Anamorphism
Beyond One-To-One Messaging:
PKE with Anamorphic Broadcast Mode

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Anamorphism

Two-View Principle

Anamorphic Art: Two Views on the Same Object

In anamorphic art, an object can be seen from many viewpoints, but the “complete” image only appears from a specific angle.

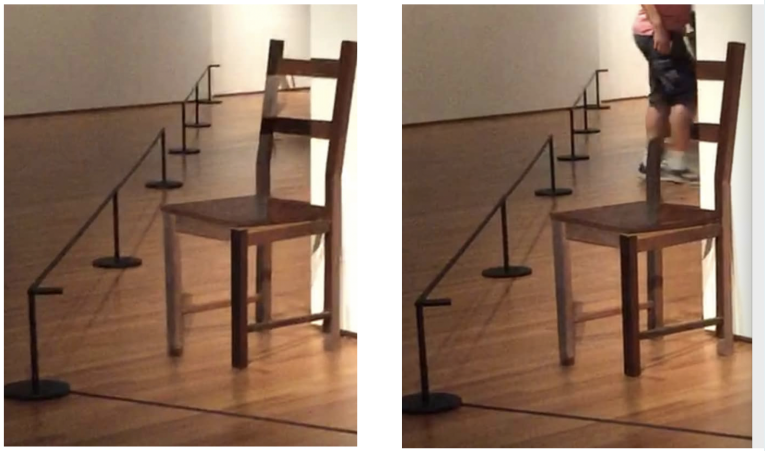
Two-View Principle

Anamorphic Art: Two Views on the Same Object

These images (*which I took at the National Gallery Singapore*) contain a proof of *anamorphism*: only from my specific position does the complete chair become visible.

Two-View Principle

Anamorphic Art: Two Views on the Same Object



Cryptography: Two Views on the "Same" Communication

- **Real view:** Actual interactions between insiders (holders of secret keys).
- **Simulated view:** Interactions are produced by a simulator.

Security: Real view $\approx$ *Simulated view* (e.g., ZKP, MPC)

Anamorphic Cryptography

[PPY22]

Anamorphic Ciphertext

$\xrightarrow{\textit{View with normal\_key}}$ a regular message

$\xrightarrow{\textit{View with double\_key}}$ *an anamorphic message*

Everything should look normal to a powerful adv. $\mathcal{D}$ (dictator):

Anamorphic Cryptography

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- **Ciphertext rule:**

  Anamorphic ciphertexts $\overset{\mathcal{D}}{\approx}$ normal ciphertexts,

  for an allowed encryption, with a specific public key.
- **Key rule:** $\mathcal{D}$ can request the secret key corresponding to any public key (unlike in *steganography*).

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- **Key rule:** $\mathcal{D}$ can request the secret key corresponding to any public key (unlike in *steganography*).
- **(New) Blocking rule:** *D* can block any party from receiving communications (the fraction of blocked parties

Crypto War: On the User's Side

The dictator enacts a law mandating weakened encryption or a built-in backdoor.

$\rightarrow$ A huge risk for **everyone**, including the dictator (e.g., Clipper chip, Dual_EC_DRBG).

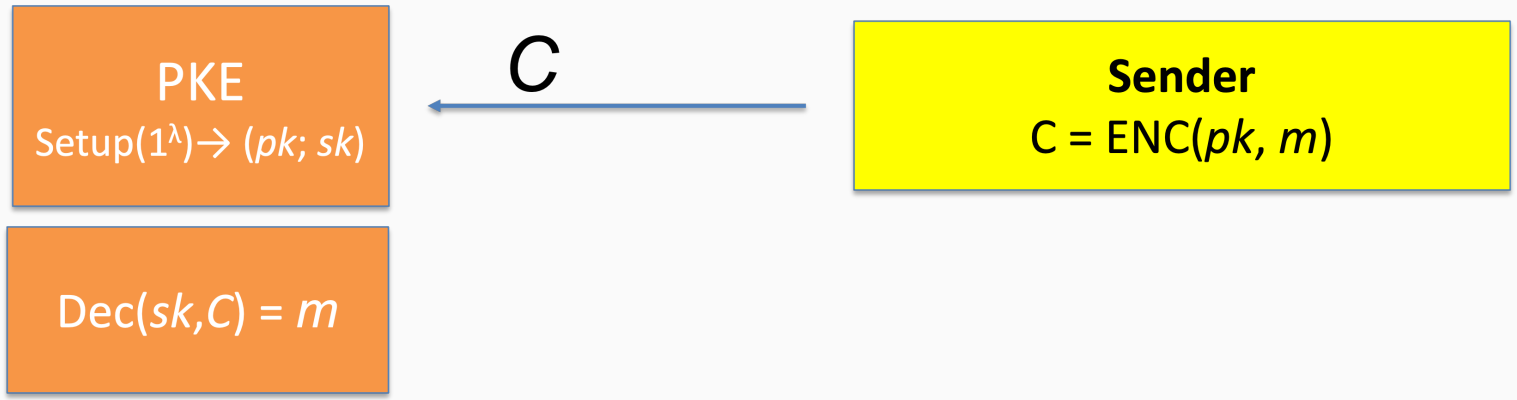
Solution: Public debate, petitions, or technical demonstrations to oppose the approval of such laws.

