Combining PAKEs for hybrid security

Merged talk based on concurrent works by

PAKE Combiners and Efficient Post-Quantum Instantiations Julia Hesse¹ Michael Rosenberg²

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Hybrid Password Authentication Key Exchange in the UC Framework

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ia.cr/2024/{1621,1630}









Two parties use a password to establish a secure shared secret





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A PAKE combiner takes 2 PAKEs and produces a new PAKE

Weird! Cryptographic statements where nothing is high entropy!







We have classical and PQ PAKEs

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But the PQ ones are new

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note

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(Appx. A.1)
   result incorrect
 (Sects. 3.2 and 3.3,
     Appx. A.4)
   result incorrect
      (Sect. 3.2)
  and Appx. A.4)
 result ambiguous,
unclear if OEKE-PRF
   or OEKE-RO;
   security proof
incorrect (Sect. 3.1);
either way result also
incorrect (Appx. A.3)
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result incorrect (Sect. 3.3, Appxs. A.1 and A.3)





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Recommended by French ANSSI and German BSI

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How do you make a hybrid PAKE?

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Parallel combiner (ParComb)



1-round PAKE + 1-round PAKE \Rightarrow 1-round PAKE

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Hybrid: can plug in an existing classical and PQ PAKEs

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Cheap: overhead of at most 2 hashes

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- Hybrid: can plug in an existing classical and PQ PAKEs
- **Yields other PAKE flavors:** PAKE \Rightarrow aPAKE, iPAKE

How not to make a hybrid PAKE

EKE w/ key conf.

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KEM combiners: just run 2 KEMs and hash the outputs

EKE w/ key conf. Diffie-Hellman with pw-encrypted shares

EKE w/ key conf. **Diffie-Hellman with** r←ℾ R' := Enc_{pw}(rG) **pw-encrypted shares**

How not to make a hybrid PAKE

KEM combiners: just run 2 KEMs and hash the outputs

 $r \leftarrow \mathbb{F}$ $R' := Enc_{pw}(rG)$ $\frac{EKE w/key conf.}{Diffie-Hellman with}$ $\frac{F}{R' := Enc_{pw}(rG)}$

 $r \leftarrow \mathbb{F} \qquad R' := Enc_{pw}(rG) \xrightarrow{R'}$

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CAKE (KEM-based) w/ key conf. (sk, pk) ← Keygen() R' R' := Enc_{pw}(pk) $pk := Dec_{pw}(R')$ $(ct, K_2) \leftarrow Encap(pk)$ $S' := Enc_{pw}(ct)$ $\tau := MAC(K_2, 0)$ **S', τ** ct := Dec_{pw}(S') K₂ := Decap(sk, ct) Verify T





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           Verify T
K := H(K_1, K_2)
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 $r \leftarrow \mathbb{F}$ R' := Enc_{pw}(rG) $\xrightarrow{R'}$ R := Dec_{pw}(R') s ← F $K_1 := DH(s, R)$ S := $Dec_{pw}(S')$ \leftarrow S', τ T := $Enc_{pw}(SG)$ τ := MAC(K₁, 0) $K_1 := DH(r, S)$ Verify τ If dlog is easy, τ lets you guess/check pw

