

# Random Oracle Combiners

Merkle-Damgård Style

Yevgeniy Dodis, Eli Goldin, Peter Hall



# (Hash) Combiners



Hey I have this great system using my all-new super hash function!



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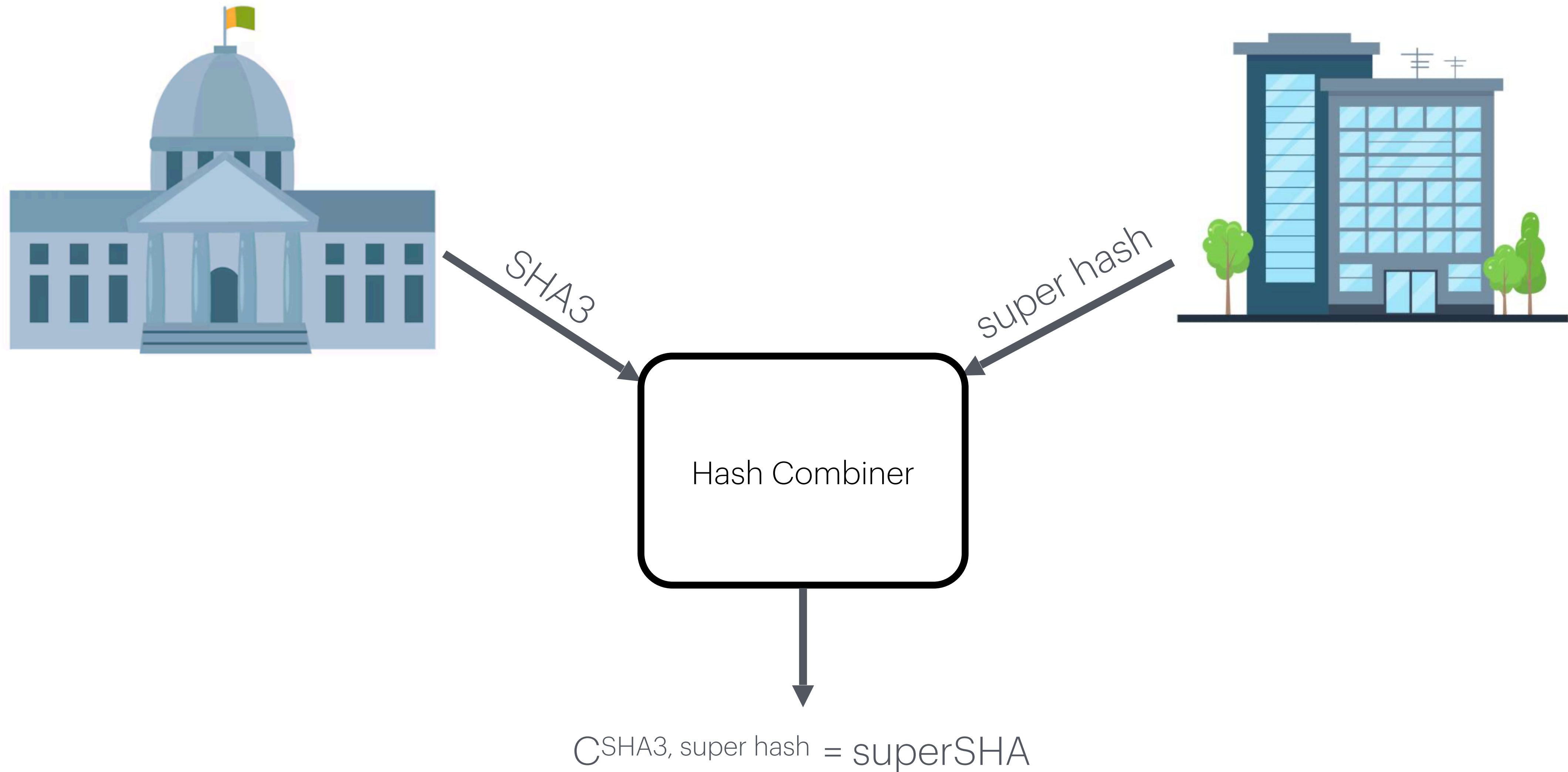
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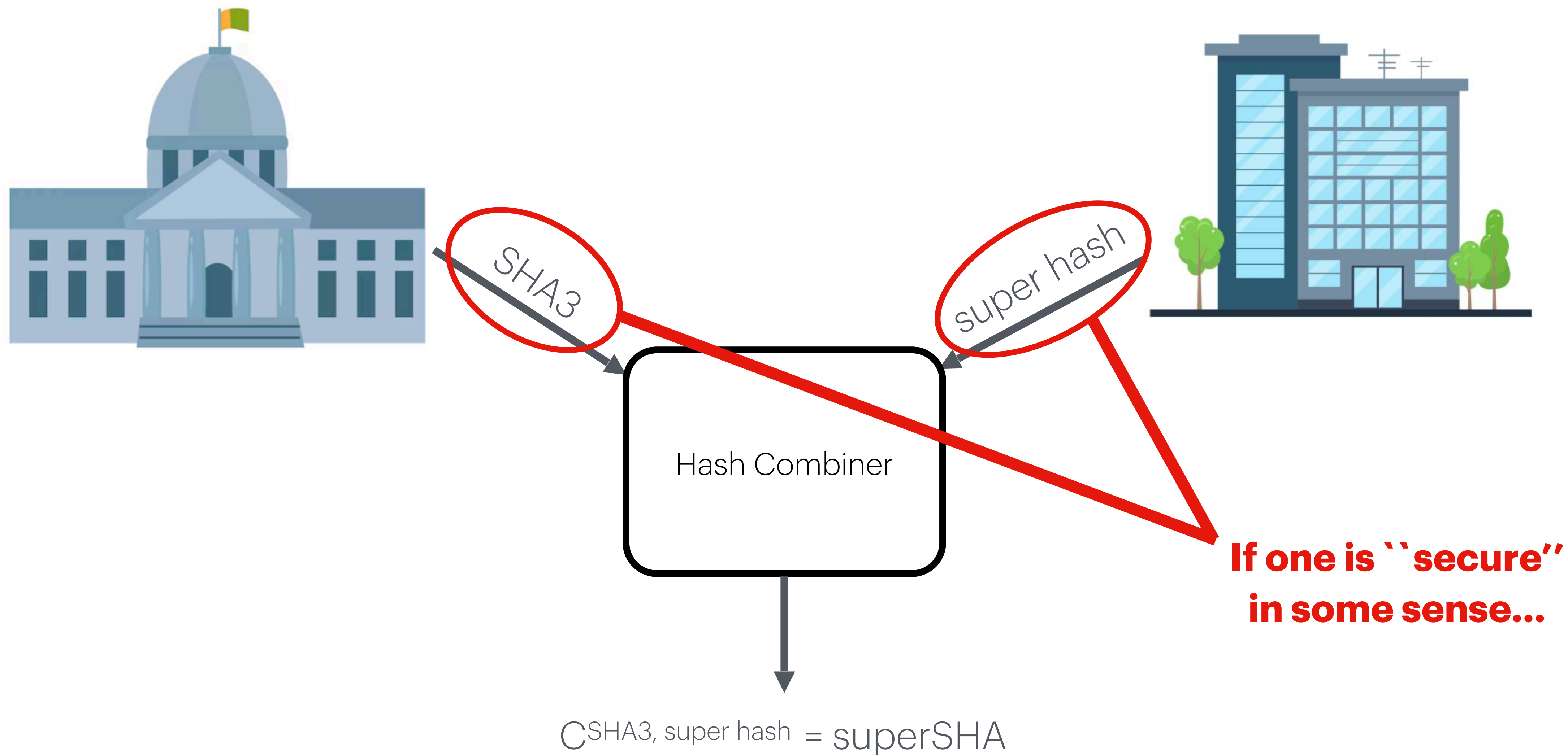
How can they both be happy??

# (Hash) Combiners

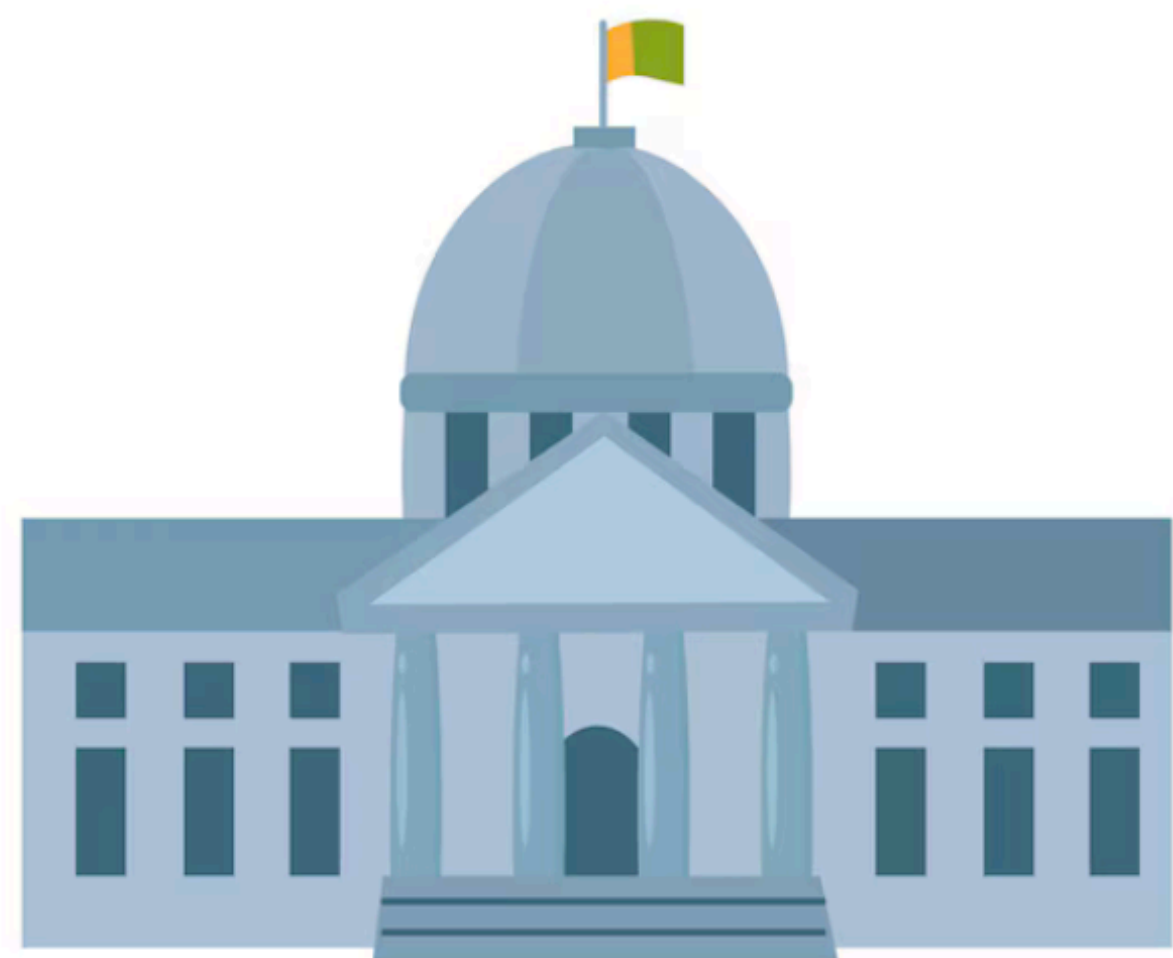




# (Hash) Combiners



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SHA3



super hash

Hash Combiner

**If one is "secure"  
in some sense...**

$\text{CSHA3, super hash} = \text{superSHA}$

**Then so is this!**



# Hash Transforms

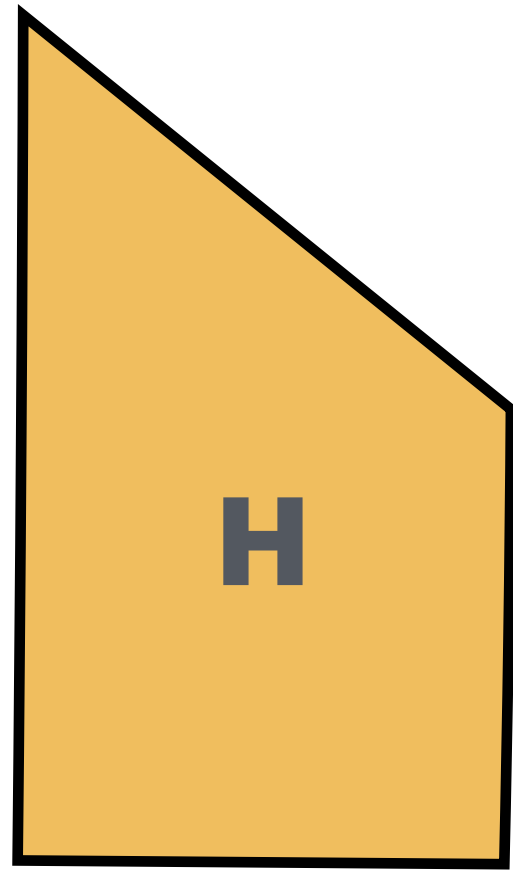
Monolithic Hash Function



# Hash Transforms

The Merkle-Damgård Approach

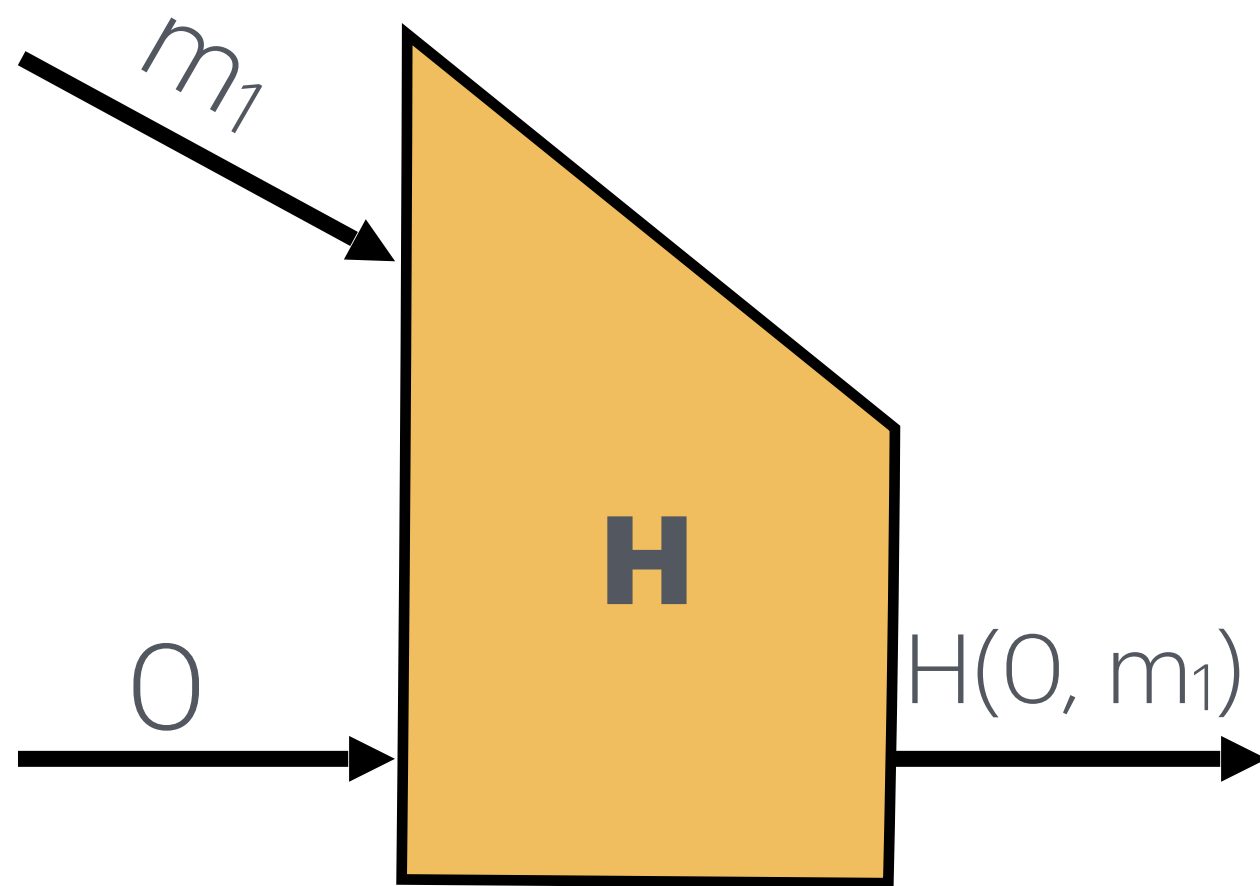
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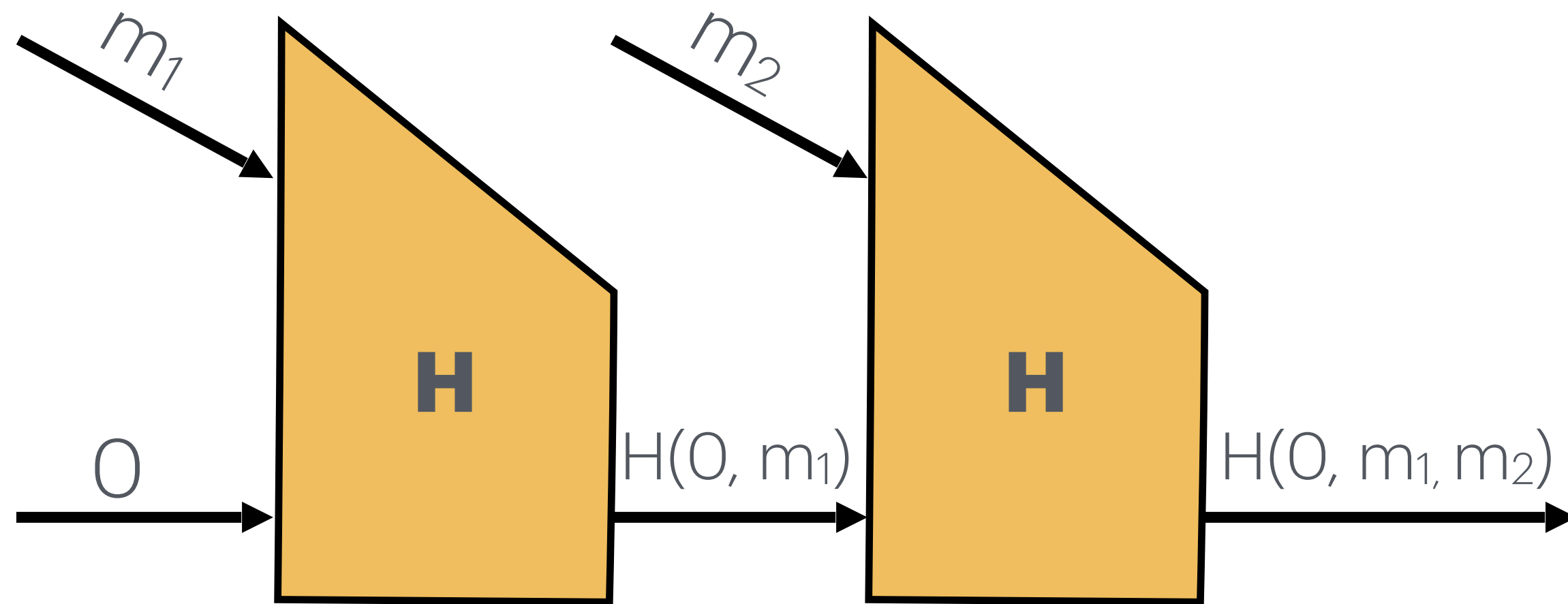
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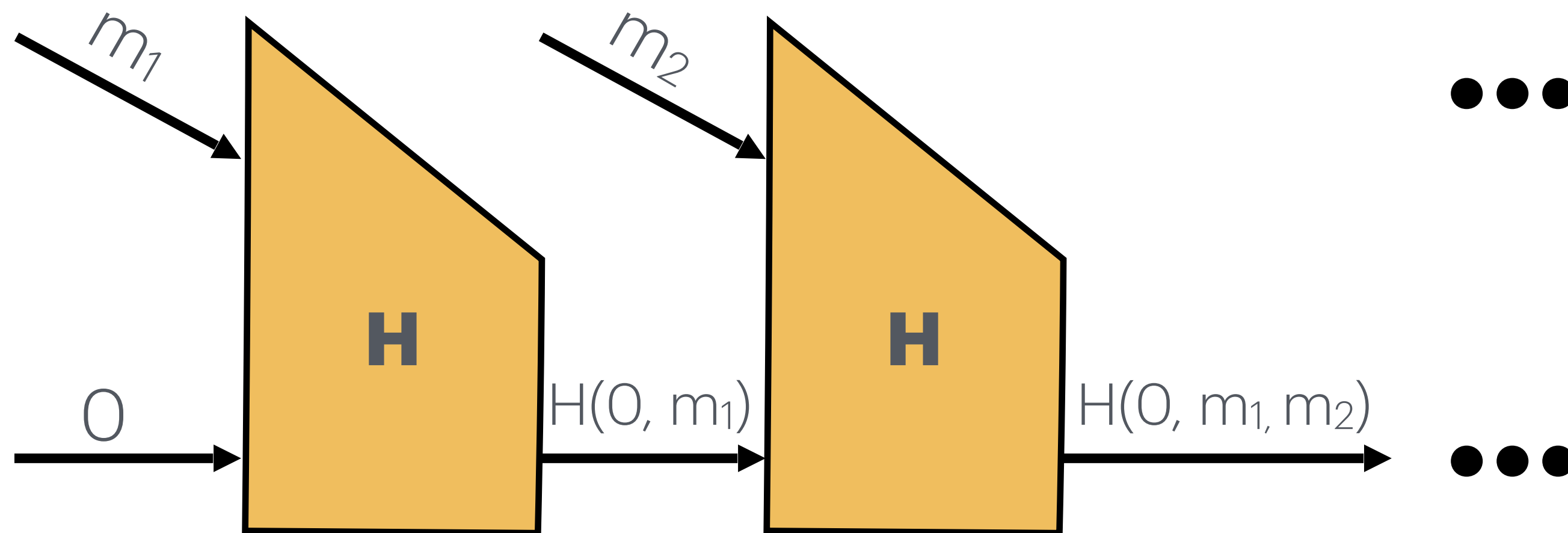
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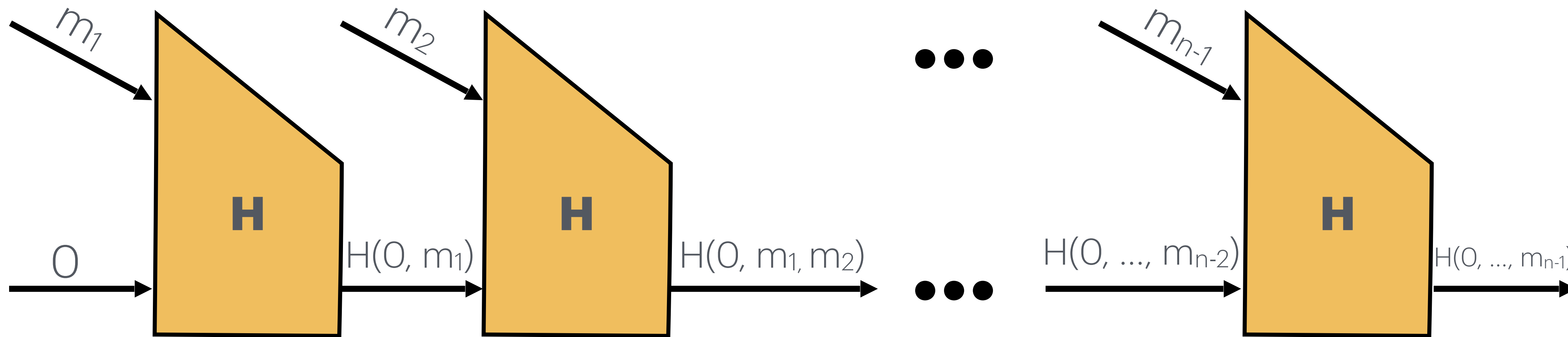




