

Generic Anamorphic Encryption, Revisited: New Limitations and Constructions

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Introduction

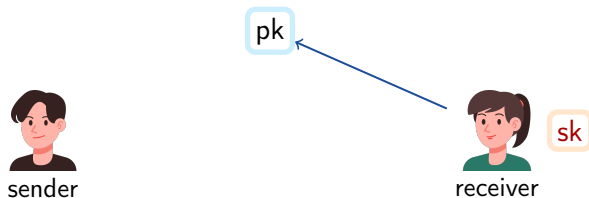


sender

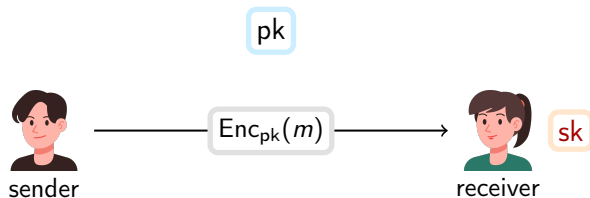


receiver

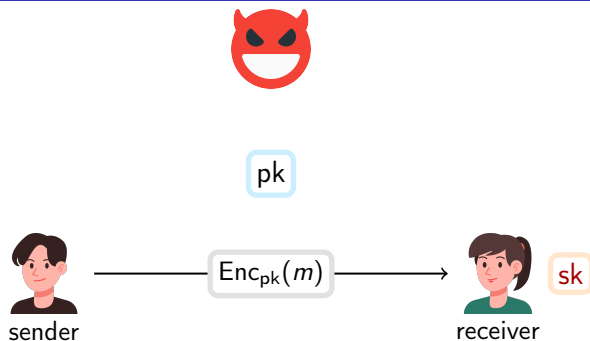
Introduction



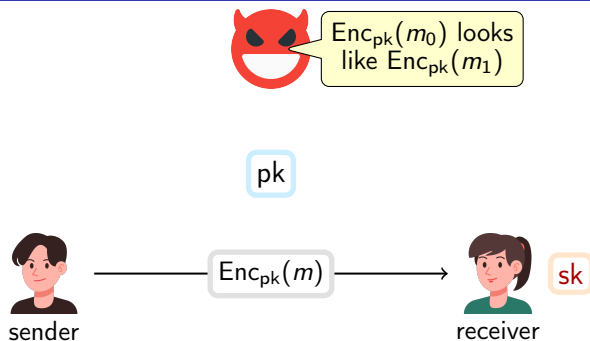
Introduction



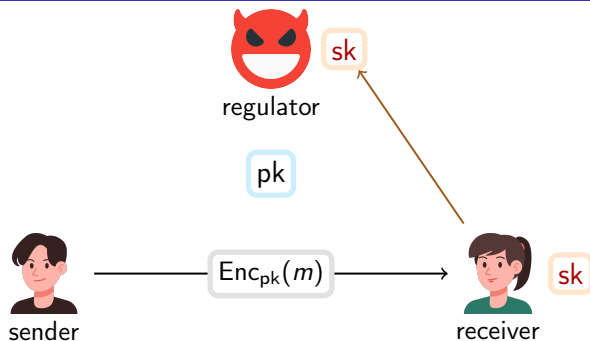
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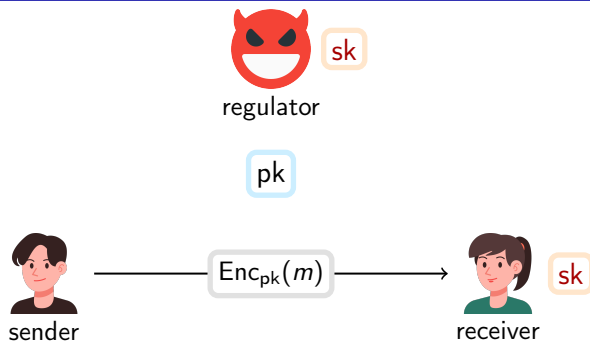
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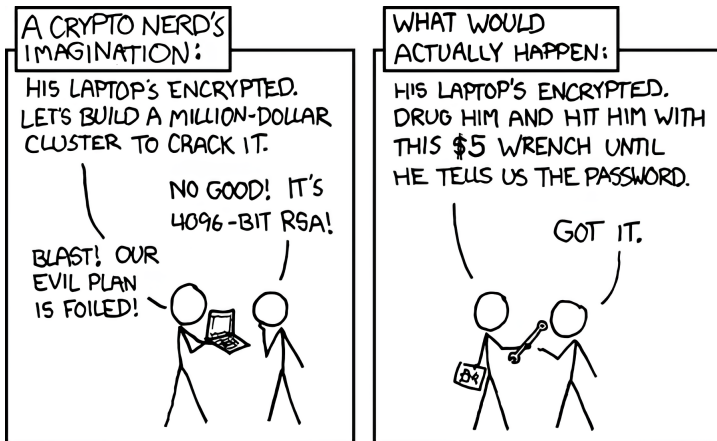
Q. Can we obtain privacy from the regulator?

