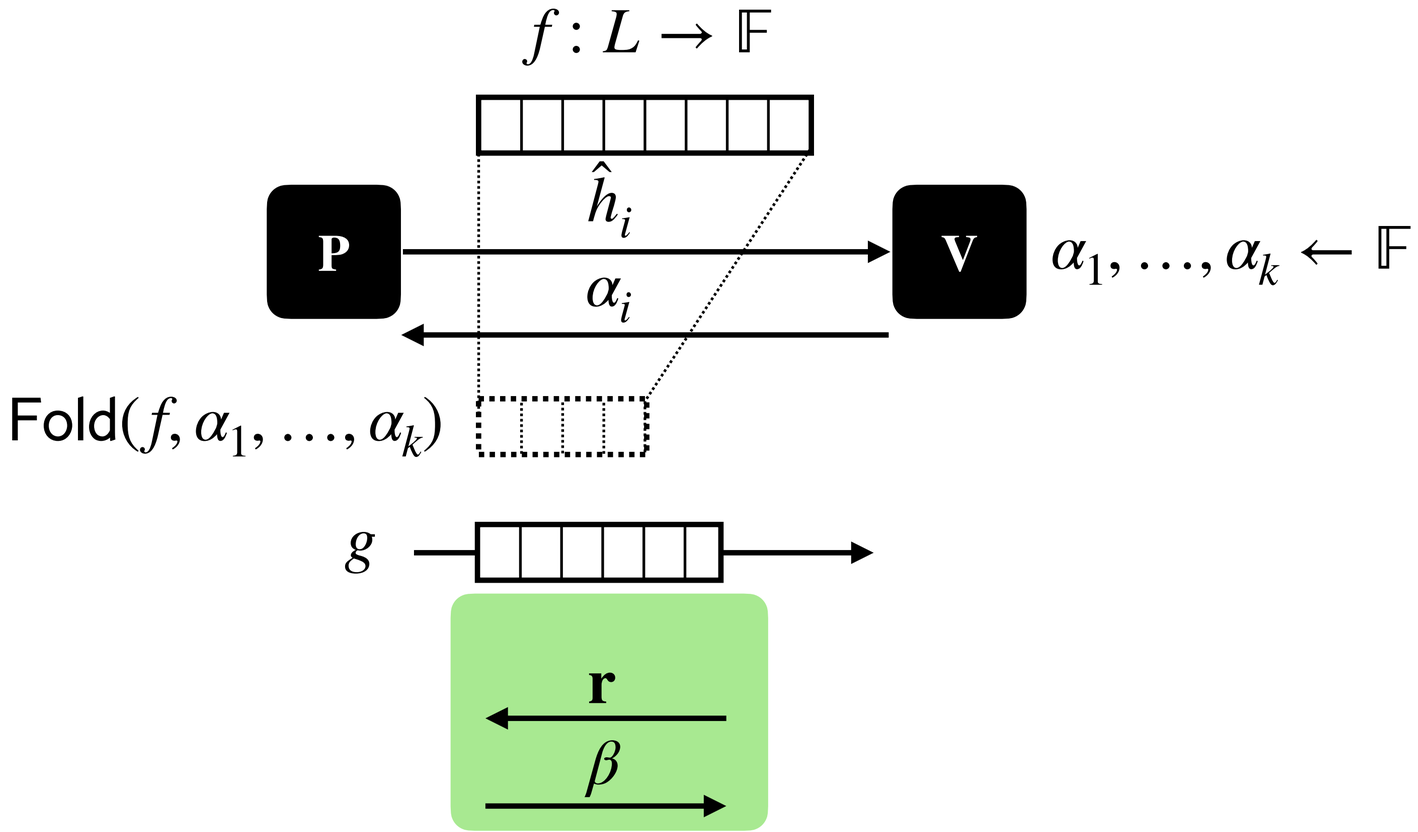
g

WHIR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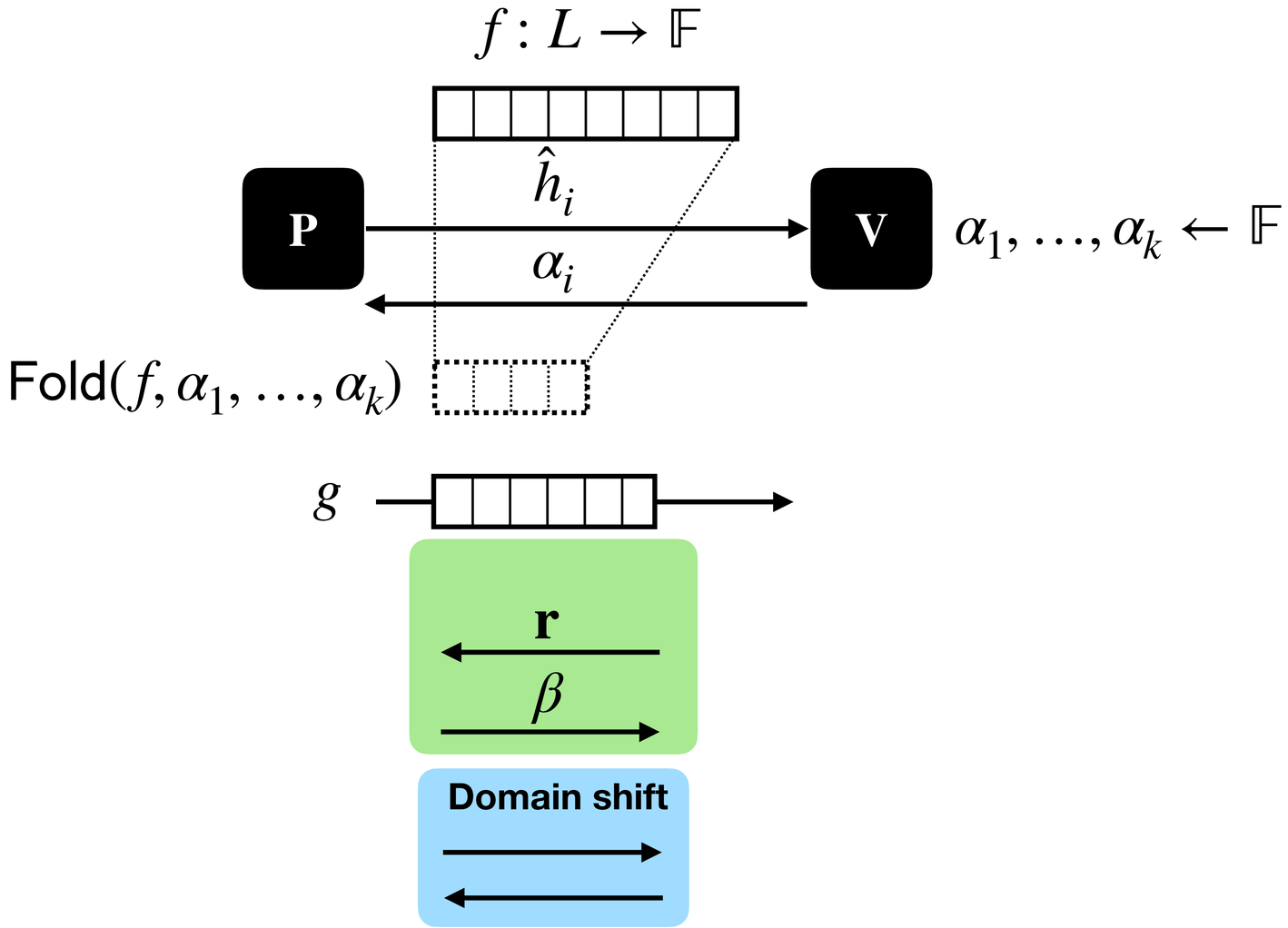
Fold$(f, \alpha_1, \ldots, \alpha_k)$
g
r
β

WHIR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g
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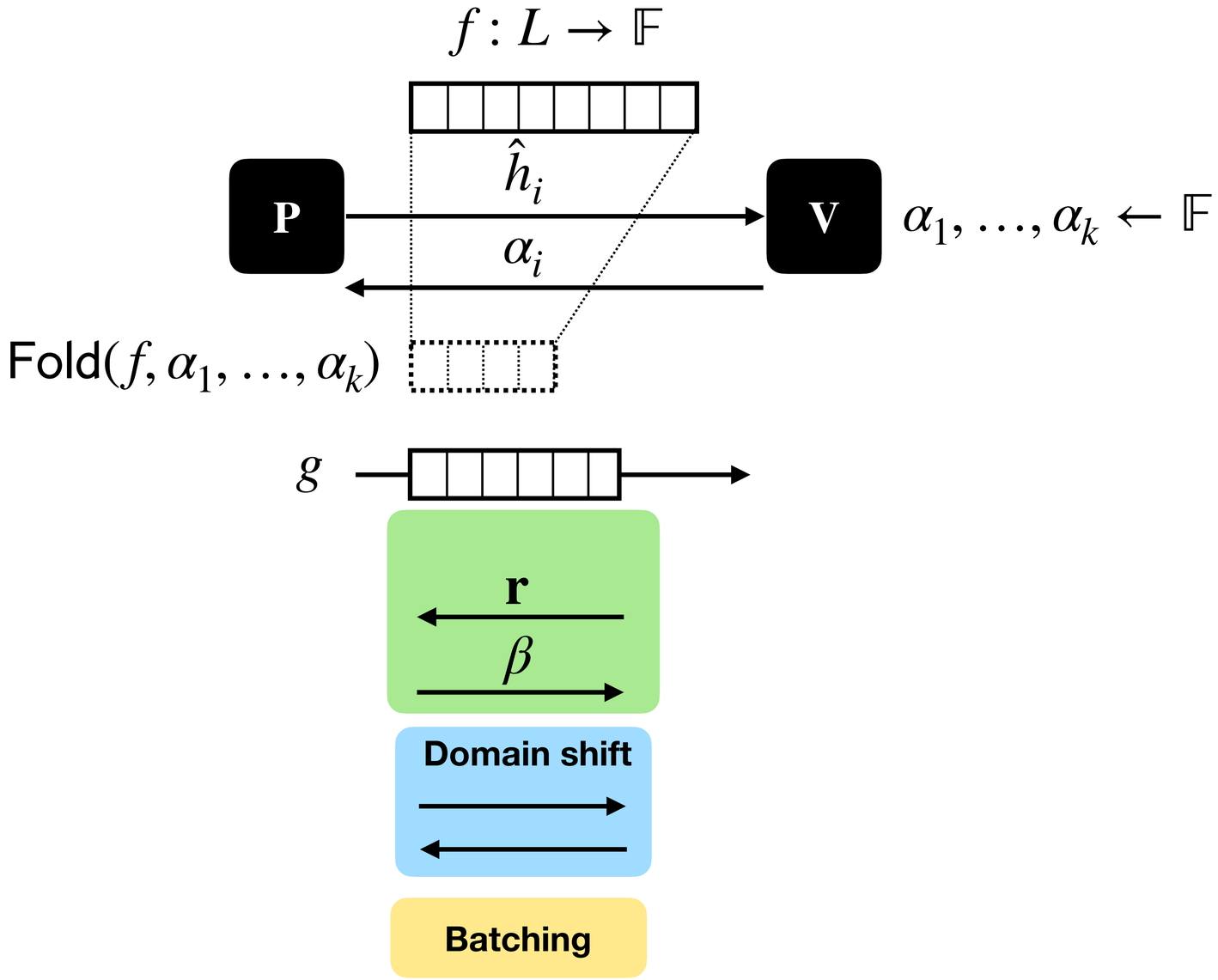
Domain shift

WHIR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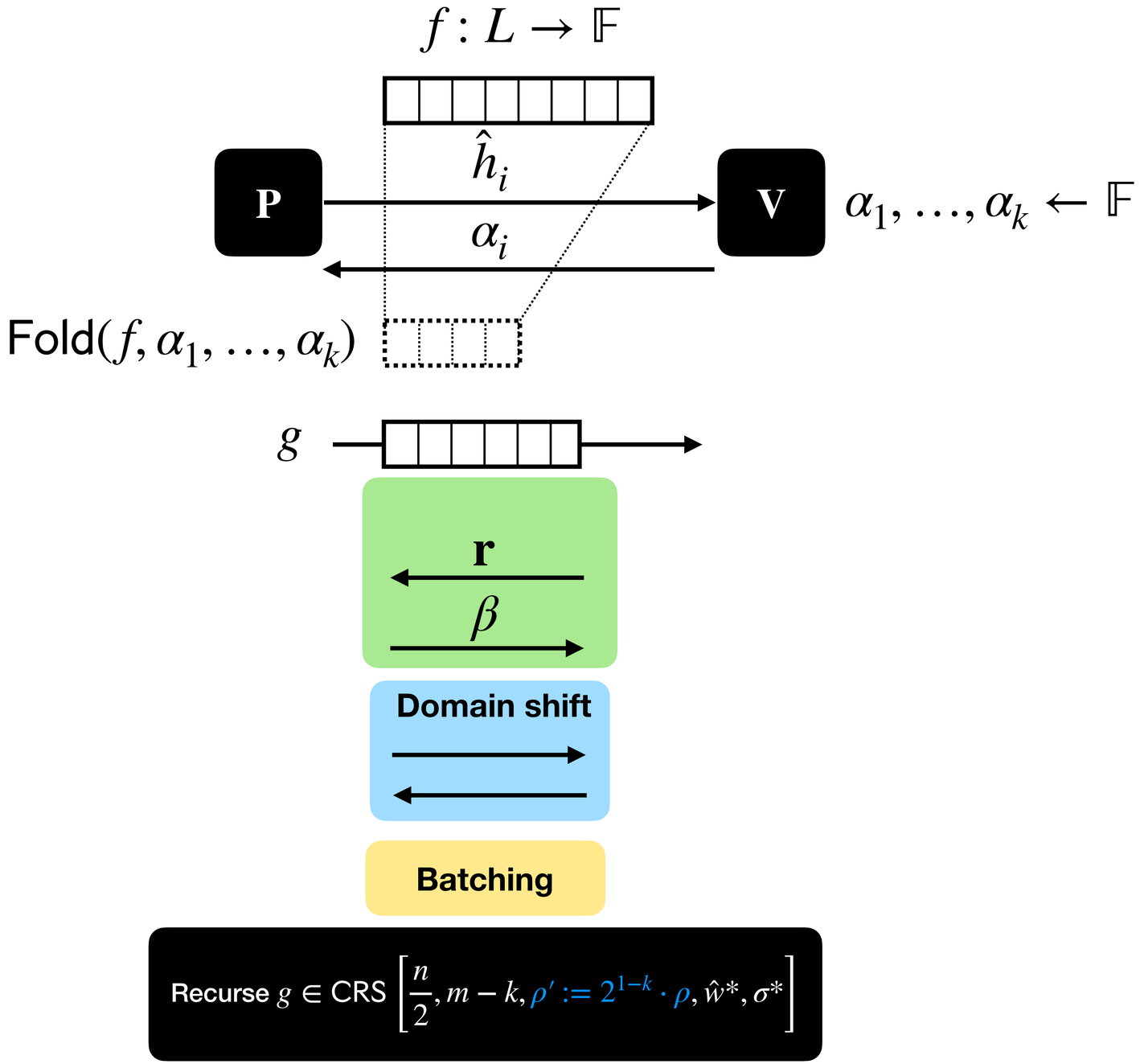
g
r
β
Domain shift
Batching

WHIR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V
α_i
$\alpha_1, \ldots, \alpha_k \leftarrow \mathbb{F}$
$\text{Fold}(f, \alpha_1, \ldots, \alpha_k)$
g
r
β
Domain shift
Batching
Recurse $g \in \text{CRS}$ $\left[\frac{n}{2}, m-k, \rho' := 2^{1-k} \cdot \rho, \hat{w}*, \sigma*\right]$

Application: Σ-IOP

High soundness compilation using constrained codes

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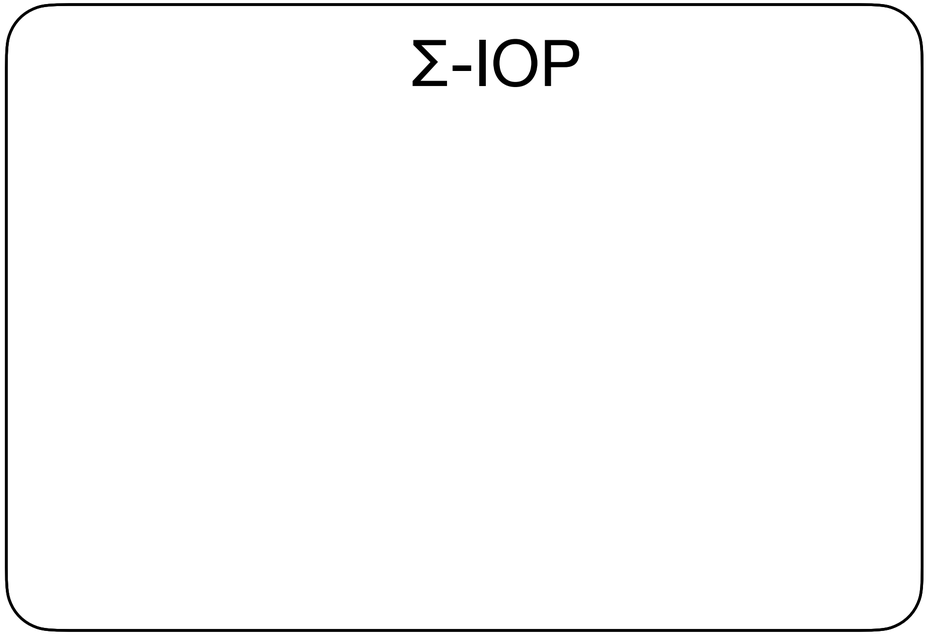
High soundness compilation using constrained codes



