TinyLabels: How to Compress Garbled Circuit Input Labels, Efficiently

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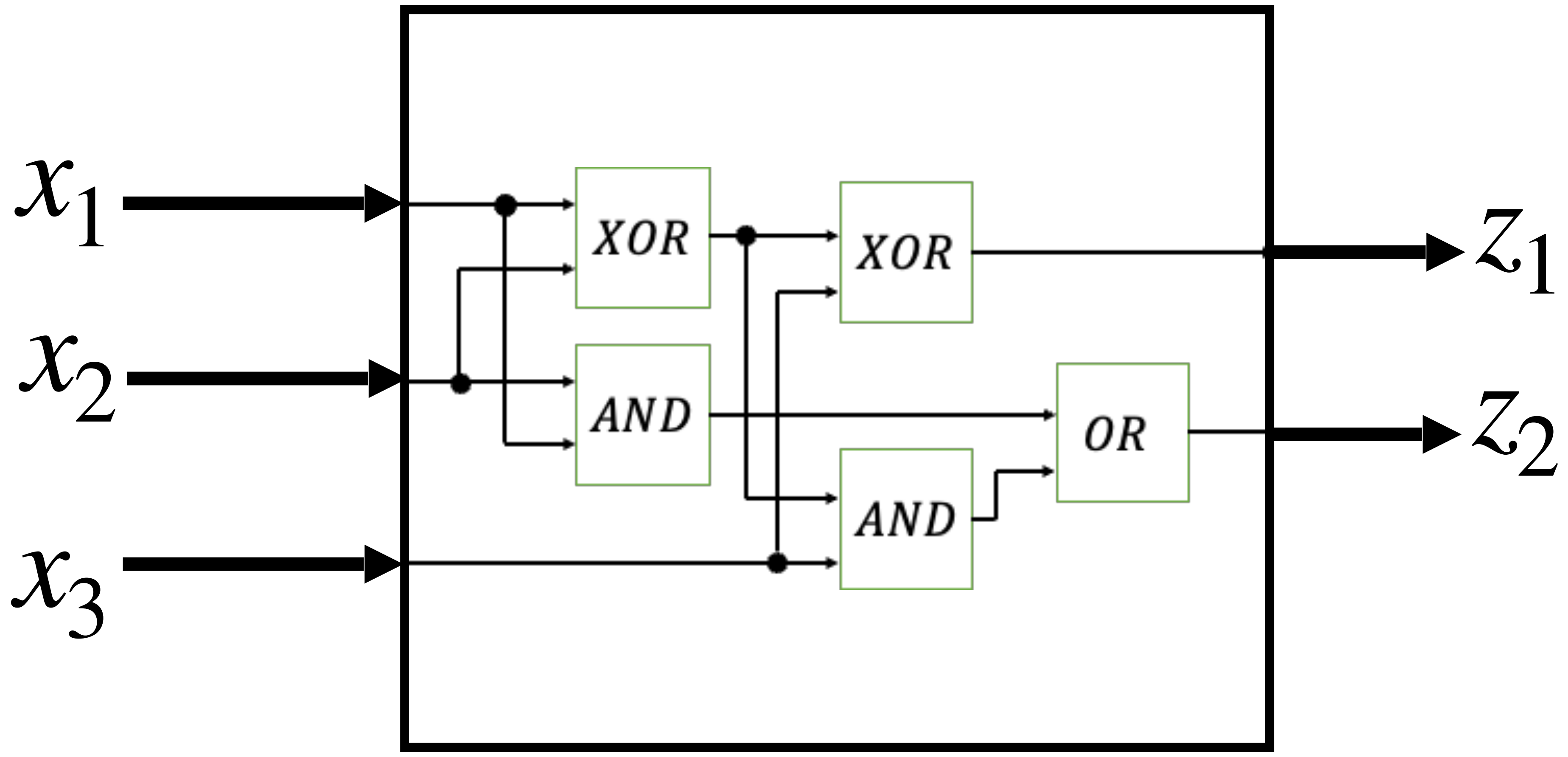
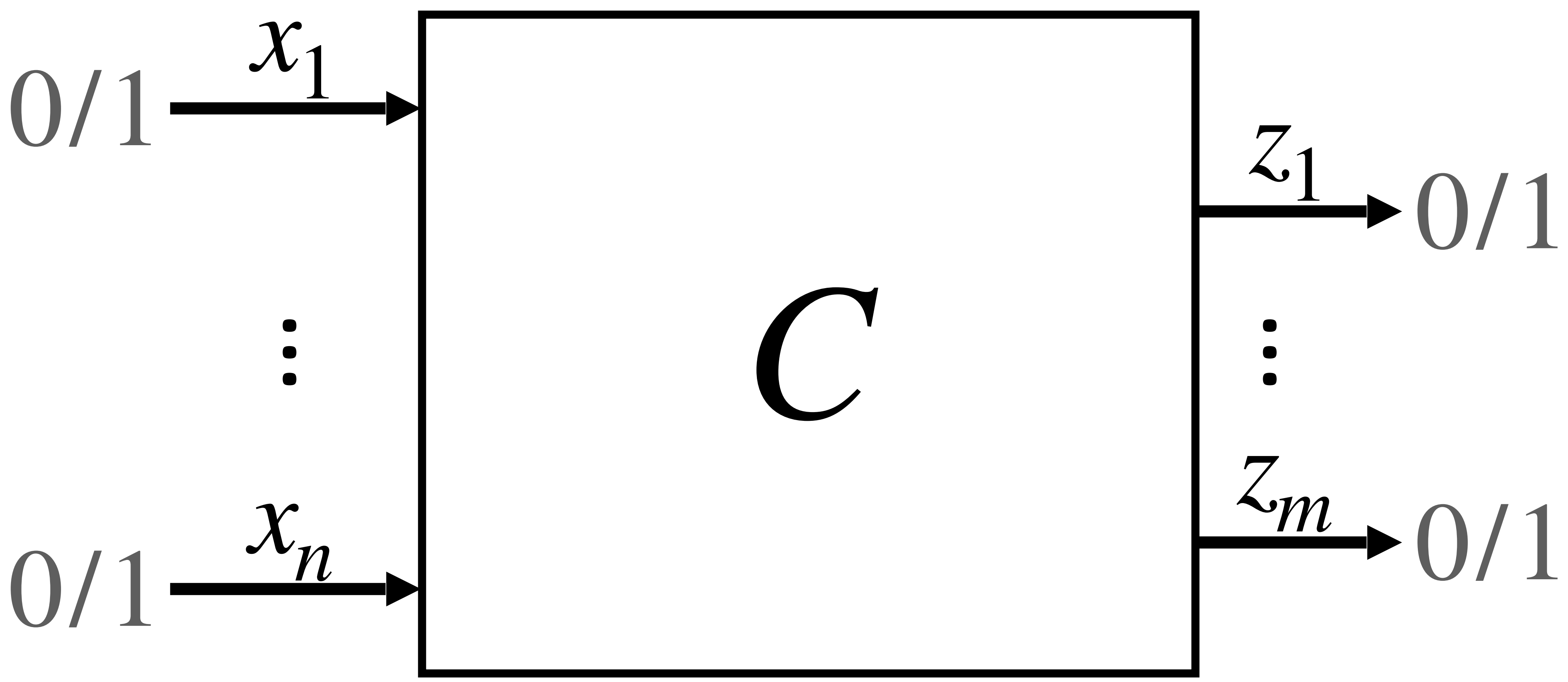
ETH Zurich University of Washington

