(inefficient Prover) ZAPs from Hard-to-Invert Functions

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30 Second Summary

one-way functions are NOT needed for a non-trivial notion of zero-knowledge proofs for NP

One Minute Summary

1. new primitive <u>below</u> one-way functions (OWF):

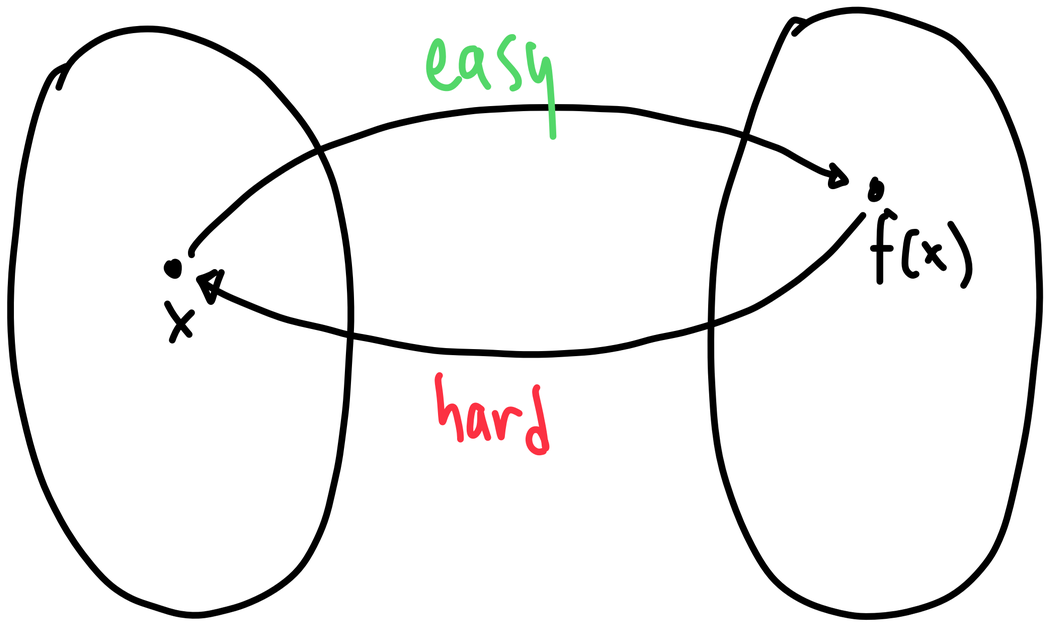
Hard-to-Invert Functions (HIF)

2. HIF $\Rightarrow$ ZAPs (2-message witness indistinguishable proofs) for NP with subexponential time prover

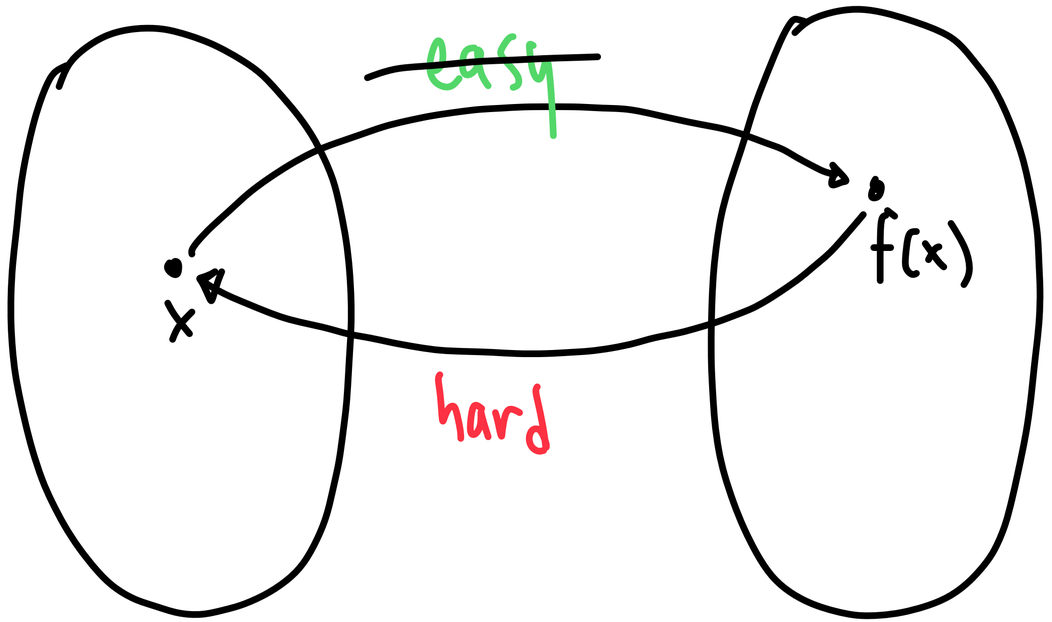
(3. HIF + more complexity assumptions $\Rightarrow$ hardness in $NP \cap coNP$, non-uniform setting)

Hard-to-Invert Function?

Recall: One-Way Function



Hard-to-Invert Function (HIF)



Can verify "$f(x) = y$" (given proof)

(i.e. $\{(x, f(x))\} \in NP$)

OWF $\Rightarrow$ HIF ($\Leftarrow$ TFUP hardness)

Zero Knowledge

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Completeness $x \in L \Rightarrow$ Verifier accepts (w.h.p.)

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soundness $x \notin L \implies$ Verifier rejects (w.h.p.), even if prover cheats

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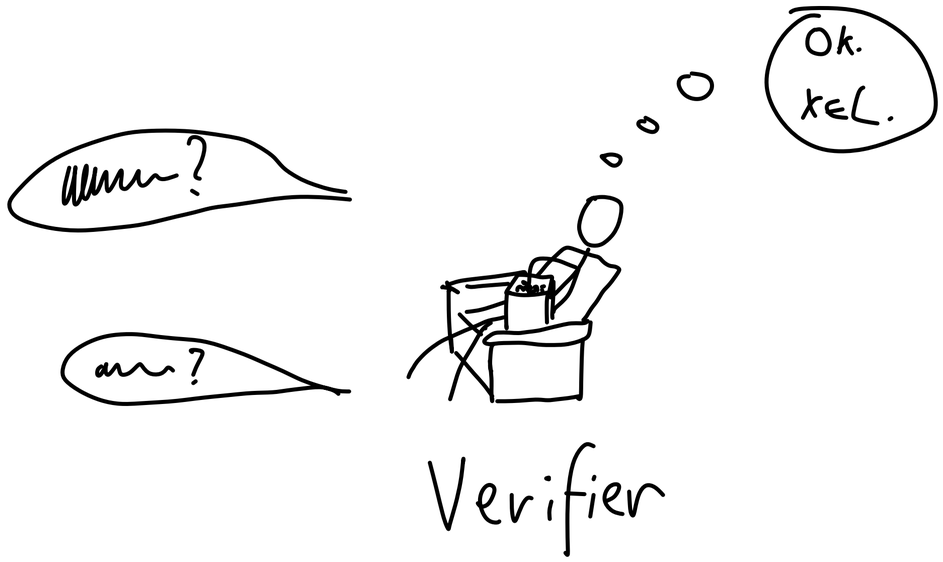
Zero-knowledge verifier only learns $x \in L$.

Zero Knowledge

Simulator

$\approx$

Prover



Completeness $x \in L \implies$ Verifier accepts (w.h.p.)

soundness $x \notin L \implies$ Verifier rejects (w.h.p.), even if prover cheats

Zero-knowledge verifier only learns $x \in L$.

($x \in L \implies$ interaction can be efficiently simulated.)

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[Goldreich Micali Wigderson '86] OWF ⇒ ZK for NP

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[Ostrovsky Wigderson '93] ¬OWF ⇒ no ZK for hard-on-average L

Zero Knowledge

Simulator Prover

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Zero Knowledge

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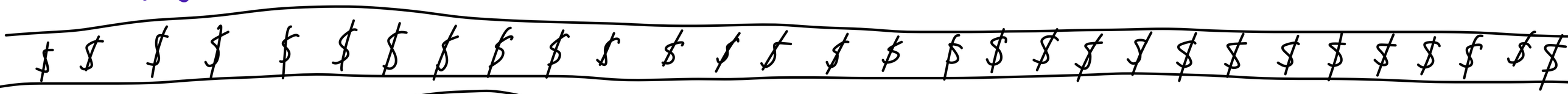
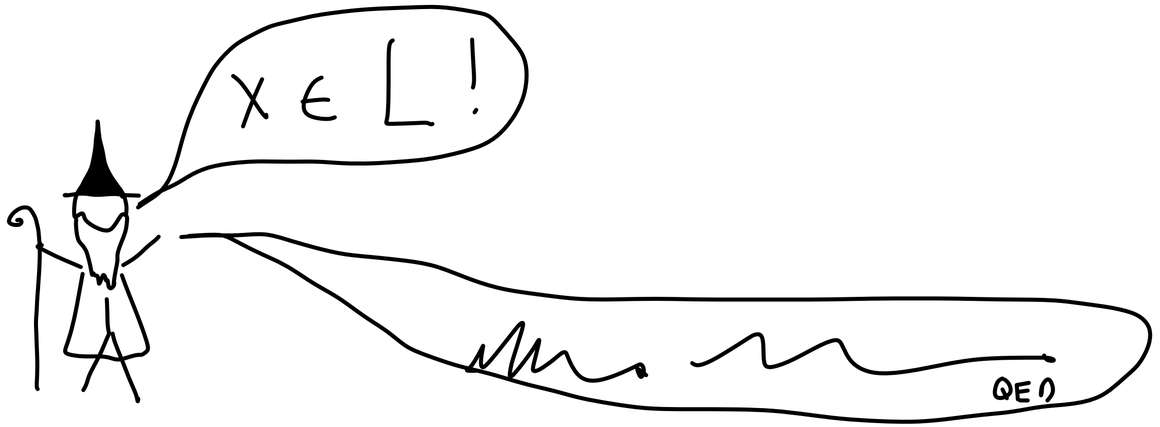
[Hirahara Nanashima '24] if P≠NP, then ZK for NP ⇒ OWF