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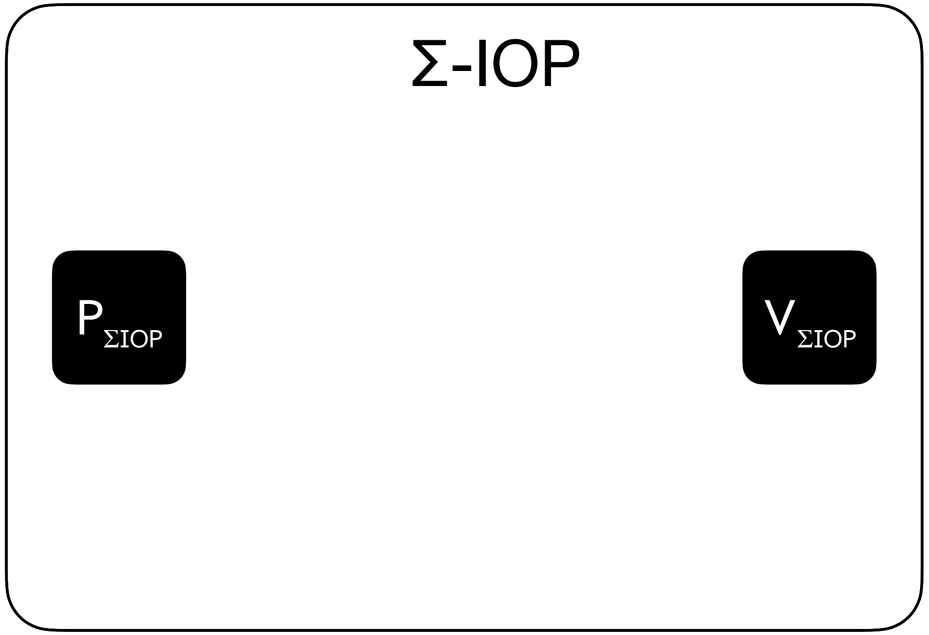

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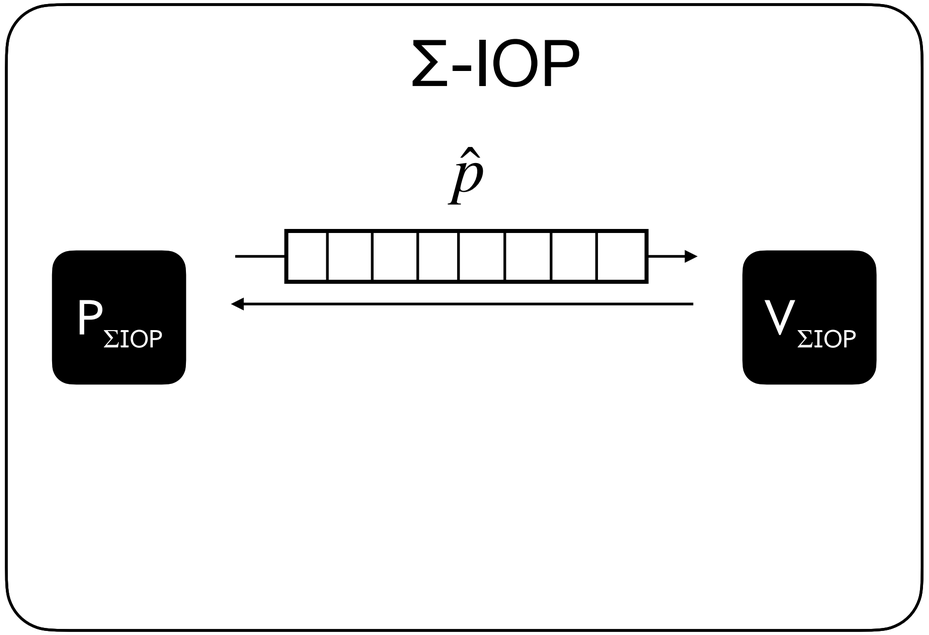

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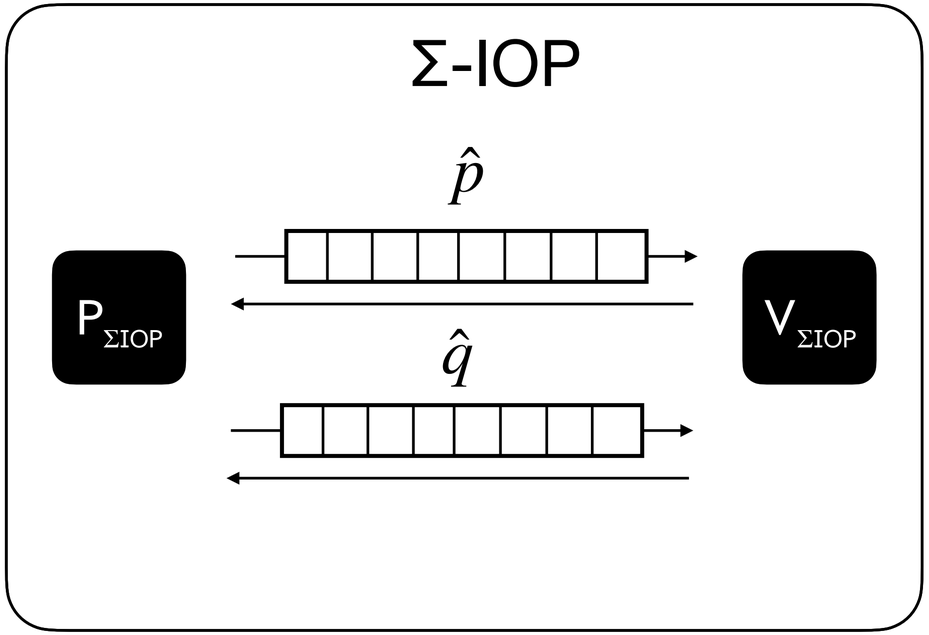
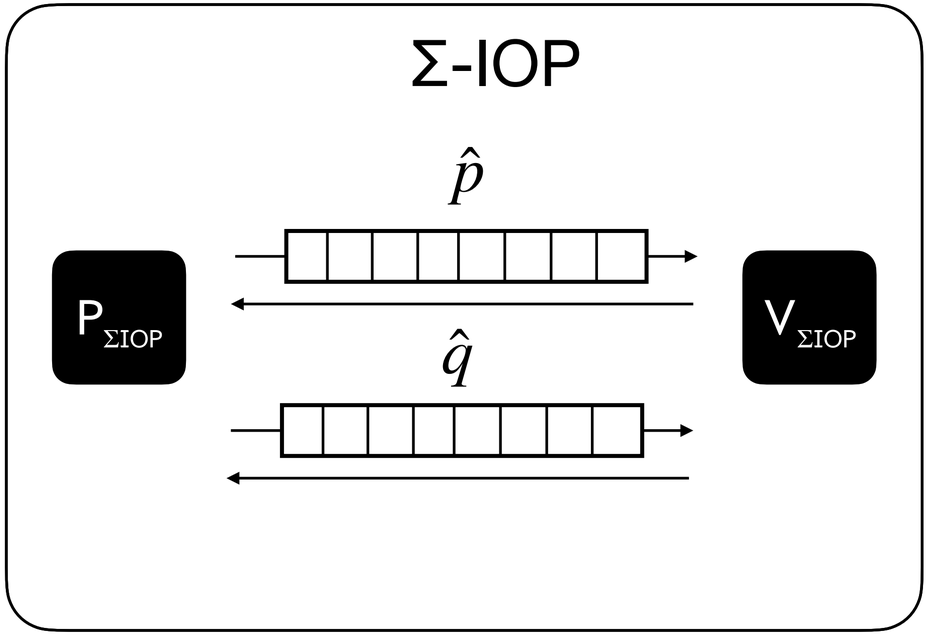


Application: Σ-IOP

High soundness compilation using constrained codes

Generalizes univariate and multilinear PIOPs at no extra cost!



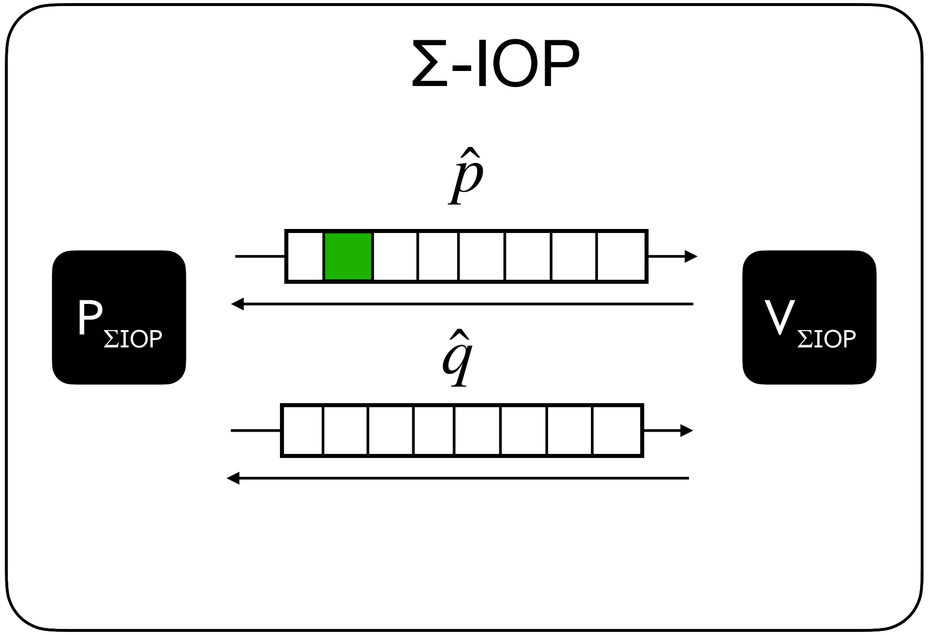
Verifier can ask **sumcheck queries**

i.e. send $\hat{w}$ and receive $\sum_{\mathbf{b}} \hat{w}(\hat{f}(\mathbf{b}), \mathbf{b})$

Generalizes univariate and multilinear PIOPs at no extra cost!

Application: Σ-IOP

High soundness compilation using constrained codes



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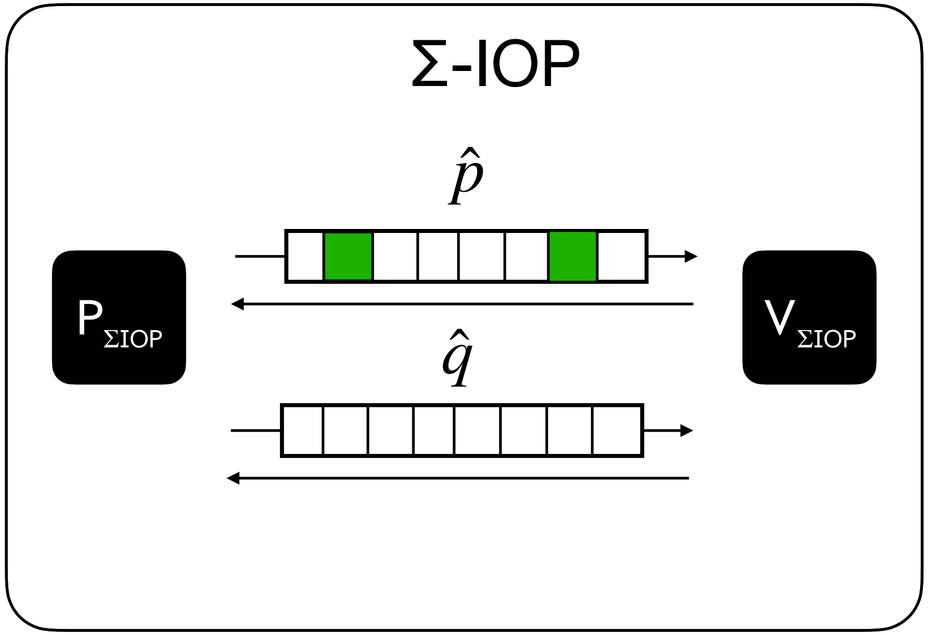

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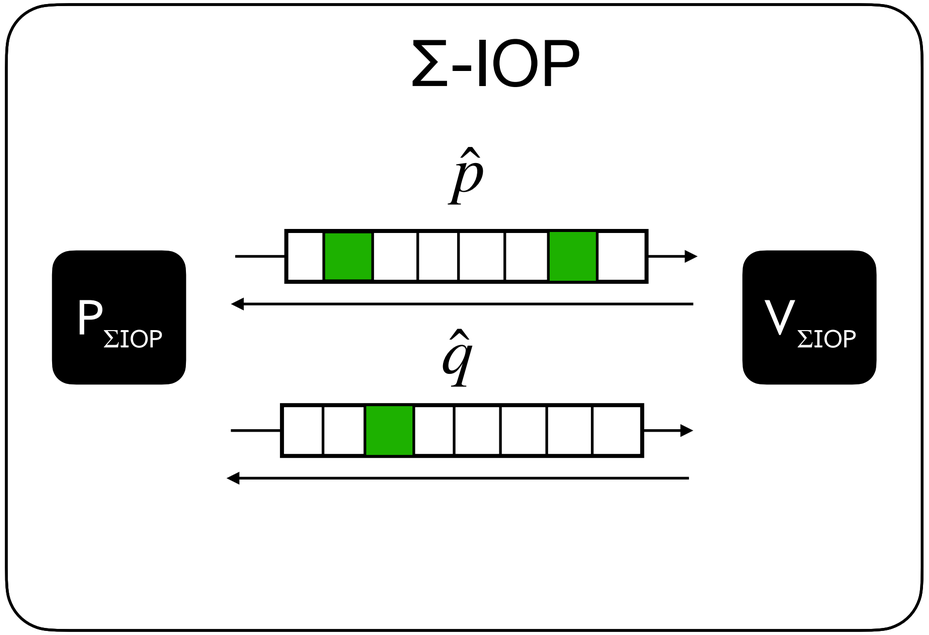

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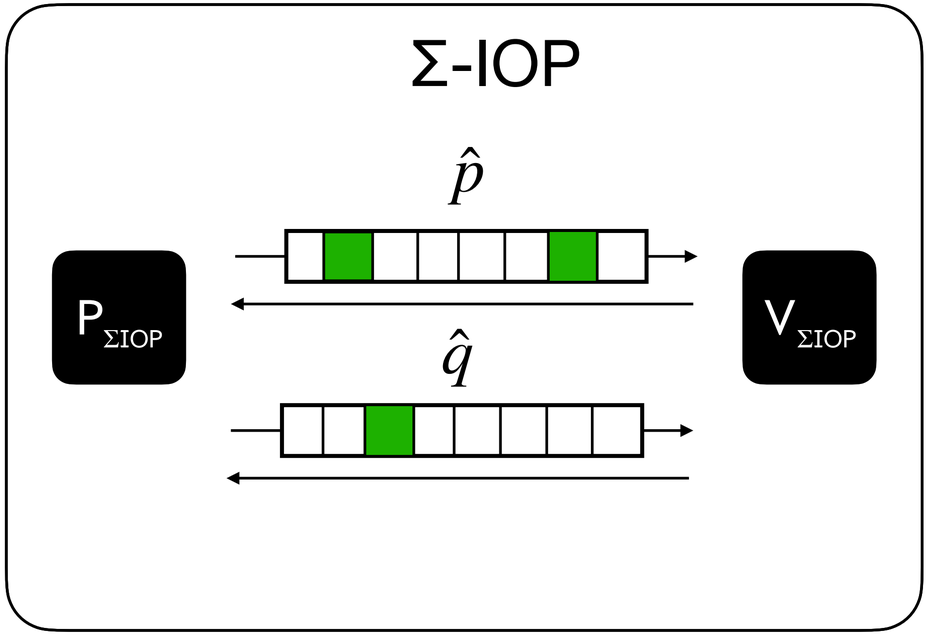