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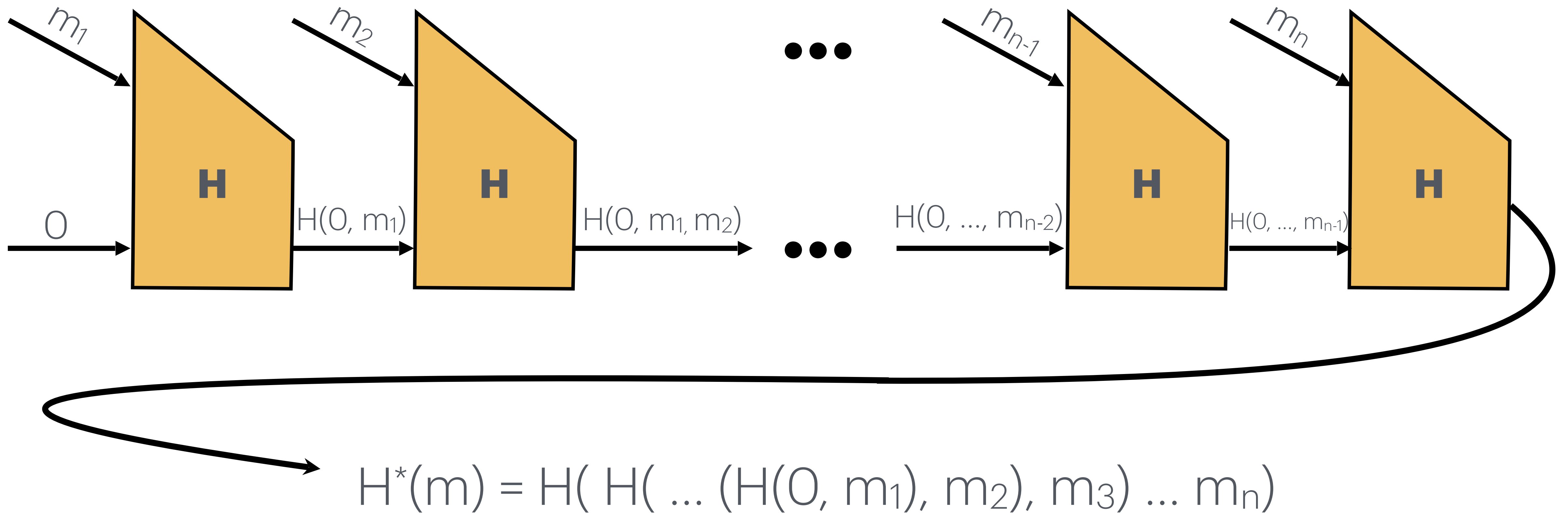
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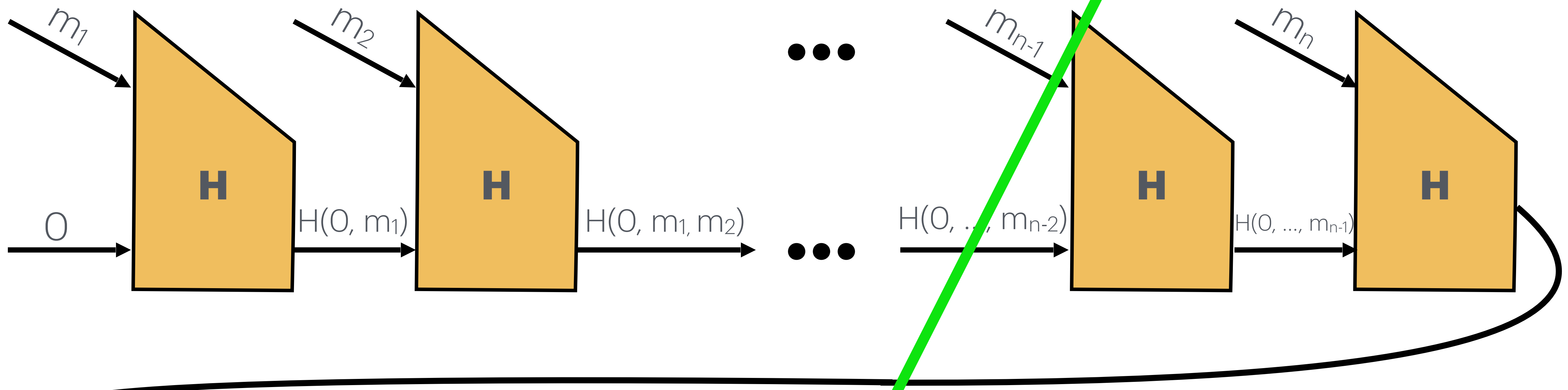
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# Hash Transforms

## The Merkle-Damgård Approach

**Advantage:** underlying hash H can be much less compressing than overall hash compression!



$$H^*(m) = H( H( \dots (H(0, m_1), m_2), m_3) \dots m_n)$$

Background

# Concatenation Barrier

- Collision Resistance barrier

$$C^{h, g}(m) = h(m) \parallel g(m)$$

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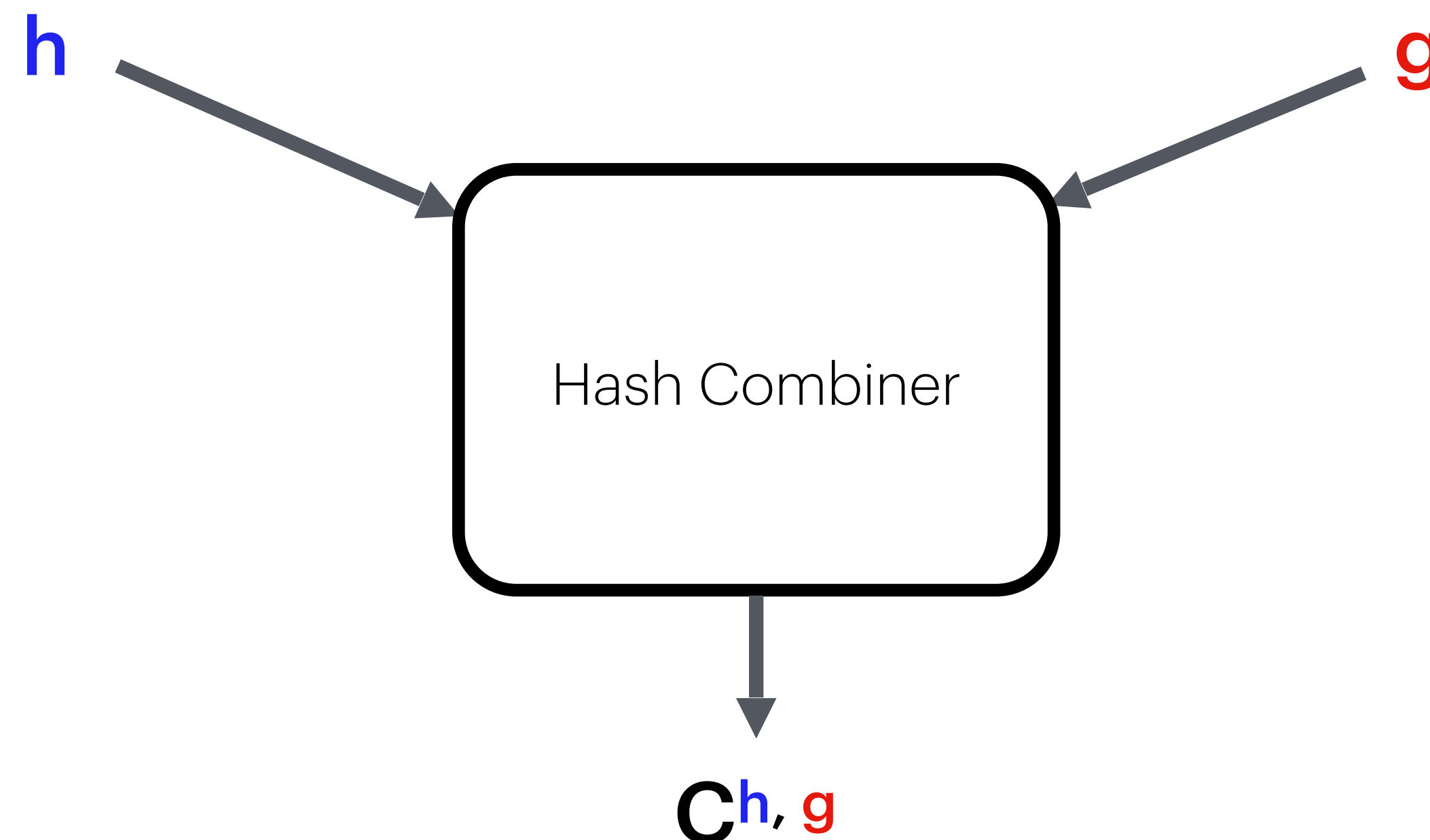
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- [Pietrzak07, 08]: Length doubling is *optimal* for collision-resistance
- [Mittelbach 13]: Cryptophia's short combiners, way around this assuming random oracles

# Random Oracle Combiners

[*DFGHP*, CRYPTO23]

- Introduced notion of Random Oracle (RO) Combiners
- *Idea*: If one of the hashes is RO, then the combiner  $C$  is *essentially* BIG-RO



# Indifferentiability Framework

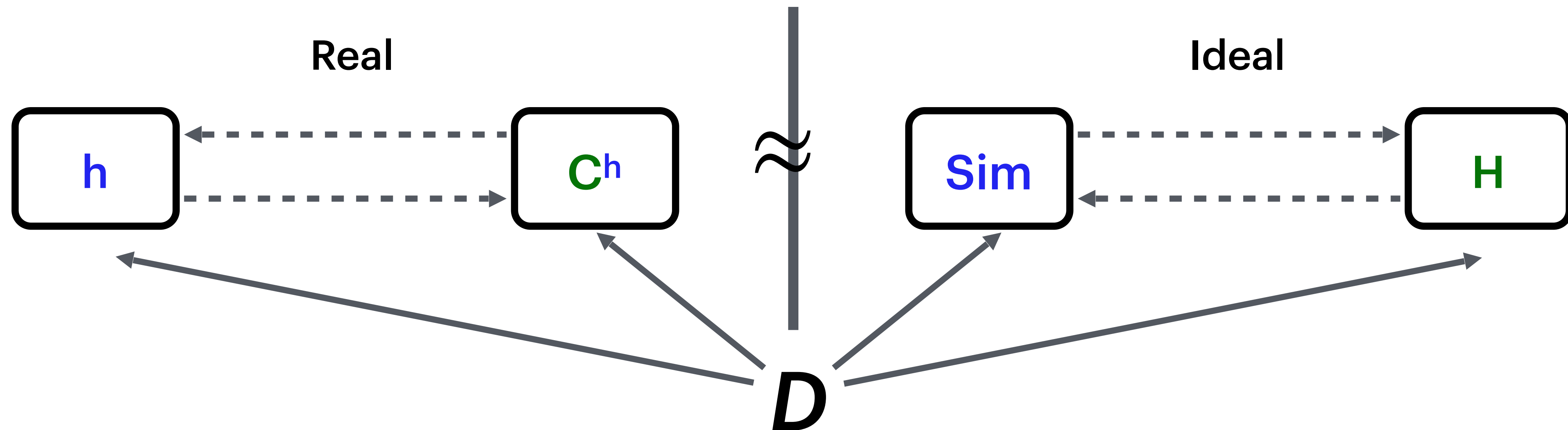
[MRH04, CDMP05]

- Real-ideal world framework, where we want indistinguishability between
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  - Interacting with a simulator  $Sim$  and RO  $H$

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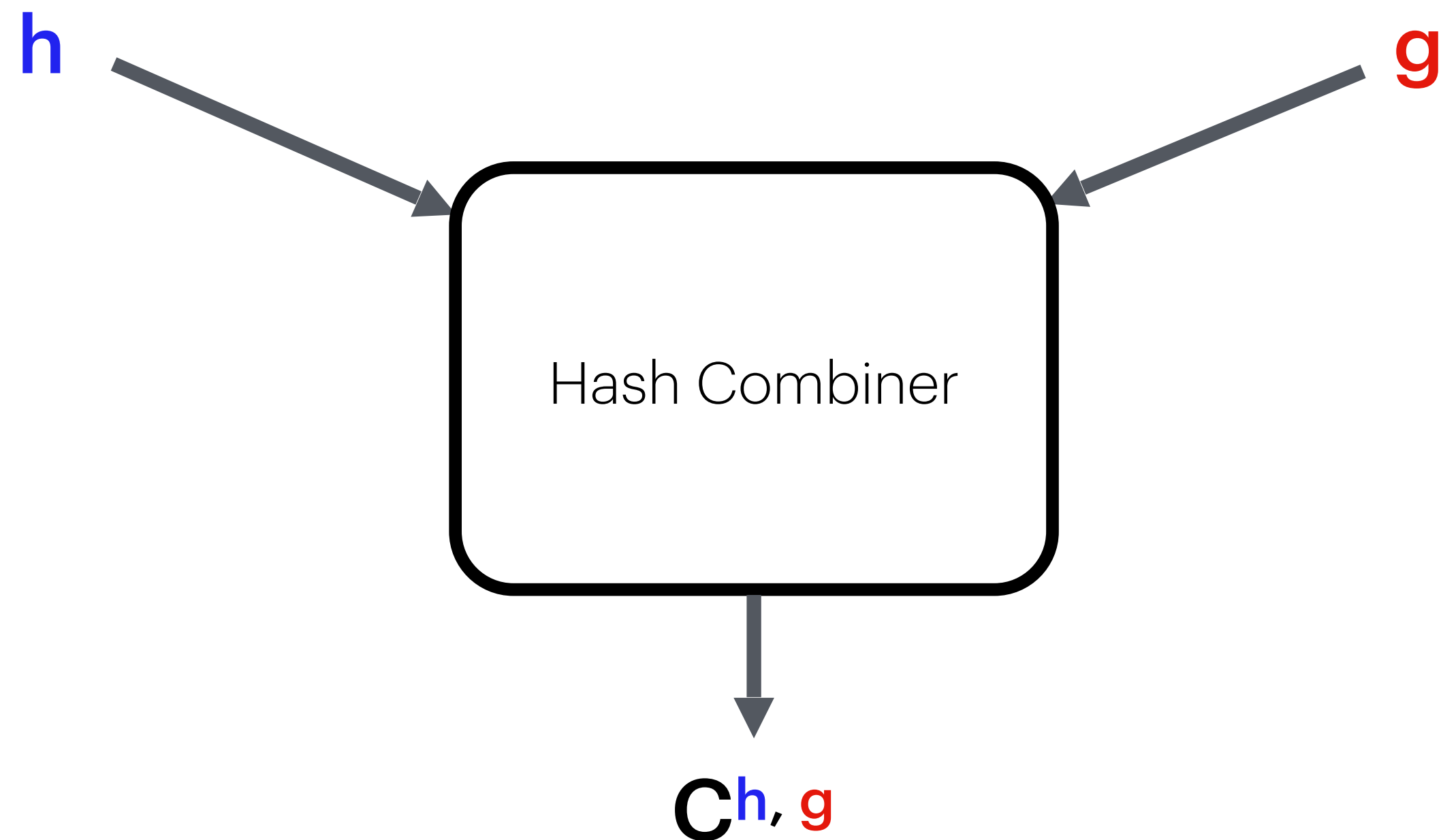




