Multi-Key Homomorphic Secret Sharing

EUROCRYPT 2025

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Sacha Servan-Schreiber

MIT

Abhishek Jain

NTT Research

JHU

Homomorphic Secret Sharing (HSS)

[Boyle-Gilboa-Ishai'16]

x

y

Homomorphic Secret Sharing (HSS)

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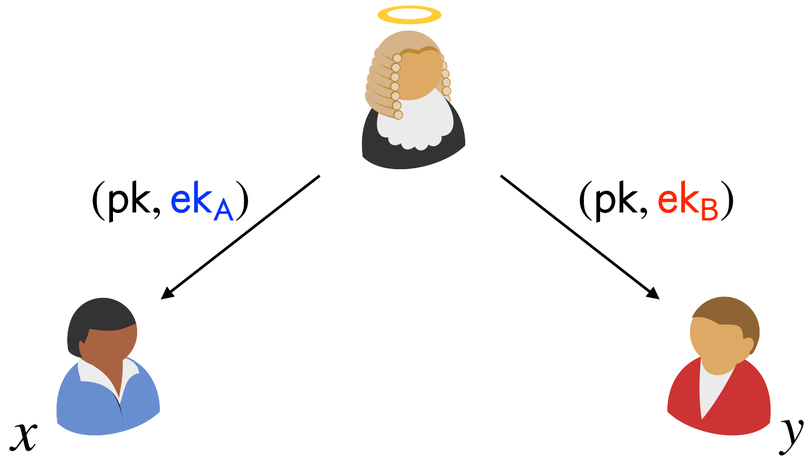

Homomorphic Secret Sharing (HSS)

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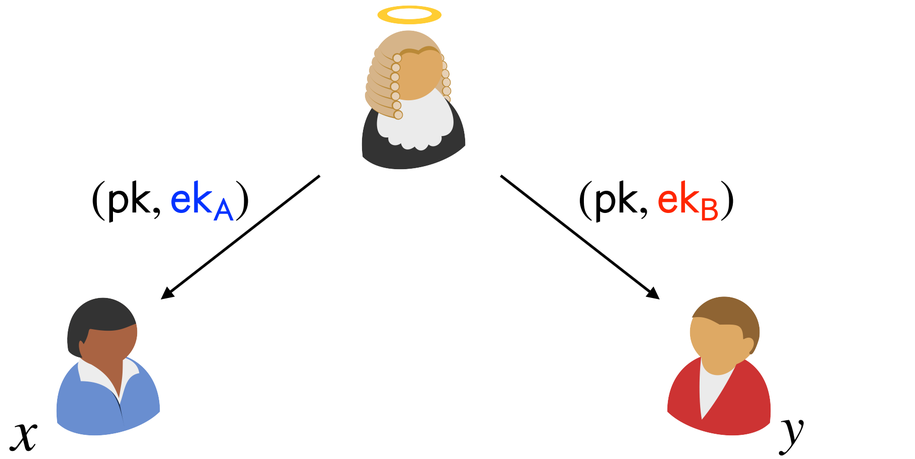

$ct_x \leftarrow \text{Encrypt}(pk, x)$

$\text{Encrypt}(pk, y) \rightarrow ct_y$

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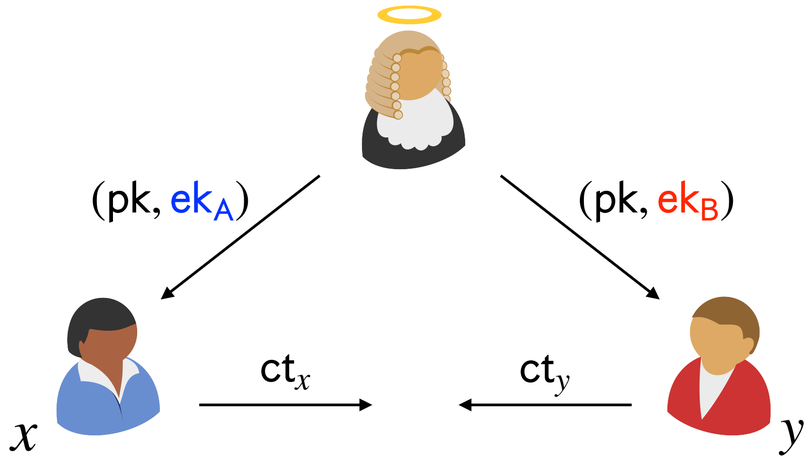

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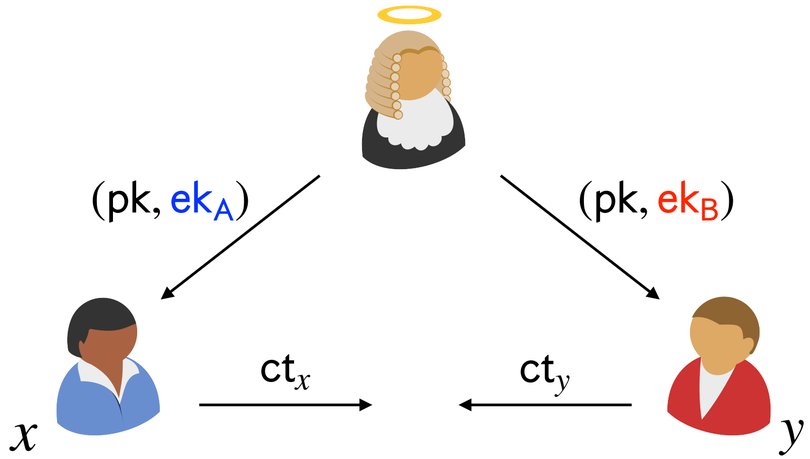

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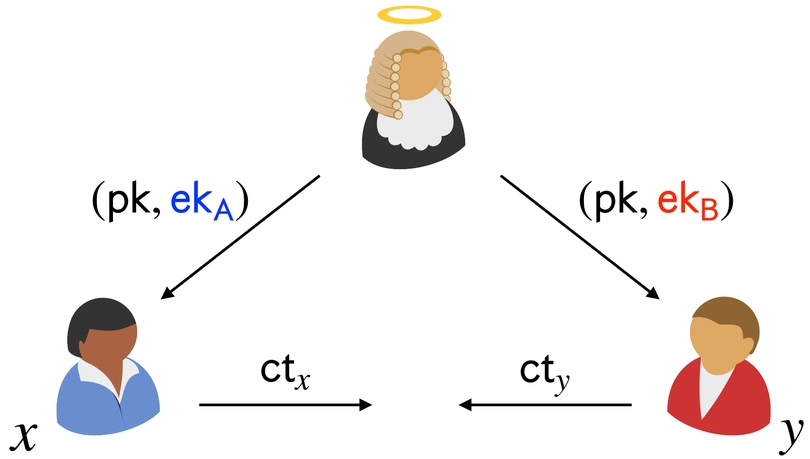

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Correctness $\quad z_A + z_B = C(x, y)$

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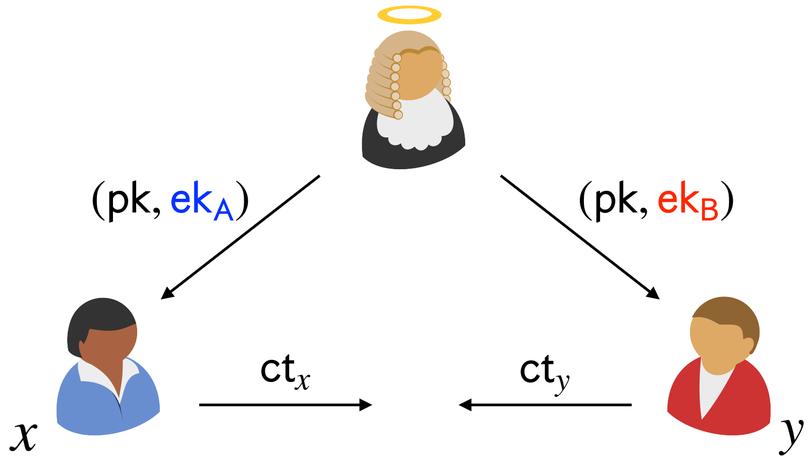

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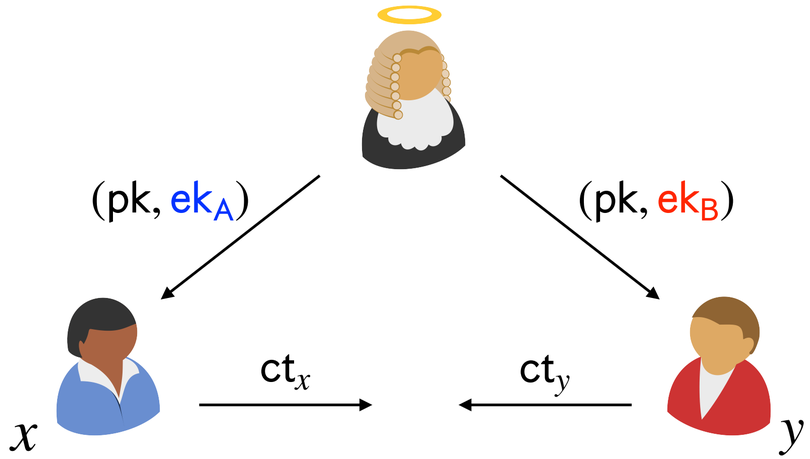

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Analogue of Threshold FHE

Correctness $z_A + z_B = C(x, y)$

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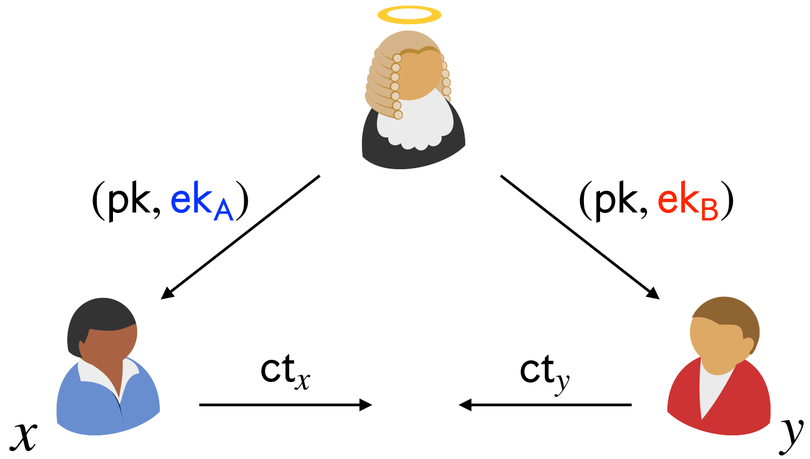

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Analogue of Threshold FHE

Only known from Lattice assumptions

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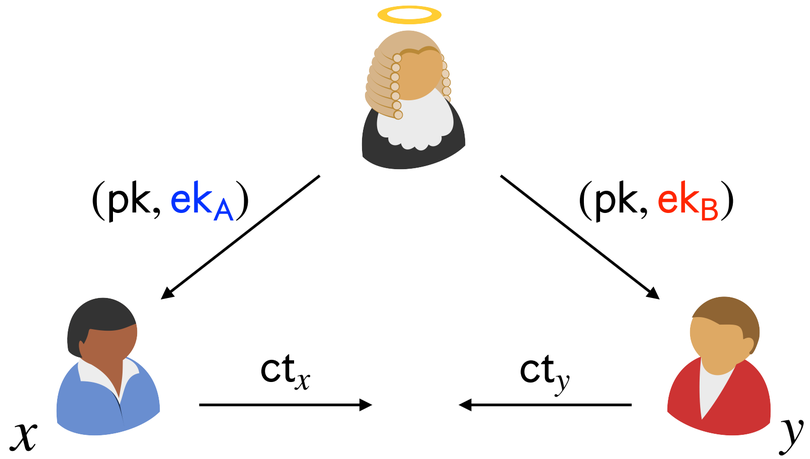

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HSS can be constructed from group-based assumptions

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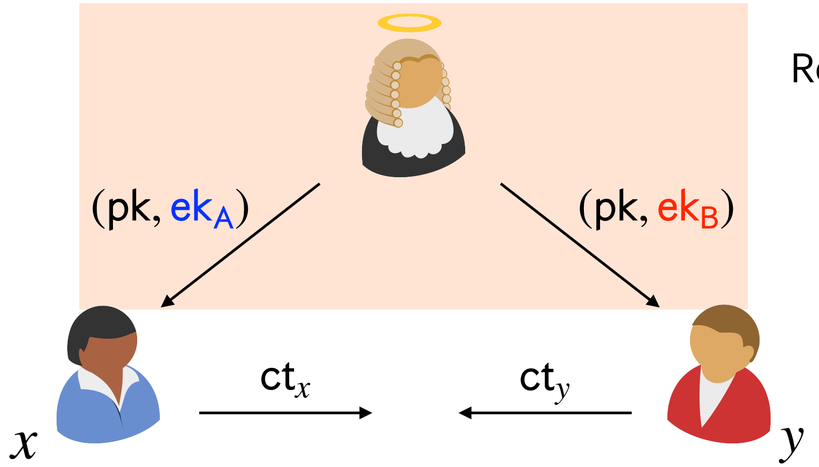

Replace correlated setup with CRS

$ct_x \leftarrow \text{Encrypt}(pk, x)$

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Analogue of Threshold FHE

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Multi-Key Homomorphic Secret Sharing

Common Reference String

x

y

Multi-Key Homomorphic Secret Sharing

Common Reference String

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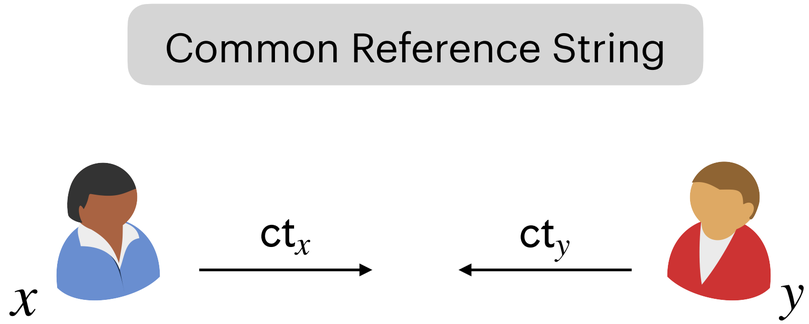
y

$(\mathsf{ct}_x, \mathsf{st}_A) \leftarrow \mathsf{Encode}(x)$

$\mathsf{Encode}(y) \rightarrow (\mathsf{ct}_y, \mathsf{st}_B)$

Multi-Key Homomorphic Secret Sharing



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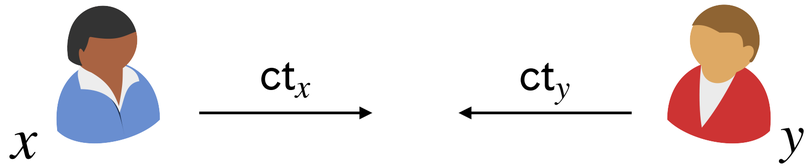

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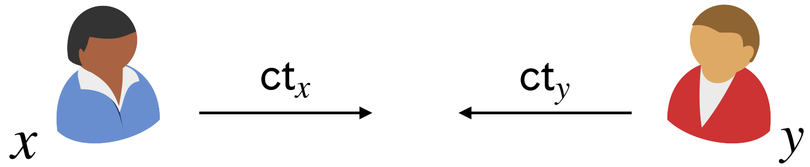

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Multi-Key Homomorphic Secret Sharing

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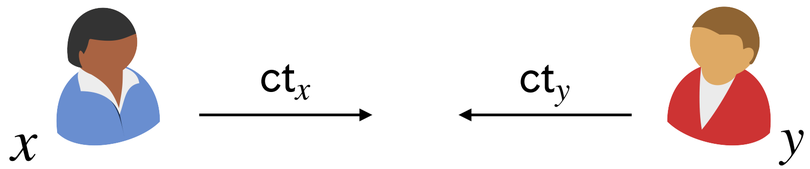

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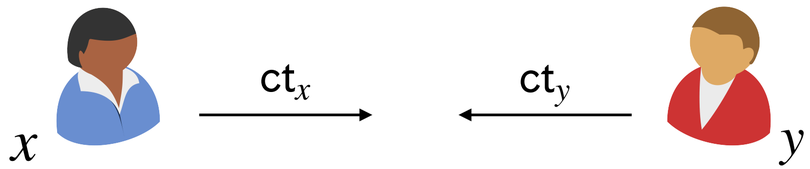

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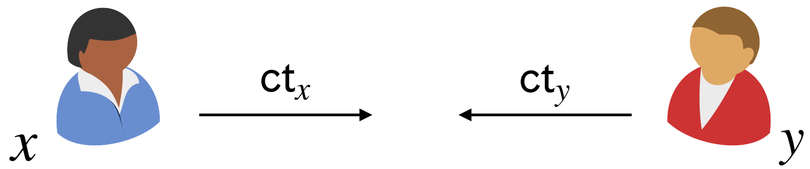

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Multi-Key HSS can be constructed from group-based assumptions

Multi-Key Homomorphic Secret Sharing

Two round secure computation with succinct communication

Public-key pseudorandom correlation functions

MPC with offline phase communication linear in number of parties

Attribute-based key exchange

Before Multi-Key HSS: From lattice-based assumptions

After Multi-Key HSS: From group-based assumptions

Outline

Applications

Our Results

Constructing Multi-Key HSS

Outline

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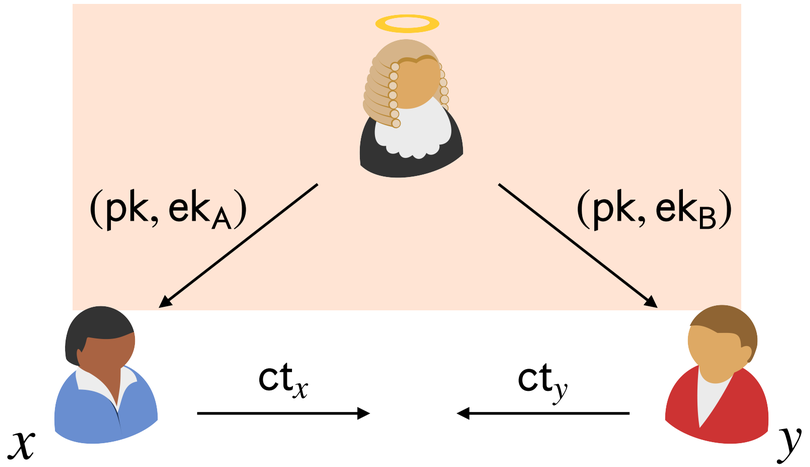
Our Results

