Pseudorandomness in the (Inverseless) Haar Random Oracle Model

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Eurocrypt, 2025

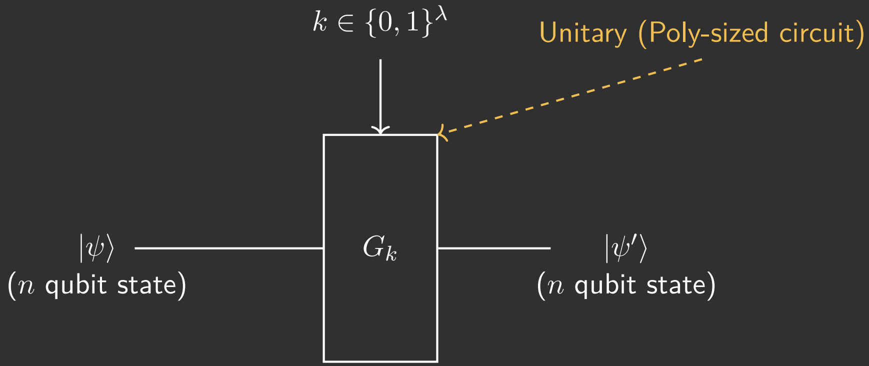
May 5, 2025

Pseudorandom Unitary (PRU)

Efficiently implementable circuits that “behave like” Haar random unitary.

Pseudorandom Unitary (PRU)

1. Efficient implementation:



Notation: n-PRU

Pseudorandom Unitary (PRU)

2. Psuedorandomness

$$\mathcal{A}^{G_k} \approx \mathcal{A}^{U}$$

Previous work

- (JLS18) defined PRU.
- (AGKL22,LQS+23,BM24) gave constructions on restricted inputs.
- (MPSY24,CBB+24) gave constructions for non-adaptive queries.
- (MH24) finally gave a construction for adaptive queries.

Defined Path-recording framework

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Why Should We Care About PRUs?

- (AQY22,MY22,AGQY22,...) builds cryptography from PRU and PRS.
- **Kretschmer (Kre21)** showed evidence that:

  Quantum pseudorandomness may exist ***even if one-way functions do not exist****.*

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Constructing PRUs Without One-Way Functions

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Modelling random circuits as Haar unitaries

- [AQY21] hypothesised random circuits to be PRUs.
- [BHH16] polydepth circuits are t-design.
- [SHH24] low depth circuits are $poly$-design as long as local gates act on $\log$ number of qubits.
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Quantum Haar Random Oracle Model

[BFV20, CM21, ABGL24]

Quantum Haar Random Oracle Model (QHROM)

Introduced by (BFV20, CM21), but were unable to get provable results.

All parties P_i as well as the adversary $\mathcal{A}$ get oracle access to a Haar Unitary and its inverse.

$$U \leftarrow \mu_n$$

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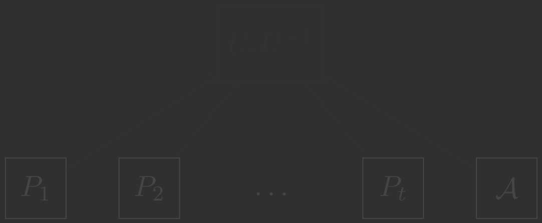

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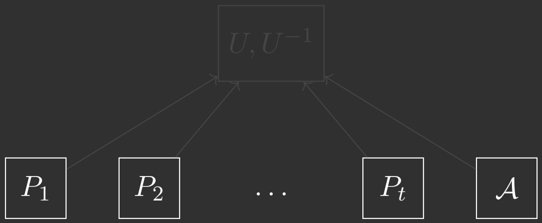

