# On the Adaptive Security of Free-XOR-based Garbling Schemes in the Plain Model

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3 - IIT Bombay, India



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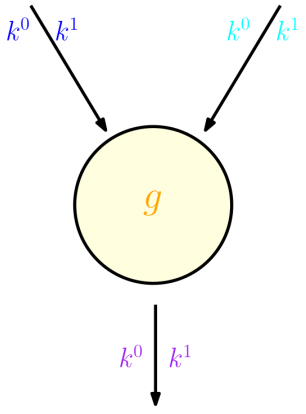
- **Plain model**: ROM gives only heuristic security, focus on standard-model security
- **Our results**: Limitations on provable security of free-XOR based garbling in this setting (applies to [App16] and [ZRE15])

# Yao's Garbling: gate-by-gate



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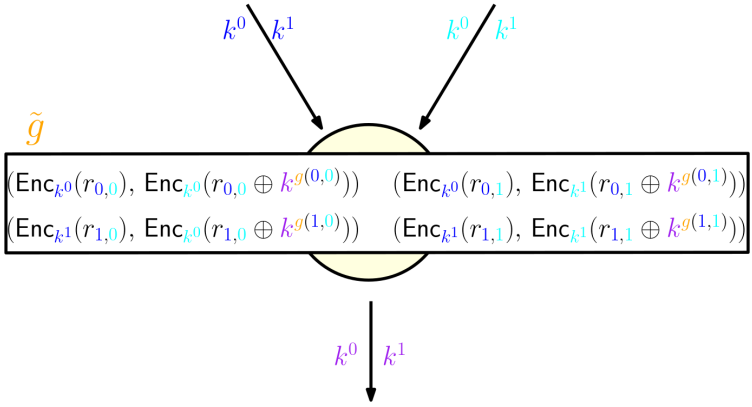

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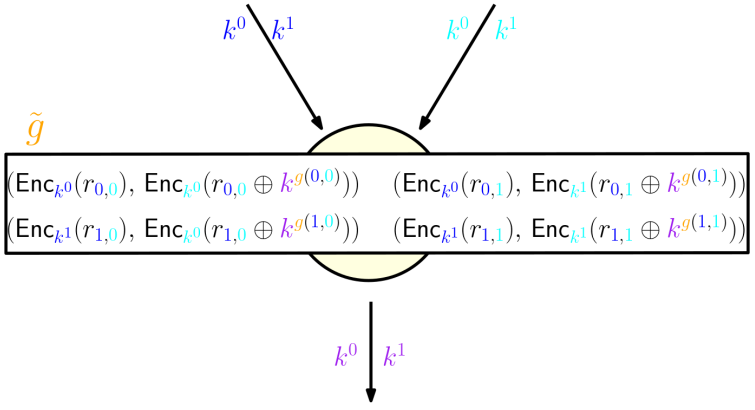


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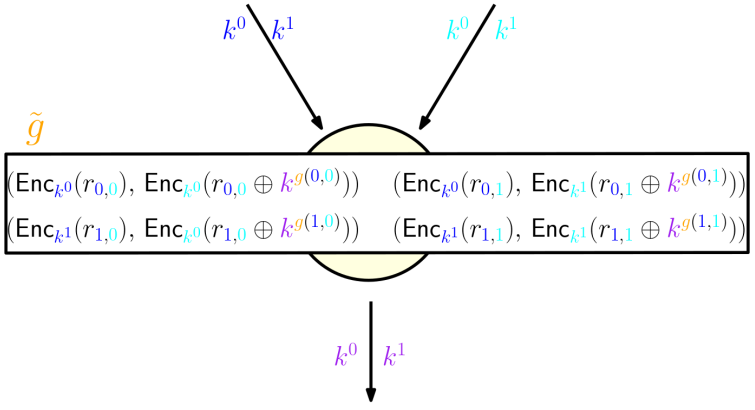


