Secret Sharing with Publicly Verifiable Deletion

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Certified deletion

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- Secure key leasing: revoking access to functionality

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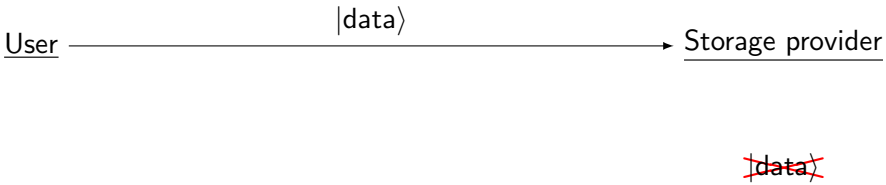

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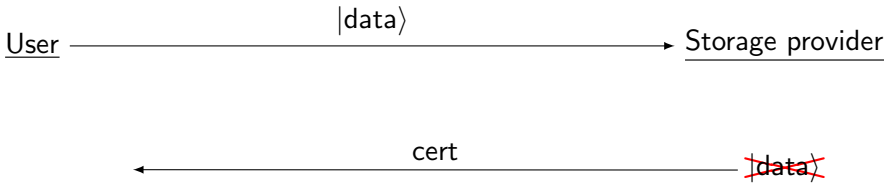

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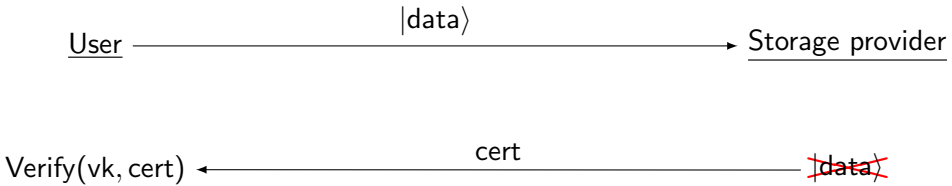

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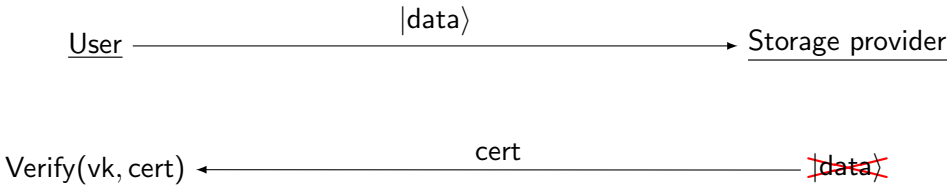

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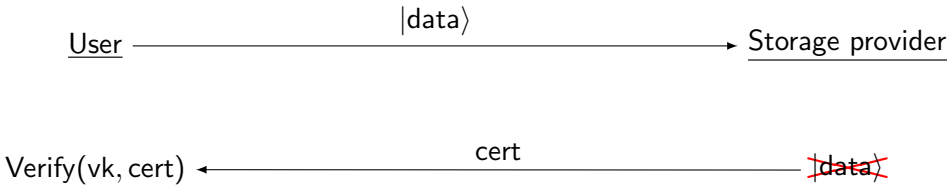

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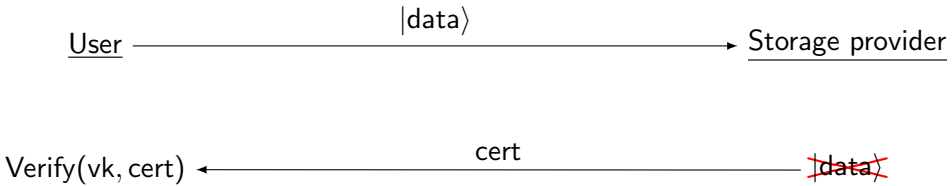

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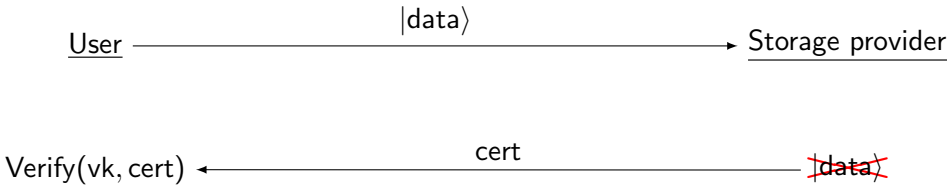