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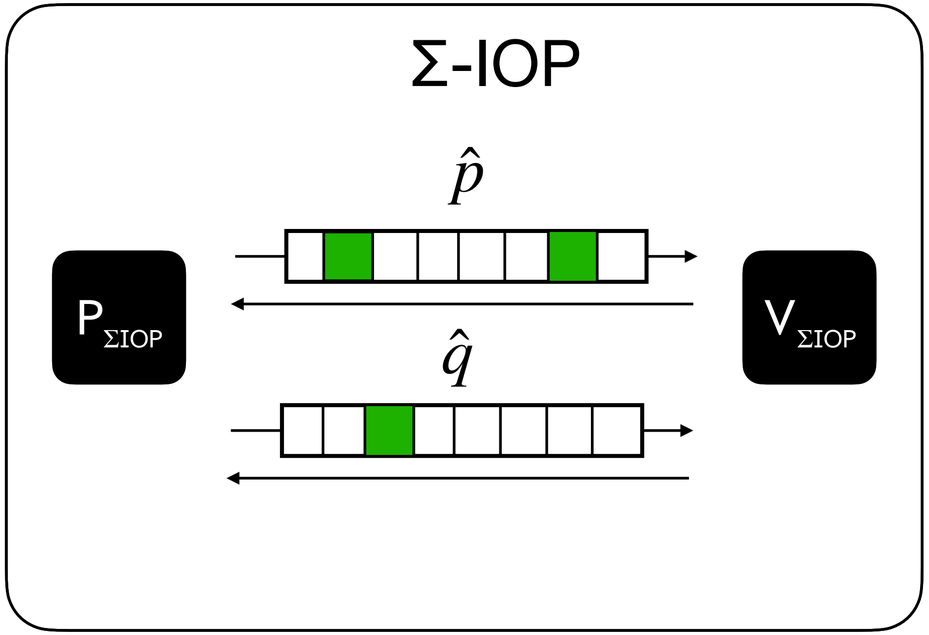

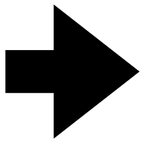

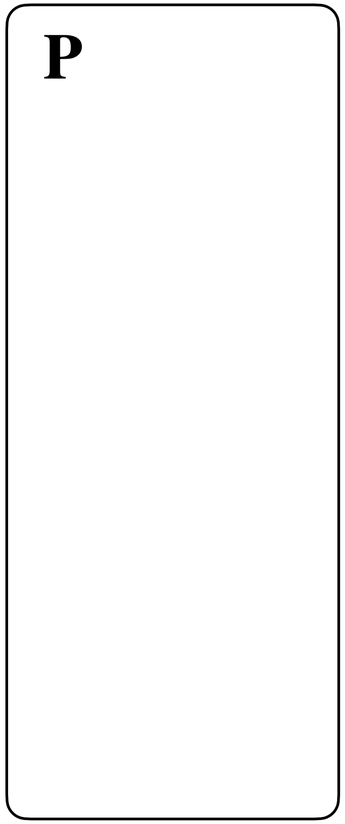

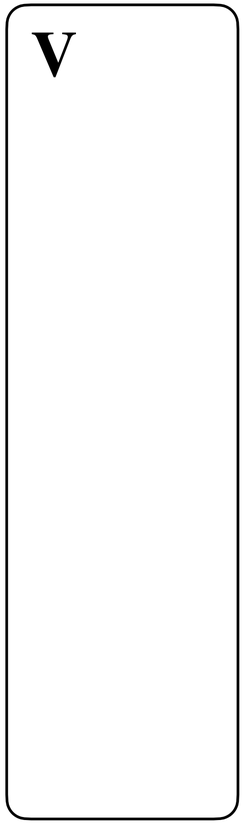

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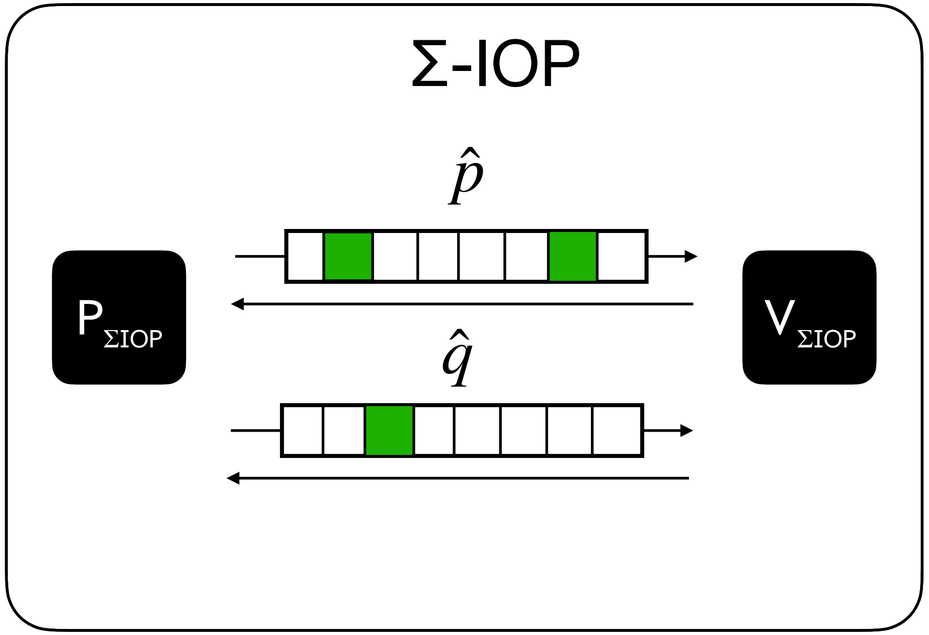

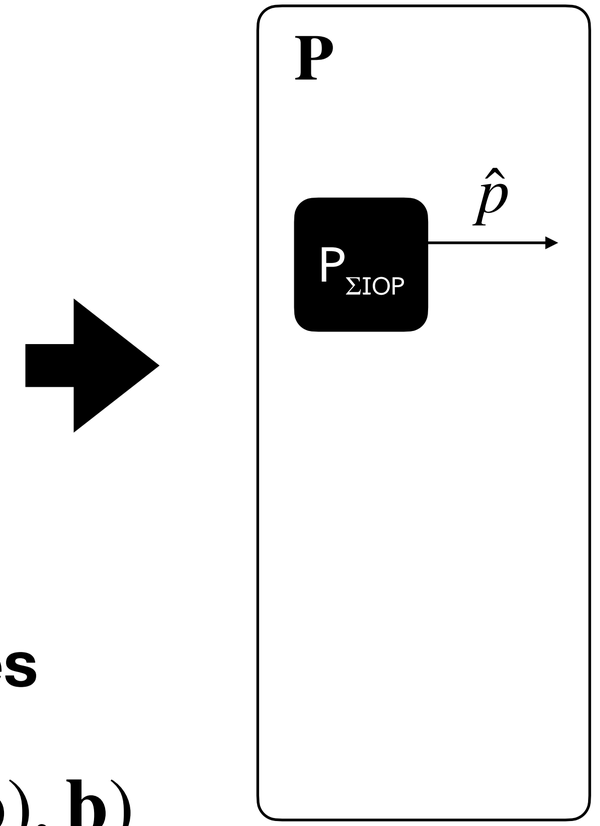

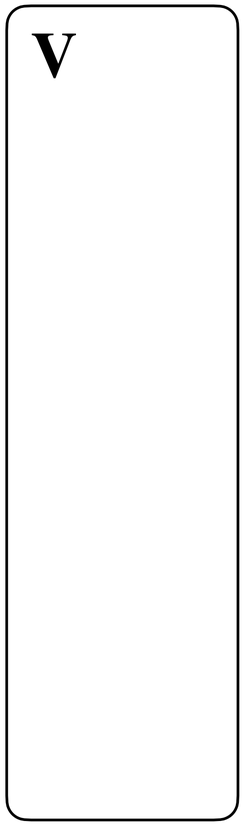

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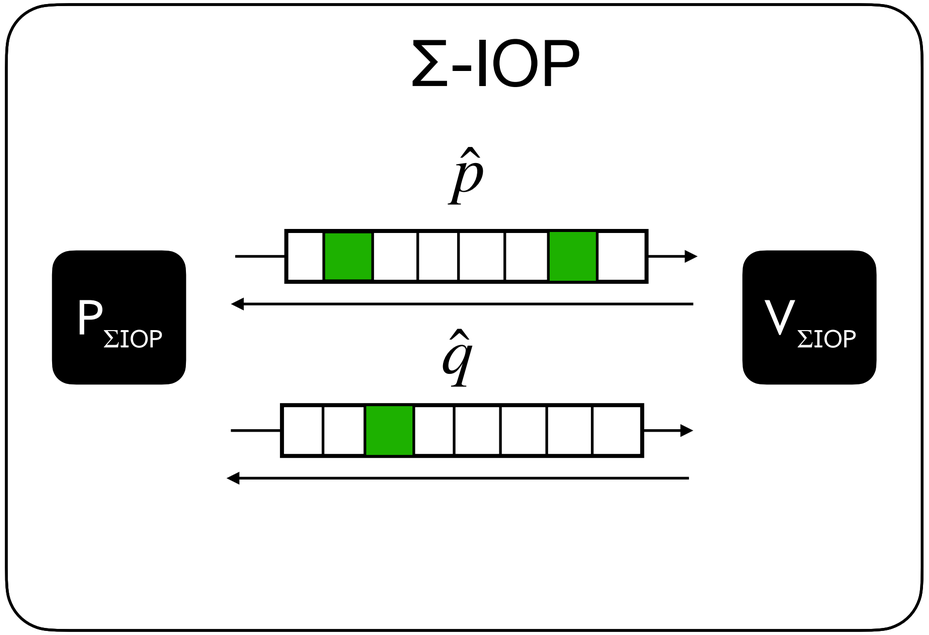

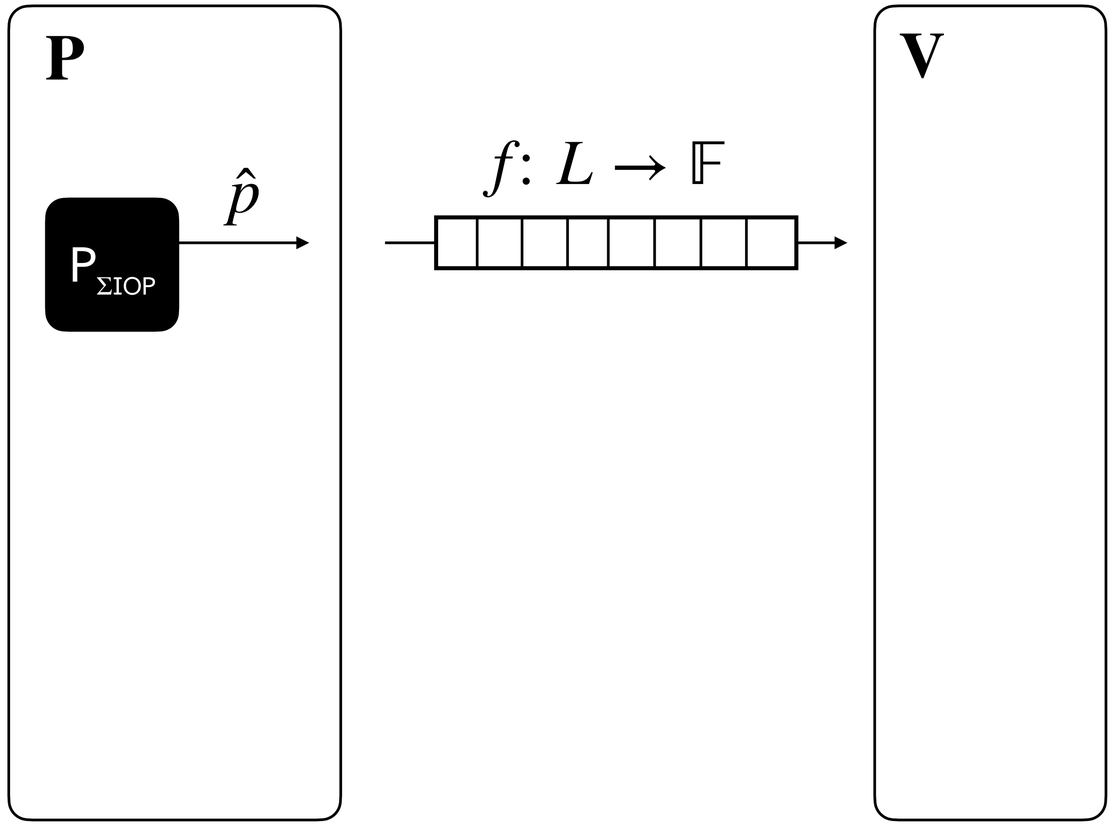

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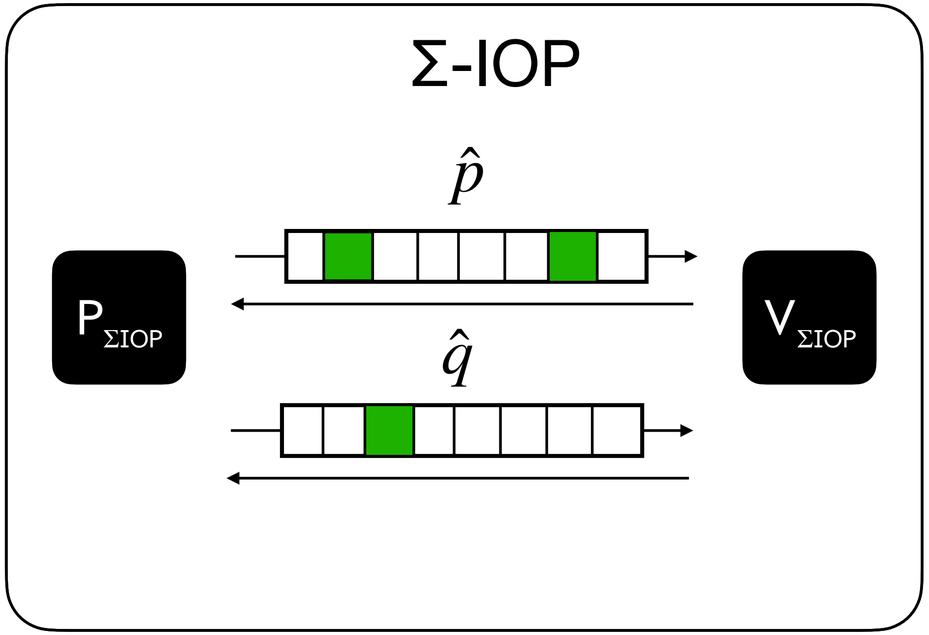

