POKÉ: A Compact and Efficient PKE from Higher-dimensional Isogenies

Andrea Basso, Luciano Maino

May 5th — EUROCRYPT 2025

A brief history of isogeny-based encryption

SIDH

First practical isogeny-based protocol

2011

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2018

CSIDH

Based on commutative group action

The (C)SIDH protocol

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The (C)SIDH protocol

The (C)SIDH protocol

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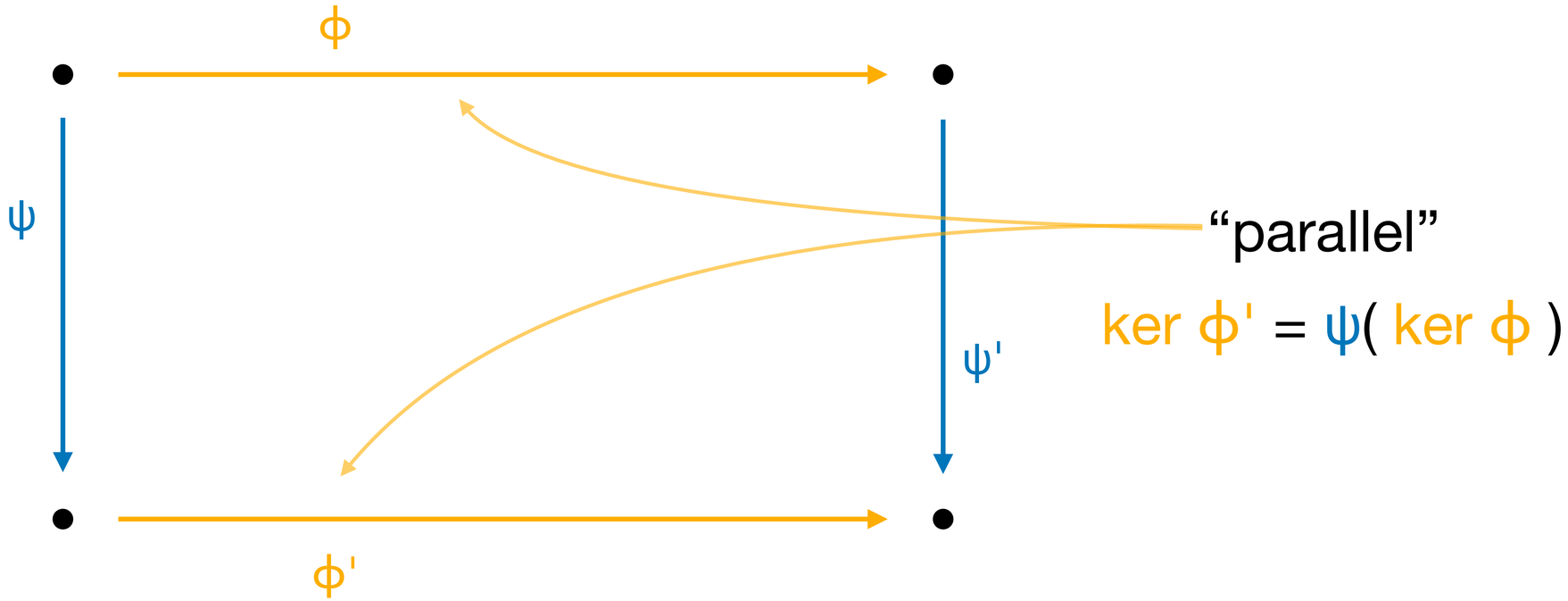
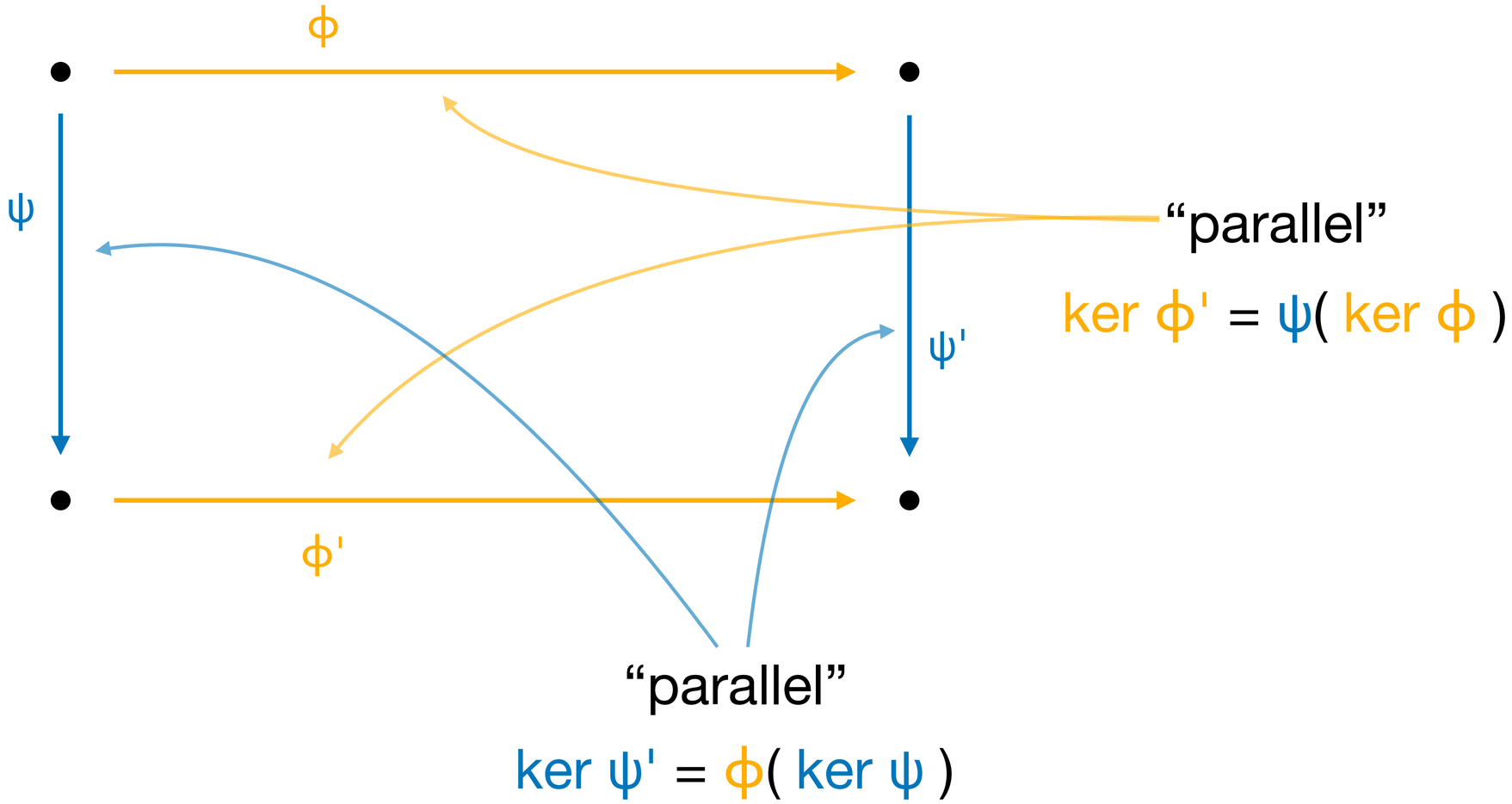


The (C)SIDH protocol



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SIDH attacks

Efficient key-recovery attacks on SIDH

2022

Higher-dimensional representations

Higher-dimensional representation

- E_0, E_1
- P, Q and $\phi(P)$, $\phi(Q)$
- deg ϕ

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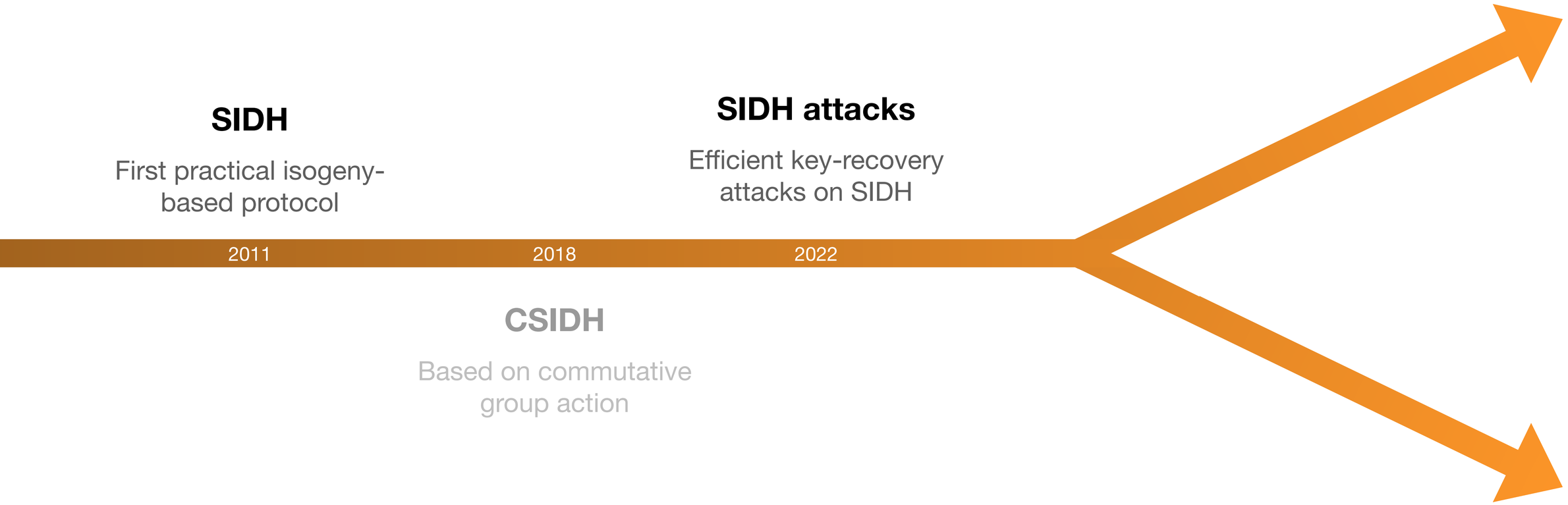

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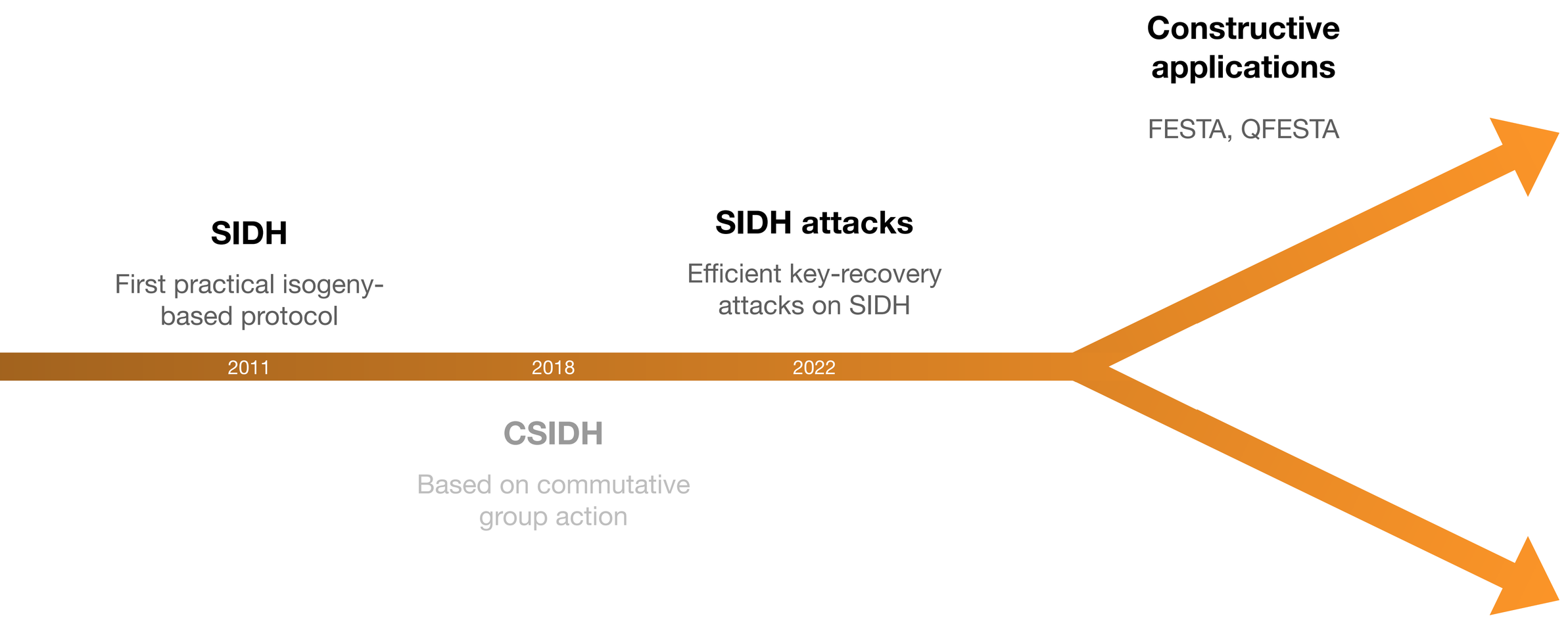