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  - Any third party can verify deletion proofs
  - Do not need to store verification keys securely

Secret sharing

Definition: Secret sharing scheme

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Classical Secret Sharing

- Parameterized by a monotone access structure defining the authorized sets

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Security: If $A \subseteq [n]$ is not authorized, then

$$\mathsf{SD}\left(\{\mathsf{sh}_i^0\}_{i \in A}, \{\mathsf{sh}_i^1\}_{i \in A}\right) \leq \epsilon.$$

Secret sharing with certified deletion [BR24]

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$$|\mathsf{State}_{s_0}\rangle \approx_c |\mathsf{State}_{s_1}\rangle \quad \forall s_0, s_1$$

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- Everlasting security: If $\mathcal{A}$ is bounded *during adaptive certified deletion experiment*, then secret s is **information-theoretically** hidden given State_s, i.e.,

$$\mathsf{TD}(|\mathsf{State}_{s_0}\rangle, |\mathsf{State}_{s_1}\rangle) \leq \mathsf{negl}(\lambda) \quad \forall s_0, s_1$$

Prior work on secret sharing with certified deletion

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- Two (incomparable) security notions:
  - No-signaling certified deletion
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- Constructions with **privately verifiable** deletion (for both security notions)
  - Secure against unbounded adversary

Summary of our contributions

	Publicly verifiable deletion	Monotone access structures
No-signaling security	• Prior work: ✗	• Prior work: ✓
Adaptive security	• Prior work: ✗	• Prior work: ✗

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Summary of our contributions

- Assume information-theoretic classical secret sharing scheme for monotone access structure $\mathbb{A}$ (with share size m)

	Security	Assumptions	Number of qubits
Construction 1	Adaptive PVD	OWF	$O(n \cdot m)$
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- Adaptive PVD with computational security $\implies$ Adaptive PVD with everlasting security.

Publicly verifiable deletion—prior work [BKM+23]

Let $f : \{0,1\}^{\ell_{\text{in}}^{\text{owf}}} \mapsto \{0,1\}^{\ell_{\text{out}}^{\text{owf}}}$ be a one-way function.

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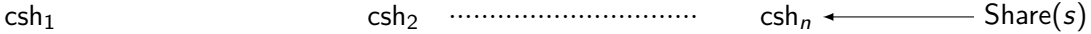
- Note: Preimages must be hidden for deletion security, but knowledge of preimages are required to extract encoded bit b.

Secret sharing with PVD from OWFs—first attempt



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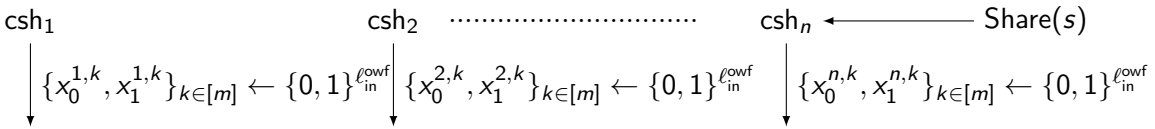

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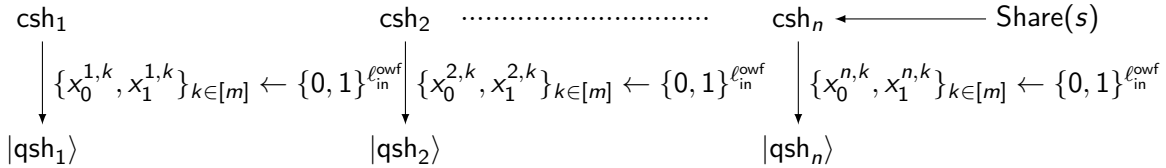