Slides prepared together with Hanjun Li

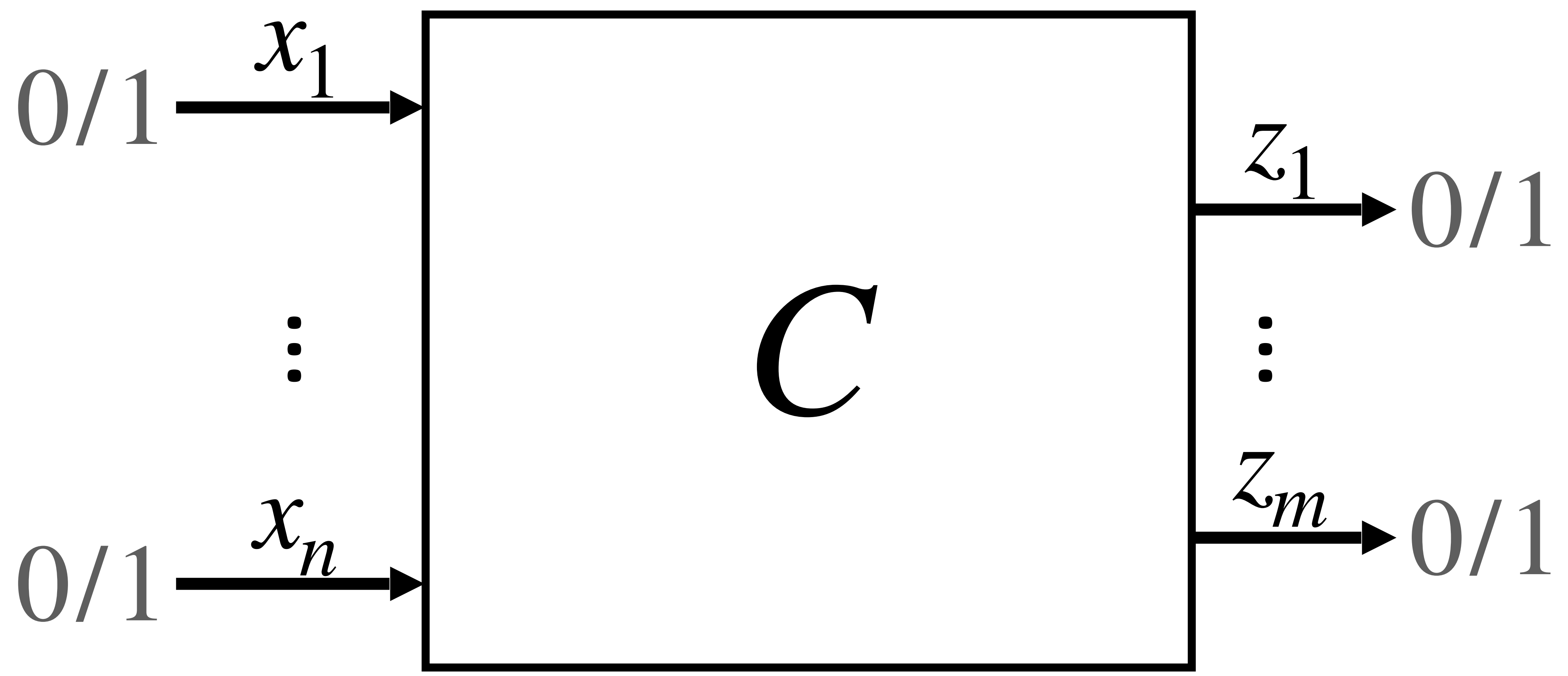
Garbling Scheme



Garbling Scheme

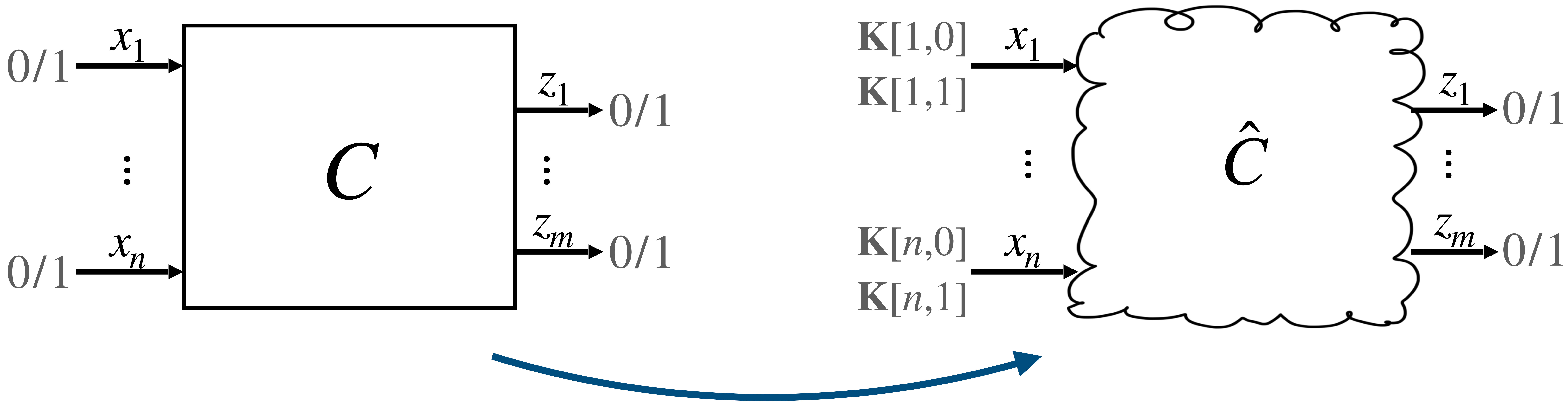


Garbling Scheme



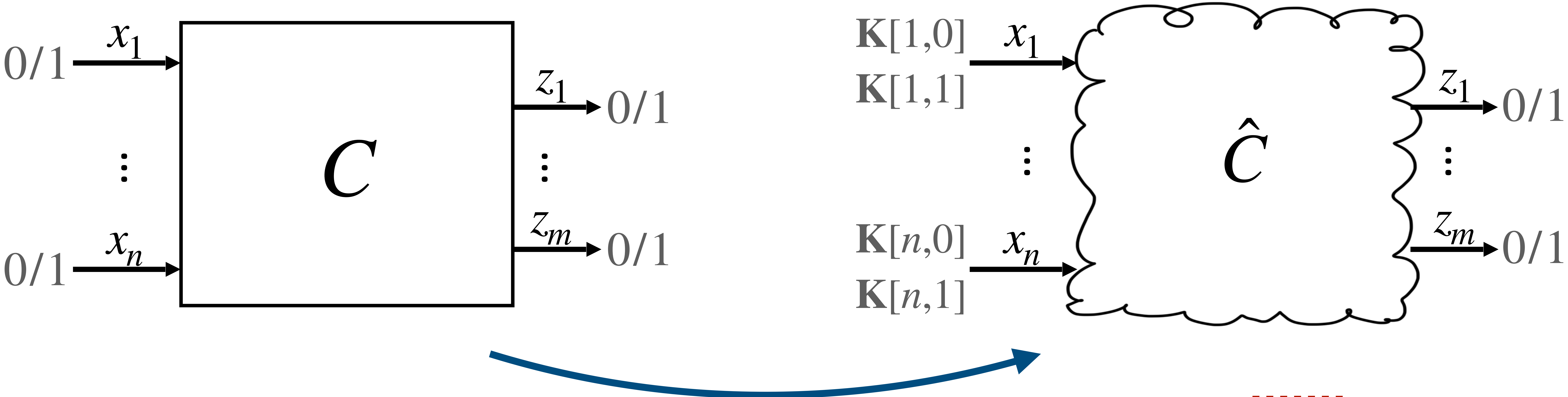
$\textsf{Garble}(C) \rightarrow \hat{C}, \mathbf{K}$

Garbling Scheme



Garble$(C) \to \hat{C}, \mathbf{K}$

Garbling Scheme

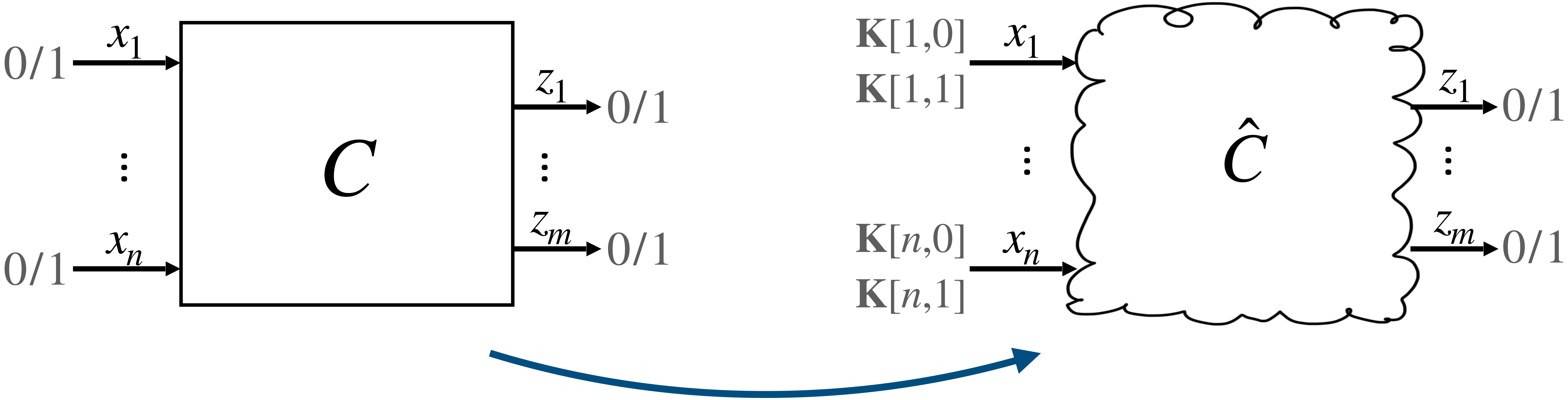


Garble$(C) \to \hat{C}, \mathbf{K}$

Eval$(\hat{C}, \boxed{\mathbf{K}[\mathbf{x}]}) \to C(\mathbf{x})$

Input Labels: One key $\mathbf{K}[i, x_i]$ per input bit

Garbling Scheme

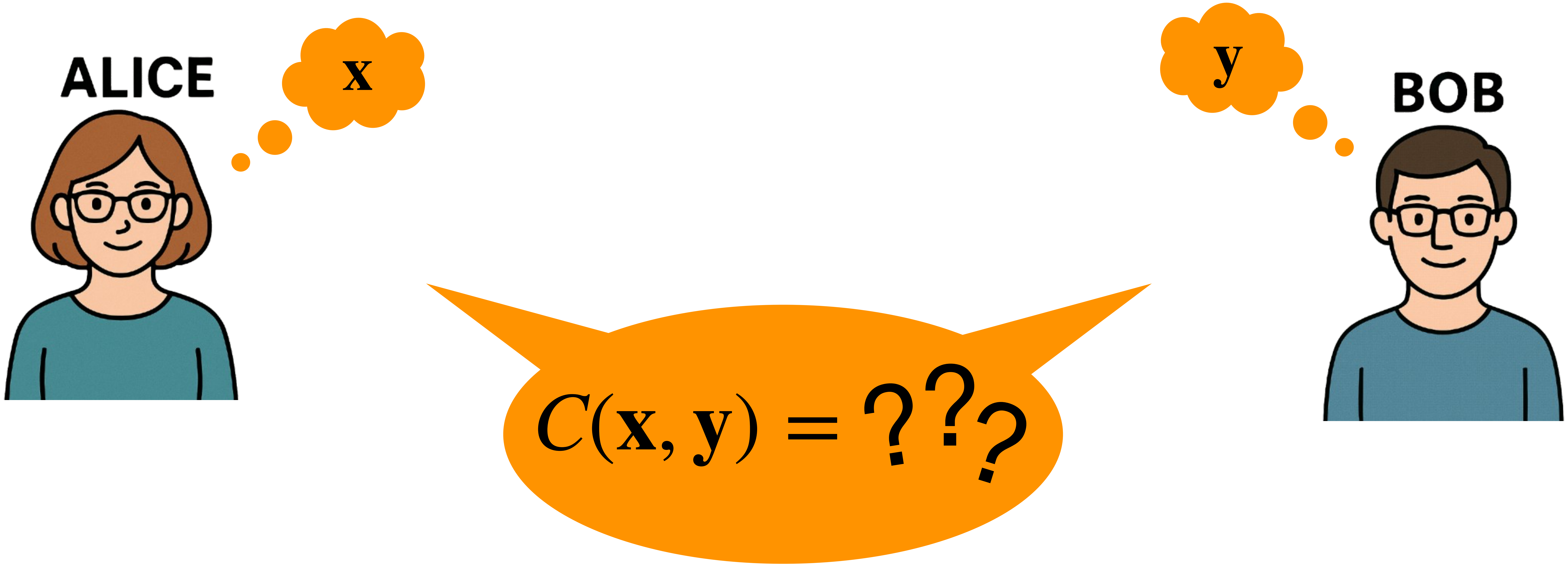


Garble$(C) \rightarrow \hat{C}, \mathbf{K}$ Eval$(\hat{C}, \mathbf{K}[\mathbf{x}]) \rightarrow C(\mathbf{x})$

Simulation Security: Sim$(C(\mathbf{x})) \rightarrow \tilde{C}, L \approx \hat{C}, \mathbf{K}[\mathbf{x}]$