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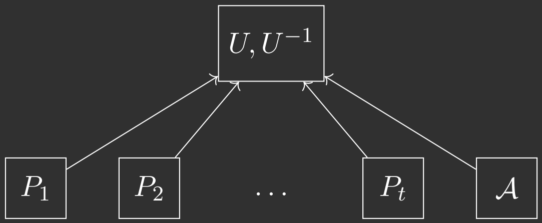
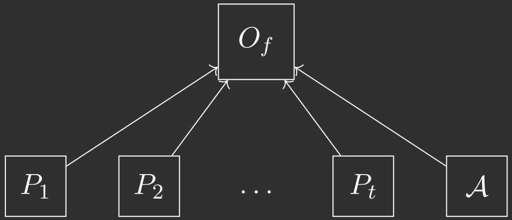


Similarity to QROM

This model is similar to the Quantum Random Oracle Model (QROM) where all parties and the adversary get access to a random function oracle.

$$f \leftarrow \mathcal{F}_n$$



All parties P_i as well as the adversary $\mathcal{A}$ get oracle access to a Haar Unitary and but not its inverse.

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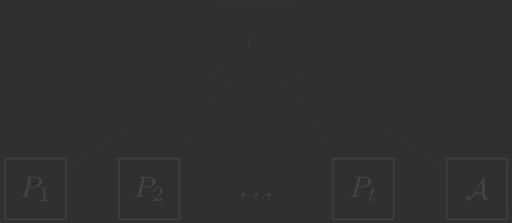

iQHROM (inverseless)

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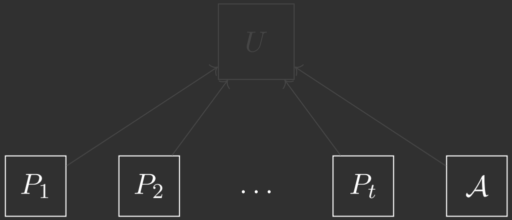

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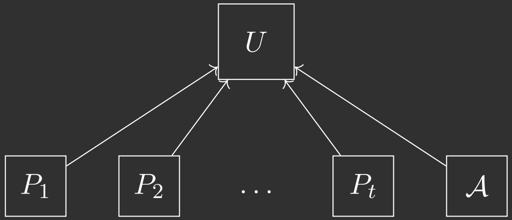

Consequences of iQHROM

- Make progress towards results in QHROM and from random circuits.
- Results give a pathway to get PRU results in the plain model.
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Results

Our Contributions

- **Unbounded-query secure PRUs in iQHROM:**
  Achieved with **two queries** to the Haar random oracle.
- **Impossibility of single-query PRUs in iQHROM:**
  No construction with **one query** to the Haar random oracle.
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Plain Model Implications

Shrinking PRU Keys for Free:
Unbounded query secure PRUs exist with keys of size $O(\lambda^{1/c})$ for any constant c, if PRU exists
Previously, GJMZ22 showed 1 query PRU with short keys exists if PRU exists.

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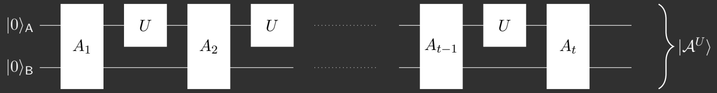
Techniques

Primitive in iQHROM

$$U \leftarrow \mu_n$$



$$\rho^{\mathcal{A}}_{\mathsf{AB}} = \mathop{\mathbb{E}}_{U \leftarrow \mu_n} \left[|\mathcal{A}^U\rangle\langle\mathcal{A}^U|_{\mathsf{AB}} \right]$$

Very hard to understand this state.

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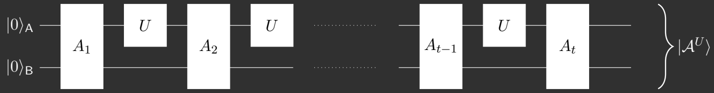

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Purification

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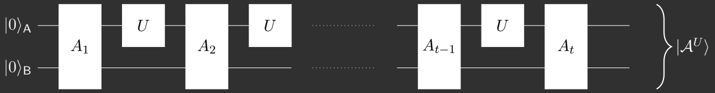

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Not unique and still hard to find

By Schmidt decomposition, for some $|\psi_{\mathcal{A}}\rangle$

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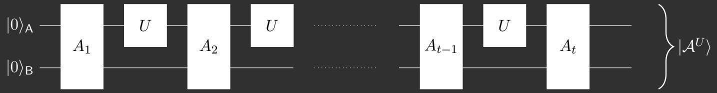