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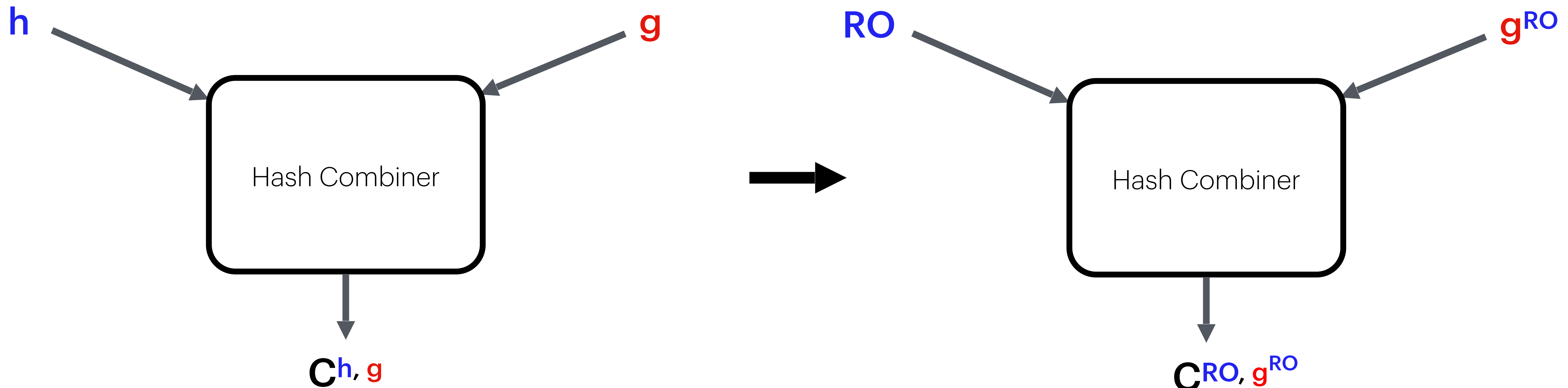
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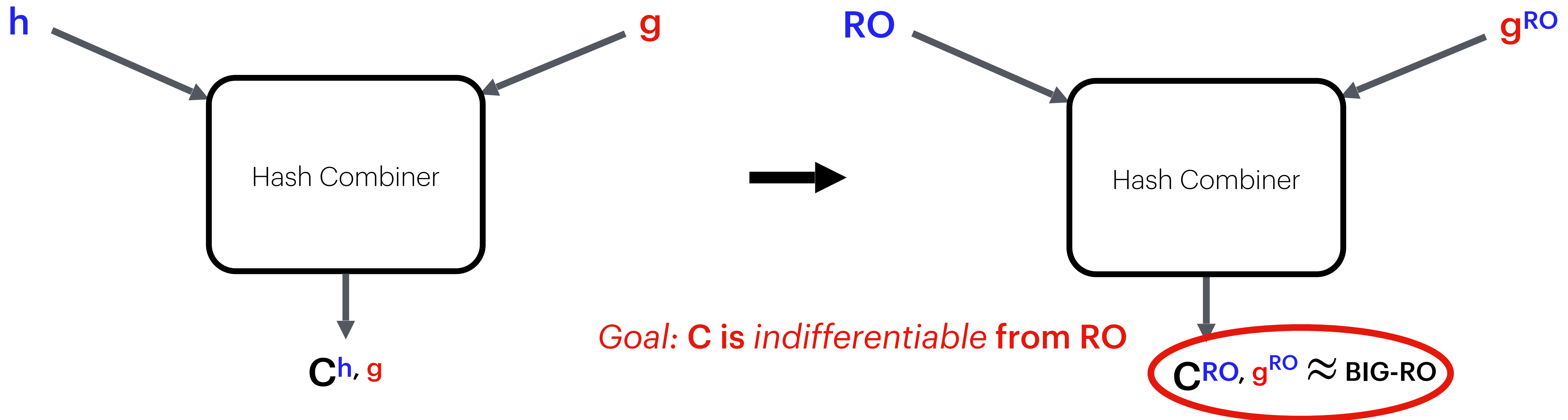
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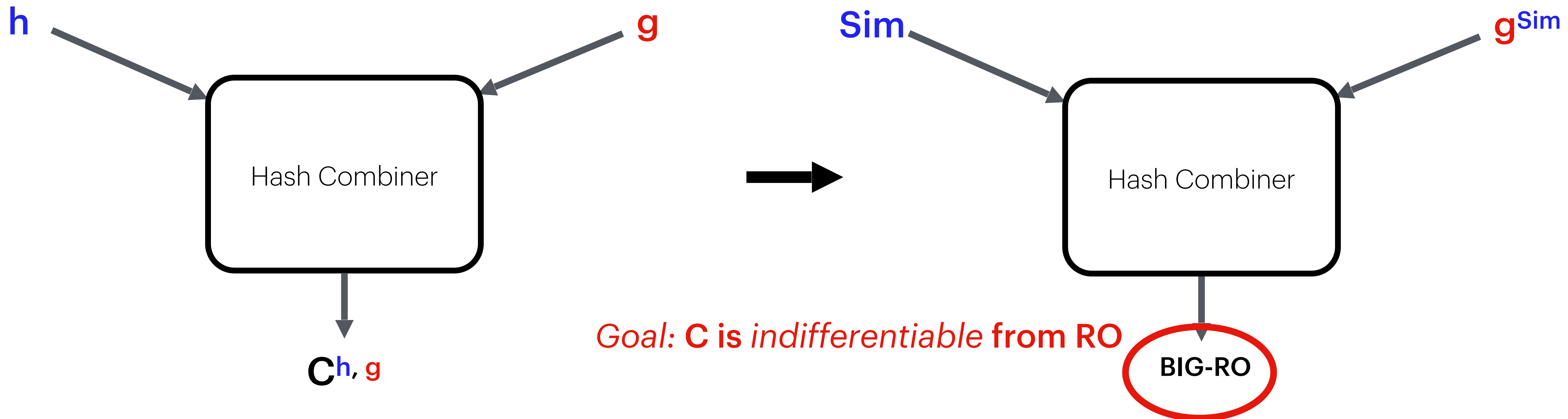
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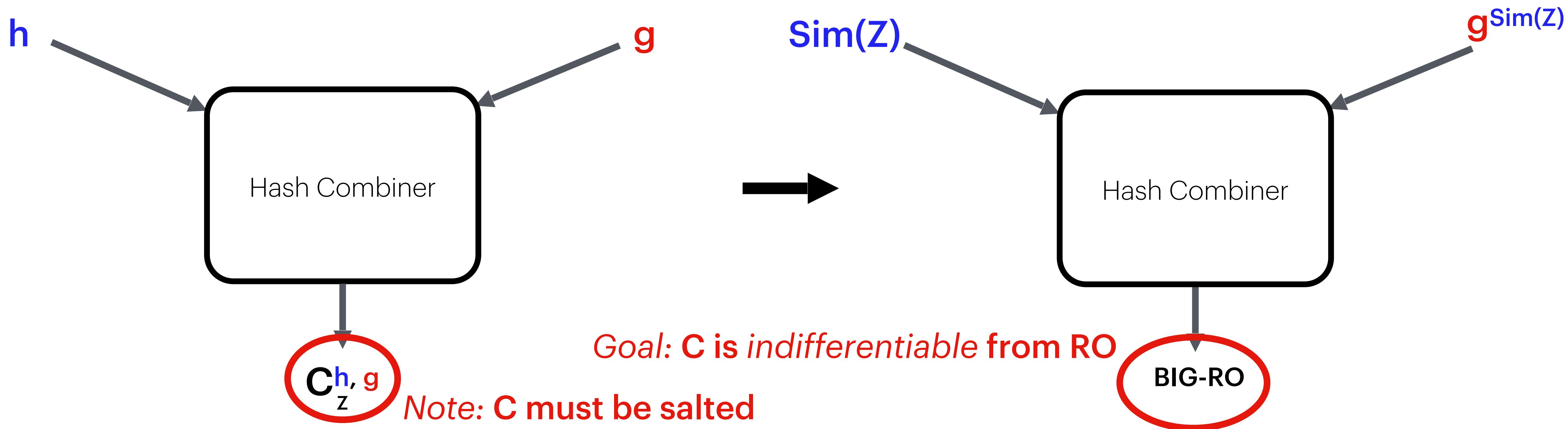
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# Hash Combiners on Long Inputs

[DFG+23] gave basically optimal construction for hashes with good compression

$$C_{Z_1, Z_2}^{h, g}(m) = h(m, Z_1) \oplus g(m, Z_2)$$

**Downside:** for applications, does not give evidence when  $h$  (and  $g$ ) Merkle-Damgård

→ *despite* the fact that the above is **indif. from RO** and **M-D** is **indif. from RO**

→ composition of the two only works for single-stage games, which **RO Comb.** is not

→ in particular,  $g$  can depend arbitrarily on underlying compression of  $h$

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Desired goals for Merkle-Damgård combiner:

1. Assumes only that one of  $h$  or  $g$  is a RO of mild compression
2. Only calls M-D transformation of  $h^*$ ,  $g^*$  on inputs
3. Supports arbitrarily long input messages, but message output is still less than 2 times output of  $h=h^*$ .

# Our Results

# Main Result

We construct the following RO Combiner:

$$C_{Z_1, Z_2}^{h, g}(m) = h^*(m, Z_1) \oplus g^*(m, Z_2)$$

## Parameters (previous):

- Compression of individual hash:  $n + \delta$
- Compression of *overall* hashes  $h, g$ :  $n + \delta$
- Length of salts  $Z_1, Z_2$ :  $|M| + \lambda$

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# What Fails with Merkle-Damgård?

- In monolithic, proof critically used the fact that  $Z$  is longer than messages to show that  $g(m', Z_2)$  cannot compute  $h(m, Z_1)$  for any messages  $m$ .
  - This way, Sim can answer  $h(m', Z_1) = H(m') \oplus g(m', Z_2)$  without any “recursion”
- Unfortunately for M-D, it is easy to construct  $(m, m')$  and define  $g^h$  such that  $(g^h)^*(m', Z_2)$  can compute completions of  $h^*(m, Z_1)$ .

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- Consider  $h : \{0,1\}^{2n} \rightarrow \{0,1\}^n$ . Then, for a message  $m$ , consider:

$$m' := \left( h^* \left( m, Z_1^{(1)}, Z_1^{(2)}, \dots, Z_1^{(k-1)} \right), Z_1^{(k)} \right)$$

Where  $k$  is the last block of  $Z_1$  and  $Z_1^{(i)}$  is the  $i$ -th block of  $Z_1$ .

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$$(g^h)^*(m', Z_2) = h(m') = h\left(\left(h^*\left(m, Z_1^{(1)}, Z_1^{(2)}, \dots, Z_1^{(k-1)}\right), Z_1^{(k)}\right)\right) = h^*(m, Z_1)$$



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**Lesson: intuitions for monolithic may not carry over to Merkle-Damgård case**

Proof

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Need to build Sim such that  $\text{Sim}^*(m, Z_1) \oplus (g^{\text{Sim}})^*(m, Z_2) = H(m)$

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- Sim must answer all queries  $(a, b) \rightarrow c$  at random while also satisfying above “global” question
- Challenges:
  1. Need to spot all such constraints
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**In essence the trickiest, will imply 1+2 if done right...**

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Got a message of length  $l+k...$   
have to be consistent here

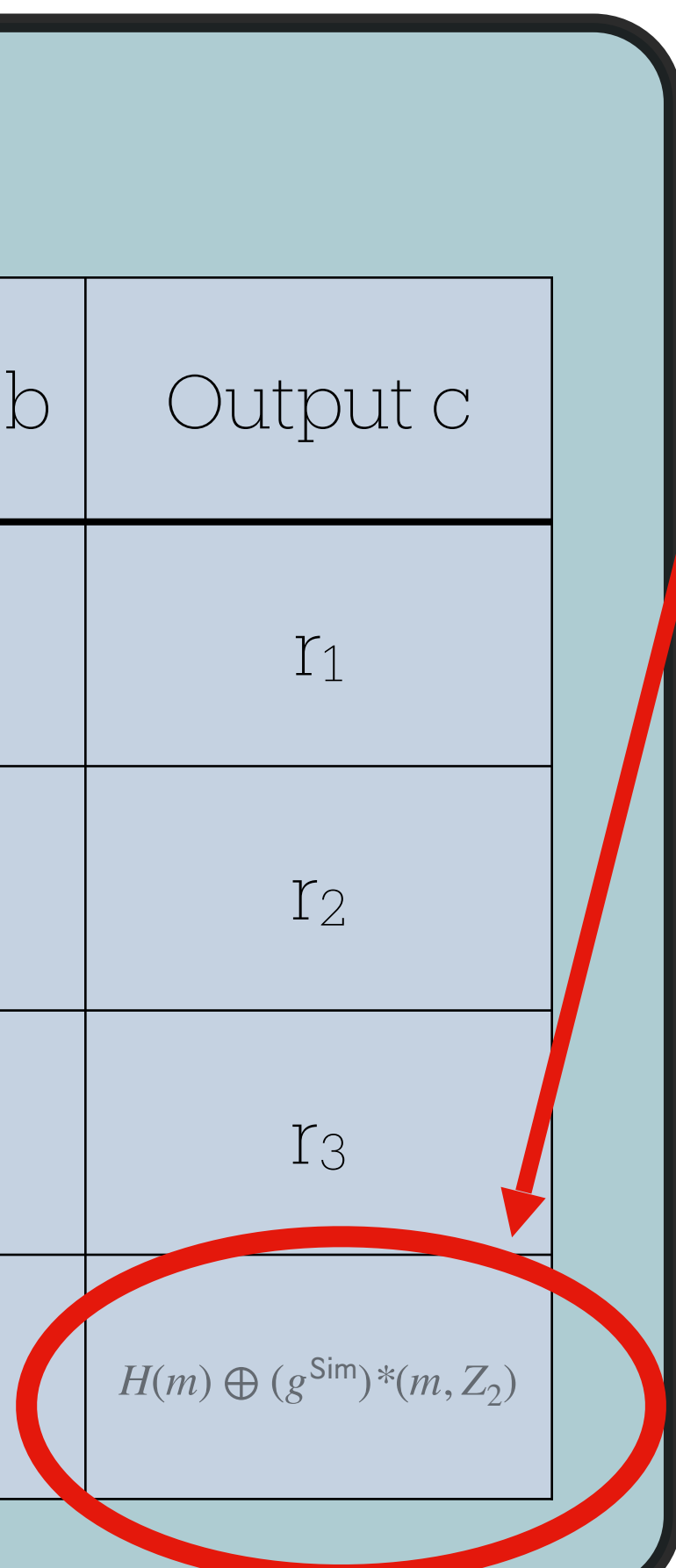
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Okay, what did  $g^*$  call here?  
I need to fill in those as well



# The worry

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**Sim:**

**Need to argue no infinite (or just overwhelming) recursion!**

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**Claim:**

**For  $D$  to find a message  $m$  such that  $(g^h)^*(m, Z_2)$  queries some  $h^*(m', Z_1)$ ,  
 $D$  must have queried  $h^*(m')$  itself.**

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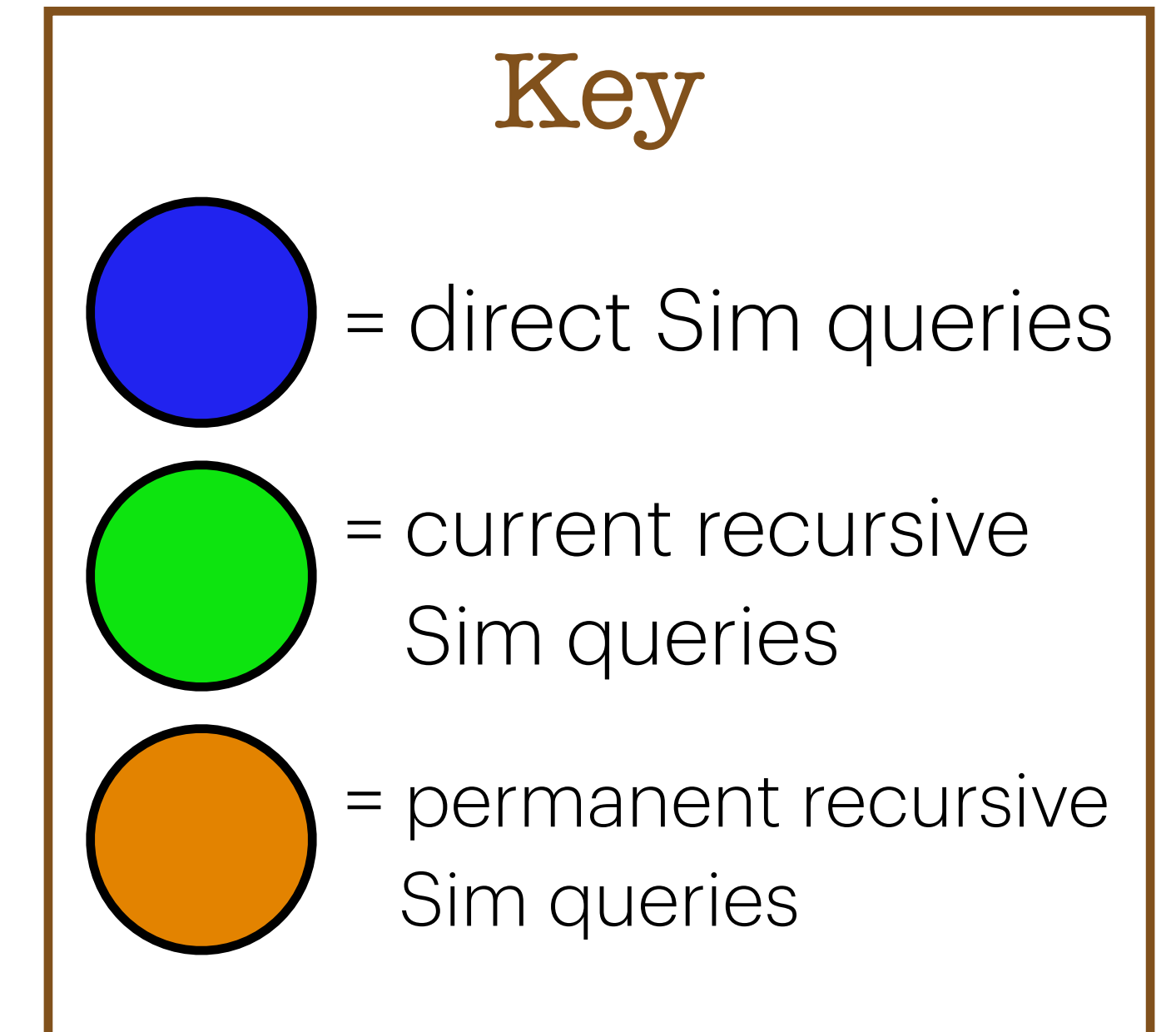
**Claim:**

**For you to find a message  $m$  such that  $(g^h)^*(m, Z_2)$  queries some  $h^*(m', Z_1)$ , then you must have queried  $h^*(m')$  yourself.**

- If this is true, then  $\#(\text{queries made by recursive } g \text{ calls}) \leq \#(\text{queries } D \text{ made})$  — bounded!