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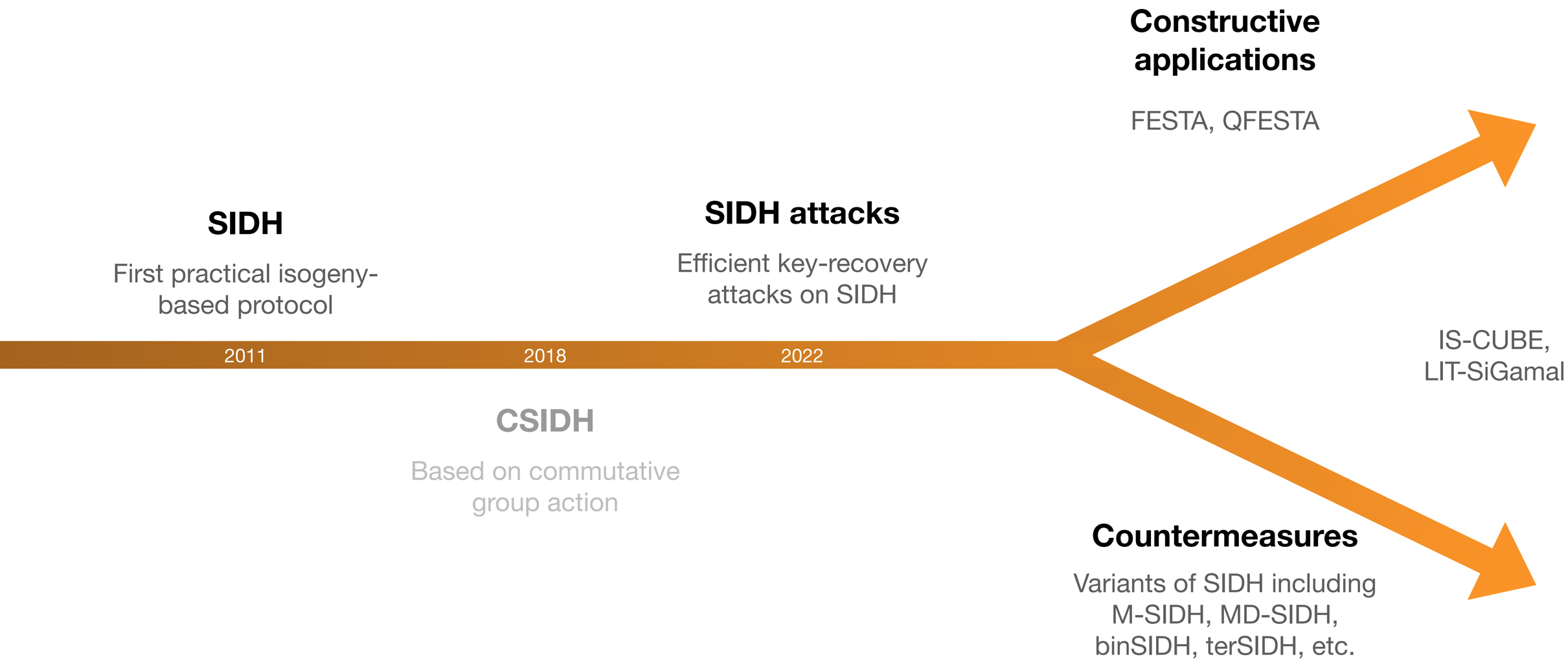

Public/private HD representations

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} public

secret

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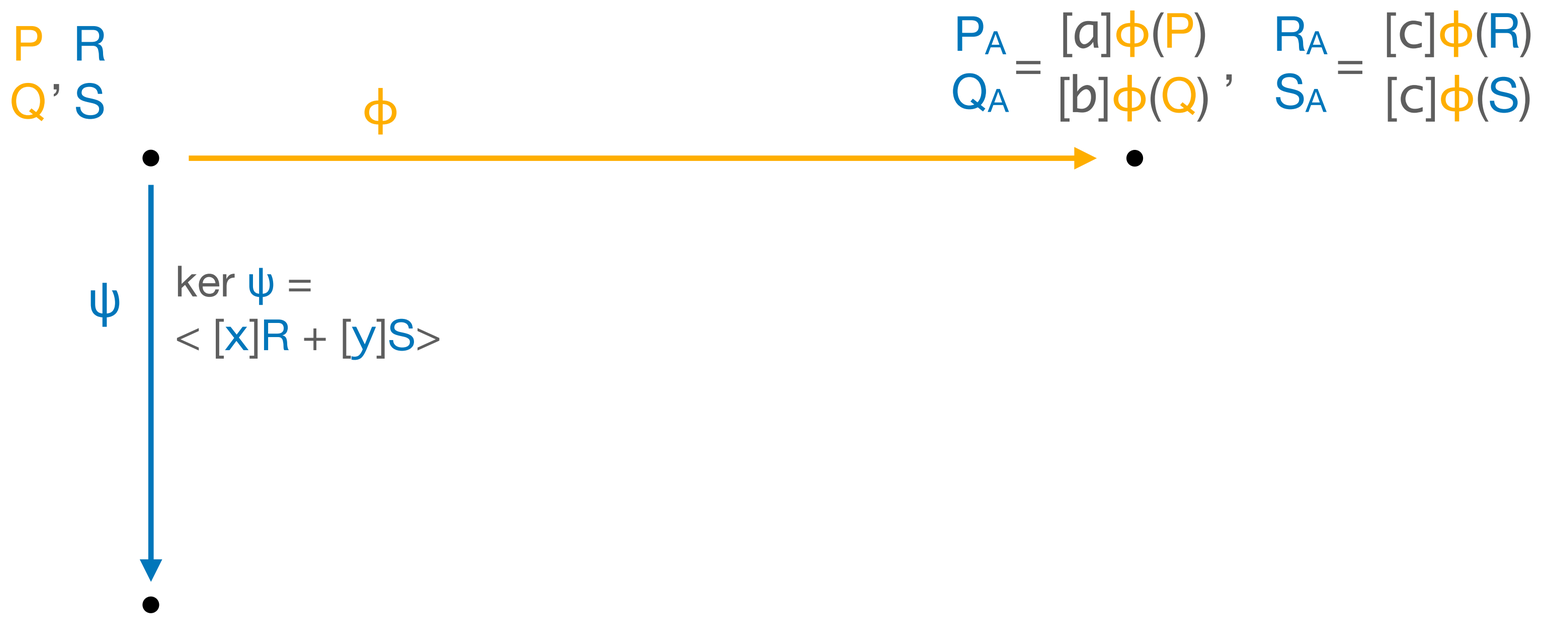

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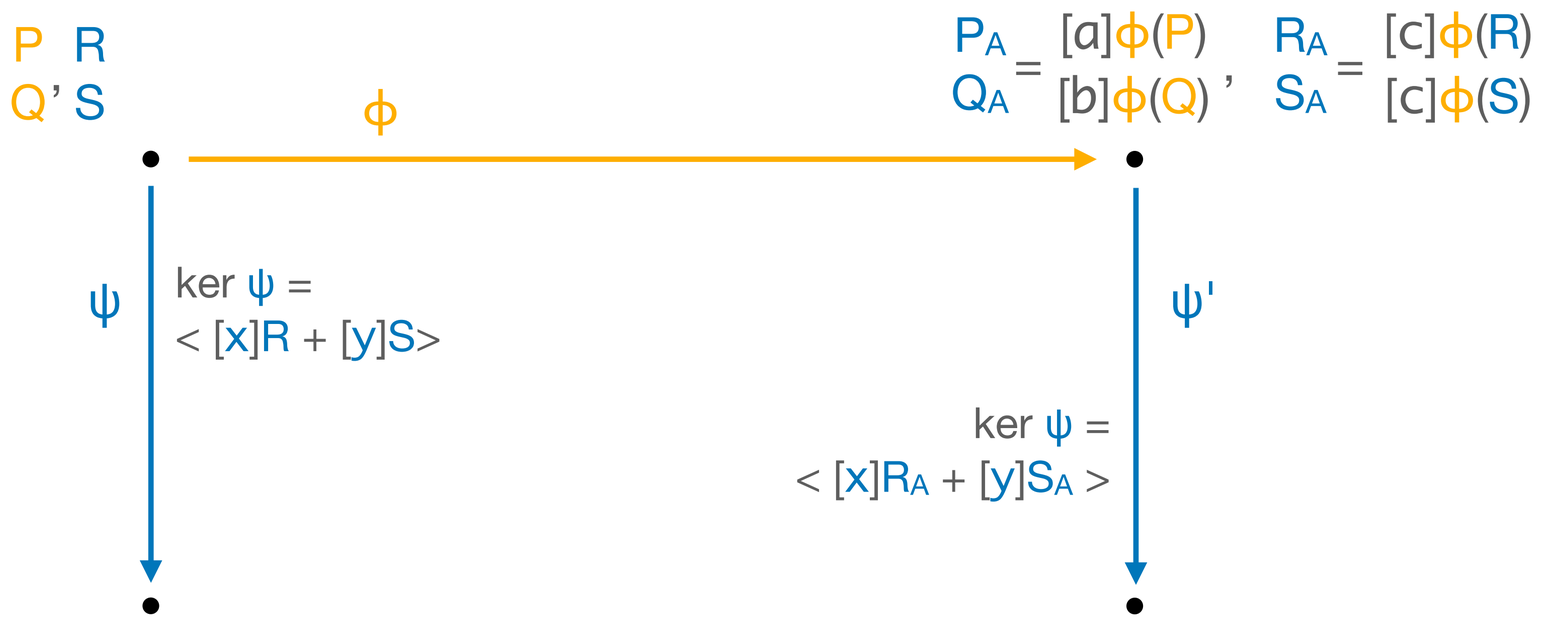

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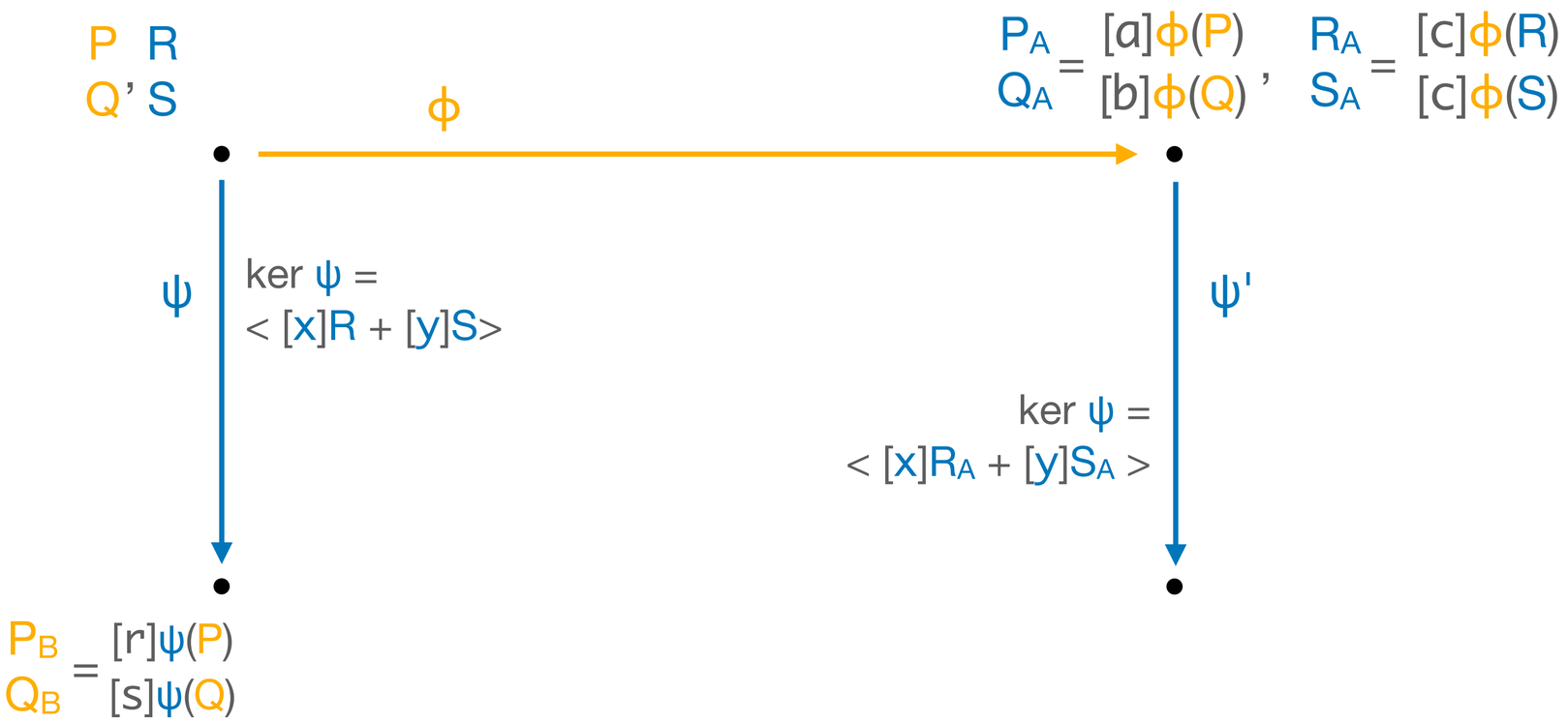