Implicit assumption

Receiver privacy assumption

Implicit assumption

Receiver privacy assumption



xkcd.com/538/

AT.Gen

AT.Enc

AT.Dec

AT.Gen

AT.Enc

AT.Dec



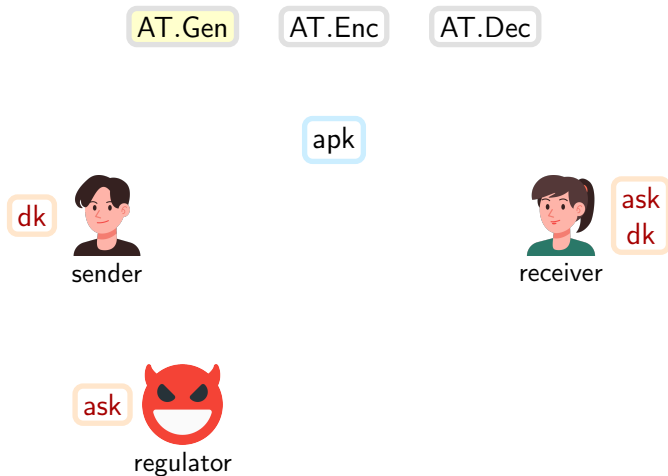
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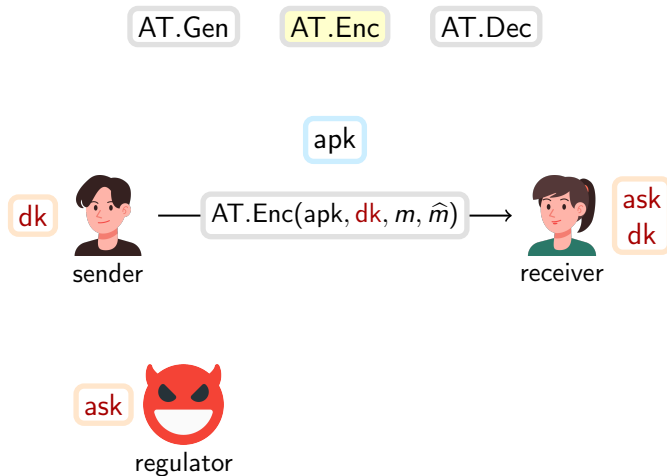