Constructing Multi-Key HSS

Key Properties of Multi-Key HSS

HSS



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Multi-Key HSS

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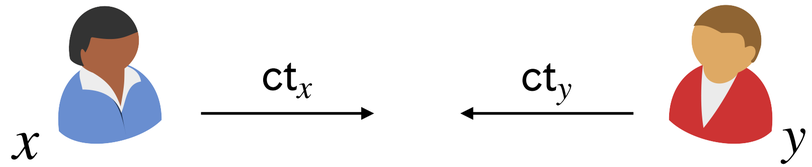

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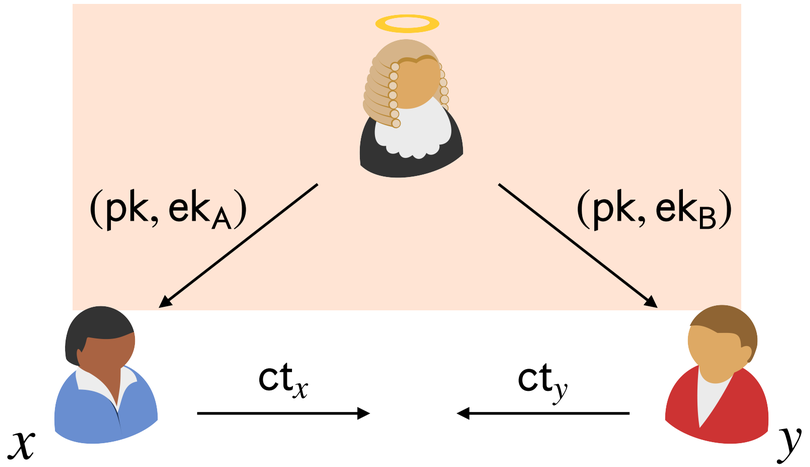
Reduces round complexity by avoiding correlated setup

Key Properties of Multi-Key HSS

HSS



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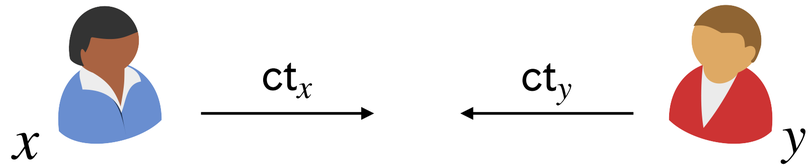

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Reusability of input encodings

Key Properties of Multi-Key HSS

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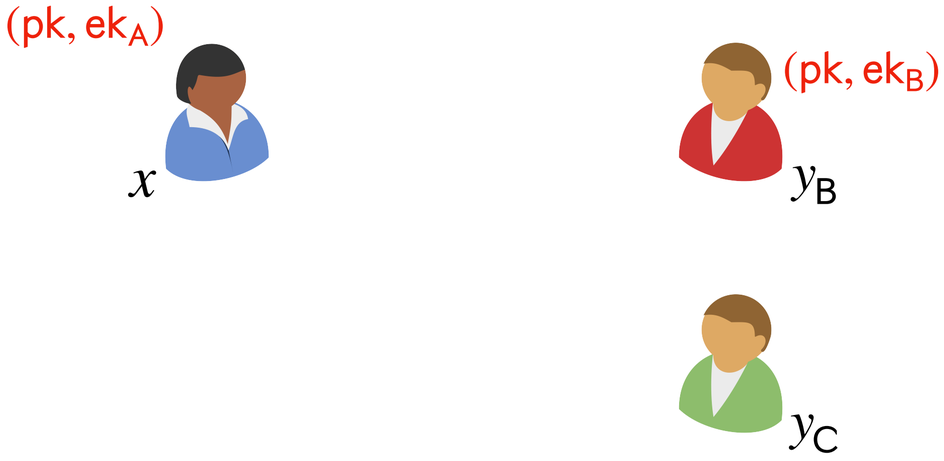

Multi-Key HSS

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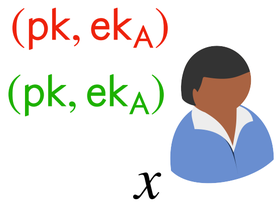

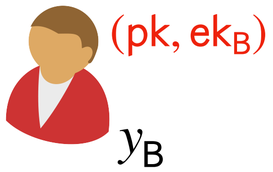

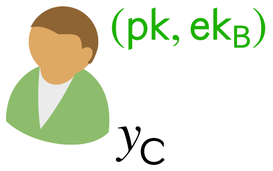

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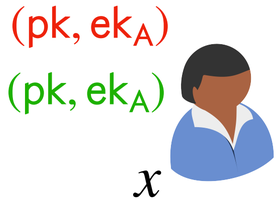

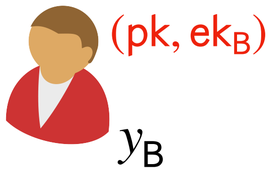

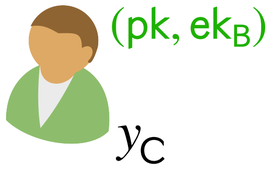

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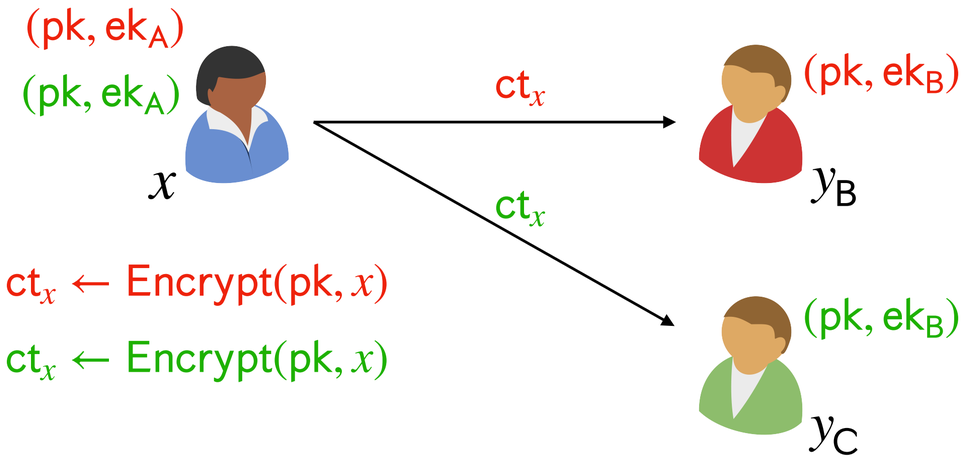

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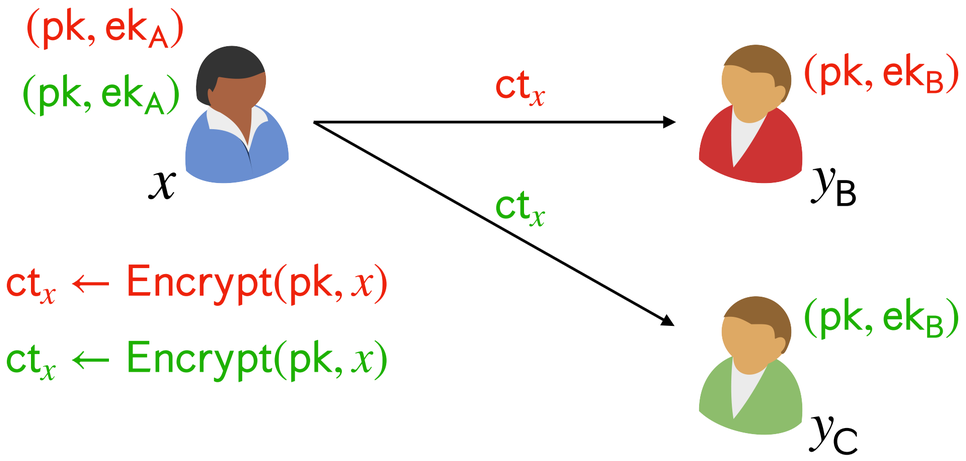

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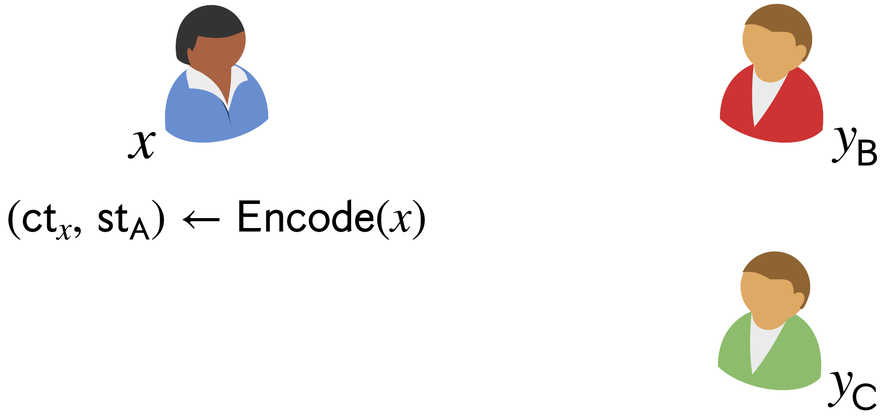

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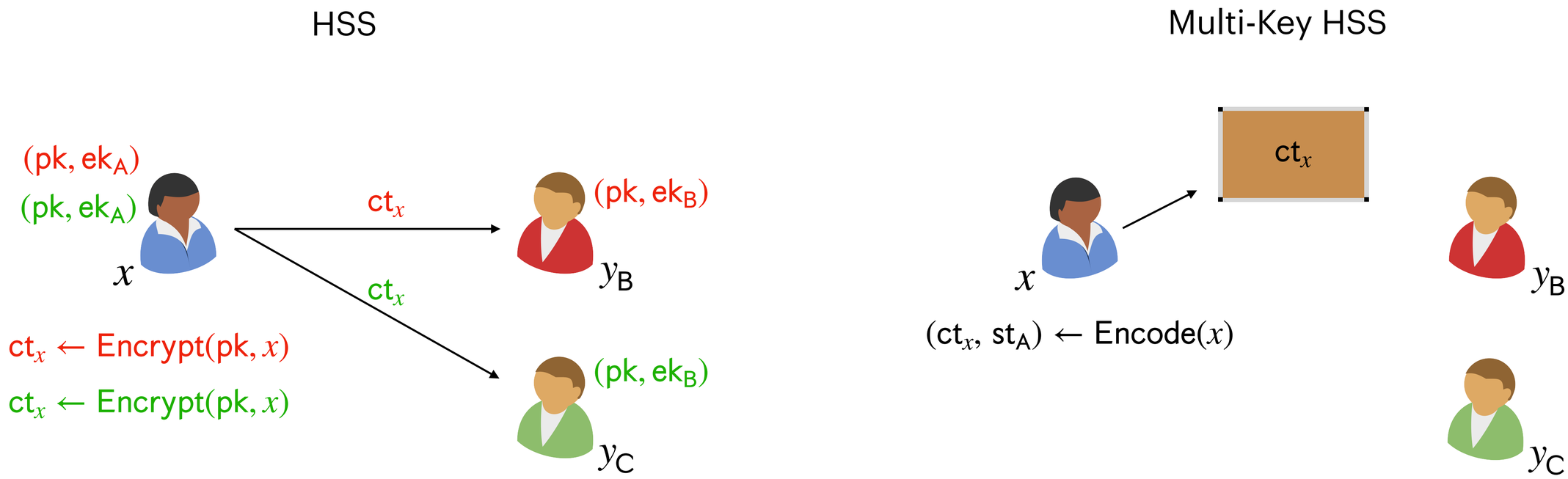

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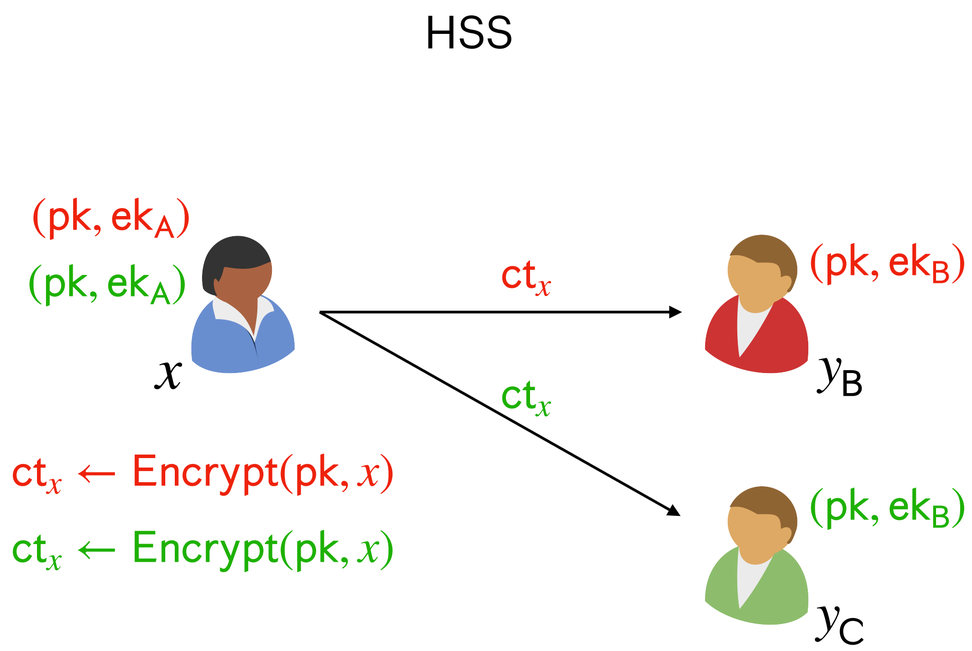

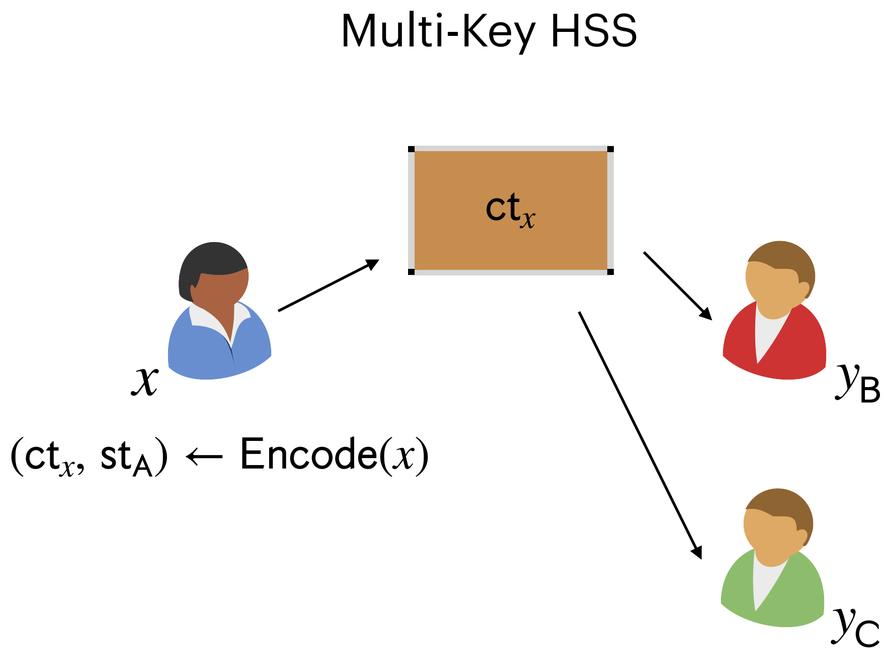

Reduces round complexity by avoiding correlated setup

Reusability of input encodings

Application 1: Succinct Two-round Secure Computation

Common Reference String

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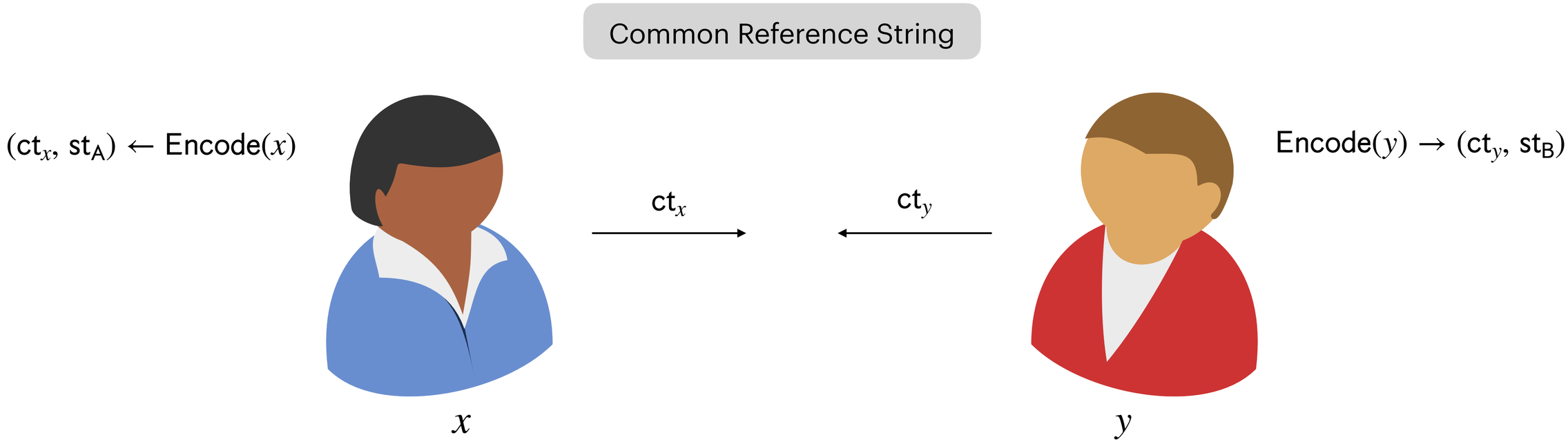

