Black Box Crypto is Useless for Doubly Efficient PIR

Wei-Kai Lin University of Virginia Ethan Mook Northeastern

Daniel Wichs Northeastern & NTT Research

Black Box Crypto is Useless for Doubly Efficient PIR

Wei-Kai Lin University of Virginia Ethan Mook Northeastern



Daniel Wichs Northeastern & NTT Research

Eurocrypt 2025

Private Information Retrieval (PIR)

Interactive protocol between a server S and a client C







Private Information Retrieval (PIR)

Interactive protocol between a server S and a client C





Private Information Retrieval (PIR)

Interactive protocol between a server S and a client C

C





input: $i \in [N]$





input: $i \in [N]$



















Doubly Efficient PIR (DEPIR) $DB \in \{0,1\}^N$







input: $i \in [N]$ output: DB[i]





Doubly Efficient PIR (DEPIR)

C

 $k \leftarrow \text{KeyGen}(1^{\lambda})$

k DB-ind. and static

input: $i \in [N], k$





 $DB \leftarrow Prep(DB, k)$

k DB-ind. and static

DB one-time preprocessed



input: $i \in [N], k$





 $DB \leftarrow Prep(DB, k)$

k DB-ind. and static

DB one-time preprocessed



input: $i \in [N], k$





 $DB \leftarrow Prep(DB, k)$

k DB-ind. and static

DB one-time preprocessed



input: $i \in [N], k$

output: DB[*i*]







 $DB \leftarrow Prep(DB, k)$

k DB-ind. and static

DB one-time preprocessed



input: $i \in [N], k$

output: DB[*i*]





Secret-Key (SK-DEPIR)

Public-Key (PK-DEPIR)

Unkeyed (UK-DEPIR) **Prior constructions**



Public-Key (PK-DEPIR)

Unkeyed (UK-DEPIR) **Prior constructions**

Secret-Key (SK-DEPIR)	$\frac{sk}{DB} \leftarrow \text{KeyGe}$
Public-Key (PK-DEPIR)	<i>pk</i> ← KeyGe ÕB ← Prep(I
Unkeyed	

Unkeyed (UK-DEPIR)

Prior constructions

- $Gen(1^{\lambda})$ C DB, sk
- (DB, pk)



Secret-Key (SK-DEPIR)	$sk \leftarrow \text{KeyGe}$ $\widetilde{\text{DB}} \leftarrow \text{Prep}(\text{I})$
Public-Key (PK-DEPIR)	<i>pk</i> ← KeyGe ÕB ← Prep(I
Unkeyed (UK-DEPIR)	$\widetilde{\text{DB}} := \text{Prep}$

Prior constructions

- Gen (1^{λ}) (DB, *sk*)
- (DB, pk)



o(DB)



Secret-Key (SK-DEPIR)	$sk \leftarrow \text{KeyGe}$ $\widetilde{\text{DB}} \leftarrow \text{Prep}(\text{I})$
Public-Key (PK-DEPIR)	<i>pk</i> ← KeyGe ÕB ← Prep(I
Unkeyed (UK-DEPIR)	$\widetilde{\text{DB}} := \text{Prep}$

Prior constructions

 $Gen(1^{\lambda})$ C DB, sk



[CHR'17,BIPW'17]: From "permuted codes with noise"

(DB, pk)



o(DB)





Secret-Key (SK-DEPIR)	$sk \leftarrow \text{KeyGe}$ $\widetilde{\text{DB}} \leftarrow \text{Prep}(\text{I})$
Public-Key (PK-DEPIR)	<i>pk</i> ← KeyGe ÕB ← Prep(I
Unkeyed (UK-DEPIR)	$\widetilde{\text{DB}} := \text{Prep}$

Prior constructions

 $ien(1^{\lambda})$ DB, <u>sk</u>)



[CHR'17,BIPW'17]: From "permuted codes with noise"

 $en(1^{\lambda})$ DB, *pk*)



[CHR'17,BIPW'17]: From SK-DEPIR + heuristic obf.

o(DB)





Secret-Key (SK-DEPIR)	$\frac{sk}{DB} \leftarrow \text{KeyGen}(1^{\lambda})$ $\widetilde{DB} \leftarrow \text{Prep}(DB, sk)$
Public-Key (PK-DEPIR)	$\frac{pk}{DB} \leftarrow \text{KeyGen}(1^{\lambda})$ $\widetilde{DB} \leftarrow \text{Prep}(\text{DB}, p)$
Unkeyed (UK-DEPIR)	$\widetilde{\text{DB}} := \text{Prep}(\text{DB})$

Prior constructions

 $den(1^{\lambda})$ DB, <u>sk</u>)



[CHR'17,BIPW'17]: From "permuted codes with noise"

 $en(1^{\lambda})$ DB, *pk*)



[CHR'17,BIPW'17]: From SK-DEPIR + heuristic obf.