Non-Interactive Zero Knowledge

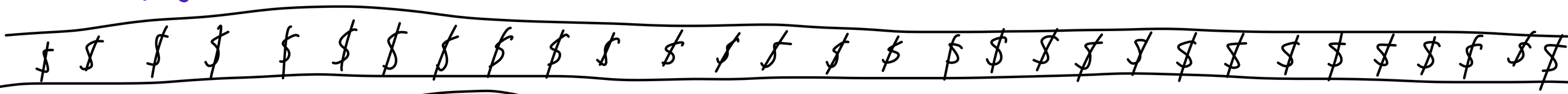


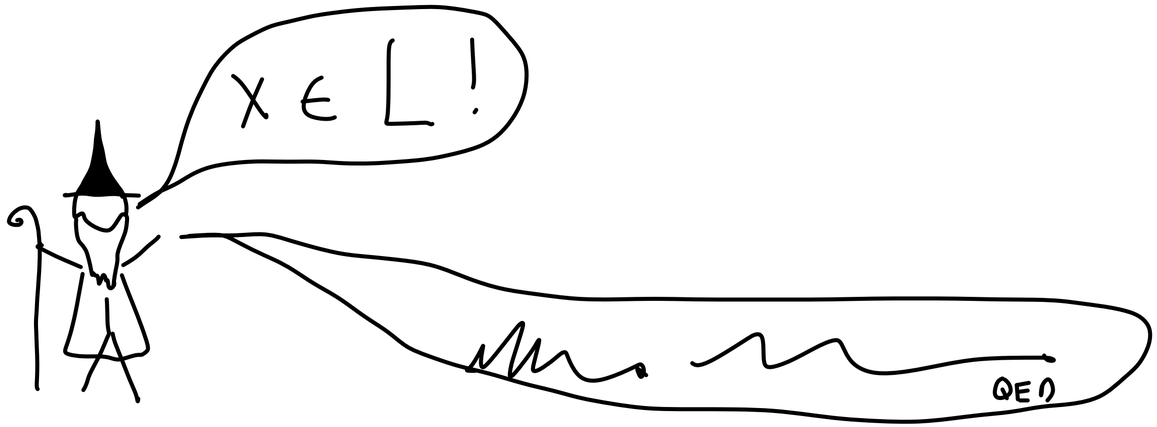


Prover

Verifier

Non-Interactive Zero Knowledge



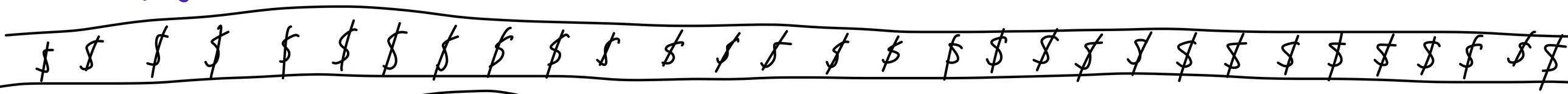


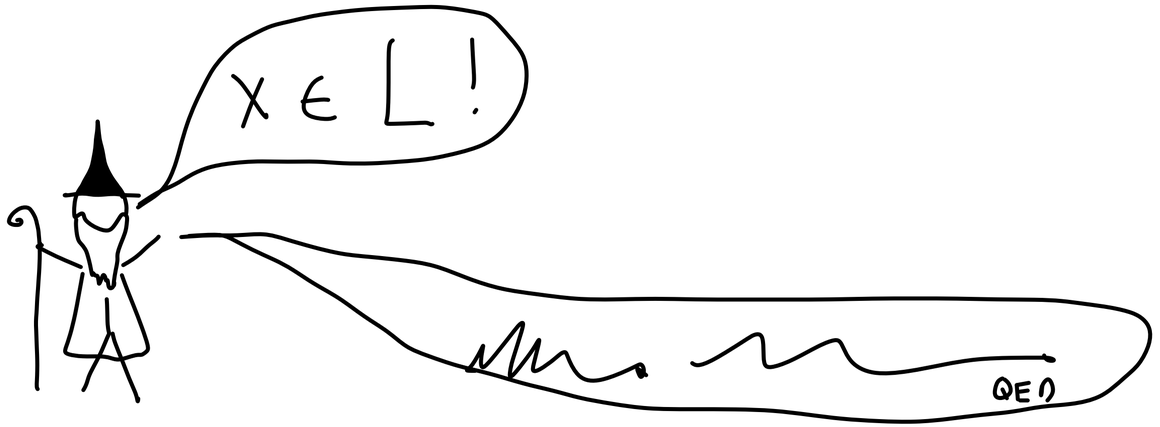
Prover

Verifier

Common "random" string (CRS) vs UNIFORM random string (URS)

Non-Interactive Zero Knowledge





Prover

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[Passshelat '05] OWF ⇒ NIZK with CRS for NP (AM)

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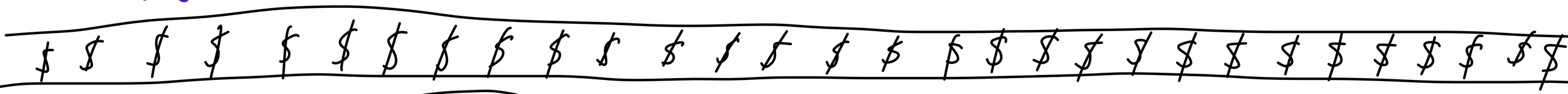
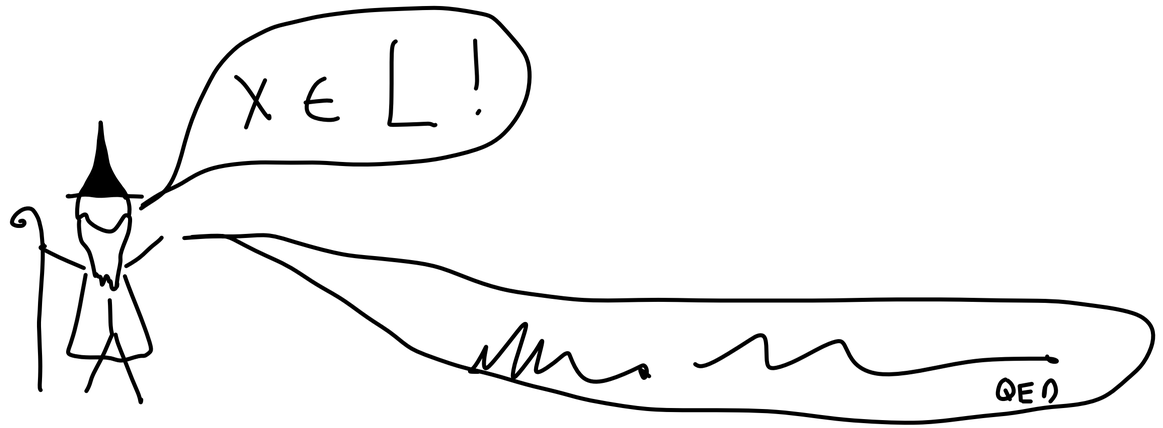




Prover

Verifier

Common "random" string (CRS)

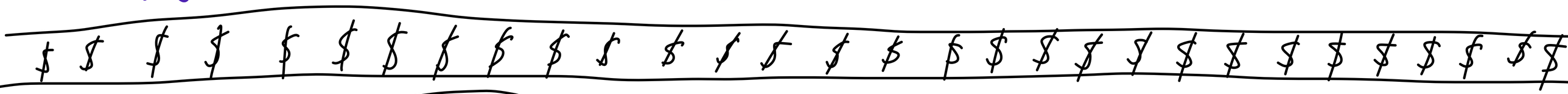
vs

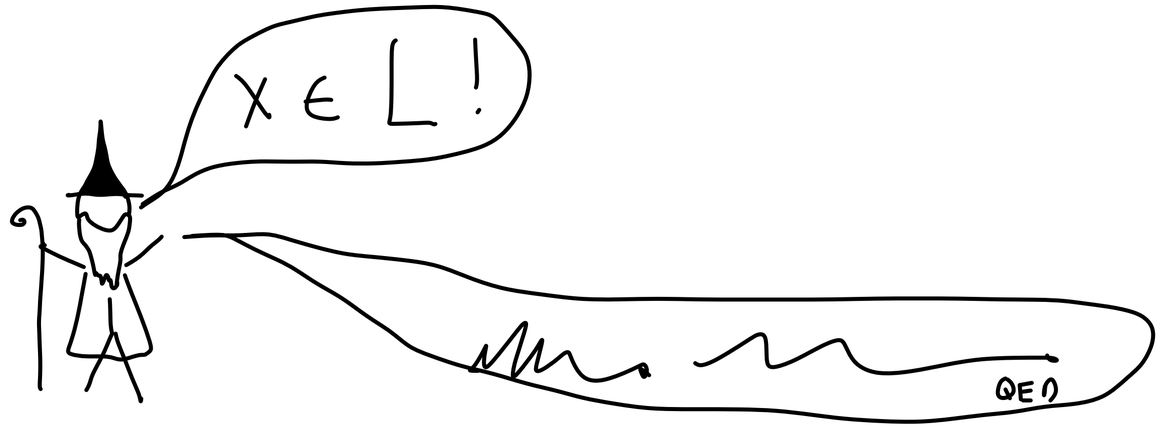
UNIFORM random string (URS)

[Passshelat '05] OWF ⇒ NIZK with CRS for NP (AM)

[FeigeLapidotShamir '99] OWP ⇒ NIZK with URS for NP (AM)

Non-Interactive Zero Knowledge





Prover

Verifier

Common "random" string (CRS) vs UNIFORM random string (URS)

[Passshelat '05] OWF ⇒ NIZK with CRS for NP (AM)

[FeigeLapidotShamir '99] OWP ⇒ NIZK with URS for NP (AM)

OPEN QUESTION:

OWF $\Rightarrow$ NIZK in URS model?

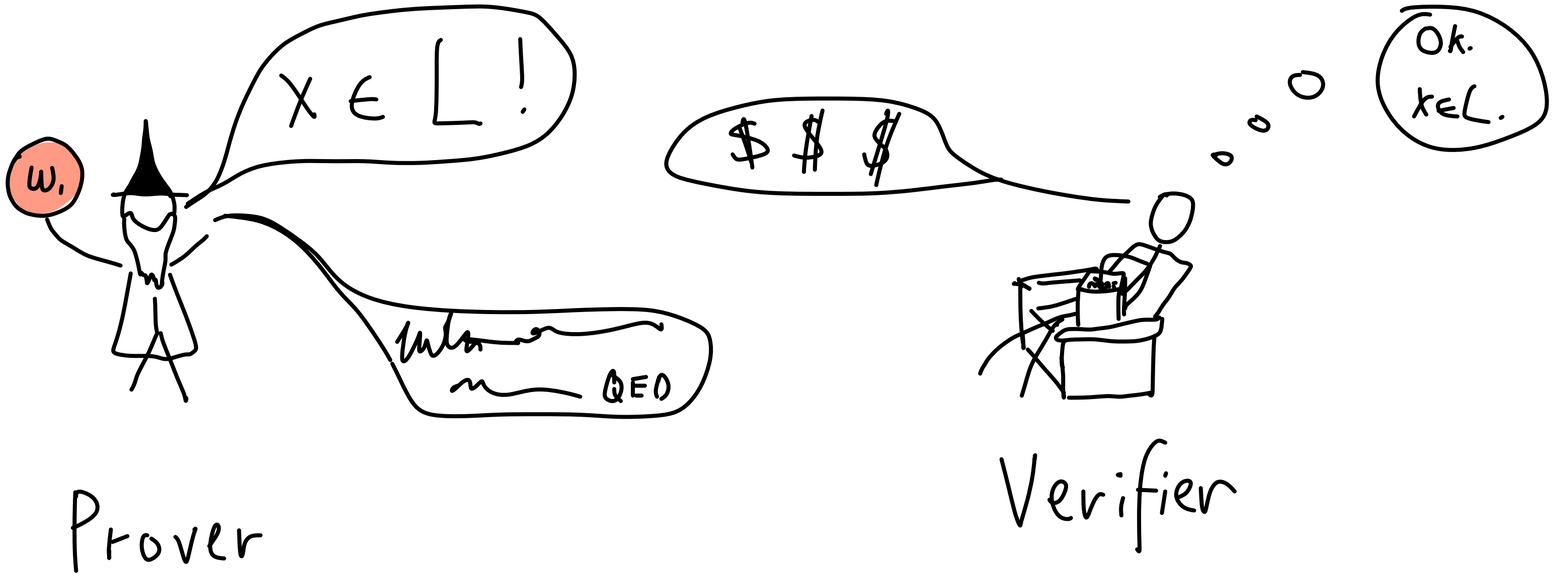
Partial progress: [Ghosal et al. '23] NIZK in ROM

this work: HIF $\Rightarrow$ ZAP (w/ inefficient prover)