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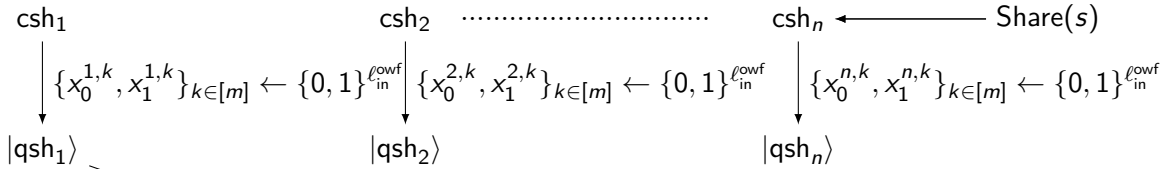

$$|\mathsf{qsh}_i\rangle := \bigotimes_{k\in[m]} \left(|x_0^{i,k}\rangle + (-1)^{\mathsf{csh}_i[k]} |x_i^{1,k}\rangle \right)$$

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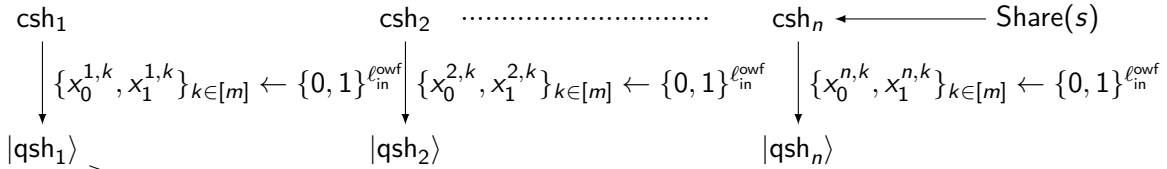

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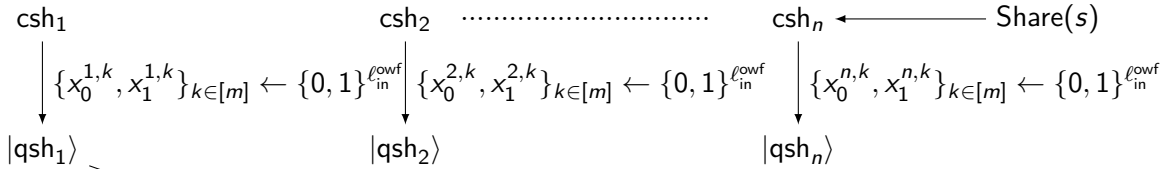

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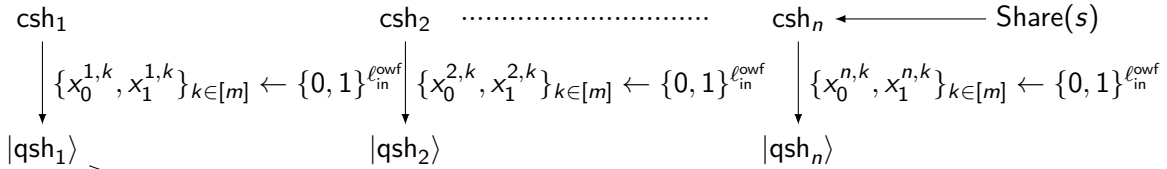

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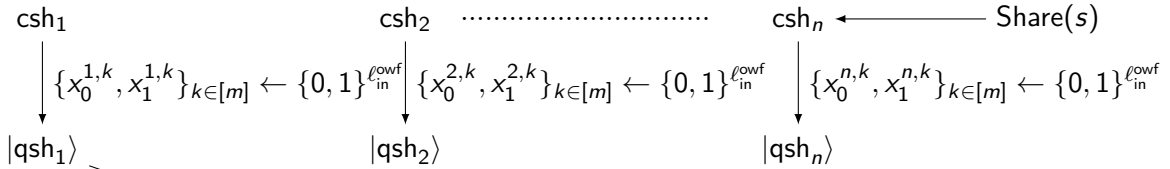

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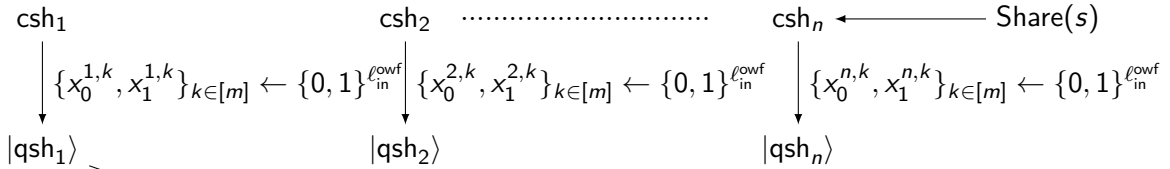