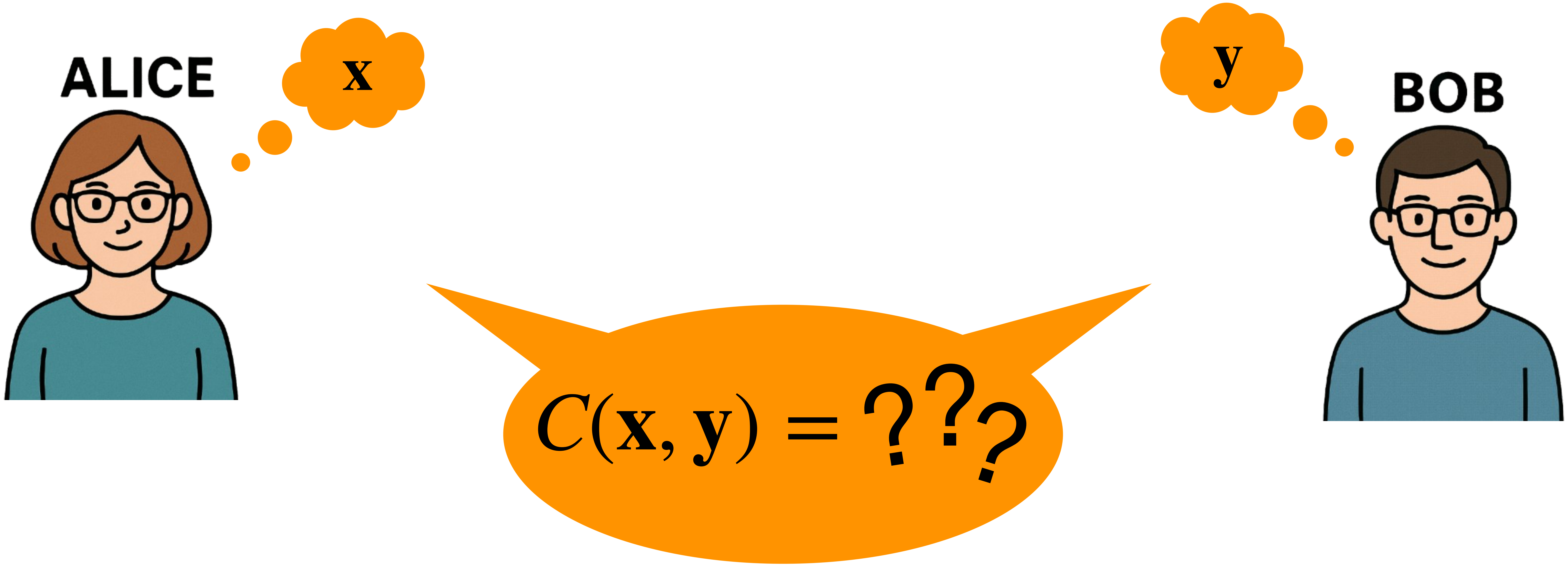
Garbling Schemes: Applications

- **Two-party computation**



Garbling Schemes: Applications

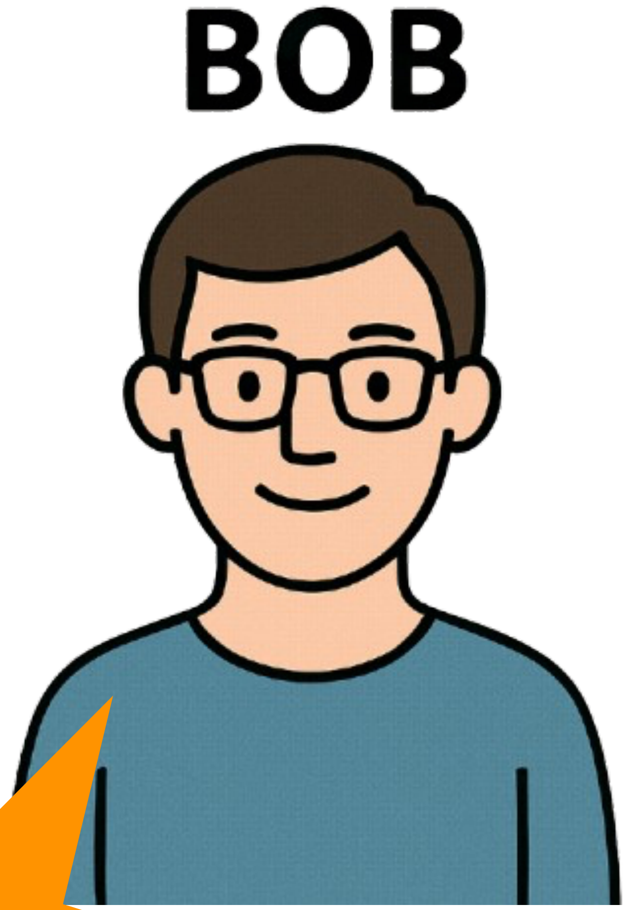
- **Two-party computation**



- **Outsourcing computation** on sensitive data

Application: Outsourcing computation

[AIKW13]



$\mathbf{x}$

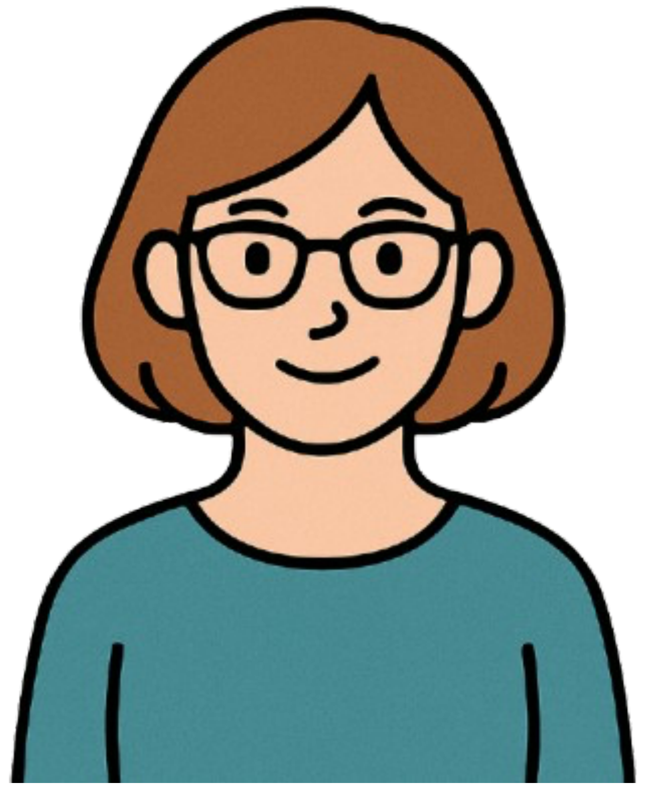
Alice has collected sensitive data $\mathbf{x}$
in a place with *limited resources*

$C(\mathbf{x}) = ???$

Application: Outsourcing computation

[AIKW13]

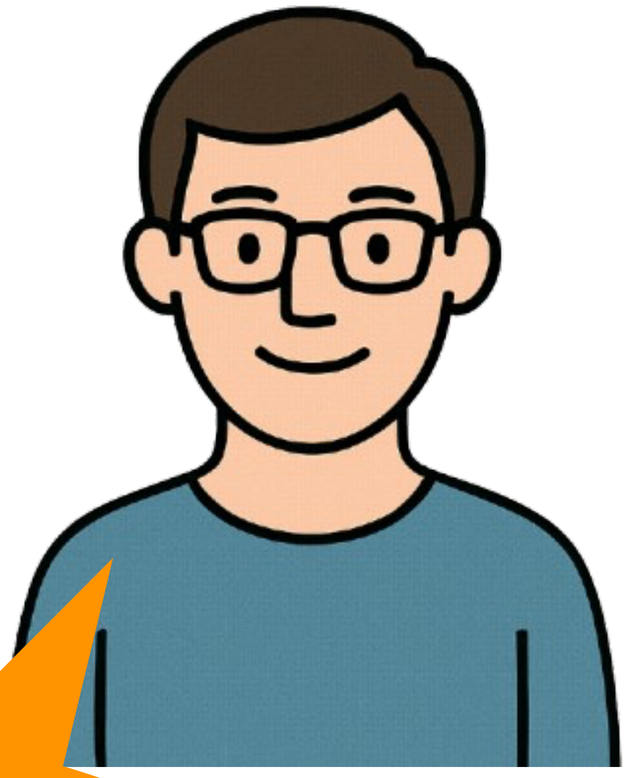
ALICE



Offline (known C)

Alice has large resources

BOB



Online

$\mathbf{x}$

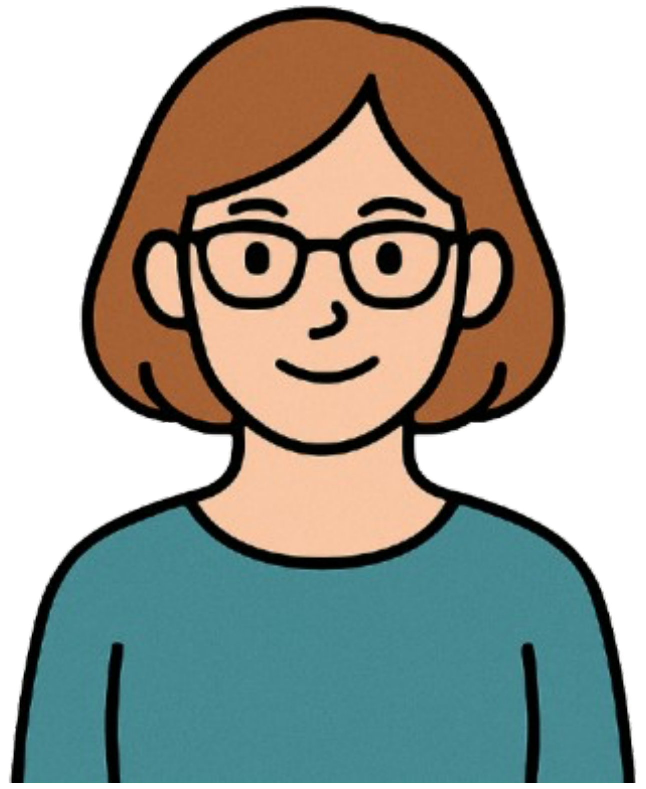
Alice has collected sensitive data $\mathbf{x}$
in a place with *limited resources*

$C(\mathbf{x}) = ???$

Application: Outsourcing computation

[AIKW13]

ALICE

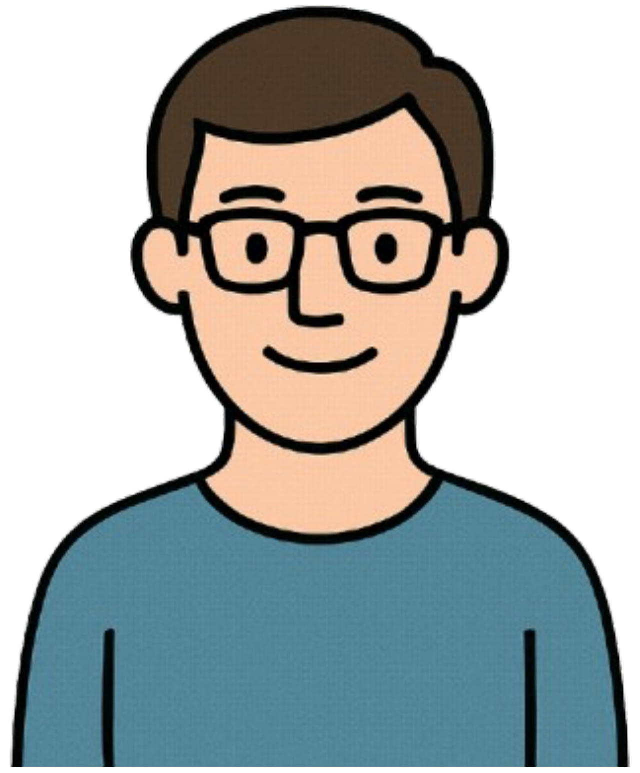


Offline (known C)

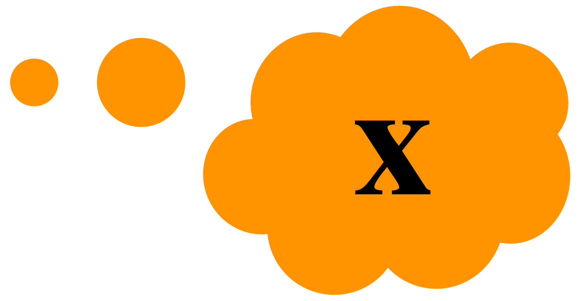
$\mathsf{Garble}(C) \to \hat{C}, \mathbf{K}$

$$\xrightarrow{\hat{C}}$$

BOB



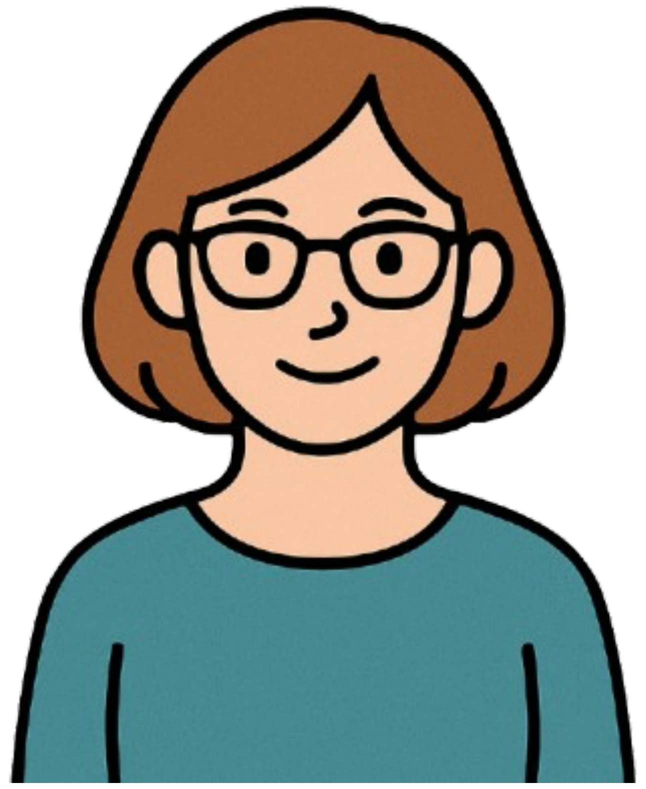
Online



Application: Outsourcing computation

[AIKW13]