(HIF < OWF) (ZAP < NIZK in URS)

ZAPs

[Dwork Naor '00]



Prover

Verifier

ZAPs

[Dwork Naor '00]

Prover

$\approx$

Prover

Verifier

WITNESS INDISTINGUISHABILITY

[Feige Shamir '90]

for any witnesses w_1, w_2 for x :

$$\text{Prover}(w_1) \rightleftarrows \text{Verifier} \approx \text{Prover}(w_2) \rightleftarrows \text{Verifier}$$

ZAPs

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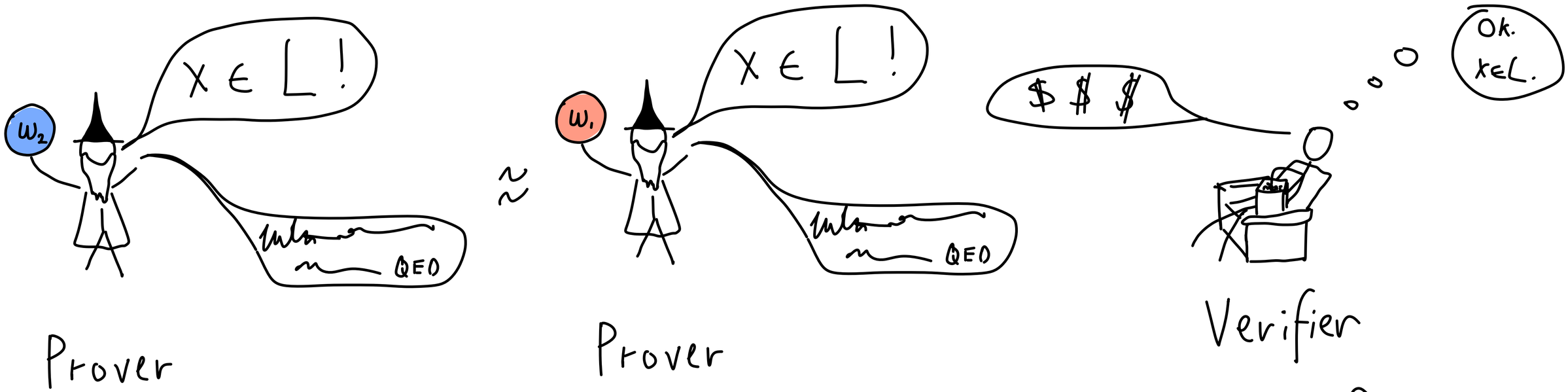

Prover ≈ Prover

Verifier

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NOTE: <u>trivial</u> if witnesses unique or prover <u>arbitrarily</u> inefficient (find & send 'canonical' witness)

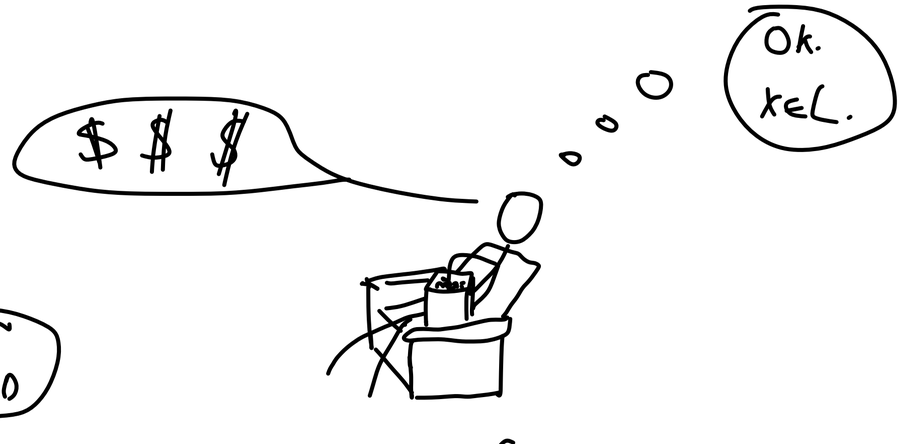
ZAPs

[Dwork Naor '00]

Prover

$\approx$

Prover



Verifier

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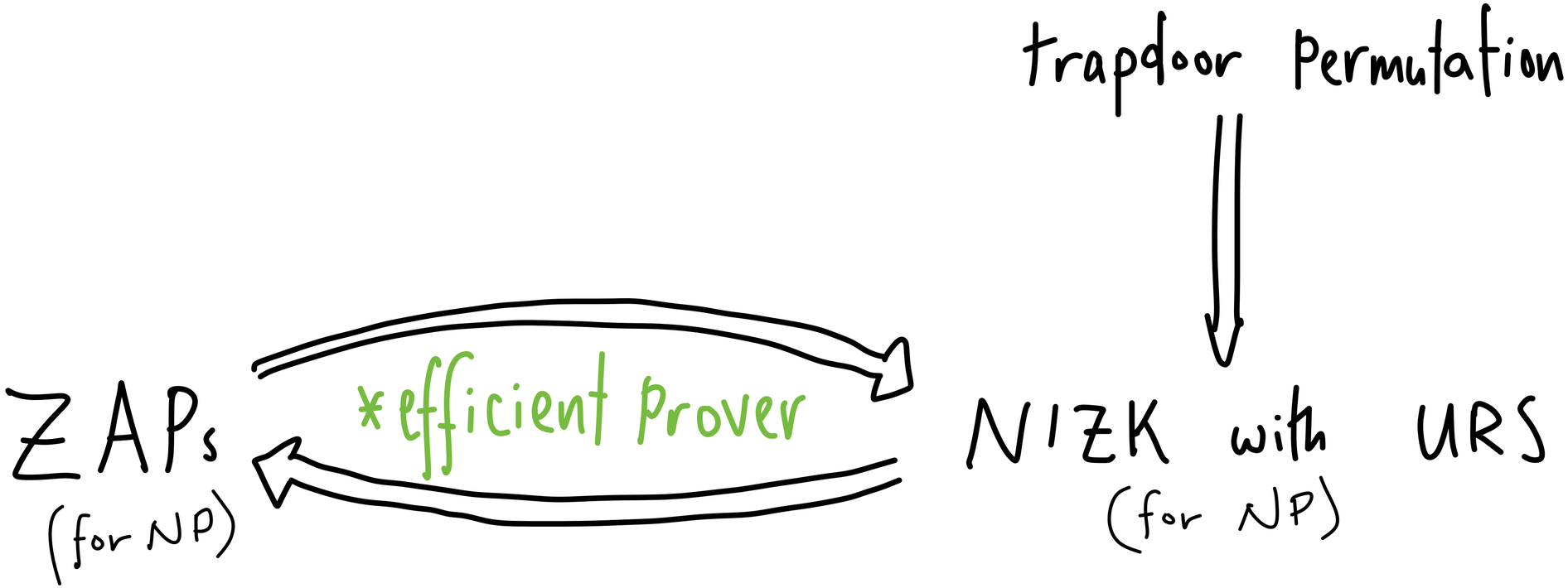
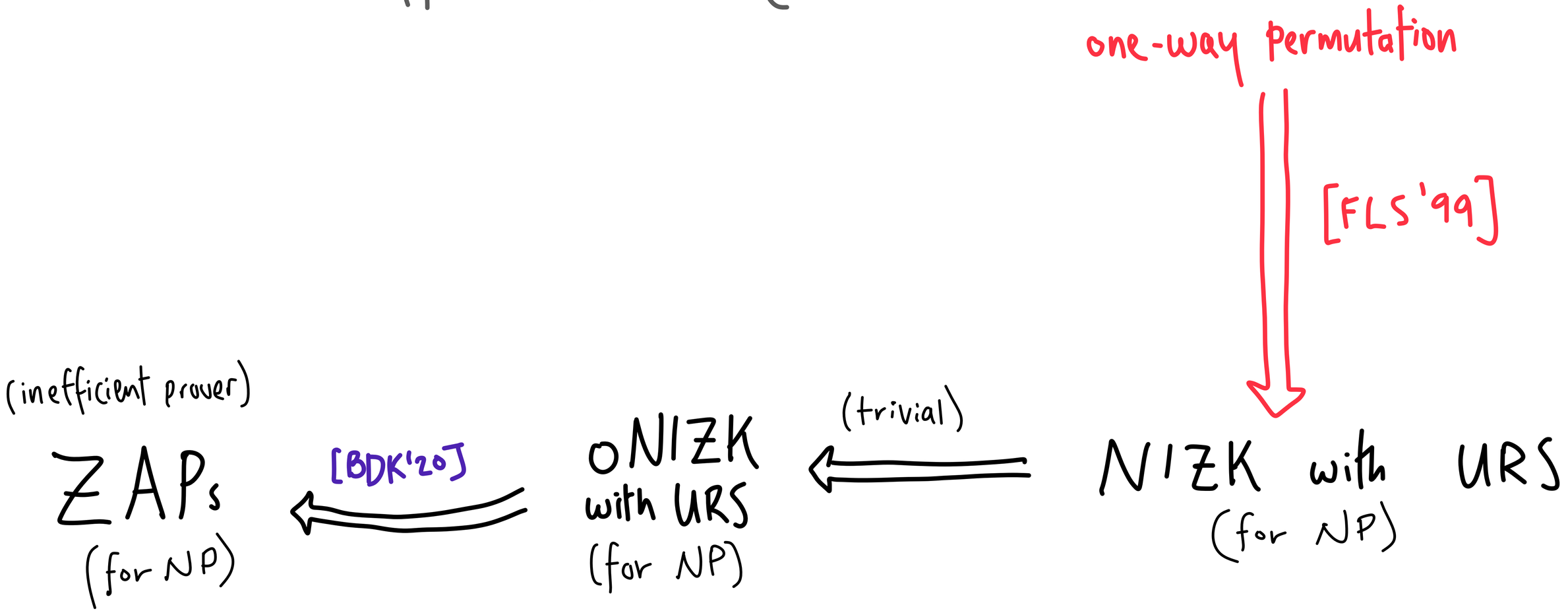
NOTE: non-trivial if prover runs in 2^{n^ε} time $(\varepsilon < 1)$.