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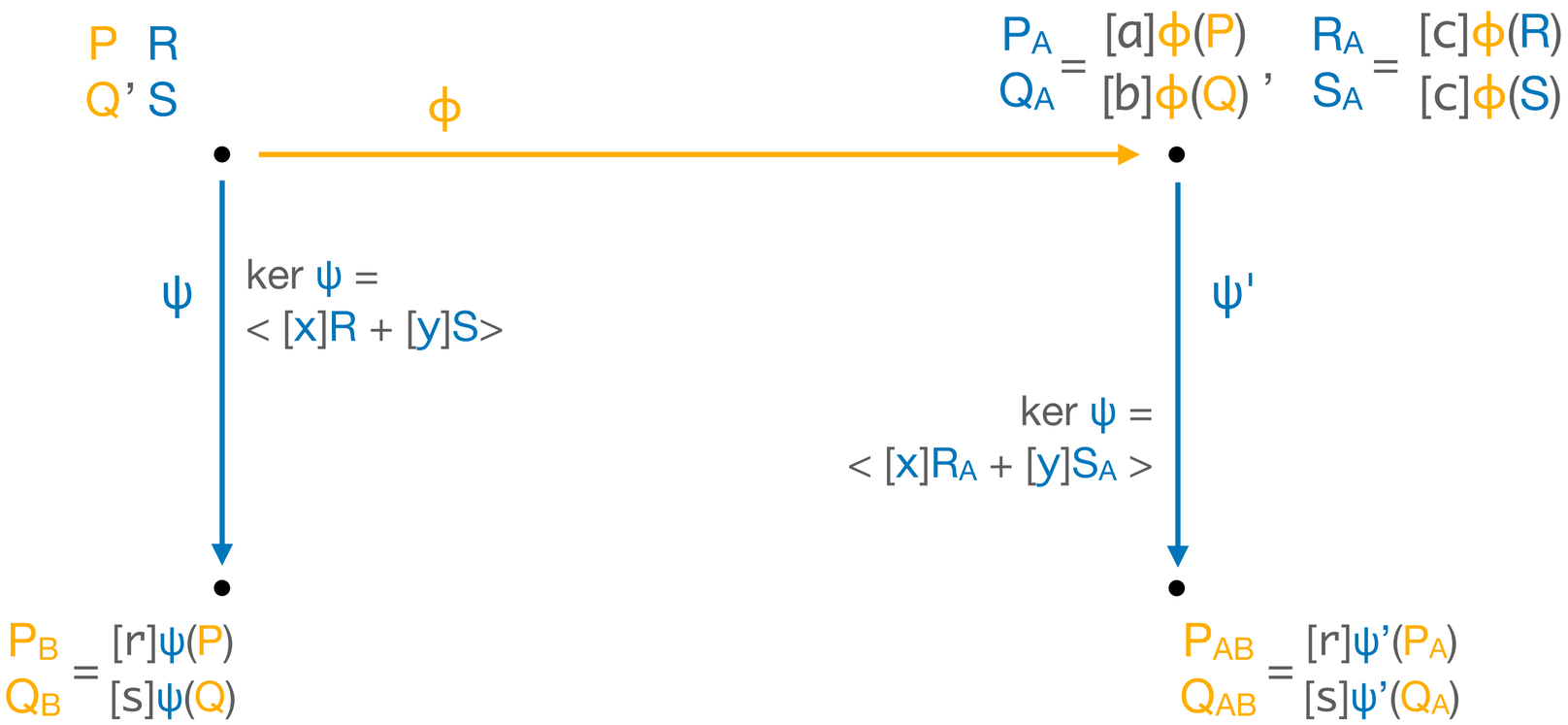

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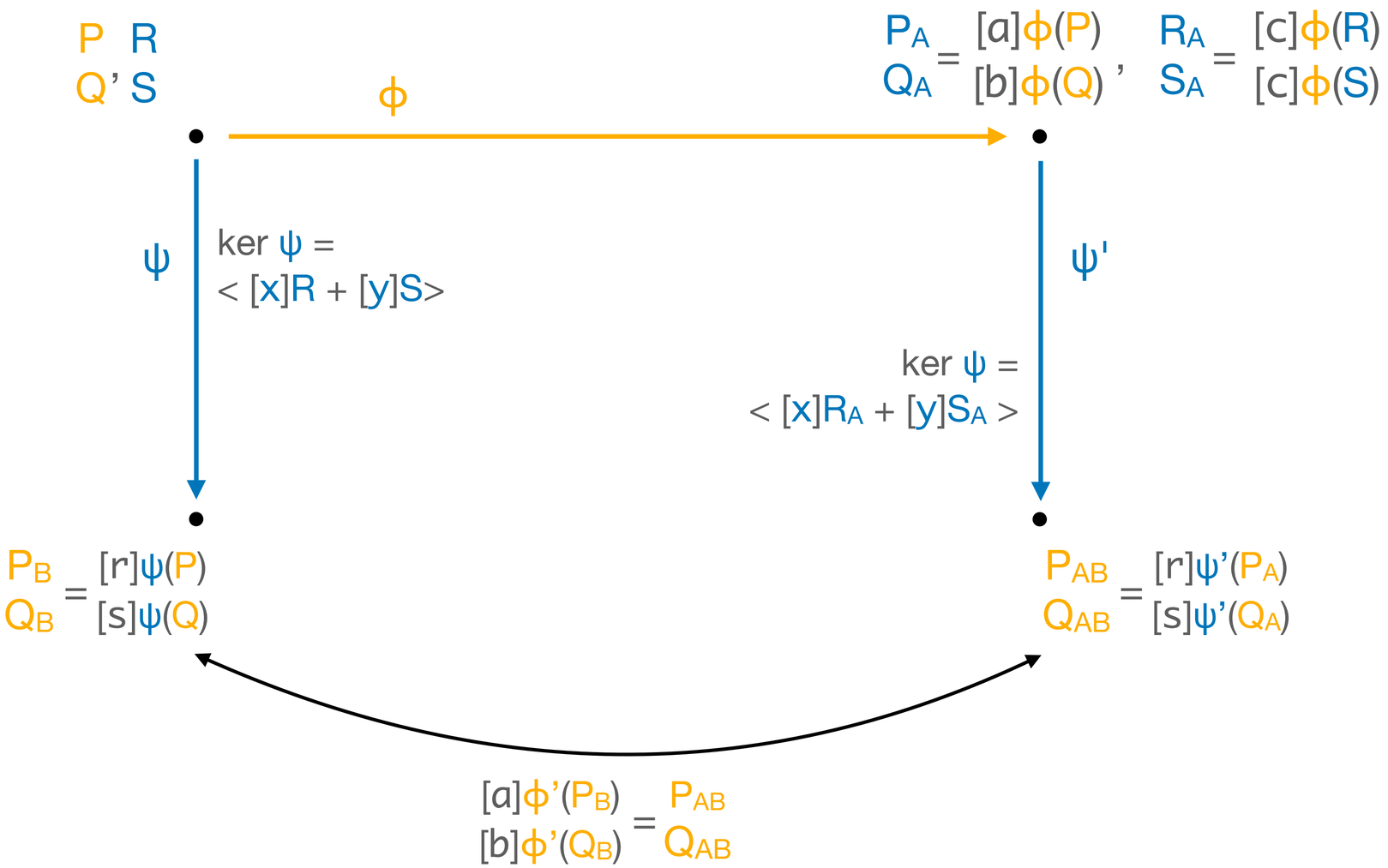

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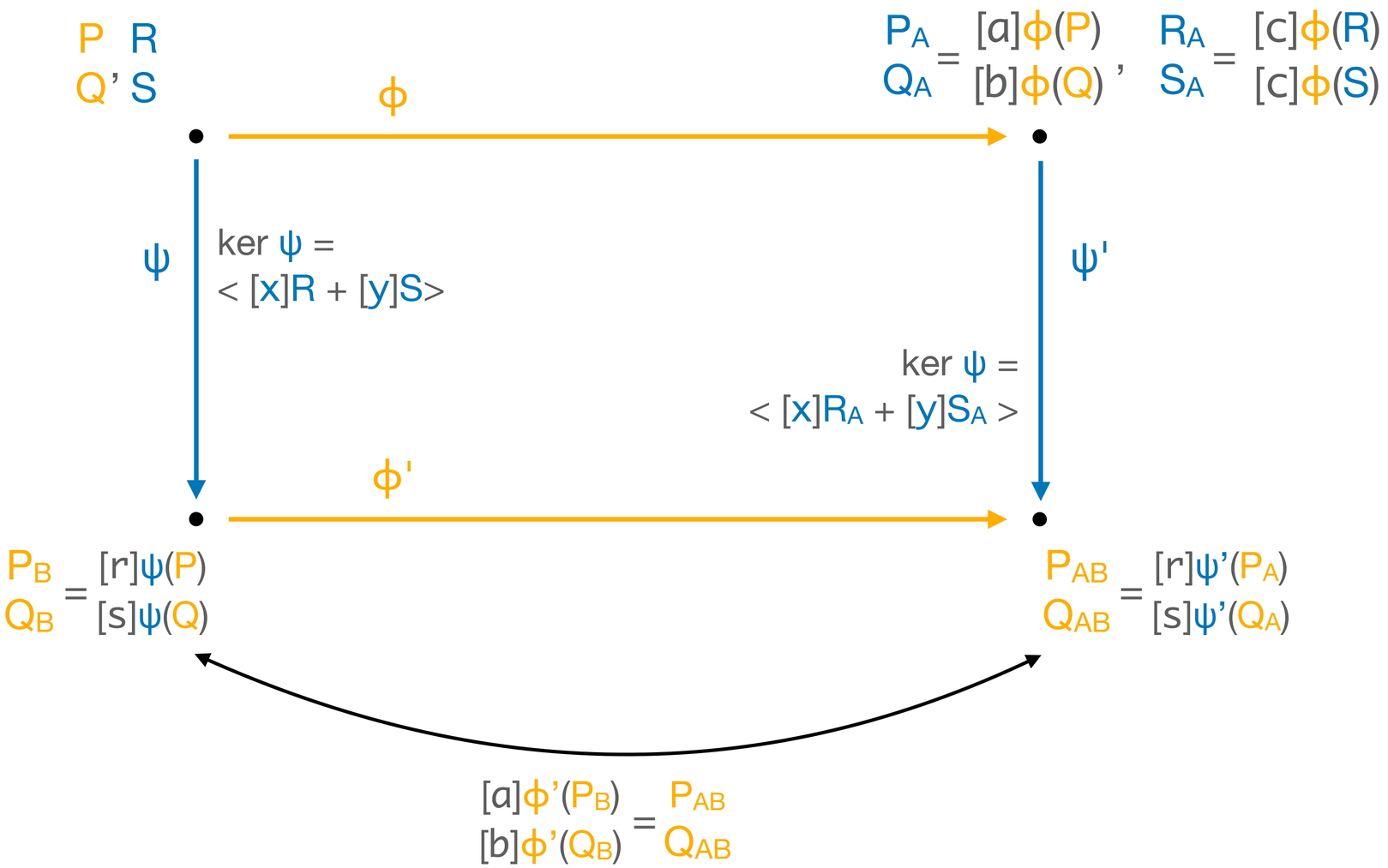