ALICE

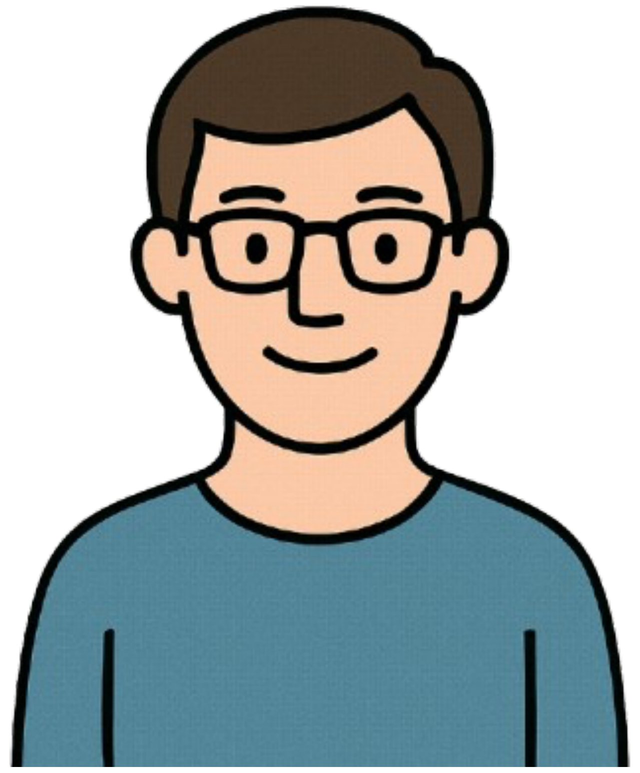


Offline (known C)

Garble(C) $\rightarrow \hat{C}, \mathbf{K}$

$\xrightarrow{\hat{C}}$

BOB



Online

$\mathbf{x}$

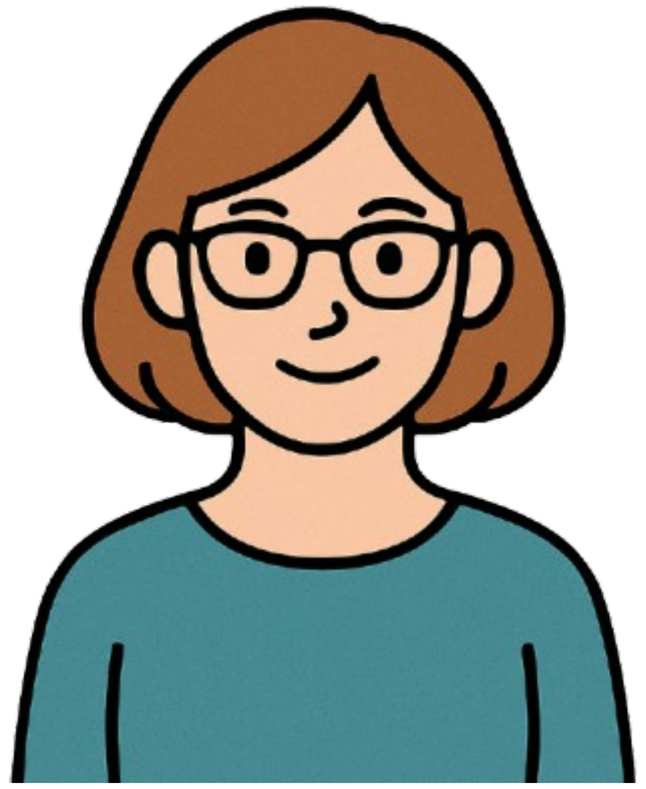
$\xrightarrow{\mathbf{L} = \mathbf{K}[\mathbf{x}]}$

Eval$(\hat{C}, \mathbf{L})$ $\rightarrow C(\mathbf{x})$

Application: Outsourcing computation

[AIKW13]

ALICE

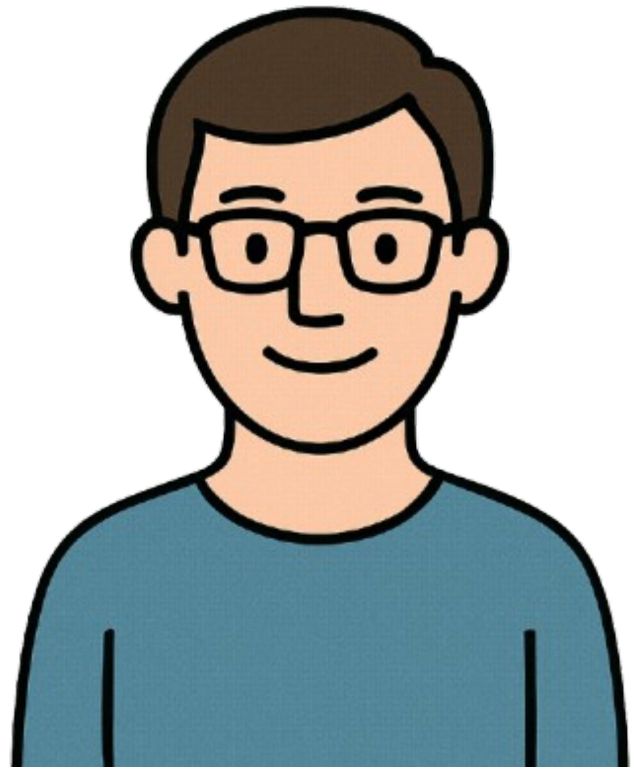


Offline (known C)

$\mathsf{Garble}(C) \to \hat{C}, \mathbf{K}$

$\xrightarrow{\hat{C}}$

BOB



Online

$\mathbf{x}$

$\xrightarrow{\mathbf{L} = \mathbf{K}[\mathbf{x}]}$

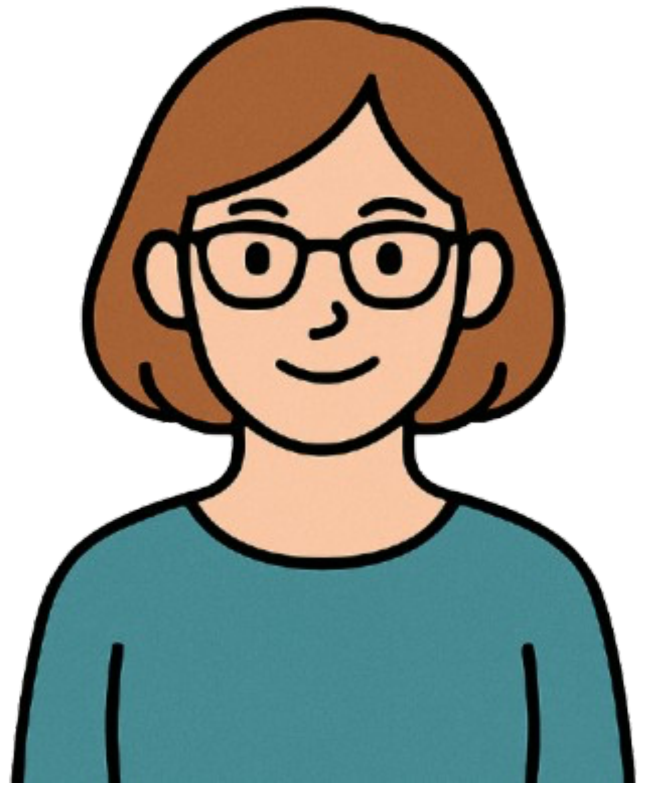
Size: $|\mathbf{x}| \cdot \lambda$

$\mathsf{Eval}(\hat{C}, \mathbf{L}) \to C(\mathbf{x})$

Application: Outsourcing computation

[AIKW13]

ALICE



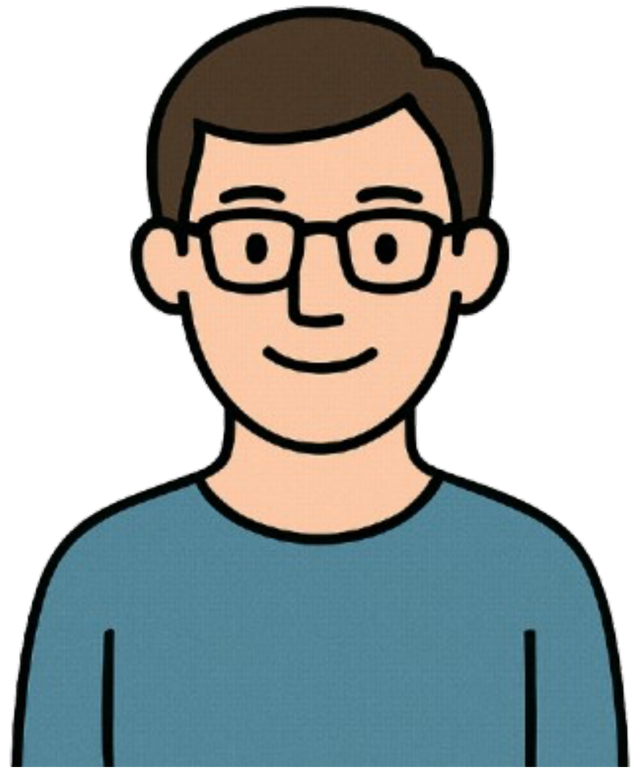
Offline (known C)

Garble(C)

$\rightarrow \hat{C}, \mathbf{K}$

$\xrightarrow{\hat{C}}$

BOB



Online

Compress$(\mathbf{K}[\mathbf{x}]), \bar{\mathbf{x}}$ $\longrightarrow$

Decompress $\rightarrow \mathbf{L}$

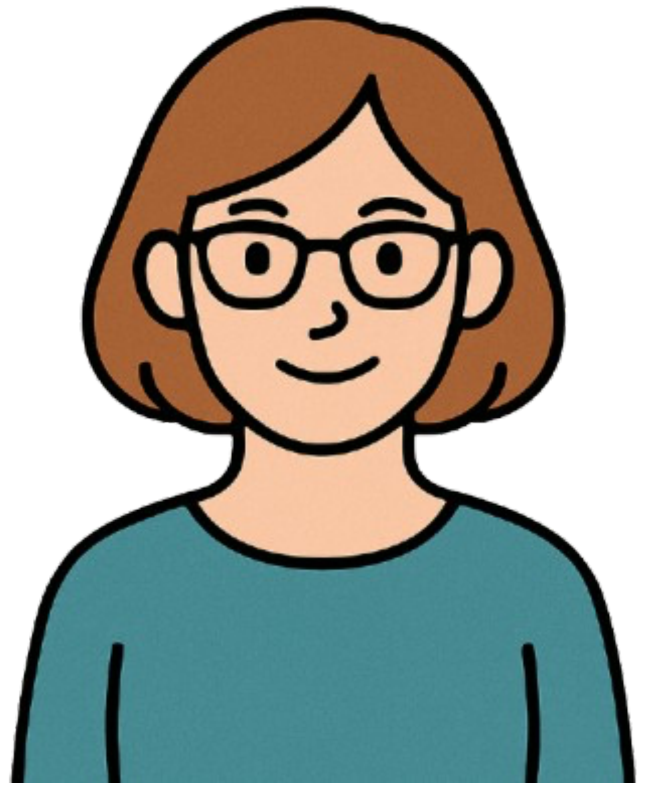
Eval$(\hat{C}, \mathbf{L})$

$\rightarrow C(\mathbf{x})$

Application: Outsourcing computation

[AIKW13]

ALICE



Offline (known C)

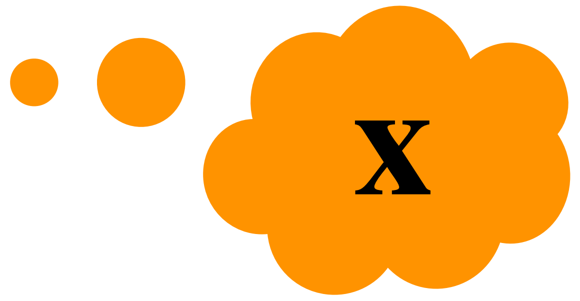
Garble(C)