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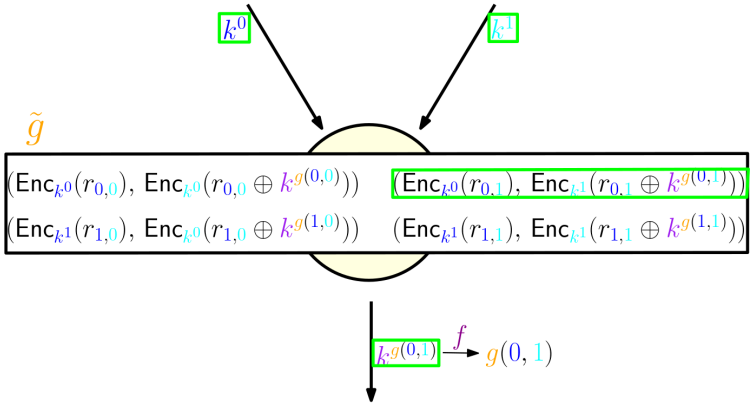


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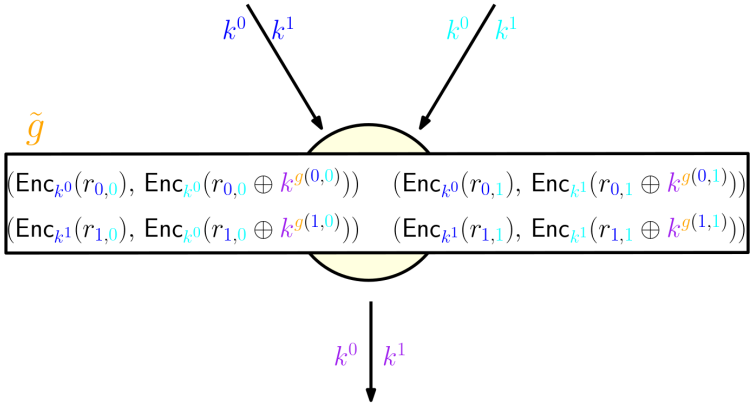


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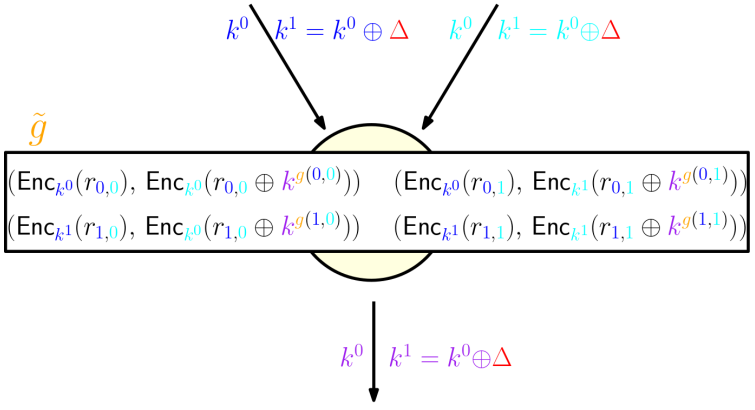


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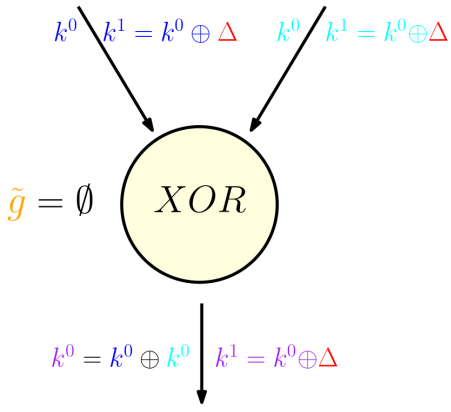


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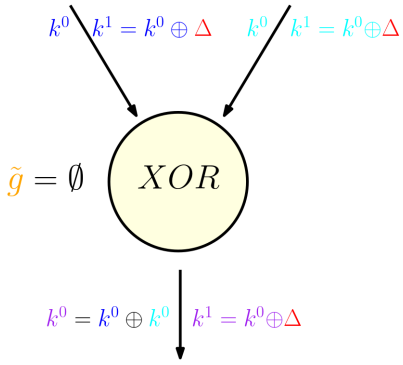


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Instantiations of free-XOR: [App16], "Half Gates" scheme [ZRE15]