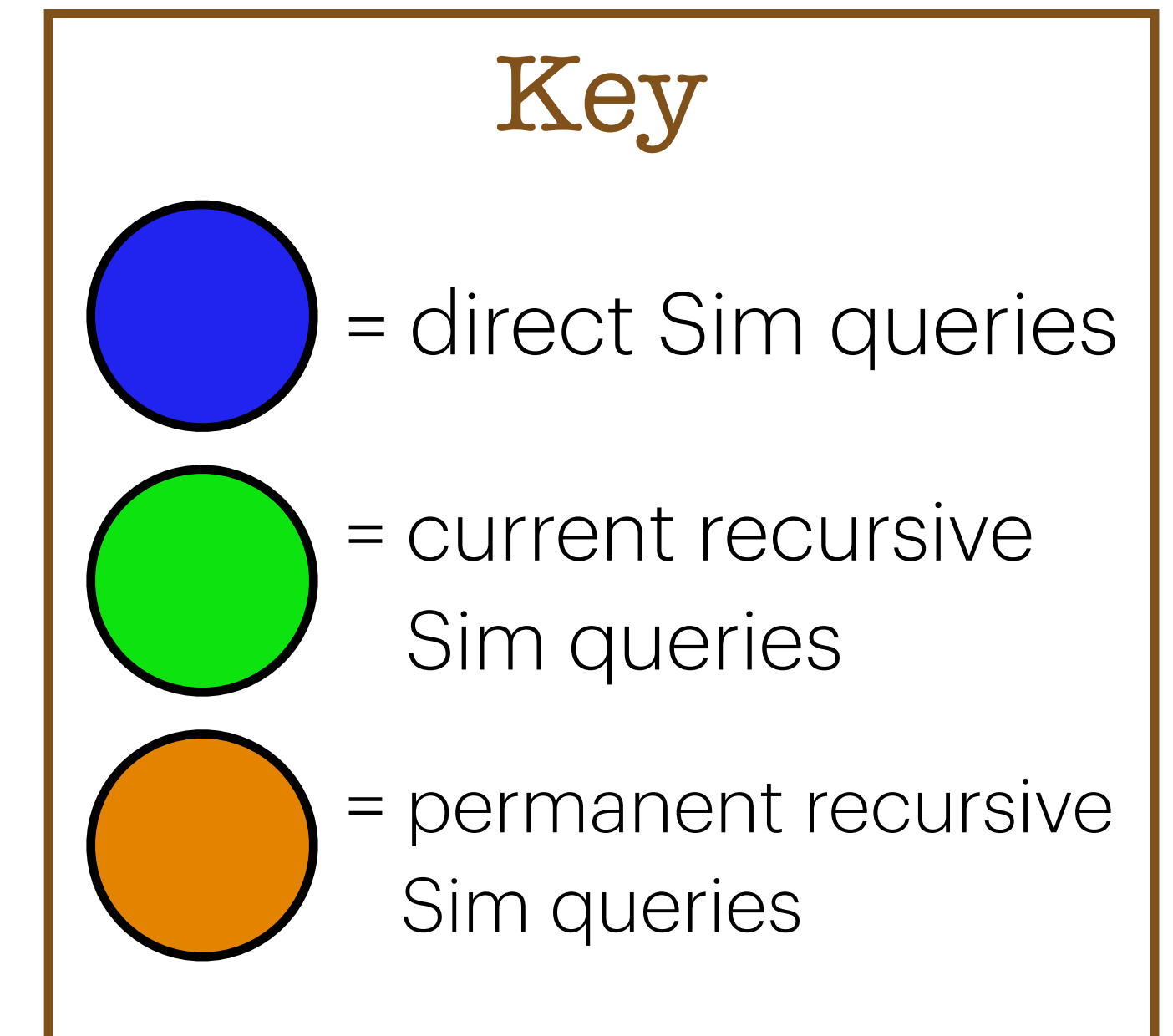


# Termination



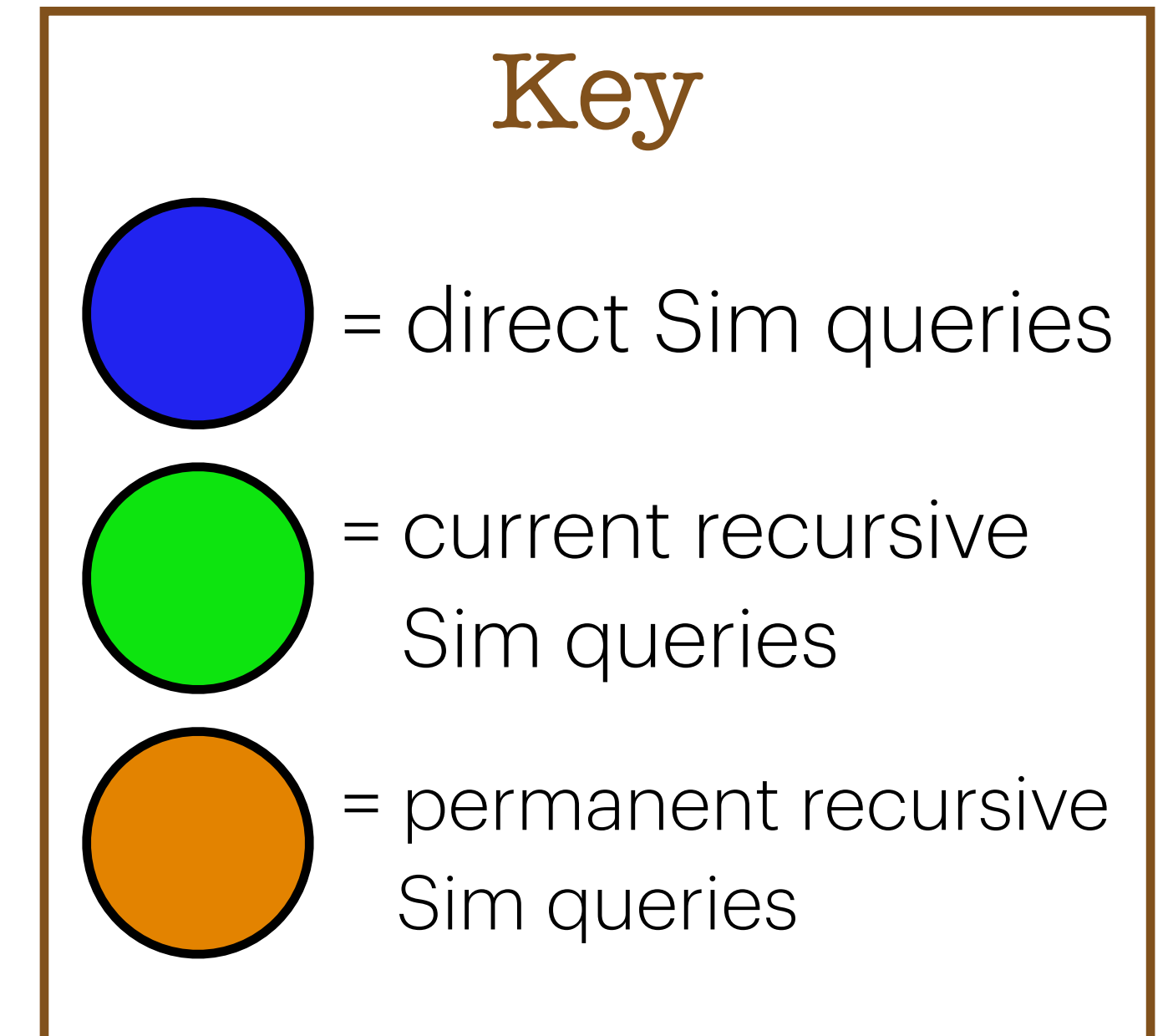
# Termination

Note: only queries of length up to  $l$  followed by some  $Z_1$  blocks "matter"



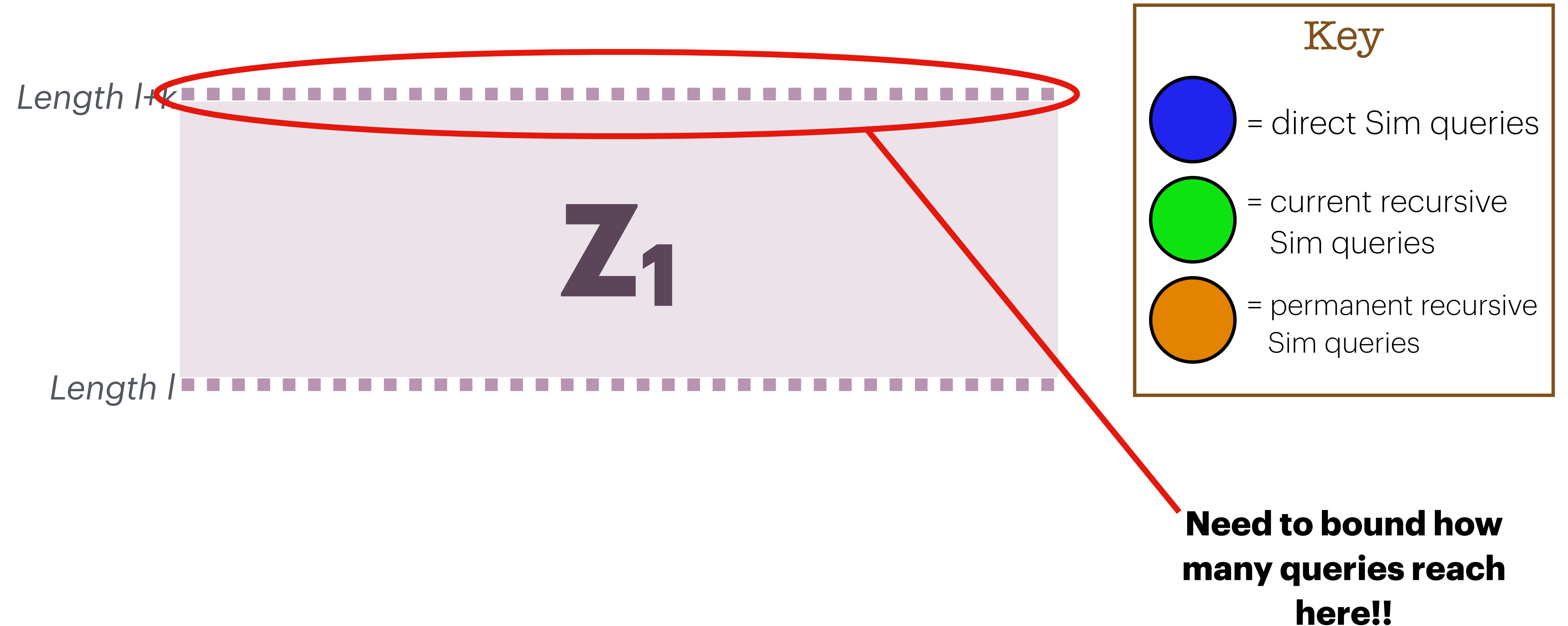
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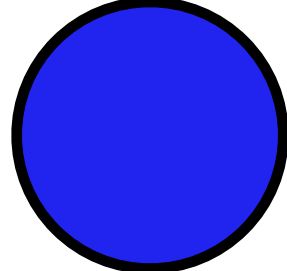
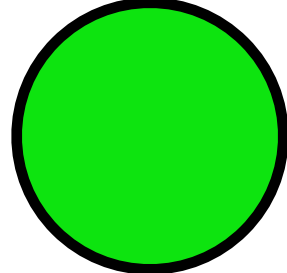
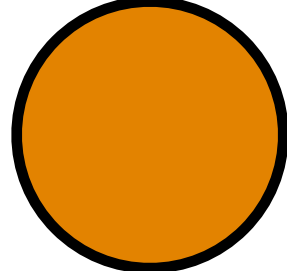


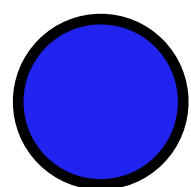
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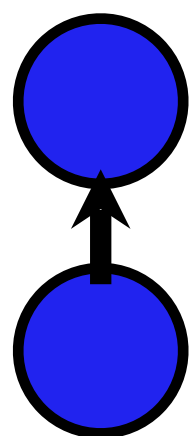
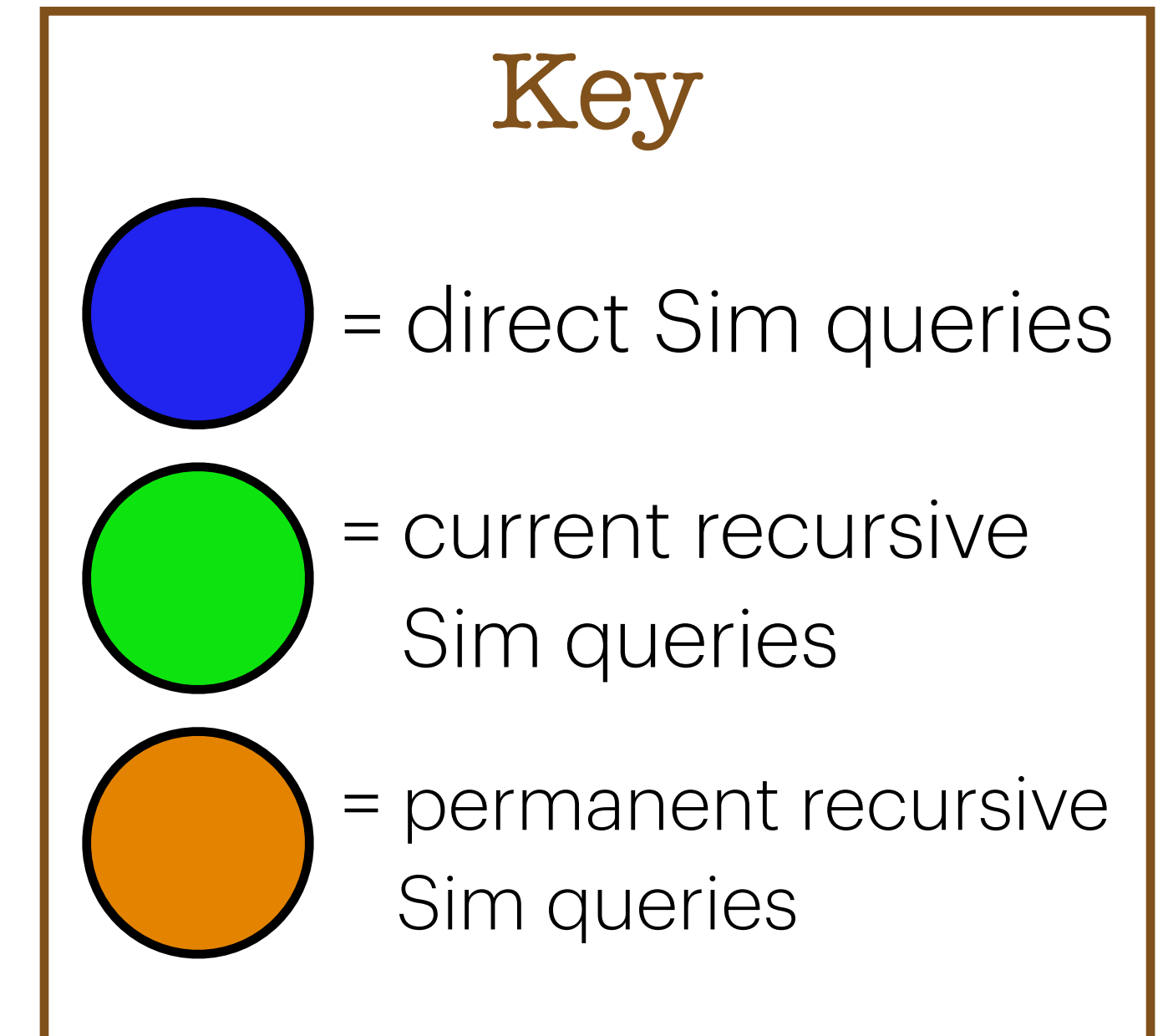
**Key**