$\rightarrow \hat{C}, \mathbf{K}$

$\hat{C}$

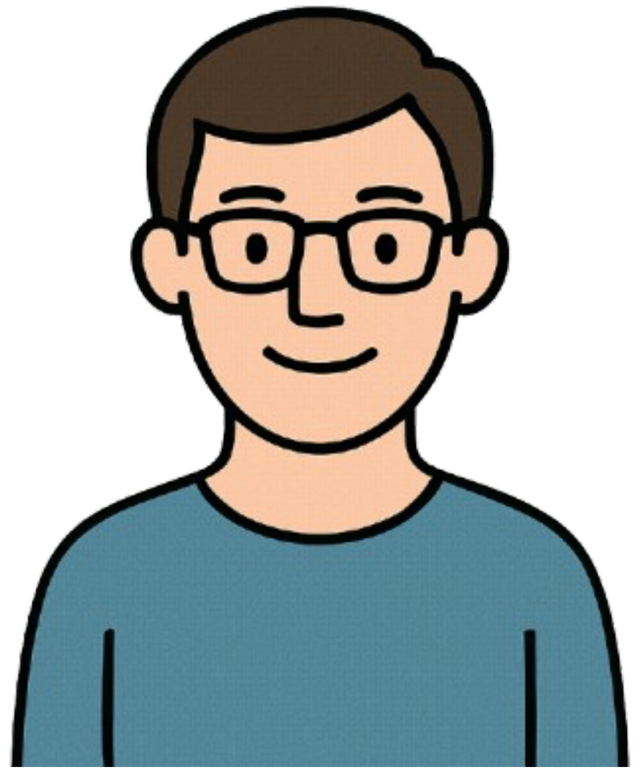
$CT(\mathbf{K})$

Online



Compress$(\mathbf{K}[\mathbf{x}]), \bar{\mathbf{x}}$

BOB



Decompress $\rightarrow \mathbf{L}$

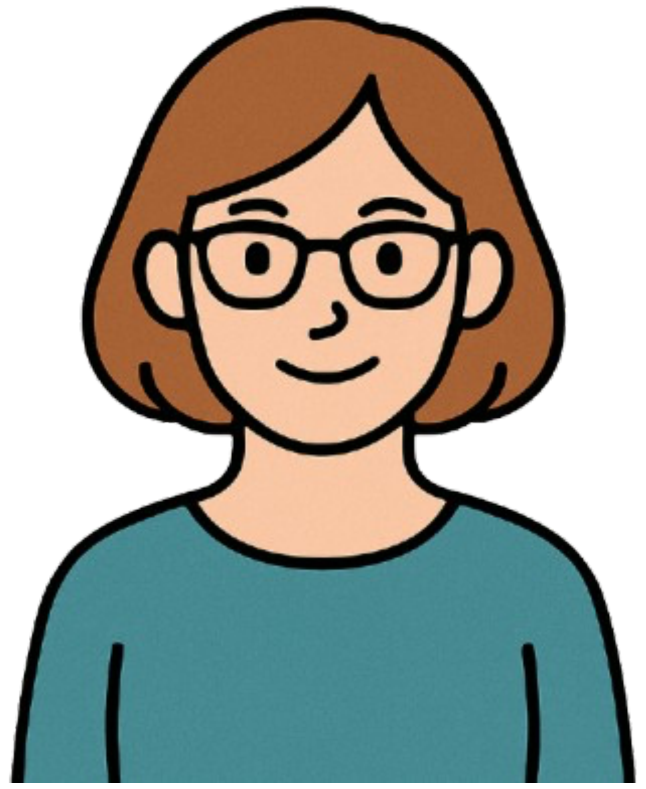
Eval$(\hat{C}, \mathbf{L})$

$\rightarrow C(\mathbf{x})$

Application: Outsourcing computation

[AIKW13]

ALICE



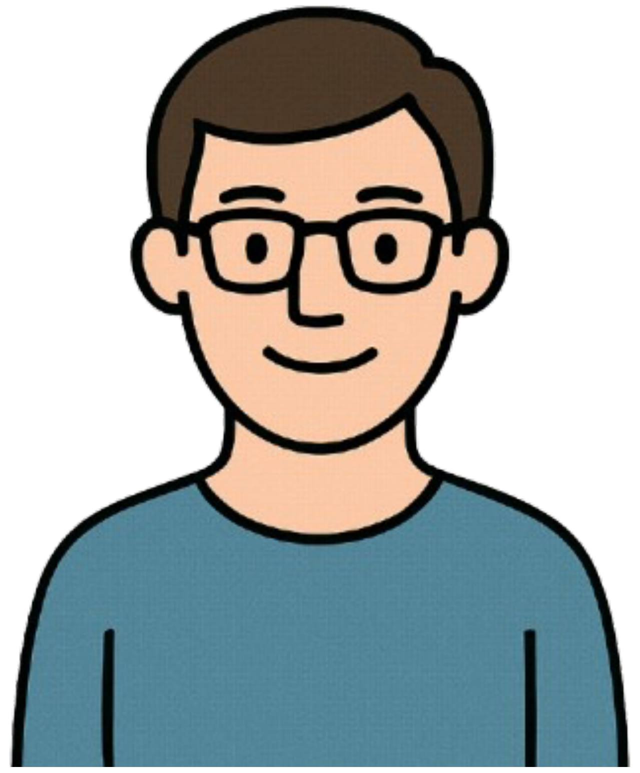
Offline (known C)

Garble(C)
$\rightarrow \hat{C}, \mathbf{K}$

$\hat{C}$

$CT(\mathbf{K})$

BOB



Online

Compress$(\mathbf{K}[\mathbf{x}]), \bar{\mathbf{x}}$

Size: $\mathrm{poly}(\lambda) + |\mathbf{x}|$

Decompress $\rightarrow \mathbf{L}$

Eval$(\hat{C}, \mathbf{L})$
$\rightarrow C(\mathbf{x})$

Existing Techniques

	Assumption	Communication Offline	Online
Naive		0	$\lvert \mathbf{x} \rvert \cdot \lambda$
[AIKW13]	RSA	$\lvert \mathbf{x} \rvert \cdot \mathrm{poly}(\lambda)$	$\lvert \mathbf{x} \rvert + \log N$
	DDH/LWE	"	$\lvert \mathbf{x} \rvert + \lvert \mathbf{x} \rvert / \lambda$

N = RSA modulus

Existing Techniques

	Assumption	Communication Offline	Communication Online
Naive		0	$\lvert\mathbf{x}\rvert \cdot \lambda$
[AIKW13]	RSA	$\lvert\mathbf{x}\rvert \cdot \mathrm{poly}(\lambda)$	$\lvert\mathbf{x}\rvert + \log N$
	DDH/LWE	"	$\lvert\mathbf{x}\rvert + \lvert\mathbf{x}\rvert/\lambda$

[GS18,GOS18]: **weaker assumptions** (factoring / CDH), but **non-black-box** cryptography

N = RSA modulus

Existing Techniques

	Assumption	Communication Offline	Communication Online
Naive		0	$\lvert \mathbf{x} \rvert \cdot \lambda$
[AIKW13]	RSA	$\lvert \mathbf{x} \rvert \cdot \mathrm{poly}(\lambda)$	$\lvert \mathbf{x} \rvert + \log N$
	DDH/LWE	"	$\lvert \mathbf{x} \rvert + \lvert \mathbf{x} \rvert / \lambda$
[ABIKLV23]	RSA/iO+SSBH	0	$\lvert \mathbf{x} \rvert + \log N$
	Bilinear DDH	0*	$\lvert \mathbf{x} \rvert + \lvert \mathbf{x} \rvert / \lambda$

N = RSA modulus

Existing Techniques

	Assumption	Communication Offline	Communication Online	Computation (Decompress)
Naive		0	$\lvert \mathbf{x} \rvert \cdot \lambda$	0
[AIKW13]	RSA DDH/LWE	$\lvert \mathbf{x} \rvert \cdot \mathrm{poly}(\lambda)$	$\lvert \mathbf{x} \rvert + \log N$	$\lvert \mathbf{x} \rvert \log \lvert \mathbf{x} \rvert$ RSA exponentiations
[ABIKLV23]	RSA/iO+SSBH Bilinear DDH	0	$\lvert \mathbf{x} \rvert + \log N$	$\lvert \mathbf{x} \rvert \log \lvert \mathbf{x} \rvert$ RSA exponentiations

N = RSA modulus

Existing Techniques

	Assumption	Communication Offline	Communication Online	Computation (Decompress)
Naive		0	$\lvert\mathbf{x}\rvert \cdot \lambda$	0
[AIKW13]	RSA DDH/LWE	$\lvert\mathbf{x}\rvert \cdot \mathrm{poly}(\lambda)$	$\lvert\mathbf{x}\rvert + \log N$	$\lvert\mathbf{x}\rvert \log \lvert\mathbf{x}\rvert$ RSA exponentiations 30 min
[ABIKLV23]	RSA/iO+SSBH Bilinear DDH	0	$\lvert\mathbf{x}\rvert + \log N$	$\lvert\mathbf{x}\rvert \log \lvert\mathbf{x}\rvert$ RSA exponentiations

(for $\lvert\mathbf{x}\rvert = 10^5$)

N = RSA modulus

Existing Techniques

* assuming 1Mbps

	Assumption	Communication		Computation
		Offline	Online	(Decompress)
Naive		0	13 sec* $\vert \mathbf{x} \vert$	0
[AIKW13]	RSA DDH/LWE	$\vert \mathbf{x} \vert \cdot \mathrm{poly}(\lambda)$	$\vert \mathbf{x} \vert + \log N$	$\vert \mathbf{x} \vert \log \vert \mathbf{x} \vert$ RSA exponentiations 30 min
[ABIKLV23]	RSA/iO+SSBH Bilinear DDH	0	$\vert \mathbf{x} \vert + \log N$	$\vert \mathbf{x} \vert \log \vert \mathbf{x} \vert$ RSA exponentiations

(for $|\mathbf{x}| = 10^5$)

N = RSA modulus

Our work — TinyLabels

	Assumption	Communication Offline	Communication Online	Computation (Decompress)
Naive		0	$\|\mathbf{x}\| \cdot \lambda$	0
[AIKW13]	RSA DDH/LWE	$\|\mathbf{x}\| \cdot \mathrm{poly}(\lambda)$	$\|\mathbf{x}\| + \log N$	$\|\mathbf{x}\| \log \|\mathbf{x}\|$ RSA exponentiations
[ABIKLV23]	RSA/iO+SSBH Bilinear DDH	0	$\|\mathbf{x}\| + \log N$	$\|\mathbf{x}\| \log \|\mathbf{x}\|$ RSA exponentiations
TinyLabels	RingLWE+RO	0*	$\|\mathbf{x}\| + \|\mathscr{R}_q\|$	$\|\mathbf{x}\| \log \|\mathbf{x}\| / n$ $\mathscr{R}_q$ multiplications