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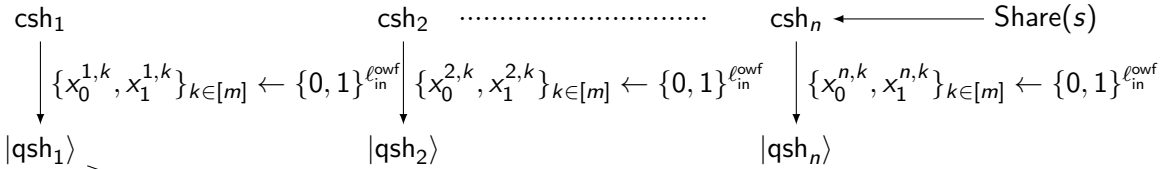

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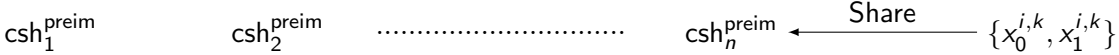
  - Problem: need preimages $x_0^{i,k} \oplus x_1^{i,k}$ to extract the classical share.

Secret sharing with PVD from OWFs—second attempt

- Idea: Give each party classical share of preimages in addition to quantum share.



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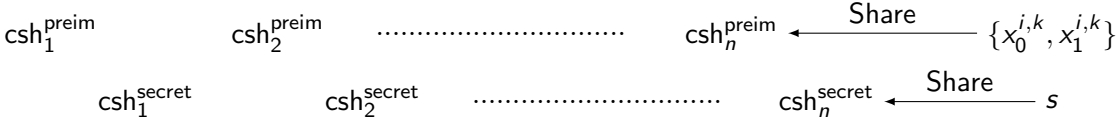

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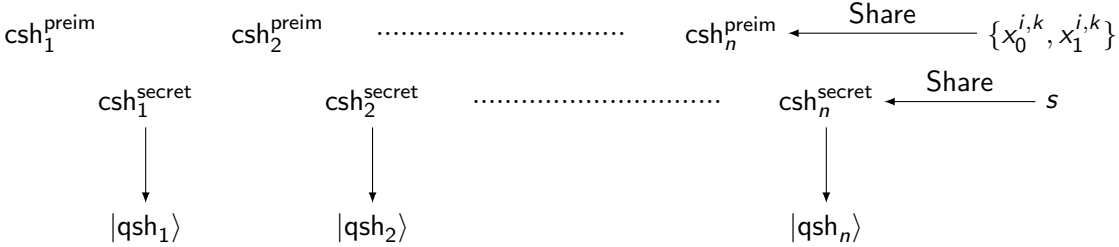

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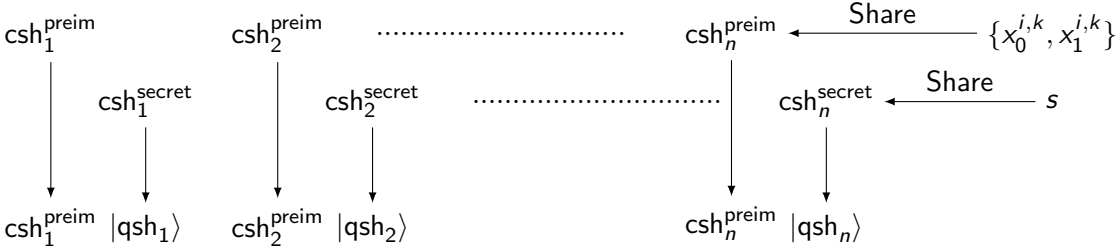

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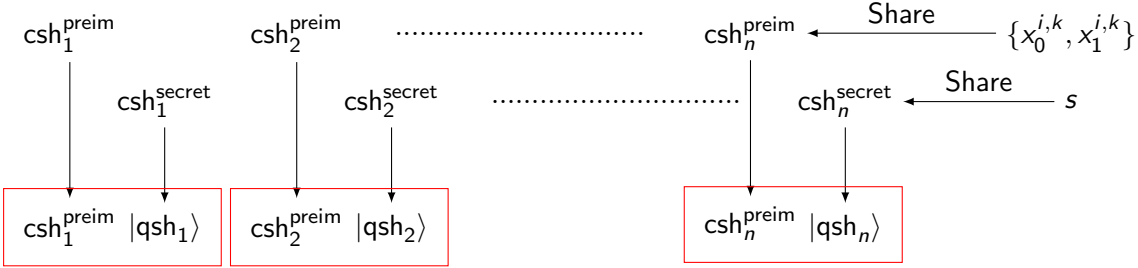