The dictator permits standard encryption but remotely and massively controls all users:

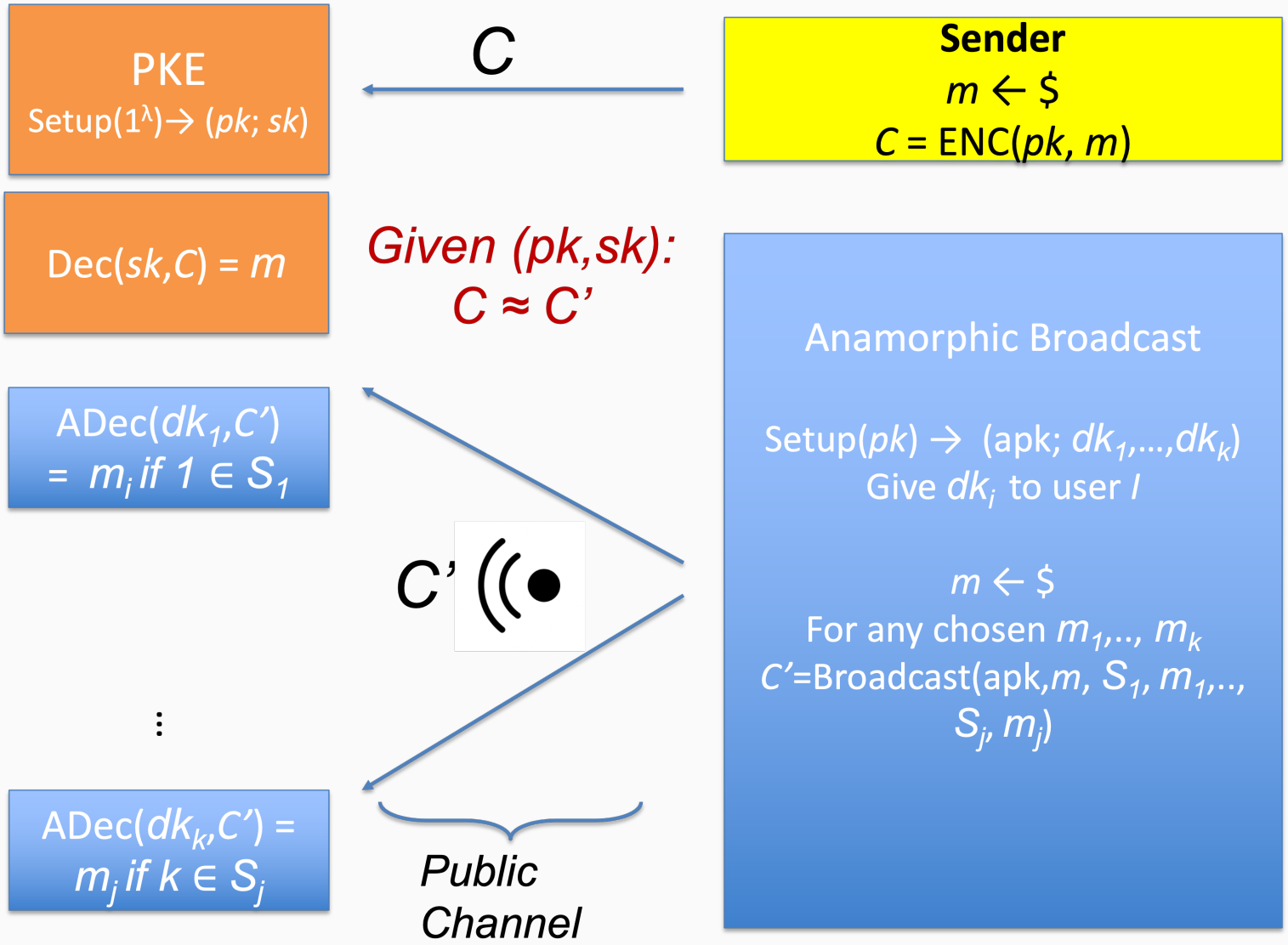
- Require receivers to surrender their **secret keys** so all messages can be decrypted.
- **Block** any suspected users.

