Faster ABE for Turing Machines from Circular Evasive LWE

Valerio Cini Hoeteck Wee

Bocconi NTTResearch

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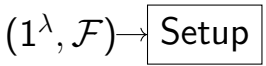

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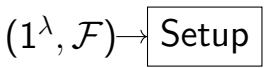

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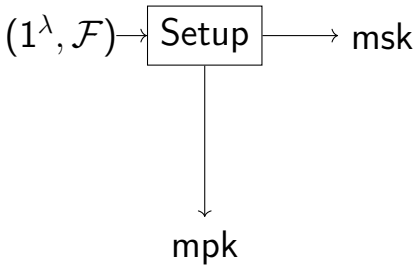

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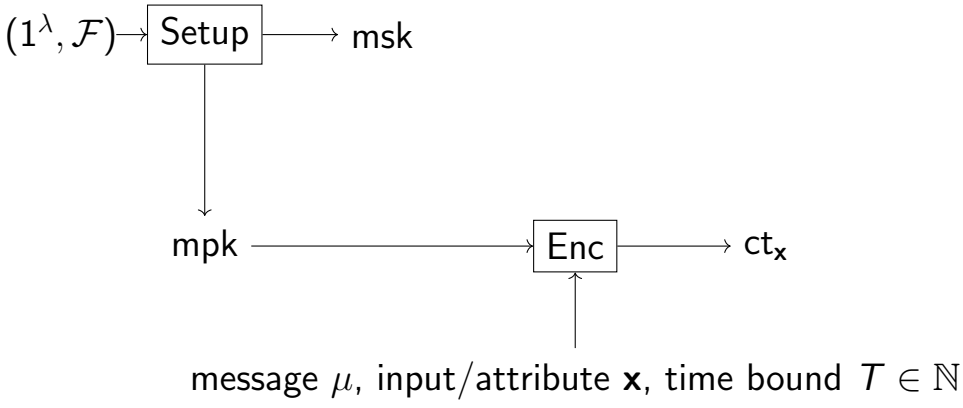

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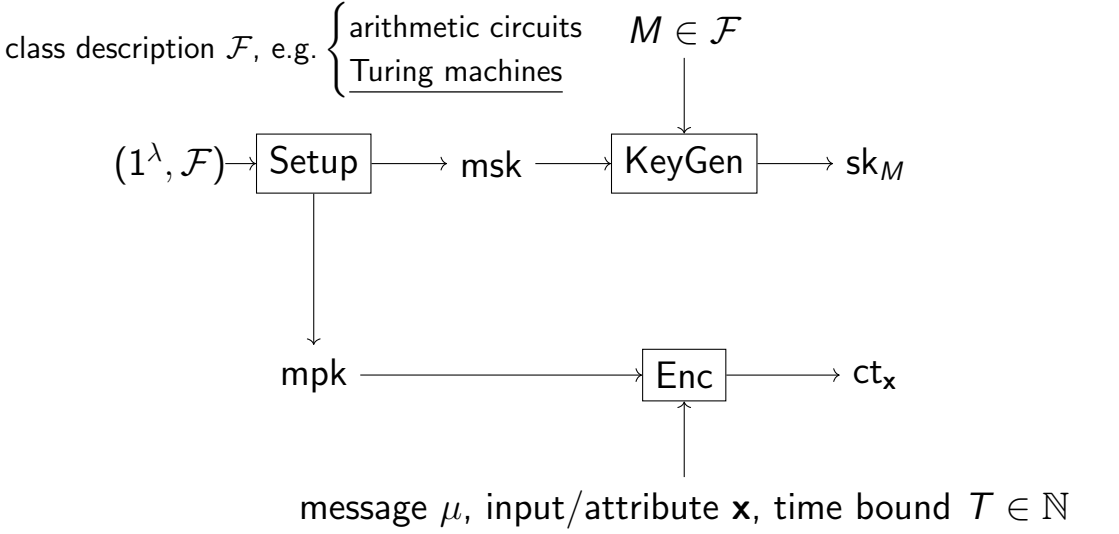