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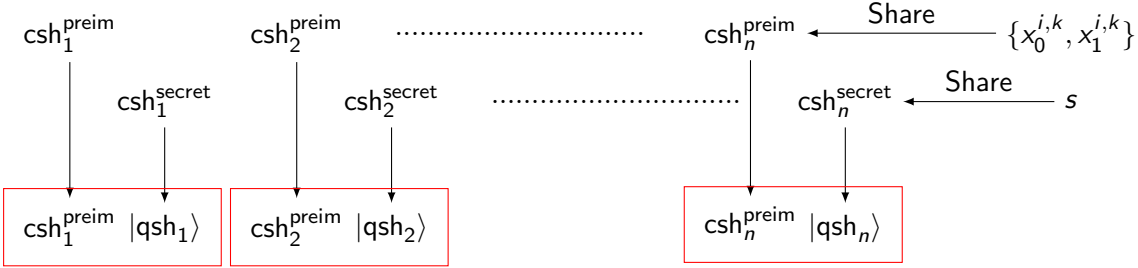

- Authorized set of parties can reconstruct preimages.

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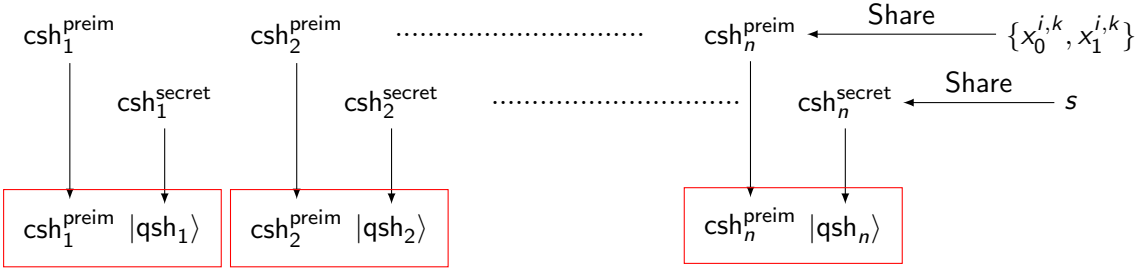

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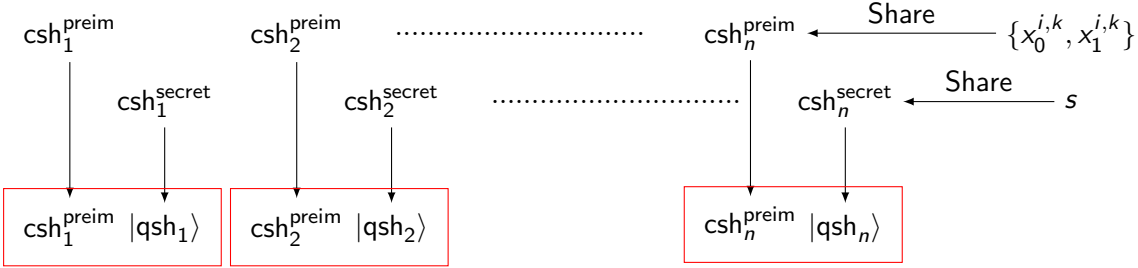

- Authorized set of parties can reconstruct preimages.
- Preimages are hidden from adversary holding an unauthorized set of shares.
- Problem: Classical part of each share does not get deleted. Adversary can accumulate an authorized set of classical preimage shares by adaptively corrupting shares

Example attack - (3,4)-secret sharing

$$\mathsf{csh}_1 \quad \mathsf{csh}_2 \quad \mathsf{csh}_3 \quad \mathsf{csh}_4 \leftarrow \mathsf{Share}_{(3,4)} \left(\{ x_0^{i,k} \oplus x_1^{i,k} \} \right)$$

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cert_1

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- Scheme is secure if it is $(4, 4)$ instead of $(3, 4)$.
- Idea: Make (3,4)-schemes secure by adding another layer to construction.