Our proposed **anamorphic model** provides a way to preserve users' privacy in this scenario.

PKE with Anamorphic Broadcast Mode



PKE with Anamorphic Broadcast Mode



Our Construction

Anonymity to Anamorphism: from Anonymous Multi-channel Broadcast (AnonMCBE)

Generic Approach

PKE + Associated Anonymous Multi-channel Broadcast such that the ciphertexts are indistinguishable.
Then we can obtain PKE with Anamorphic Broadcast Mode.

How does it work?

- The sender sets up the AnonMCBE.
- Instead of sending a PKE ciphertext, they send a AnonMCBE's ciphertext.
- Anonymity + PKE ciphertexts $\approx$ AnonMCBE ciphertexts
  $\rightarrow$ Everything appears normal to the dictator.
- Double keys are decryption keys of AnonMCBE.
- Each anamorphic user decrypts to a message depending on the channel they belong to.

Anamorphic Broadcast: Instantiation

Generic Approach

PKE + Associated Anonymous Multi-channel Broadcast
such that: the ciphertexts are indistinguishable
then we can get PKE with Anamorphic Broadcast Mdoe.

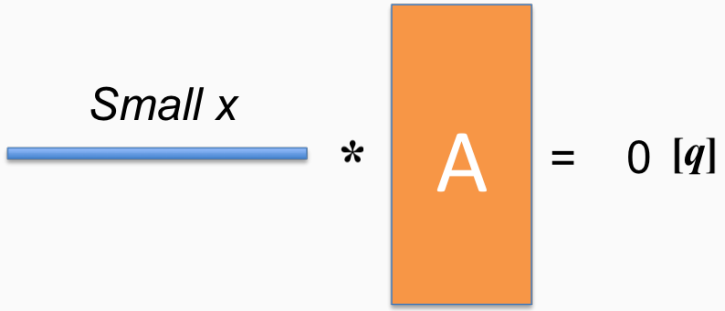
Concrete Construction based on previous works:

- Multi-channel Broadcast [PPT12]: not anonymous.
- Multi-receiver Encryption [LPSS14]: 1 channel, ciphertext is of the same form as in PKE.
- Anonymous Broadcast [DPY20]: 1 channel, ciphertext is of the same form as in PKE.

Our Scheme: Anonymous Multi-channel Broadcast: many channels, ciphertext is of the same form as in PKE.

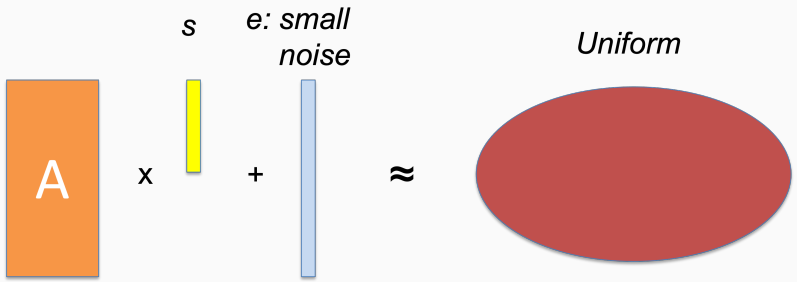
Short Integer Solution [Ajtai96] and Learning With Errors [Regev05] problems

- Params: $m, n, q \geq 0$, $A \hookleftarrow U(\mathbb{Z}_q^{m \times n})$



SIS

Find small $\mathbf{x} \in \mathbb{Z}^m \setminus \mathbf{0}$ s.t. $\mathbf{x}^t A = \mathbf{0}\ [q]$