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Path Recording framework

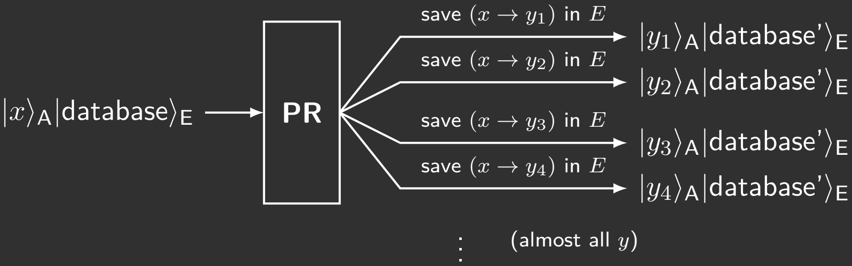
[MH24]

Compressed Purification

AIM: Find a state close to the purification:

$$\underset{U\leftarrow\mu_n}{\mathbb{E}}\left[|\mathcal{A}^U\rangle\langle\mathcal{A}^U|_{\mathsf{AB}}\right] \approx Tr_{\mathsf{E}}\left(|\mathcal{A}^{PR}\rangle\langle\mathcal{A}^{PR}|_{\mathsf{ABE}}\right)$$

Path Recording



$$\mathrm{PR}_{\mathsf{AE}} : |x\rangle_{\mathsf{A}}|R\rangle_{\mathsf{E}} \mapsto \frac{1}{\sqrt{N-|R|}} \sum_{\substack{y\in[N],\\ y\notin \mathrm{Im}(R)}} |y\rangle_{\mathsf{A}} \underbrace{|R\cup\{(x,y)\}\rangle_{\mathsf{E}}}_{\text{Path}} .$$

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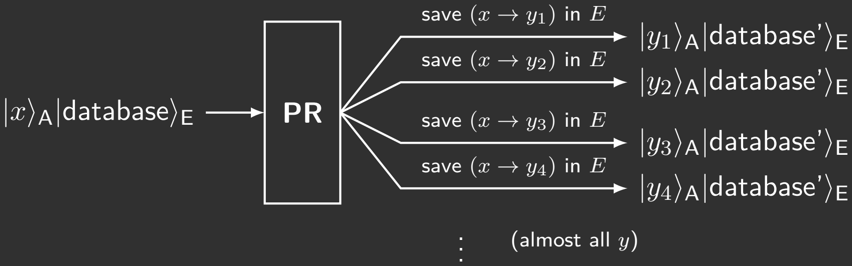

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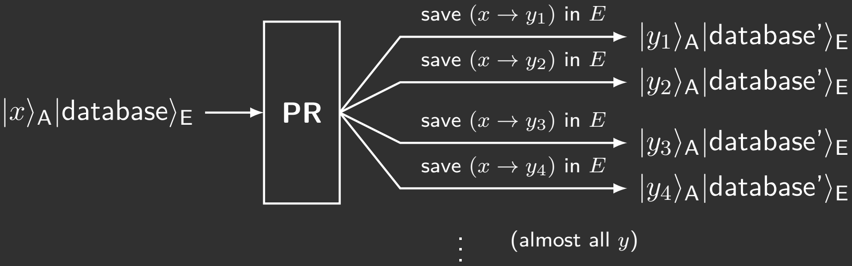

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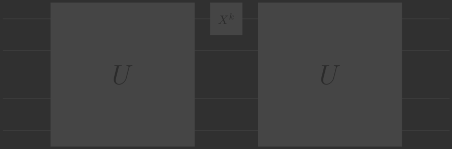
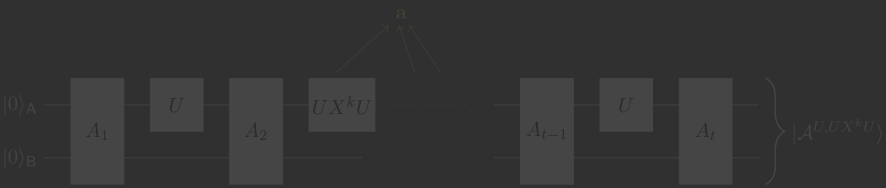
PRU in iQHROM