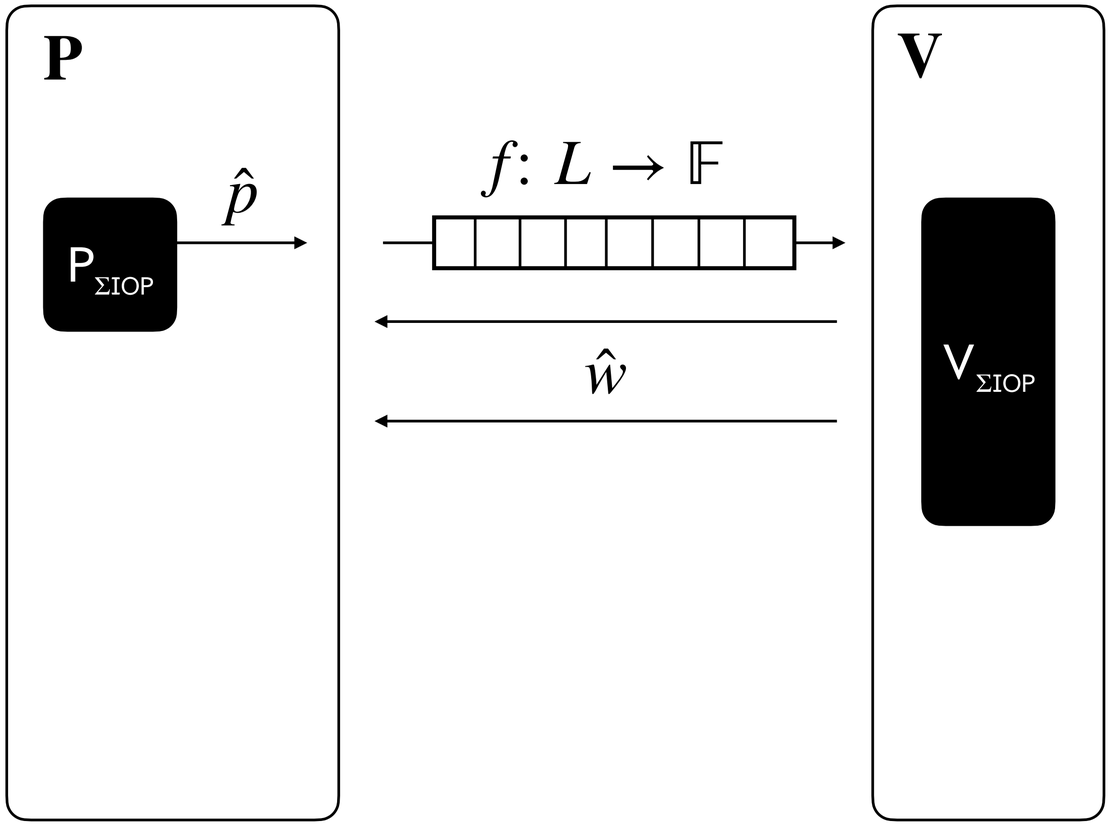

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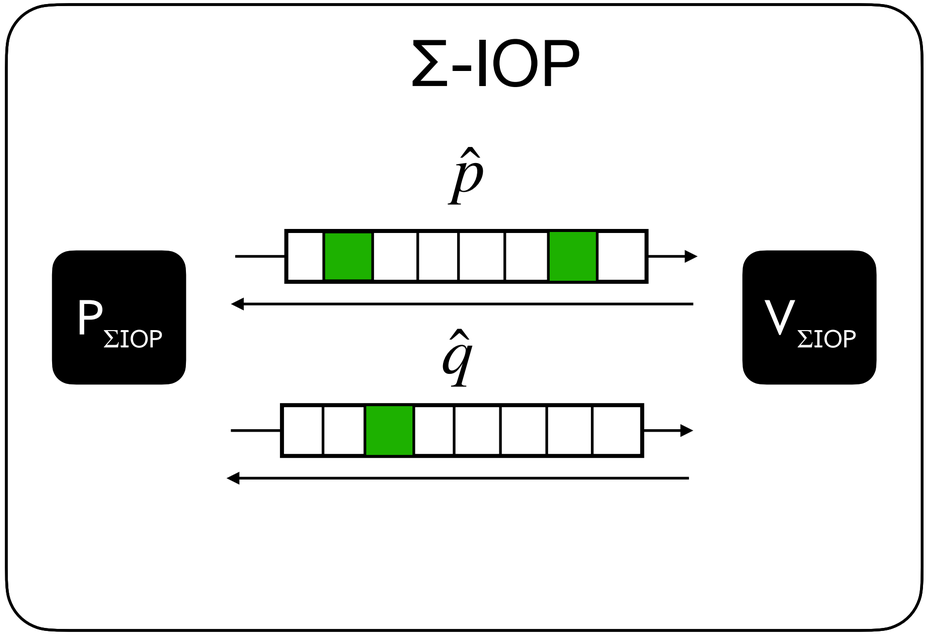

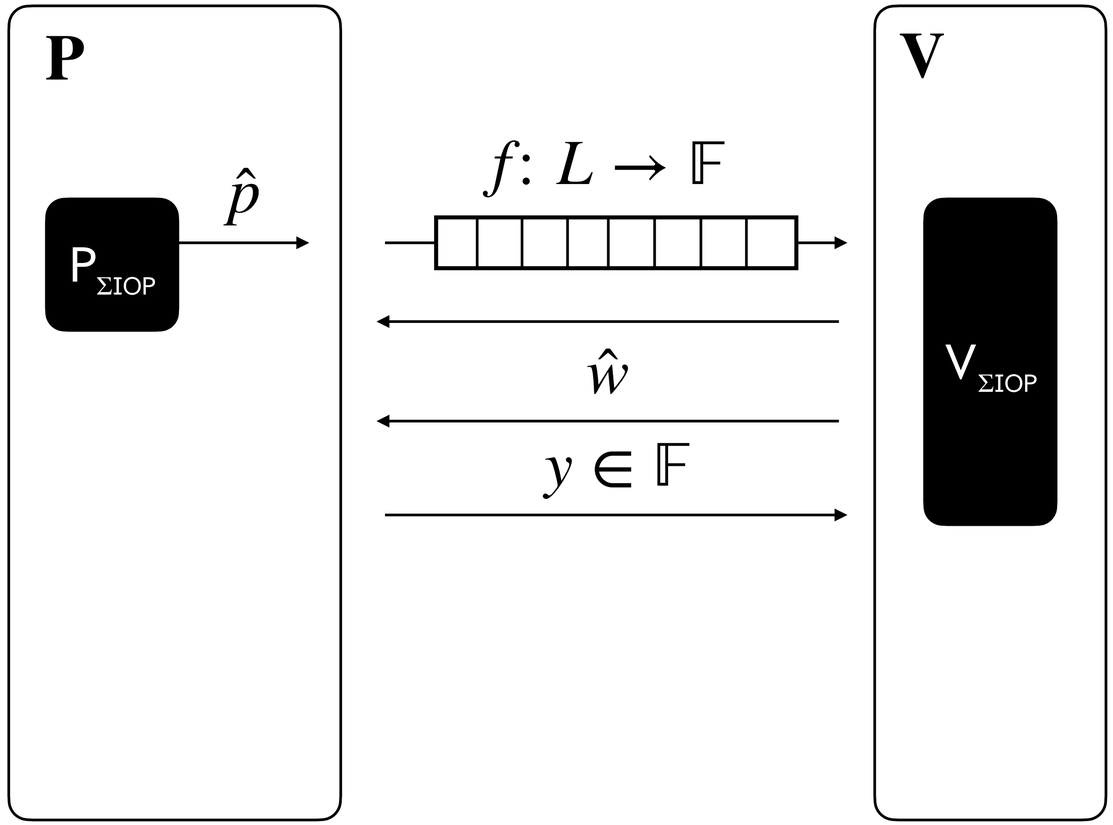

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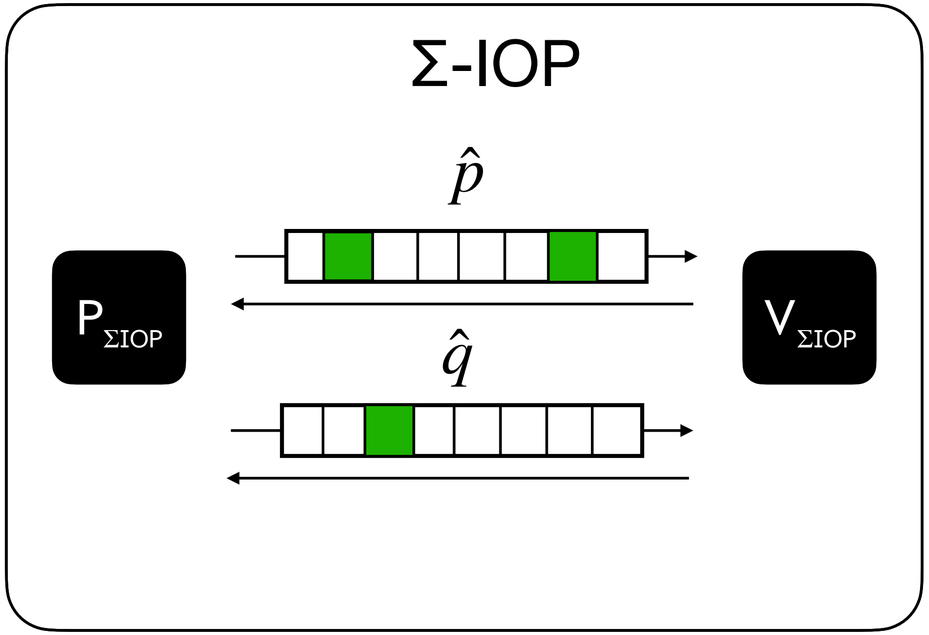

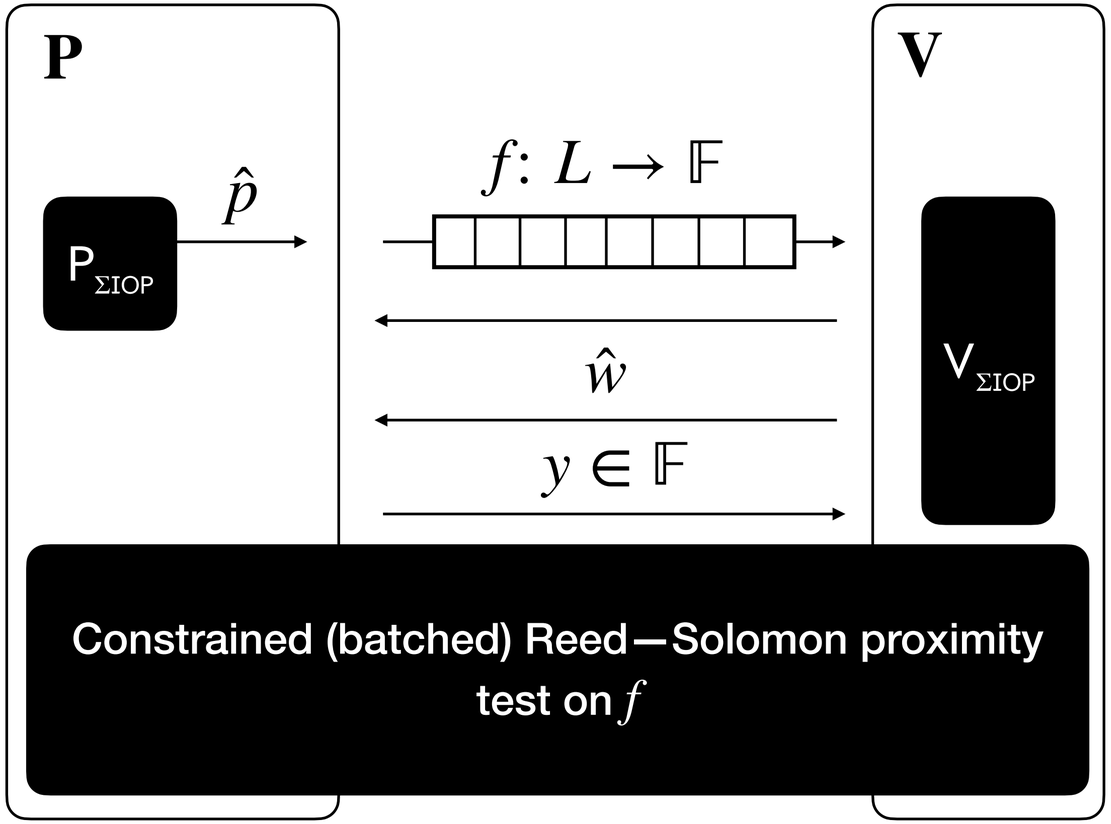

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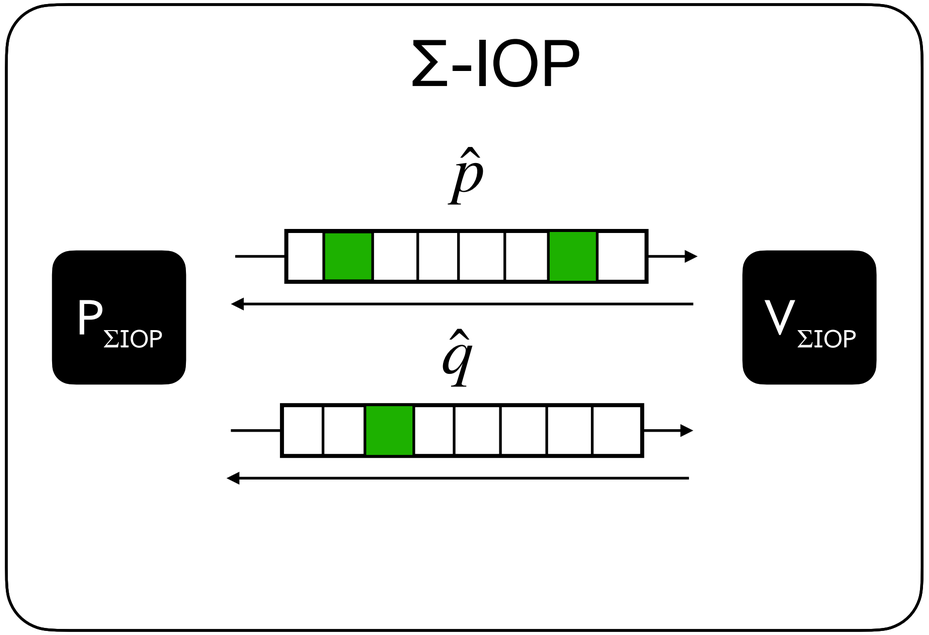
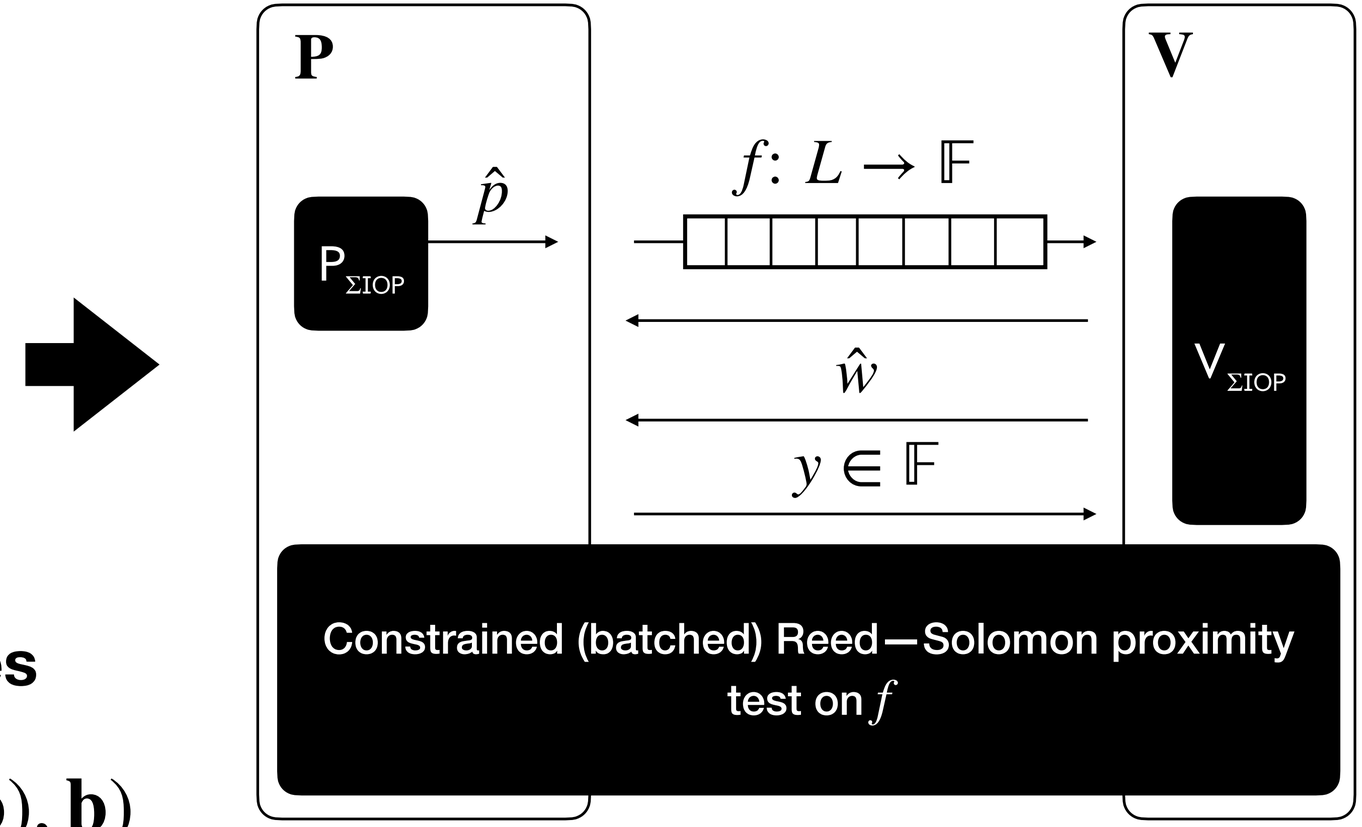
Q: Can we use this to do more **efficient** arithmetizations?