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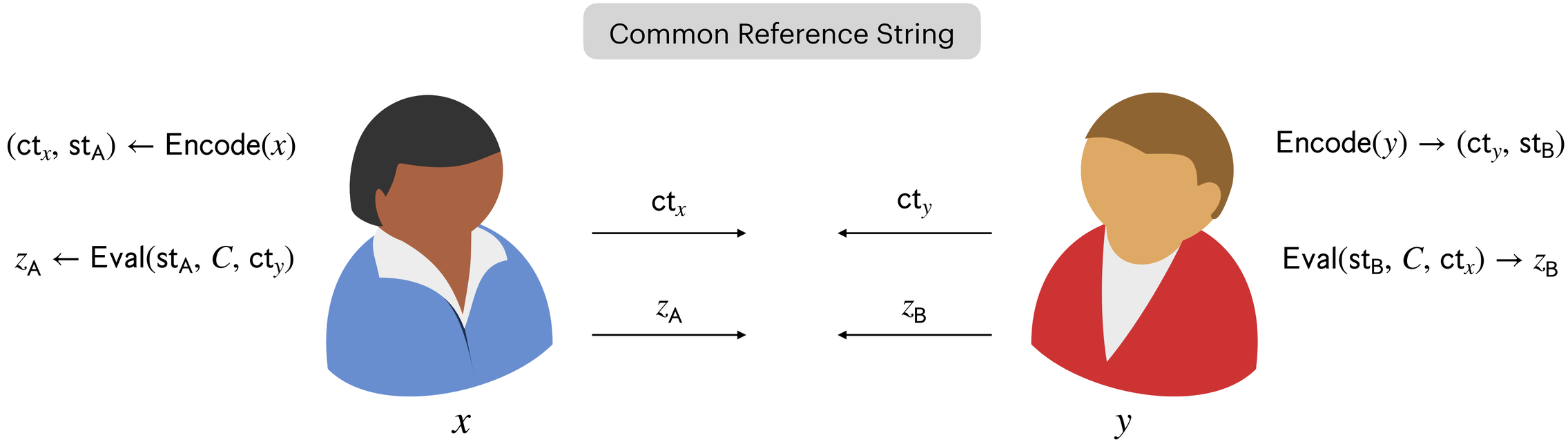

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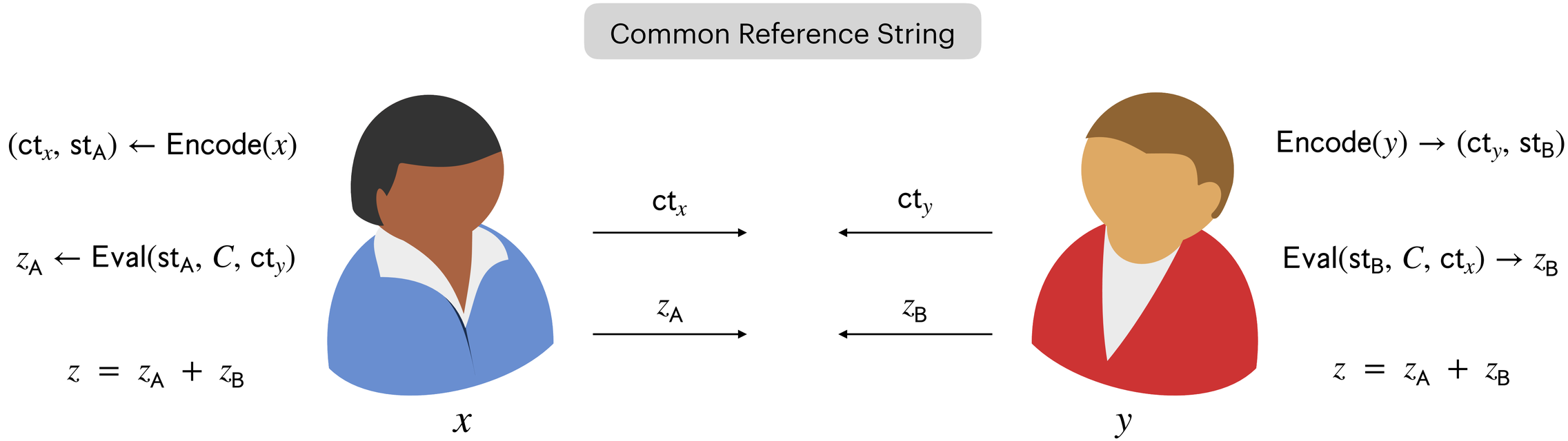

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Common Reference String

$(\mathsf{ct}_x, \mathsf{st}_A) \leftarrow \mathsf{Encode}(x)$

$z_A \leftarrow \mathsf{Eval}(\mathsf{st}_A, C, \mathsf{ct}_y)$

$z = z_A + z_B$

x

ct_x →

z_A →

← ct_y

← z_B

y

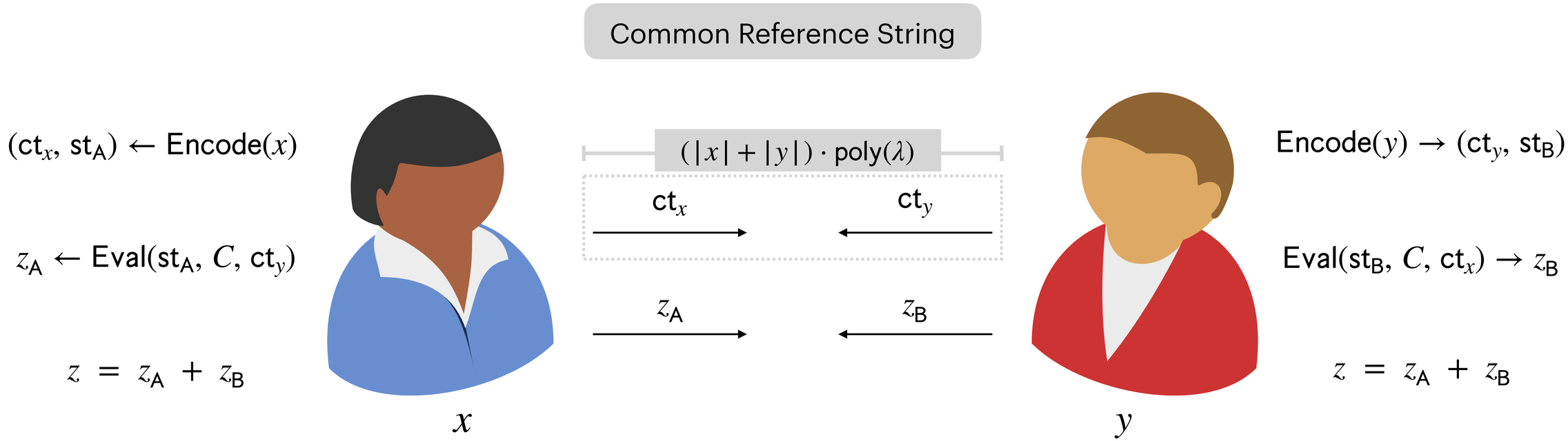
$\mathsf{Encode}(y) \rightarrow (\mathsf{ct}_y, \mathsf{st}_B)$

$\mathsf{Eval}(\mathsf{st}_B, C, \mathsf{ct}_x) \rightarrow z_B$

$z = z_A + z_B$

Communication is independent of evaluated circuit

Application 1: Succinct Two-round Secure Computation



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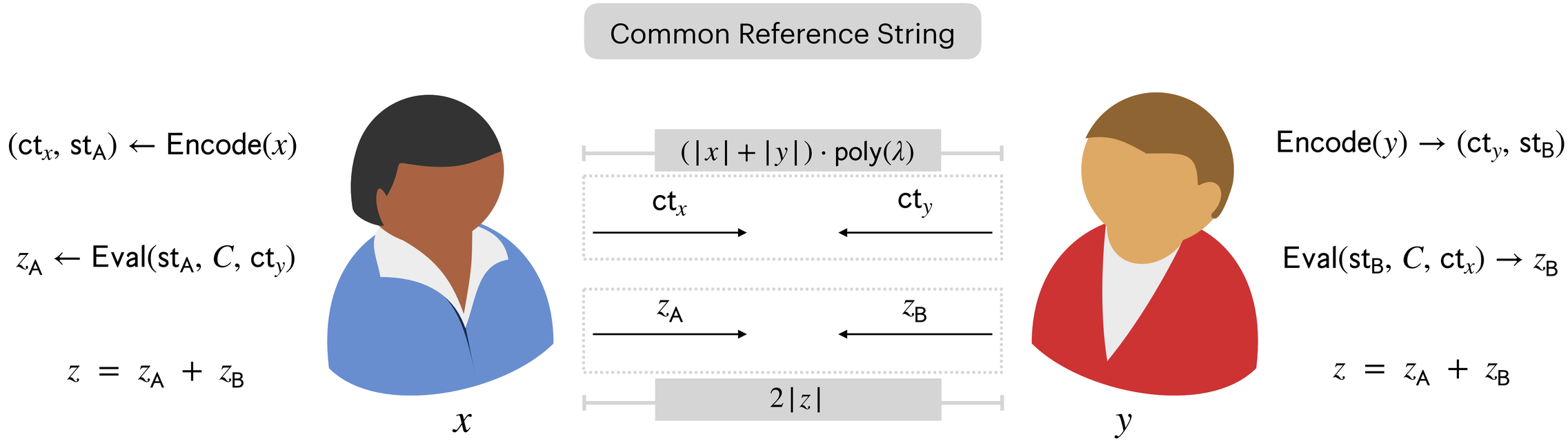
Application 1: Succinct Two-round Secure Computation



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Application 1: Succinct Two-round Secure Computation

Common Reference String

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$z_A \leftarrow \mathsf{Eval}(\mathsf{st}_A, C, \mathsf{ct}_y)$

$z = z_A + z_B$

x

$(|x| + |y|) \cdot \mathsf{poly}(\lambda)$

ct_x →

← ct_y

z_A →

← z_B

$2|z|$

y

$\mathsf{Encode}(y) \rightarrow (\mathsf{ct}_y, \mathsf{st}_B)$

$\mathsf{Eval}(\mathsf{st}_B, C, \mathsf{ct}_x) \rightarrow z_B$

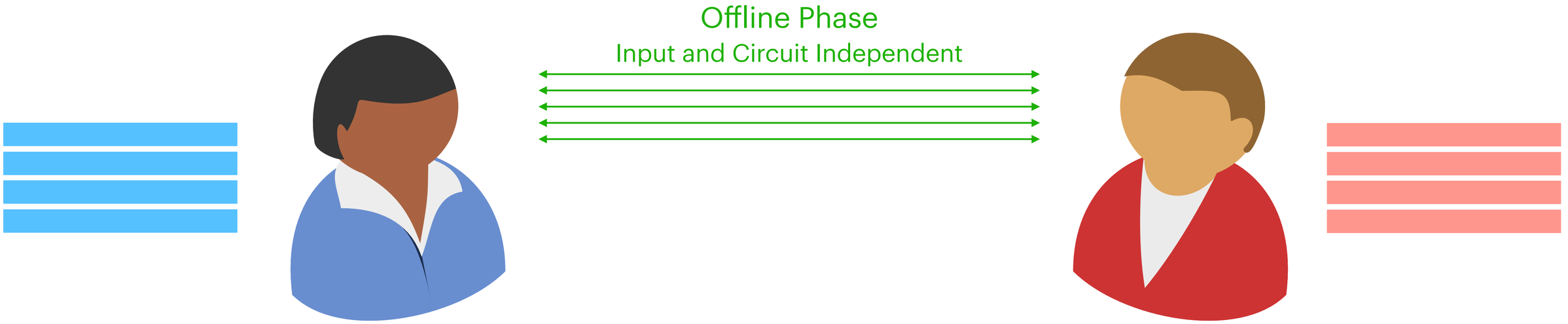
$z = z_A + z_B$

Communication is independent of evaluated circuit

Two-round protocol in the CRS model

Preprocessing Model for Secure Computation

Preprocessing Model for Secure Computation



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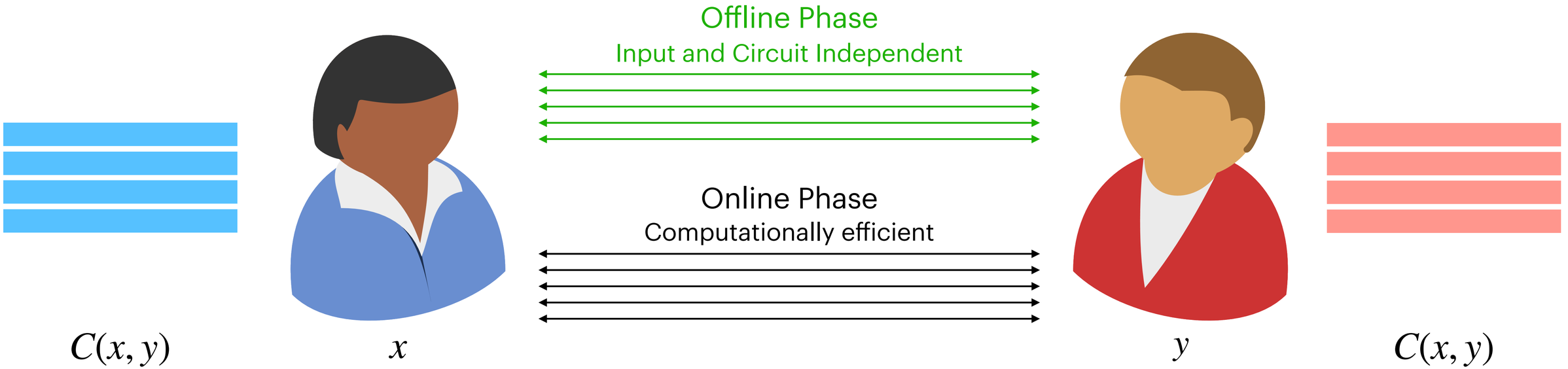
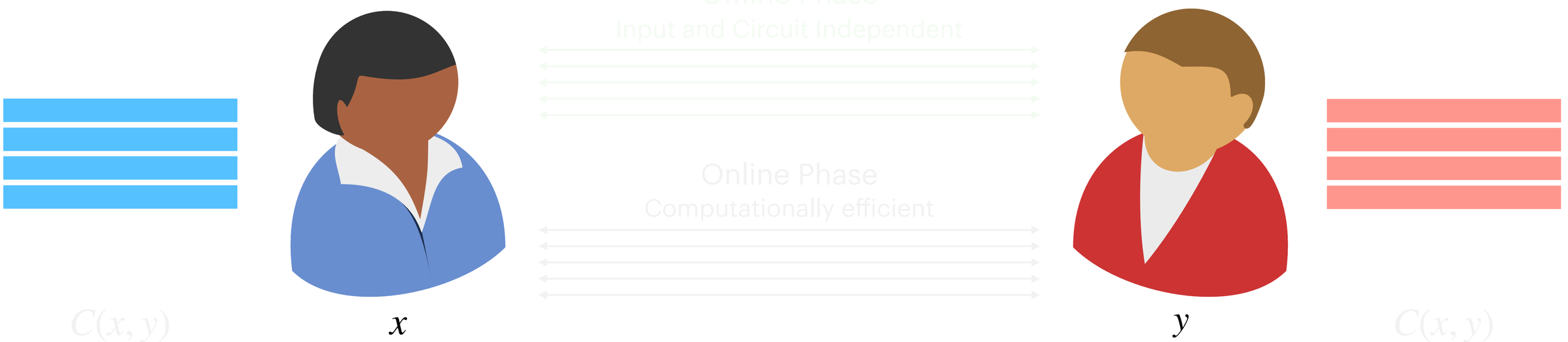
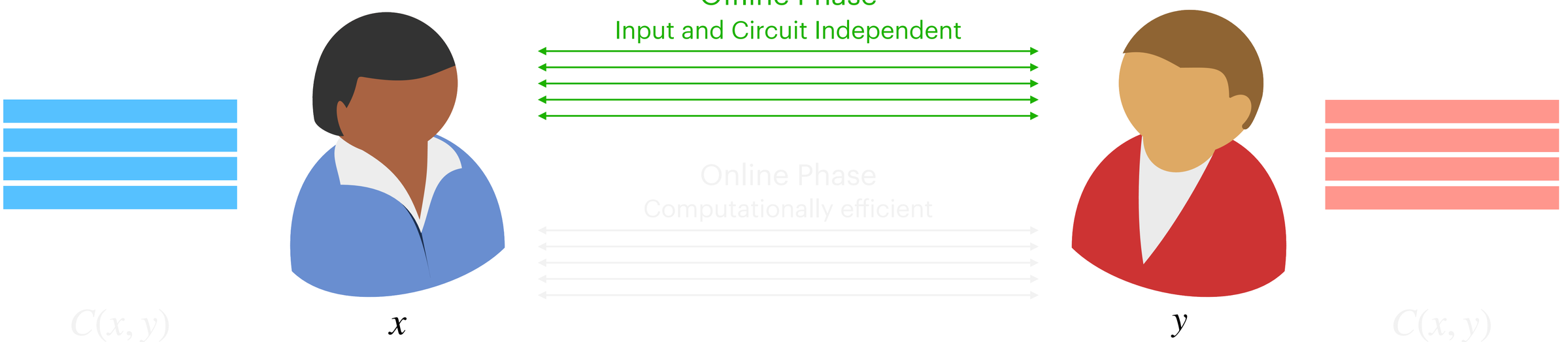