[Dwork Naor '00]

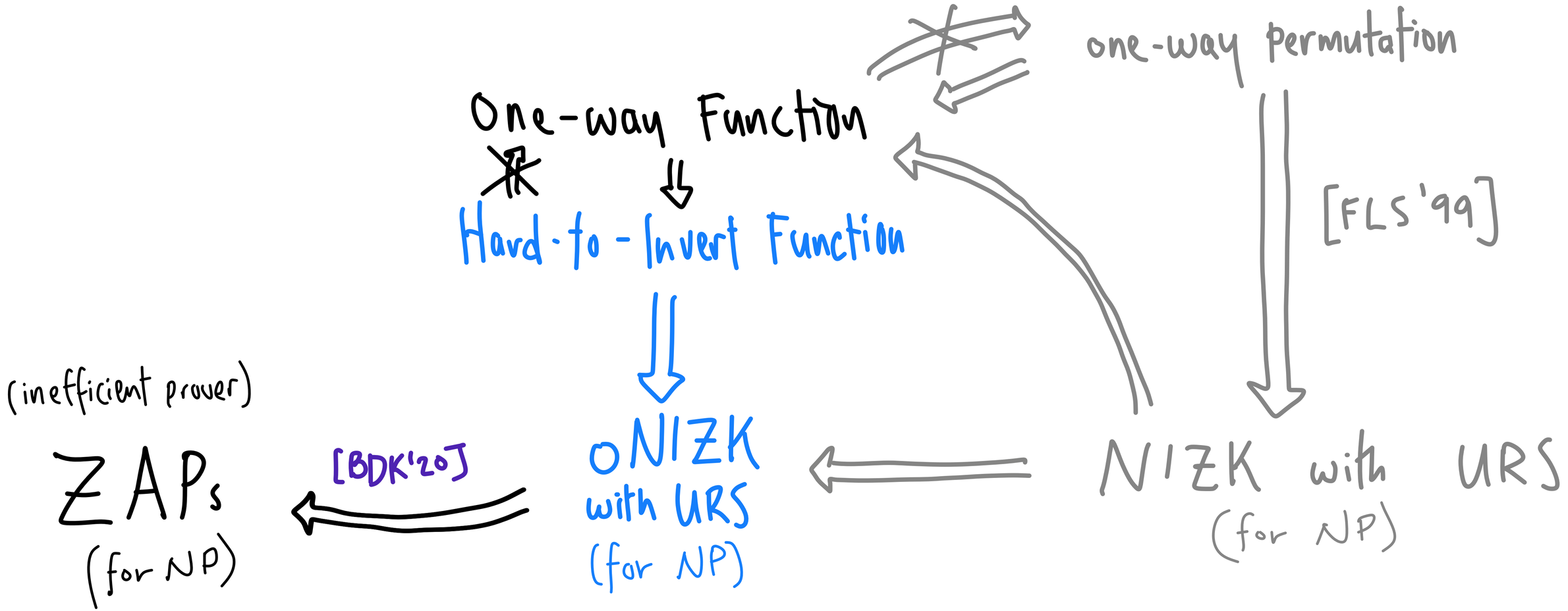


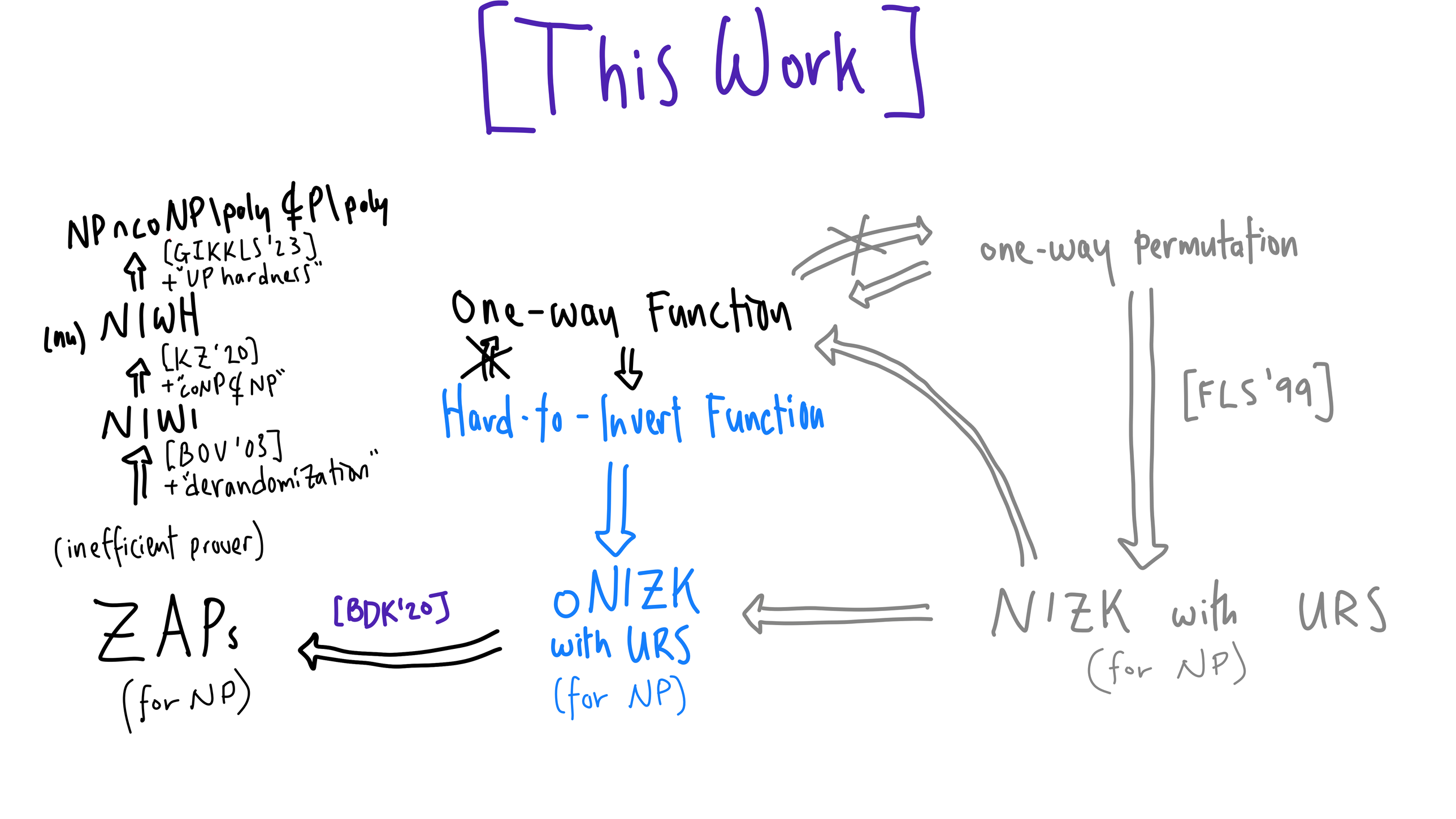
[Ball Dachman-Soled Kulkarni '20]

inefficient provers (for better assumptions)?



[This Work]


one-way permutation
One-way Function
Hard-to-Invert Function
[FLS'99]
(inefficient prover)
ZAPs
(for NP)
[BDK'20]
oNIZK
with URS
(for NP)
NIZK with URS
(for NP)


[This Work]
NP∩coNP/poly ⊄ P/poly
⇑ [GIKKLS'23] + "UP hardness"
(nu) NIWH
⇑ [KZ'20] + "coNP ⊄ NP"
NIWI
⇑ [BOV'03] + "derandomization"
(inefficient prover)
ZAPs
(for NP)
[BDK'20]
One-way Function
Hard-to-Invert Function
oNIZK with URS
(for NP)
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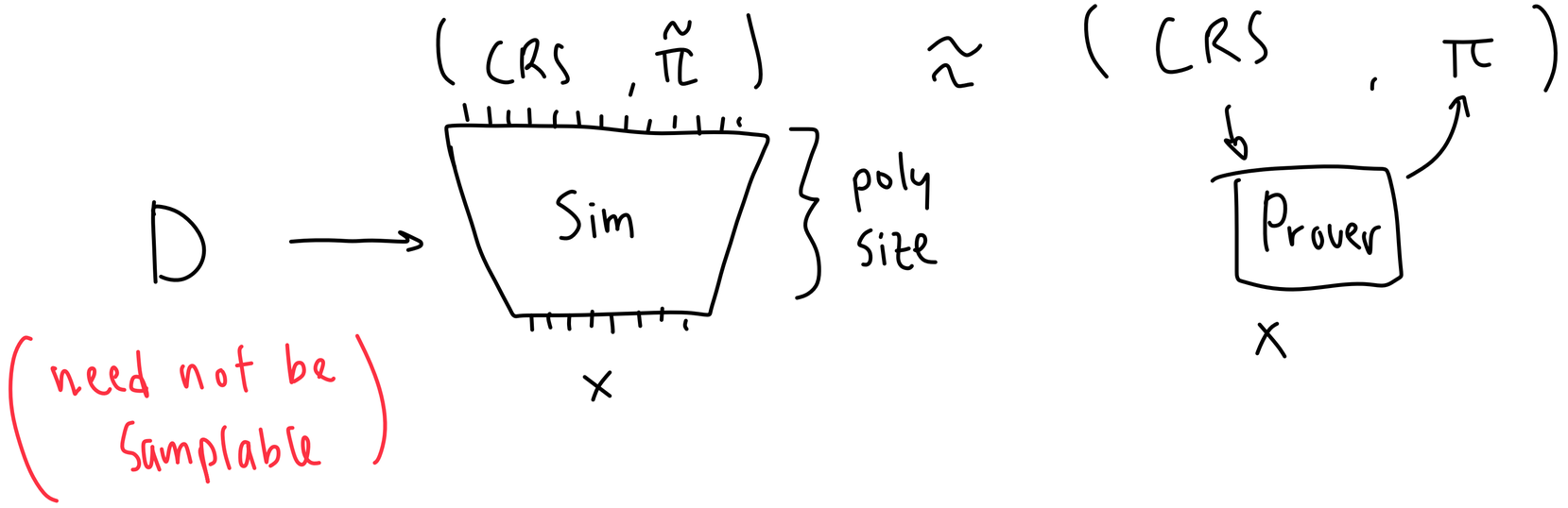
oNIZK?

"Offline Simulation"

$\forall x \in L$.



oNIZK?

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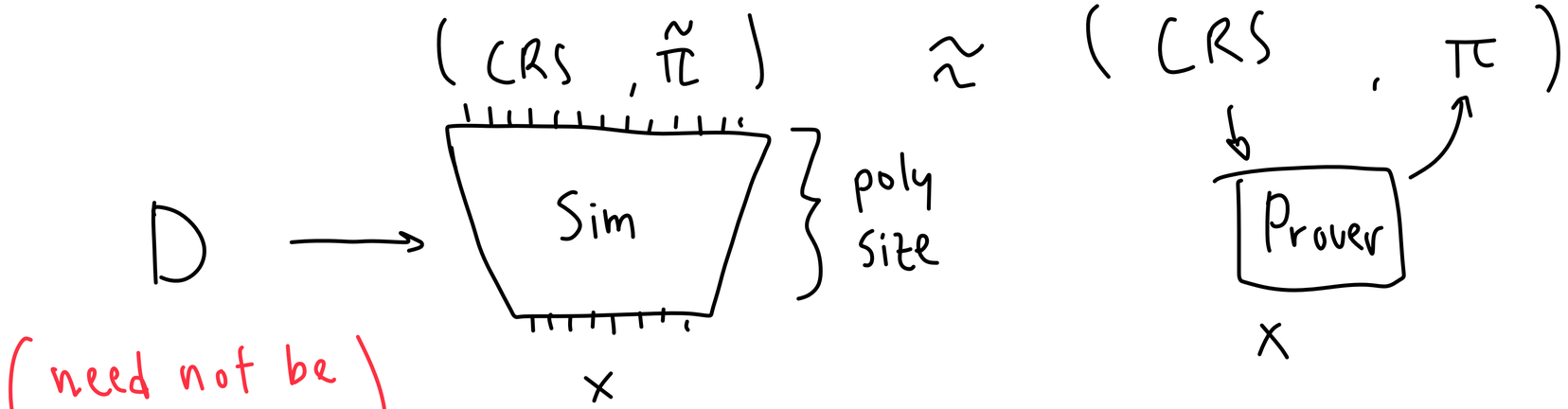

(need not be Samplable)

(NIZK: D → Sim (Samplable))

Technical Overview

how to build oNIZK from HIF?