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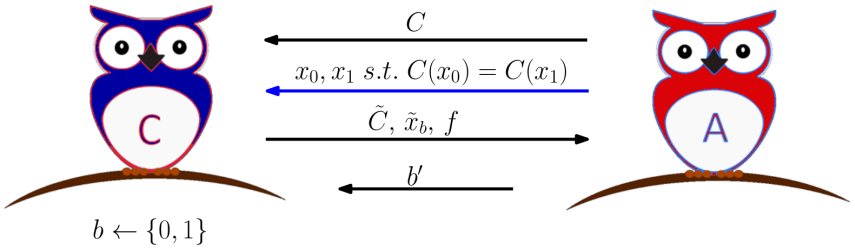
**selective indistinguishability**

(weaker than simulation-based security)



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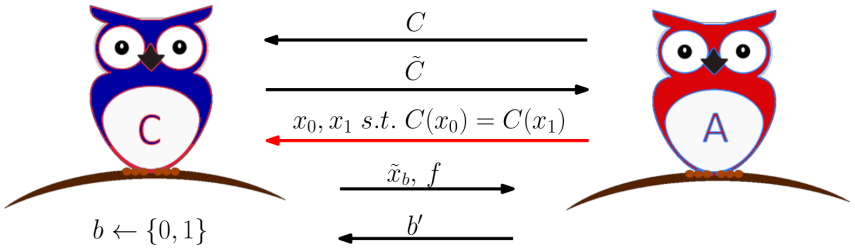


Required for offline precomputation

# Security of Yao's Garbling

- LP09: **selective** security proof based on IND-CPA security of SKE
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- KKPW21: Any **black-box proof** of **adaptive** security for Yao's garbling scheme for circuits with $n$-bit input and depth $D \leq 2n$ based on **IND-CPA secure SKE** incurs a security loss $2^{\Omega(\sqrt{D})}$.

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(applies only to *black-box* proofs for *specific* constructions from *specific* assumptions)

# Proof Idea: oracle separation

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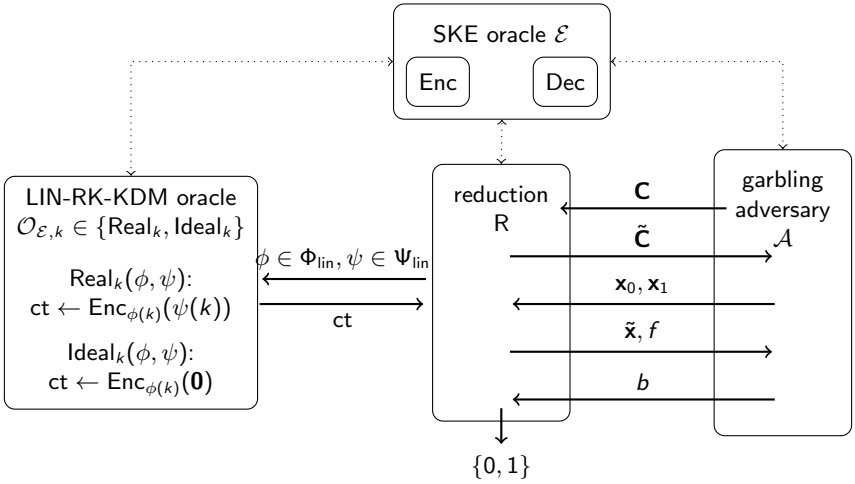
Define oracles $\mathcal{E}$ and $\mathcal{A}$ such that

- $\mathcal{E} = (\mathsf{Enc}, \mathsf{Dec})$ is an **ideal SKE scheme**
- $\mathcal{A}$ is an (inefficient) **adversary** breaking adaptive security of the free-XOR scheme, but "not too helpful" in breaking $\mathcal{E}$.

# Proof Idea: oracle separation



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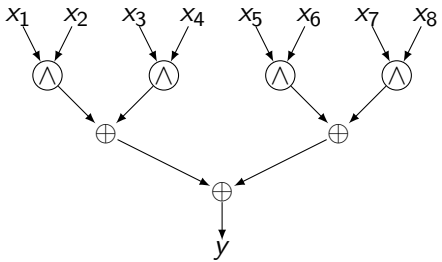


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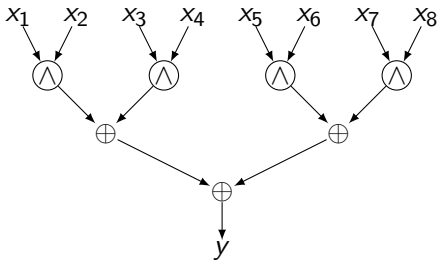