-  = direct Sim queries
-  = current recursive Sim queries
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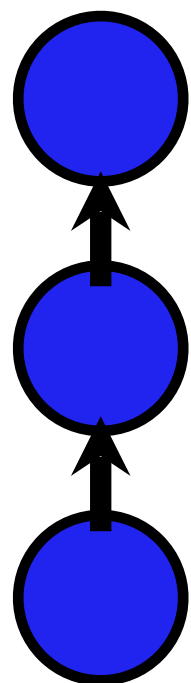
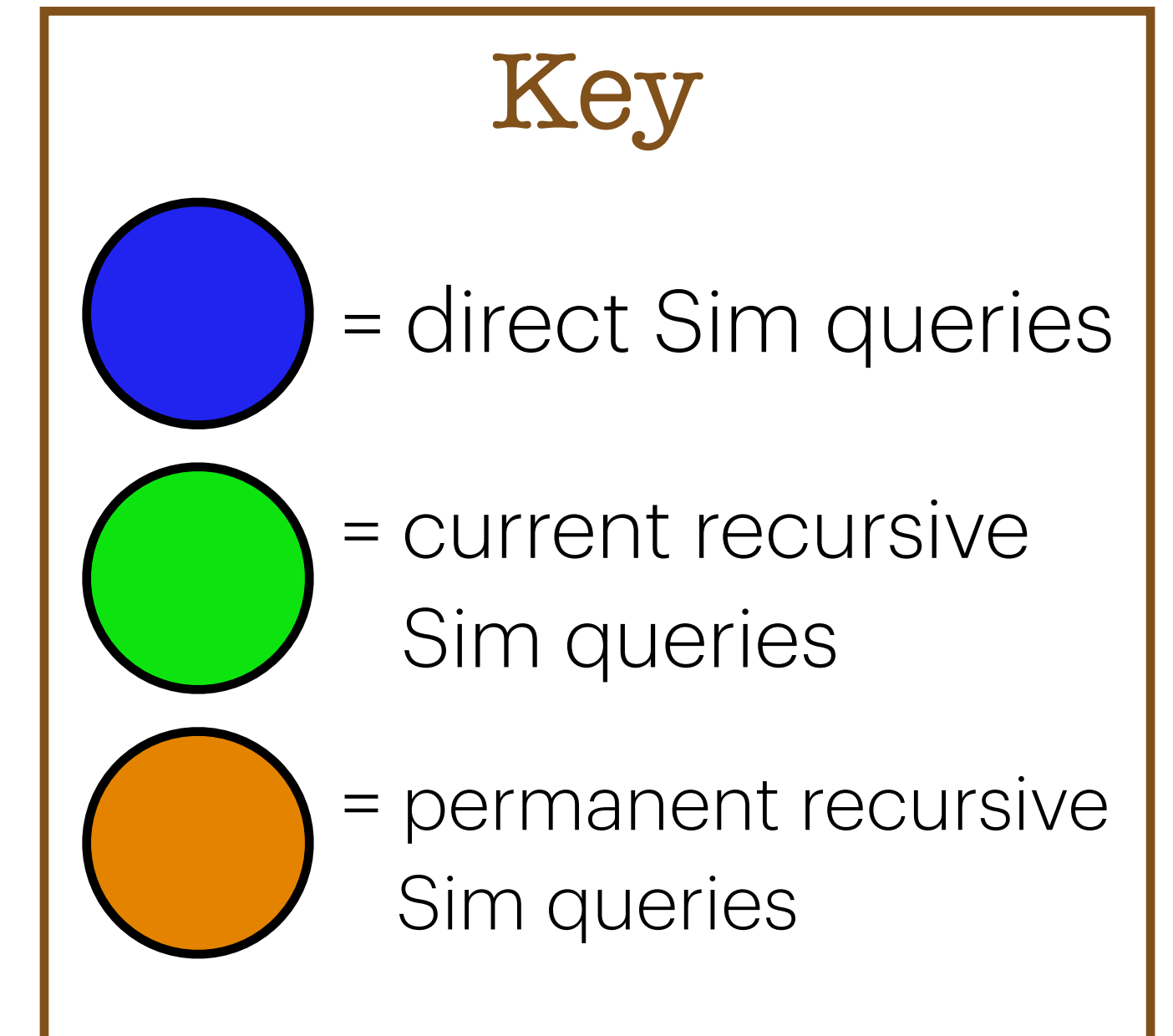
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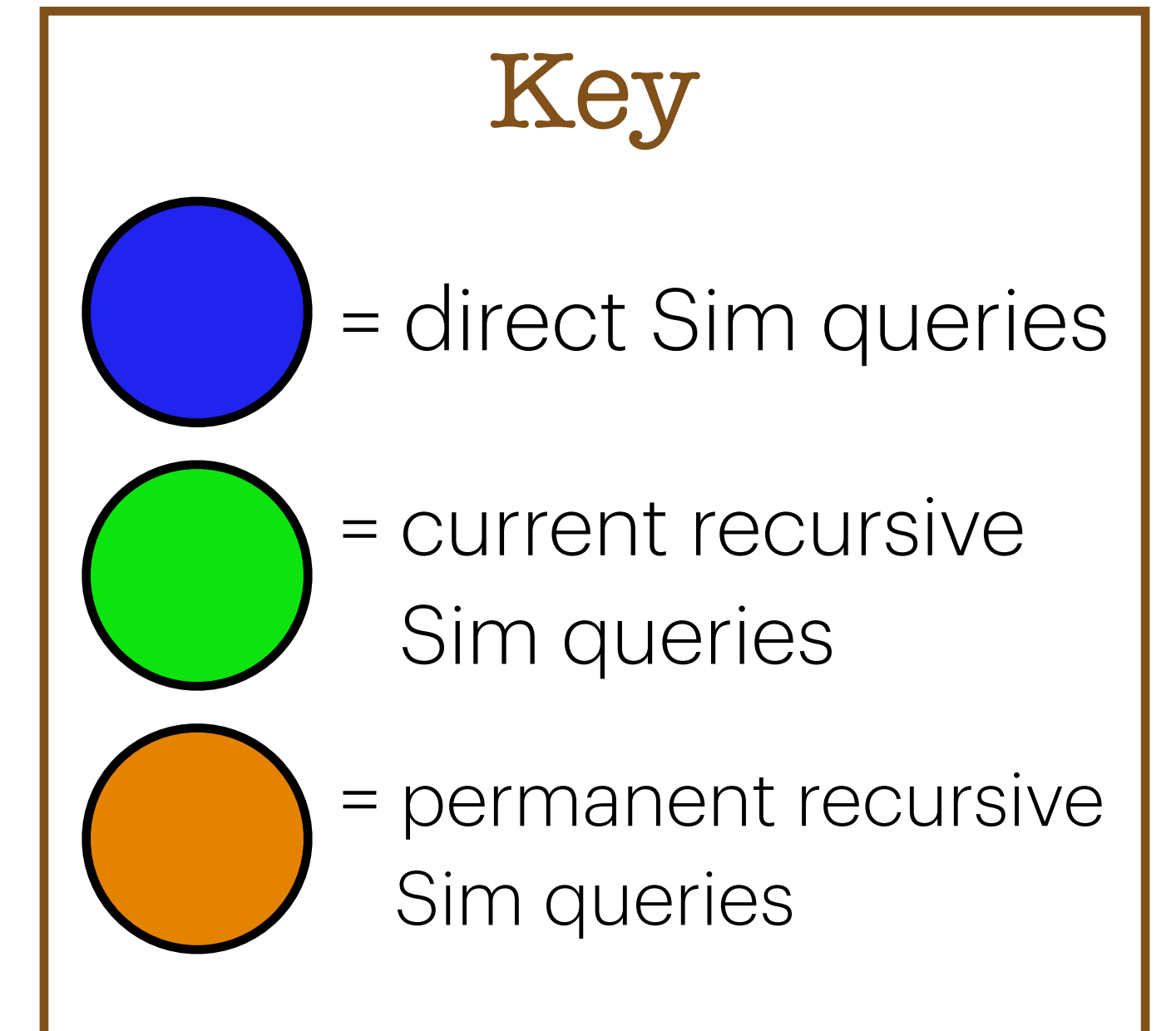
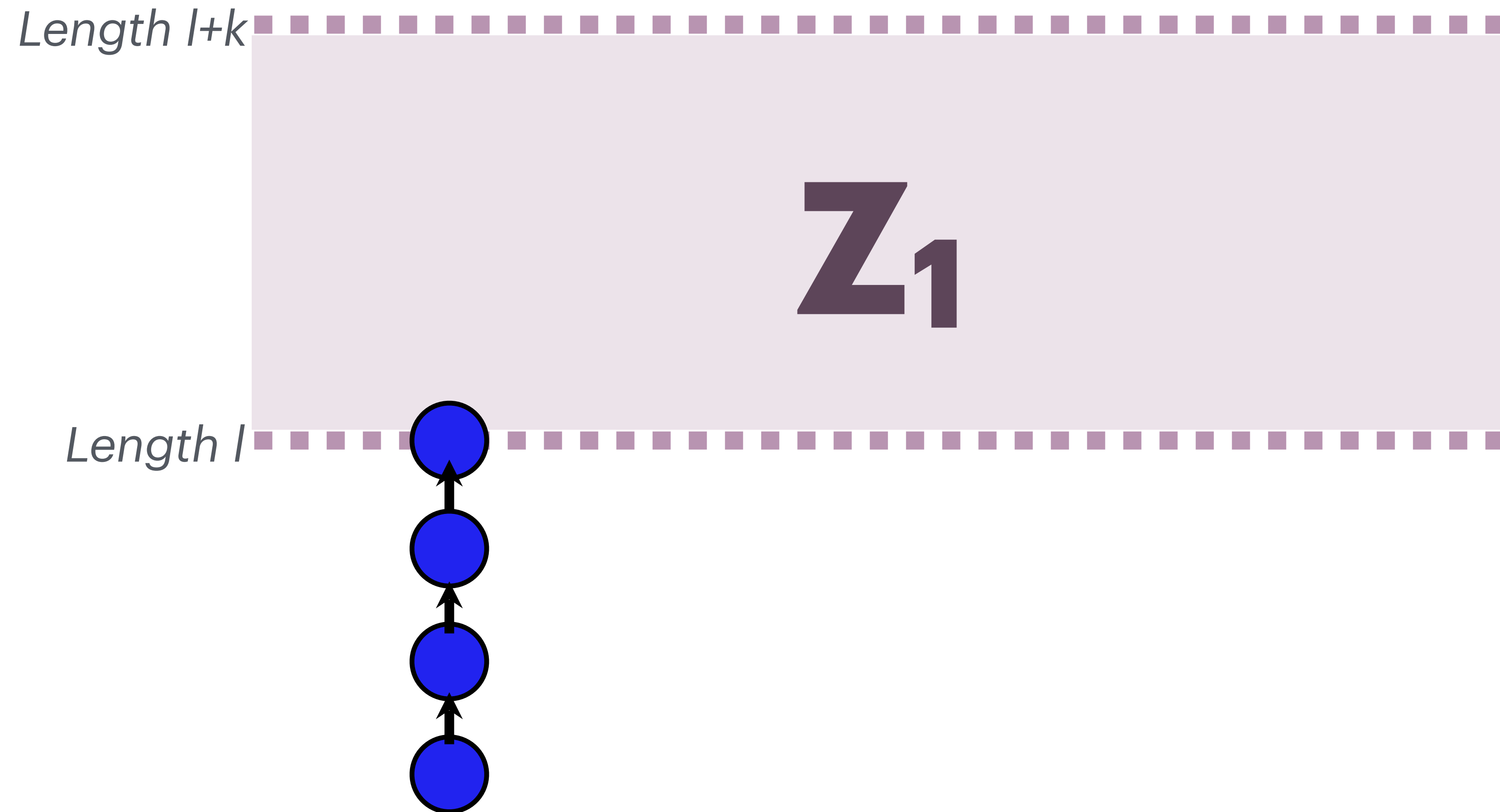
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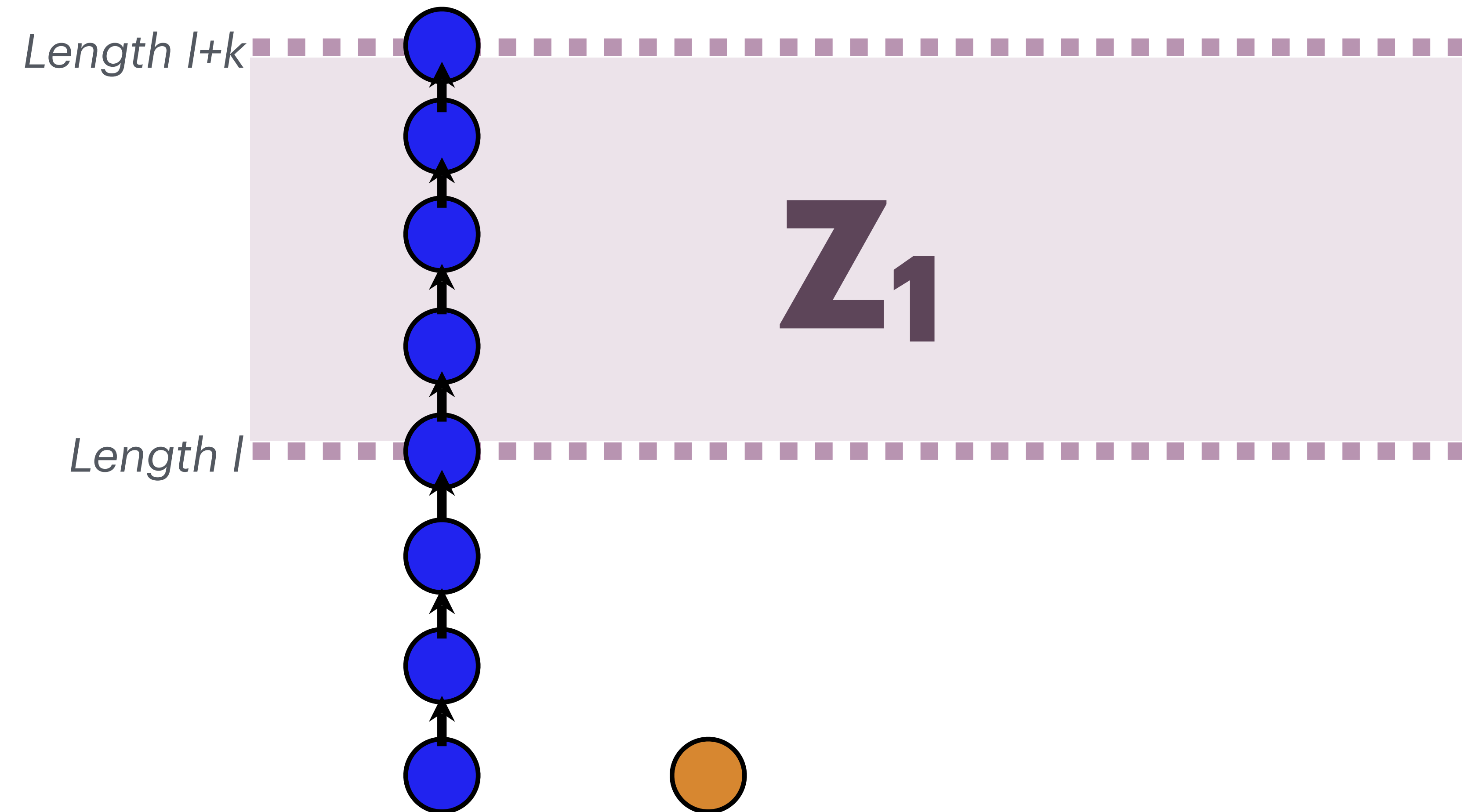
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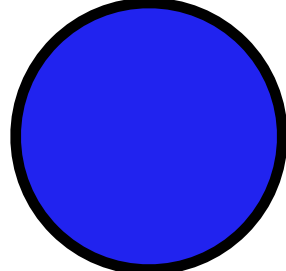
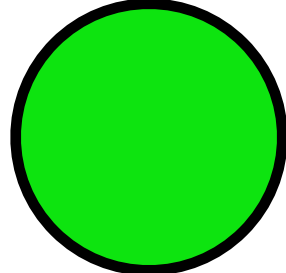
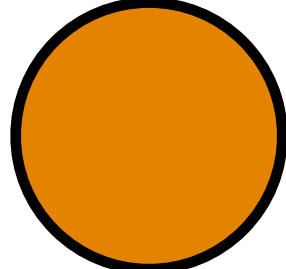


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Note: only queries of length up to  $l$  followed by some  $Z_1$  blocks "matter"