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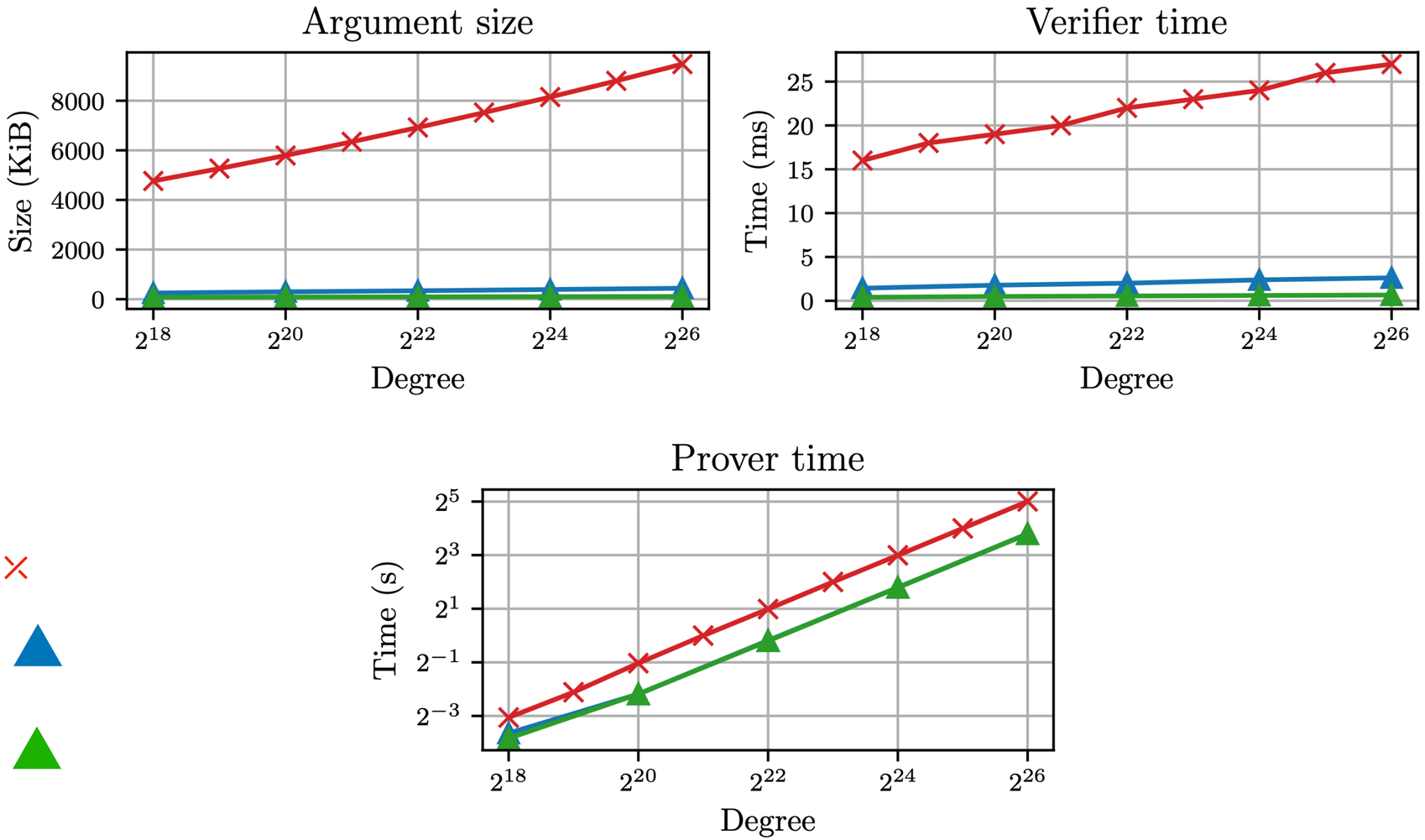
Extra slides

Comparison with BaseFold



BaseFold: ×

WHIR-UD: ▲

WHIR-CB: ▲

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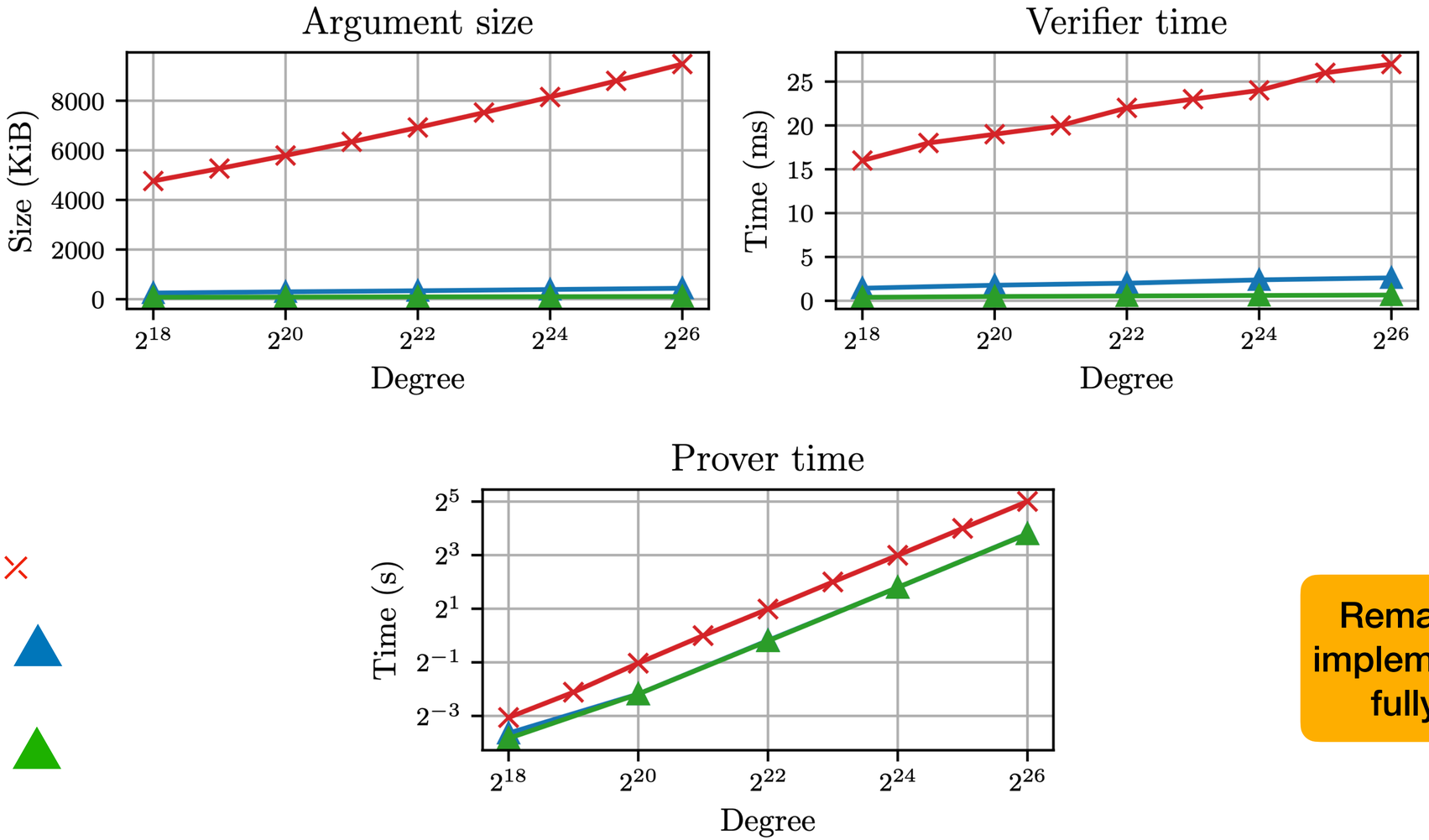

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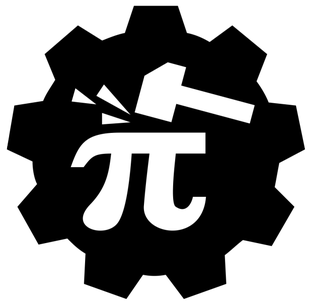
Remark: BaseFold implementation is not fully optimised

Implementation



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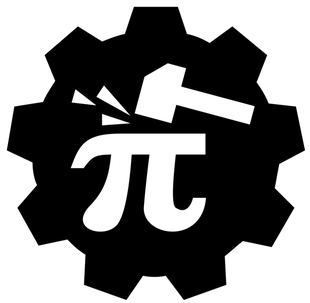

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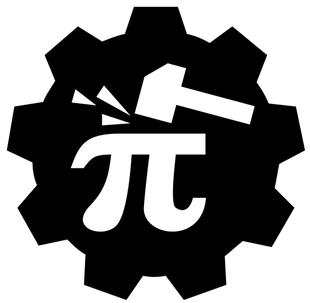

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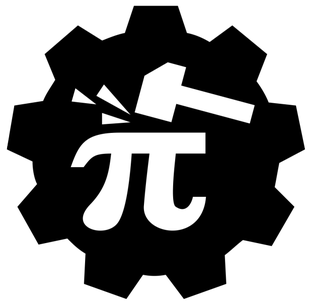

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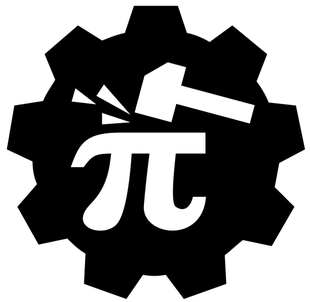

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  - Can be used in compiler for Σ-IOP

Comparison to STIR and FRI

FRI: $O\left(\frac{\lambda}{k} \cdot m\right)$

STIR & WHIR $O\left(\frac{\lambda}{k} \cdot \log m\right)$

- **Drop-in** replacement of FRI and STIR (when used for $\mathsf{CRS}[\mathbb{F}, m, \rho, 0, 0]$)
- **Same** benefits as STIR over FRI, and **faster** prover time.
- **Additionally, richer proximity tests means that:**
  - Can be used as a **multilinear** PCS (instead of BaseFold, FRI-Binius, etc)
  - Additionally, **bivariate** PCS (and anything in between)
  - Can be used in compiler for Σ-IOP
- **Further,** super-fast verification (next)

Batching

Pick your favourite sumcheck batching

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$g : L \to \mathbb{F}$

Sumcheck claims on g:
$(\hat{w}_1, \sigma_1), \ldots, (\hat{w}_\ell, \sigma_\ell)$

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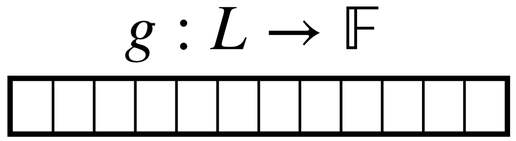

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Many ways this can be done: **we chose random linear combination.**

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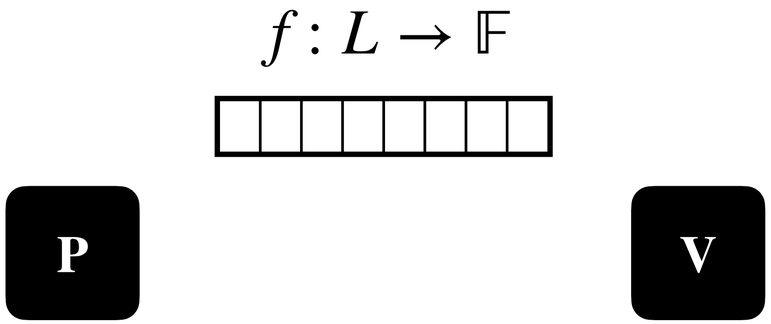

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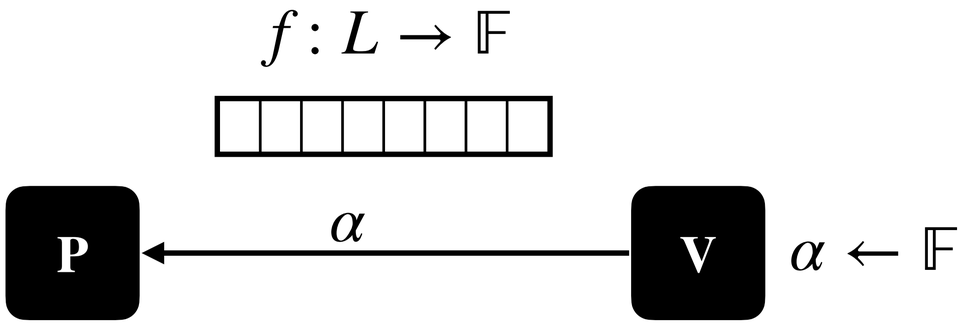

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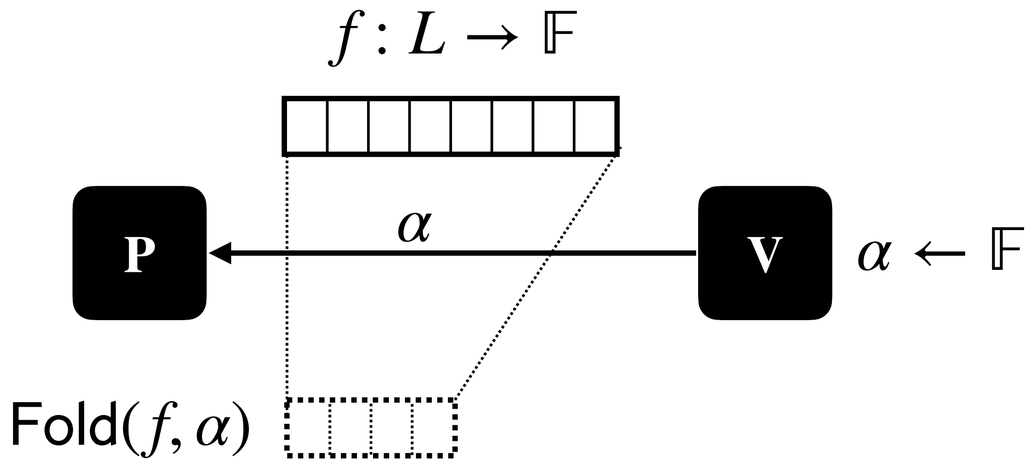

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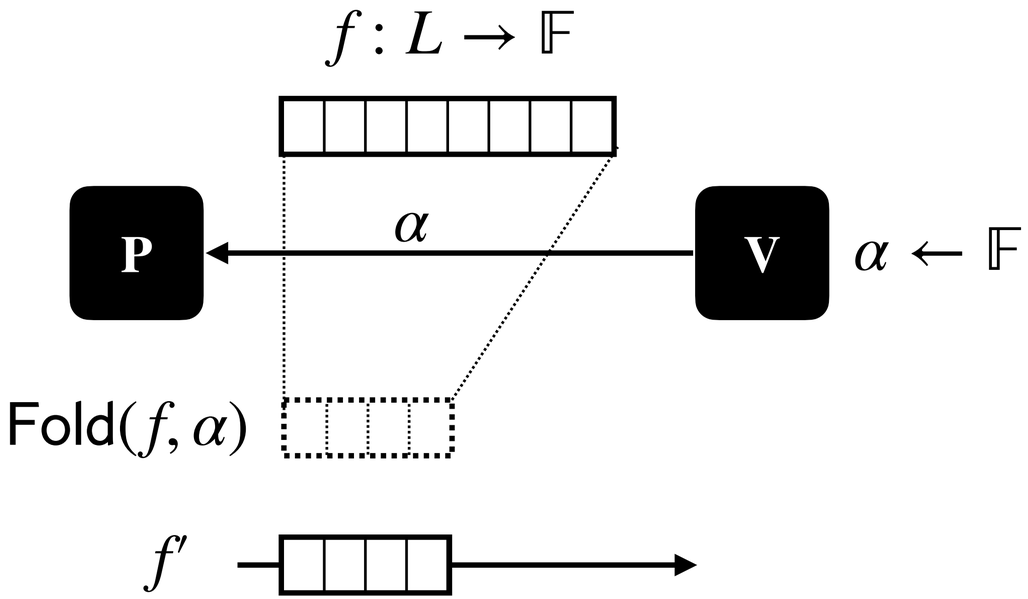

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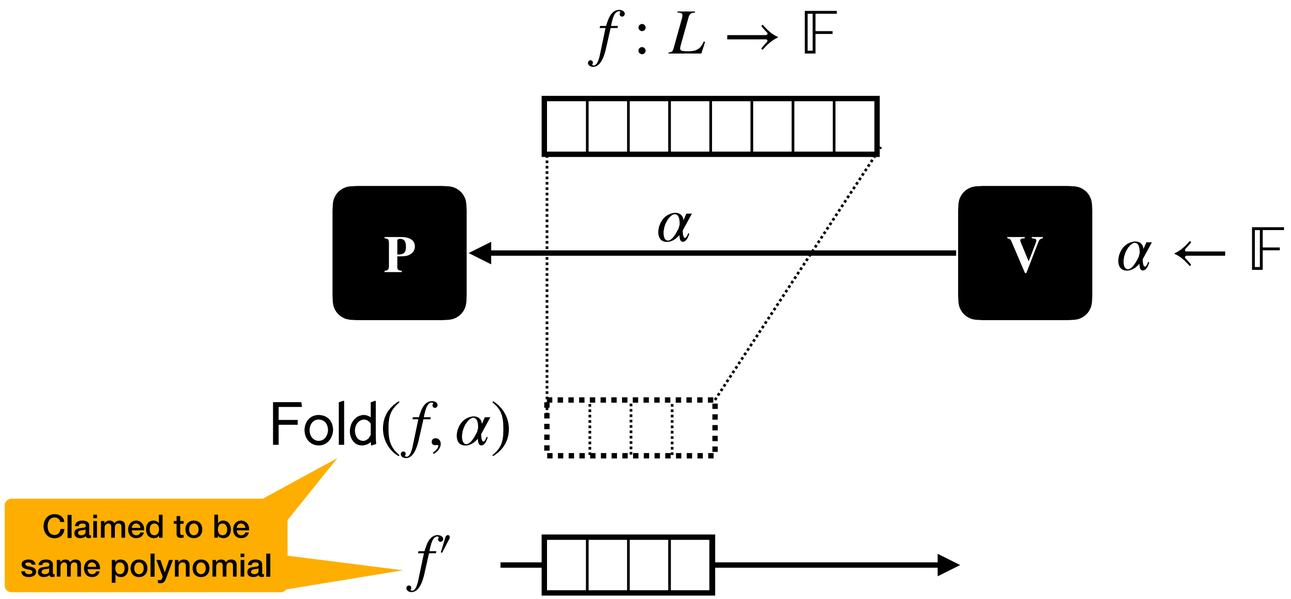