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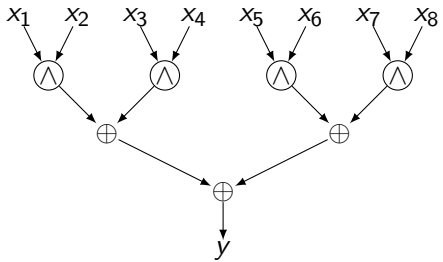


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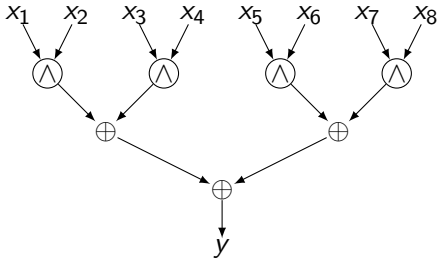


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(here: consider *non-rewinding* reduction that runs $\mathcal{A}$ *once*, general case: $q$-wise independent hash functions)

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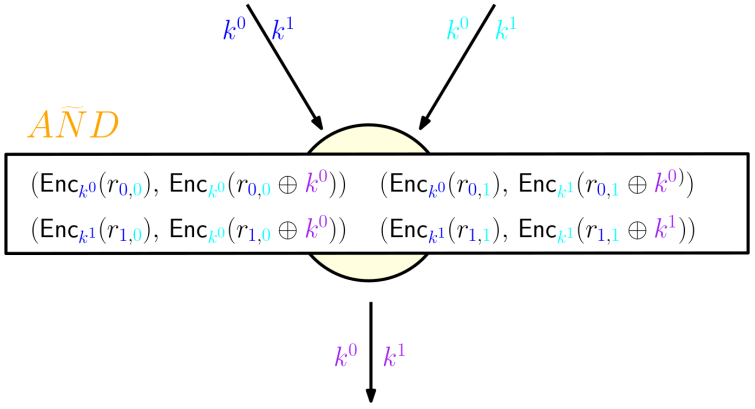
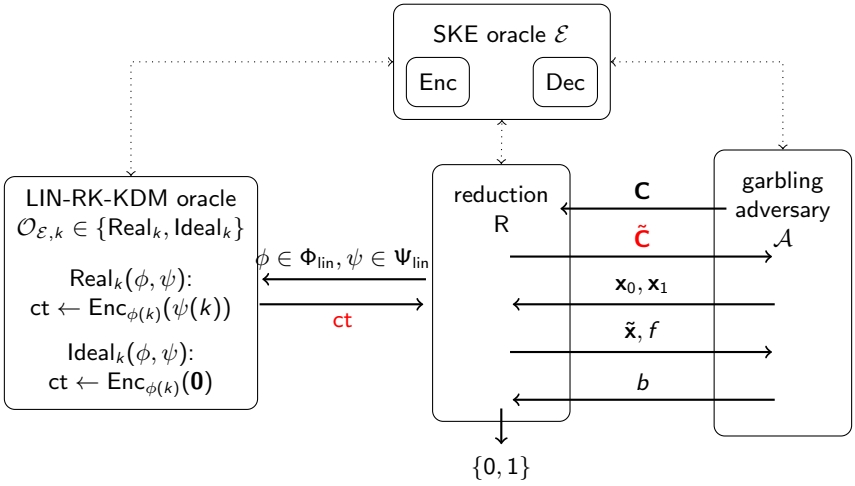
Wellformedness of $\tilde{C}$ by brute-force:




Allows to map keys to bits $\Rightarrow$ check consistency of this map with $(\tilde{\mathbf{x}}, \mathbf{x_1})$