[Feige Lapidot Shamir '99] framework for NIZK

① NIZK for NP in "hidden bits" model

② hidden bits from URS + OWP

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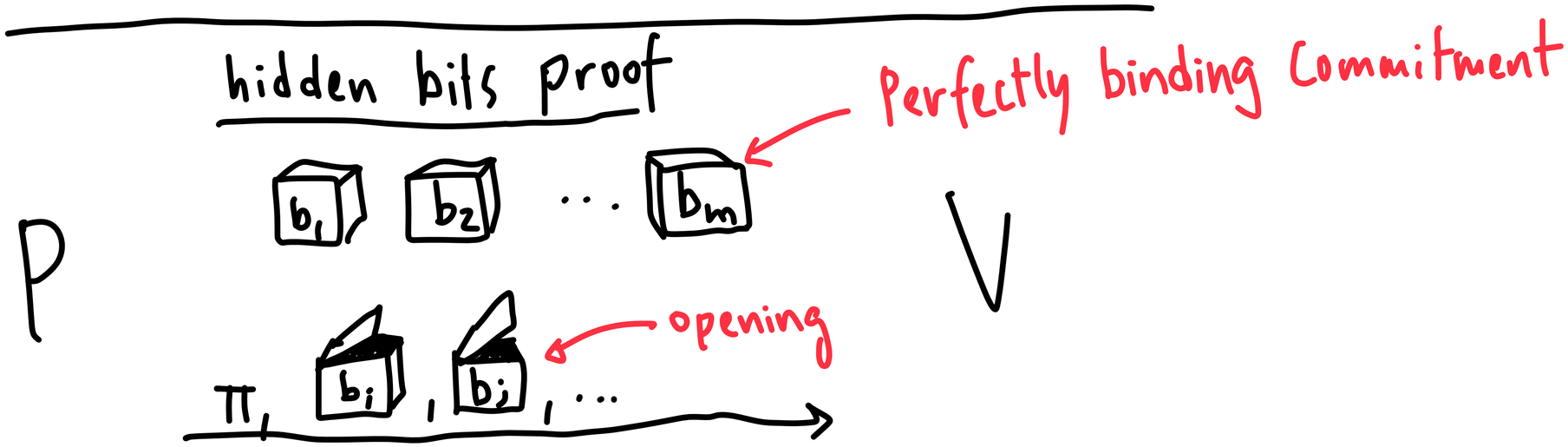

how to build ∘NIZK from HIF?

[Feige Lapidot Shamir '99] framework for ∘NIZK

① ∘NIZK for NP in "relaxed hidden bits" model (no change) ✓

② relaxed hidden bits from URS + HIF ← main contribution

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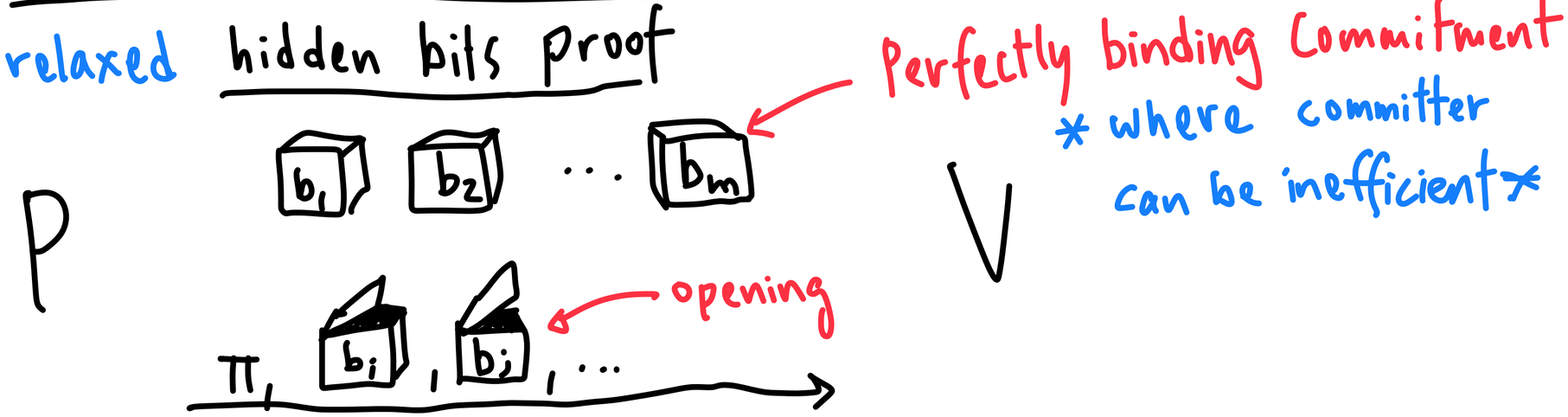
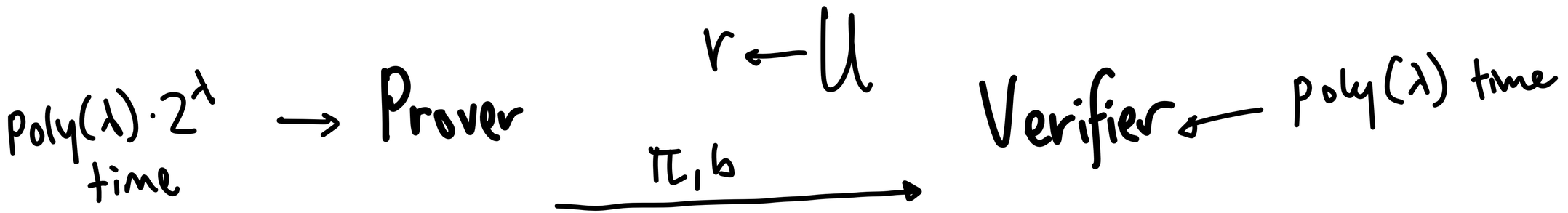
relaxed hidden bits proof



Relaxed Hidden Bits in URS model (informal)

"Verifiable Hardcore of Random String"



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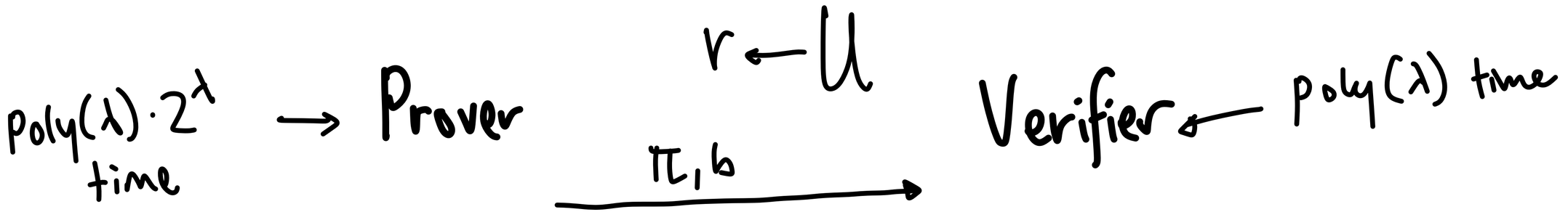

with high probability over r:

1. (total) perfect binding: $\exists$ unique b that verifier will accept (w/ π)

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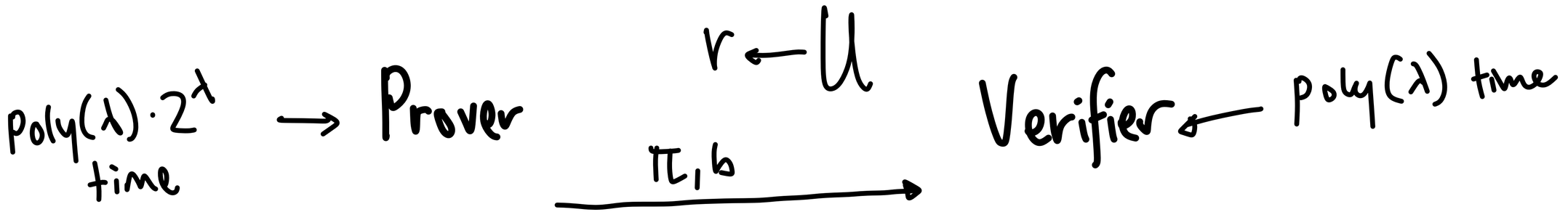
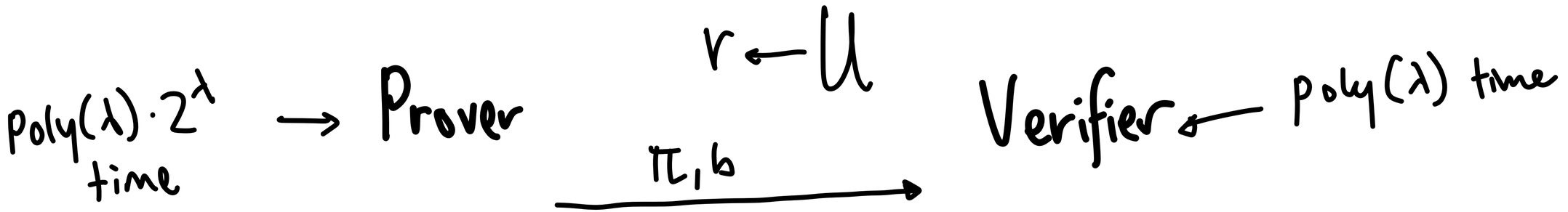


with high probability over r:

1. (total) perfect binding: ∃ unique b that verifier will accept (w/ π)
2. hiding: $\left(r \mid \text{Prover}(r)=(\pi,0)\right) \approx \left(r \mid \text{Prover}(r)=(\pi,1)\right)$

Relaxed Hidden Bits in URS model (informal)

"Verifiable Hardcore of Random String"



with high probability over r:

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non-relaxed version: can efficiently sample (r, π, b)

Recall: [FLS'99] hidden bits

tools:
- one-way permutation: $f : \{0,1\}^\lambda \xrightarrow{\sim} \{0,1\}^\lambda$
- hardcore predicate for f: $\phi : \{0,1\}^\lambda \to \{0,1\}$

Recall: [FLS'99] hidden bits

tools: • one-way permutation: $f : \{0,1\}^\lambda \xrightarrow{\sim} \{0,1\}^\lambda$

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Construction:

$r \leftarrow U_\lambda$

Prover		Verifier
compute: 1. $x = f^{-1}(r)$	$\xrightarrow{x, b}$	check $f(x) = r$
2. $b = \phi(x)$		& $\phi(x) = b$