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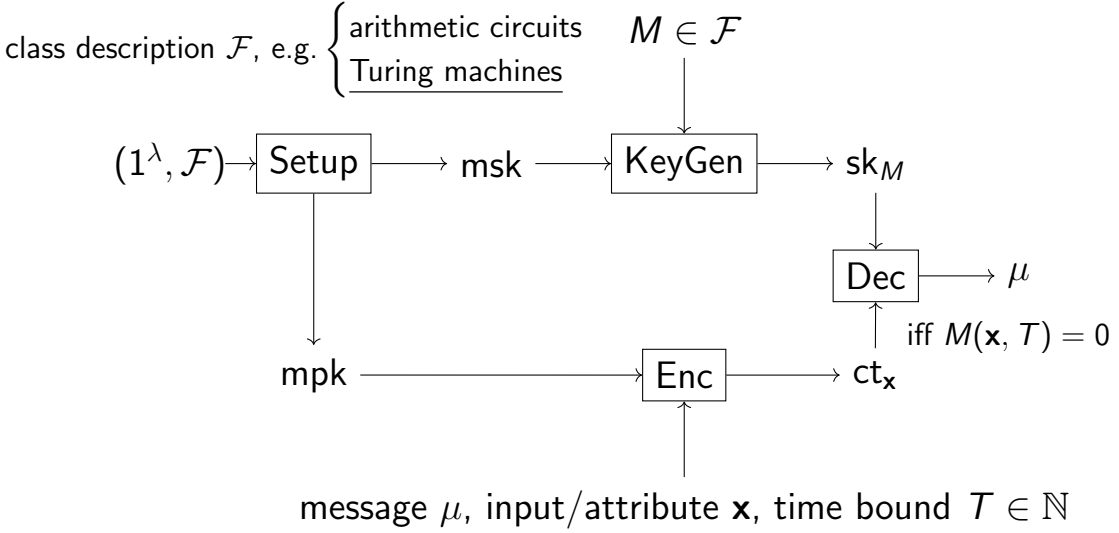

ABE for Turing Machines

[AgrawalKumariYamada24]

- $|\mathsf{ct}| = O(T^2)$
- $|\mathsf{sk}| = O(|M|^2)$
- from Circular Evasive LWE [HLL23] + Tensor LWE [W22]

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Our Results

- $|\mathsf{ct}| = O(T)$ and $\mathsf{Time}(\mathsf{Enc}) = O(T)$
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new encoding and techniques to switch b/w them

Attribute Encodings

[BonehGentryGorbunovHaleviNikolaenko SegevVaikuntanathanVinayagamurthy14] : BGG^+ encodings of $\mathbf{z} \in \{0,1\}^\ell$

$$\mathbf{s}(\mathbf{A} - \mathbf{z} \otimes \mathbf{G}) = [\mathbf{s}(\mathbf{A}_1 - z_1\mathbf{G}) | \ldots | \mathbf{s}(\mathbf{A}_\ell - z_\ell\mathbf{G})]$$

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✗ only support projections ✓ $|\mathbf{A}_0| = O(1)$

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Main Idea:
- dual encoding for (global) work tape
- BGG^{+} encodings for (local) computation

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how to switch between the two types of encodings?

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$|(\mathbf{C}, \mathbf{C}')| = O(|\mathbf{z}|) \longleftarrow$ linear in size of attribute / depend on $\mathbf{S}, \mathbf{S}'$

$|\mathbf{K}| = O(1) \longleftarrow$ constant size / independent

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$|\text{ciphertext component}| \longleftarrow$ linear in size of attribute

$|\text{key component}| \longleftarrow$ constant size

similar results hold for all other possible recodings
1) dual-to-dual
2) BGG^+-to-dual
3) dual-to-BGG^+

Iterated Computation

ABE for Iterated Computation with locality L, space S, and time T

$$f : \{0,1\}^L \longrightarrow \{0,1\}^L, \quad \mathsf{read}, \mathsf{write} : [T] \longrightarrow \binom{S}{L}, \quad L \ll S \leq T$$

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Arbitrary TM with $O(\log T)$ overhead [PippengerFischer79]

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- final: each $\mathbf{S}_t$ fresh random in U_t, with $|U_t| = O(L)$.

Warm-Up Construction

step t computation

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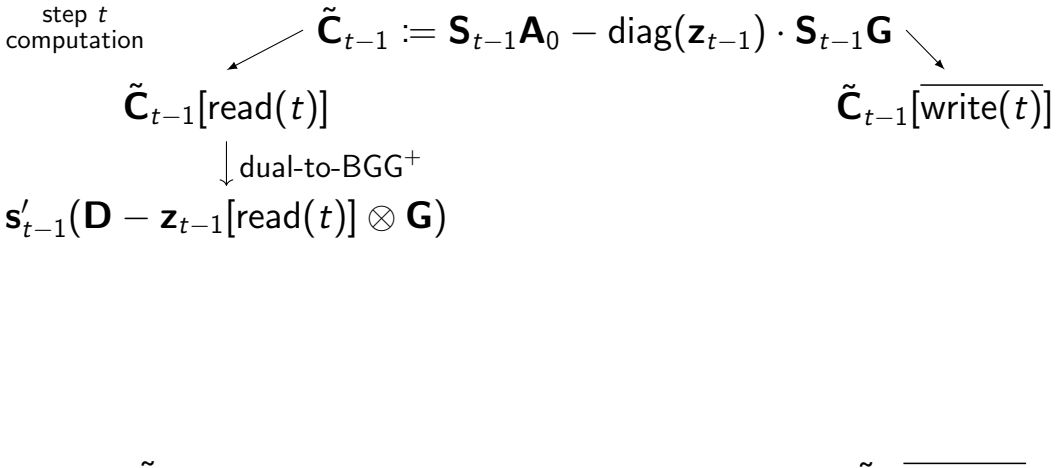