Key Idea for 1-bit PKE:
w.o secret key x : U(Im(A)) + noise ≈ *Uniform*
with secret key x : x * U(Im(A)) + noise → *small*

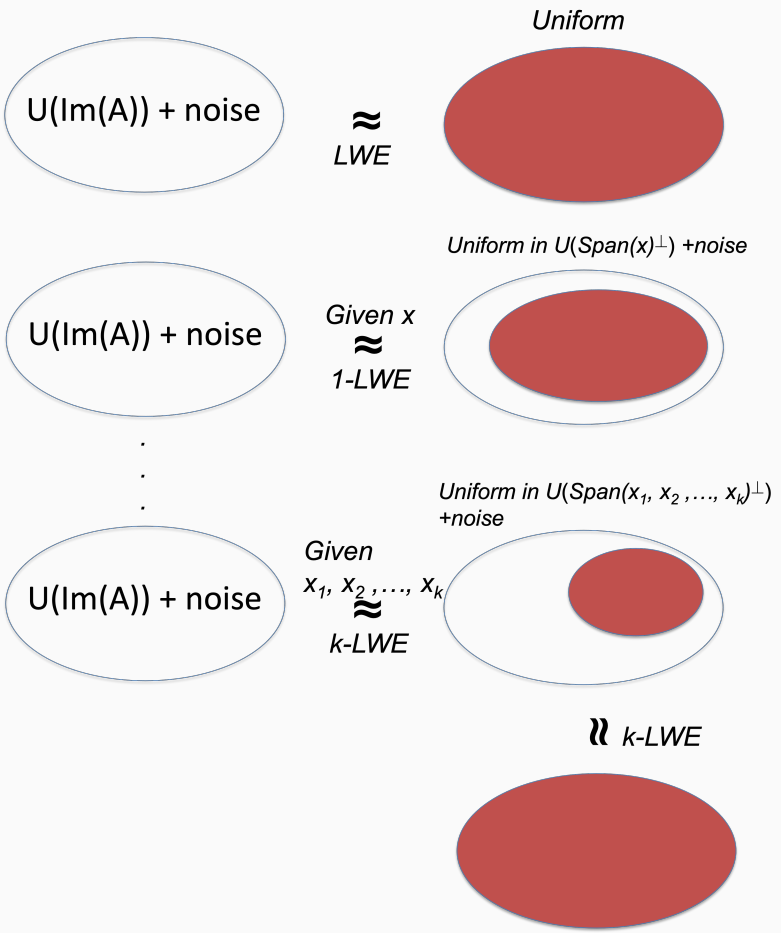
(Here, A *denotes* $\frac{\text{A}}{\text{u}}$ *in Dual Regev LWE Encryption*)

LWE

Dist. $A\mathbf{s} + \mathbf{e}\ [q]$ and $U(\mathbb{Z}_q^m)$, for $\mathbf{s} \hookleftarrow U(\mathbb{Z}_q^n)$, noise $\mathbf{e} \in \mathbb{Z}^m$

k-LWE and Anonymous Broadcast

- Params: $m, n, q \geq 0$, $A \hookleftarrow U(\mathbb{Z}_q^{m \times n})$
- Given k small hints $(\mathbf{x}_i)_{i \leq k}$ s.t. $\mathbf{x}_i^t A = \mathbf{0}\ [q]$ (from [GPV08])



k-SIS [BF11]

Find small $\mathbf{x} \in \mathbb{Z}^m$ s.t.

- $\mathbf{x}^t A = \mathbf{0}\ [q]$
- $\mathbf{x} \notin \mathrm{Span}_{i \leq k}(\mathbf{x}_i)$

k-LWE [LPSS14]

Distinguish $A\mathbf{s} + \mathbf{e}$ and $U(\mathrm{Span}_{i \leq k}(\mathbf{x}_i)^\perp) + \mathbf{e}'$ for $\mathbf{s} \hookleftarrow U(\mathbb{Z}_q^n)$ and small noises $\mathbf{e}, \mathbf{e}' \in \mathbb{Z}^m$

Anon-Broadcast [DPY20]