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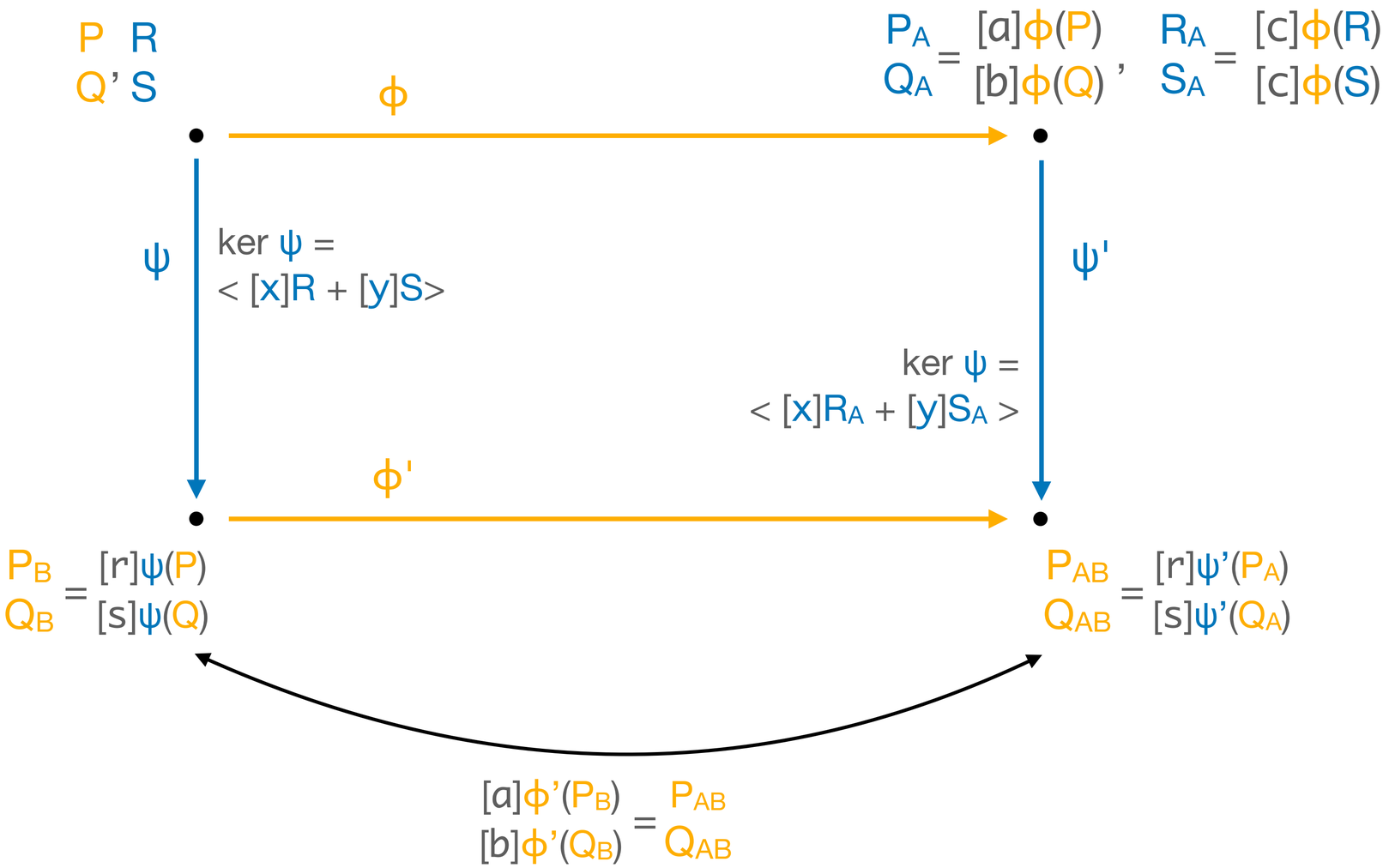

public/private HD representations

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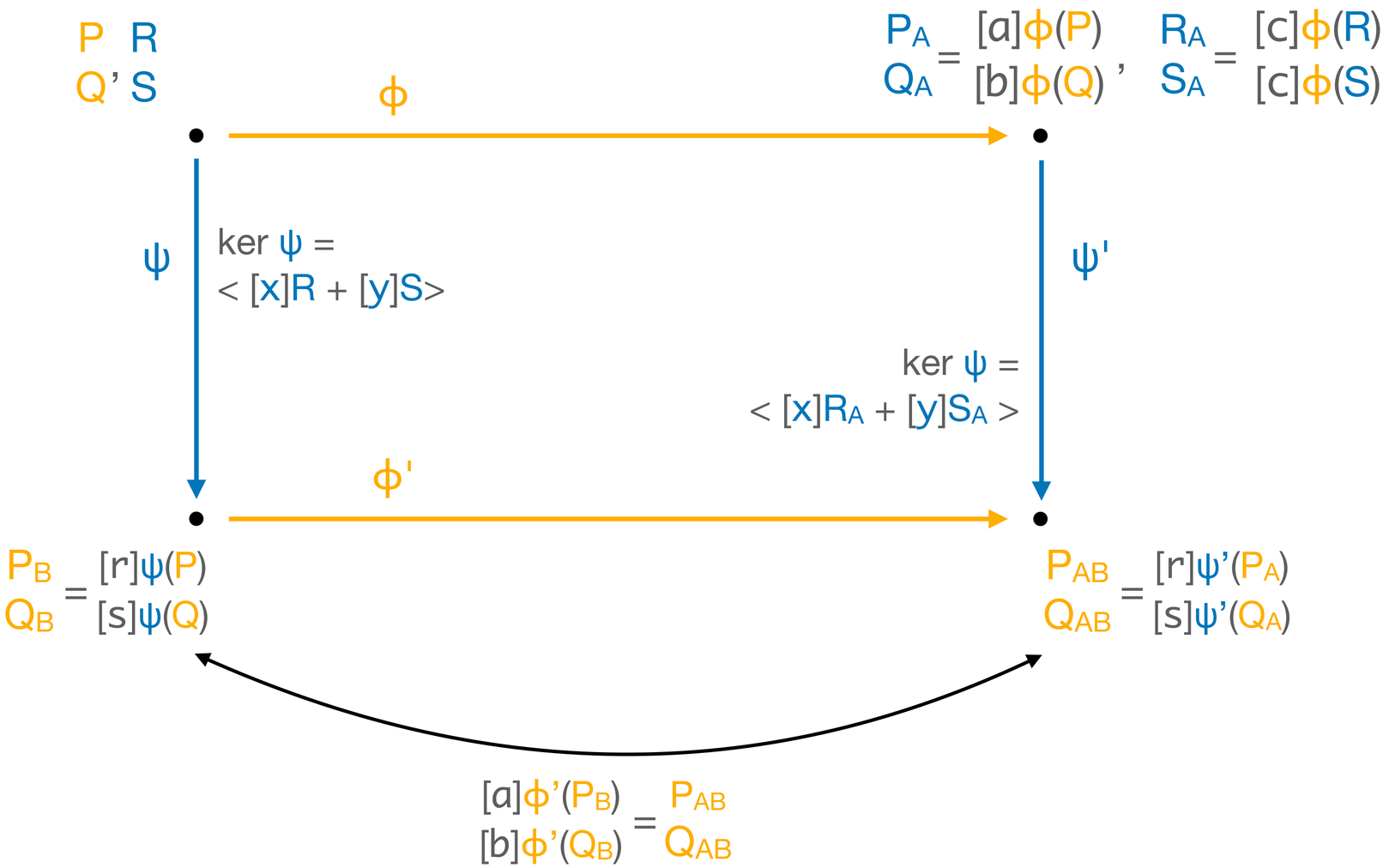

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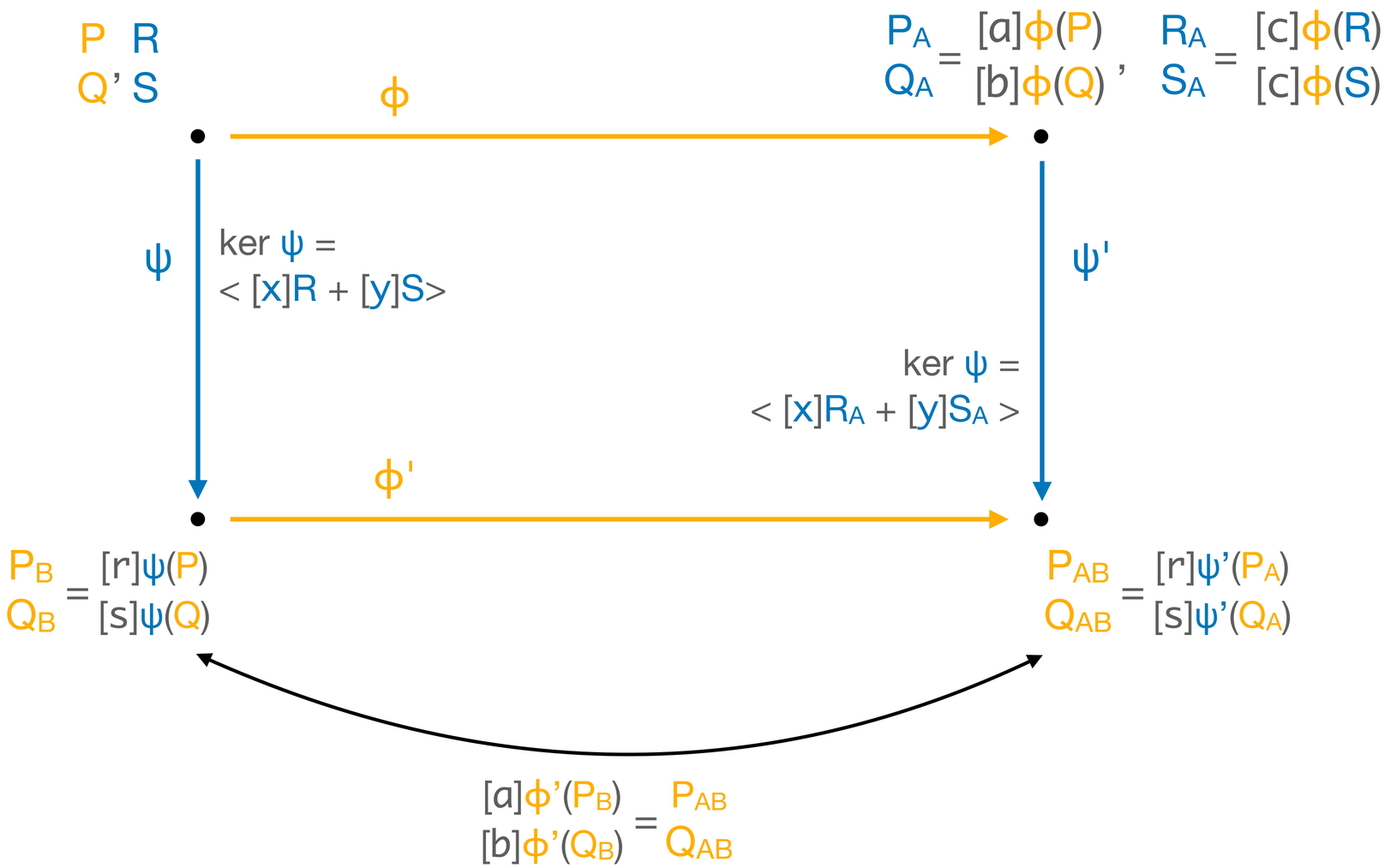

public/private HD representations

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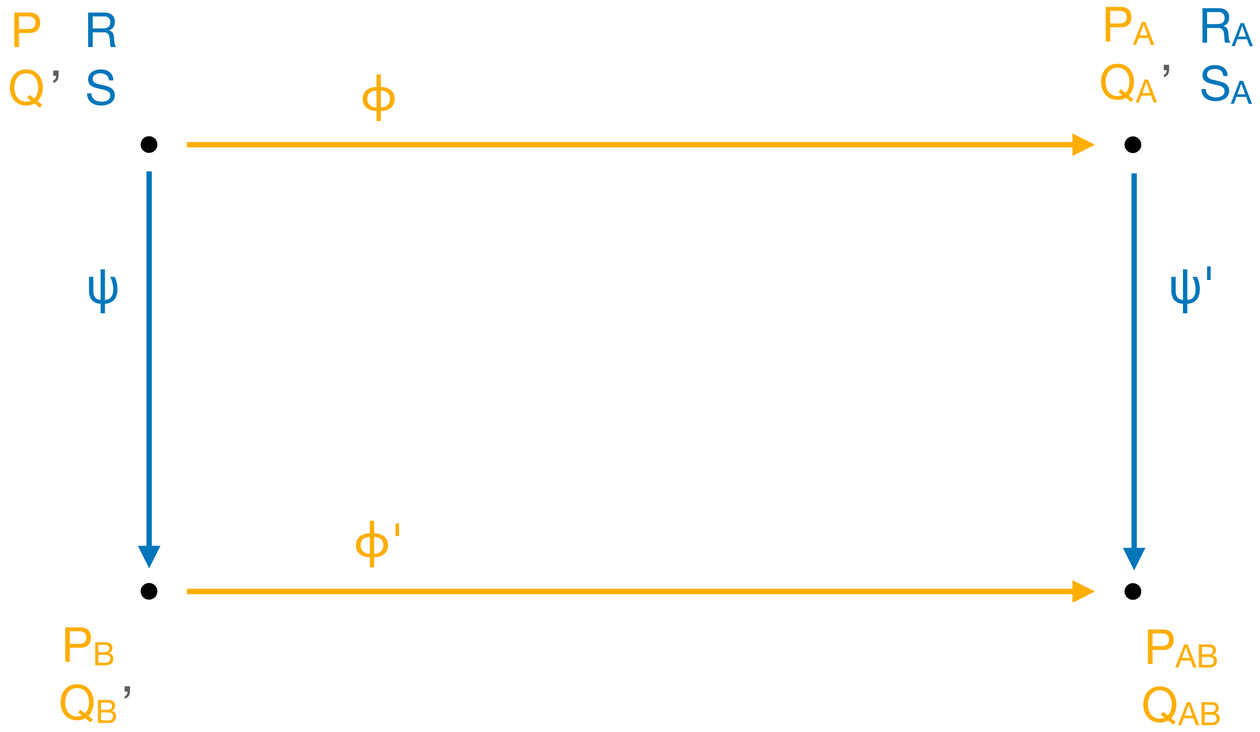
parallel isogenies like in (C)SIDH

A shared secret?