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$\downarrow$ dual-to-BGG$^+$

$$\mathbf{s}'_{t-1}(\mathbf{D} - \mathbf{z}_{t-1}[\mathsf{read}(t)] \otimes \mathbf{G})$$

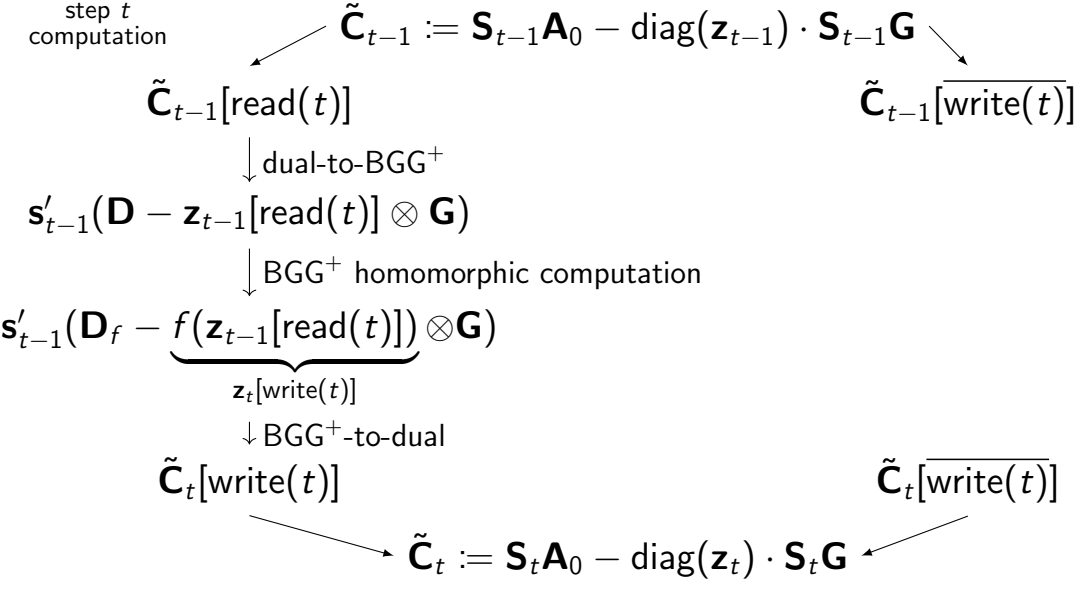
$\downarrow$ BGG$^+$ homomorphic computation

$$\mathbf{s}'_{t-1}(\mathbf{D}_f - \underbrace{f(\mathbf{z}_{t-1}[\mathsf{read}(t)])}_{\mathbf{z}_t[\mathsf{write}(t)]} \otimes \mathbf{G})$$

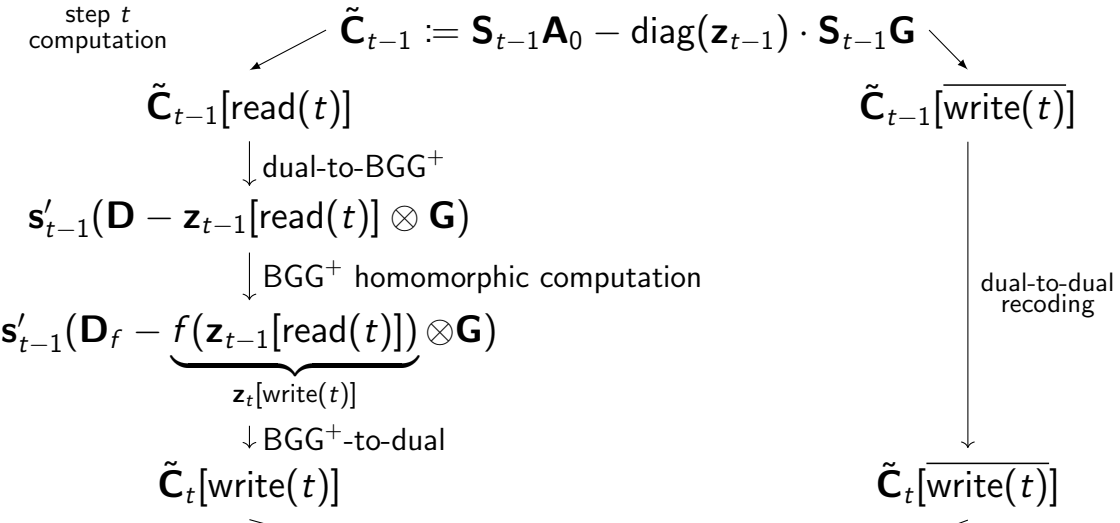
$$\tilde{\mathbf{C}}_t[\mathsf{write}(t)] \qquad \tilde{\mathbf{C}}_t[\overline{\mathsf{write}(t)}]$$