Using $U(\mathrm{Span}_{i \leq k}(\mathbf{x}_i)^\perp) + \mathbf{e}'$

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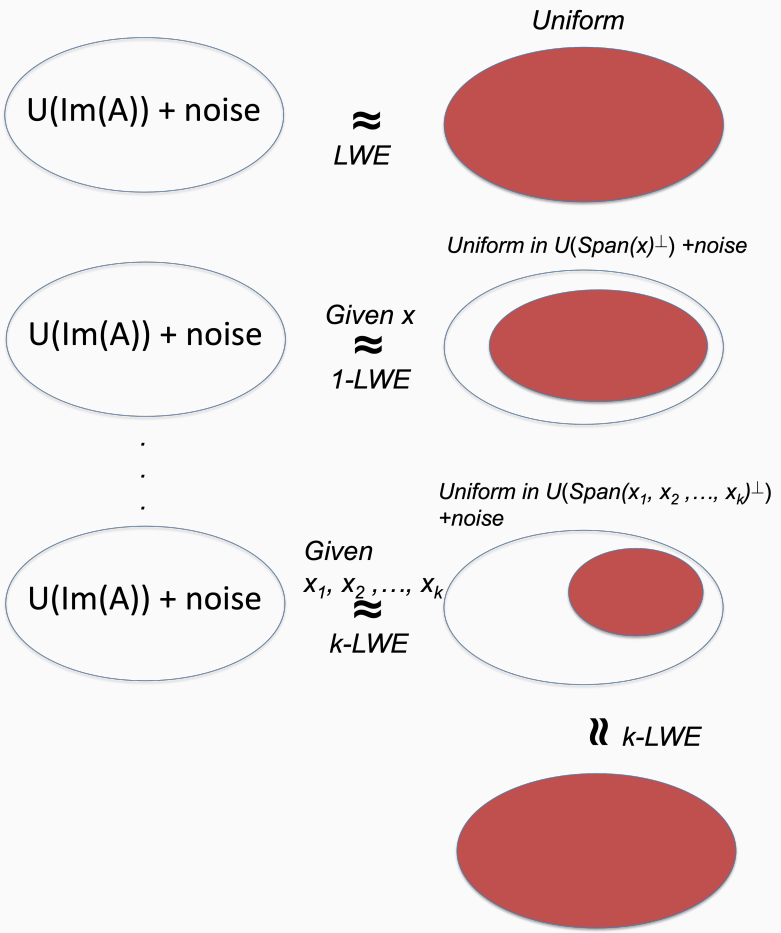