S

[LMW'23]: From RingLWE



Secret-Key (SK-DEPIR)	$\frac{sk}{DB} \leftarrow \text{KeyGen}(1^{\lambda})$
Public-Key (PK-DEPIR)	$\frac{pk}{DB} \leftarrow \text{KeyGen}(1^{\lambda})$ $\widetilde{DB} \leftarrow \text{Prep}(DB, p)$
Unkeyed (UK-DFPIR)	$\widetilde{\text{DB}} := \text{Prep}(\text{DB})$

Prior constructions

 $ben(1^{\lambda})$ DB, sk



[CHR'17,BIPW'17]: From "permuted codes with noise"

 $en(1^{\lambda})$ DB, *pk*)



[CHR'17,BIPW'17]: From SK-DEPIR + heuristic obf.

S

[LMW'23]: From RingLWE



Secret-Key (SK-DEPIR)	$\frac{sk}{\text{DB}} \leftarrow \text{KeyGen}(1^{\lambda})$ $\widetilde{\text{DB}} \leftarrow \text{Prep}(\text{DB}, \frac{sk}{sk})$
Public-Key (PK-DEPIR)	$\begin{array}{l} pk \leftarrow KeyGen(1^{\lambda}) \\ \widetilde{DB} \leftarrow Prep(DB, p) \end{array}$
Unkeyed (UK-DEPIR)	$\widetilde{\text{DB}} := \text{Prep}(\text{DB})$
↓ (Standard) PIR	

Prior constructions

 $ben(1^{\lambda})$ DB, sk)



[CHR'17,BIPW'17]: From "permuted codes with noise"

 $en(1^{\lambda})$ DB, *pk*)



[CHR'17,BIPW'17]: From SK-DEPIR + heuristic obf.

S

[LMW'23]: From RingLWE



How does (SK,PK,UK)–DEPIR relate to other crypto primitives?

How does (SK,PK,UK)-DEPIR relate to other crypto primitives?

<u>Theorem 1</u>: Let *P* be a crypto primitive that *large class (to be specified)*



<u>Theorem 1</u>: Let *P* be a crypto primitive that *large class (to be specified)* • If: there is a BB construction of SK-DEPIR from P



Theorem 1: Let *P* be a crypto primitive that *large class (to be specified) If:* there is a BB construction of SK-DEPIR from *P* Then: there is a BB construction of SK-DEPIR from OWFs



- *If:* there is a BB construction of SK-DEPIR from P
- Then: there is a BB construction of SK-DEPIR from OWFs

Partial progress ruling out SK-DEPIR from OWFs

Our Results

Black-box use of advanced crypto is *useless* for constructing DEPIR

Theorem 1: Let *P* be a crypto primitive that *large class (to be specified)*



Theorem 1: Let *P* be a crypto primitive that *large class (to be specified)* • *If:* there is a BB construction of SK-DEPIR from *P*

• Then: there is a BB construction of SK-DEPIR from OWFs

Partial progress ruling out SK-DEPIR from OWFs

<u>Theorem 2</u>: There is no BB construction of **2-round**, **passive server SK-DEPIR** from OWFs

Our Results

Black-box use of advanced crypto is *useless* for constructing DEPIR





Techniques

Starting point: An SK-DEPIR that exists in an idealized world DEPIR = (Prep^{\mathcal{O}_P}, $C^{\mathcal{O}_P}$, $S^{\mathcal{O}_P}$)



Techniques $\mathcal{O}_P = \text{oracle implementing } P$

Starting point: An SK-DEPIR that exists in an idealized world DEPIR = (Prep^{\mathcal{O}_P}, $C^{\mathcal{O}_P}$, $S^{\mathcal{O}_P}$)


Starting point: An SK-DEPIR that exists in an idealized world

Key insight: No hiding property from the client \Rightarrow client can make the server's oracle calls for it

 $\mathsf{DEPIR} = (\mathsf{Prep}^{\mathcal{O}_P}, C^{\mathcal{O}_P}, S^{\mathcal{O}_P})$



Starting point: An SK-DEPIR that exists in an idealized world DEPIR = (Prep^{\mathcal{O}_P}, $C^{\mathcal{O}_P}$, $S^{\mathcal{O}_P}$)