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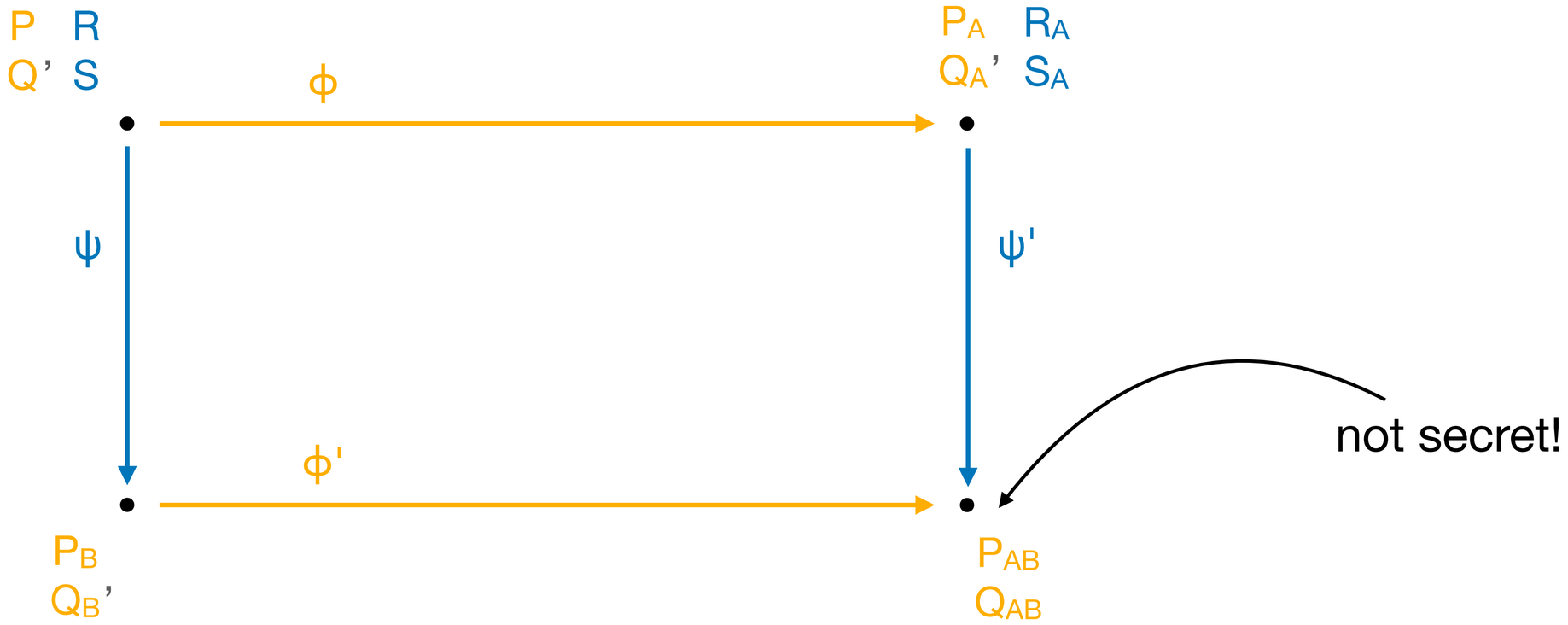

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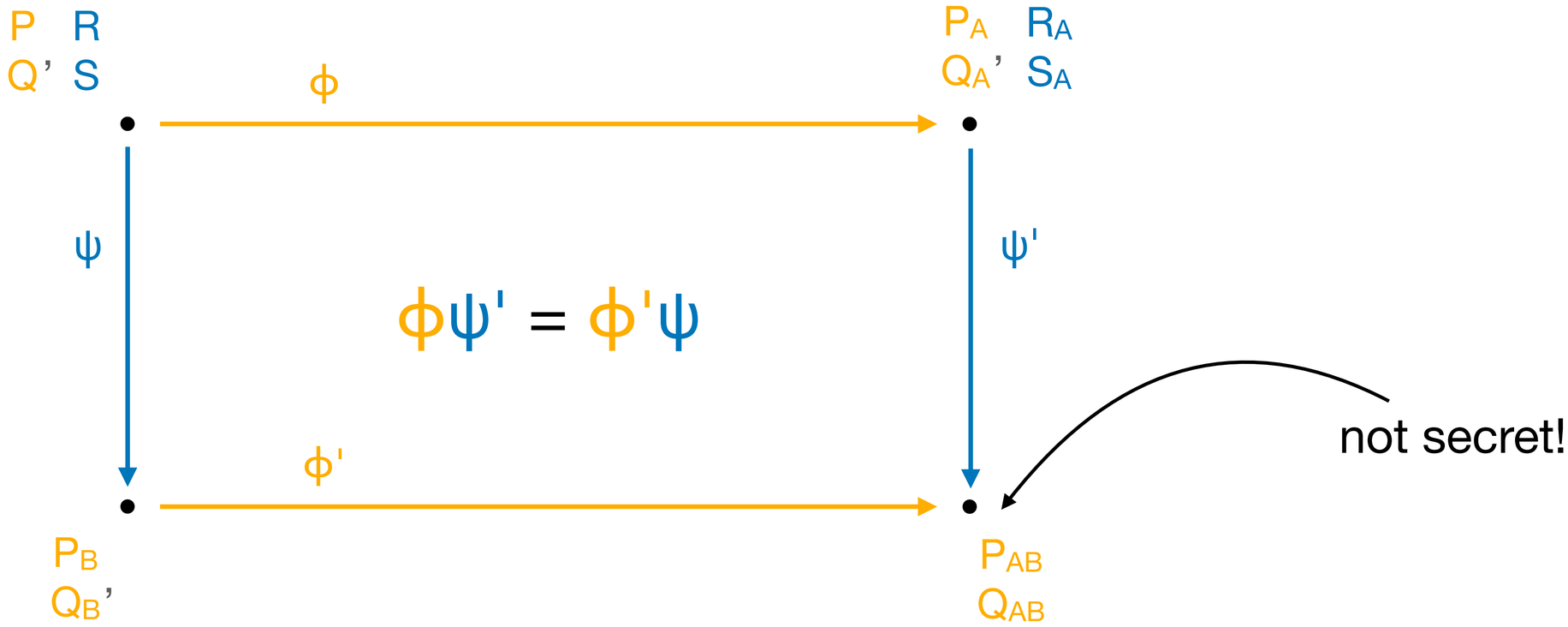

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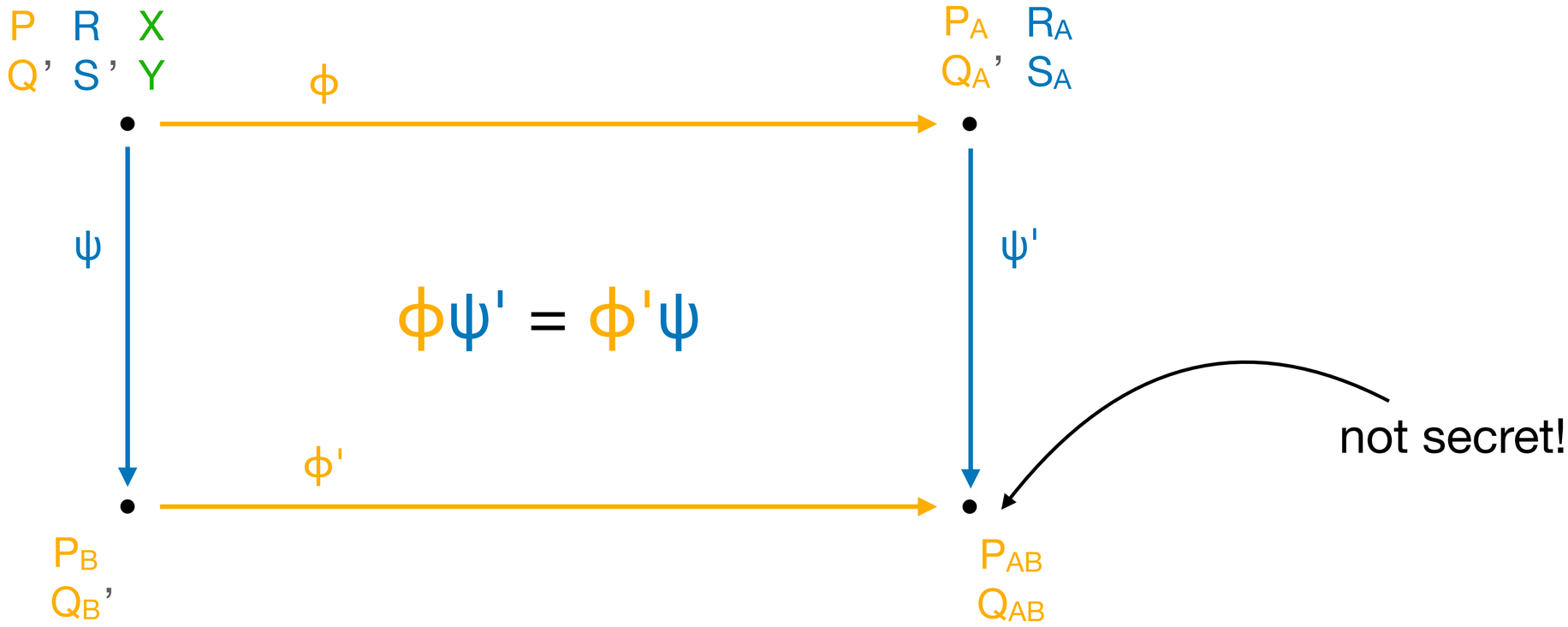

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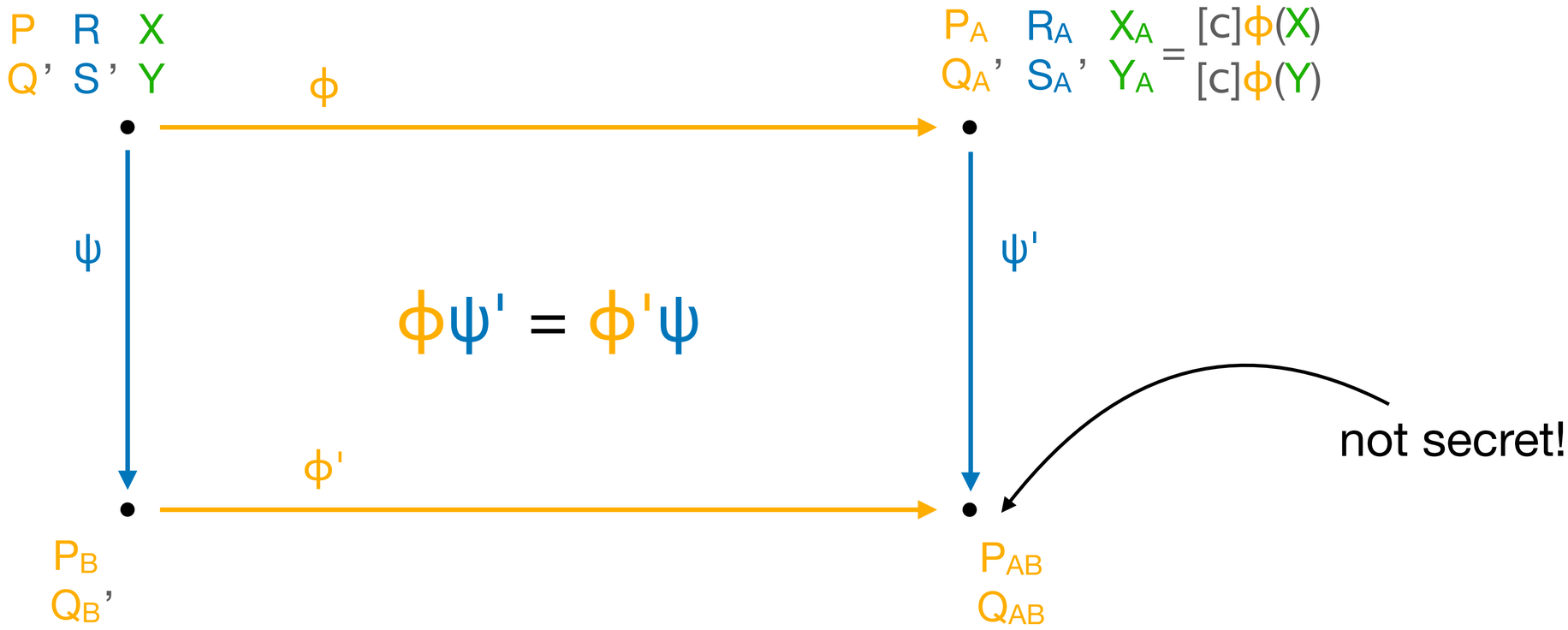

