A Meta-Complexity Characterisation of Quantum Cryptography

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Joint work with
Eli Goldin (NYU), Matthew Gray (Oxford), Peter Hall (NYU)



EUROCRYPT 2025

The holy grail of cryptography?

Minimal complexity-theoretic assumption required for cryptography?

- $P \neq NP$: necessary; sufficient?
  - Specific constructions: Factoring, LWE, etc.
- Quantum complication: PRSs may exist even if $BQP = QMA$ (in particular, $\nexists$ pqOWFs) [Kretschmer, TQC 2021]
  - The classical "holy grail" is no longer relevant
- Meta-complexity: classical characterisations!

This talk: an equivalence between **quantum** cryptography and complexity theory *via meta-complexity*.

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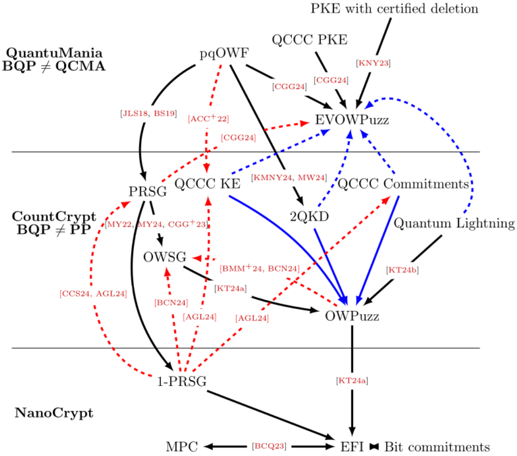

Credits: Goldin, Morimae, Mutreja, and Yamakawa [2024]

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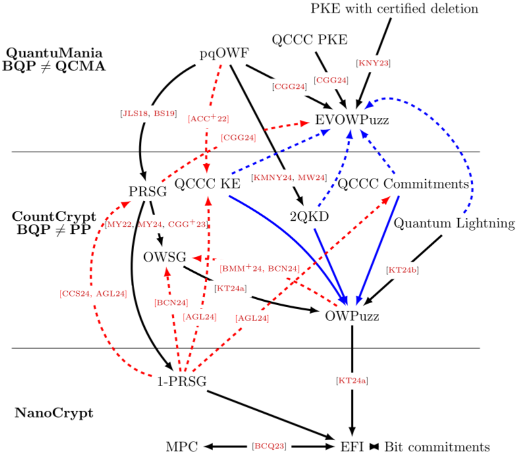

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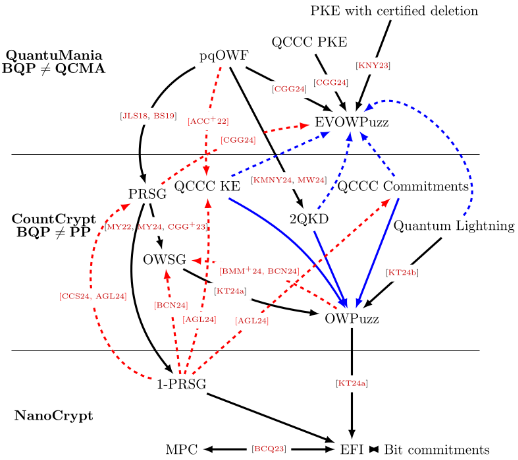

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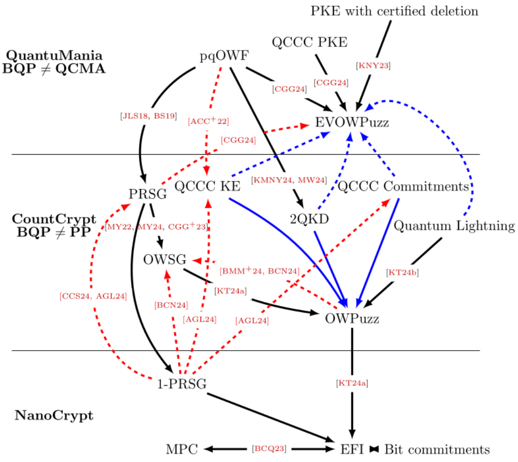