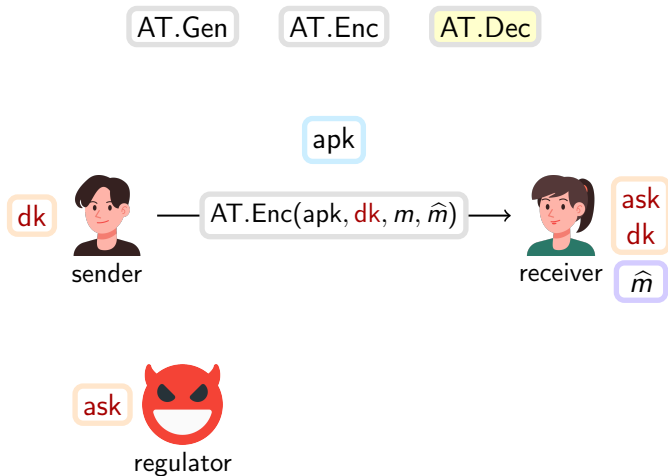
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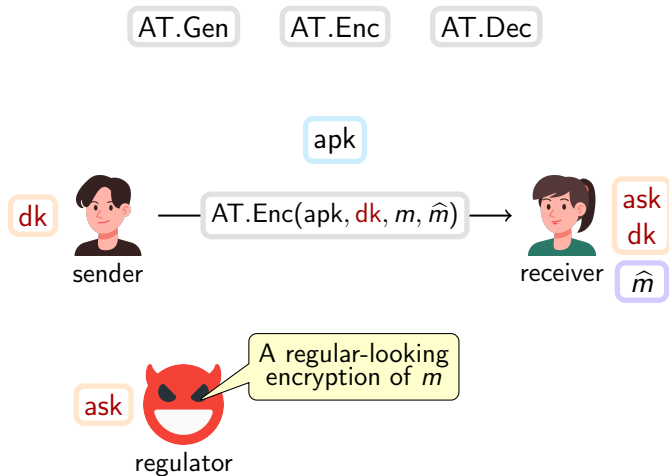


regulator









Security of an Anamorphic Triplet (AT.Gen, AT.Enc, AT.Dec) is defined with respect to a PKE (E.Gen, E.Enc, E.Dec).

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Real Game $pk, sk \leftarrow E.Gen(\lambda)$ pk, sk $(m_i, \hat{m}_i)$ $c_i \leftarrow E.Enc(pk, m_i)$ c_i 

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Anamorphic Game

$apk, ask, dk \leftarrow AT.Gen(\lambda)$

apk, ask

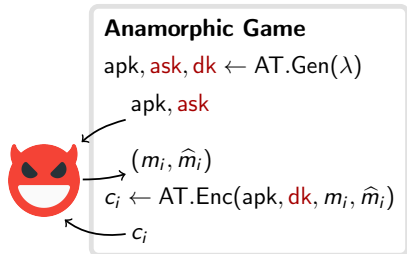
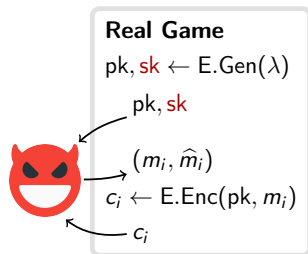
$(m_i, \hat{m}_i)$

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c_i



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Real Game $\approx^c$ Anamorphic Game

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Generic:

- Rejection Sampling (RS) [PPY22]

AT.Gen(λ) :

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✓ Supports $O(\log \lambda)$ bits long messages. [CGM24b]

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regulator

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Q. There exists an Anamorphic Triplet for any PKE?

Weak PKE - example

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- Injective OWF F
- A set B s.t. $|B| = \text{poly}(\lambda)$ disjoint from C

Weak PKE - example

$E.Gen^*(\lambda) :$

1. $pk, sk \xleftarrow{\$} E.Gen(\lambda)$
2. $m^* \xleftarrow{\$} M, y^* \leftarrow F(m^*)$
3. $pk^* \leftarrow (pk, y^*), sk^* \leftarrow (sk, m^*)$
4. **return** (pk^*, sk^*)

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1. Parse $pk^* = (pk, y^*)$
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3. **return** $c \xleftarrow{\$} B$
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$E.Dec^*(sk^*, c) :$