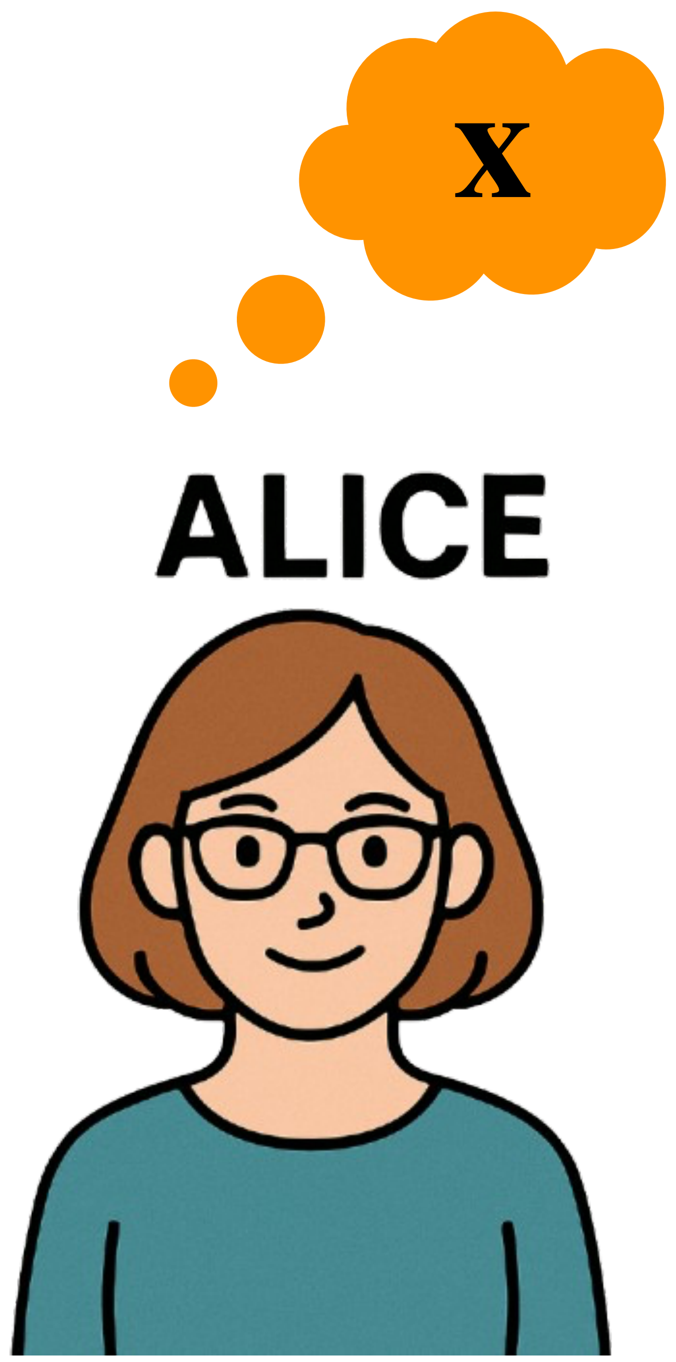
N = RSA modulus $\quad\quad$ $\mathscr{R}_q$ = ring with degree n and modulus q

Our work — TinyLabels

	Assumption	Communication: Offline	Communication: Online	Computation (Decompress)
Naive		0	$\lvert \mathbf{x} \rvert$ 13 sec*	0
[AIKW13]	RSA DDH/LWE	$\lvert \mathbf{x} \rvert \cdot \mathrm{poly}(\lambda)$	$\lvert \mathbf{x} \rvert + \log N$	$\lvert \mathbf{x} \rvert \log \lvert \mathbf{x} \rvert$ RSA exponentiations 30 min
[ABIKLV23]	RSA/iO+SSBH Bilinear DDH	0	$\lvert \mathbf{x} \rvert + \log N$	$\lvert \mathbf{x} \rvert \log \lvert \mathbf{x} \rvert$ RSA exponentiations
TinyLabels	RingLWE+RO	0*	$\lvert \mathbf{x} \rvert + \lvert \mathscr{R}_q \rvert$	$\lvert \mathbf{x} \rvert \log \lvert \mathbf{x} \rvert / n$ $\mathscr{R}_q$ multiplications 0.2 sec

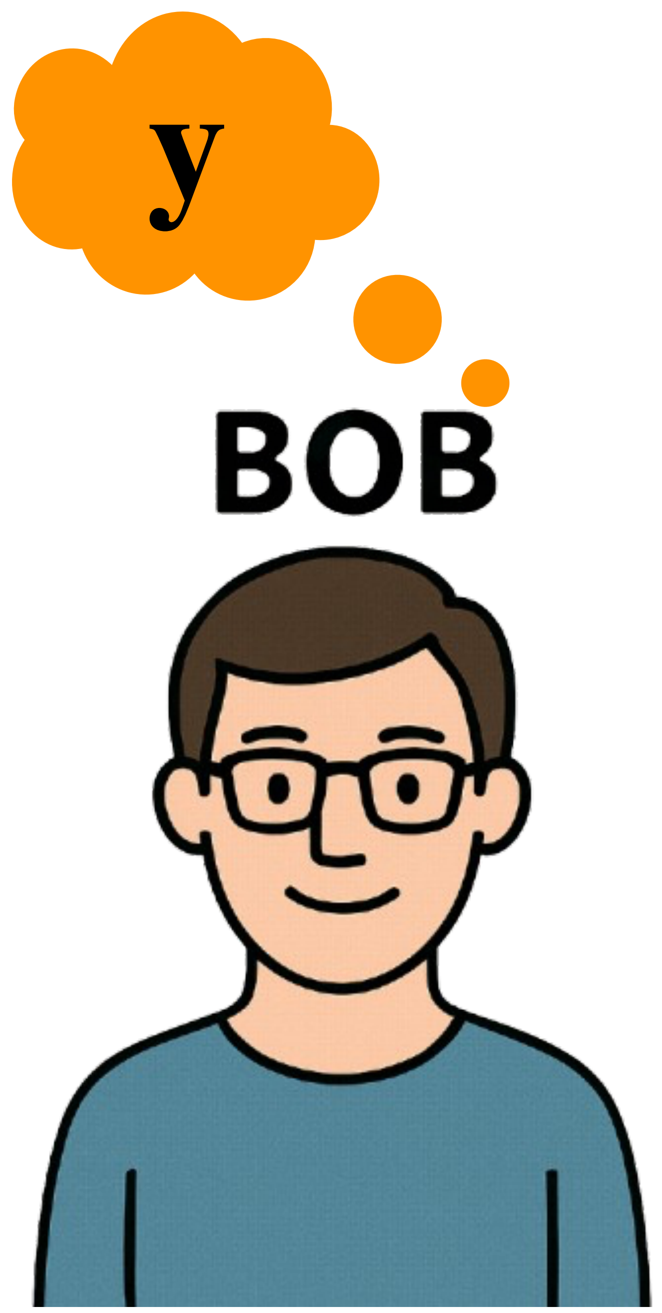
N = RSA modulus $\mathscr{R}_q$ = ring with degree n and modulus q

Application: Garbling with preprocessing

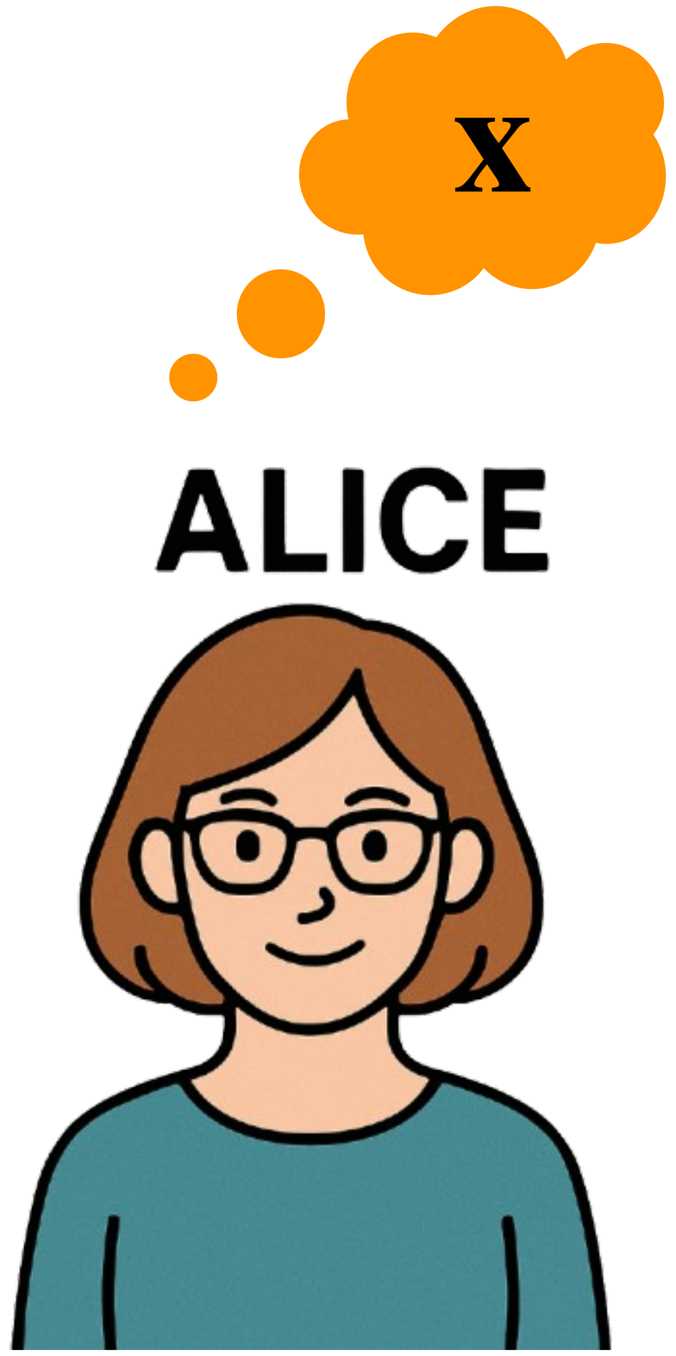


Let’s analyze our joint data securely!

Okay, but using which circuit C?