$$\tilde{\mathbf{C}}_t \coloneqq \mathbf{S}_t\mathbf{A}_0 - \mathsf{diag}(\mathbf{z}_t) \cdot \mathbf{S}_t\mathbf{G}$$

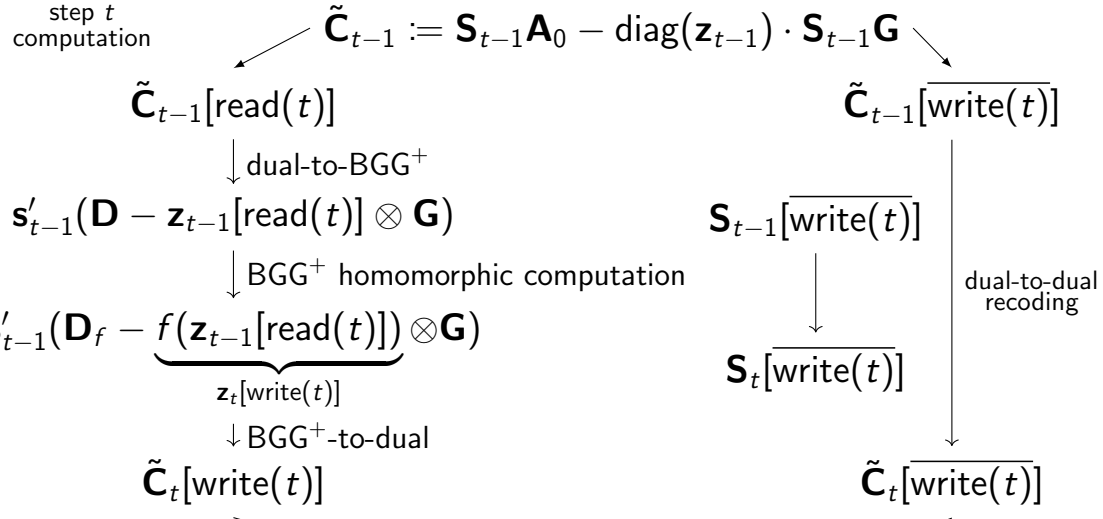
Warm-Up Construction



Warm-Up Construction



Warm-Up Construction



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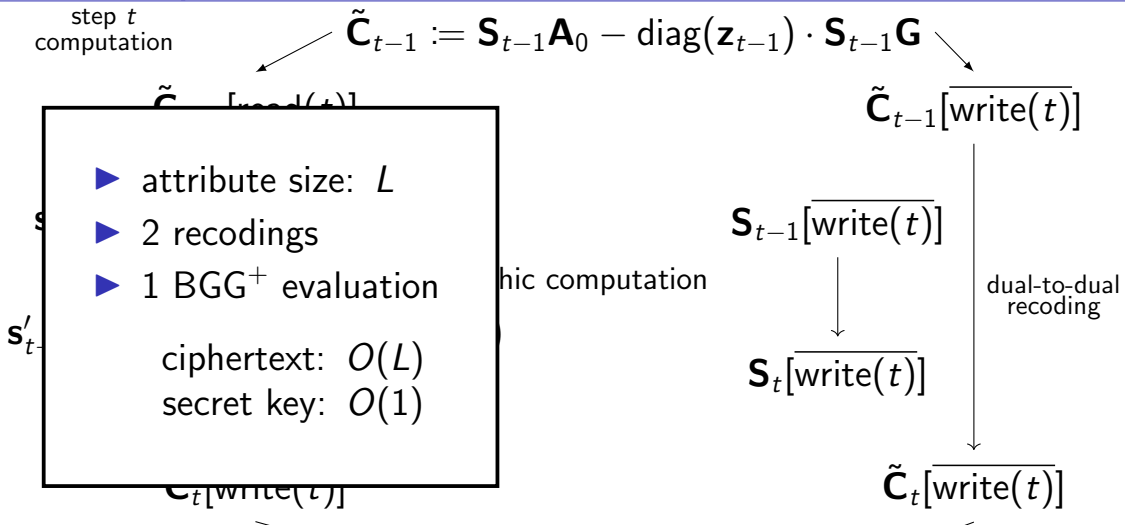