1. Parse $sk^* = (sk, m^*)$
2. **if** $c \in B$: **return** m^*
3. **else:** **return** $E.Dec(sk, c)$

Weak PKE - example

$$\text{ask} = (\text{sk}^*, m^*)$$

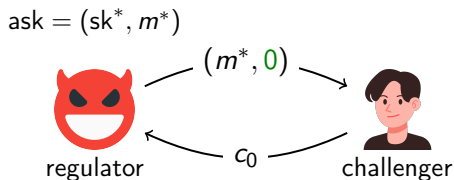


regulator



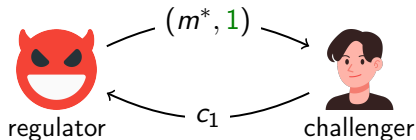
challenger

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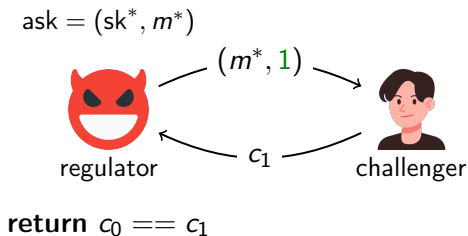


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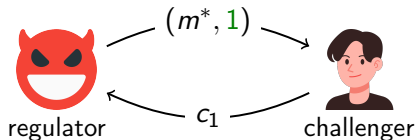


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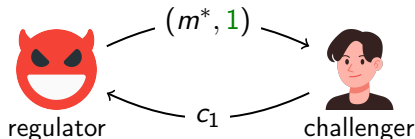


return $c_0 == c_1$

$$\Pr[c_0 = c_1 \mid \text{Real Game}] = 1/|B|$$

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$$\Pr[c_0 = c_1 \mid \text{Real Game}] = 1/|B|$$

$$\Pr[c_0 = c_1 \mid \text{Anamorphic Game}] = 1/|B|(1/2 + \text{negl}(\lambda))$$

Black-Box Anamorphic Encryption

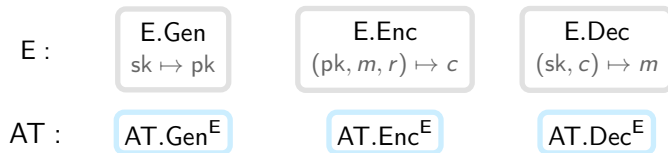
E :

E.Gen
 $sk \mapsto pk$

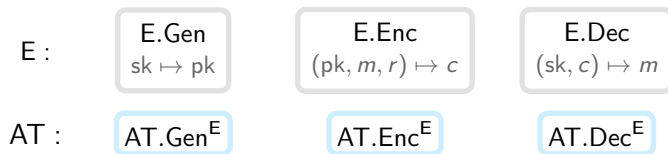
E.Enc
 $(pk, m, r) \mapsto c$

E.Dec
 $(sk, c) \mapsto m$

Black-Box Anamorphic Encryption

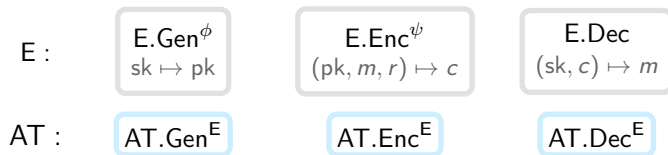


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Assumption: If the oracles in E form a correct and secure encryption scheme then (E, AT^E) is a secure AE.

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Ideal PKE: Take $E.Gen$ and $E.Enc$ as truly random functions.

$$\phi : SK \rightarrow PK \qquad \psi : PK \times M \times R \rightarrow C$$

Weak Messages

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- m^* is hard to find given only pk

E.Gen^ϕ
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 $(\text{sk}, c) \mapsto m$

$\text{E.Enc}^{\psi, \tau}$
 $(\text{pk}, m, r) \mapsto c$

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E.Find
 $(\text{sk}, i) \mapsto m_i^*(\text{sk})$

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Ideal PKE: Take E.Gen, E.Enc and m_i^* as truly random functions.