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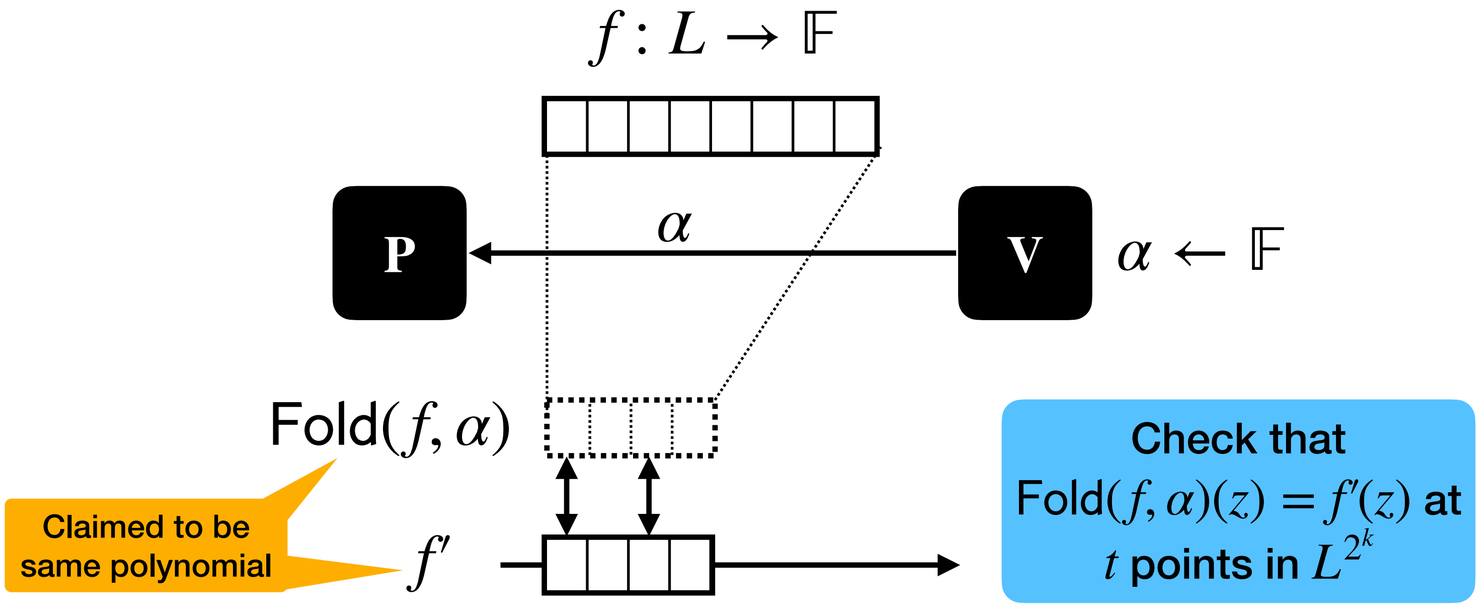

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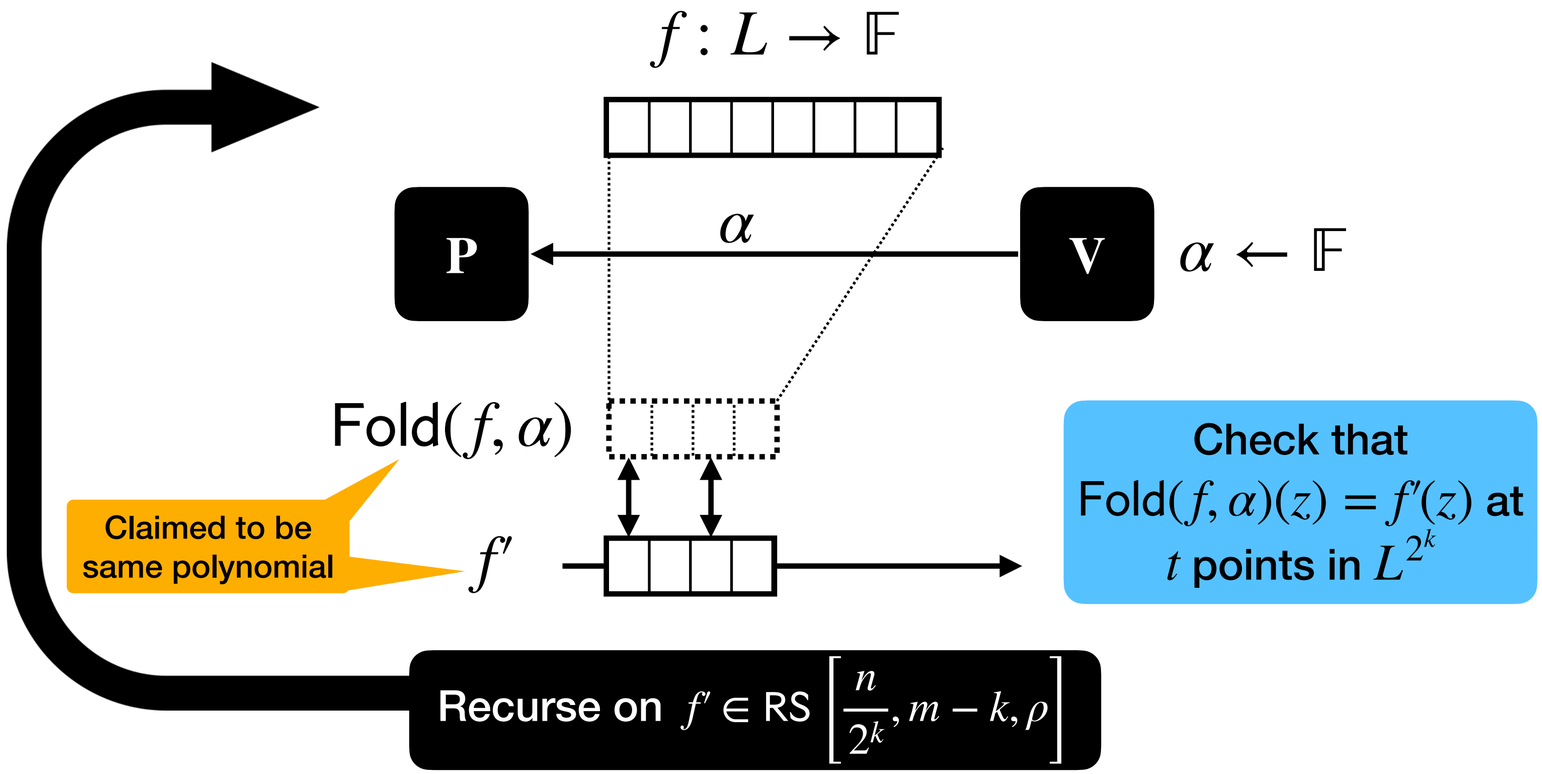

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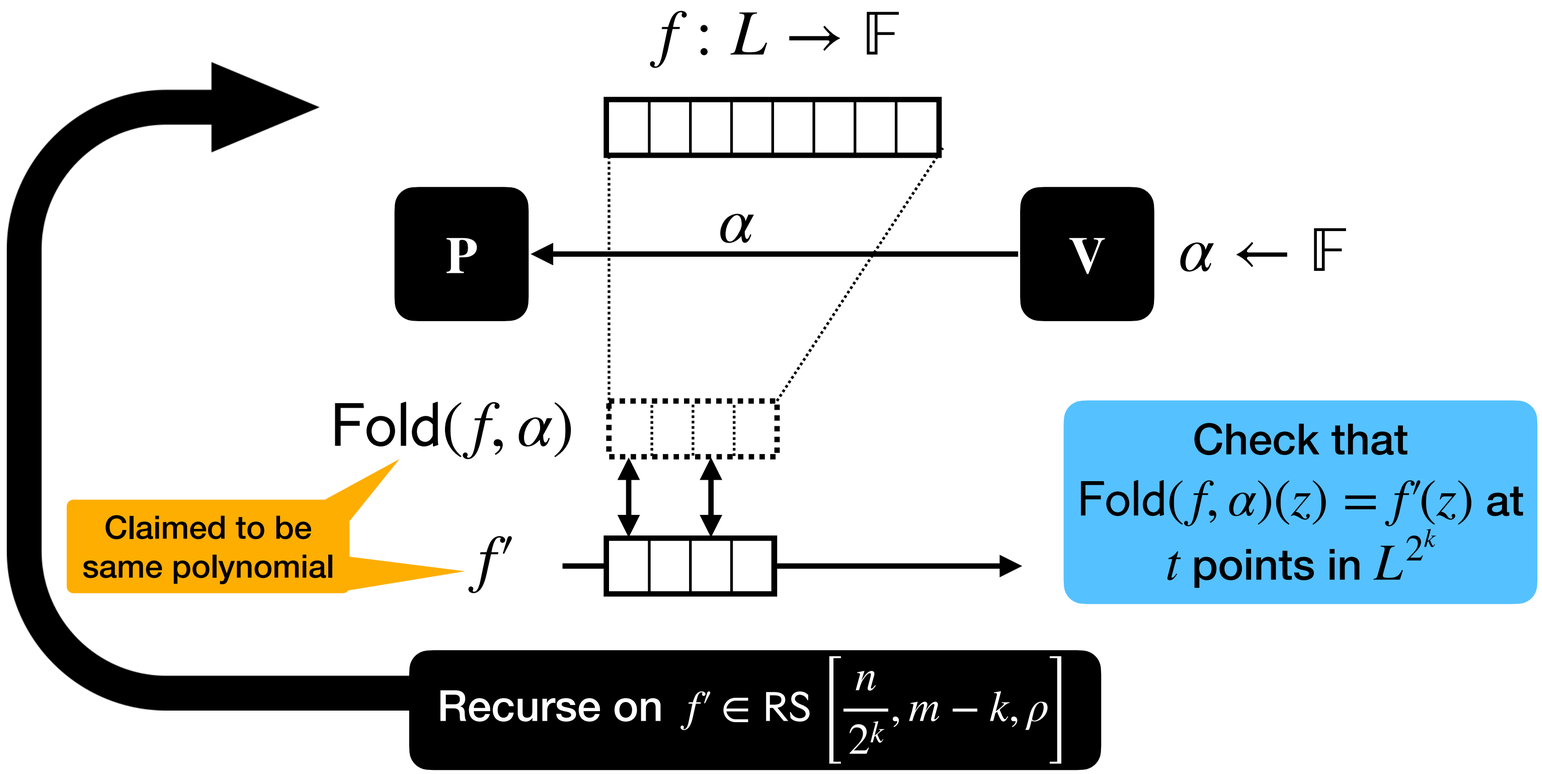

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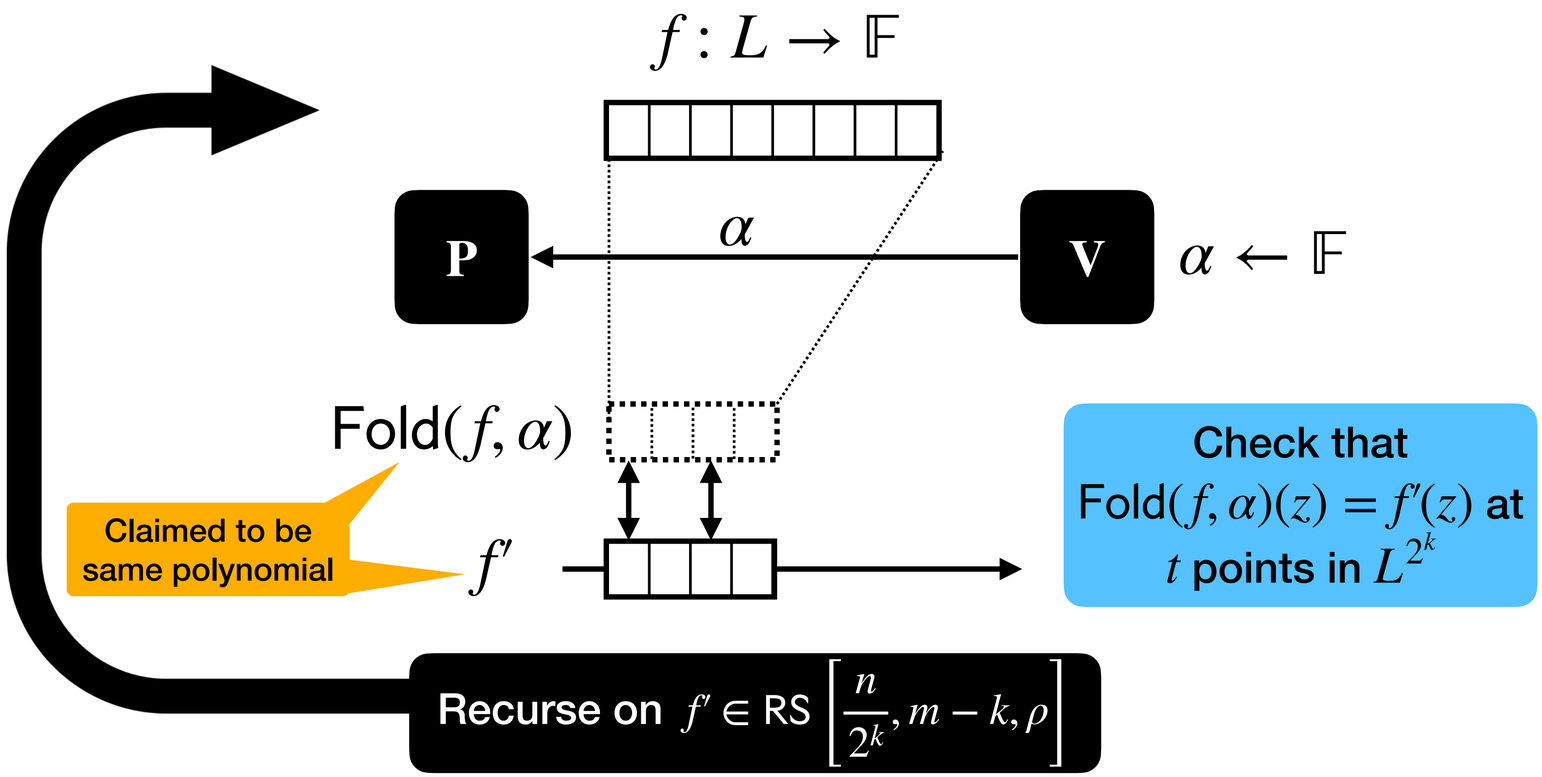

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If f is δ-far from $\mathrm{RS}[n, m, \rho]$,