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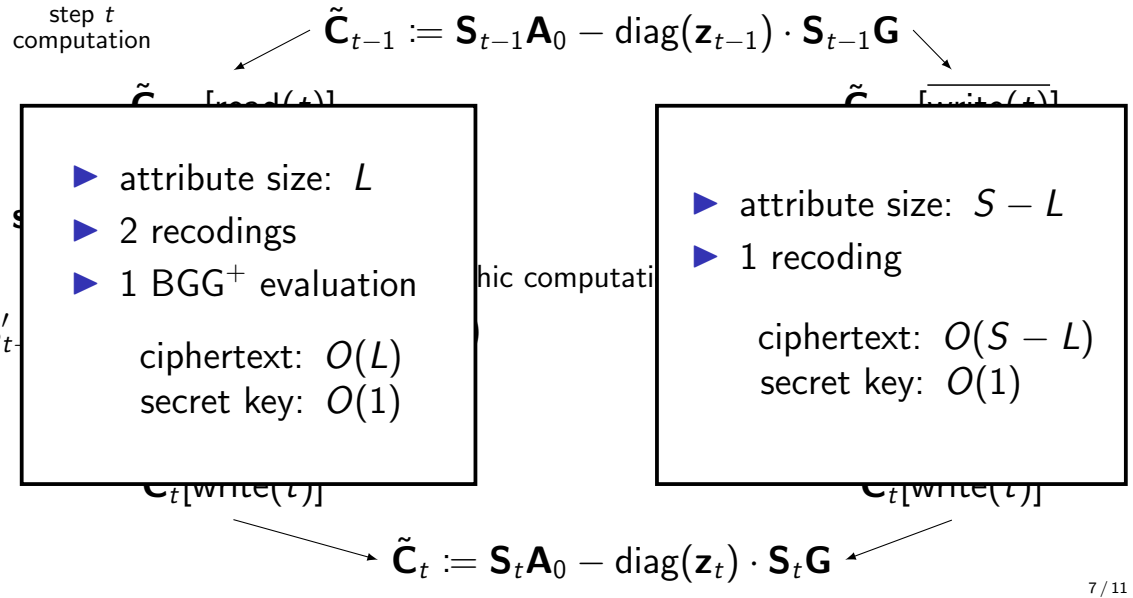

Warm-Up Construction

step t computation

$$\tilde{\mathbf{C}}_{t-1} := \mathbf{S}_{t-1}\mathbf{A}_0 - \mathrm{diag}(\mathbf{z}_{t-1}) \cdot \mathbf{S}_{t-1}\mathbf{G}$$

$\tilde{\mathbf{C}}$ [read(t)]

$\tilde{\mathbf{C}}$ [write(t)]

$\mathbf{s}$

hic computati

$\mathbf{s}'_{t-}$

- attribute size: L
- 2 recodings
- 1 BGG$^+$ evaluation

ciphertext: $O(L)$
secret key: $O(1)$

- attribute size: $S - L$
- 1 recoding

ciphertext: $O(S - L)$
secret key: $O(1)$

over T computation steps $\begin{cases} \text{ciphertext: } O(S \cdot T) \\ \text{secret key: } O(1) \end{cases}$

Improving Efficiency (1/2)

Can we set $\mathbf{S}_0 = \mathbf{S}_1 = \cdots = \mathbf{S}_T$?

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$\Longrightarrow$ it leaks partial information about $\mathbf{S}_0$ whenever $\mathbf{z}_0 \neq \mathbf{z}_1$ ✗

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Instead, we use fresh randomness for

$$\mathbf{S}_1[U_1], \mathbf{S}_2[U_2], \ldots, \mathbf{S}_T[U_T]$$

for some sets $U_1, U_2, \ldots, U_T$.