τ is a randomness biasing function.

Acts as identity on *good* messages, compresses on *weak* messages.

IND-CPA proof idea

- sk is random $\implies sk$ is hard to find

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- if sk is unknown then $m_i^*(sk)$ is random $\implies m_b$ is not weak

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- if sk is unknown then $m_i^*(sk)$ is random $\implies m_b$ is not weak
- if c^* is never decrypted and m_b is not weak $\implies \mathcal{A}$ cannot query E.Enc with the same randomness as used to generate the challenge ciphertext

Weak PKE - adversary

$$m^* \leftarrow \text{E.Find}(\text{sk}, \log \nu)$$



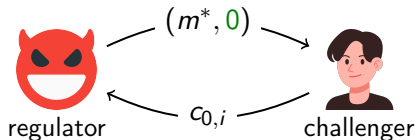
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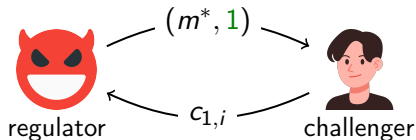
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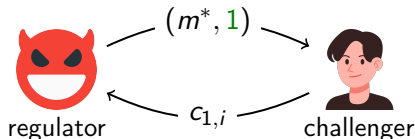
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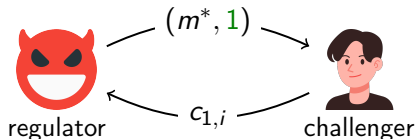


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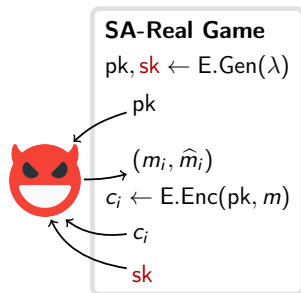
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$$\Pr[c_0 = c_1 \mid \text{Anamorphic Game}] \leq 1/(\lambda\sqrt{2})$$

Semi-Adaptive security.

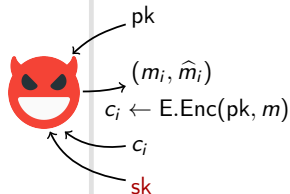
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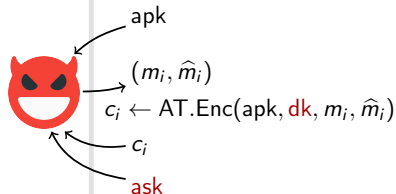
SA-Real Game

$pk, sk \leftarrow E.Gen(\lambda)$

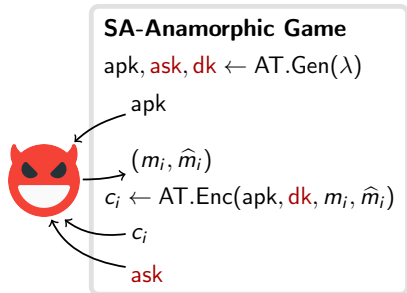
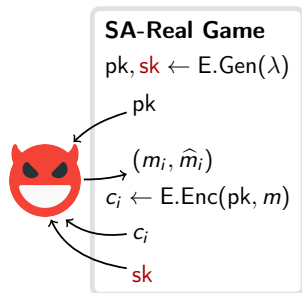


SA-Anamorphic Game

$apk, ask, dk \leftarrow AT.Gen(\lambda)$



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SA-Real Game $\approx^c$ SA-Anamorphic Game

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- Black-box Asymmetric SA-AE is impossible

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⚠* If for all m , $H_{\infty}(\text{E.Enc}(\text{pk}, m)) = \omega(\log \lambda)$