Fold(f, α) must be δ-far from $\mathrm{RS}[n/2^k, m-k, \rho]$

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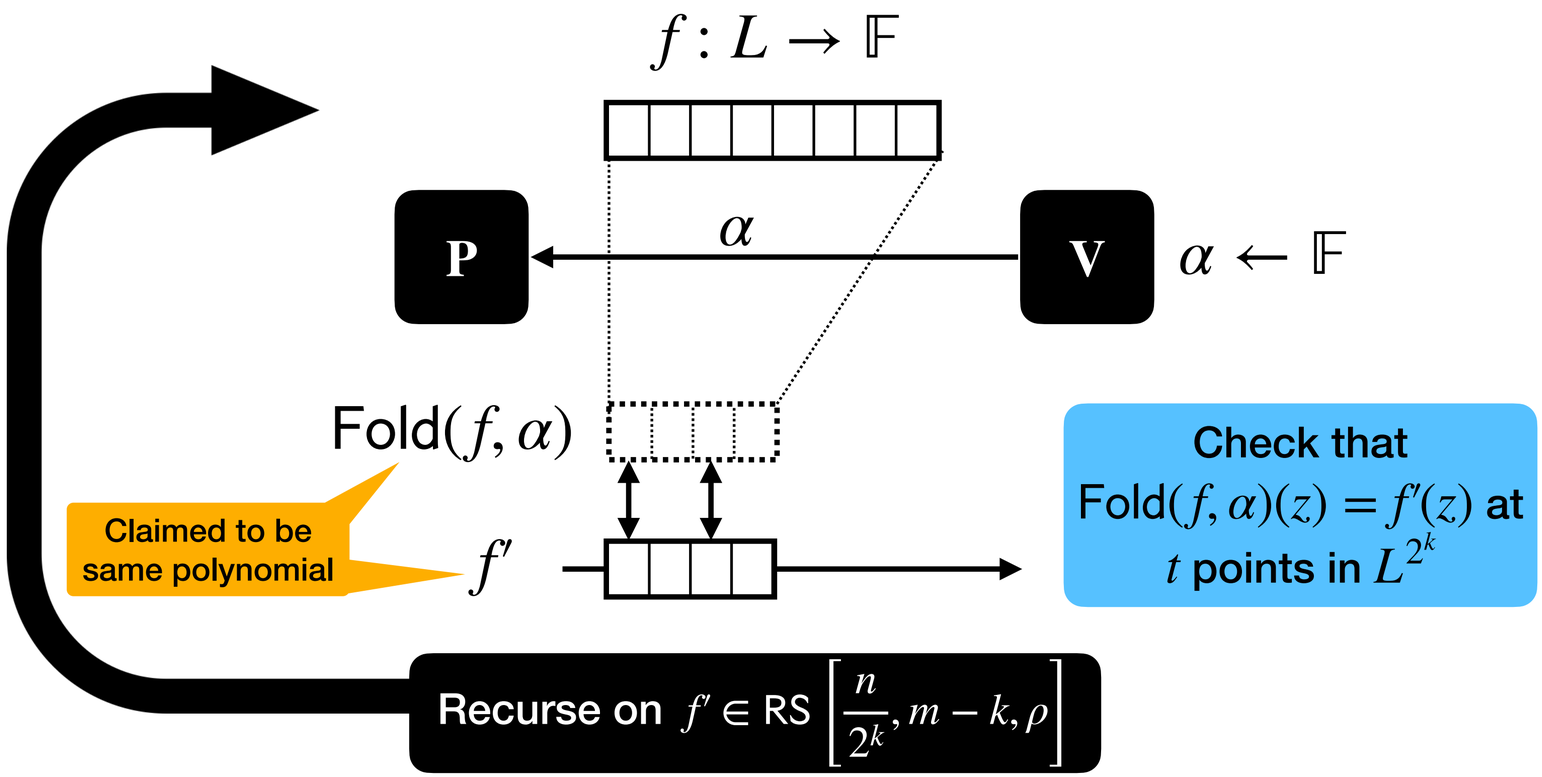

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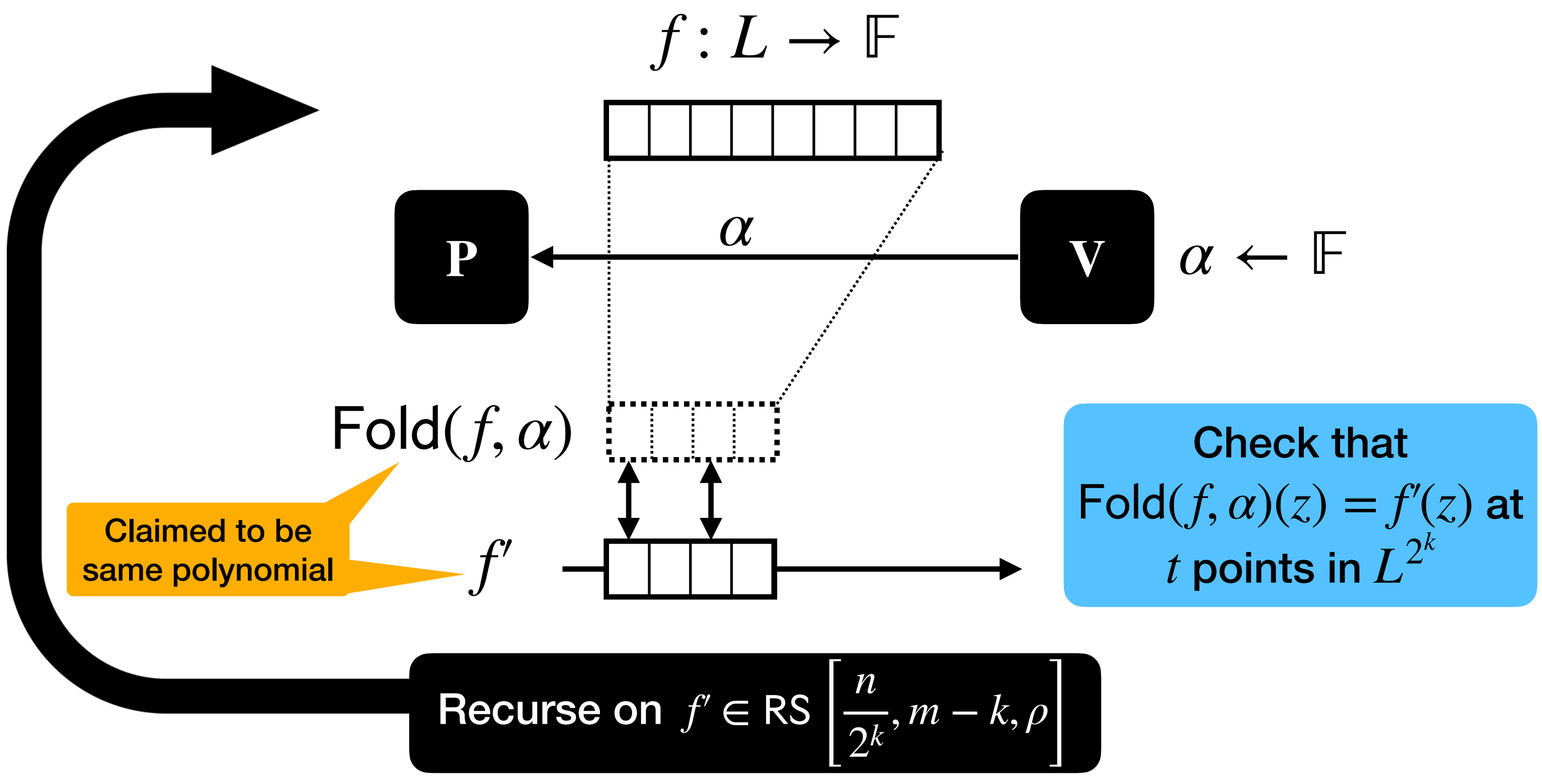

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To get soundness error $\varepsilon_{\mathrm{RBR}} \leq 2^{-\lambda}$:

set $\delta := 1 - \sqrt{\rho}$ and $t := \dfrac{\lambda}{-\log\sqrt{\rho}}$

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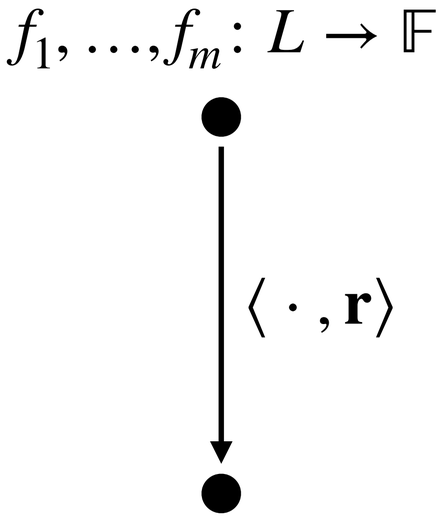

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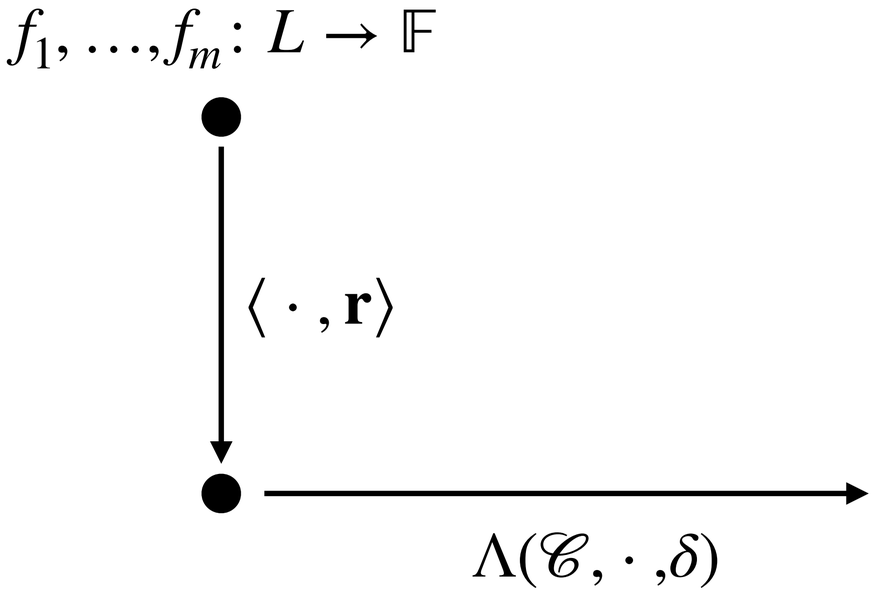

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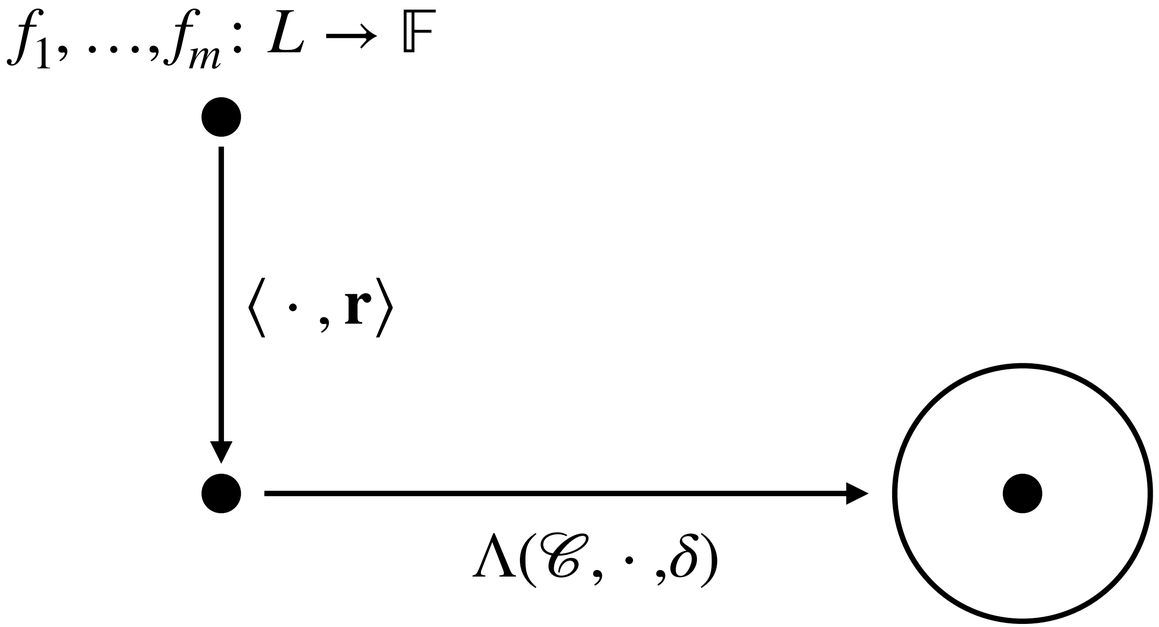

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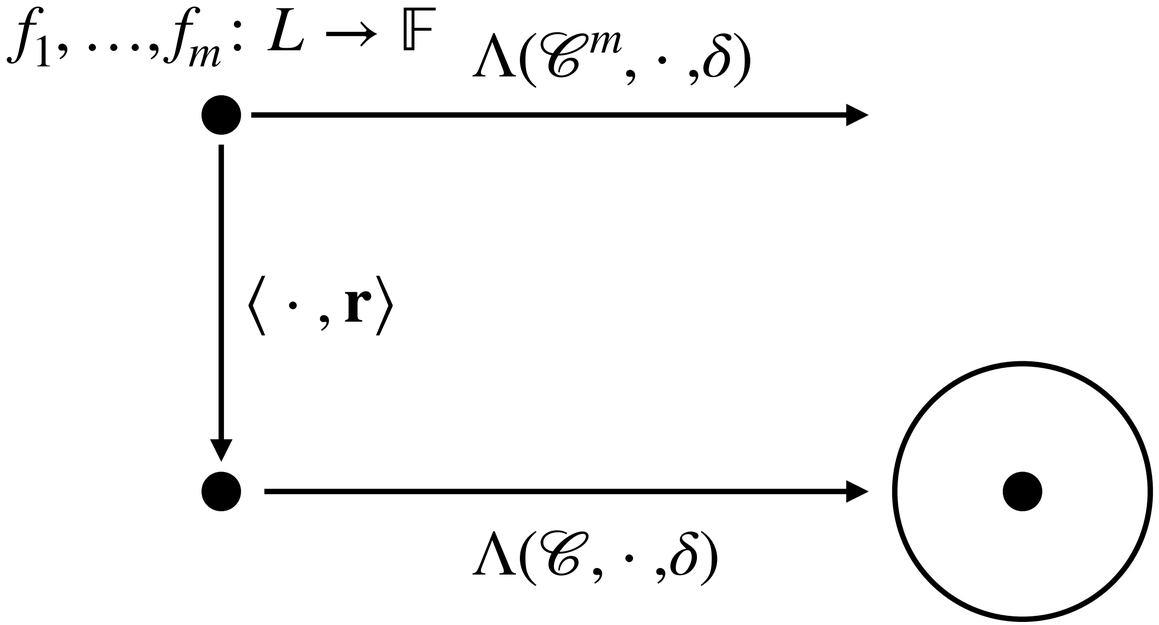

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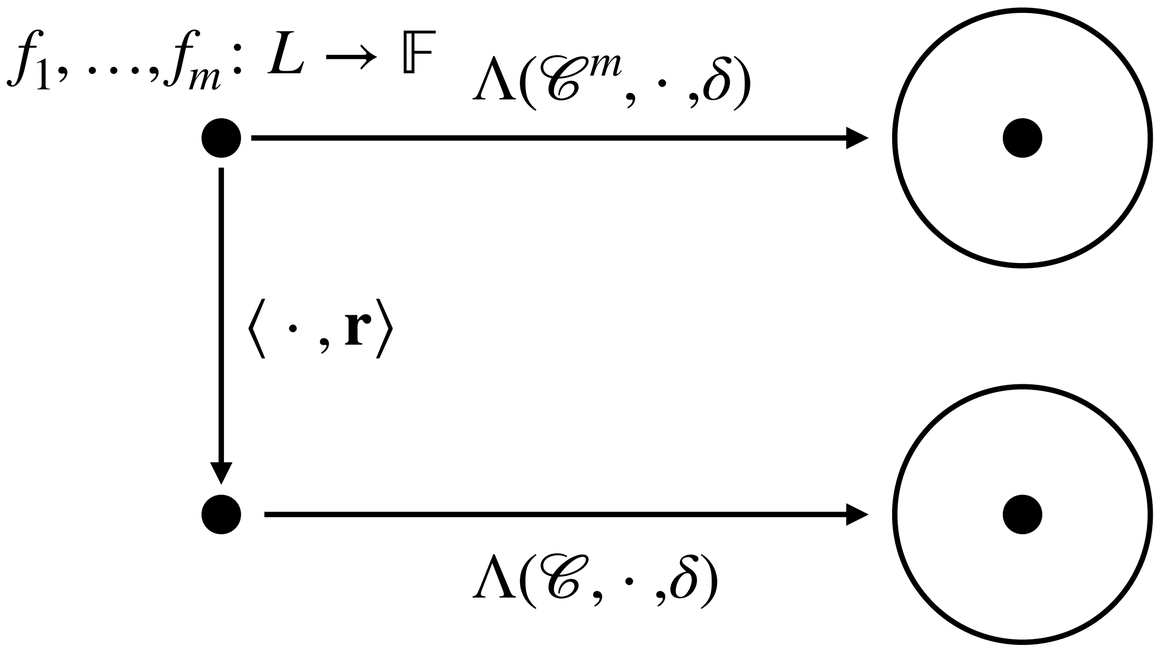