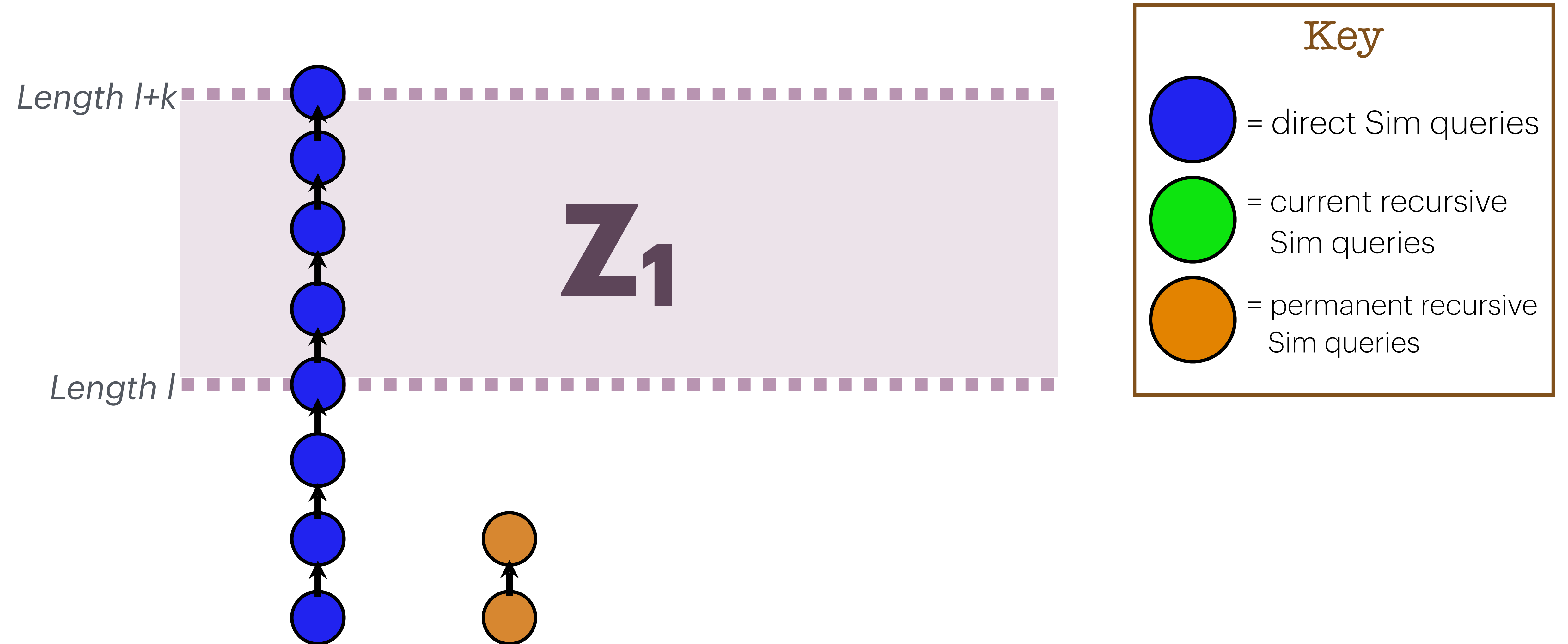


## Key

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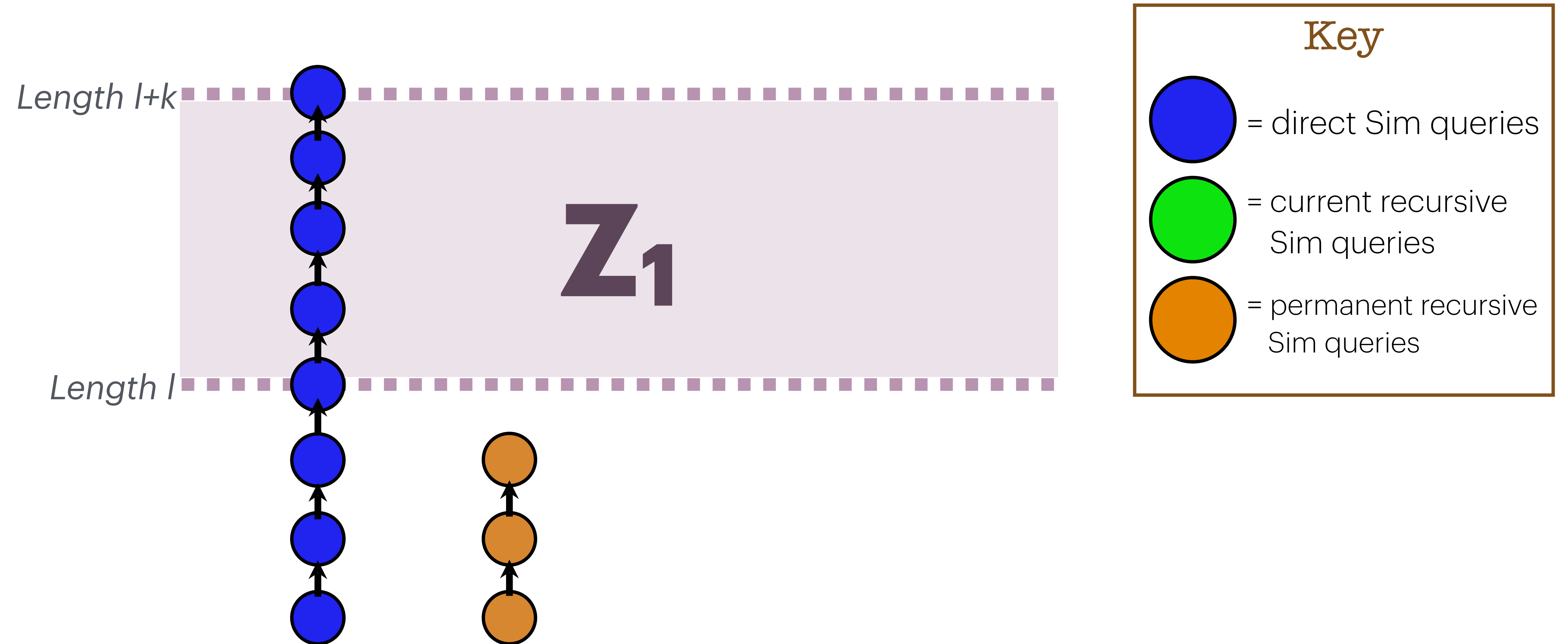
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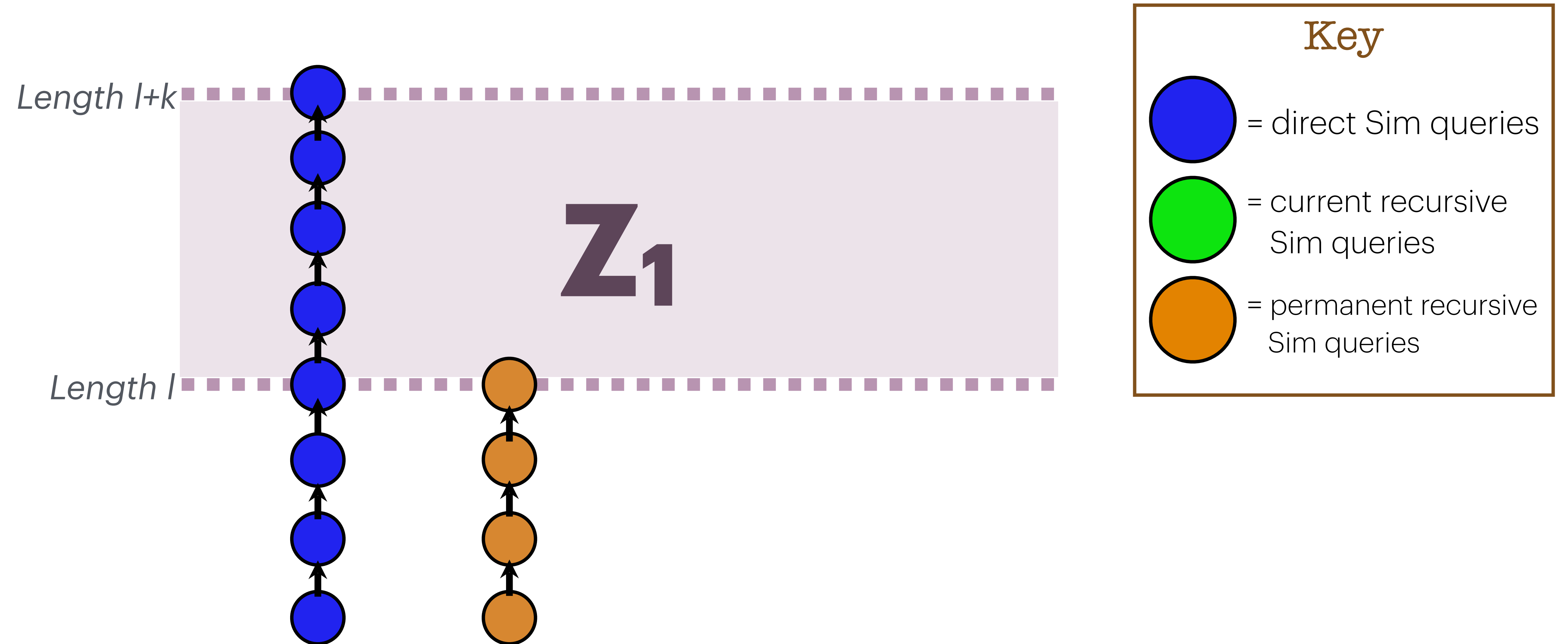
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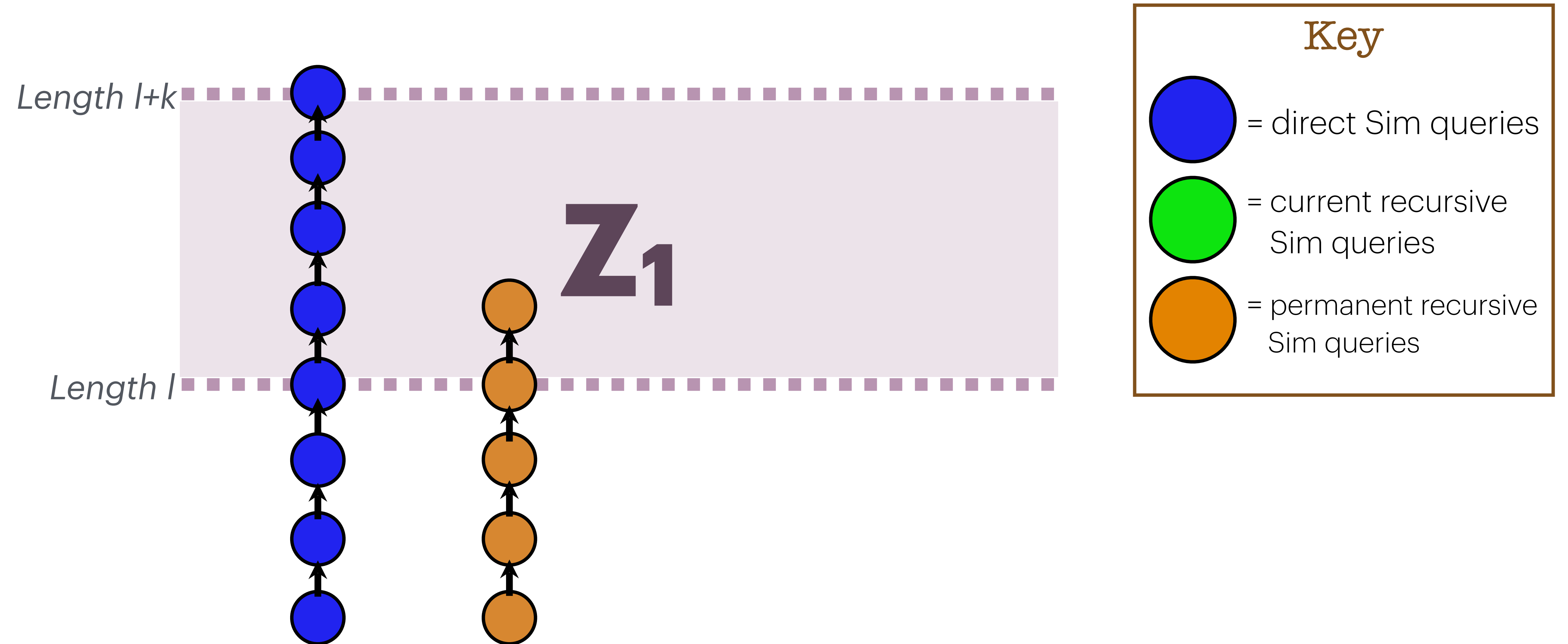
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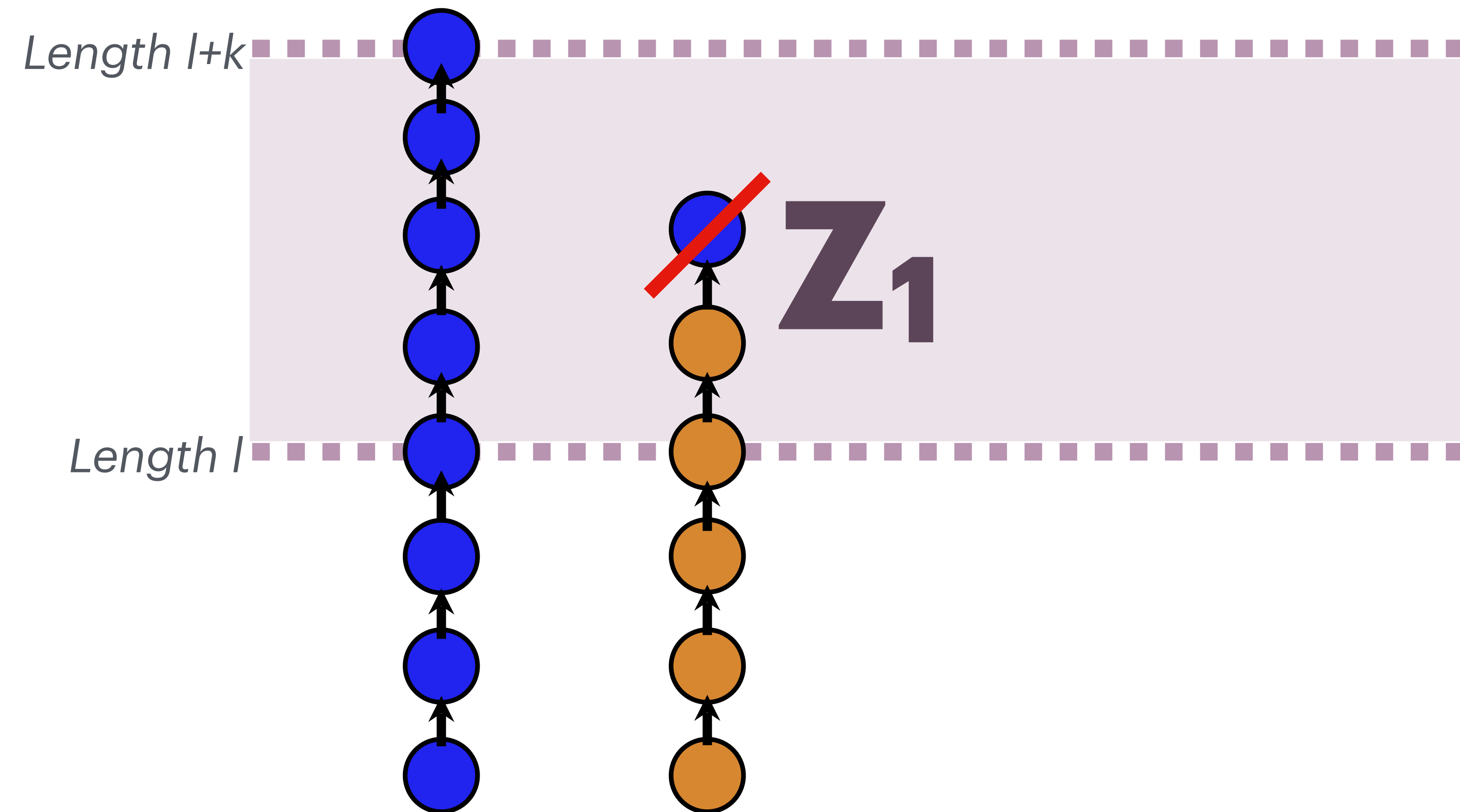
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Note: only queries of length up to  $l$  followed by some  $Z_1$  blocks "matter"

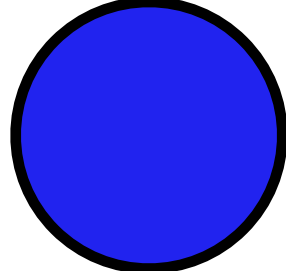
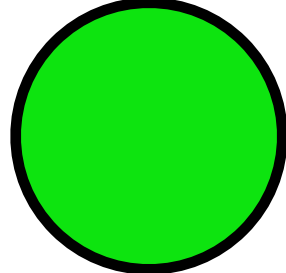
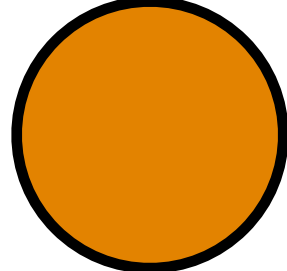


# Termination

Property: blue query cannot follow orange query

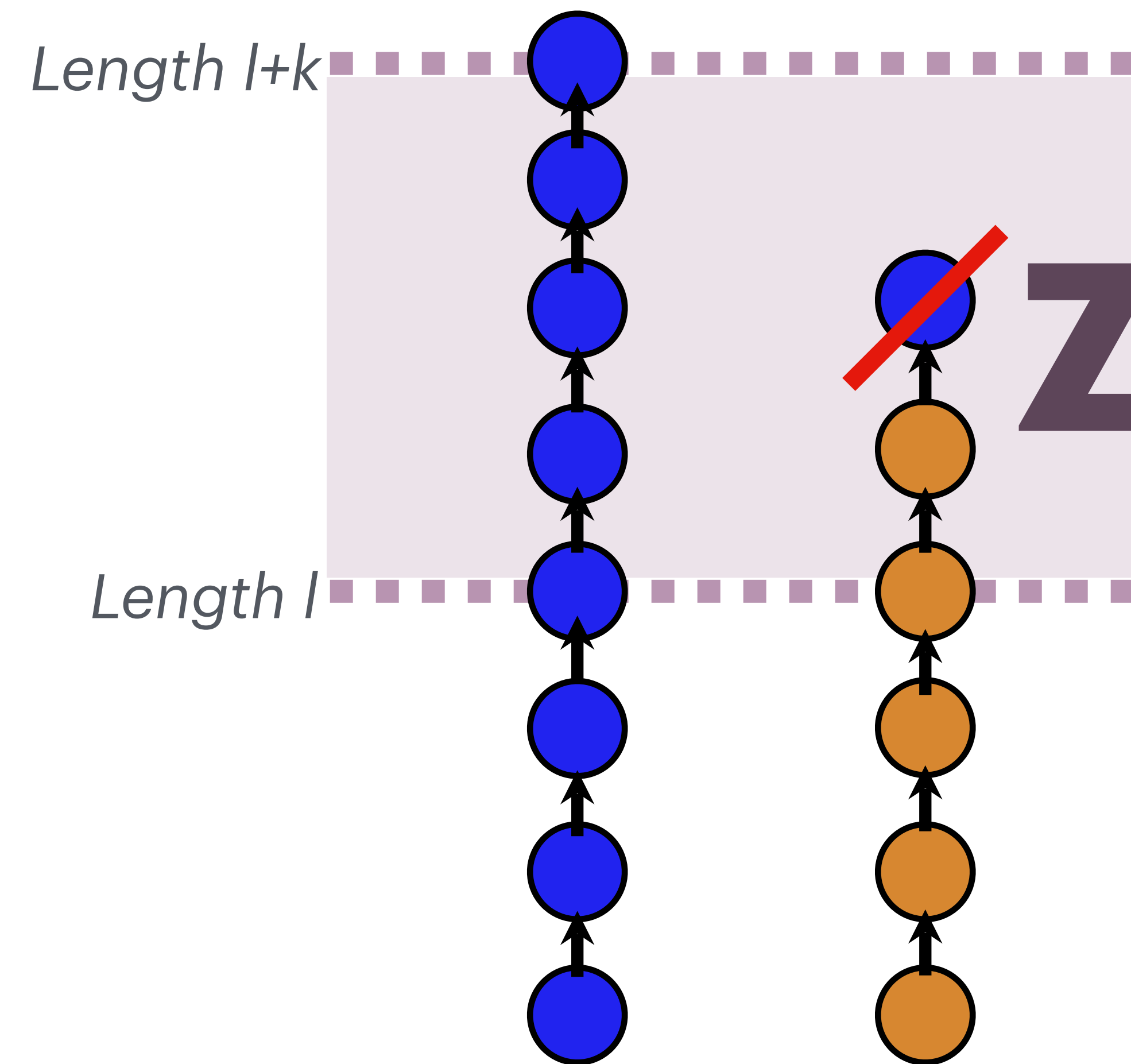


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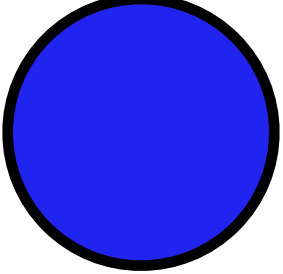
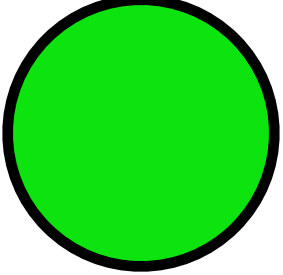
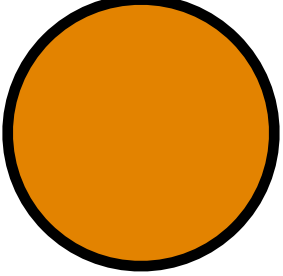
# Termination

Property: blue query cannot follow orange query



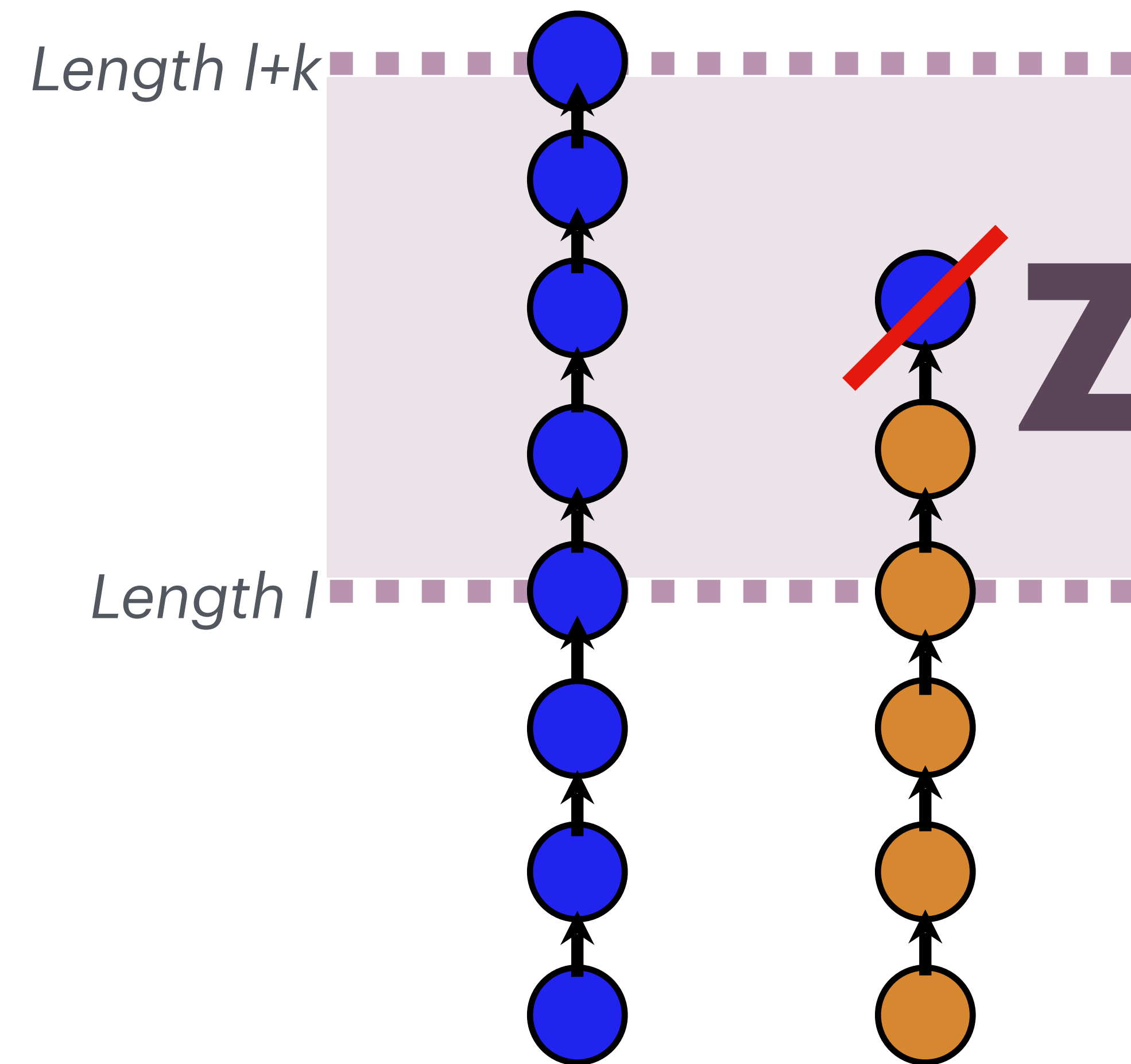
Why?  
If it could, we could generate all blue queries without orange queries

**Key**

-  = direct Sim queries
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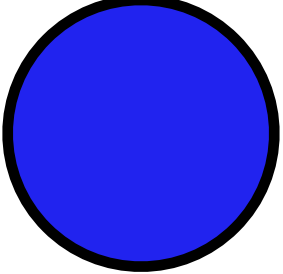
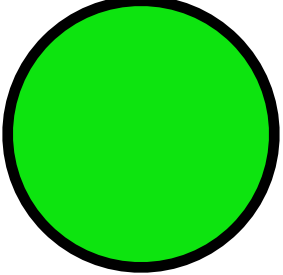
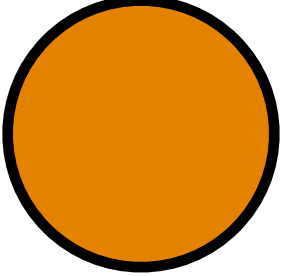
# Termination