Secure $(3,4)$-secret sharing scheme

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cert_1

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Secure $(3,4)$-secret sharing scheme

$$\mathsf{csh}_1 \quad \mathsf{csh}_2 \quad \mathsf{csh}_3 \quad \mathsf{csh}_4 \leftarrow \mathsf{Share}_{(3,4)}\left(\{x_{0,2}^{i,k} \oplus x_{1,2}^{i,k}\}\right)$$

$$\cancel{|\mathsf{qsh}_1^1\rangle} \quad |\mathsf{qsh}_2^1\rangle \quad |\mathsf{qsh}_3^1\rangle \quad |\mathsf{qsh}_4^1\rangle \leftarrow \mathsf{Share}_{(3,4)}\left(\{x_{0,1}^{i,k} \oplus x_{1,1}^{i,k}\}\right)$$

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Final construction from one-way functions

- $\mathsf{Share}_{\mathbb{A}}$: Classical secret sharing scheme for access structure $\mathbb{A}$.

$$
\begin{array}{cccc}
\mathsf{csh}_1 & \dots & \dots & \mathsf{csh}_n \\
|\mathsf{qsh}_1^{n-1}\rangle & \ddots & & |\mathsf{qsh}_n^{n-1}\rangle \\
\vdots & & \ddots & \vdots \\
|\mathsf{qsh}_1^{0}\rangle & \dots & \dots & |\mathsf{qsh}_n^{0}\rangle
\end{array}
\quad
\begin{array}{l}
\leftarrow \mathsf{Share}_{\mathbb{A}}\left(\{x_{0,n}^{i,k}, x_{1,n}^{i,k}\}\right) \\
\leftarrow \mathsf{Share}_{\mathbb{A}}\left(\{x_{0,n-1}^{i,k}, x_{1,n-1}^{i,k}\}\right) \\
\leftarrow \mathsf{Share}_{\mathbb{A}}\left(\{x_{0,1}^{i,k}, x_{1,1}^{i,k}\}\right) \\
\leftarrow \mathsf{Share}_{\mathbb{A}}(s)
\end{array}
$$

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\vdots & & \ddots & \vdots & \leftarrow \mathsf{Share}_{\mathbb{A}}\left(\{x_{0,1}^{i,k}, x_{1,1}^{i,k}\}\right) \\
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\end{array}$$

- Problem: The size of the secret being shared at each level is a multiple of the size of the shares in the previous level.

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- Problem: The size of the secret being shared at each level is a multiple of the size of the shares in the previous level.
- Solution: Use a PRF to generate the preimages, and share the PRF key at each level

Final construction from one-way functions

- $\mathsf{Share}_{\mathbb{A}}$: Classical secret sharing scheme for access structure $\mathbb{A}$.

csh_1	$\dots$	$\dots$	csh_n	$\leftarrow \mathsf{Share}_{\mathbb{A}}(k_n)$	
$\lvert\mathsf{qsh}_1^{n-1}\rangle$	$\ddots$		$\lvert\mathsf{qsh}_1^{n-1}\rangle$	$\leftarrow \mathsf{Share}_{\mathbb{A}}(k_{n-1})$	$\{x_{b,n-1}^{i,k}\} = \{\mathsf{PRF}(k_n, b\|\|i\|\|k)\}$
$\vdots$		$\ddots$	$\vdots$	$\leftarrow \mathsf{Share}_{\mathbb{A}}(k_1)$	$\{x_{b,1}^{i,k}\} = \{\mathsf{PRF}(k_2, b\|\|i\|\|k)\}$
$\lvert\mathsf{qsh}_1^{0}\rangle$	$\dots$	$\dots$	$\lvert\mathsf{qsh}_n^{0}\rangle$	$\leftarrow \mathsf{Share}_{\mathbb{A}}(s)$	$\{x_{b,0}^{i,k}\} = \{\mathsf{PRF}(k_1, b\|\|i\|\|k)\}$

- Problem: The size of the secret being shared at each level is a multiple of the size of the shares in the previous level.
- Solution: Use a PRF to generate the preimages, and share a seperate PRF key at each level.