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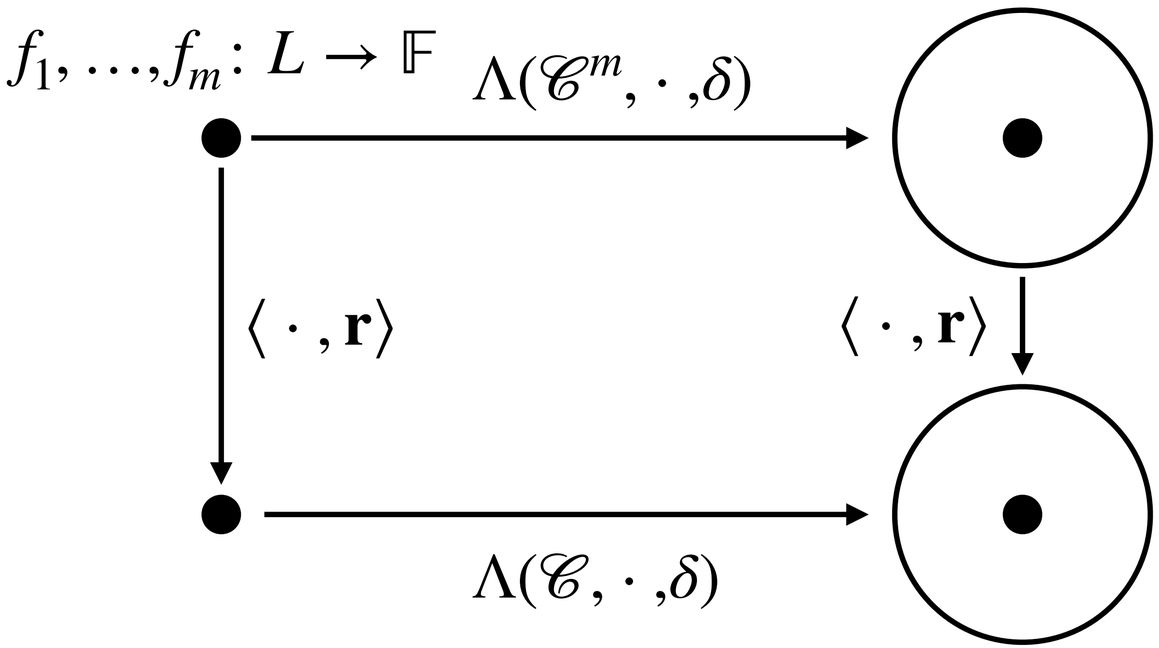

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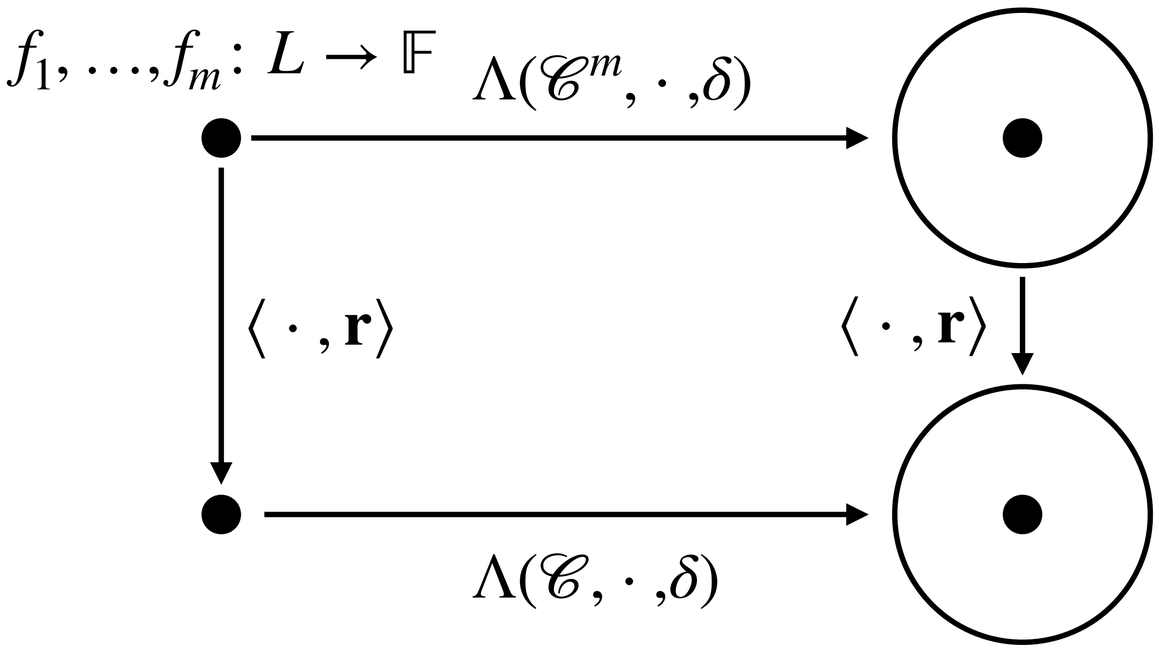

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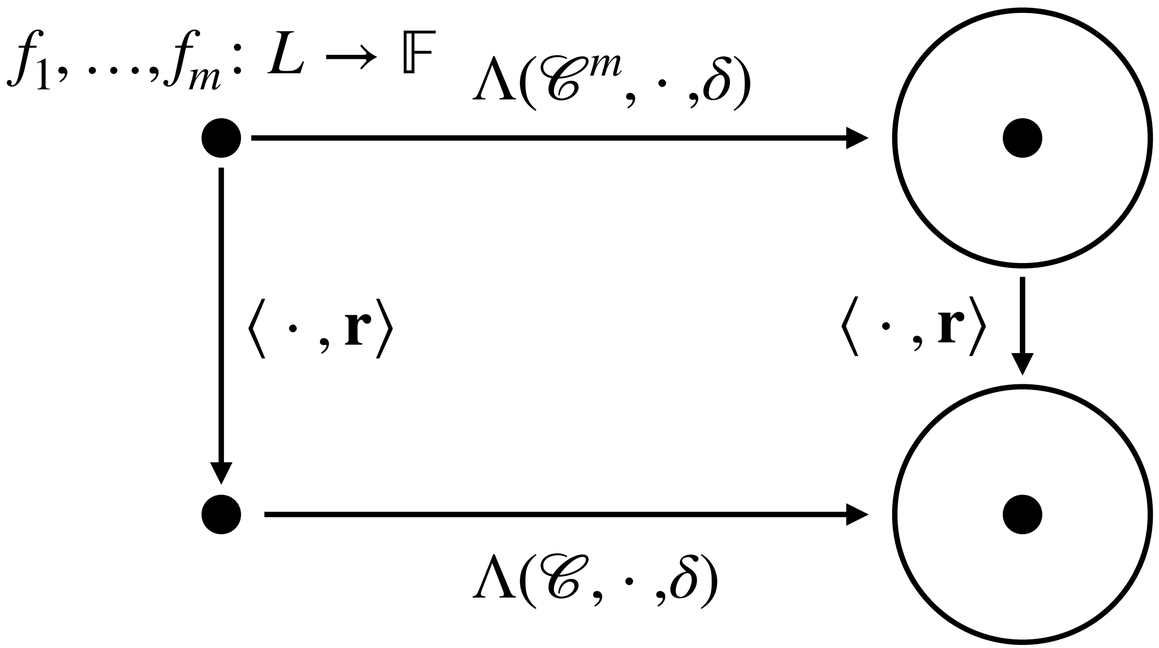

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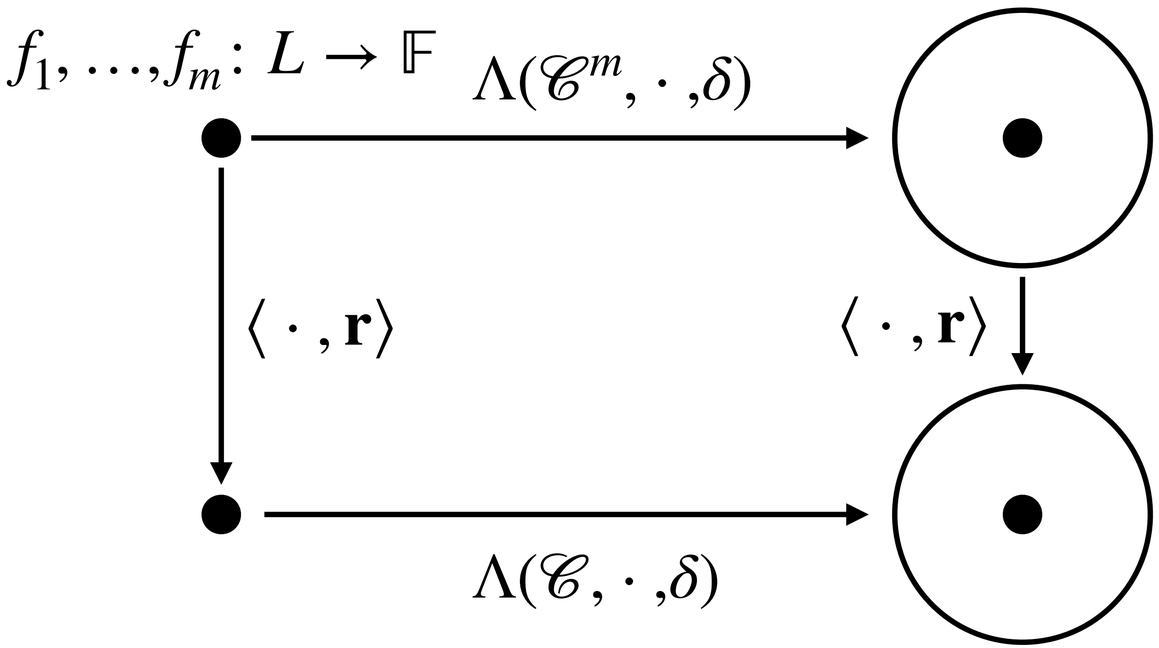

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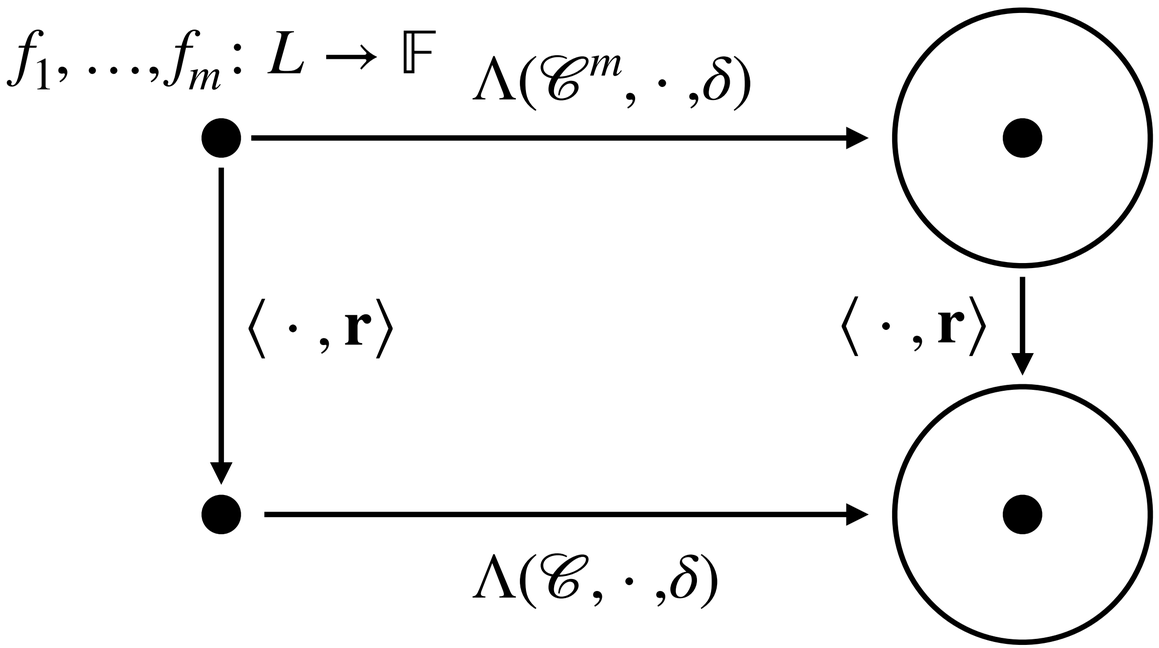

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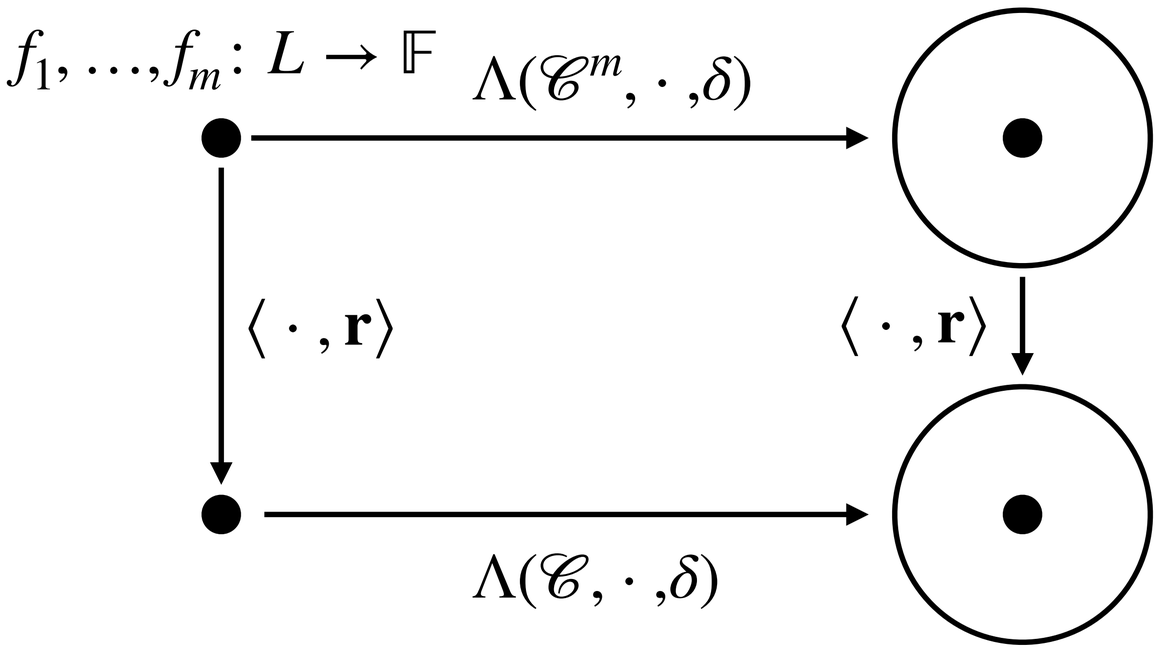

Stronger than what is required for STIR's soundness

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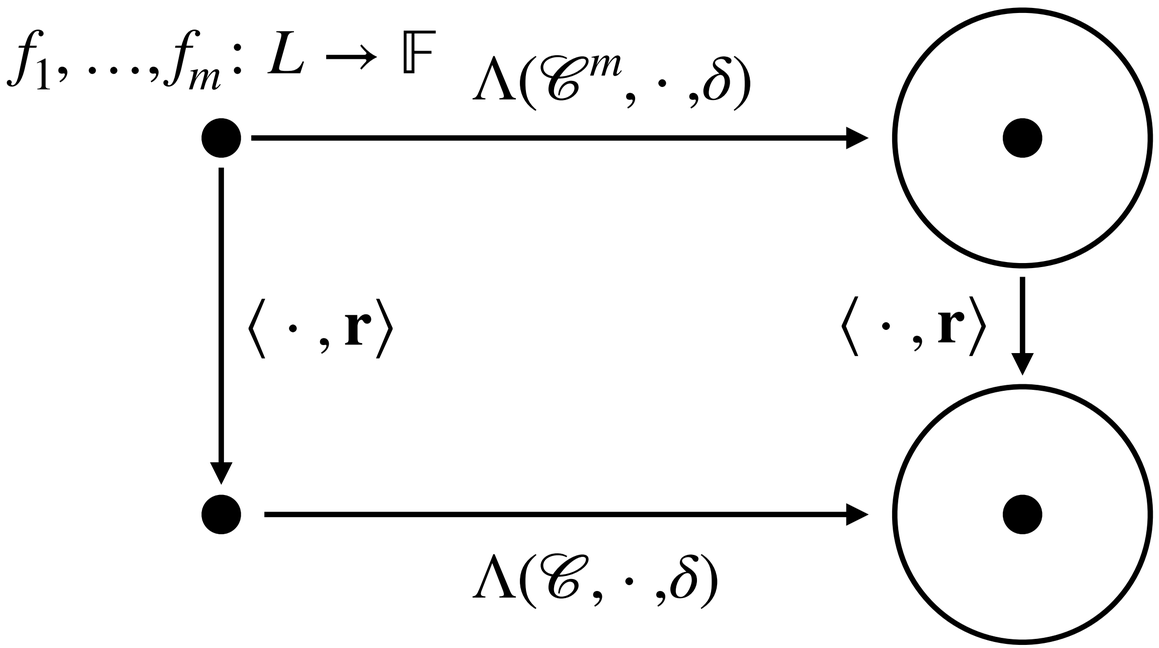

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Recent results show it holds up to 1.5 Johnson for general linear codes!