Property: blue query cannot follow orange query



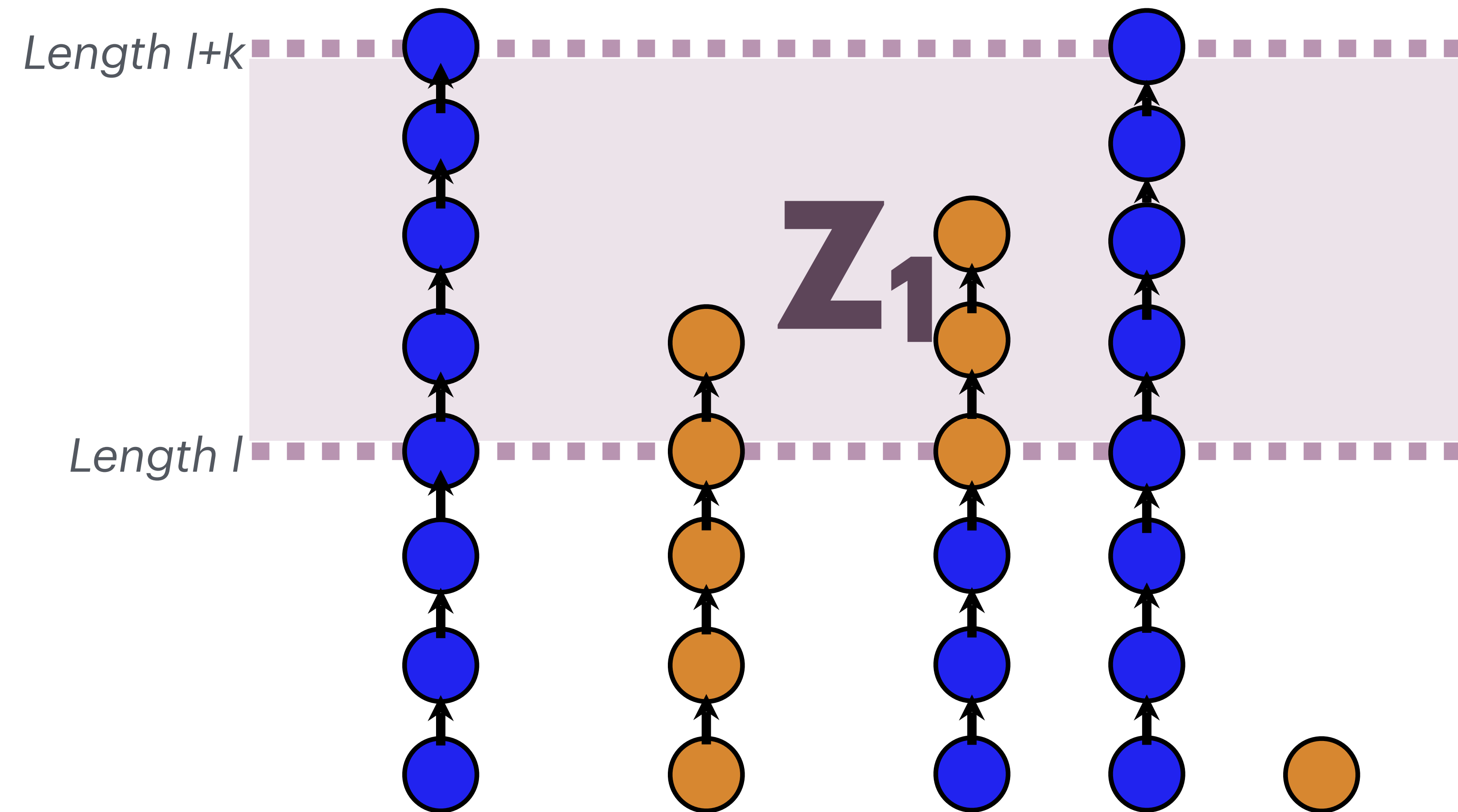
Why?  
If it could, we could generate all blue queries without orange queries  
Then, we could predict random oracle outputs

**Key**

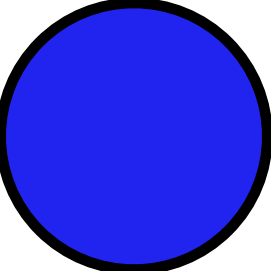
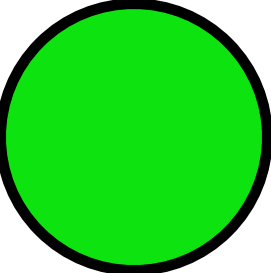
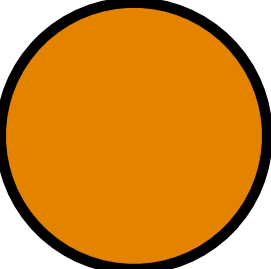
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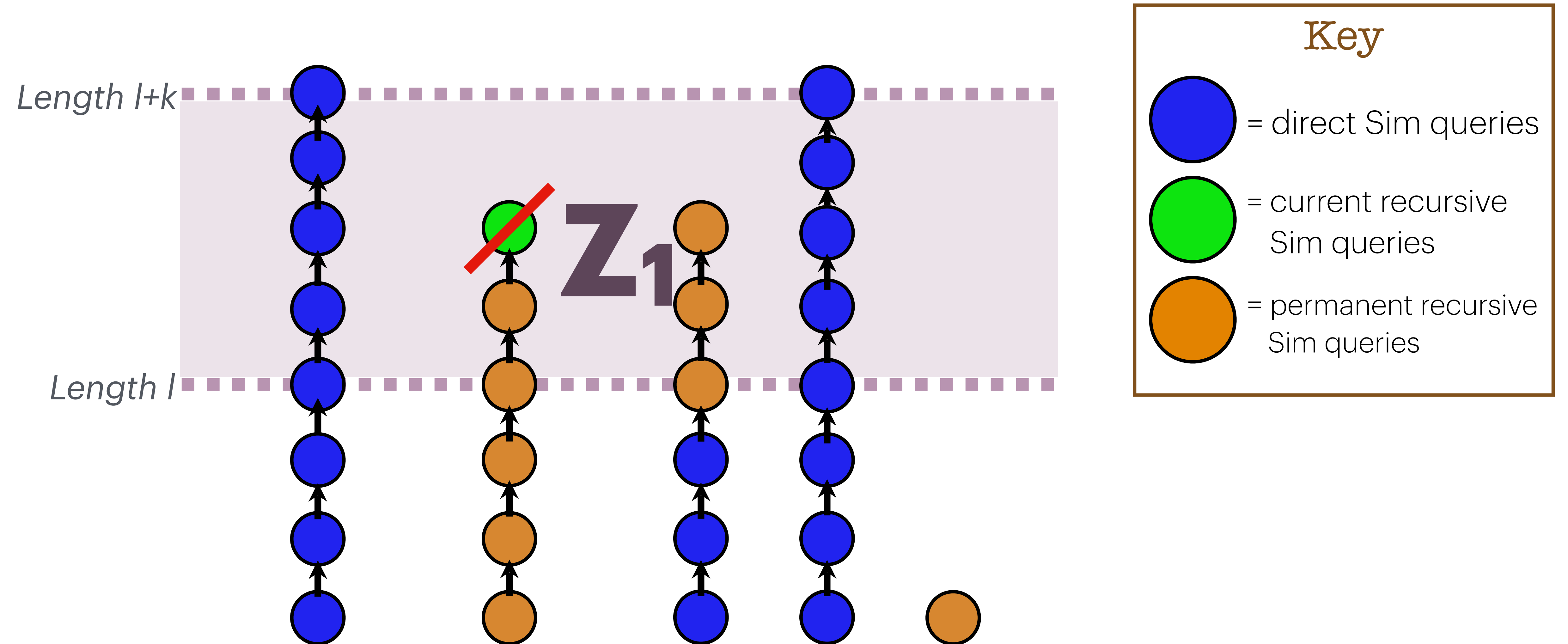


## Key

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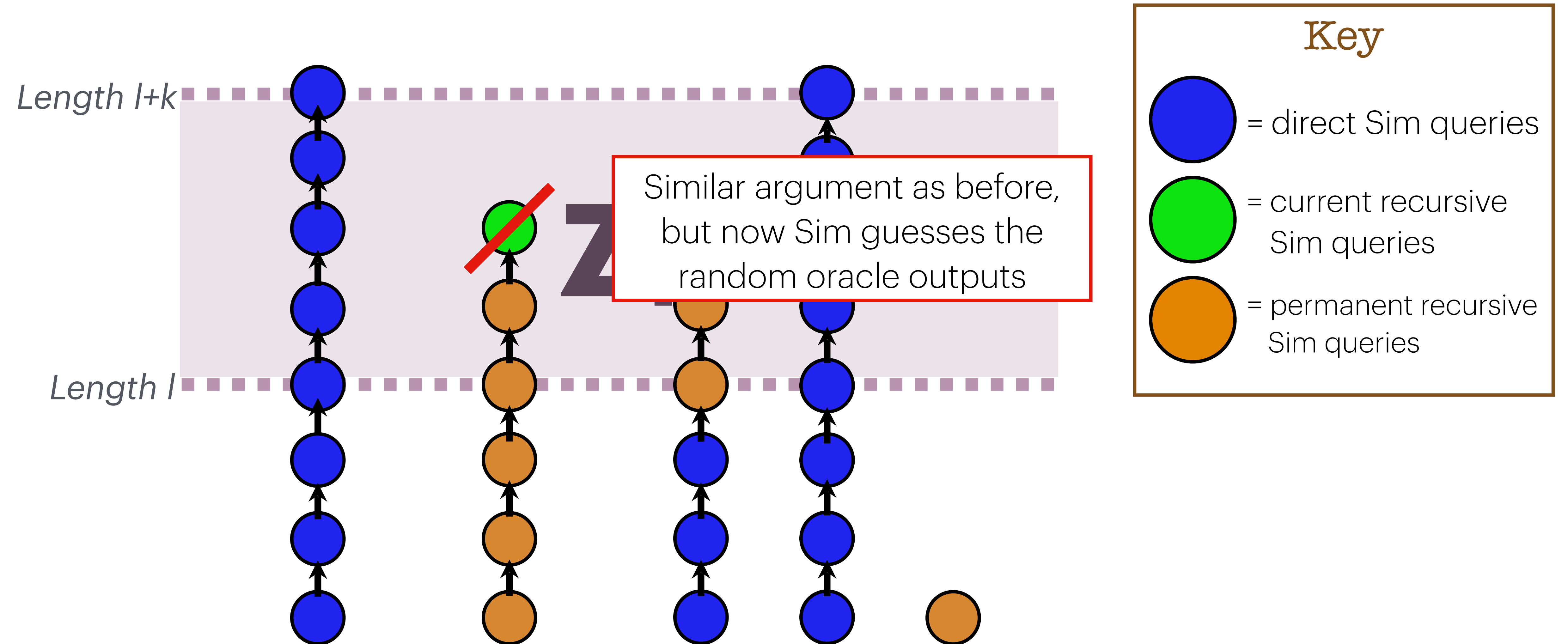
# Termination

Property: green query cannot follow orange query

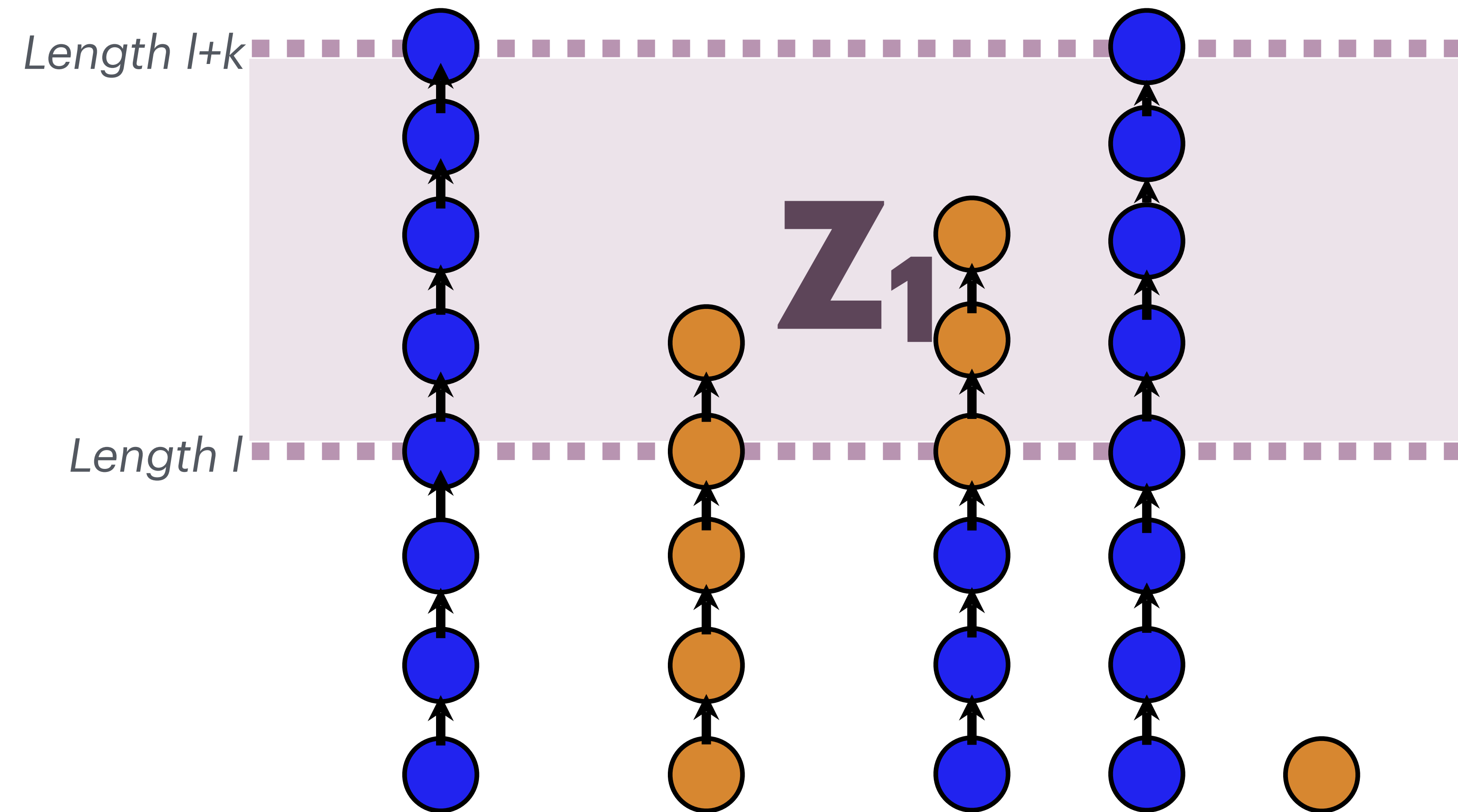


# Termination

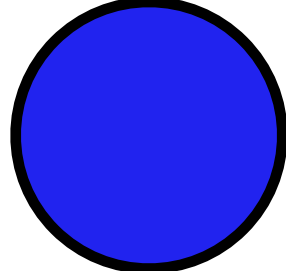
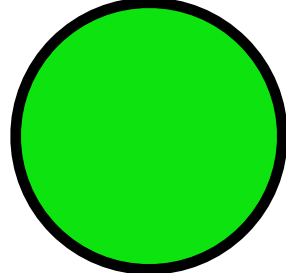
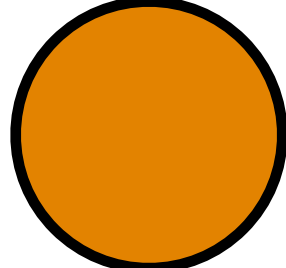
Property: green query cannot follow orange query