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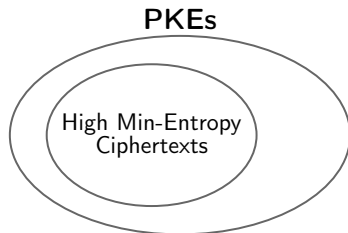
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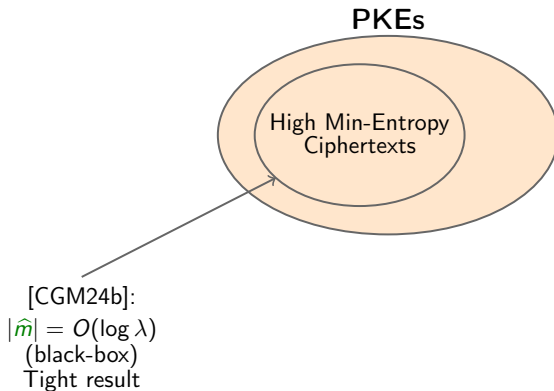
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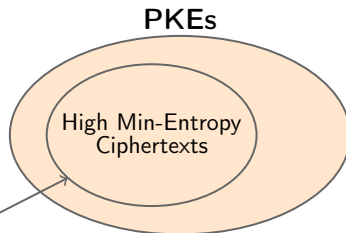
Anamorphic Limits overview



Anamorphic Limits overview



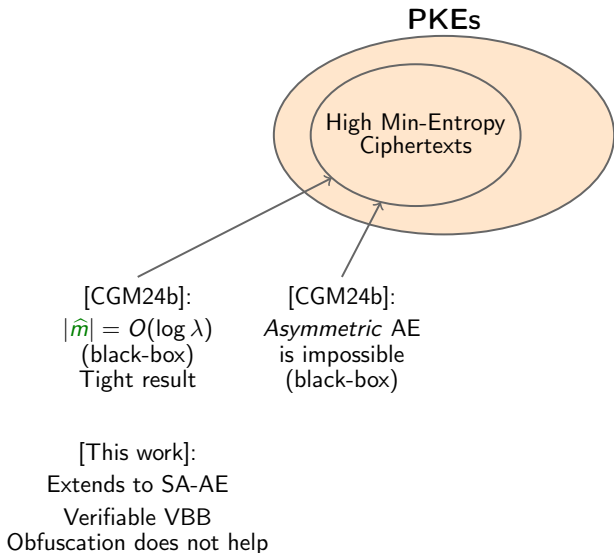
Anamorphic Limits overview



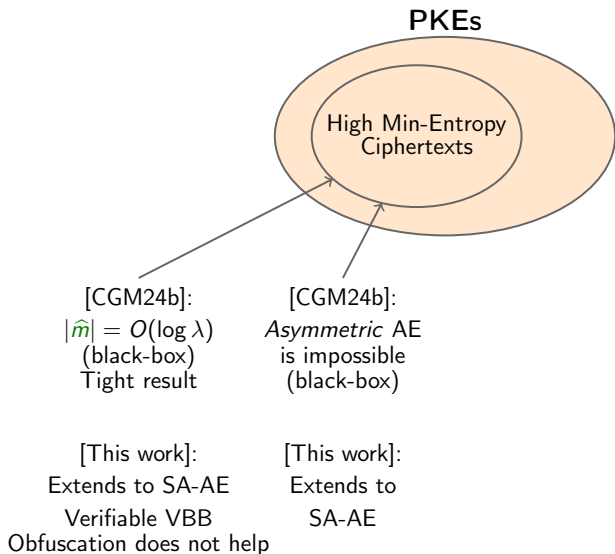
[CGM24b]:
 $|\hat{m}| = O(\log \lambda)$
(black-box)
Tight result