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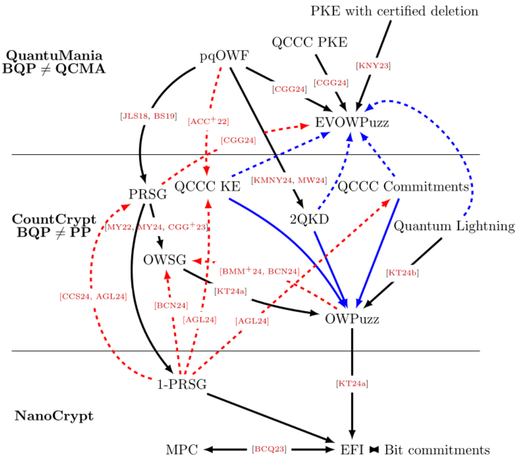

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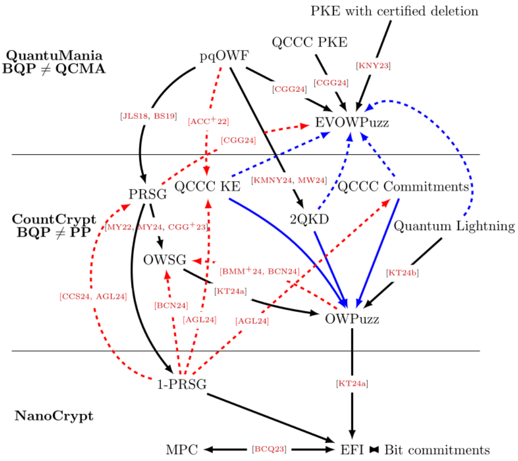

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One-way puzzles

- OWPuzz = (Samp, Ver). [Khurana-Tomer, STOC 2024]
- $\mathsf{Samp}(1^\lambda) \to (k, s) \in \{0,1\}^*$ in quantum polynomial-time (QPT).
  - *k : key*; *s: puzzle*.
- *Soundness:* $\mathbb{P}_{(k,s)\leftarrow \mathsf{Samp}(1^\lambda)}[\mathsf{Ver}(k,s) = \top] = 1 - \mathsf{negl}(n)$
- *Correctness:* $\forall$QPT$\mathcal{A}$, $\mathbb{P}_{(k,s)\leftarrow \mathsf{Samp}(1^\lambda)}[\mathsf{Ver}(\mathcal{A}(s), s) = \top] = \mathsf{negl}(n)$
- Notably: Ver can be inefficient!
- Arises naturally from *shadow tomography*.

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Kolmogorov Complexity

How much information is in a text?

- Example 1: “1010101010101010” (Print “10" 8 times)
- Example 2: “0001010100001011” (???)

Kolmogorov complexity $\mathrm{K}(x)$ of a string $x \in \{0,1\}^*$:
minimum length of a program that outputs x.

PROPERTIES

1. $\mathrm{K}(x) \leqslant |x|$.
2. Random strings have near maximum Kolmogorov complexity.
3. Computing $\mathrm{K}(x)$ is impossible!

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Approximating Kolmogorov Complexity (GapK)

Let GapK$[s, s+\Delta]$ be the "promise" problem of distinguishing between strings of K.c. at most s and those with K.c. at least $s+\Delta$.

IRS'20: GapK can be solved in exponential-time *on average* (PP = PostBQP).

For every PPT-samplable distribution $\mathcal{D}$ and $\Delta = \omega(\log n)$, it's possible to compute GapK$[s, s+\Delta]$ on $\mathcal{D}$ with error at most $n^{-O(1)}$ in exponential time.
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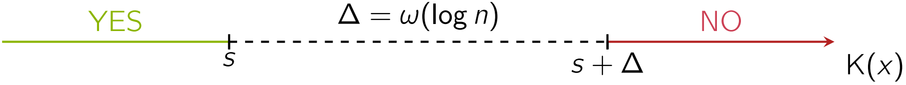

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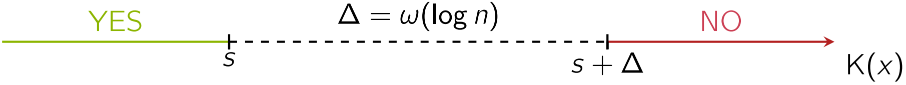

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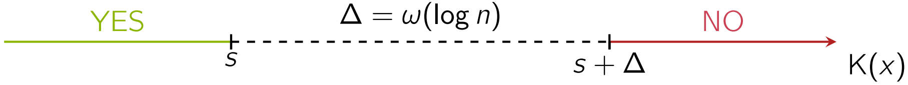

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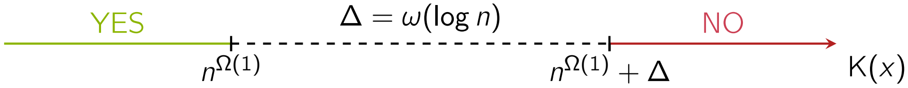
Cryptography vs. Kolmogorov complexity

Theorem (Ilango-Ren-Santhanam [IRS], STOC 2020)

The following are equivalent:

- *One-way functions exist.*
- *For some* $s = n^{\Omega(1)}$ *and* $\Delta = \omega(\log n)$, *there exists a samplable distribution* $\mathcal{D}$ *such that* $\mathrm{GapK}[s, s+\Delta]$ *is average-case hard on* $\mathcal{D}$.