# Proof Idea: intuition for R

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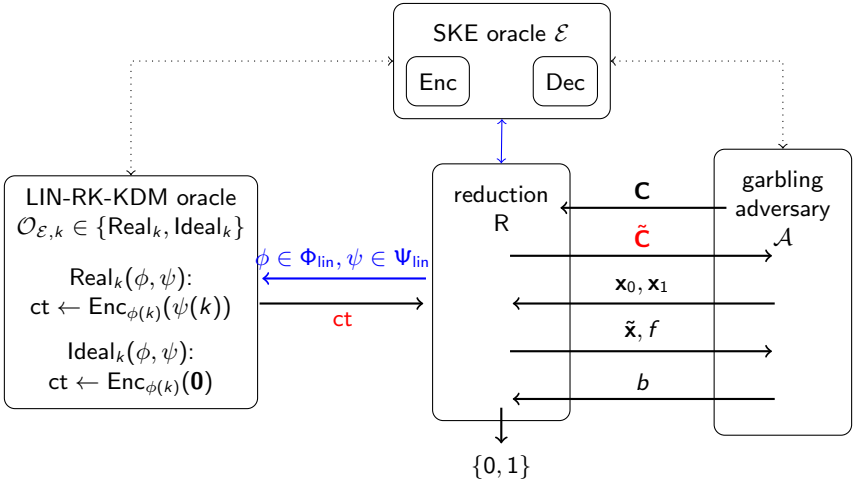


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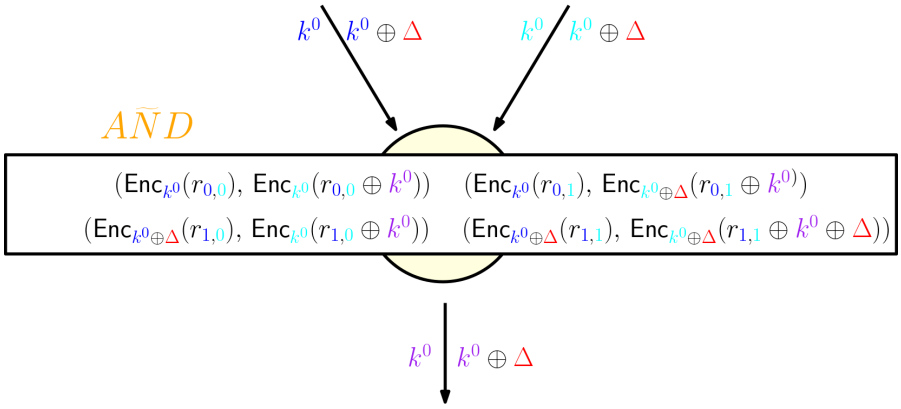


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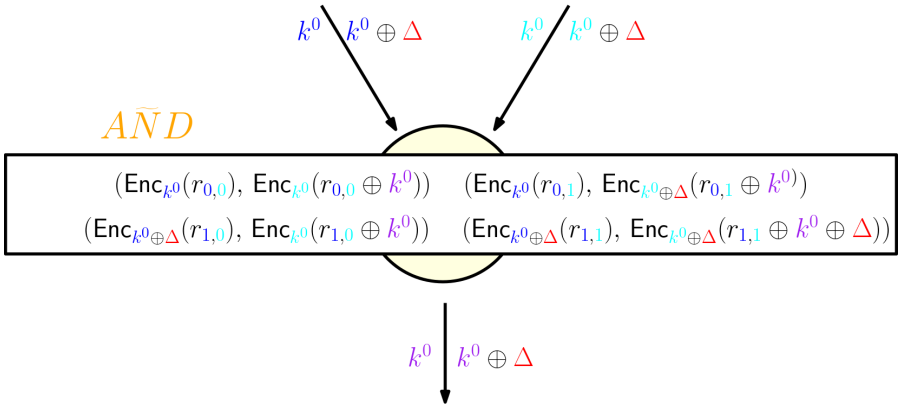


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$\Rightarrow$ either $(\tilde{\mathbf{C}}, \tilde{\mathbf{x}})$ malformed w.r.t. $\mathbf{x_1}$, or can extract $\Delta$ from queries

(except for negl chance of embedding LIN-RK-KDM challenge key as $\Delta$ *consistent* with $\mathbf{x_1}$ (req. guessing $\mathbf{x_1}$))

# Conclusion

**Theorem (Our results (informal))**

*Any* **black-box proof** *of* **adaptive** *security for* **free-XOR / "Half Gates"** *based on* **LIN-RK-KDM secure SKE / CCR secure hash function** *incurs an* **exponential** *security loss (even for NC1 circuits).*

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