Application: Garbling with preprocessing



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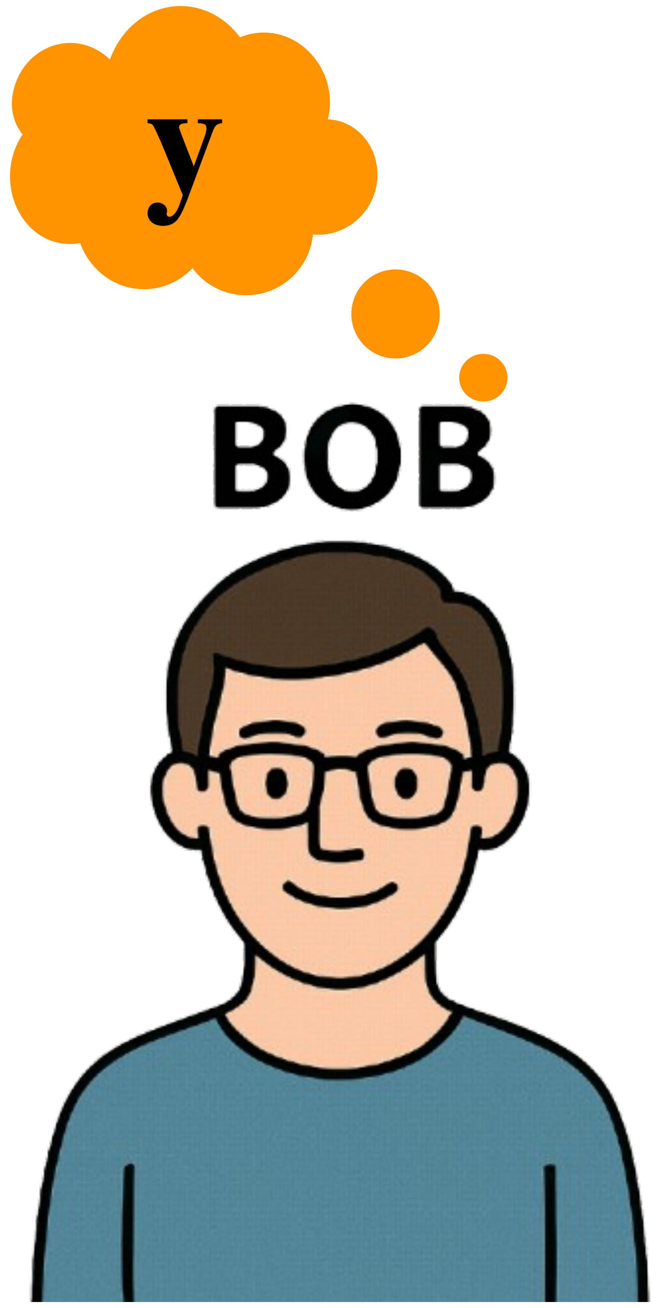
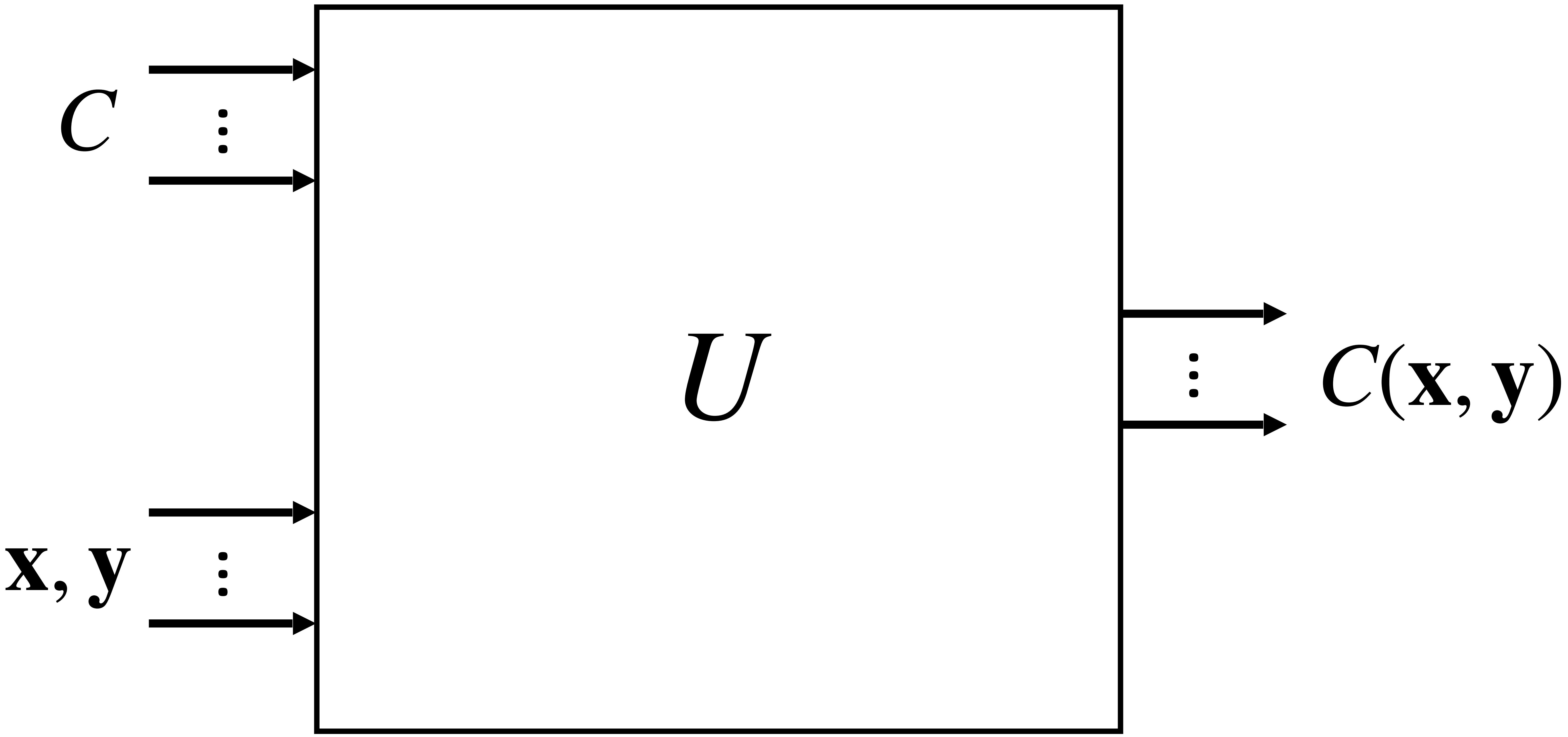


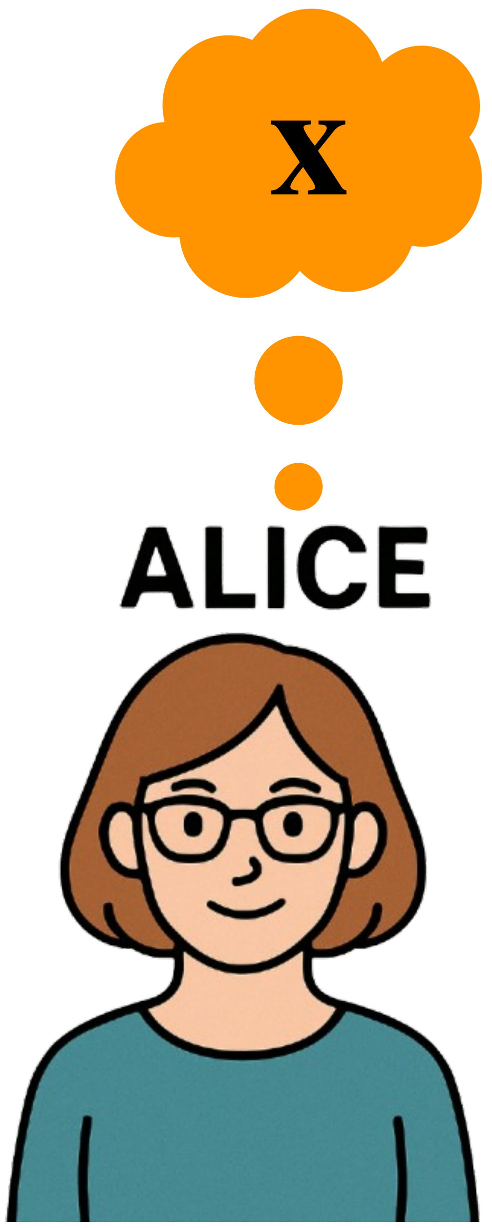
Let's discuss. In the meantime, we should preprocess with TinyLabels.

Universal Circuits



[ZYZL19]: $|U| \approx 4.5\,|C| \log |C|$ AND gates

Application: Garbling with preprocessing

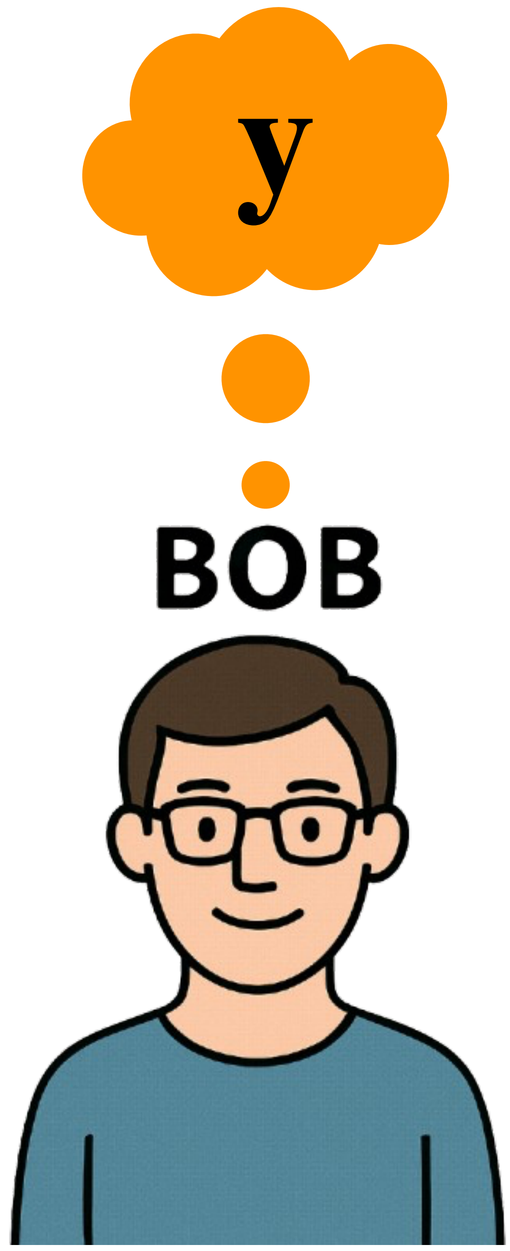


Garble(C)

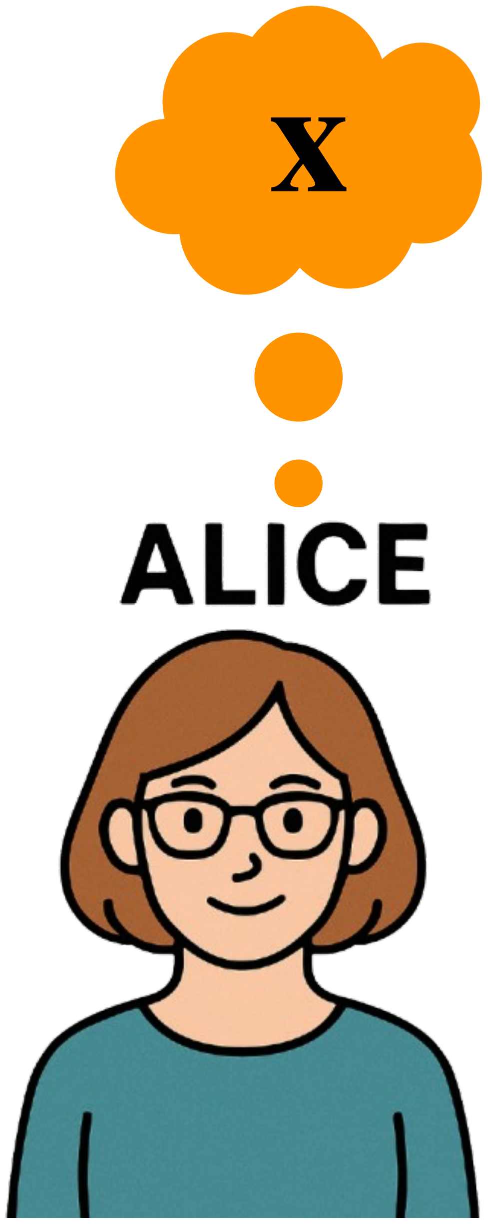
$\rightarrow \hat{C}, \mathbf{K}$

$\xrightarrow{\hat{C}}$

Size: $O(|C| \cdot \lambda)$



Application: Garbling with preprocessing

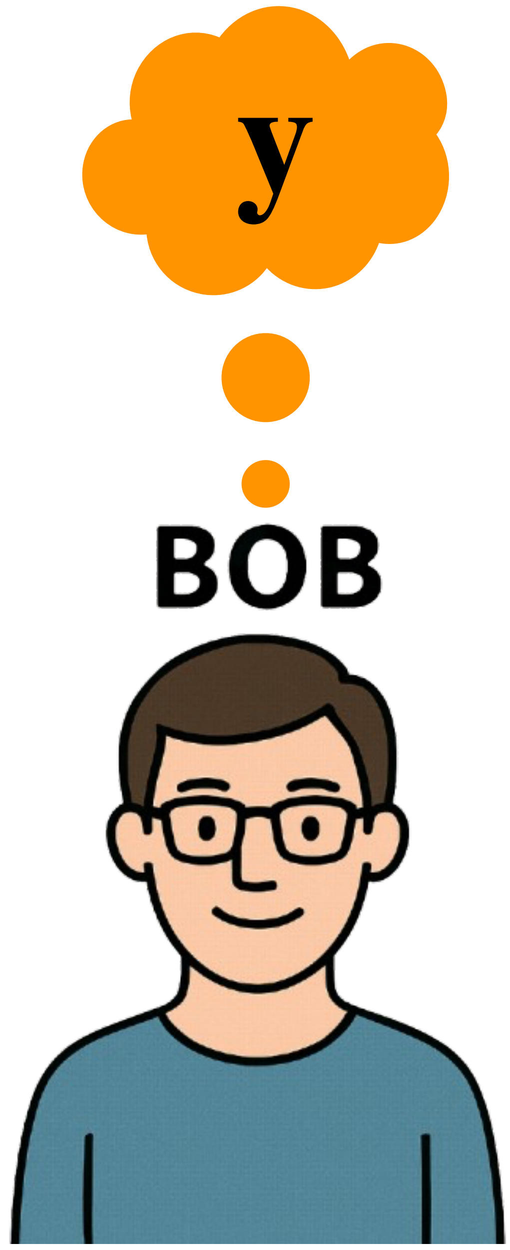


$\mathsf{Garble}(C)$

$\rightarrow \hat{C}, \mathbf{K}$

$\xrightarrow{\hat{C}}$

Size: $O(|C| \cdot \lambda)$



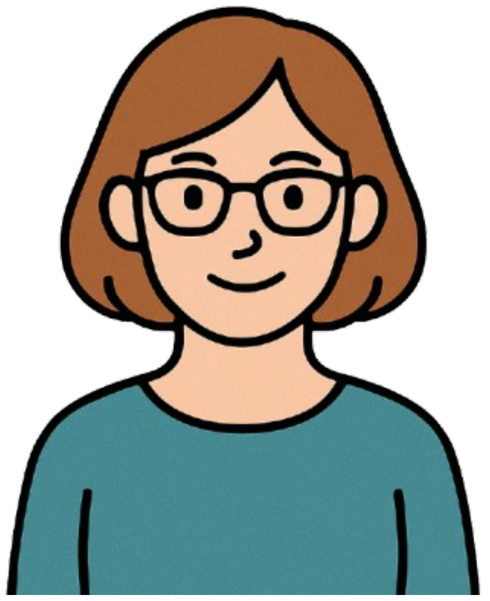
Transferring $\mathbf{K}^x[\mathbf{x}], \mathbf{K}^y[\mathbf{y}]$