Preprocessing Model for Secure Computation



Size of preprocessing is linear in the size of circuit evaluated $|C|$

Preprocessing Model for Secure Computation



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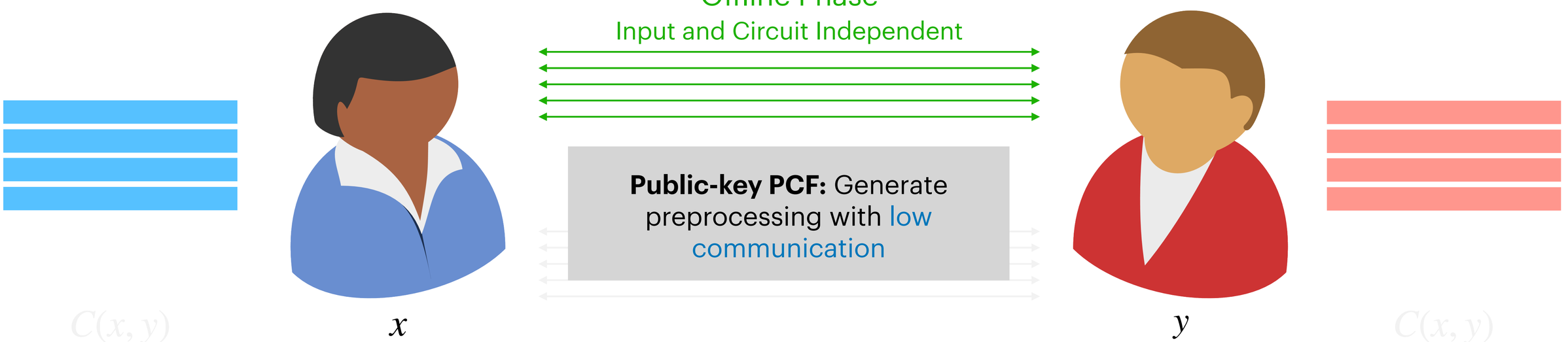
Preprocessing Model for Secure Computation



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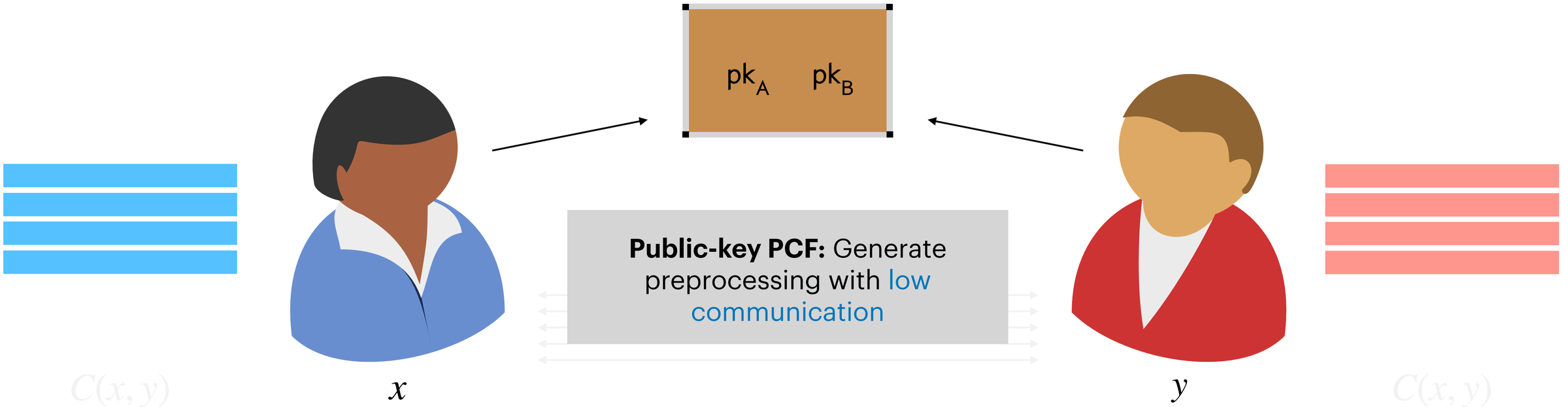
Preprocessing Model for Secure Computation



Public-key PCF: Generate preprocessing with low communication

Size of preprocessing is linear in the size of circuit evaluated $|C|$

Public-Key Pseudorandom Correlation Function (PCF)

[Orlandi-Scholl-Yakoubov'21] [Bui-Couteau-Meyer-Passalègue-Riahinia'24]

Public-Key Pseudorandom Correlation Function (PCF)

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$(\mathsf{pk}_A, \mathsf{sk}_A) \leftarrow \mathsf{KeyGen}(1^\lambda)$

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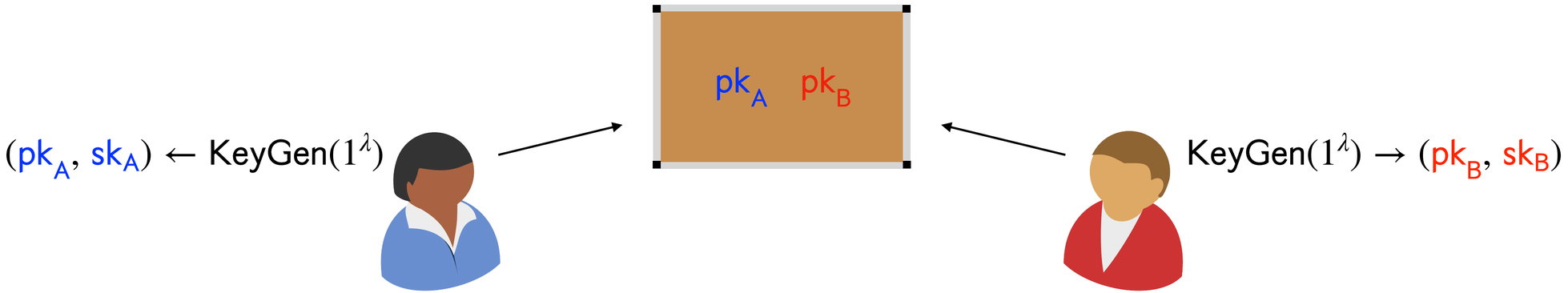

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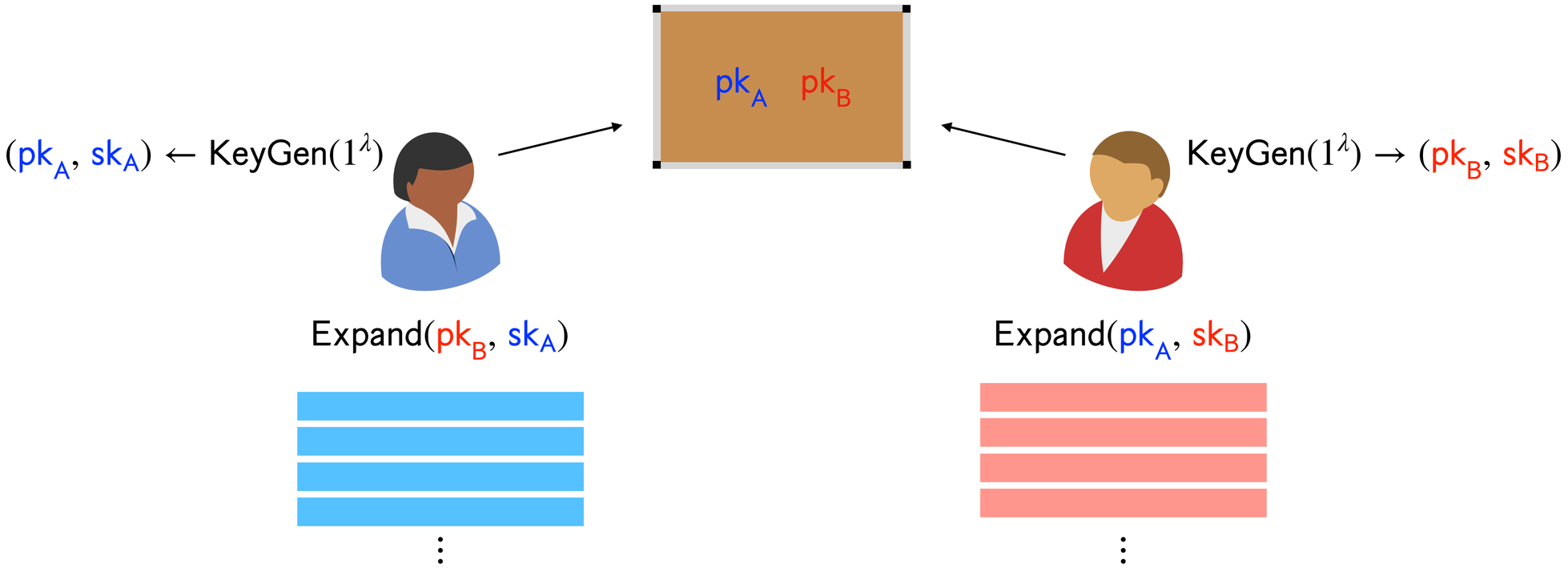

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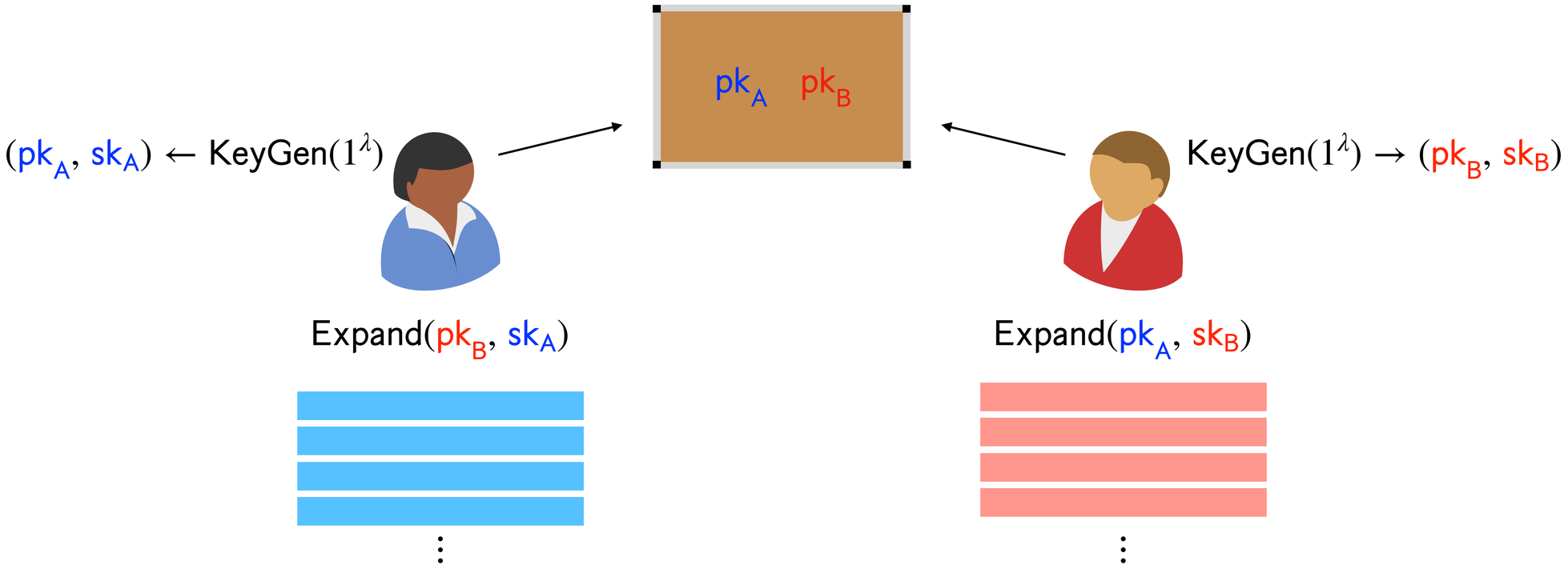
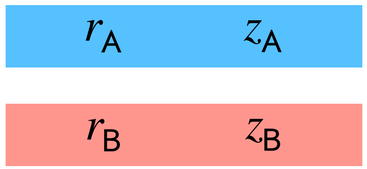


Additive Correlations:



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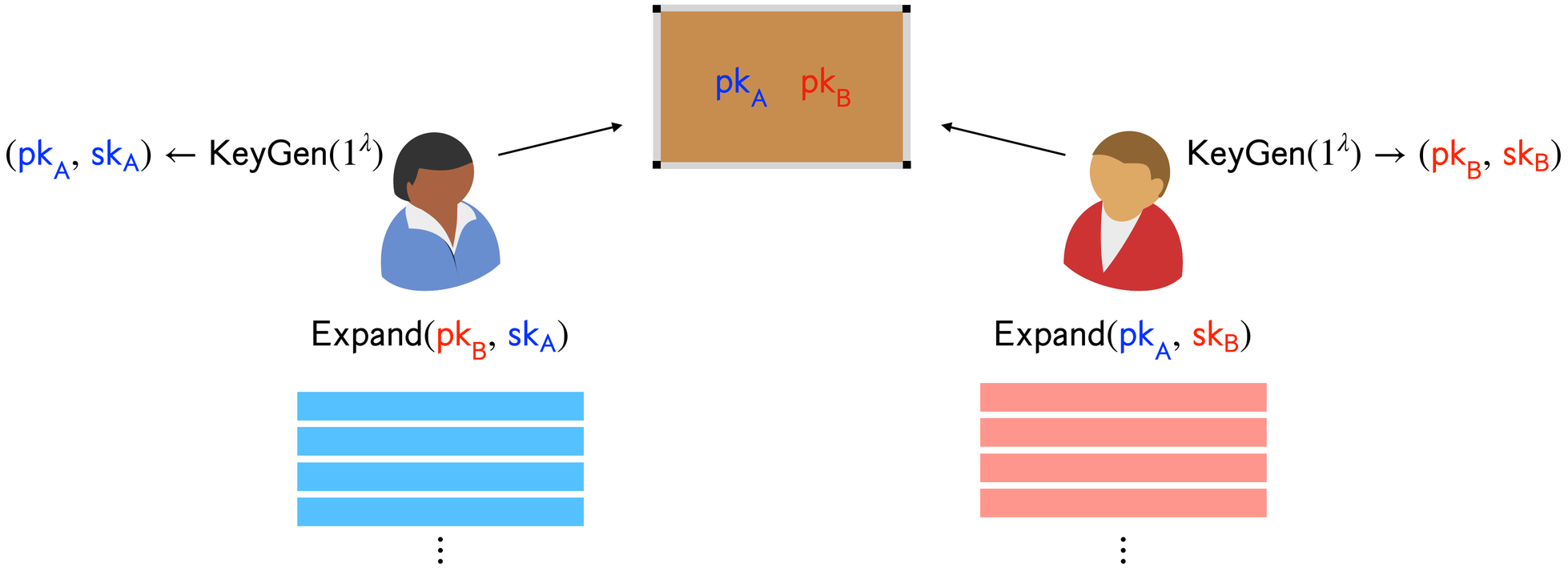
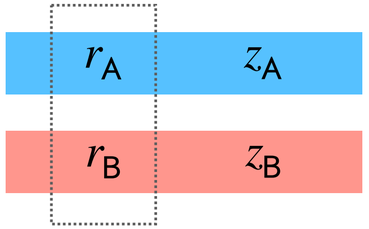


Private pseudorandom value

Additive Correlations:



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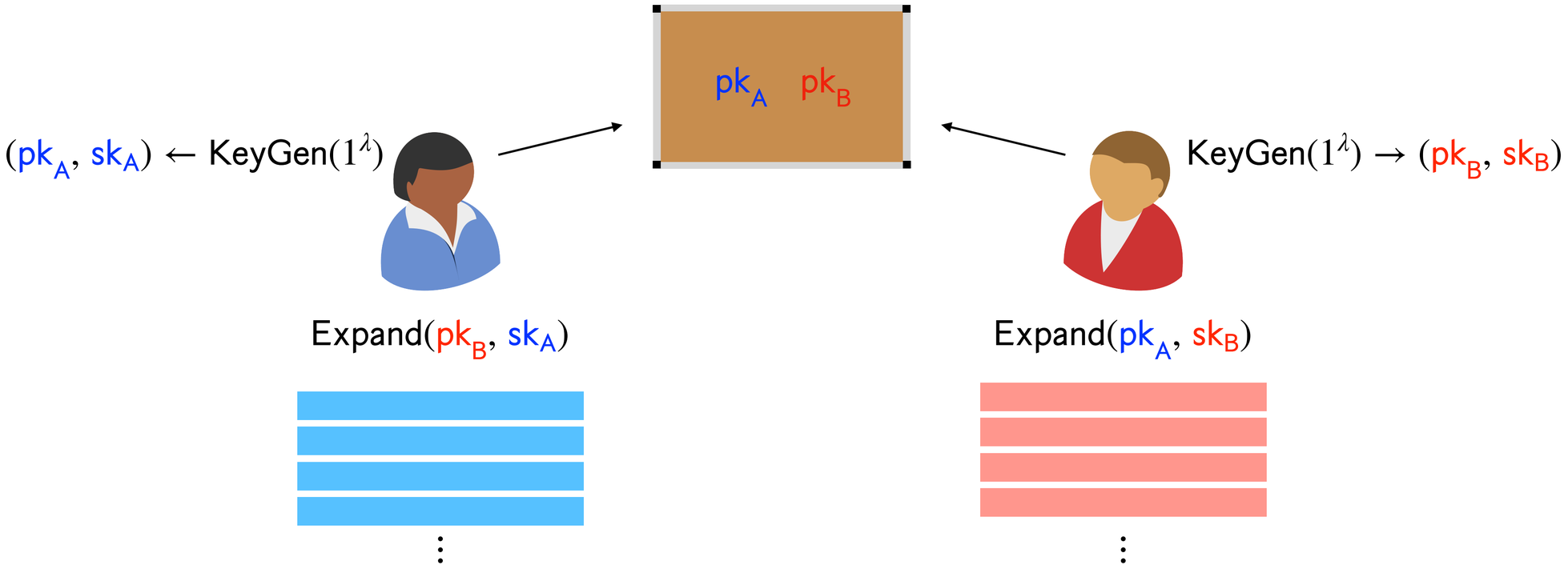
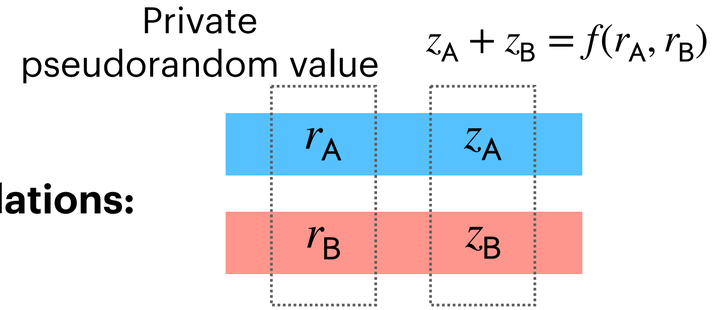