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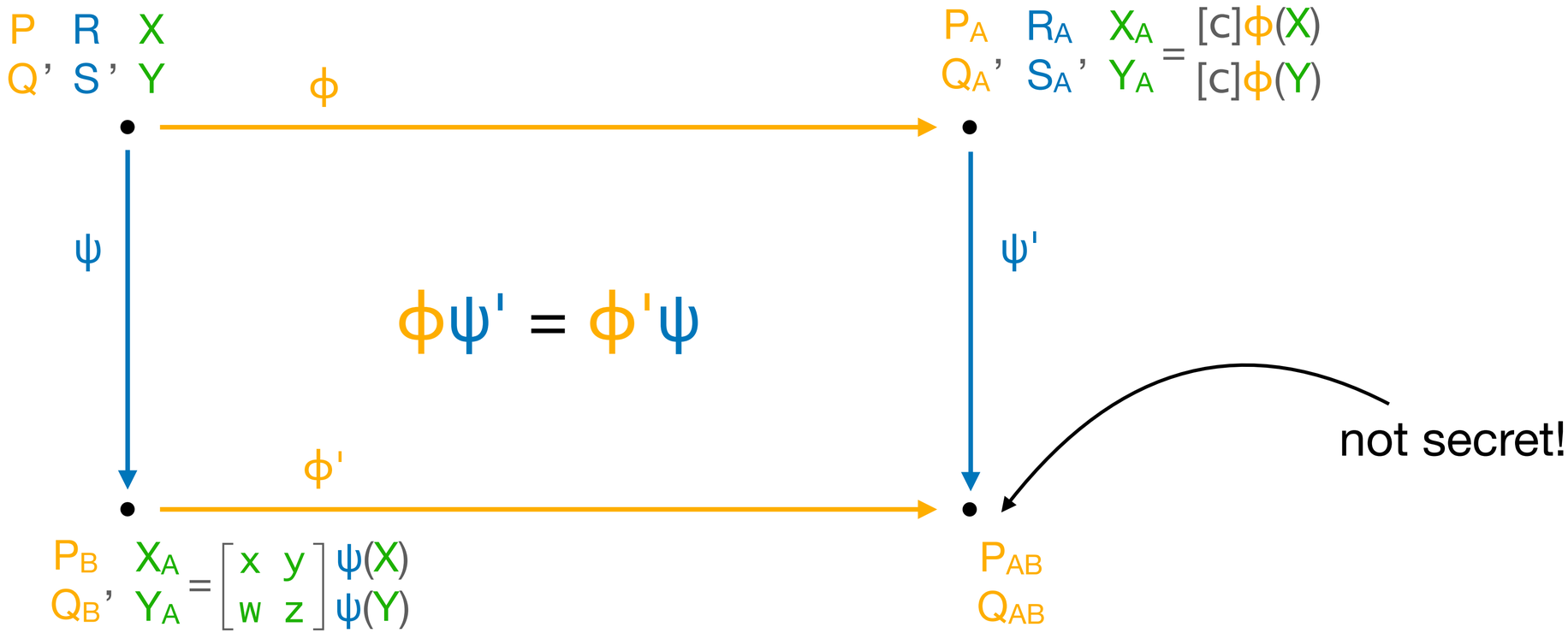

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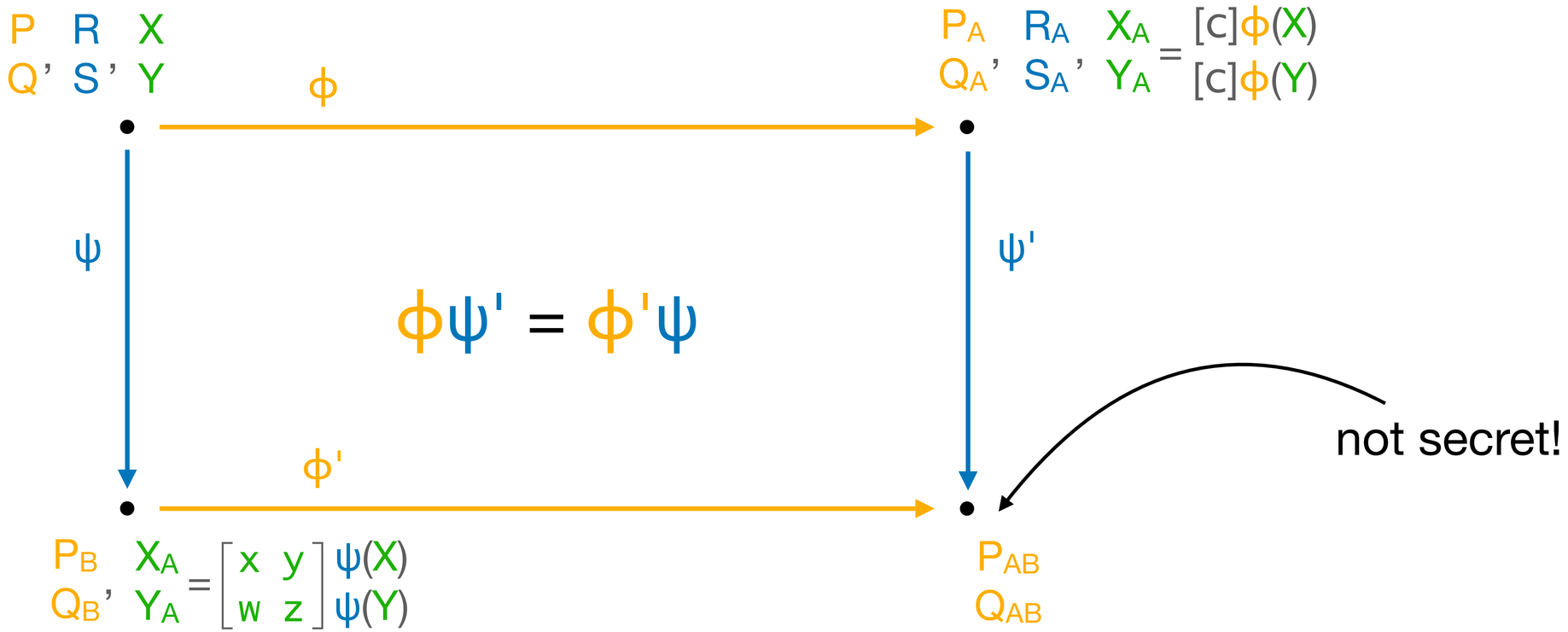
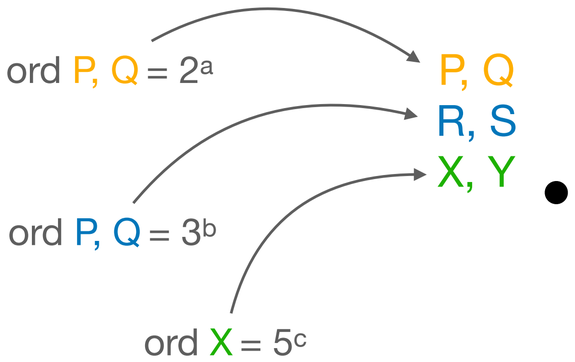


$$X_{AB} = \begin{bmatrix} x & y \\ w & z \end{bmatrix} \psi'(X_A) = [c]\phi'(X_B)$$
$$Y_{AB} = \begin{bmatrix} x & y \\ w & z \end{bmatrix} \psi'(Y_A) = [c]\phi'(Y_B)$$

The POKE PKE

P, Q
R, S
X, Y .

The POKE PKE



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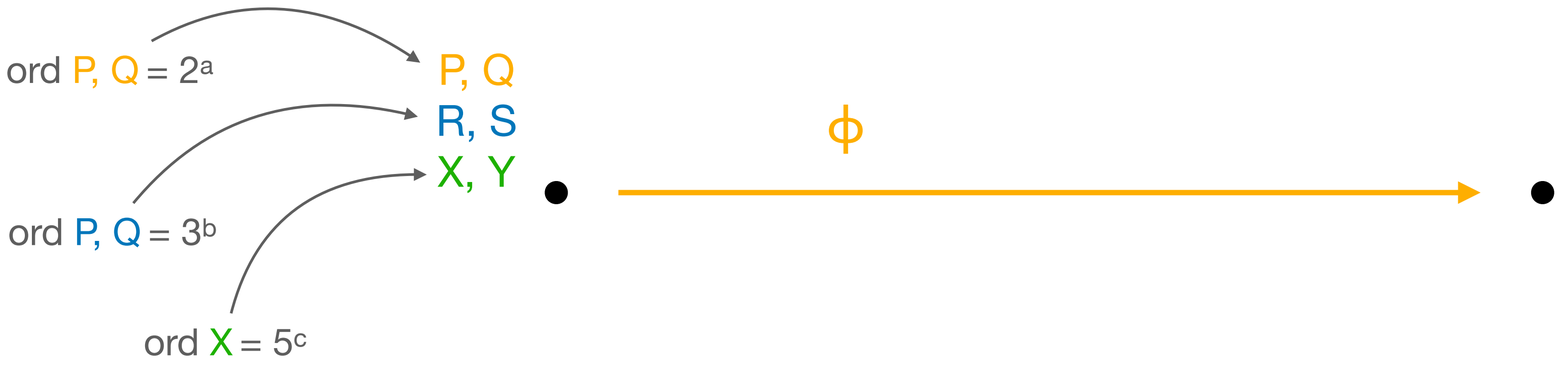

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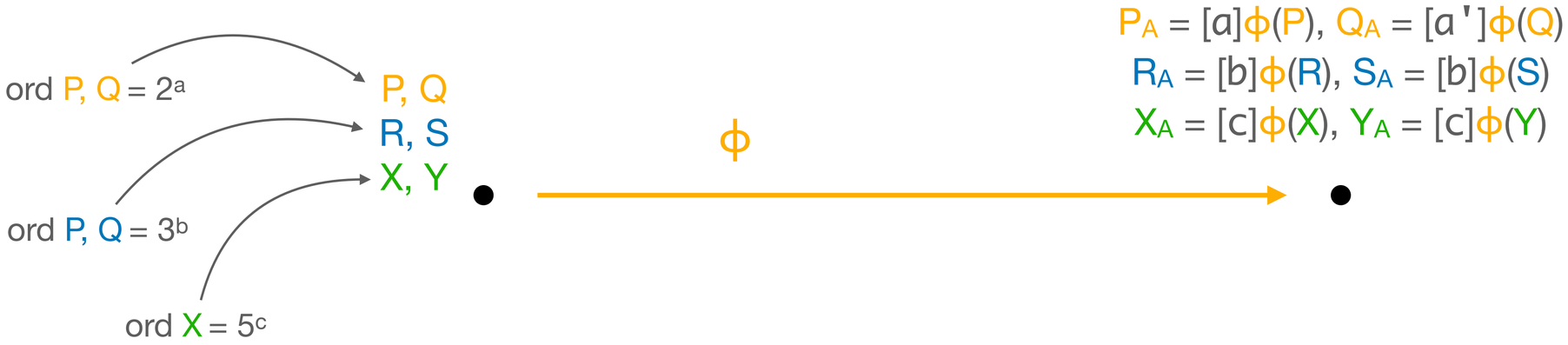
ord P, Q = 2^a
ord P, Q = 3^b
ord X = 5^c
P, Q
R, S
X, Y
ϕ
P_A = [a]ϕ(P), Q_A = [a']ϕ(Q)
R_A = [b]ϕ(R), S_A = [b]ϕ(S)
X_A = [c]ϕ(X), Y_A = [c]ϕ(Y)