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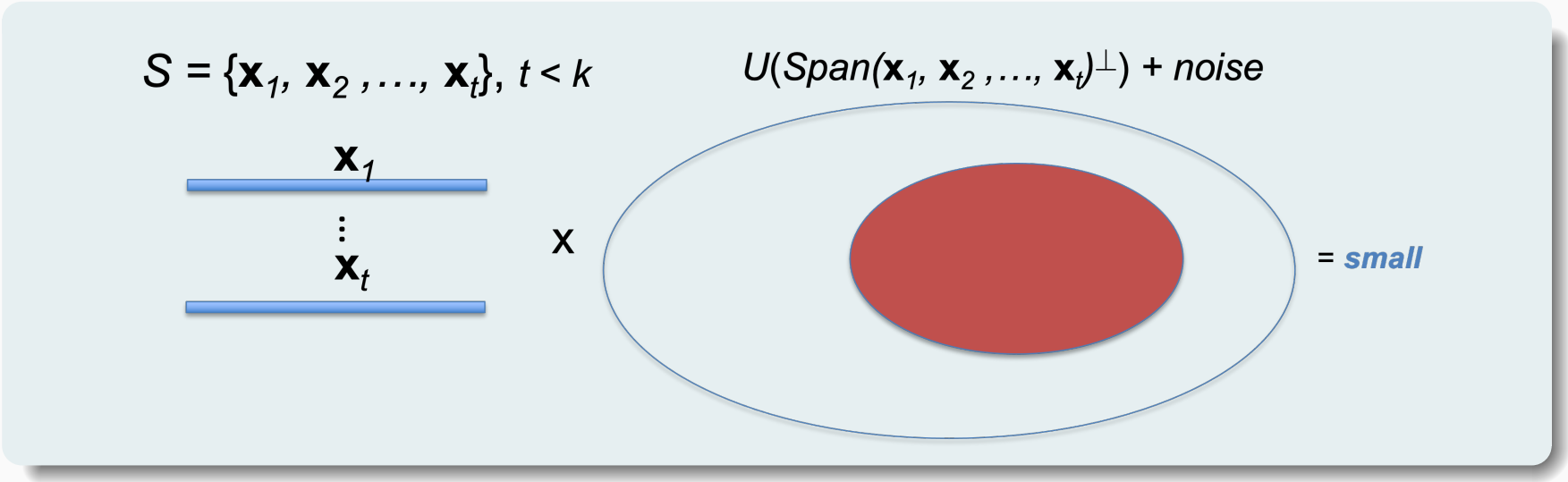
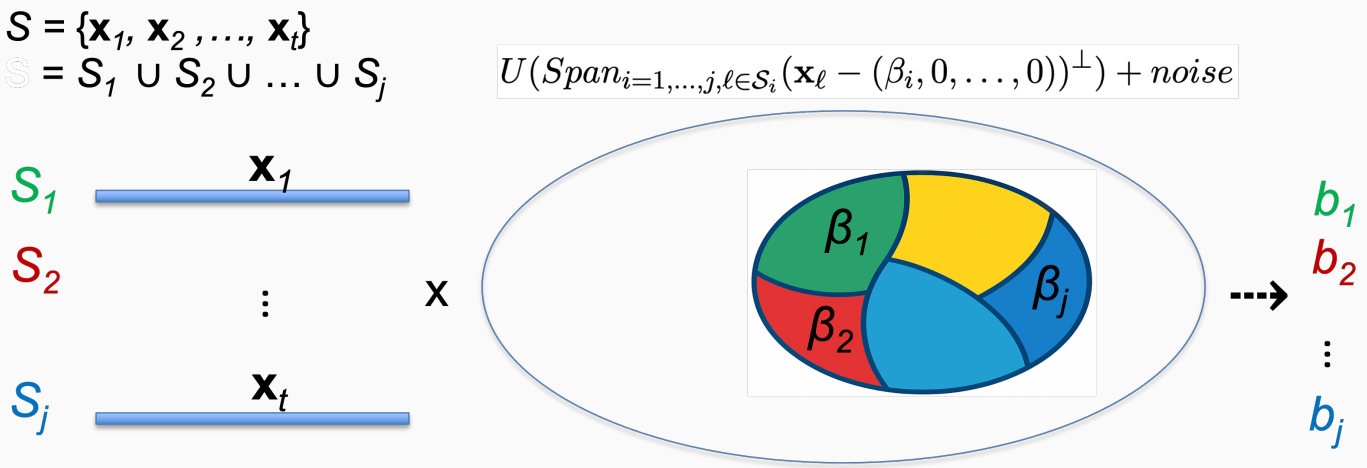
Anonymous Broadcast Encryption from k-LWE in a Bounded Universe of k Users



- Twist on [LPSS14]: Using $U(\mathrm{span}_{i \le t}(\mathbf{x}_i)^{\perp}) + \mathbf{e}$ for broadcasting (instead of tracing).
- Any user $\mathbf{x}_i$ can decrypt (while all others receive a random bit), and the ciphertext remains indistinguishable from standard PKE ciphertexts, under the k-LWE assumption.

Anonymous Multi-channel Broadcast Encryption



- 1-channel: Using $\mathbf{y} \hookleftarrow U(\mathrm{Span}_{i\leq t}(\mathbf{x}_i - (\beta, 0, \ldots, 0))^\perp) + \mathbf{e}$ to broadcast, then the decrypted result is modified, depending on βy_1 (all users in S still get the same result).
- **Multi-channel**: Partition the set into $S_1, \ldots, S_j$, where each set S_i is associated with $\beta_i \rightarrow$ anyone in the same subset decrypts to the same bit (depending on $\beta_i y_1$).

Discussion and Open Questions

Anamorphic Crypto: Recent Active Directions

- Sender Anamorphism: The dictator can ask the sender to encrypt a specific message and provide proof of doing so (by giving him the randomness).
- Robustness; Anamorphic Extensions;
- Anamorphic-Resistance Encryption (ARE),
- Anamorphism in other primitives such as Signature, FHE.
- Public-key Anamorphism: No shared double keys.
- Generic constructions for any scheme? ($\rightarrow$ next talk.)

Open questions

- Extend the one-to-many channel model along the above directions, in particular public-key anamorphism.
- Anamorphic broadcast mode for more schemes?
  $\rightarrow$ Bounded-Collusion Anonymous Broadcast Encryption.