Additive Correlations:



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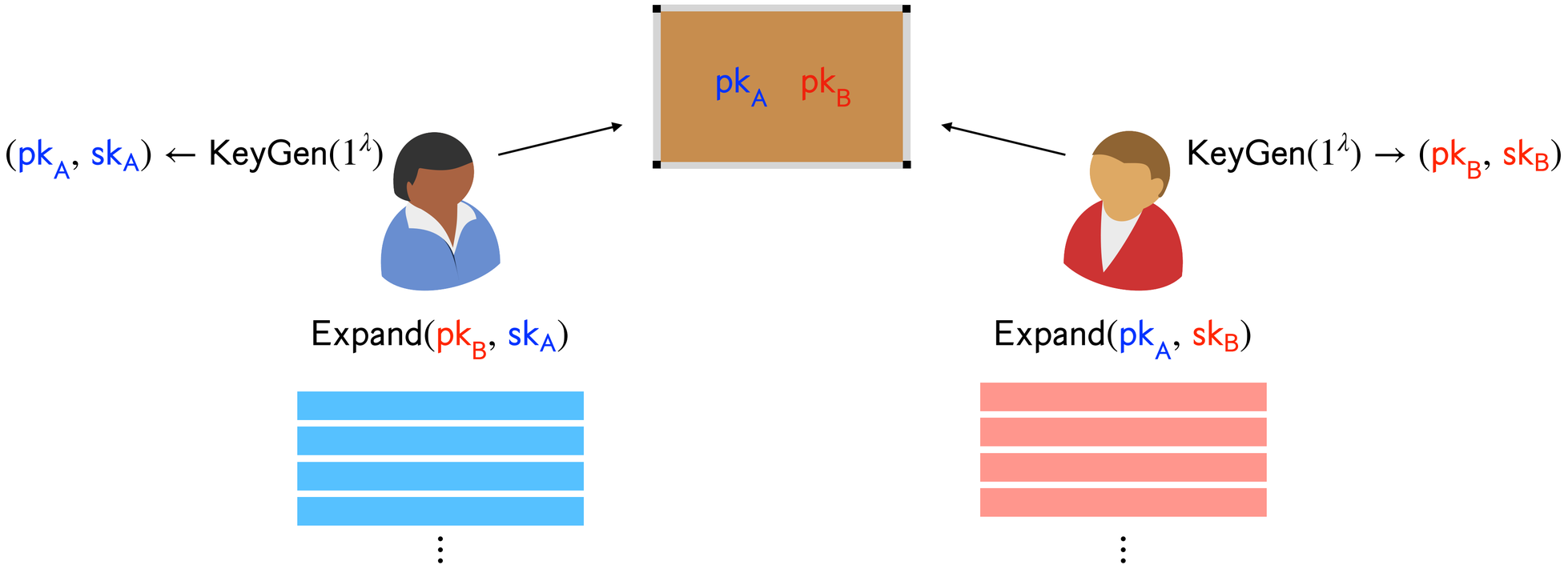
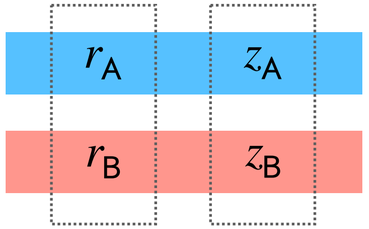


Captures most MPC correlations
e.g., correlated OT, OLE, and
Beaver triples

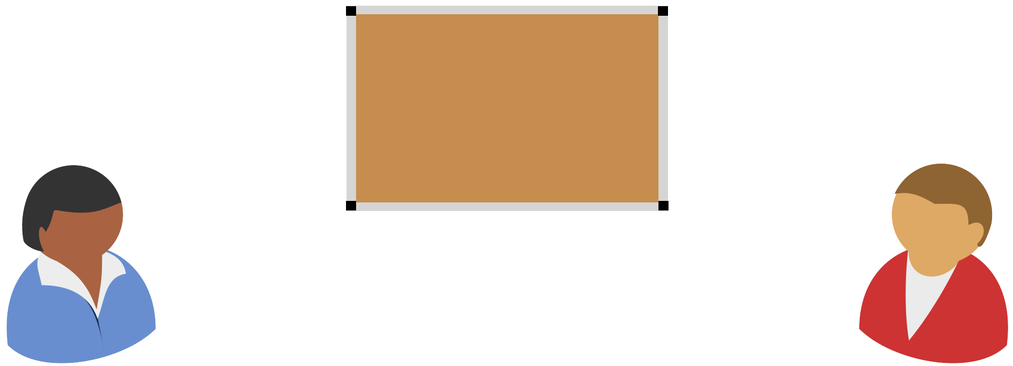
Additive Correlations:

Private
pseudorandom value

$z_A + z_B = f(r_A, r_B)$



Application 2: Public-Key PCFs for Additive Correlations



Application 2: Public-Key PCFs for Additive Correlations

$k_A \leftarrow \{0,1\}^\lambda$

$k_B \leftarrow \{0,1\}^\lambda$

Application 2: Public-Key PCFs for Additive Correlations

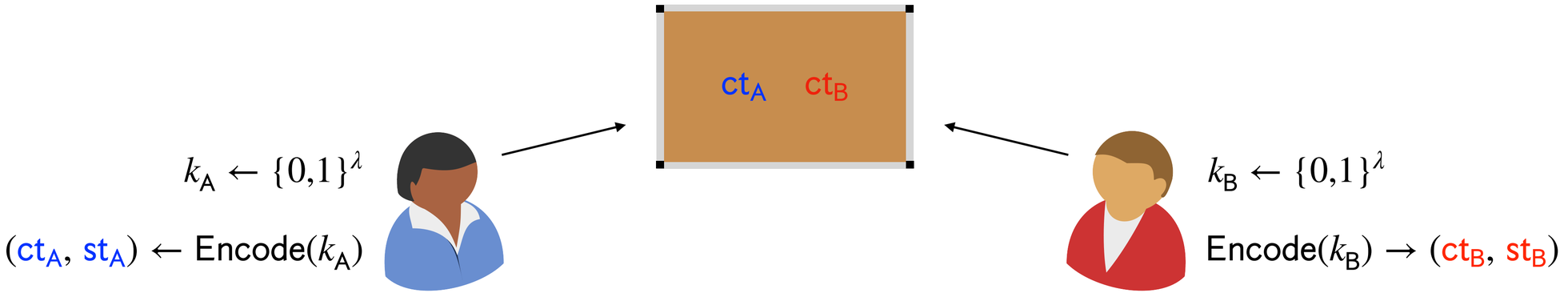
$k_A \leftarrow \{0,1\}^\lambda$

$(ct_A, st_A) \leftarrow \text{Encode}(k_A)$

$k_B \leftarrow \{0,1\}^\lambda$

$\text{Encode}(k_B) \rightarrow (ct_B, st_B)$

Application 2: Public-Key PCFs for Additive Correlations



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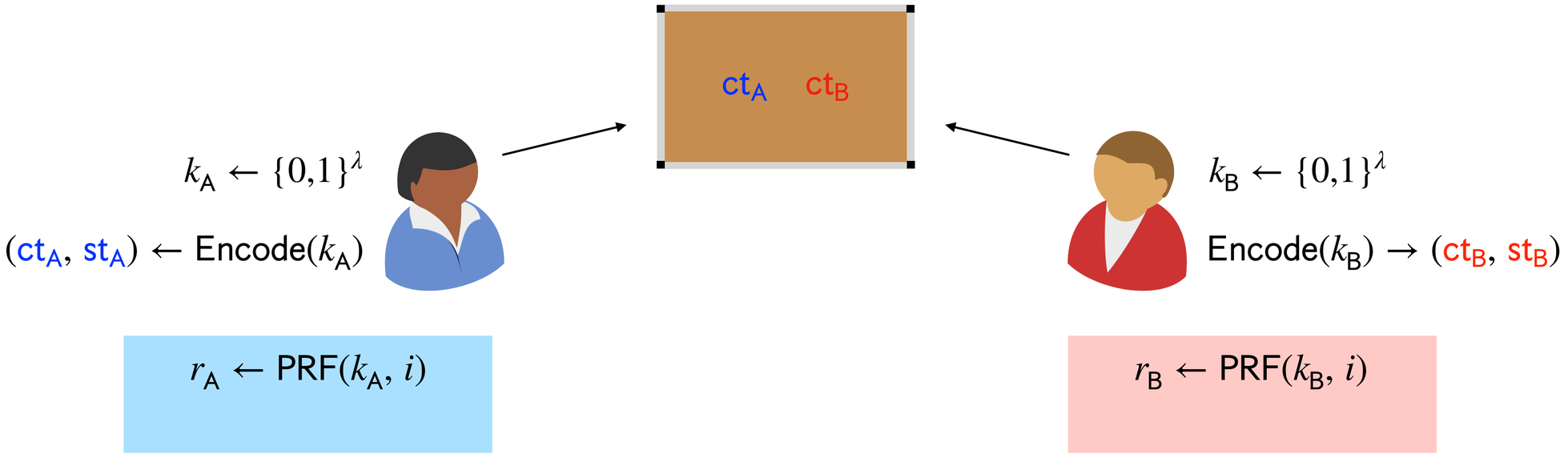

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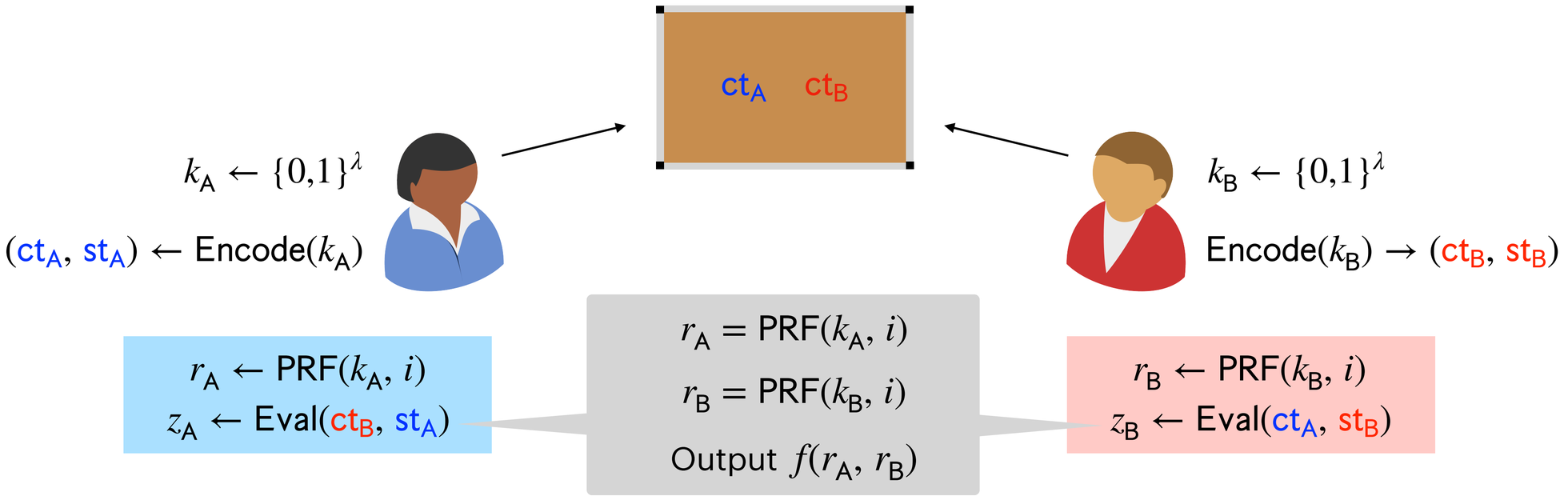

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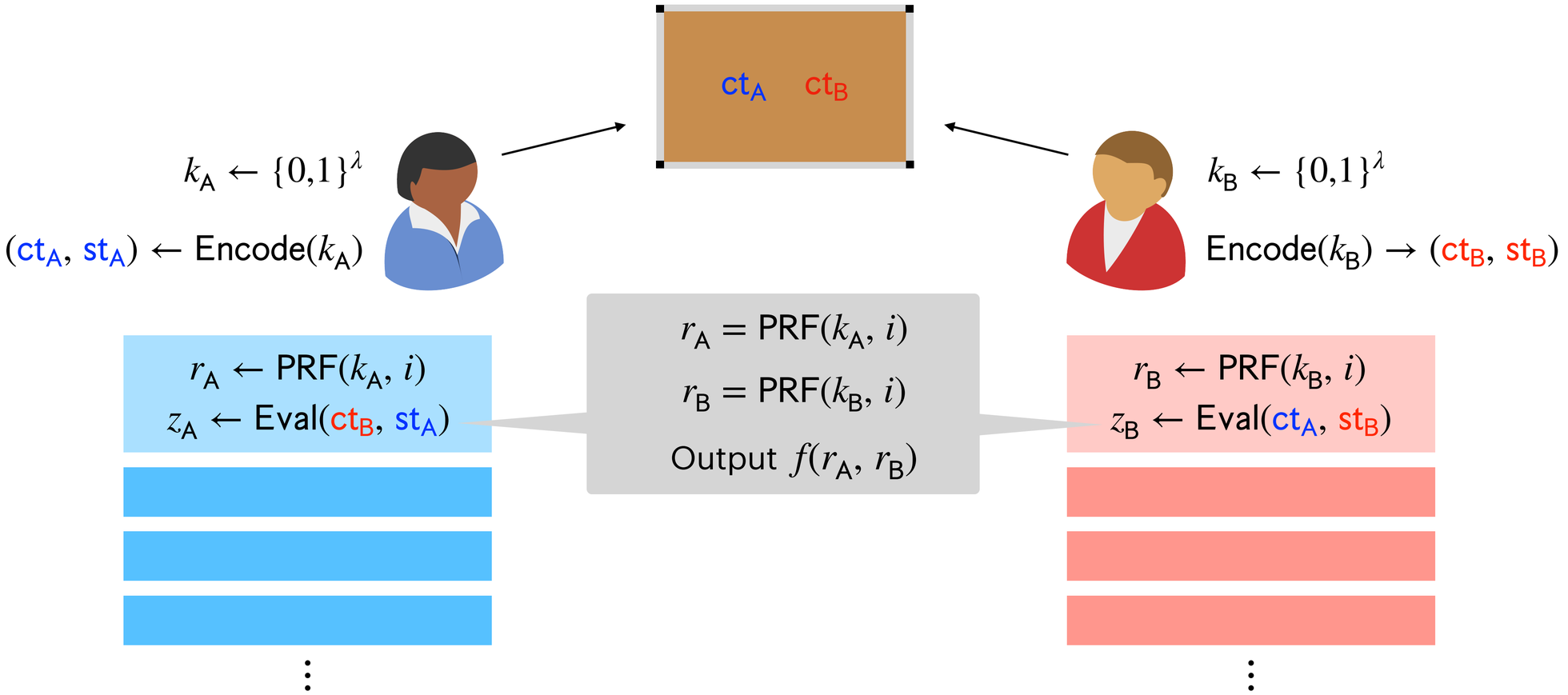