Key insight: No hiding property from the client ⇒ client can make the server's oracle calls for it

Inspired by techniques of [Dujmovic-Hajiabadi'24]



Starting point: An SK-DEPIR that exists in an idealized world

Key insight: No hiding property from the client \Rightarrow client can make the server's oracle calls for it

 $\mathsf{DEPIR} = (\mathsf{Prep}^{\mathcal{O}_P}, C^{\mathcal{O}_P}, S^{\mathcal{O}_P})$

Inspired by techniques of [Dujmovic-Hajiabadi'24]

Compile into an SK-DEPIR where the server doesn't use the oracle $\widetilde{\mathsf{DEPIR}} = (\mathsf{Prep}^{\mathcal{O}_P}, \widetilde{C}^{\mathcal{O}_P}, \widetilde{S})$







Starting point: An SK-DEPIR that exists in an idealized world Key insight: No hiding property from the client \Rightarrow client can make the server's oracle calls for it If \mathcal{O}_P is "nice", then the client can emulate \mathcal{O}_P with just a PRF

 $\mathsf{DEPIR} = (\mathsf{Prep}^{\mathcal{O}_P}, C^{\mathcal{O}_P}, S^{\mathcal{O}_P})$

Inspired by techniques of [Dujmovic-Hajiabadi'24]

Compile into an SK-DEPIR where the server doesn't use the oracle $\widetilde{\mathsf{DEPIR}} = (\mathsf{Prep}^{\mathcal{O}_P}, \widetilde{C}^{\mathcal{O}_P}, \widetilde{S})$

 $\overline{\text{DEPIR}} = (\overline{\text{Prep}}^{\text{PRF}}, \overline{C}^{\text{PRF}}, \tilde{S})$







































Removing the Server's Oracle \mathcal{O}_P DB DB 0_p \tilde{S} CS S



Removing the Server's Oracle \mathcal{O}_P DB DB \mathcal{O}_{P} $\widetilde{\text{DB}}[j_1]$ Ŝ CS S









Removing the Server's Oracle



Communication & \tilde{S} efficiency: O(# locations read) = o(N)





Removing the Server's Oracle \mathcal{O}_P DB DB \mathcal{O}_{P} $\widetilde{\text{DB}}[j_1]$ \tilde{S} S S $DB[j_{e}]$





Communication & \tilde{S} efficiency: \tilde{S} is purely passive O(# locations read) = o(N)





Definition: A *crypto oracle* is an oracle of the form B^R where

• R is a private random function available only to B

• R is a private random function available only to BSuppose $\mathcal{O}_P = B^R$ is a crypto oracle

• R is a private random function available only to BSuppose $\mathcal{O}_P = B^R$ is a crypto oracle



• R is a private random function available only to BSuppose $\mathcal{O}_P = B^R$ is a crypto oracle



• R is a private random function available only to BSuppose $\mathcal{O}_P = B^R$ is a crypto oracle



• R is a private random function available only to BSuppose $\mathcal{O}_P = B^R$ is a crypto oracle



• R is a private random function available only to BSuppose $\mathcal{O}_P = B^R$ is a crypto oracle



- **Definition:** A crypto oracle is an oracle of the form B^R where • *B* is a poly time (stateless, deterministic) Turing machine

Prep	\overline{C}



• R is a private random function available only to BSuppose $\mathcal{O}_P = B^R$ is a crypto oracle



- **Definition:** A crypto oracle is an oracle of the form B^R where • *B* is a poly time (stateless, deterministic) Turing machine

Prep	\overline{C}	
Prep		Ĉ



• R is a private random function available only to BSuppose $\mathcal{O}_P = B^R$ is a crypto oracle









• R is a private random function available only to B




Theorem 1: Let *P* be a crypto primitive that *large class (to be specified)* • *If:* there is a BB construction of SK-DEPIR from *P*

• Then: there is a BB construction of SK-DEPIR from OWFs

Partial progress ruling out SK-DEPIR from OWFs

<u>Theorem 2</u>: There is no BB construction of **2-round**, **passive server SK-DEPIR** from OWFs

Our Results

Black-box use of advanced crypto is *useless* for constructing DEPIR





Black-box use of advanced crypto is *useless* for constructing DEPIR

<u>Theorem 1</u>: Let P be a crypto primitive that exists wrt a crypto oracle • If: there is a BB construction of SK-DEPIR from P • Then: there is a BB construction of SK-DEPIR from OWFs