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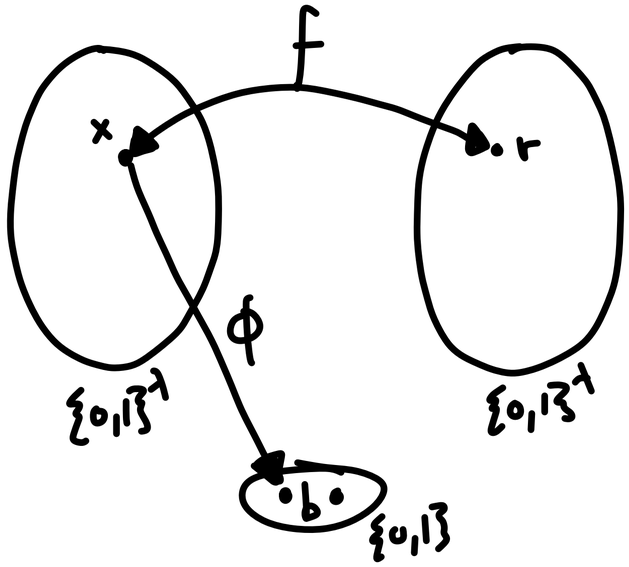

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(total) perfect binding: $x = f^{-1}(r)$ always exists and is unique (sim. $\phi(x) = b$)

Recall: [FLS'99] hidden bits

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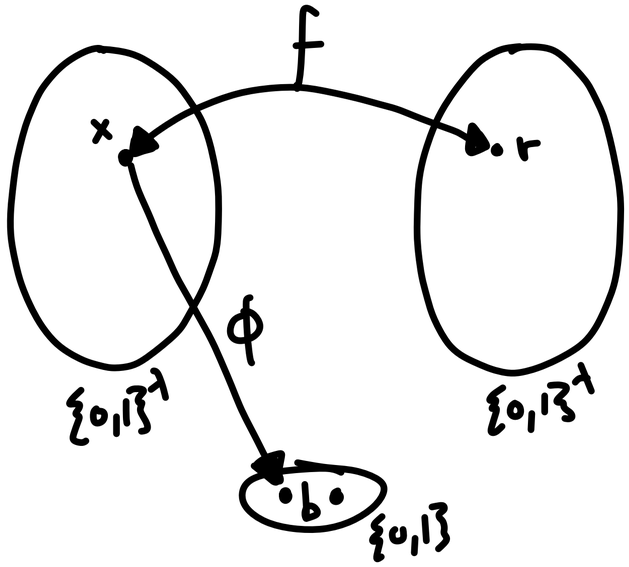

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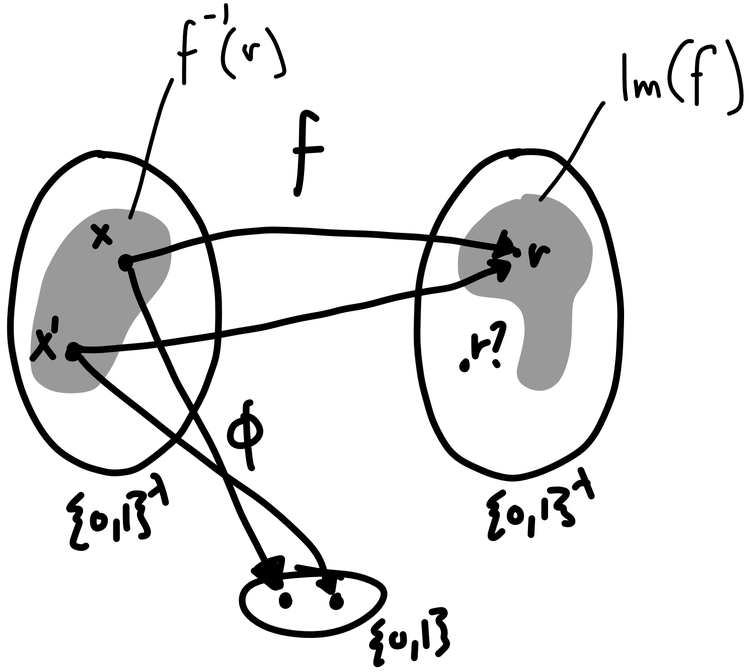
hiding: ϕ is hardcore predicate

Attempt from OWF (HIF same)

tools: • one-way ~~permutation~~ function: $f : \{0,1\}^\lambda \xrightarrow{\sim} \{0,1\}^\lambda$

• hardcore predicate for f: $\phi : \{0,1\}^\lambda \to \{0,1\}$



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(total) perfect binding: $x = f^{-1}(r)$ ~~always exists~~ and is ~~unique~~ (sim. $\phi(x) = b$) ☹

hiding: ϕ is hardcore predicate ✓

Attempt 2: making total

given (H, r)

Prover

compute: 1. $x = (H \circ f)^{-1}(r)$

2. $b = \phi(x)$

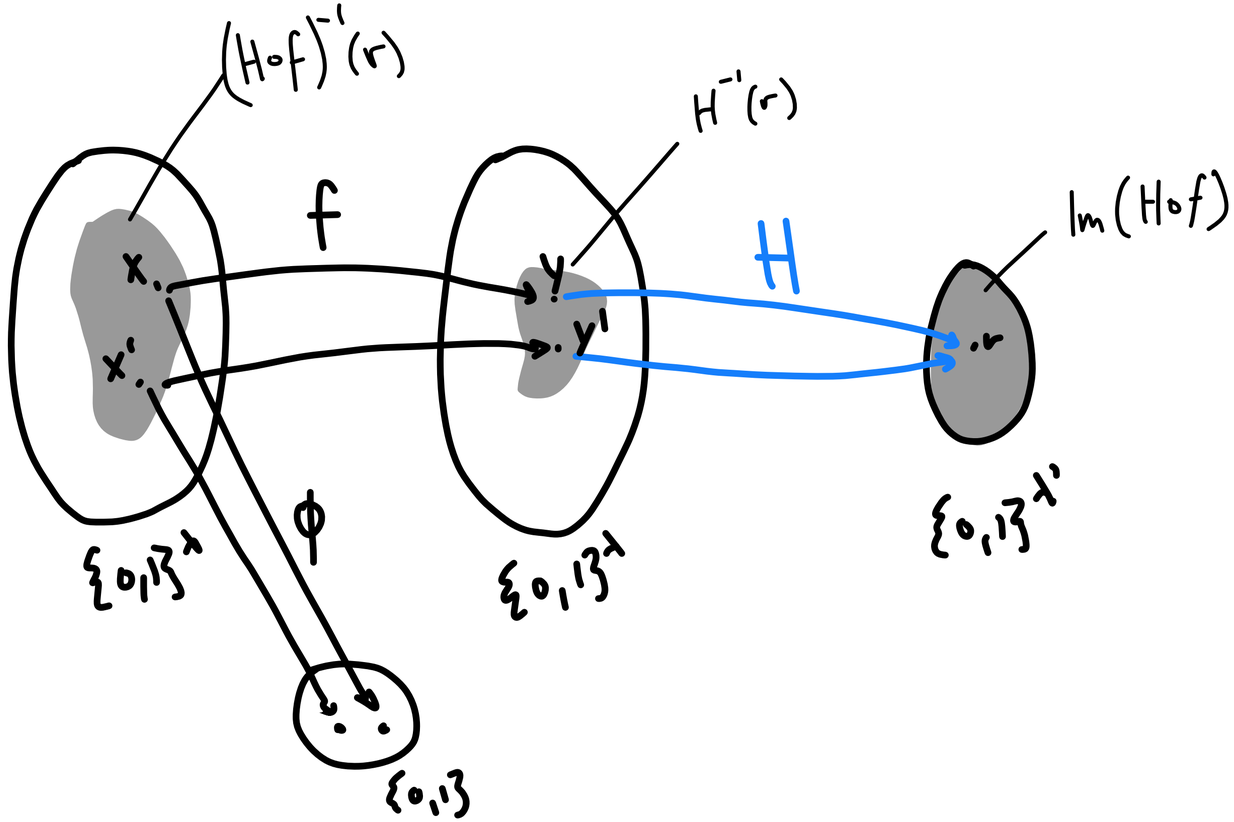
send (x, b) to open

Verifier

check $H \circ f(x) = r$

& $\phi(x) = b$

H: t-wise independent hash



(total) perfect binding: $x \in (H \circ f)^{-1}(r)$ always exists and is ~~unique~~

hiding: ϕ is hardcore predicate ✓

Attempt 3: making unique?

given (H, r, h)

Prover

compute: 1. $x \in (H \circ f)^{-1}(r) \cap h^{-1}(0)$

2. $b = \phi(x)$

send (x, b) to open

Verifier

check $H \circ f(x) = r$

& $\phi(x) = b$

& $h(x) = 0$

H: t-wise independent hash

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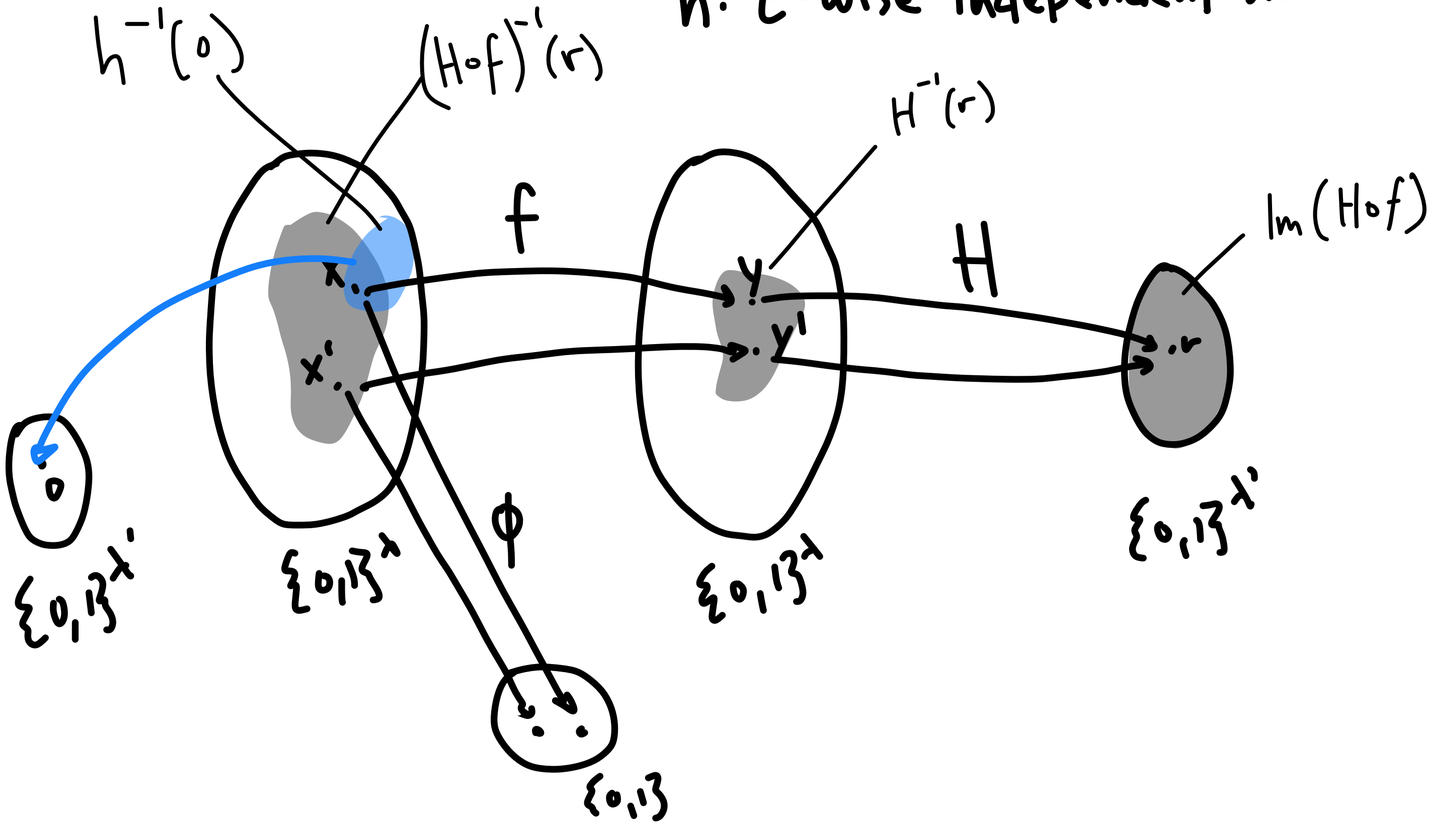