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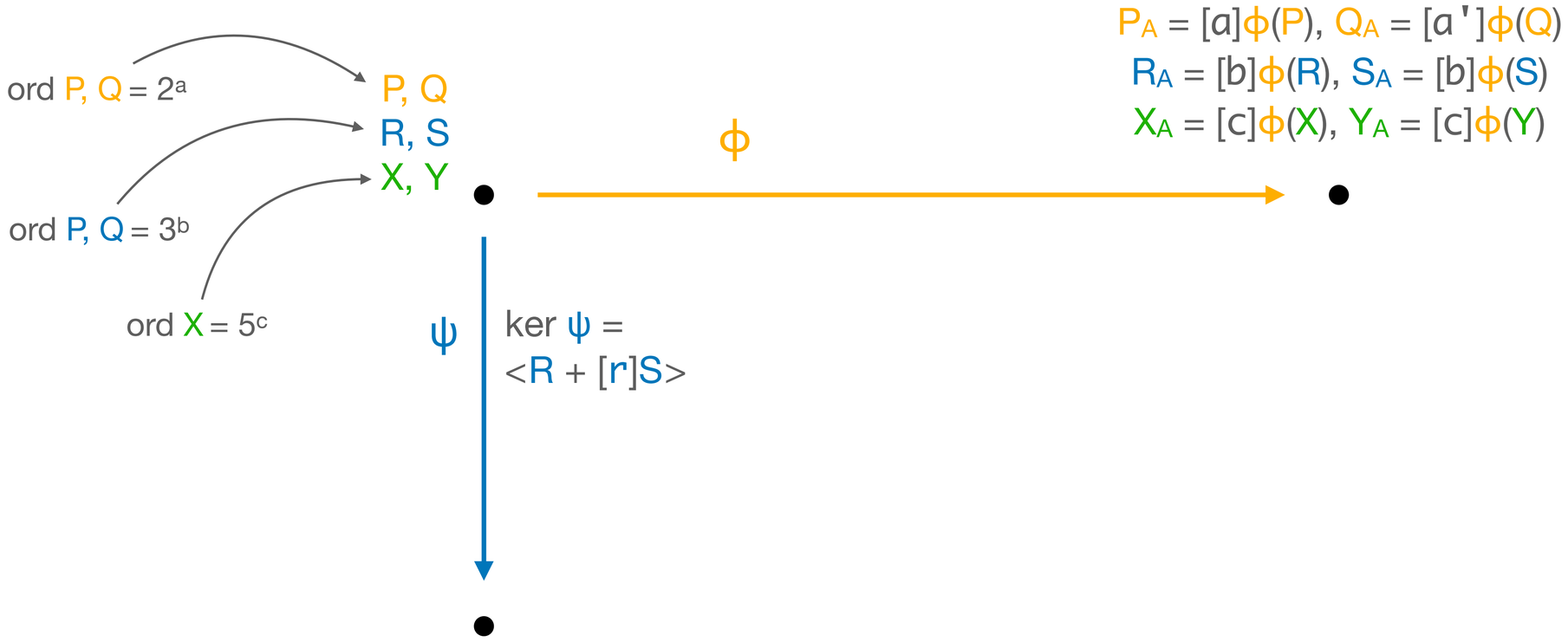
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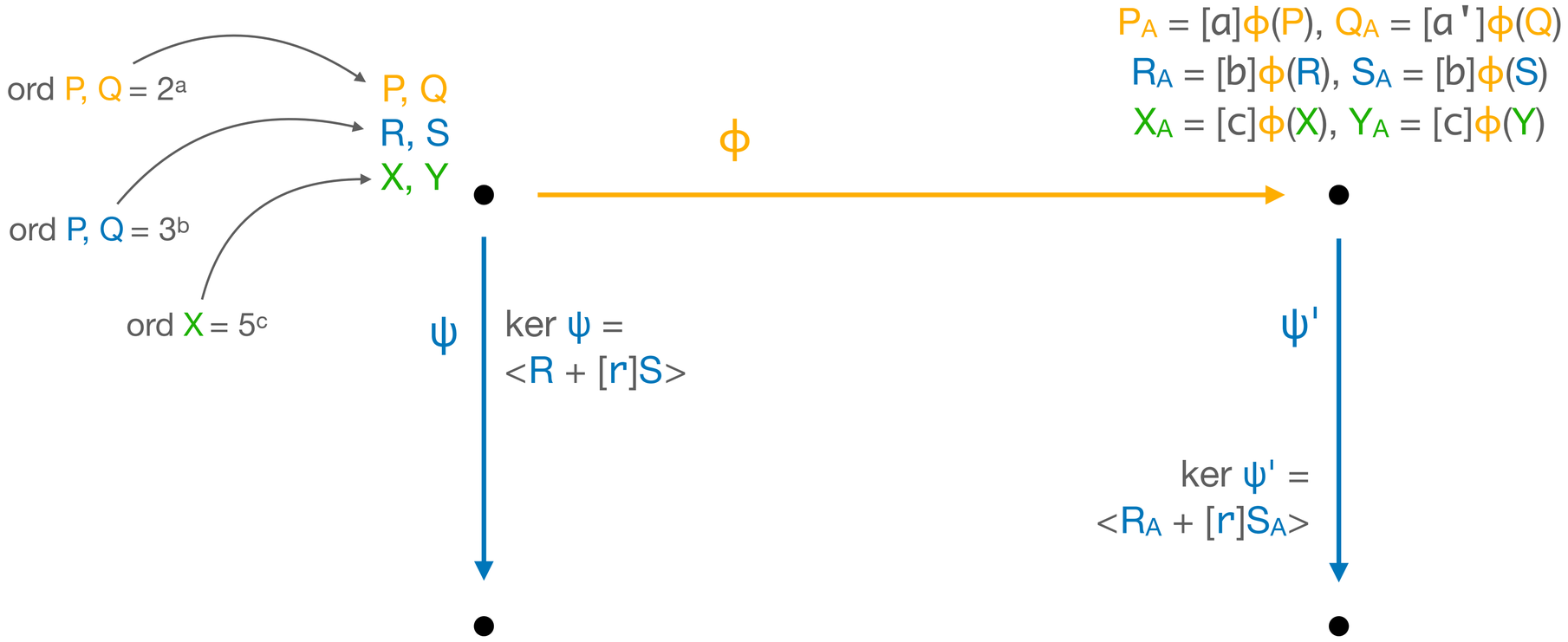
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ψ'
ker ψ' =
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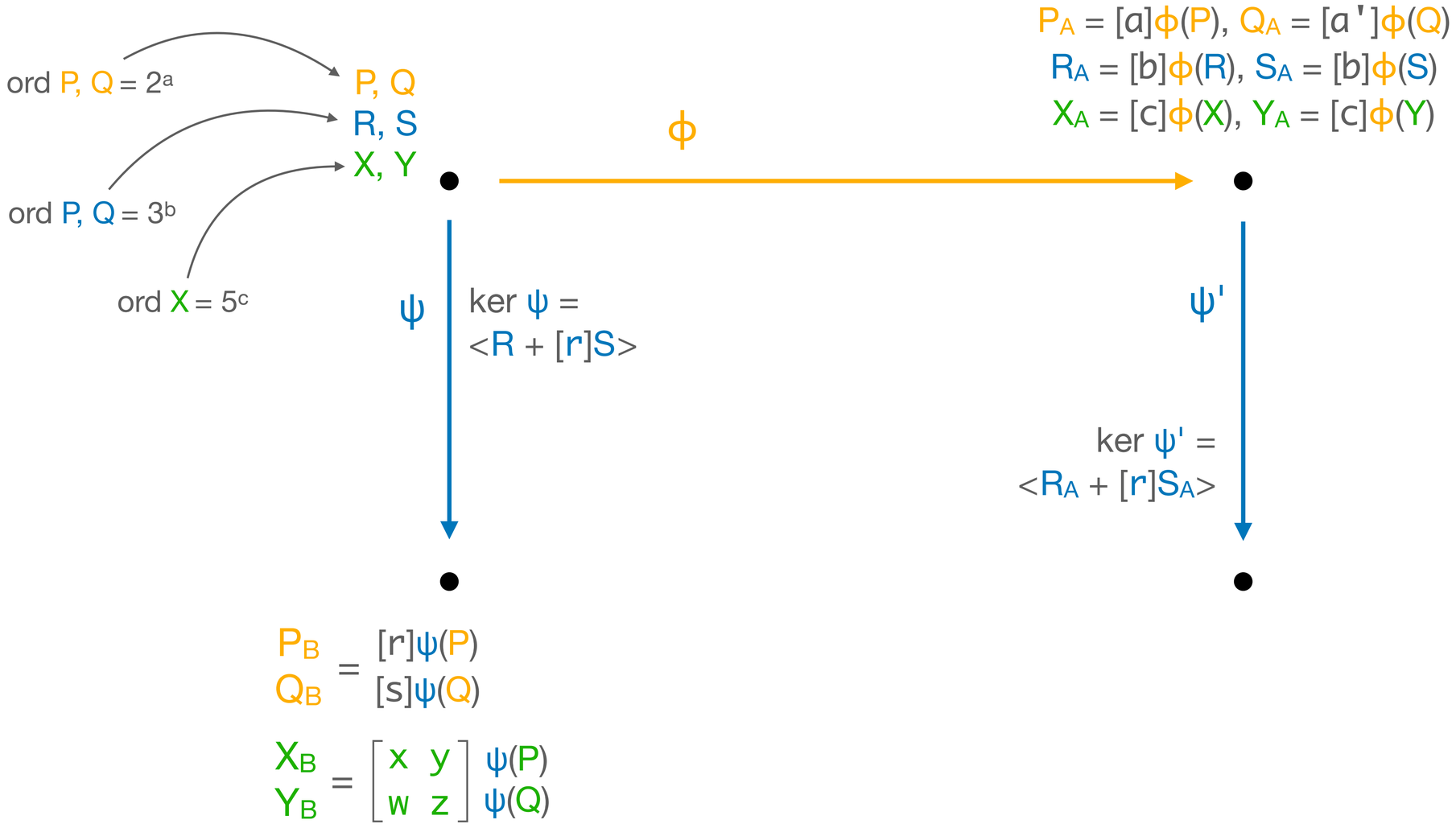
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Q_B = [s]ψ(Q)
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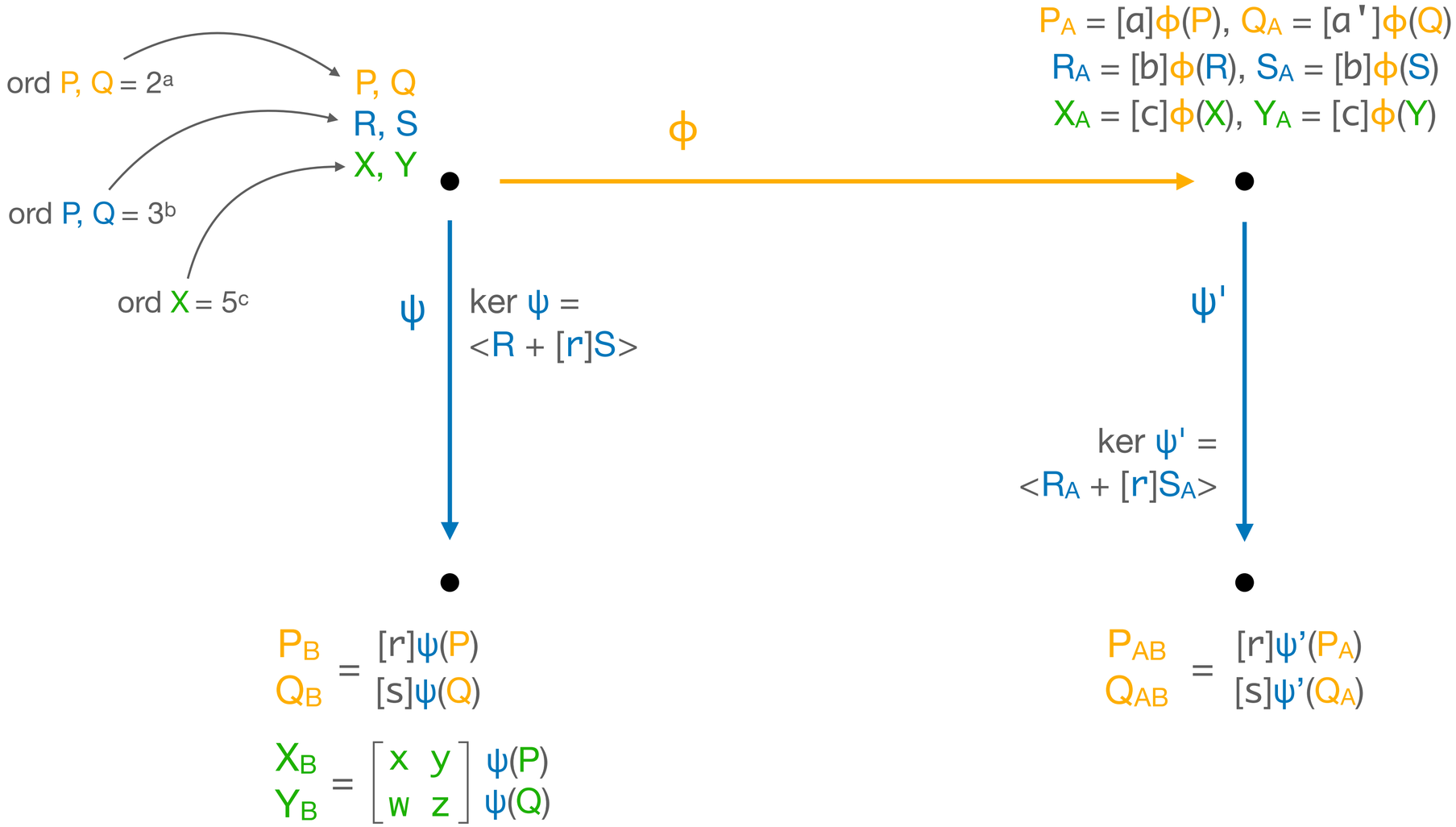
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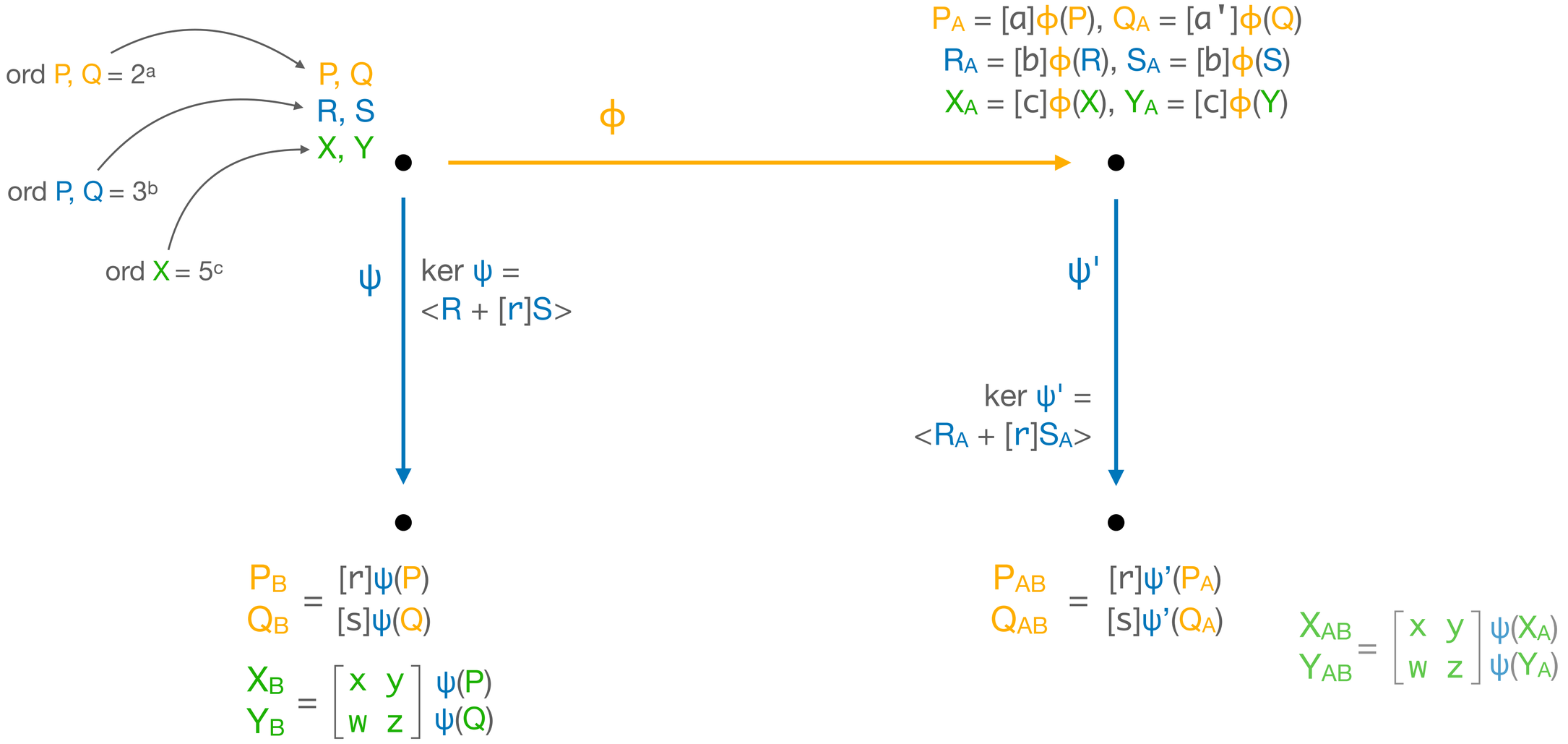
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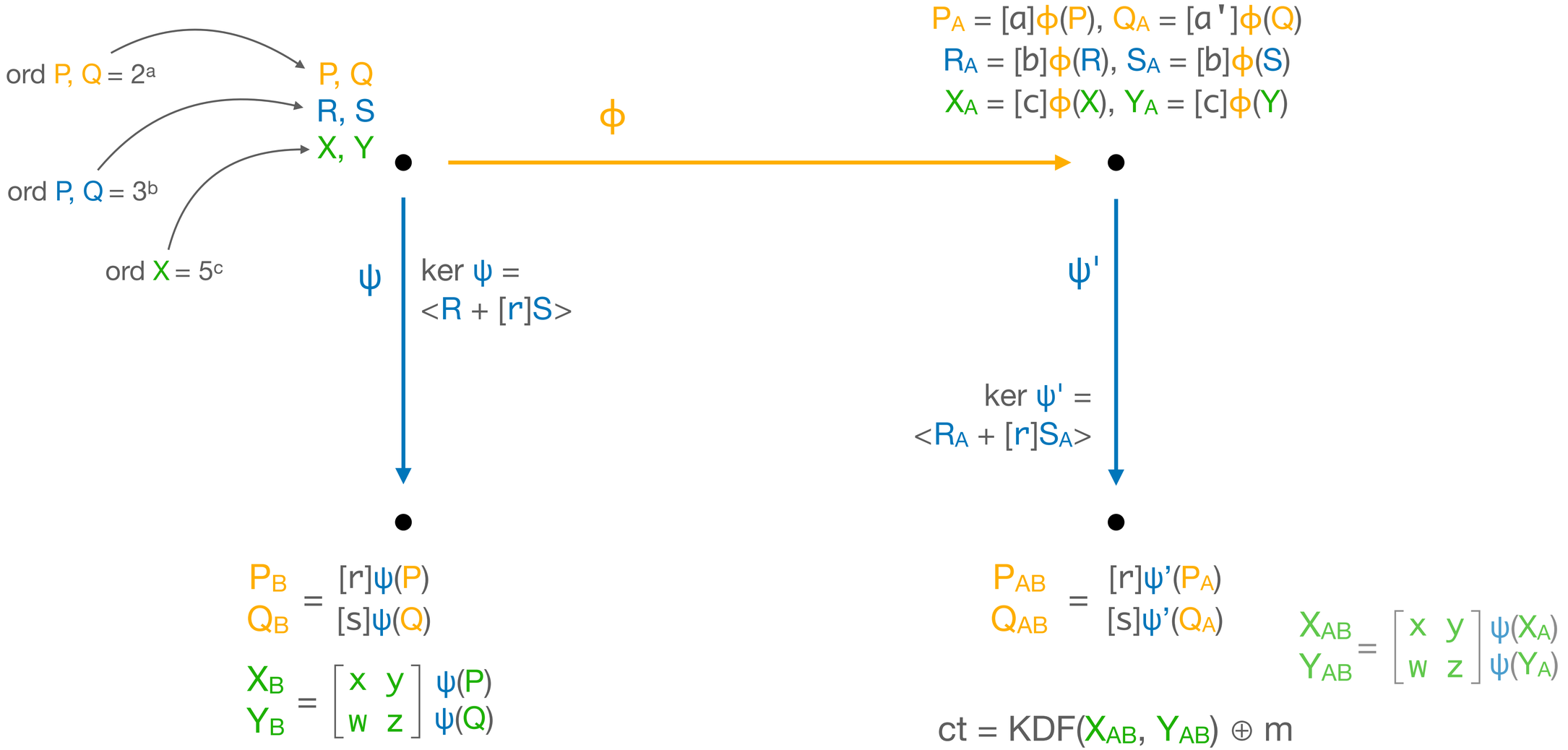
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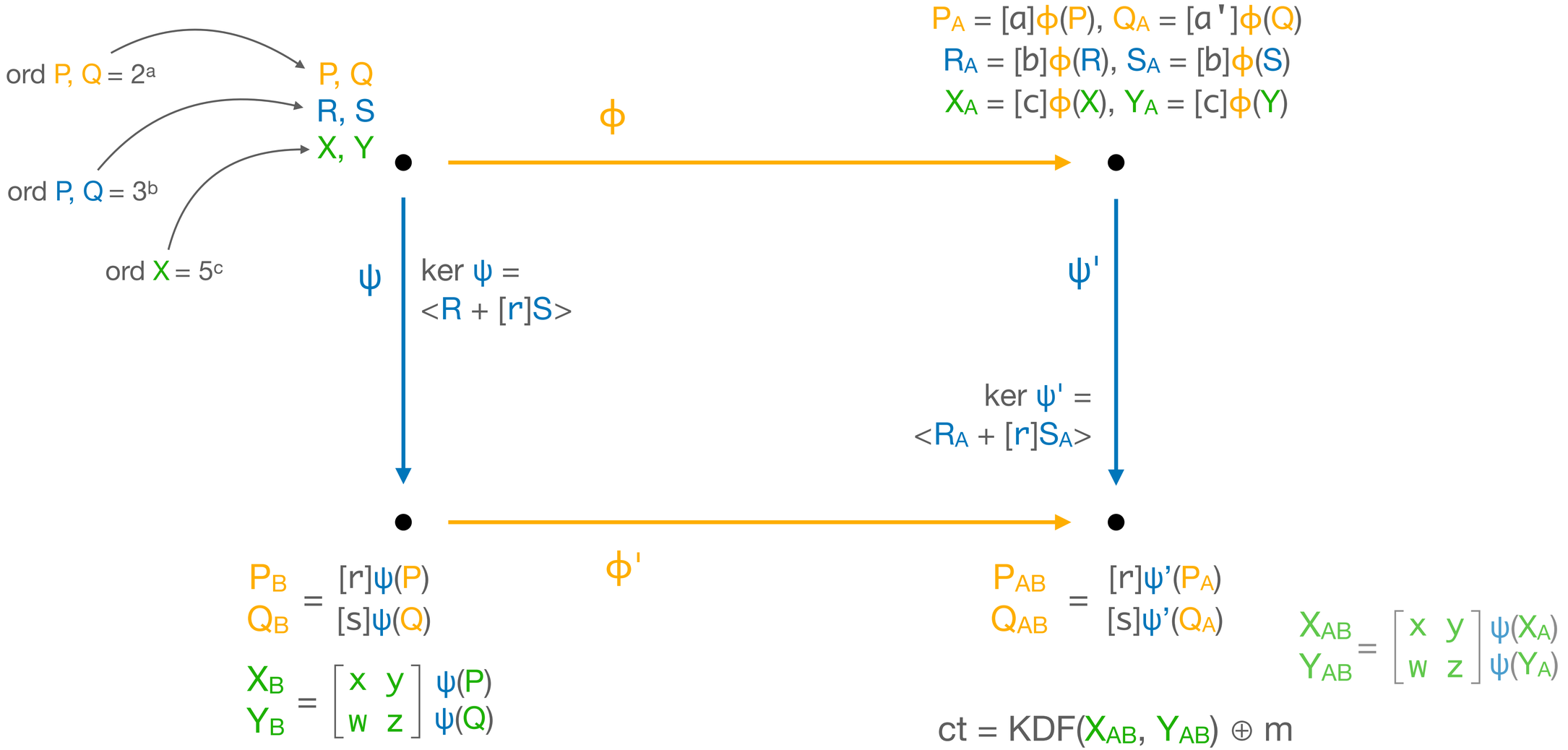
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Security