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Instead, we use fresh randomness for $\boxed{U_t := \mathsf{write}(t) \cup \mathsf{read}(t)}$

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for some sets $U_1, U_2, \ldots, U_T$. Need $|U_i| = O(L)$ for efficiency

Improving Efficiency (2/2)

before:

$\tilde{\mathbf{C}}_{t-1}[\overline{\mathsf{write}(t)}]$

dual-to-dual recoding

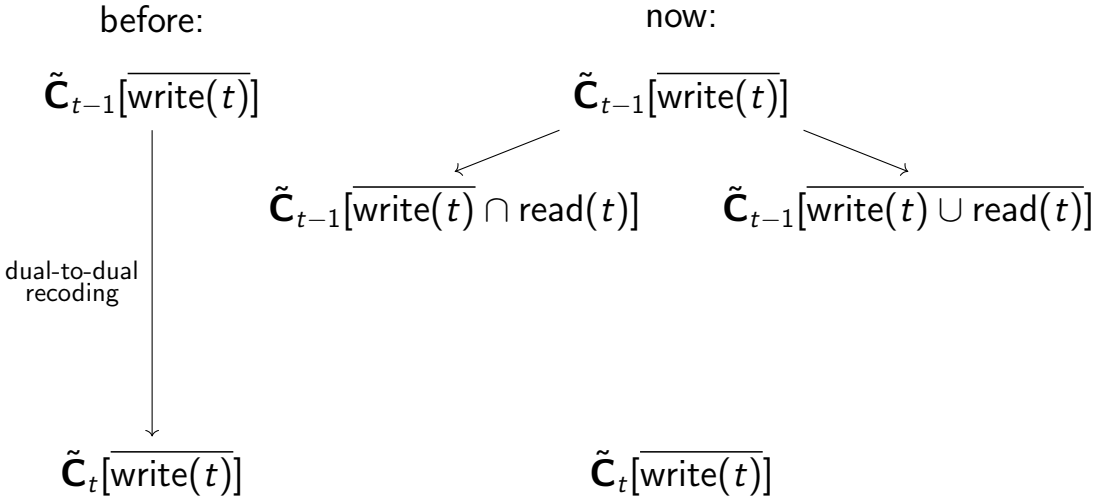
$\tilde{\mathbf{C}}_{t}[\overline{\mathsf{write}(t)}]$

now:

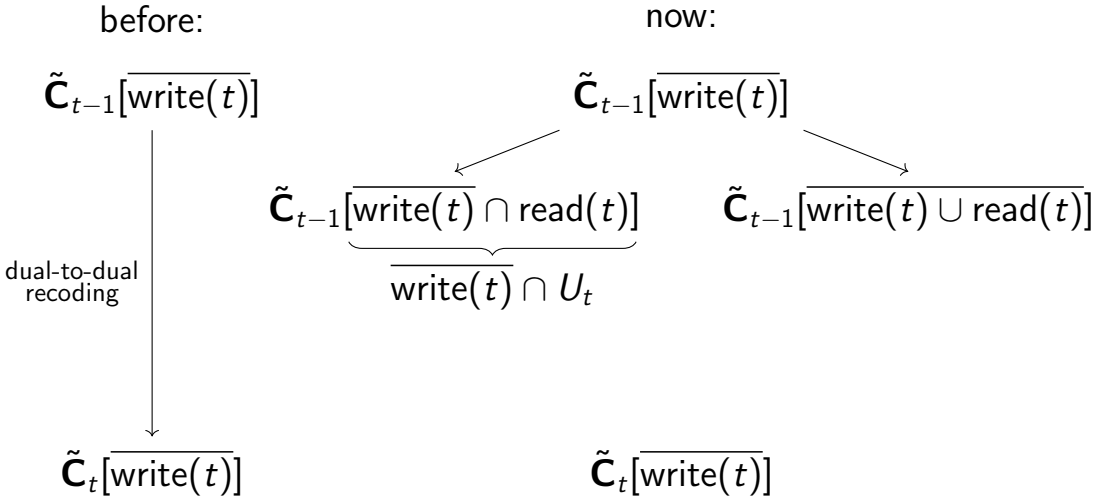
$\tilde{\mathbf{C}}_{t-1}[\overline{\mathsf{write}(t)}]$

$\tilde{\mathbf{C}}_{t}[\overline{\mathsf{write}(t)}]$

Improving Efficiency (2/2)



Improving Efficiency (2/2)