Summary of first result

Theorem 1

There exists a secret sharing scheme satisfying **adaptive publicly verifiable deletion** *for any monotone access structure* $\mathbb{A}$ *using* $O(n \cdot m)$ *qubits per share assuming:*

- *OWFs*
- *classical (information-theoretic) secret sharing scheme for access structure* $\mathbb{A}$ *(with share size* m*)*

Secret sharing with smaller qubit complexity

- Construction from OWFs required $O(n \cdot (\text{classical share size}))$ qubits for n parties.

Secret sharing with smaller qubit complexity

- Construction from OWFs required $O(n \cdot (\text{classical share size}))$ qubits for n parties.
- Can we eliminate dependence on n?

Construction from compute-and-compare obfuscation

Starting point for construction:

- $\mathsf{csh}_1, \ldots, \mathsf{csh}_n \leftarrow \mathsf{Share}(s)$

$$|\mathsf{qsh}_i\rangle := \bigotimes_{k \in [m]} \left(|x_0^{i,k}\rangle + (-1)^{\mathsf{csh}_i[k]} |x_1^{i,k}\rangle \right)$$

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  - Need to hide $x_0 \oplus x_1$ for valid deletion.

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$\mathsf{Rec}(\{d_{i,k}\}_{i\in A})$

- Hardcode preimages $\{x_0^{i,k}, x_1^{i,k}\}_{i\in[n],k\in[m]}$

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- Output $\mathsf{Reconstruct}(\{\mathsf{csh}'_i\}_{i\in A})$

Instantiating the reconstruction oracle

- Consider the compute-and-compare program:

$$\mathsf{CC}[P, \mathsf{lock}, z](x) := \begin{cases} z & P(x) = \mathsf{lock} \\ \perp & \text{otherwise.} \end{cases}$$

Instantiating the reconstruction oracle

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- **Functionality**: $\widetilde{\mathsf{CC}} \leftarrow \mathsf{CC.Obf}(\mathsf{CC})$, where $\widetilde{\mathsf{CC}}(x) = \mathsf{CC}(x)$

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- Can be constructed assuming LWE [WZ17]
- Idea: Set $P = \mathsf{Rec}$ and $\mathsf{lock} = z =$ secret

Secret sharing from compute-and-compare obfuscation

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- Compute $\mathsf{csh}'_i[k] = d_{i,k} \cdot (x_0^{i,k} \oplus x_1^{i,k})$
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- Solution: Use a uniform value independent of s as the lock

Secret sharing from compute-and-compare obfuscation

Share(s):

- Generate classical shares $\{\mathsf{csh}_i\}_{i\in[n]} \leftarrow$ ~~Share(s)~~ Share(lock) (lock is uniform)
- Generate corresponding quantum shares $\{|\mathsf{qsh}_i\rangle\}_{i\in[n]} \leftarrow \{\mathsf{csh}_i\}_{i\in[n]}$.
- ~~$\widetilde{\mathsf{Rec}} \leftarrow \mathsf{Obf}(\mathsf{CC}[\mathsf{Rec}, s, s])$~~ $\widetilde{\mathsf{Rec}} \leftarrow \mathsf{Obf}(\mathsf{CC}[\mathsf{Rec}, \mathsf{lock}, s])$

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- Output Reconstruct$(\{\mathsf{csh}'_i\}_{i\in A})$

- Problem: The secret s is not unpredictable.
- Solution: Use a uniform value independent of s as the lock

Summary of second result

Theorem 2

There exists a secret sharing scheme with **adaptive publicly verifiable deletion** *for any monotone access structure* $\mathbb{A}$ *using* $O(m)$ *qubits per share assuming:*

- *LWE*
- *classical (information-theoretic) secret sharing scheme for access structure* $\mathbb{A}$

Open problems

- Secret sharing with PVD from weaker assumptions than OWF (e.g., from hard quantum planted problems [KNY23])?
- No-signaling security (with PVD) without sub-exponentially secure OWF?
- **Information-theoretic** secret sharing with adaptive certified deletion for any monotone access structure in the privately verifiable setting?

Thank You!