Formally: security depends on CDH-type assumption

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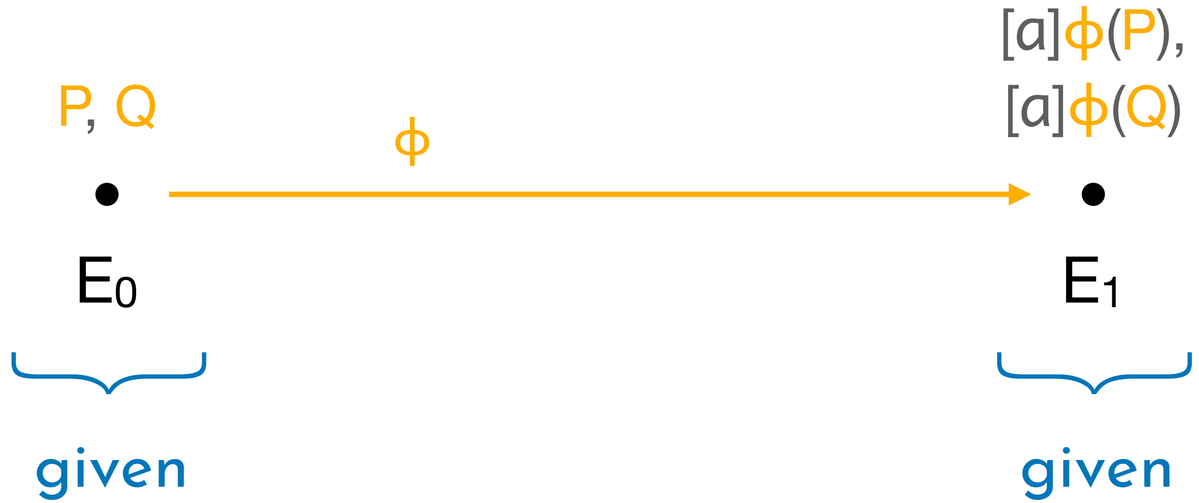
Unknown Degree Isogeny Problem:



Results

Parameters

- 2^{λ} : order of torsion points for HD repr (P, Q)
- $3^{b} \approx 2^{2\lambda}$: degree of smooth isogenies (R, S)
- $5^{c} \approx 2^{\lambda/3}$: order of validation points (X, Y)

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Sizes

- Public key: 6 log p (≈324 bytes)
- Ciphertext: 12 log p (≈648 bytes)

- Both (comp.): 5 log p (≈270 bytes)

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POKÉ is the fastest isogeny-based PKE

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Paper

https://ia.cr/2024/624

POC source code

https://github.com/andreavico/POKE-PKE