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                          ct := Dec<sub>pw</sub>(S')
                          K<sub>2</sub> := Decap(sk, ct)
                          Verify \tau
                                          Ditto if KEM is broken
This hybrid is the WEAKER of the two!
```





How else not to make a hybrid PAKE

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CAKE (KEM-based) with key conf.

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How else not to make a hybrid PAKE

CAKE is KEM-based. Just use a hybrid KEM

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EKE w/ key conf.

 $r \leftarrow \mathbb{F}$ $R' := Enc_{pw}(rG) \xrightarrow{R'} R := Dec_{pw}(R')$ $s \leftarrow \mathbb{F}$ $K_1 := DH(s, R)$ $S' := Enc_{pw}(sG)$ T := DH(r, S) $Verify \tau$

K := H(

$$CAKE (KEM-based) w/ key co$$

$$(sk, pk) \leftarrow Keygen()$$

$$R' := Enc_{pw}(pk) \xrightarrow{R'} pk := Dec_{pw}(R')$$

$$(ct, K_2) \leftarrow Encap()$$

$$S' := Enc_{pw}(ct)$$

$$Ct := Dec_{pw}(S') \xleftarrow{S', \tau} \tau := MAC(K_2, 0)$$

$$K_2 := Decap(sk, ct)$$

$$Verify \tau$$

$$K_1, K_2)$$







K := H

$$CAKE (KEM-based) w/ key consistent of the second structure of the second str$$





EKE w/ key conf. r ← F $R' := Enc_{pw}(rG) \xrightarrow{R'} R := Dec_{pw}(R')$ s ← F $K_1 := DH(s, R)$ $S' := Enc_{pw}(sG)$ $S := Dec_{pw}(S') \leftarrow S', \tau$ τ := MAC(K₁, 0) $K_1 := DH(r, S)$ Verify τ

Observation: it seems **τ was the only issue** with the parallel example

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$$\mathbf{R}' := \operatorname{Enc}_{pw}(\mathbf{r}G) \xrightarrow{\mathbf{R}'} \mathbf{R} := \operatorname{Dec}_{pw}(\mathbf{R}')$$

$$\mathbf{s} \leftarrow \mathbb{F}$$

$$K_1 := \operatorname{DH}(\mathbf{s}, \mathbf{R})$$

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$$unif$$

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We'll call this **statistical password hiding**, or (full) DHtype PAKE

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EKE, SPAKE2, CPace are stat. pw-hiding

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 $K := H(K_1, K_2, tr)$

Requirement: PAKE₁ and PAKE₂ are statistically passwordhiding



 $K := H(K_1, K_2, tr)$

Requirement: PAKE₁ and PAKE₂ are statistically passwordhiding



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Requirement: PAKE₁ and PAKE₂ are statistically passwordhiding



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Requirement: PAKE₁ and PAKE₂ are statistically passwordhiding



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Requirement: PAKE₁ and PAKE₂ are statistically passwordhiding


Building ParComb

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PAKE is still secure



Theorem 1: Let PAKE₁ and PAKE₂ be 1-round statistically password-hiding, then



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- **Theorem 1:** Let PAKE₁ and PAKE₂ be 1-round statistically password-hiding, then
- 1. **PAKE**₁ is $\mathcal{F}_{PAKE} \Rightarrow ParComb[PAKE_1, PAKE_2]$ is \mathcal{F}_{PAKE}
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No 1-round statistically password-hiding PQ scheme realizing $\mathcal{F}_{\text{PAKE}}$ exists

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Theorem 2:



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Theorem 2:

1. **PAKE**₁ is $\mathcal{F}_{lePAKE} \Rightarrow ParComb[PAKE_1, PAKE_2]$ is \mathcal{F}_{lePAKE}



No 1-round statistically passwordhiding PQ scheme realizing \mathcal{F}_{PAKE} exists PAKE₂] is \mathcal{F}_{IePAKE}

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Theorem 2:

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No 1-round statistically passwordhiding PQ scheme realizing $\mathcal{F}_{\text{PAKE}}$ exists

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No 1-round statistically passwordhiding PQ scheme realizing \mathcal{F}_{PAKE} exists

PAKE₁































K := H(0, K₂, tr)











Session key is predictable! Input more into **PAKE**₂













 $Z := H(pw, K_1, tr)$





Z := H(pw, K₁, tr)






 $K := H(K_1, K_2, tr)$



 $K := H(K_1, K_2, tr)$





Combined PAKE is still secure!













 $Z := H(pw, K_1, tr)$



 $K := H(K_1, K_2, tr)$















Session key is secure



 $Z := H(pw, K_1, tr)$





Session key is secure If Z leaks, the **success of PAKE**₁ leaks





- - New PAKE₂ property: Z := H(pw, K₁, tr) statistical preshared Z key equality hiding.



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is statistically hidden

- **Session key is secure** If Z leaks, the success of PAKE₁ leaks
 - New PAKE₂ property: Z := H(pw, K₁, tr) statistical preshared key equality hiding. **K**₂
 - If inputs are high entropy, their equality is statistically hidden

EKE-PRF and CAKE are stat. PSK-equality-hiding

pw





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Theorem 4: Let PAKE₁ be 1-round stat. pw-hiding, and PAKE₂ be stat. PSK equality-hiding. Then



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Bonus lemma: PAKE₁ $\mathcal{F}_{\text{lePAKE}} \Rightarrow \mathcal{F}_{\text{rlePAKE}}$

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Theorem 5: Let PAKE₁ be 1-round stat. pw-hiding, and **PAKE₂ be any PAKE**. Then



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 $K := H(K_1, K_2, tr)$

Bonus lemma: $PAKE_1 \mathcal{F}_{lePAKE} \Rightarrow \mathcal{F}_{rlePAKE}$

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 $K := H(K_1, K_2, tr)$

Bonus lemma: $PAKE_1 \mathcal{F}_{lePAKE} \Rightarrow \mathcal{F}_{rlePAKE}$



First construction of a hybrid PAKE

First construction of a hybrid PAKE

Presented **two methods**, parallel and sequential

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Sequential has **mild assumptions**, working with efficient existing PAKEs

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Combiner	PAKE ₁	PAKE ₂	Combined	Мс
ParComb	\mathcal{F}_{PAKE} +PH	\mathcal{F}_{PAKE} +PH	FPAKE	R
	F blePAKE+PH	Flepake+PH	FblePAKE	R
SeqComb	<i>Э_{РАКЕ}+РН</i>	<i> F</i> РАКЕ+РЕН	FPAKE	R
	ℱ _{lePAKE} +PH	<i>Э_{РАКЕ}+РЕН</i>	FrlePAKE	R
	<i></i> F _{РАКЕ} +РН	FPAKE	(F _{PAKE})	Q

+PH means stat. pw-hiding +PEH means stat. PSK-equality-hiding (F) means nearly UC-realizes



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SeqComb	<i>Э_{РАКЕ}+РН</i>	<i> F</i> РАКЕ+РЕН	FPAKE	R
	ℱ _{lePAKE} +PH	\mathcal{F}_{PAKE} +PEH	FrlePAKE	R
	<i></i> F _{РАКЕ} +РН	FPAKE	(F _{PAKE})	QI

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https://www.ietf.org/archive/id/ draft-vos-cfrg-pqpake-00.html



First construction of a **hybrid PAKE**

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Come find us!

Combiner	PAKE ₁	PAKE ₂	Combined	Мс
ParComb	ℱ _{РАКЕ} +РН	ℱ _{РАКЕ} +РН	FPAKE	R
	F blePAKE+PH	Flepake+PH	FblePAKE	R
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	Flepake+PH	<i> F</i> _{РАКЕ} +РЕН	FrlePAKE	R
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