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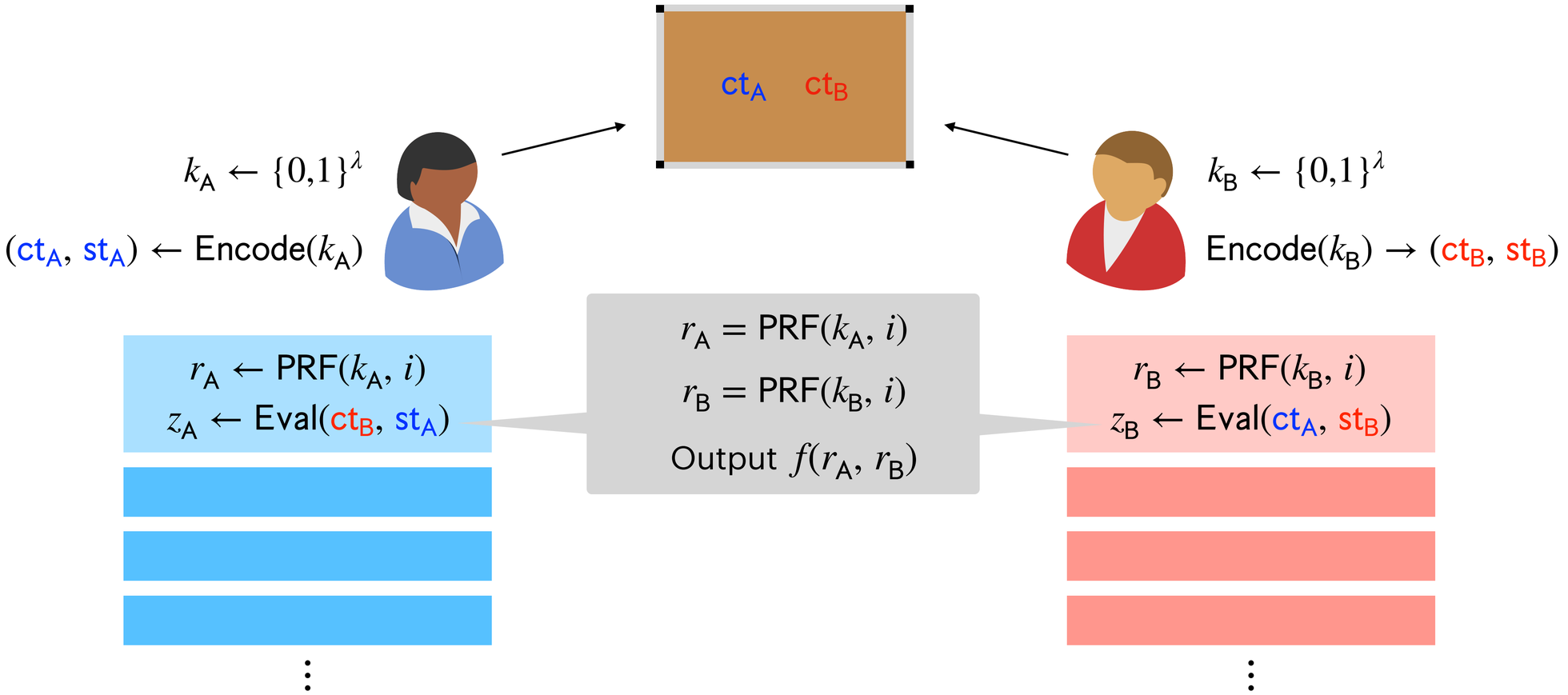

Reusability of input encodings $\Longrightarrow$ public-key can be used by multiple parties

Outline

Applications

Our Results

Constructing Multi-Key HSS

Our Results: Multi-Key HSS

Two-party multi-key HSS schemes for evaluating NC^1 functions

DDH

DCR

DDH over class groups

Previously known only from LWE and $i\mathcal{O}$ + DDH [Dodis-Halevi-Rothblum-Wichs'16]

Our Results: Multi-Key HSS

Two-party multi-key HSS schemes for evaluating NC^1 functions

	HSS Schemes from Prior Works (Require Correlated Setup)
DDH	[Boyle-Gilboa-Ishai'16]
DCR	[Orlandi-Scholl-Yakoubov'21] [Roy-Singh'21]
DDH over class groups	[Abram-Damgård-Orlandi-Scholl'22]

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Our Results: Multi-Key HSS

Two-party multi-key HSS schemes for evaluating NC^1 functions

Inverse polynomial correctness error

	HSS Schemes from Prior Works (Require Correlated Setup)
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Our Results: Multi-Key HSS

Two-party multi-key HSS schemes for evaluating NC^1 functions

		HSS Schemes from Prior Works (Require Correlated Setup)
Transparent setup	DDH	[Boyle-Gilboa-Ishai'16]
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Previously known only from LWE and $i\mathcal{O}$ + DDH [Dodis-Halevi-Rothblum-Wichs'16]

Our Results: **Applications** of Multi-Key HSS

Two-round succinct 2PC for NC^1 circuits in the CRS model

DDH

DCR

DDH over class groups

Previously known only from multi-key FHE [Mukherjee-Wichs'16]

Our Results: **Applications** of Multi-Key HSS

Two-round succinct 2PC for NC^1 circuits in the CRS model

DDH

DCR

DDH over class groups

Previously from group-based assumptions

3 round protocol in the CRS model

Previously known only from multi-key FHE [Mukherjee-Wichs'16]

Our Results: **Applications** of Multi-Key HSS

Two-round succinct 2PC for NC^1 circuits in the CRS model

DDH | DCR | DDH over class groups

Previously from group-based assumptions

3 round protocol in the CRS model

Previously known only from multi-key FHE [Mukherjee-Wichs'16]

Attribute-based NIKE supporting NC^1 predicates

DCR | DDH over class groups

Our Results: **Applications** of Multi-Key HSS

Public-key PCFs for NC^1 additive correlations

DCR

DDH over class groups

Our Results: **Applications** of Multi-Key HSS

Public-key PCFs for NC^1 additive correlations

Includes Beaver triples, correlated OT, OLE etc.,

DCR

DDH over class groups

Our Results: **Applications** of Multi-Key HSS

Public-key PCFs for NC^1 additive correlations

DCR

DDH over class groups

Previously from group-based assumptions

Public-key PCFs for OT and Vector-OLE correlations

Our Results: **Applications** of Multi-Key HSS

Public-key PCFs for NC^1 additive correlations

DCR

DDH over class groups

Previously from group-based assumptions

Public-key PCFs for OT and Vector-OLE correlations

n-party secure computation protocol in the preprocessing model with communication complexity

- Offline phase: $\mathsf{poly}(\lambda) \cdot n$
- Online phase: $O(|C| \cdot n)$

DCR

DDH over class groups

Previously from group-based assumptions

Offline communication complexity $\mathsf{poly}(\lambda) \cdot n^2$

Outline

Applications

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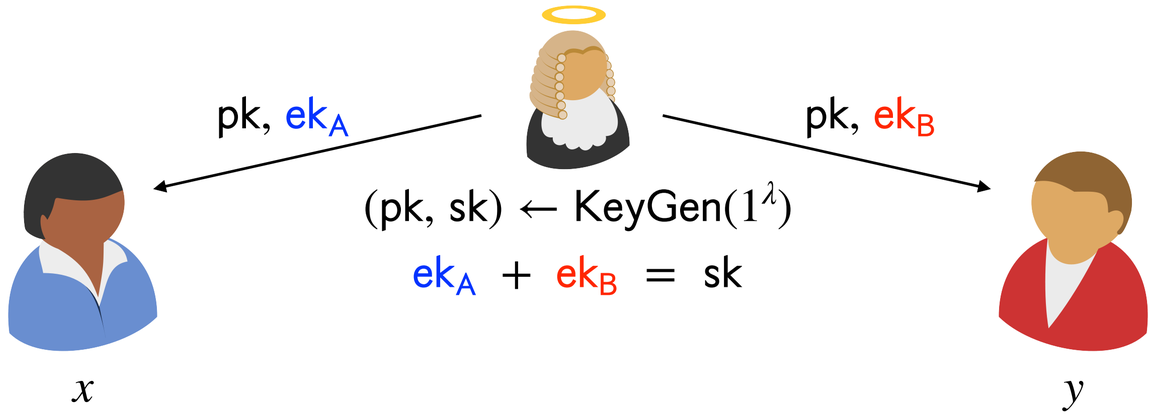
Constructing Multi-Key HSS

Group-based HSS Schemes

[Boyle-Gilboa-Ishai'16][Orlandi-Scholl-Yakoubov'21][Roy-Singh'21][Abram-Damgård-Orlandi-Scholl'22]



Group-based HSS Schemes

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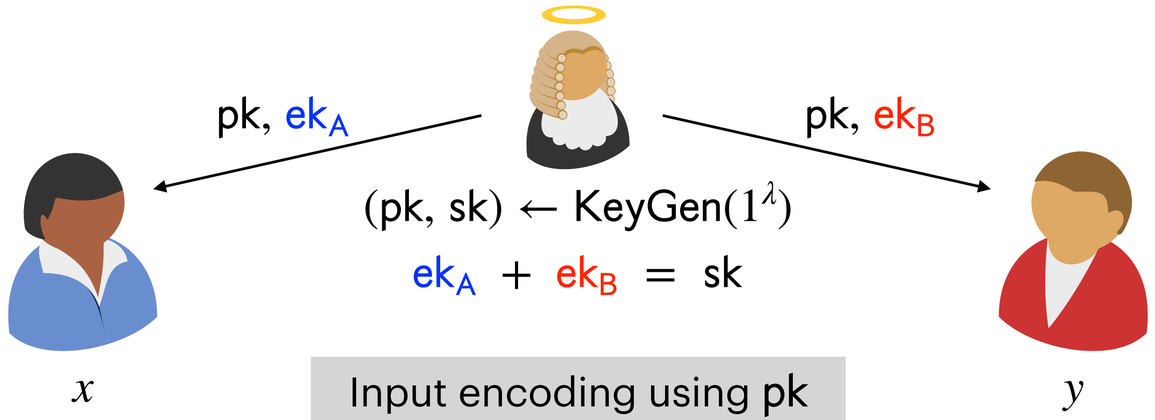

Input encoding using pk

$$ct_x = \begin{pmatrix} Enc(pk, x) \\ Enc(pk, sk \cdot x) \end{pmatrix}$$

Group-based HSS Schemes

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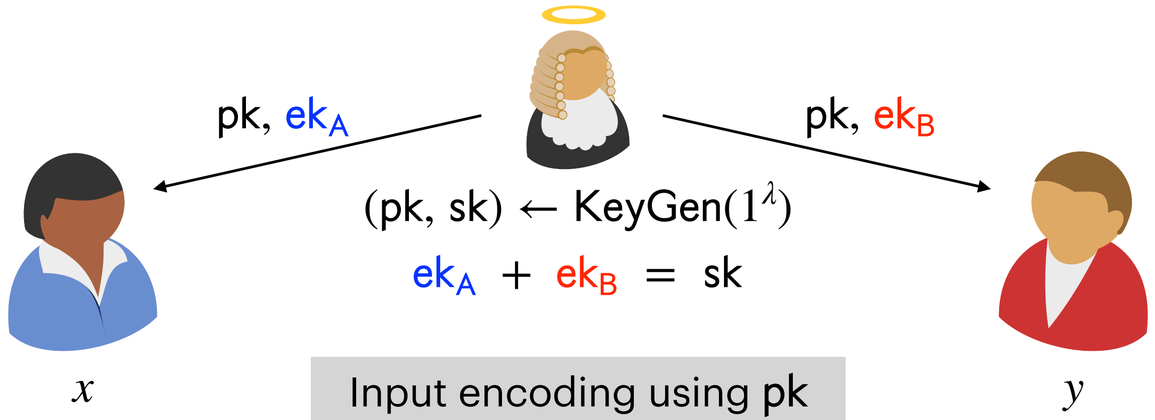

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$Enc(pk, x)$
$Enc(pk, sk \cdot x)$

$Enc(pk, y)$
$Enc(pk, sk \cdot y)$

$Enc(pk, x)$
$Enc(pk, sk \cdot x)$

$Enc(pk, y)$
$Enc(pk, sk \cdot y)$

Group-based HSS Schemes

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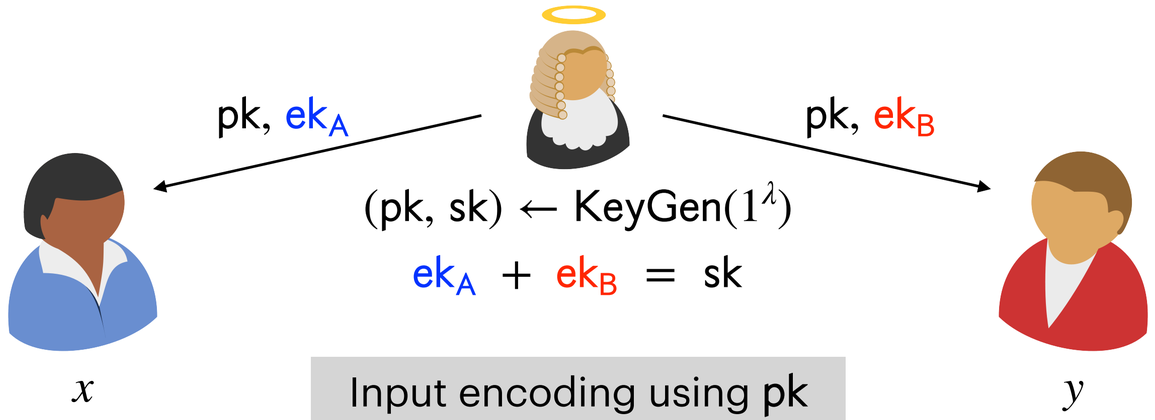

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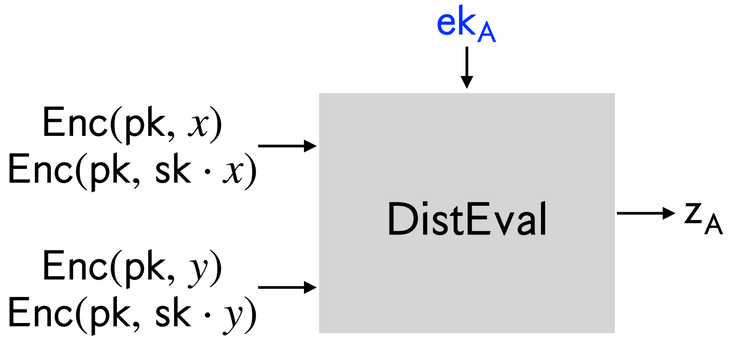
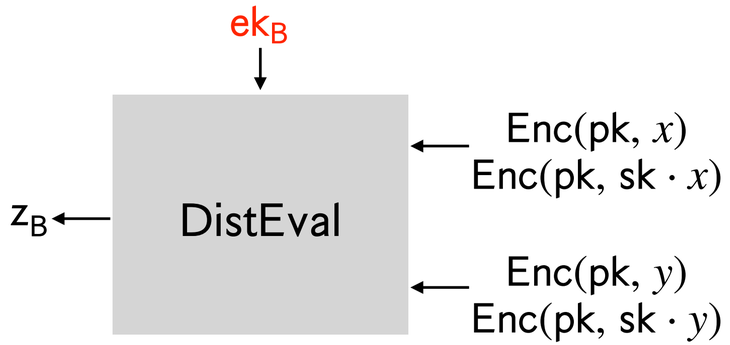


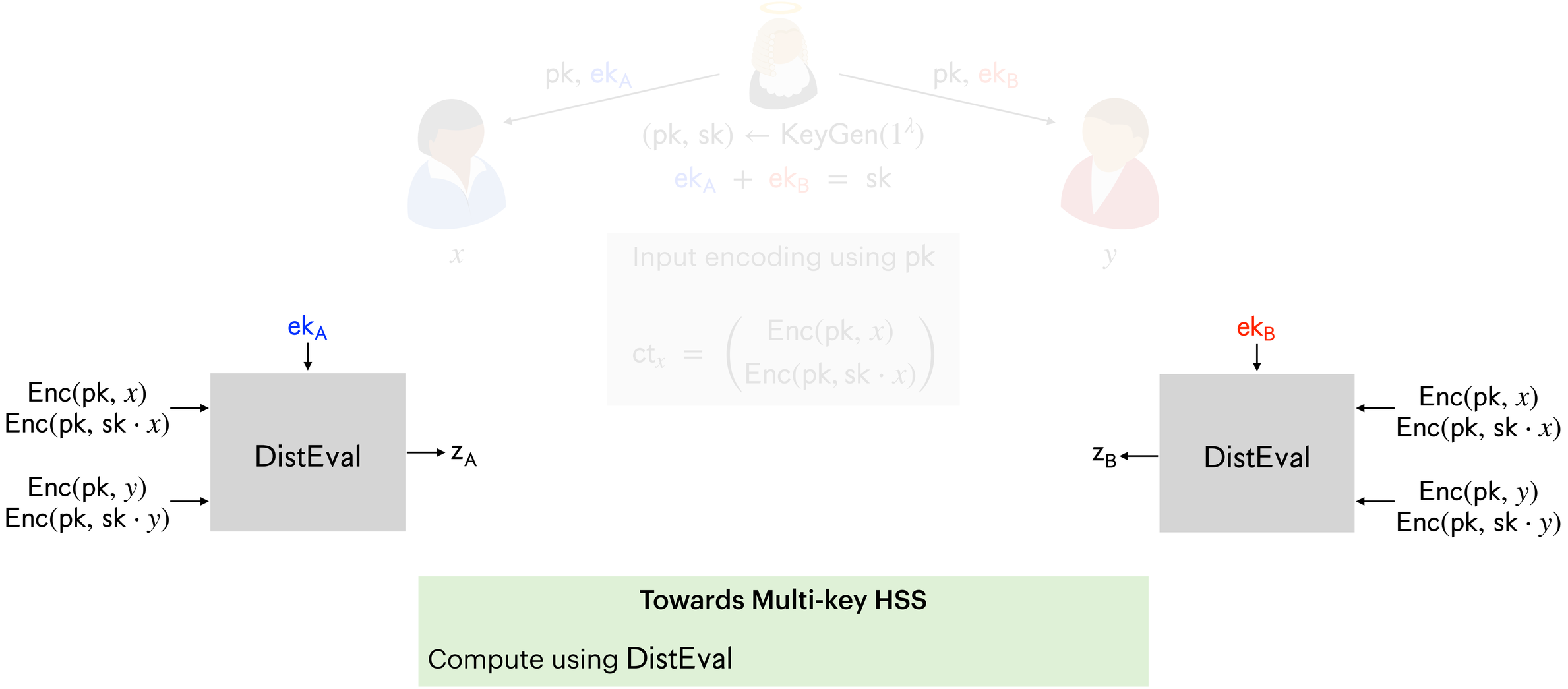


$$z_A + z_B = C(x, y)$$

Any NC^1 circuit

Group-based HSS Schemes

[Boyle-Gilboa-Ishai'16][Orlandi-Scholl-Yakoubov'21][Roy-Singh'21][Abram-Damgård-Orlandi-Scholl'22]



Towards Multi-key HSS

Compute using DistEval

Group-based HSS Schemes

[Boyle-Gilboa-Ishai'16][Orlandi-Scholl-Yakoubov'21][Roy-Singh'21][Abram-Damgård-Orlandi-Scholl'22]