Partial progress ruling out SK-DEPIR from OWFs

<u>Theorem 2</u>: There is no BB construction of **2-round**, **passive server SK-DEPIR** from OWFs

Our Results





- Then: there is a BB construction of SK-DEPIR from OWFs

Partial progress ruling out SK-DEPIR from OWFs

<u>Theorem 2</u>: There is no BB construction of **2-round**, **passive server SK-DEPIR** from OWFs Adapted from [CHR'17] info. theoretic lower bound

 \rightarrow see paper for details

Our Results

Black-box use of advanced crypto is *useless* for constructing DEPIR

<u>Theorem 1</u>: Let P be a crypto primitive that exists wrt a crypto oracle • If: there is a BB construction of SK-DEPIR from P





Lemma: Virtual black-box obfuscation exists relative to a crypto oracle



Lemma: Virtual black-box obfuscation exists relative to a crypto oracle



Lemma: Virtual black-box obfuscation exists relative to a crypto oracle

$$B^R = (Obf^R, Eval^R)$$



Lemma: Virtual black-box obfuscation exists relative to a crypto oracle

$$B^{R} = (Obf^{R}, Eval^{R})$$
$$Obf^{R}$$





Lemma: Virtual black-box obfuscation exists relative to a crypto oracle

$$B^{R} = (Obf^{R}, Eval^{R})$$
$$Obf^{R}$$
$$Eval^{R}$$





Proof sketch:

$$B^{R} = (Obf^{R}, Eval^{R})$$
$$Obf^{R}$$
$$Eval^{R}$$

Lemma: Virtual black-box obfuscation exists relative to a crypto oracle





Lemma: Virtual black-box obfuscation exists relative to a crypto oracle

$$B^{R} = (Obf^{R}, Eval^{R})$$
$$Obf^{R}$$
$$Eval^{R}$$





Lemma: Virtual black-box obfuscation exists relative to a crypto oracle

$$B^{R} = (Obf^{R}, Eval^{R})$$
$$Obf^{R}$$
$$Eval^{R}$$





Lemma: Virtual black-box obfuscation exists relative to a crypto oracle

$$B^{R} = (Obf^{R}, Eval^{R})$$
$$Obf^{R}$$
$$Eval^{R}$$
$$C := Dec_{R}$$





Lemma: Virtual black-box obfuscation exists relative to a crypto oracle

$$B^{R} = (Obf^{R}, Eval^{R})$$
$$Obf^{R}$$
$$Eval^{R}$$
$$C := Dec_{R}$$





Lemma: Virtual black-box obfuscation exists relative to a crypto oracle

Proof sketch:

$$B^{R} = (Obf^{R}, Eval^{R})$$
$$Obf^{R}$$
$$Eval^{R}$$
$$C := Dec_{R}$$

Lemma: The generic multi-linear group model can be realized as a crypto oracle \Rightarrow OT, FHE, ... can as well







Crypto oracles capture a wide array of primitives, even combinations

Corollary:

- If: there is a BB construction of SK-DEPIR from P
- Then: there is a BB construction of SK-DEPIR from OWFs

<u>Corollary</u>: There is no BB construction of 2-round, passive server **SK-DEPIR** from OWFs



Crypto oracles capture a wide array of primitives, even combinations

<u>Corollary</u>:

- If: there is a BB construction of SK-DEPIR from FHE
- Then: there is a BB construction of SK-DEPIR from OWFs

<u>Corollary</u>: There is no BB construction of **2-round, passive server** SK-DEPIR from FHE

of SK-DEPIR from FHE on of SK-DEPIR from OWFs



Crypto oracles capture a wide array of primitives, even combinations

Corollary:

- Then: there is a BB construction of SK-DEPIR from OWFs

<u>Corollary</u>: There is no BB construction of **2-round, passive server** SK-DEPIR from





Crypto oracles capture a wide array of primitives, even combinations

Corollary:

- Then: there is a BB construction of SK-DEPIR from OWFs

Corollary: There is no BB construction of 2-round, passive server SK-DEPIR from IO+FHE





Crypto oracles capture a wide array of primitives, even combinations

Corollary:

Corollary: There is no BB construction of **2-round, passive server** SK-DEPIR from O+FHE

Prior lemma gives VBB for oracle-aided circuits \Rightarrow capture non-BB techniques like "eval crypto under obfuscation"