[This work]:
Extends to SA-AE
Verifiable VBB
Obfuscation does not help

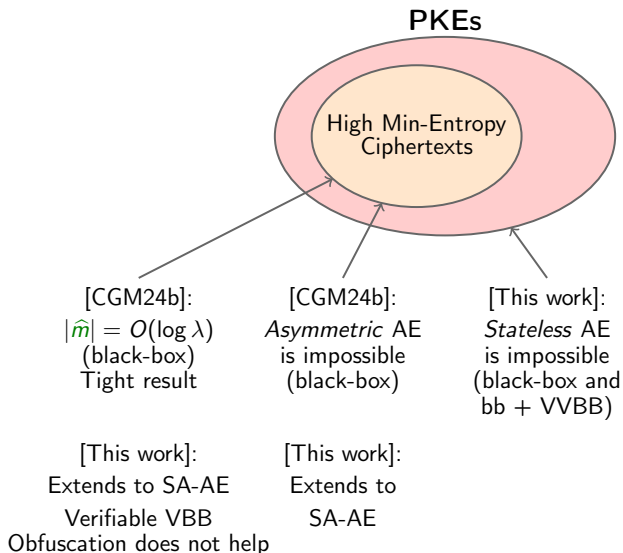
Anamorphic Limits overview



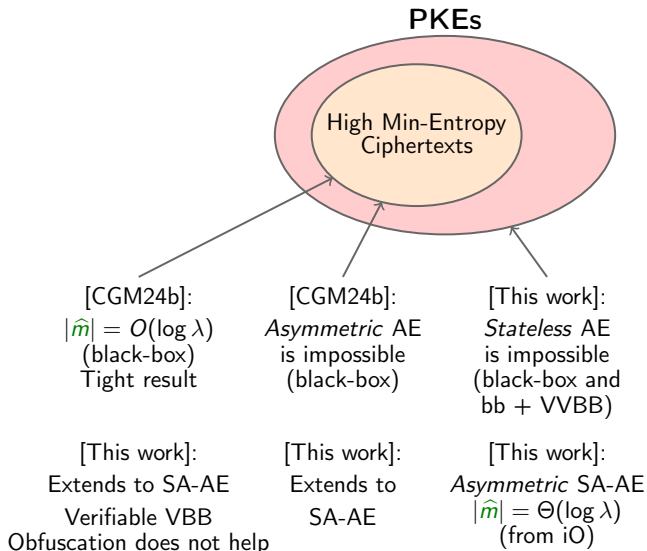
Anamorphic Limits overview



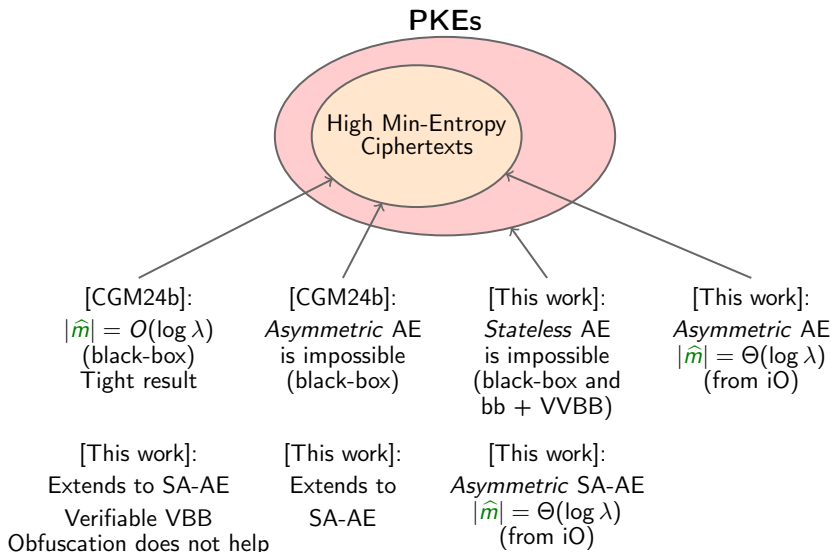
Anamorphic Limits overview



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Thanks for your attention

Any questions?

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