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hiding: ϕ is hard core predicate ✓

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Verifier

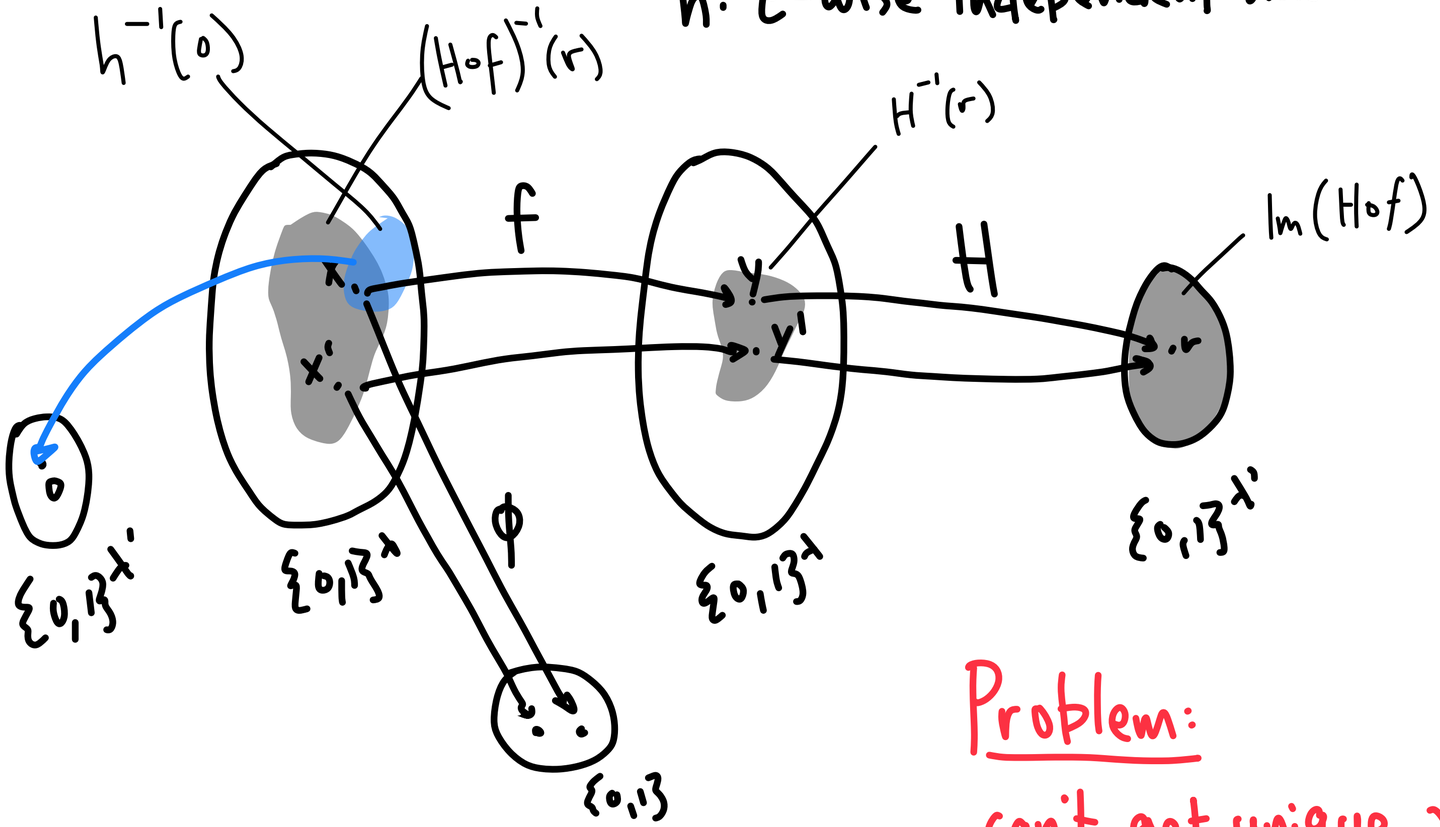
check $H \circ f(x) = r$

& $\phi(x) = b$

& $h(x) = 0$

H: t-wise independent hash

h: t-wise independent hash



Problem:

can't get unique x w.h.p.

(total) perfect binding: $x \in (H \circ f)^{-1}(r) \cap h^{-1}(0)$ always exists and is unique?

hiding: ϕ is hard core predicate ✓

Attempt 4: use everything

given (H, r, h)

Prover

compute: 1. $S = (H \circ f)^{-1}(r) \cap h^{-1}(0)$

2. $b = \phi(S)$

send (S, b) to open

Verifier

check $\forall x \in S$,

$H \circ f(x) = r$

& $h(x) = 0$,

& $\phi(S) = b$

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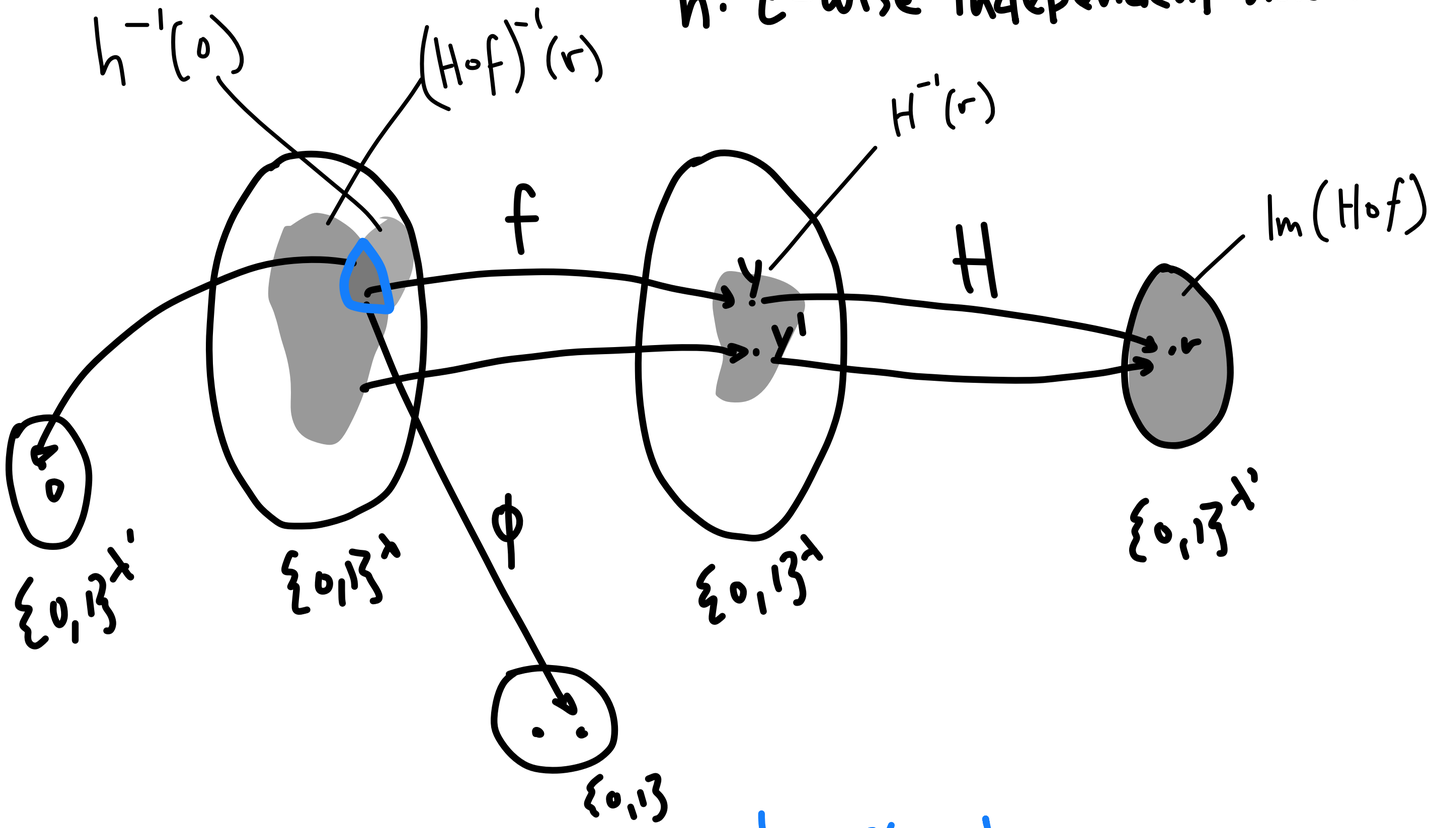