Our compiler is agnostic to the presence of other assumptions

Our compiler is agnostic to the presence of other assumptions

<u>Corollary (e.g.)</u>: Let P be a primitive that exists wrt a crypto oracle



Our compiler is agnostic to the presence of other assumptions

Corollary (e.g.): Let P be a primitive that exists wrt a crypto oracle
If: there is a construction of SK-DEPIR making BB use of P and assuming DDH



Our compiler is agnostic to the presence of other assumptions

<u>Corollary (e.g.)</u>: Let P be a primitive that exists wrt a crypto oracle • If: there is a construction of SK-DEPIR making BB use of P and

- assuming DDH

• Then: there is a construction of SK-DEPIR assuming just DDH



Our compiler is agnostic to the presence of other assumptions

<u>Corollary (e.g.)</u>: Let P be a primitive that exists wrt a crypto oracle • If: there is a construction of SK-DEPIR making BB use of P and

- assuming DDH

Crucial: need DDH over a *concrete group* (e.g. \mathbb{Z}_p^*), the generic group can be realized as crypto oracle

• Then: there is a construction of SK-DEPIR assuming just DDH



Recall: Prior positive results • UK-DEPIR from RingLWE [LMW'23]

SK-DEPIR from "permuted codes with noise" [CHR'17, BIPW'17]



Recall: Prior positive results • UK-DEPIR from RingLWE [LMW'23]

assumption

- SK-DEPIR from "permuted codes with noise" [CHR'17, BIPW'17]
- Both make use of concrete algebraic structure of the underlying



Recall: Prior positive results • UK-DEPIR from RingLWE [LMW'23]

- Both make use of concrete algebraic structure of the underlying assumption
- [CHR'17,BIPW'17]: Hardness over a locally decodable code

SK-DEPIR from "permuted codes with noise" [CHR'17, BIPW'17]



Recall: Prior positive results • UK-DEPIR from RingLWE [LMW'23]

- Both make use of concrete algebraic structure of the underlying assumption
- [CHR'17,BIPW'17]: Hardness over a locally decodable code • [LMW'23]: RingLWE ring is conducive to some preprocessing

SK-DEPIR from "permuted codes with noise" [CHR'17, BIPW'17]



Recall: Prior positive results • UK-DEPIR from RingLWE [LMW'23]

assumption

- [CHR'17,BIPW'17]: Hardness over a locally decodable code • [LMW'23]: RingLWE ring is conducive to some preprocessing

- SK-DEPIR from "permuted codes with noise" [CHR'17, BIPW'17]
- Both make use of concrete algebraic structure of the underlying

Our result: This concreteness is *inherent*



Theorem 1: For a large class of primitives P

- If: BB construction of SK-DEPIR from P
- *Then:* BB construction of SK-DEPIR from OWFs

The class: constructible relative to crypto oracle

<u>Theorem 2</u>: There is no BB construction of 2-round, passive server SK-DEPIR from OWFs

P om OWFs pto oracle f

<u>Theorem 1</u>: For a large class of primitives P

- If: BB construction of SK-DEPIR from P
- *Then:* BB construction of SK-DEPIR from OWFs

The class: constructible relative to crypto oracle

<u>Theorem 2</u>: There is no BB construction of 2-round, passive server SK-DEPIR from OWFs



Open Questions:

<u>Theorem 1</u>: For a large class of primitives P

- If: BB construction of SK-DEPIR from P
- Then: BB construction of SK-DEPIR from OWFs

The class: constructible relative to crypto oracle

<u>Theorem 2</u>: There is no BB construction of 2-round, passive server SK-DEPIR from OWFs



Open Questions:

• Fully rule out SK-DEPIR from OWFs

Theorem 1: For a large class of primitives *P*

- *If:* BB construction of SK-DEPIR from *P*
- **Then:** BB construction of SK-DEPIR from OWFs

The class: constructible relative to crypto oracle

Theorem 2: There is no BB construction of **2-round, passive server** SK-DEPIR from OWFs



Open Questions:

- Fully rule out SK-DEPIR from **OWFs**
- DEPIR from new (concrete) assumptions
 - Weaker than RingLWE, more standard than permuted codes with noise



Theorem 1: For a large class of primitives *P*

- *If:* BB construction of SK-DEPIR from *P*
- **Then:** BB construction of SK-DEPIR from OWFs

The class: constructible relative to crypto oracle

Theorem 2: There is no BB construction of **2-round, passive server** SK-DEPIR from OWFs





Open Questions:

- Fully rule out SK-DEPIR from **OWFs**
- DEPIR from new (concrete) assumptions
 - Weaker than RingLWE, more standard than permuted codes with noise

eprint: 2025/552