# Termination

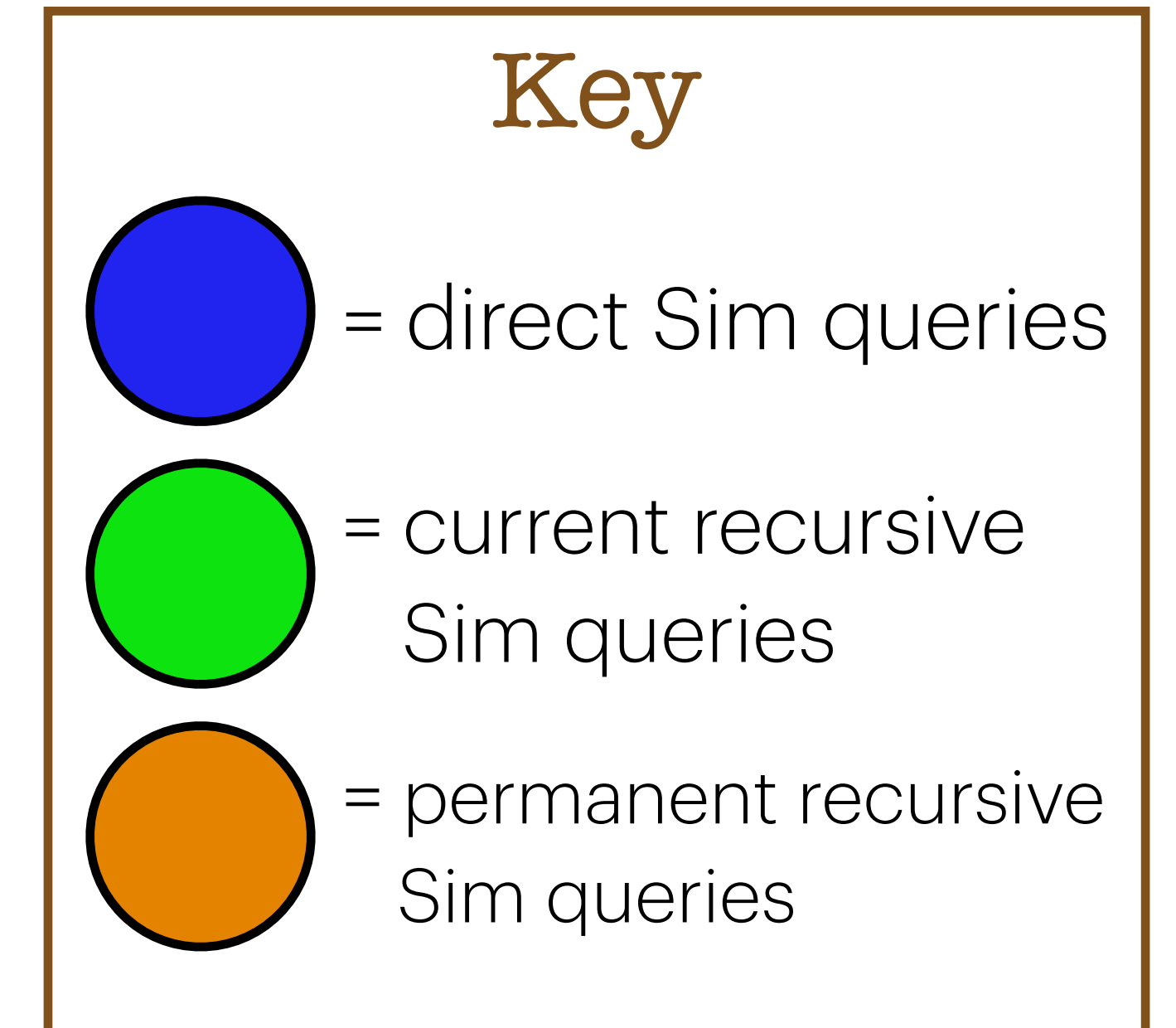
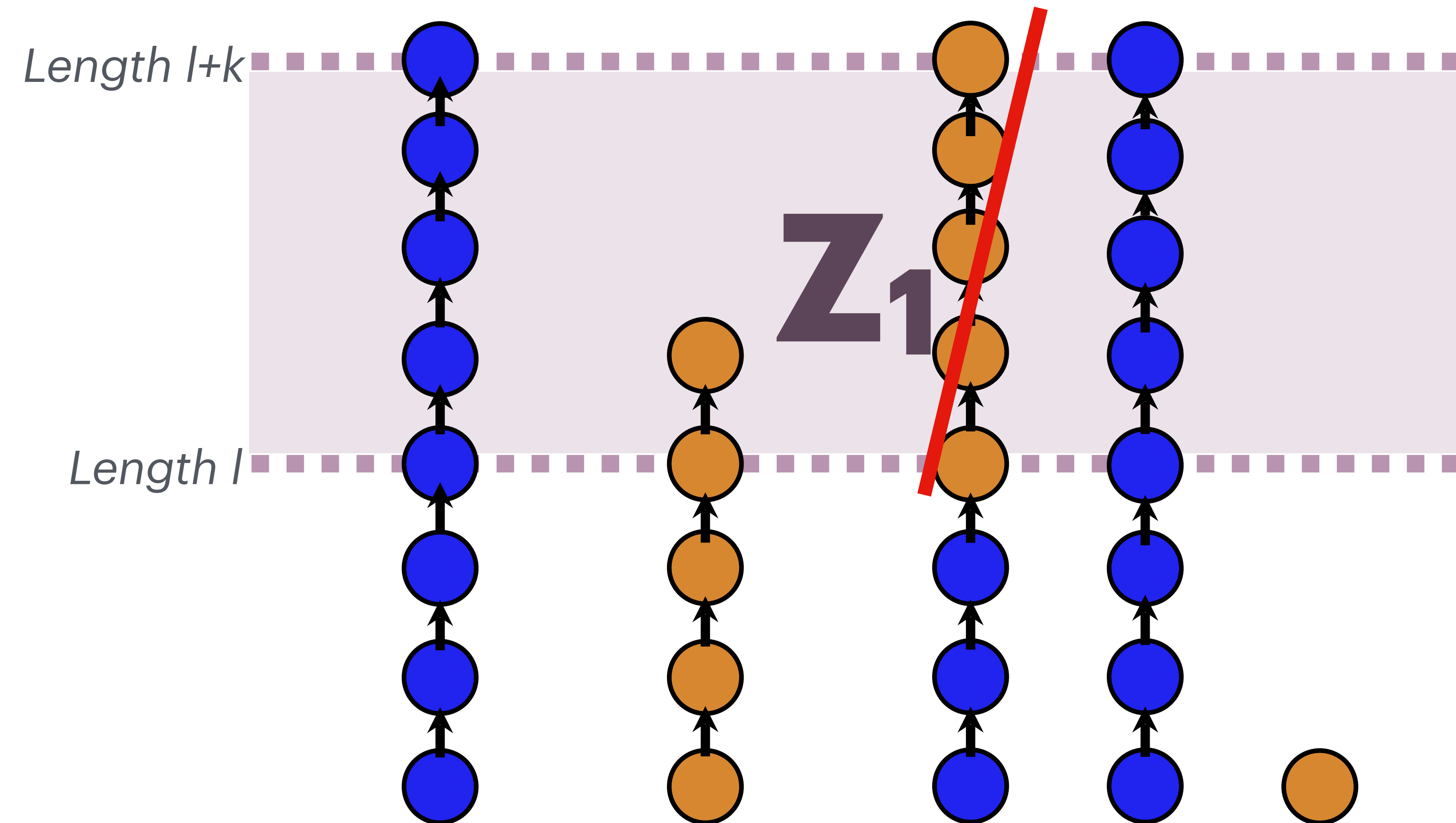


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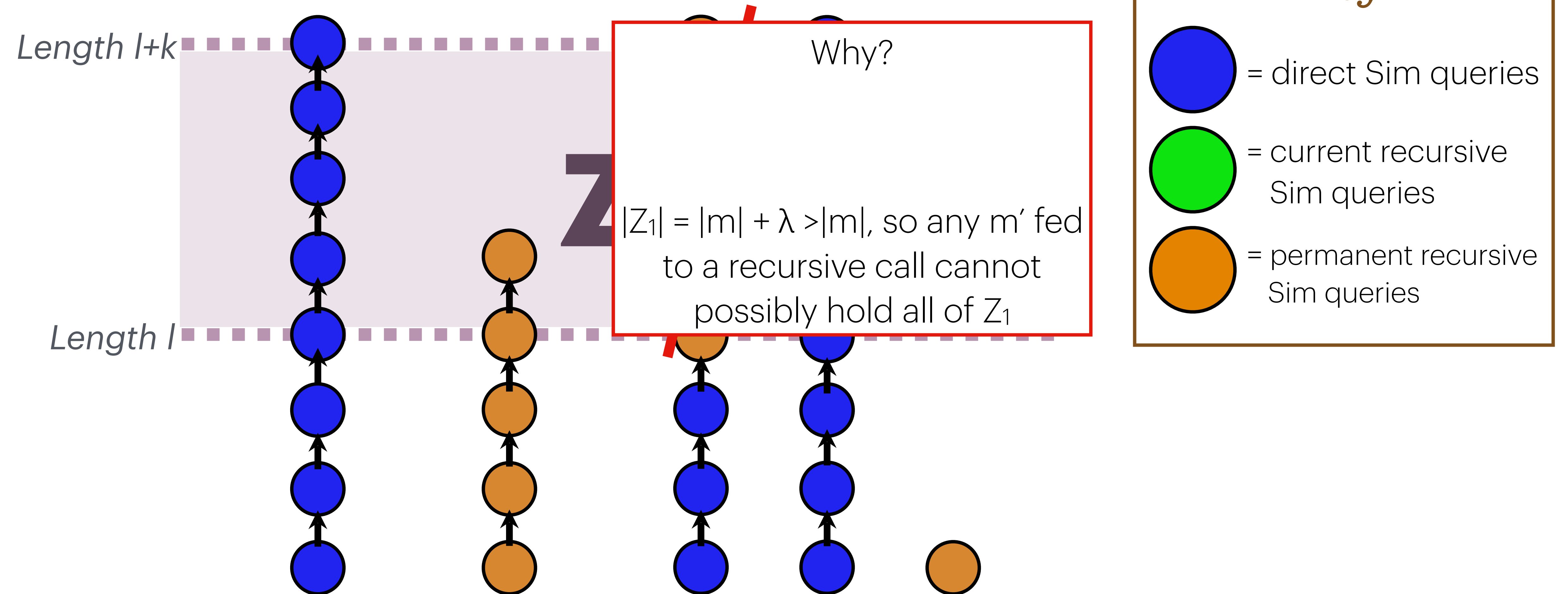
# Termination

Property: orange query can't find all of  $Z_1$



# Termination

Property: orange query can't find all of  $Z_1$

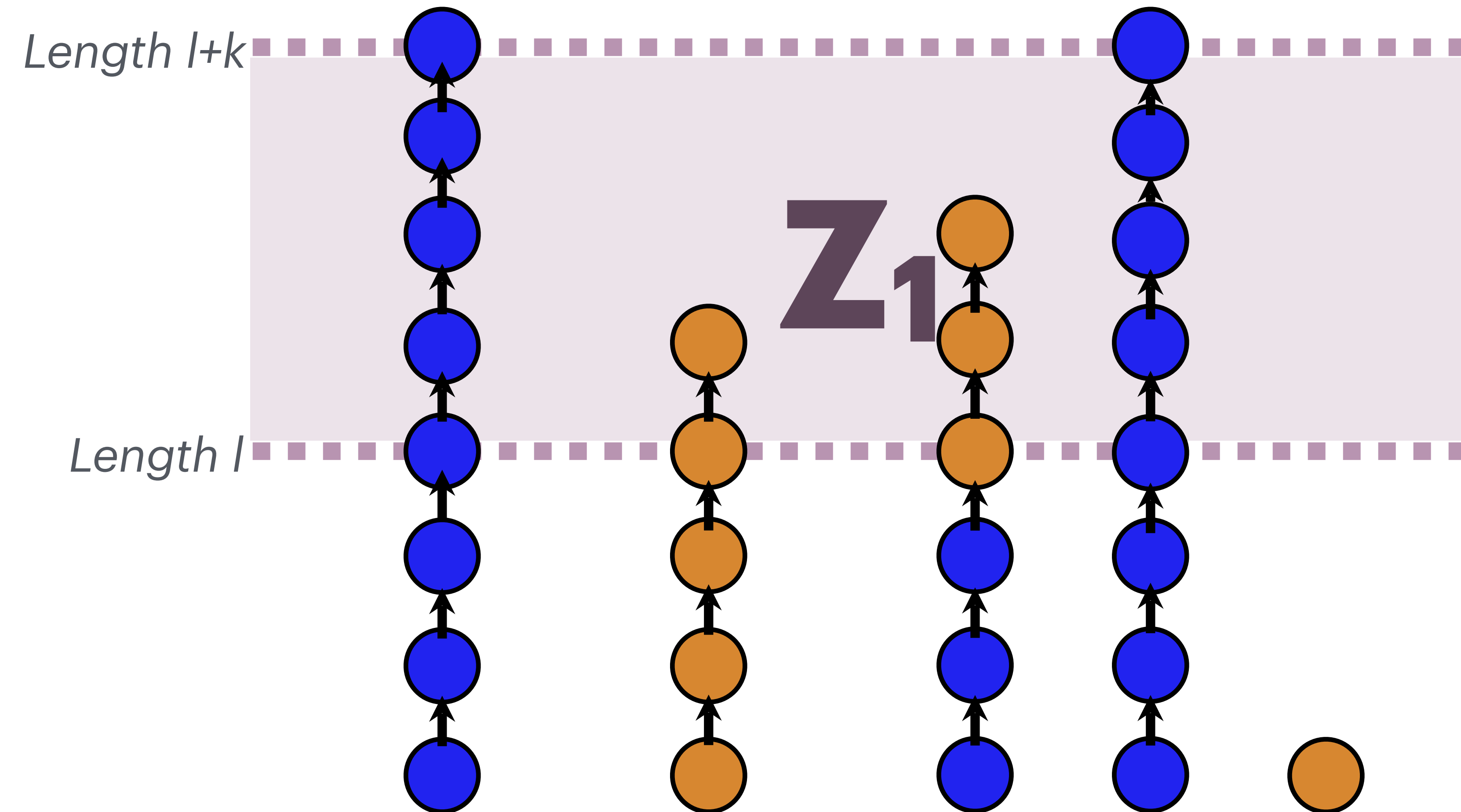


# Termination

Property 1: blue query cannot follow orange query

Property 2: green query cannot follow orange query

Property 3: orange query can't find all of  $Z_1$



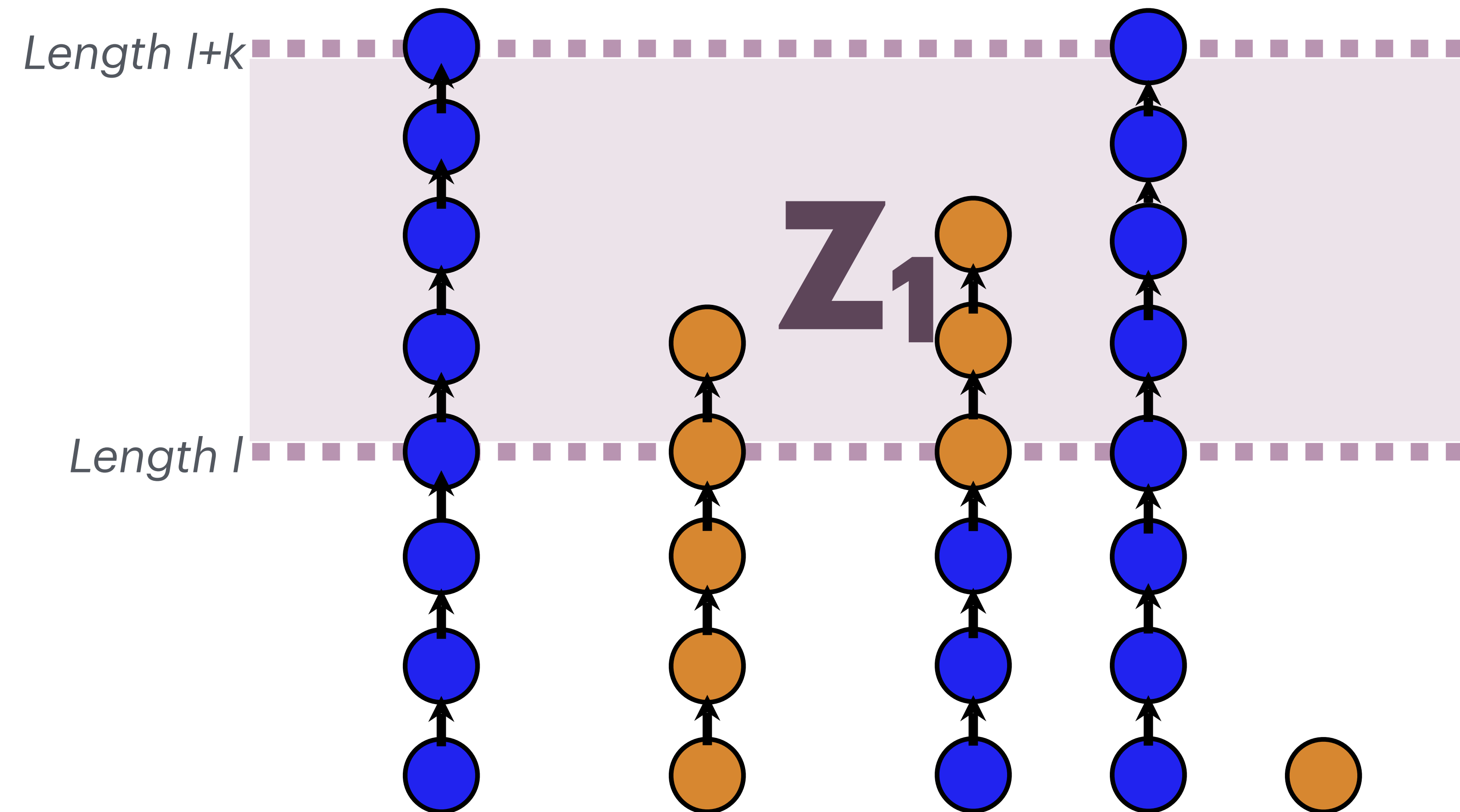


# Termination

Property 1: blue query cannot follow orange query

Property 2: green query cannot follow orange query

Property 3: orange query can't find all of  $Z_1$



Putting these properties together, we see that orange queries can only complete blue paths (triggering recursion) that extend past  $l=|m|$ , so there must be fewer recursive paths than blue ones!

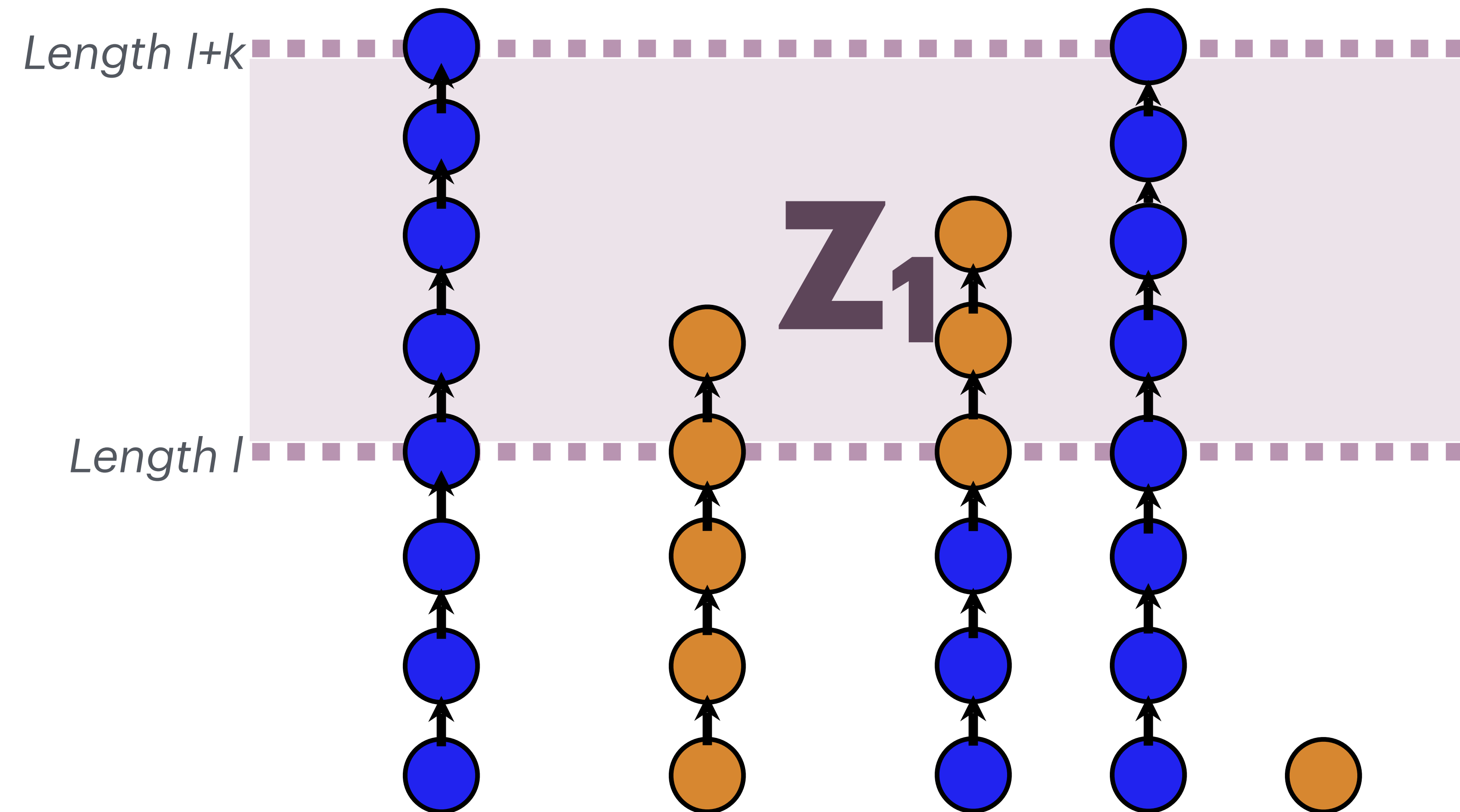


# Termination

Property 1: blue query cannot follow orange query

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Putting these properties together, we see that orange queries can only complete blue paths (triggering recursion) that extend past  $l=|m|$ , so there must be fewer recursive paths than blue ones!

Completes termination argument - hard part about showing Sim works in ideal

# Conclusions

# Conclusion and Open Questions

## Conclusions

- **We expand the practical utility of RO Combiners by constructing a RO Combiner for standard, Merkle-Damgård-based constructions**
- **We only rely on underlying compression functions being a RO using delicate termination argument**

# Conclusion and Open Questions

## Conclusions

**1. Can we construct a RO Combiner for Merkle-Damgård hashes with:**

**A. Small  $O(\lambda)$  salts  $Z_1, Z_2$**

**B. A constant number of calls to  $h^*, g^*$ ?**

**4. Alternatively, can we show such a RO Combiner cannot exist? Tradeoffs?**

Thank you!



2025/609