Towards Multi-key HSS

Compute using DistEval $\Longrightarrow$ compatible input encoding

Group-based HSS Schemes

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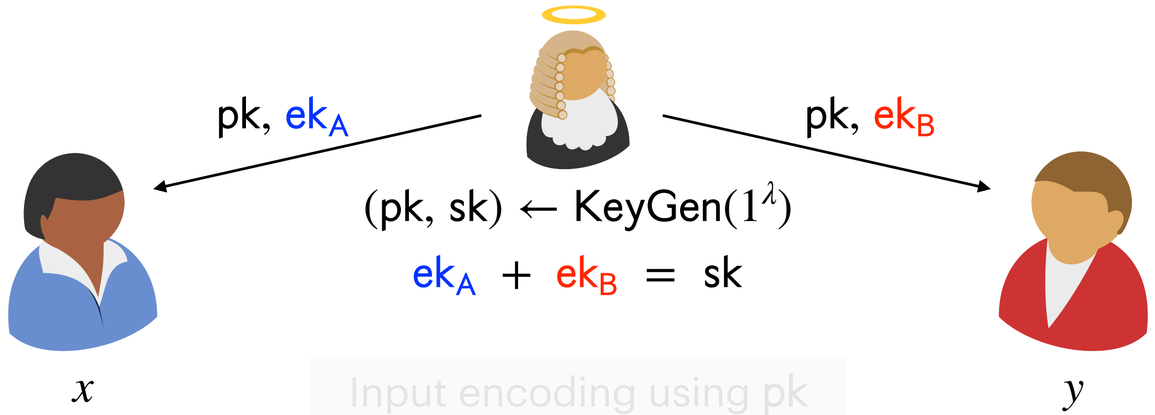
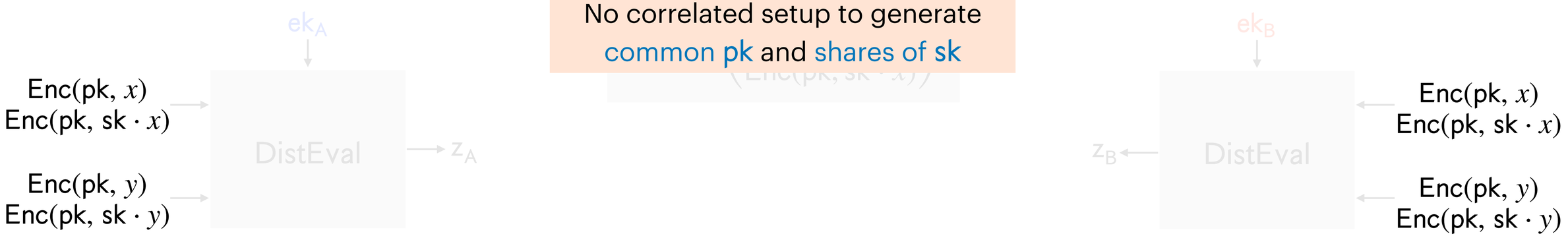


No correlated setup to generate common pk and shares of sk



Towards Multi-key HSS

Compute using DistEval $\Longrightarrow$ compatible input encoding

Constructing Multi-Key HSS

Common Reference String

x

y

Constructing Multi-Key HSS

Common Reference String

$(\mathsf{pk}_A, \mathsf{sk}_A) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_B, \mathsf{sk}_B)$

x

y

Constructing Multi-Key HSS

Common Reference String

$(\mathsf{pk}_A, \mathsf{sk}_A) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_B, \mathsf{sk}_B)$

$$\mathsf{ct}_x = \begin{pmatrix} \mathsf{Enc}(\mathsf{pk}_A, x) \\ \mathsf{Enc}(\mathsf{pk}_A, \mathsf{sk}_A \cdot x) \end{pmatrix}$$

x

y

$$\mathsf{ct}_y = \begin{pmatrix} \mathsf{Enc}(\mathsf{pk}_B, y) \\ \mathsf{Enc}(\mathsf{pk}_B, \mathsf{sk}_B \cdot y) \end{pmatrix}$$

Constructing Multi-Key HSS

Common Reference String

$(\mathsf{pk}_A, \mathsf{sk}_A) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_B, \mathsf{sk}_B)$

$$\mathsf{ct}_x = \begin{pmatrix} \mathsf{Enc}(\mathsf{pk}_A, x) \\ \mathsf{Enc}(\mathsf{pk}_A, \mathsf{sk}_A \cdot x) \end{pmatrix}$$

x

$\mathsf{ct}_x, \mathsf{pk}_A$

$\mathsf{ct}_y, \mathsf{pk}_B$

y

$$\mathsf{ct}_y = \begin{pmatrix} \mathsf{Enc}(\mathsf{pk}_B, y) \\ \mathsf{Enc}(\mathsf{pk}_B, \mathsf{sk}_B \cdot y) \end{pmatrix}$$

$\mathsf{ct}_x, \mathsf{pk}_A$

Constructing Multi-Key HSS

Common Reference String

$(\mathsf{pk}_A, \mathsf{sk}_A) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_B, \mathsf{sk}_B)$

$$\mathsf{ct}_x = \begin{pmatrix} \mathsf{Enc}(\mathsf{pk}_A, x) \\ \mathsf{Enc}(\mathsf{pk}_A, \mathsf{sk}_A \cdot x) \end{pmatrix}$$

x

$\mathsf{ct}_x, \mathsf{pk}_A$ →

← $\mathsf{ct}_y, \mathsf{pk}_B$

y

$$\mathsf{ct}_y = \begin{pmatrix} \mathsf{Enc}(\mathsf{pk}_B, y) \\ \mathsf{Enc}(\mathsf{pk}_B, \mathsf{sk}_B \cdot y) \end{pmatrix}$$

$\mathsf{ct}_y, \mathsf{pk}_B$

$\mathsf{ct}_x, \mathsf{pk}_A$

Cannot be used with DistEval since inputs are encrypted under different keys

$\mathsf{Enc}(\mathsf{pk}_A, x)$ →
$\mathsf{Enc}(\mathsf{pk}_A, \mathsf{sk}_A \cdot x)$

$\mathsf{Enc}(\mathsf{pk}_B, y)$ →
$\mathsf{Enc}(\mathsf{pk}_B, \mathsf{sk}_B \cdot y)$

DistEval

Constructing Multi-Key HSS

Common Reference String

$(\mathsf{pk}_A, \mathsf{sk}_A) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_B, \mathsf{sk}_B)$

$$\mathsf{ct}_x = \begin{pmatrix} \mathsf{Enc}(\mathsf{pk}_A, x) \\ \mathsf{Enc}(\mathsf{pk}_A, \mathsf{sk}_A \cdot x) \end{pmatrix}$$

x

$\mathsf{ct}_x, \mathsf{pk}_A$

$\mathsf{ct}_y, \mathsf{pk}_B$

y

$$\mathsf{ct}_y = \begin{pmatrix} \mathsf{Enc}(\mathsf{pk}_B, y) \\ \mathsf{Enc}(\mathsf{pk}_B, \mathsf{sk}_B \cdot y) \end{pmatrix}$$

$\mathsf{ct}_x, \mathsf{pk}_A$

Key Ingredient: Synchronize ciphertexts under different keys to a common public key pk

Constructing Multi-Key HSS

Common Reference String

$(\mathsf{pk}_A, \mathsf{sk}_A) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$$\mathsf{ct}_x = \begin{pmatrix} \mathsf{Enc}(\mathsf{pk}_A, x) \\ \mathsf{Enc}(\mathsf{pk}_A, \mathsf{sk}_A \cdot x) \end{pmatrix}$$

$\mathsf{ct}_y, \mathsf{pk}_B$

x

$\mathsf{ct}_x, \mathsf{pk}_A$ →

← $\mathsf{ct}_y, \mathsf{pk}_B$

y

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_B, \mathsf{sk}_B)$

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$\mathsf{ct}_x, \mathsf{pk}_A$

Key Ingredient: Synchronize ciphertexts under different keys to a common public key pk

↓

$\mathsf{Enc}(\mathsf{pk}, x)$
$\mathsf{Enc}(\mathsf{pk}, \mathsf{sk} \cdot x)$

$\mathsf{Enc}(\mathsf{pk}, y)$
$\mathsf{Enc}(\mathsf{pk}, \mathsf{sk} \cdot y)$

↓

$\mathsf{Enc}(\mathsf{pk}, x)$
$\mathsf{Enc}(\mathsf{pk}, \mathsf{sk} \cdot x)$

$\mathsf{Enc}(\mathsf{pk}, y)$
$\mathsf{Enc}(\mathsf{pk}, \mathsf{sk} \cdot y)$

Constructing Multi-Key HSS

Common Reference String

$(\mathsf{pk}_A, \mathsf{sk}_A) \leftarrow \mathsf{KeyGen}(1^\lambda)$

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$\mathsf{ct}_y, \mathsf{pk}_B$

x

$\mathsf{ct}_x, \mathsf{pk}_A$ →

← $\mathsf{ct}_y, \mathsf{pk}_B$

y

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_B, \mathsf{sk}_B)$

$$\mathsf{ct}_y = \begin{pmatrix} \mathsf{Enc}(\mathsf{pk}_B, y) \\ \mathsf{Enc}(\mathsf{pk}_B, \mathsf{sk}_B \cdot y) \end{pmatrix}$$

$\mathsf{ct}_x, \mathsf{pk}_A$

Key Ingredient: Synchronize ciphertexts under different keys to a common public key pk

ek_A $\quad$ $\mathsf{ek}_A + \mathsf{ek}_B = \mathsf{sk}$ $\quad$ ek_B

$\mathsf{Enc}(\mathsf{pk}, x)$
$\mathsf{Enc}(\mathsf{pk}, \mathsf{sk} \cdot x)$

$\mathsf{Enc}(\mathsf{pk}, y)$
$\mathsf{Enc}(\mathsf{pk}, \mathsf{sk} \cdot y)$

$\mathsf{Enc}(\mathsf{pk}, x)$
$\mathsf{Enc}(\mathsf{pk}, \mathsf{sk} \cdot x)$

$\mathsf{Enc}(\mathsf{pk}, y)$
$\mathsf{Enc}(\mathsf{pk}, \mathsf{sk} \cdot y)$

Constructing Multi-Key HSS

Common Reference String

$(\mathsf{pk}_A, \mathsf{sk}_A) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$$\mathsf{ct}_x = \begin{pmatrix} \mathsf{Enc}(\mathsf{pk}_A, x) \\ \mathsf{Enc}(\mathsf{pk}_A, \mathsf{sk}_A \cdot x) \end{pmatrix}$$

$\mathsf{ct}_y, \mathsf{pk}_B$

x

$\mathsf{ct}_x, \mathsf{pk}_A$ →

← $\mathsf{ct}_y, \mathsf{pk}_B$

y

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_B, \mathsf{sk}_B)$

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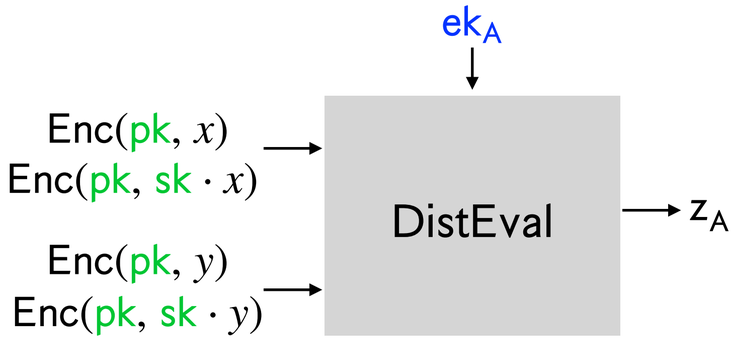
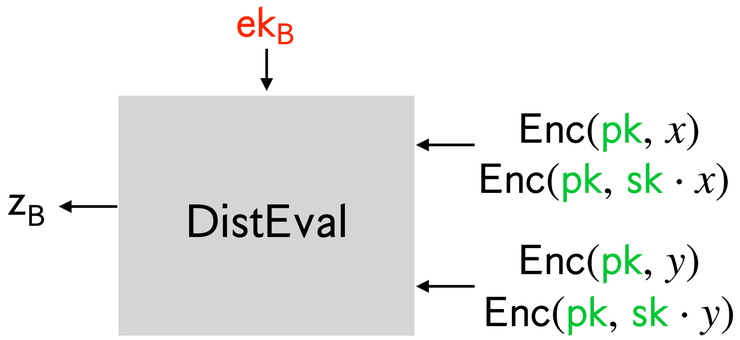
$\mathsf{ct}_x, \mathsf{pk}_A$

Key Ingredient: Synchronize ciphertexts under different keys to a common public key pk

$\mathsf{ek}_A + \mathsf{ek}_B = \mathsf{sk}$





Constructing Multi-Key HSS

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$$\mathsf{ct}_x = \begin{pmatrix} \mathsf{Enc}(\mathsf{pk}_A, x) \\ \mathsf{Enc}(\mathsf{pk}_A, \mathsf{sk}_A \cdot x) \end{pmatrix}$$

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$\mathsf{ct}_x, \mathsf{pk}_A$ →

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$\mathsf{ct}_x, \mathsf{pk}_A$

Key Ingredient: Synchronize ciphertexts under different keys to a common public key pk

$\mathsf{ek}_A + \mathsf{ek}_B = \mathsf{sk}$

ek_A

$\mathsf{Enc}(\mathsf{pk}, x)$
$\mathsf{Enc}(\mathsf{pk}, \mathsf{sk} \cdot x)$

$\mathsf{Enc}(\mathsf{pk}, y)$
$\mathsf{Enc}(\mathsf{pk}, \mathsf{sk} \cdot y)$

DistEval → z_A

ek_B

z_B ← DistEval

← $\mathsf{Enc}(\mathsf{pk}, x)$
$\mathsf{Enc}(\mathsf{pk}, \mathsf{sk} \cdot x)$

← $\mathsf{Enc}(\mathsf{pk}, y)$
$\mathsf{Enc}(\mathsf{pk}, \mathsf{sk} \cdot y)$

Constructing Multi-Key HSS

Structure of Synchronized Key

Common Reference String: $\mathbb{G}$, g

$\mathsf{sk}_A \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_A = g^{\mathsf{sk}_A}$

x

$\mathsf{sk}_B \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_B = g^{\mathsf{sk}_B}$

Constructing Multi-Key HSS

Structure of Synchronized Key

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$\mathsf{sk}_A \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_A = g^{\mathsf{sk}_A}$

x

$\mathsf{sk}_B \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_B = g^{\mathsf{sk}_B}$

Inspired by Multi-Key FHE
[Mukherjee-Wichs'16]

$\mathsf{sk} = (\mathsf{sk}_A, \mathsf{sk}_B)$

$\mathsf{pk} = (g^{\mathsf{sk}_A}, g^{\mathsf{sk}_B})$

Constructing Multi-Key HSS

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Inspired by Multi-Key FHE
[Mukherjee-Wichs'16]

$\mathsf{sk} = (\mathsf{sk}_A, \mathsf{sk}_B)$ $\quad$ $\mathsf{pk} = (g^{\mathsf{sk}_A}, g^{\mathsf{sk}_B})$

$\mathsf{Enc}(\mathsf{pk}, \mathsf{sk} \cdot x) \in \mathbb{G}^{2\times 3} =$

Constructing Multi-Key HSS

Structure of Synchronized Key

Common Reference String: $\mathbb{G}, g$

$\mathsf{sk}_A \leftarrow \mathbb{Z}_p$

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$\mathsf{sk}_B \leftarrow \mathbb{Z}_p$

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Inspired by Multi-Key FHE
[Mukherjee-Wichs'16]

$\mathsf{sk} = (\mathsf{sk}_A, \mathsf{sk}_B)$ $\quad$ $\mathsf{pk} = (g^{\mathsf{sk}_A}, g^{\mathsf{sk}_B})$

Decryption with sk

$$\mathsf{Enc}(\mathsf{pk}, \mathsf{sk} \cdot x) \in \mathbb{G}^{2\times 3} = \begin{pmatrix} & & \\ & & \end{pmatrix} \begin{pmatrix} \mathsf{sk}_A \\ \mathsf{sk}_B \\ 1 \end{pmatrix} = \begin{pmatrix} g^{\mathsf{sk}_A \cdot x} \\ g^{\mathsf{sk}_B \cdot x} \end{pmatrix}$$

Constructing Multi-Key HSS

Synchronizing to Enc(pk, sk $\cdot\, x$)

Common Reference String: $\mathbb{G}$, g

$\mathsf{sk}_A \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_A = g^{\mathsf{sk}_A}$

x

$\mathsf{sk} = (\mathsf{sk}_A, \mathsf{sk}_B)$

$\mathsf{pk} = (g^{\mathsf{sk}_A}, g^{\mathsf{sk}_B})$

$\mathsf{sk}_B \leftarrow \mathbb{Z}_p$

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Constructing Multi-Key HSS

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x

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$\mathsf{pk} = (g^{\mathsf{sk}_A}, g^{\mathsf{sk}_B})$

$\mathsf{sk}_B \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_B = g^{\mathsf{sk}_B}$

Flipped ElGamal Encryption:

[Abram-Damgård-Orlandi-Scholl'22]

$$\mathsf{FlipEG}(\mathsf{pk}_A, x) = \left(g^{-r} \cdot g^{x}, \mathsf{pk}_A^{r}\right) = \left(g^{-r} \cdot g^{x}, g^{\mathsf{sk}_A \cdot r}\right)$$

Constructing Multi-Key HSS

Synchronizing to Enc(pk, sk $\cdot\, x$)

Common Reference String: $\mathbb{G}$, g

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$\mathsf{pk} = (g^{\mathsf{sk}_A}, g^{\mathsf{sk}_B})$

$\mathsf{sk}_B \leftarrow \mathbb{Z}_p$

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Decryption with sk_A: $\left(g^{-r} \cdot g^{x}\right)^{\mathsf{sk}_A} \cdot g^{\mathsf{sk}_A \cdot r} = g^{\mathsf{sk}_A \cdot x}$

Constructing Multi-Key HSS

Synchronizing to Enc(pk, sk $\cdot\, x$)

Common Reference String: $\mathbb{G}, g$

$\mathsf{sk}_A \leftarrow \mathbb{Z}_p$

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$\mathsf{sk} = (\mathsf{sk}_A, \mathsf{sk}_B)$

$\mathsf{pk} = (g^{\mathsf{sk}_A}, g^{\mathsf{sk}_B})$

$\mathsf{sk}_B \leftarrow \mathbb{Z}_p$

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Flipped ElGamal Encryption:

[Abram-Damgård-Orlandi-Scholl'22]

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Decryption with sk_A: $\left(g^{-r} \cdot g^{x}\right)^{\mathsf{sk}_A} \cdot g^{\mathsf{sk}_A \cdot r} = g^{\mathsf{sk}_A \cdot x}$

Decryption with sk

$$\mathsf{Enc}(\mathsf{pk}, \mathsf{sk} \cdot x) \in \mathbb{G}^{2\times 3} = \begin{pmatrix} & & \\ & & \end{pmatrix} \begin{pmatrix} \mathsf{sk}_A \\ \mathsf{sk}_B \\ 1 \end{pmatrix} = \begin{pmatrix} g^{\mathsf{sk}_A \cdot x} \\ g^{\mathsf{sk}_B \cdot x} \end{pmatrix}$$

Constructing Multi-Key HSS

Synchronizing to Enc(pk, sk $\cdot\, x$)

Common Reference String: $\mathbb{G}$, g

$\mathsf{sk}_A \leftarrow \mathbb{Z}_p$

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x

$\mathsf{sk} = (\mathsf{sk}_A, \mathsf{sk}_B)$

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$\mathsf{pk}_B = g^{\mathsf{sk}_B}$

Flipped ElGamal Encryption:

[Abram-Damgård-Orlandi-Scholl'22]

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Decryption with sk_A: $\left(g^{-r} \cdot g^{x}\right)^{\mathsf{sk}_A} \cdot g^{\mathsf{sk}_A \cdot r} = g^{\mathsf{sk}_A \cdot x}$

How to synchronize to an encryption of $\mathsf{sk}_B \cdot x$ under sk?