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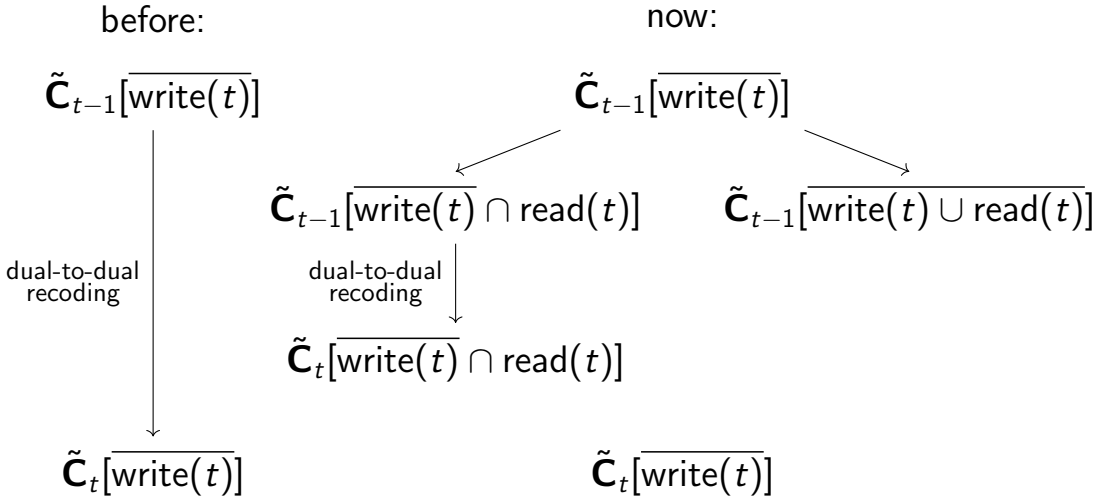

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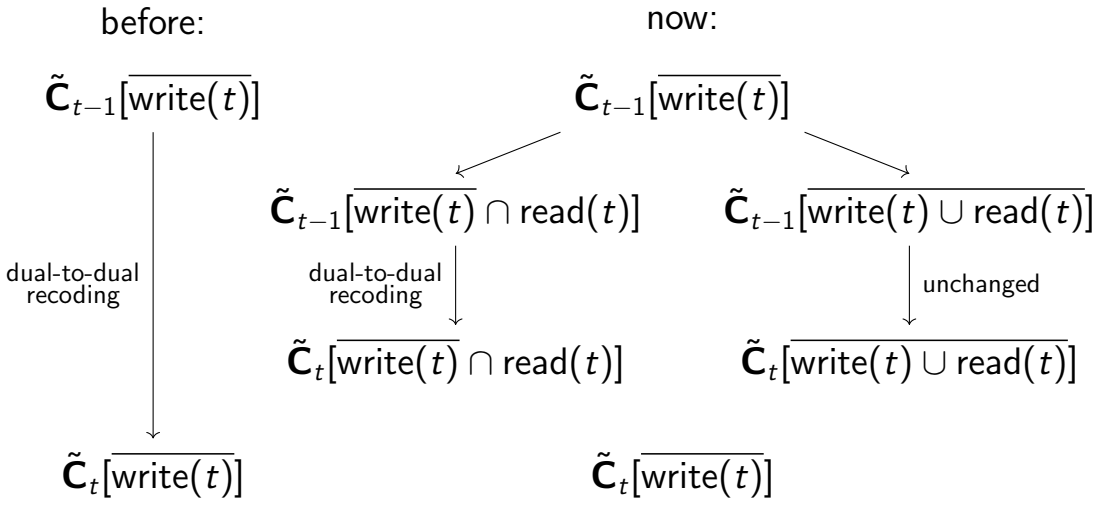
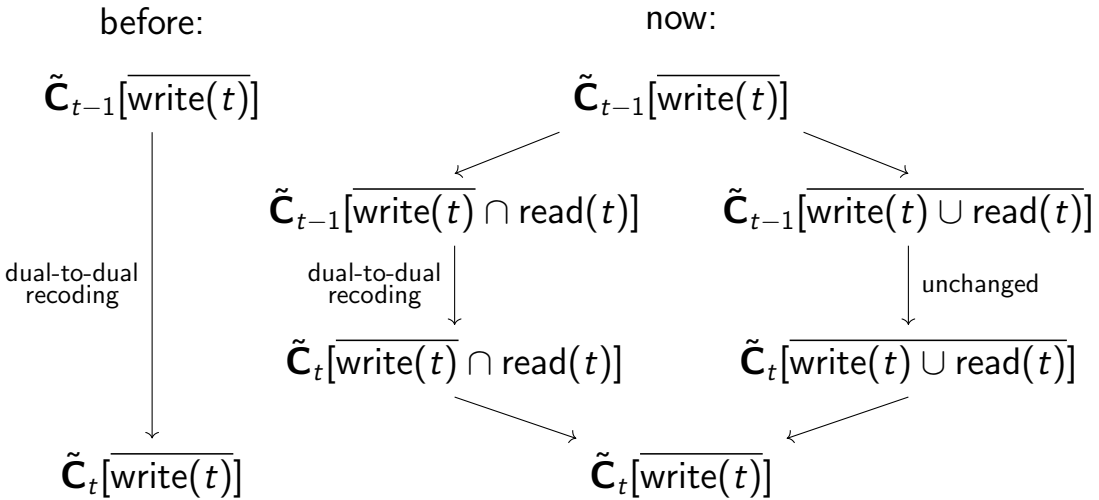