The OWF encodes a hard probability distribution, and vice-versa.

Quantum Cryptography vs. Kolmogorov complexity

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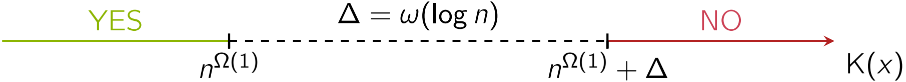

The OWPuzz encodes a hard probability distribution, and vice-versa.

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- Further evidence of OWPuzz's centrality in QCCC!
  - Besides combiners, hardness amplification, etc.
  - Natural generalisation of OWFs
  - Embodies some fundamental hardness via meta-complexity

Probability estimation: crucial intermediate step

Let $\mathcal{D}$ be a sampler. Let $p_x := \mathbb{P}[x \text{ is sampled by } \mathcal{D}]$.
δ-Prob. estimation: given $x \leftarrow \mathcal{D}$, output $\mathcal{A}(x) \in [\delta \cdot p_x, p_x]$ w.h.p..

$$\text{Breaking OWPuzz} \implies \text{Prob. estimation}$$

Classically: use the random bits of the sampler to construct a OWF candidate:

$$f_k(h, r) = \mathcal{D}(r), h, h(r),$$

where h is a hash function mapping to k bits.
Estimate p_x by sampling h, z and inverting (x, h, z) (many times).
(Intuition: select a random hash bucket and see if r "is there")

Quantumly: randomness is inherent to the hard distribution!

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(Intuition: select a random hash bucket and see if r "is there")

Quantumly: randomness is inherent to the hard distribution!

Distributional one-way puzzles

On Central Primitives for Quantum Cryptography with Classical Communication

Kai-Min Chung[1], Eli Goldin[2], and Matthew Gray[3]

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- One-way puzzles $\iff$ Distributional one-way puzzles
- **Takeaway:** $\nexists$ OWPuzz $\implies$ sample from (k, s) *conditioned* on s.

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- **Takeaway:** $\nexists$ OWPuzz $\Longrightarrow$ sample from (k, s) *conditioned* on s.

Amplitude estimation with a distributional inverter

One-way puzzle: sample $x \leftarrow \mathcal{D}$, h hash function mapping to k bits.
Key: x, Puzzle: $(h, h(x))$.
Distributionally invert [CGG]: as k increases, x is isolated by $(h, h(x))$.
(Intuition: fix the "right" random hash bucket, and see if anything else besides x is there.)
The threshold when x starts to become isolated is our estimate for p_x.

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Computing GapK with probability estimation

Just like the classical case [IRS].
Requires a "quantum coding theorem":

$$\mathrm{K}(x) \lessapprox \log\left(\frac{1}{\mathbb{P}[\mathcal{D} \to x]}\right) + |\mathcal{D}|\,.$$

We then use probability as a proxy for Kolmogorov complexity.
(In other words, low probability $\approx$ high Kolmogorov complexity.)

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Breaking one-way puzzles with GapK

CGG: OWPuzz $\implies$ "non-uniform entropy-gap generator".



A **QPT** distribution $\mathcal{D}$ whose entropy is sufficiently far from maximum and which is **QPT**-indistinguishable from uniform.

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Concurrent work

1. Khurana-Tomer (STOC 2025): Shows equivalence between probability estimation and one-way puzzles (with tighter approximation factor);
2. Hiroka-Morimae (2024): Shows how to construct one-way puzzles from average-case hardness of GapK (with a different proof).

Open questions

1. Characterising with a problem with worst-case complexity bound? (Classical: AM).
2. Characterise other primitives? E.g., EFIs, OWSGs.
3. Strong average-case hardness?
4. Oracle separation does not rule out: construct OWPuzzles from the hardness of a (meta-complexity?) problem in QMA.

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Thanks!