Decryption with sk

$$\mathsf{Enc}(\mathsf{pk}, \mathsf{sk} \cdot x) \in \mathbb{G}^{2\times 3} = \begin{pmatrix} & & \\ & & \end{pmatrix} \begin{pmatrix} \mathsf{sk}_A \\ \mathsf{sk}_B \\ 1 \end{pmatrix} = \begin{pmatrix} g^{\mathsf{sk}_A \cdot x} \\ g^{\mathsf{sk}_B \cdot x} \end{pmatrix}$$

Constructing Multi-Key HSS

Synchronizing to Enc(pk, sk $\cdot\, x$)

Common Reference String: $\mathbb{G}$, g

$\mathsf{sk}_A \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_A = g^{\mathsf{sk}_A}$

x

$\mathsf{sk} = (\mathsf{sk}_A, \mathsf{sk}_B)$

$\mathsf{pk} = (g^{\mathsf{sk}_A}, g^{\mathsf{sk}_B})$

$\mathsf{sk}_B \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_B = g^{\mathsf{sk}_B}$

Flipped ElGamal Encryption:

[Abram-Damgård-Orlandi-Scholl'22]

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Decryption with sk_A: $\left(g^{-r} \cdot g^{x}\right)^{\mathsf{sk}_A} \cdot g^{\mathsf{sk}_A \cdot r} = g^{\mathsf{sk}_A \cdot x}$

Decryption with sk_B: $\left(g^{-r} \cdot g^{x}\right)^{\mathsf{sk}_B} = g^{\mathsf{sk}_B \cdot x} \cdot g^{-\mathsf{sk}_B \cdot r}$

Constructing Multi-Key HSS

Synchronizing to Enc(pk, sk $\cdot\, x$)

Common Reference String: $\mathbb{G}, g$

$\mathsf{sk}_A \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_A = g^{\mathsf{sk}_A}$

x

$\mathsf{sk} = (\mathsf{sk}_A, \mathsf{sk}_B)$

$\mathsf{pk} = (g^{\mathsf{sk}_A}, g^{\mathsf{sk}_B})$

$\mathsf{sk}_B \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_B = g^{\mathsf{sk}_B}$

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Constructing Multi-Key HSS

Synchronizing to Enc(pk, sk $\cdot\, x$)

Common Reference String: $\mathbb{G}, g$

$\mathsf{sk}_A \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_A = g^{\mathsf{sk}_A}$

x

$\mathsf{sk} = (\mathsf{sk}_A, \mathsf{sk}_B)$

$\mathsf{pk} = (g^{\mathsf{sk}_A}, g^{\mathsf{sk}_B})$

$\mathsf{sk}_B \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_B = g^{\mathsf{sk}_B}$

Flipped ElGamal Encryption:

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Decryption with sk_B: $\left(g^{-r} \cdot g^{x}\right)^{\mathsf{sk}_B} = g^{\mathsf{sk}_B \cdot x} \cdot g^{-\mathsf{sk}_B \cdot r}$

Constructing Multi-Key HSS

Synchronizing to Enc(pk, sk $\cdot\, x$)

Common Reference String: $\mathbb{G}, g$

$\mathsf{sk}_A \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_A = g^{\mathsf{sk}_A}$

x

$\mathsf{sk} = (\mathsf{sk}_A, \mathsf{sk}_B)$

$\mathsf{pk} = (g^{\mathsf{sk}_A}, g^{\mathsf{sk}_B})$

$\mathsf{sk}_B \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_B = g^{\mathsf{sk}_B}$

Flipped ElGamal Encryption:

[Abram-Damgård-Orlandi-Scholl'22]

$$\mathsf{FlipEG}(\mathsf{pk}_A, x) = \left(g^{-r} \cdot g^{x},\ \mathsf{pk}_A^{r}\right) = \left(g^{-r} \cdot g^{x},\ g^{\mathsf{sk}_A \cdot r}\right)$$

Decryption with sk_A: $\left(g^{-r} \cdot g^{x}\right)^{\mathsf{sk}_A} \cdot g^{\mathsf{sk}_A \cdot r} = g^{\mathsf{sk}_A \cdot x}$

Decryption with sk_B: $\left(g^{-r} \cdot g^{x}\right)^{\mathsf{sk}_B} = g^{\mathsf{sk}_B \cdot x} \cdot g^{-\mathsf{sk}_B \cdot r}$

Convert Enc(pk_A, r) into Enc(pk_A, $\mathsf{sk}_B \cdot r$)

Constructing Multi-Key HSS

Synchronizing to Enc(pk, sk · x)

Common Reference String: $\mathbb{G}$, g

$\mathsf{sk}_A \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_A = g^{\mathsf{sk}_A}$

x

$\mathsf{sk} = (\mathsf{sk}_A, \mathsf{sk}_B)$

$\mathsf{pk} = (g^{\mathsf{sk}_A}, g^{\mathsf{sk}_B})$

$\mathsf{sk}_B \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_B = g^{\mathsf{sk}_B}$

Flipped ElGamal Encryption:

[Abram-Damgård-Orlandi-Scholl'22]

$$\mathsf{FlipEG}(\mathsf{pk}_A, x) = \left(g^{-r} \cdot g^{x}, \mathsf{pk}_A^{r}\right) = \left(g^{-r} \cdot g^{x}, g^{\mathsf{sk}_A \cdot r}\right)$$

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Decryption with sk_B: $\left(g^{-r} \cdot g^{x}\right)^{\mathsf{sk}_B} = g^{\mathsf{sk}_B \cdot x} \cdot g^{-\mathsf{sk}_B \cdot r}$

From key synchronization to homomorphism

Convert $\mathsf{Enc}(\mathsf{pk}_A, r)$ into $\mathsf{Enc}(\mathsf{pk}_A, \mathsf{sk}_B \cdot r)$

Constructing Multi-Key HSS

Synchronizing to Enc(pk, sk $\cdot\, x$)

Common Reference String: $\mathbb{G}$, g

$\mathsf{sk}_A \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_A = g^{\mathsf{sk}_A}$

x

$\mathsf{sk} = (\mathsf{sk}_A, \mathsf{sk}_B)$

$\mathsf{pk} = (g^{\mathsf{sk}_A}, g^{\mathsf{sk}_B})$

$\mathsf{sk}_B \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_B = g^{\mathsf{sk}_B}$

$\mathsf{FlipEG}(\mathsf{pk}_A, x; r), \quad \mathsf{EG}(\mathsf{pk}_A, r) = \left(g^r \cdot g^x, g^{-\mathsf{sk}_A \cdot r}\right), \quad \left(g^u, g^{-\mathsf{sk}_A \cdot u} \cdot g^r\right)$

Constructing Multi-Key HSS

Synchronizing to Enc(pk, sk · x)

Common Reference String: $\mathbb{G}$, g

$\mathsf{sk}_A \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_A = g^{\mathsf{sk}_A}$

x

$\mathsf{sk} = (\mathsf{sk}_A, \mathsf{sk}_B)$

$\mathsf{pk} = (g^{\mathsf{sk}_A}, g^{\mathsf{sk}_B})$

$\mathsf{sk}_B \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_B = g^{\mathsf{sk}_B}$

$\mathsf{FlipEG}(\mathsf{pk}_A, x; r), \quad \mathsf{EG}(\mathsf{pk}_A, r) = (g^r \cdot g^x, g^{-\mathsf{sk}_A \cdot r}), \quad (g^u, g^{-\mathsf{sk}_A \cdot u} \cdot g^r)$

Convert $\mathsf{EG}(\mathsf{pk}_A, r)$**:**

Raise each component to sk_B

$$(g^u, g^{-\mathsf{sk}_A \cdot u} \cdot g^r)^{\mathsf{sk}_B}$$

$$= (g^{u \cdot \mathsf{sk}_B}, g^{-\mathsf{sk}_A \cdot u \cdot \mathsf{sk}_B} \cdot g^{\mathsf{sk}_B \cdot r})$$

$$= (g^{u'}, g^{-\mathsf{sk}_A \cdot u'} \cdot g^{\mathsf{sk}_B \cdot r})$$

$$= \mathsf{Enc}(\mathsf{pk}_A, \mathsf{sk}_B \cdot r)$$

Constructing Multi-Key HSS

Synchronizing to Enc(pk, sk · x)

Common Reference String: $\mathbb{G}$, g

$\mathsf{sk}_A \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_A = g^{\mathsf{sk}_A}$

x

$\mathsf{sk} = (\mathsf{sk}_A, \mathsf{sk}_B)$

$\mathsf{pk} = (g^{\mathsf{sk}_A}, g^{\mathsf{sk}_B})$

$\mathsf{sk}_B \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_B = g^{\mathsf{sk}_B}$

$\mathsf{FlipEG}(\mathsf{pk}_A, x; r), \quad \mathsf{EG}(\mathsf{pk}_A, r) = (g^r \cdot g^x, g^{-\mathsf{sk}_A \cdot r}), \quad (g^u, g^{-\mathsf{sk}_A \cdot u} \cdot g^r)$

Convert EG(pk_A, r):

Re-encrypt r using pk_B

$$\left(\mathsf{pk}_B^u, \mathsf{pk}_B^{-\mathsf{sk}_A \cdot u} \cdot \mathsf{pk}_B^r\right)$$

$$= \left(g^{u \cdot \mathsf{sk}_B}, g^{-\mathsf{sk}_A \cdot u \cdot \mathsf{sk}_B} \cdot g^{\mathsf{sk}_B \cdot r}\right)$$

$$= \left(g^{u'}, g^{-\mathsf{sk}_A \cdot u'} \cdot g^{\mathsf{sk}_B \cdot r}\right)$$

$$= \mathsf{Enc}(\mathsf{pk}_A, \mathsf{sk}_B \cdot r)$$

Convert EG(pk_A, r):

Raise each component to sk_B

$$\left(g^u, g^{-\mathsf{sk}_A \cdot u} \cdot g^r\right)^{\mathsf{sk}_B}$$

$$= \left(g^{u \cdot \mathsf{sk}_B}, g^{-\mathsf{sk}_A \cdot u \cdot \mathsf{sk}_B} \cdot g^{\mathsf{sk}_B \cdot r}\right)$$

$$= \left(g^{u'}, g^{-\mathsf{sk}_A \cdot u'} \cdot g^{\mathsf{sk}_B \cdot r}\right)$$

$$= \mathsf{Enc}(\mathsf{pk}_A, \mathsf{sk}_B \cdot r)$$

Constructing Multi-Key HSS

Synchronizing to Enc(pk, sk · x)

Common Reference String: $\mathbb{G}$, g

$\mathsf{sk}_A \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_A = g^{\mathsf{sk}_A}$

x

$\mathsf{sk} = (\mathsf{sk}_A, \mathsf{sk}_B)$

$\mathsf{pk} = (g^{\mathsf{sk}_A}, g^{\mathsf{sk}_B})$

$\mathsf{sk}_B \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_B = g^{\mathsf{sk}_B}$

$\mathsf{FlipEG}(\mathsf{pk}_A, x; r), \quad \mathsf{EG}(\mathsf{pk}_A, r) = (g^r \cdot g^x, g^{-\mathsf{sk}_A \cdot r}), \quad (g^u, g^{-\mathsf{sk}_A \cdot u} \cdot g^r)$

Convert $\mathsf{EG}(\mathsf{pk}_A, r)$**:**

Re-encrypt r using pk_B

$$\left(\mathsf{pk}_B^u, \mathsf{pk}_B^{-\mathsf{sk}_A \cdot u} \cdot \mathsf{pk}_B^r\right)$$

$$= \left(g^{u \cdot \mathsf{sk}_B}, g^{-\mathsf{sk}_A \cdot u \cdot \mathsf{sk}_B} \cdot g^{\mathsf{sk}_B \cdot r}\right)$$

$$= \left(g^{u'}, g^{-\mathsf{sk}_A \cdot u'} \cdot g^{\mathsf{sk}_B \cdot r}\right)$$

$$= \mathsf{Enc}(\mathsf{pk}_A, \mathsf{sk}_B \cdot r)$$

$$\left(\mathsf{Enc}(\mathsf{pk}_A, \mathsf{sk}_B \cdot r), \ \mathsf{FlipEG}(\mathsf{pk}_A, x; r)\right) \begin{pmatrix} \mathsf{sk}_A \\ \mathsf{sk}_B \\ 1 \end{pmatrix}$$

$$= g^{\mathsf{sk}_B \cdot r} \cdot g^{\mathsf{sk}_B \cdot x} \cdot g^{-\mathsf{sk}_B \cdot r}$$

$$= g^{\mathsf{sk}_B \cdot x}$$

Convert $\mathsf{EG}(\mathsf{pk}_A, r)$**:**

Raise each component to sk_B

$$\left(g^u, g^{-\mathsf{sk}_A \cdot u} \cdot g^r\right)^{\mathsf{sk}_B}$$

$$= \left(g^{u \cdot \mathsf{sk}_B}, g^{-\mathsf{sk}_A \cdot u \cdot \mathsf{sk}_B} \cdot g^{\mathsf{sk}_B \cdot r}\right)$$

$$= \left(g^{u'}, g^{-\mathsf{sk}_A \cdot u'} \cdot g^{\mathsf{sk}_B \cdot r}\right)$$

$$= \mathsf{Enc}(\mathsf{pk}_A, \mathsf{sk}_B \cdot r)$$

Constructing Multi-Key HSS

Synchronizing to Enc(pk, sk · x)

Decryption with sk

$$\mathsf{Enc}(\mathsf{pk}, \mathsf{sk} \cdot x) \in \mathbb{G}^{2\times 3} = \begin{pmatrix} \mathsf{sk}_A & \mathsf{sk}_B & 1 \end{pmatrix} = \begin{pmatrix} g^{\mathsf{sk}_A \cdot x} \\ g^{\mathsf{sk}_B \cdot x} \end{pmatrix}$$

$$\left(\mathsf{Enc}(\mathsf{pk}_A, \mathsf{sk}_B \cdot r),\ \mathsf{FlipEG}(\mathsf{pk}_A, x; r)\right) \begin{pmatrix} \mathsf{sk}_A \\ \mathsf{sk}_B \\ 1 \end{pmatrix}$$

$$= g^{\mathsf{sk}_B \cdot r} \cdot g^{\mathsf{sk}_B \cdot x} \cdot g^{-\mathsf{sk}_B \cdot r}$$

$$= g^{\mathsf{sk}_B \cdot x}$$

Conclusion

- Discussed approach works for Multi-Key HSS from class groups

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- **DDH over prime-order groups:** Extension to synchronize to encryption of "small values"

Synchronized ciphertexts: $\mathsf{Enc}(\mathsf{pk}, \mathsf{s}_0 \cdot x) \quad \ldots \quad \mathsf{Enc}(\mathsf{pk}, \mathsf{s}_\lambda \cdot x)$

$$\mathsf{sk} = \sum_{i=0}^{\lambda} 2^i \cdot s_i$$

Conclusion

- Discussed approach works for Multi-Key HSS from class groups
- **DDH over prime-order groups:** Extension to synchronize to encryption of "small values"

Synchronized ciphertexts: $\mathsf{Enc}(\mathsf{pk}, s_0 \cdot x) \quad \ldots \quad \mathsf{Enc}(\mathsf{pk}, s_\lambda \cdot x)$

$$\mathsf{sk} = \sum_{i=0}^{\lambda} 2^i \cdot s_i$$

- **DCR:** Requires a different synchronization approach

$$\mathsf{pk}_A = g^{\mathsf{sk}_A}$$
$$\mathsf{pk}_B = g^{\mathsf{sk}_B}$$
$$\longrightarrow \quad \mathsf{pk} = g^{\mathsf{sk}_A \cdot \mathsf{sk}_B}$$

Thank You

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