Improving Efficiency (2/2)



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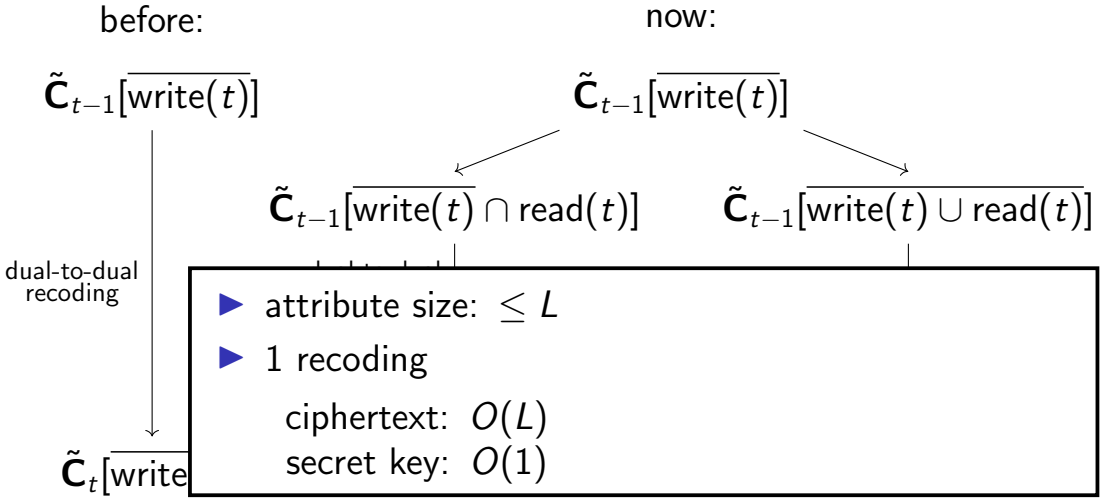
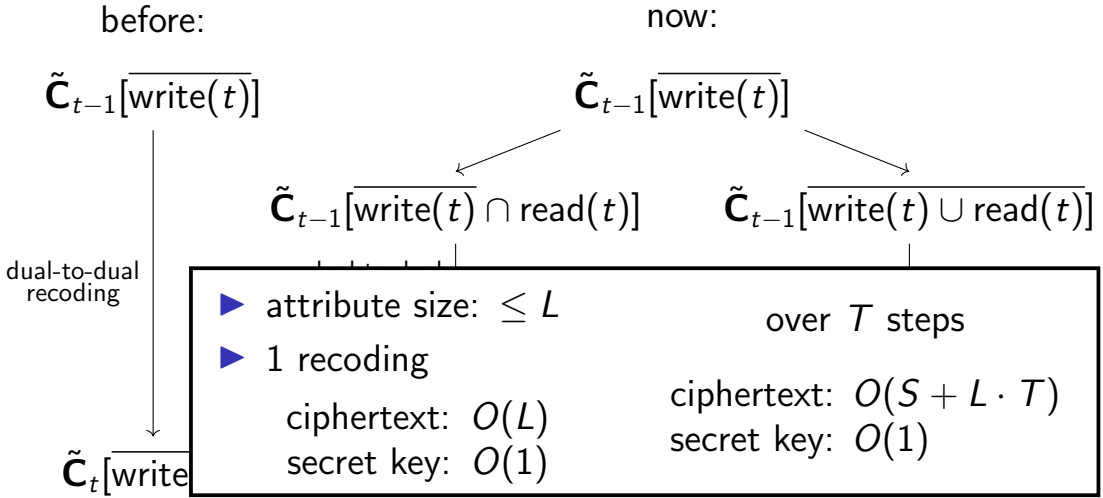


Improving Efficiency (2/2)



Security

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need to show that all the intermediate quantities computed during decryption are jointly pseudorandom

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LWE

What about noise-growth?

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$\Uparrow$ LWE

What about noise-growth?

$\Downarrow$

techniques from [HseihLinLuo23]
to handle unbounded depth computation

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techniques from [HseihLinLuo23] to handle unbounded depth computation

$\Downarrow$

need to use their circular-secure variant of evasive LWE

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need to show that all the intermediate quantities computed during decryption are jointly pseudorandom

$\Uparrow$ LWE

What about noise-growth?

$\Downarrow$

techniques from [HseihLinLuo23] to handle unbounded depth computation

$\Downarrow$

need to use their circular-secure variant of evasive LWE

- construction gets modified accordingly

(need to add circular encryptions)

Conclusion

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Thank you!