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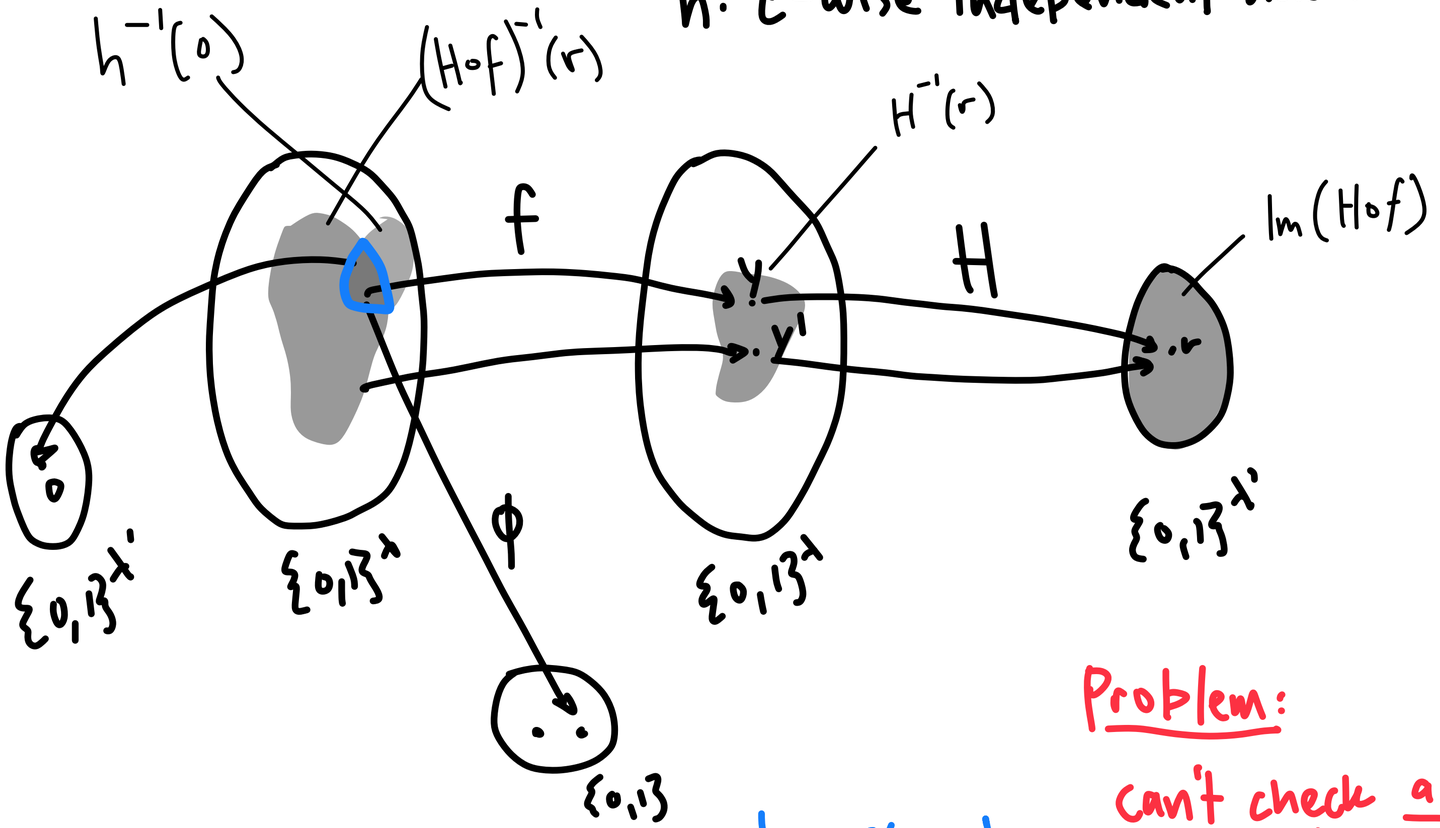

Problem:

can't check all of S sent!

$\phi(S) \neq \phi(S \setminus \{x\})$

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Attempt 5: repeat & use statistics

given $(H_1, r_1, h_1) \cdots (H_m, r_m, h_m)$

Prover

compute: for $i = 1 \cdots m$:

1. $S_i = (H_i \circ f_i)^{-1}(r_i) \cap h^{-1}(0)$
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set $b = MAJ(b_1, \cdots b_m)$

send $(S_1, \cdots, S_m, b)$

Verifier

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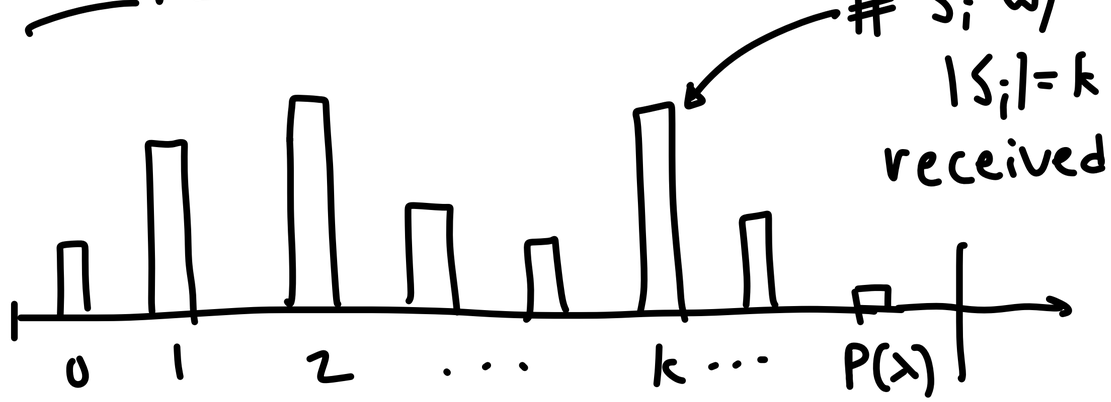
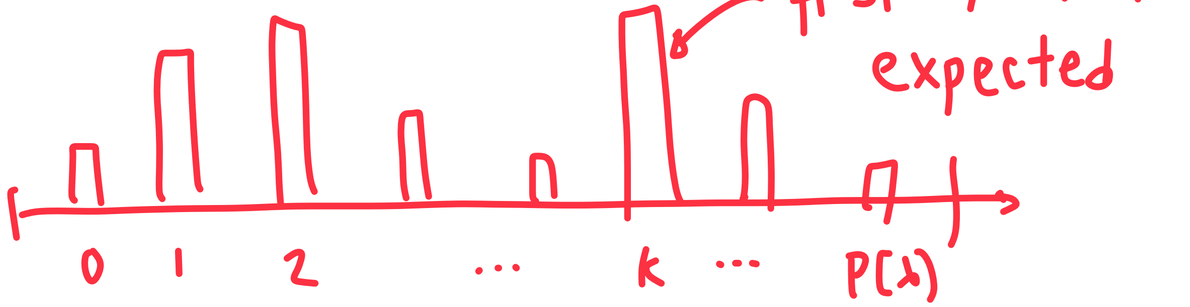
Compare (acceptif close):





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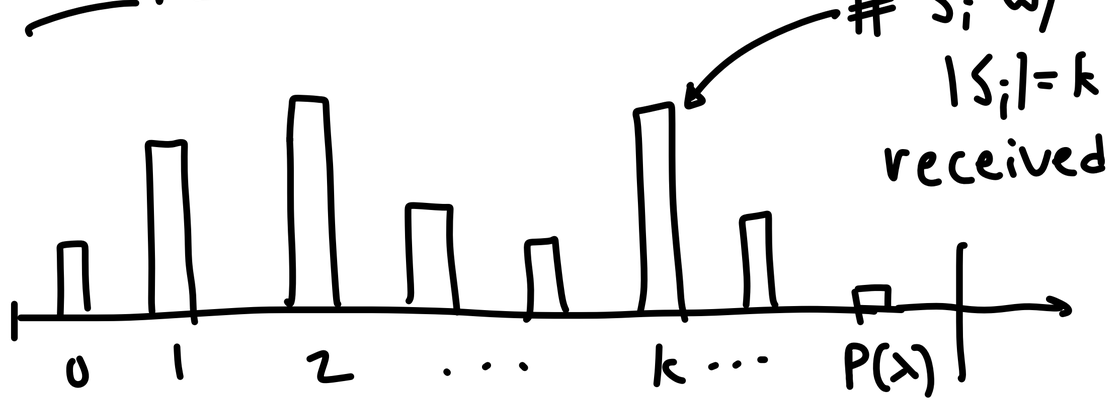

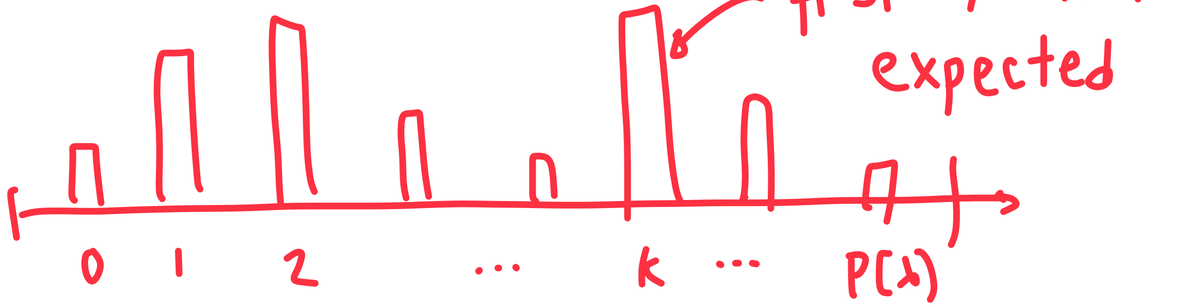

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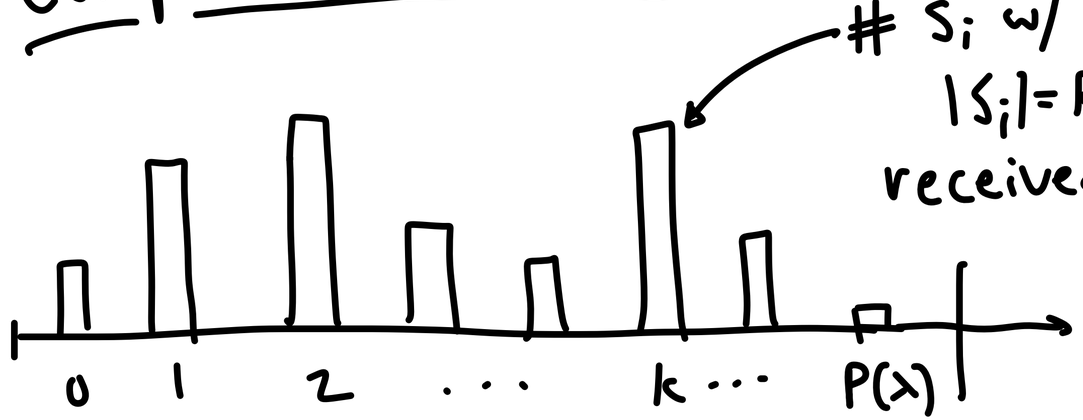

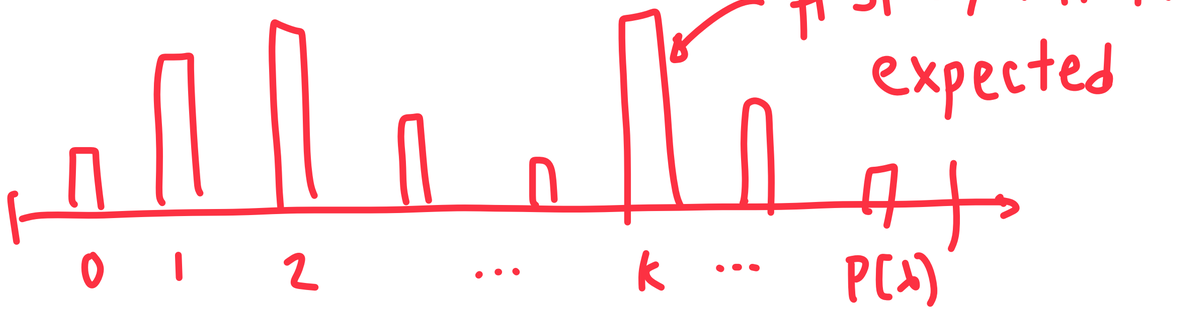

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Prover (w/ proof) [Haitner Mahmoody Xiao '10]

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(Sketch: ① $\Rightarrow$ ZAP $\overset{+②}{\Rightarrow}$ NIWI [BOV'03] $\overset{+③}{\Rightarrow}$ NIWH [KZ'20] $\overset{+④}{\Rightarrow}$ $P \neq NP \cap coNP$ [GIKKLS'23])

Open Questions

1. minimal assumptions for...
   - (a) hard-to-invert functions?
   - (b) oNIZK?
   - (c) inefficient prover ZAPs?

2. OWF $\Rightarrow$ NIZK in URS model?

3. explicit hardness in NP $\cap$ coNP without OWP?

thanks!