Analysis in iQHROM

PRU in iQHROM : $G^U(k) = UX^kU$



Adversaries state :



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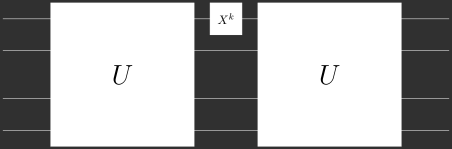

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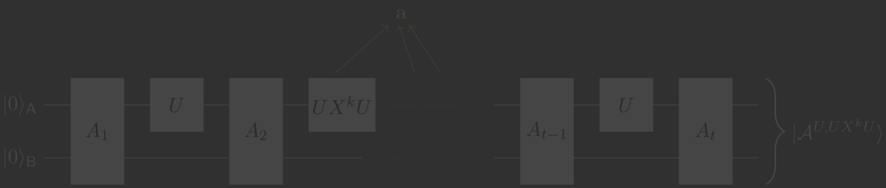

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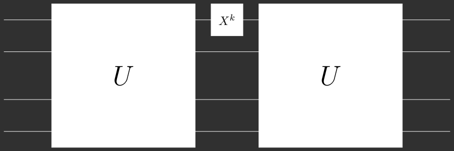

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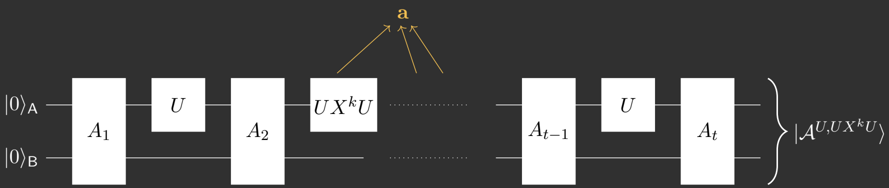

What does the purification look like?

Ideal

- U_1, U_2
- Separate "paths"
- (x^1, y^1) added to the first path
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- $|\{(x^1, y^1)\}\rangle|\{(x^2, y^2)\}\rangle$

Real

- U, UX^kU
- Single combined path
- (x^1, y^1) added to the combined path
- $(x^2, z^2), (z^2 \oplus k, y^2)$ added to the combined path
- $|\{(x^1, y^1)\} \cup \{(x^2, z^2), (z^2 \oplus k, y^2)\}\rangle$

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Real experiment close to Ideal experiment

Ideal:

$$\sum_{\vec{x},\vec{y}} |\phi_{\vec{x},\vec{y}}\rangle_{\mathsf{AB}} \otimes$$

$$|\{(x_i^1, y_i^1)\}_{i\in\mathbf{a}}\rangle_{\mathsf{E}_1} |\{(x_i^2, y_i^2)\}_{i\in\mathbf{b}}\rangle_{\mathsf{E}_2}$$

Isometry I for most keys

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$$\sum_{\vec{x},\vec{y}} |\phi_{\vec{x},\vec{y}}\rangle_{\mathsf{AB}} \otimes$$

Any isometry on purification doesn't change state

Ideal$\approx$ Real

$$\sum_{k,\vec{z}} |\{(x_i^1, y_i^1)\}_{i\in\mathbf{b}} \cup \{(x_i^2, z_i^2), (z_i^2 \oplus k, y_i^2)\}_{i\in\mathbf{a}}\rangle_{\mathsf{E}_1} |k\rangle_{\mathsf{K}}$$

Real experiment close to Ideal experiment

Ideal:

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Results and open-problems

Our Contributions

- **Unbounded-query secure PRUs in iQHROM**
- **Impossibility of single-query PRUs in iQHROM**
- **Constructing PRSGs and PRFSs in iQHROM**
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  Follow-up [ABGL25] shows strong PRU exists in QHROM

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Thank You