Size: $|\mathbf{x}| + |\mathbf{y}| \cdot \mathrm{poly}(\lambda)$

Application: Garbling with preprocessing

$\mathbf{x}$

ALICE



Garble(C)

$\rightarrow \hat{C}, \mathbf{K}$

$\hat{C}$

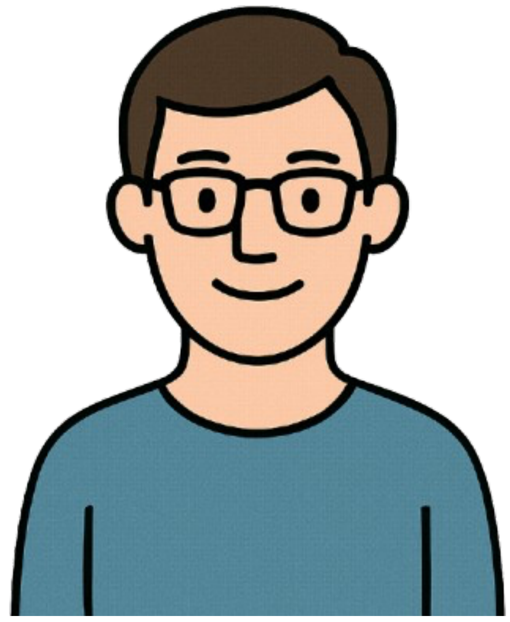
Size: $O(|C| \cdot \lambda)$

Transferring $\mathbf{K}^x[\mathbf{x}], \mathbf{K}^y[\mathbf{y}]$

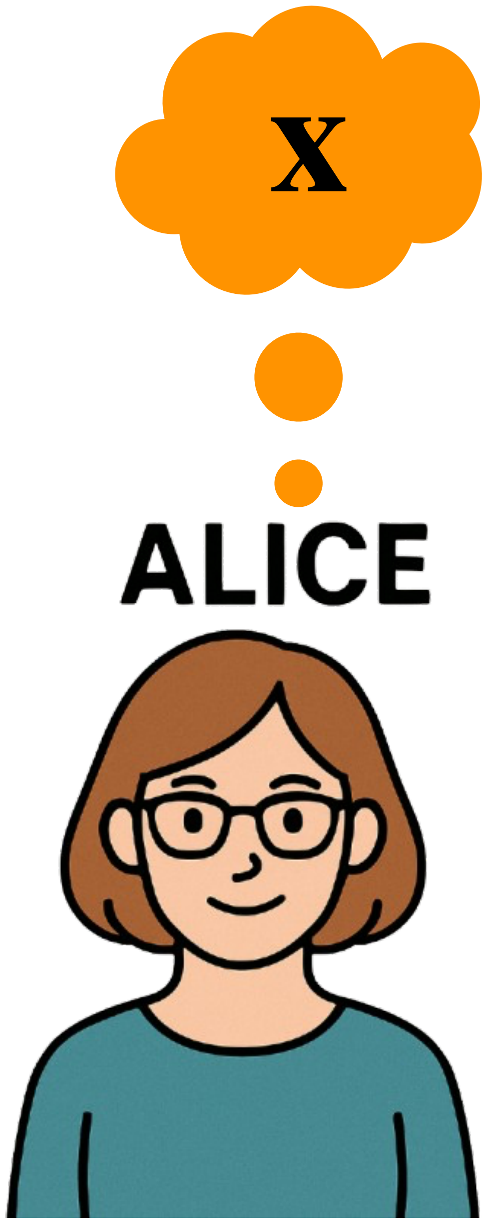
Size: $|\mathbf{x}| + |\mathbf{y}| \cdot \mathrm{poly}(\lambda)$

$\mathbf{y}$

BOB



Application: Garbling with preprocessing



Offline

Garble(U)

$\rightarrow \hat{U}, (\mathbf{K}^C, \mathbf{K}^x, \mathbf{K}^y)$

$\xrightarrow{\hat{U}}$

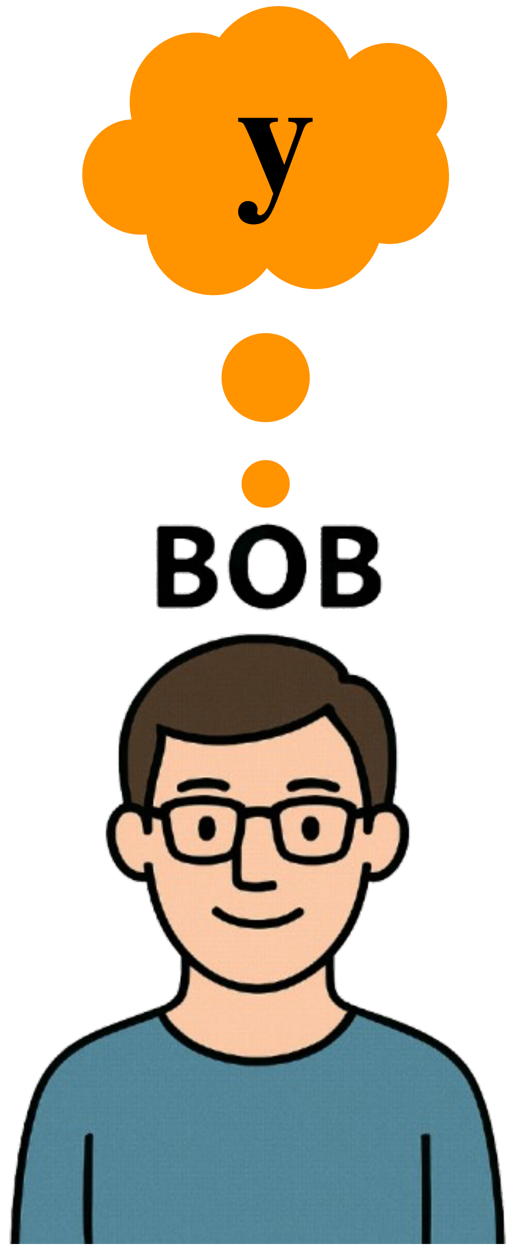
Online (C known)

$\xrightarrow{\text{Compress}(\mathbf{K}^C[C])}$

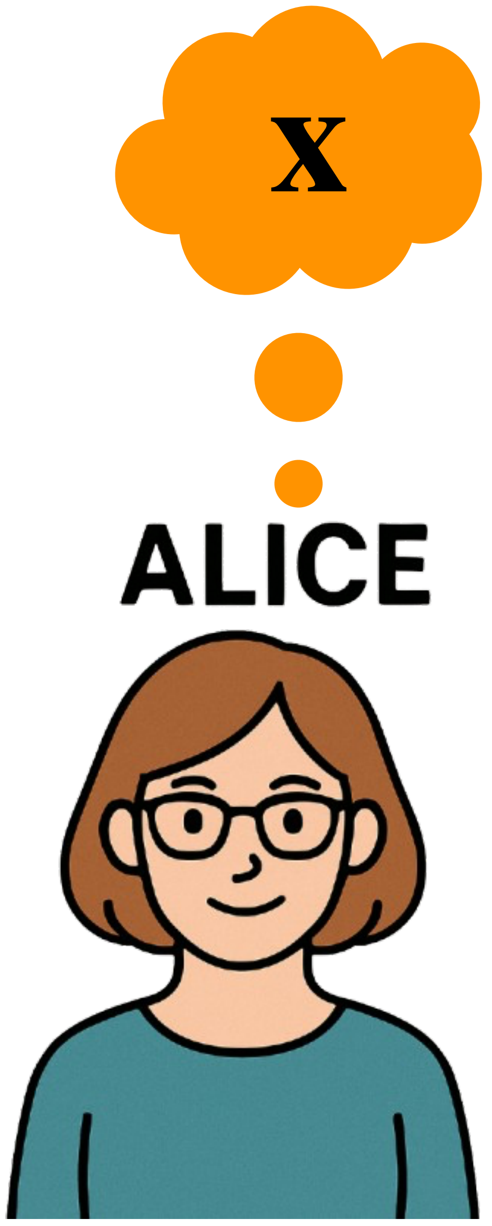
Decompress $\rightarrow \mathbf{L}^C$

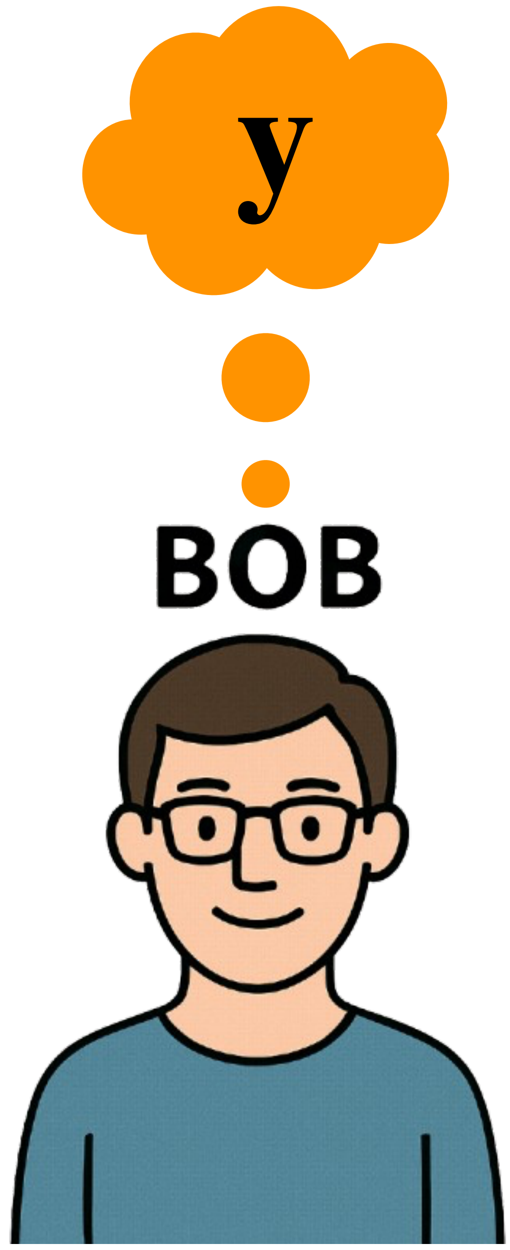
Transferring $\mathbf{K}^x[\mathbf{x}], \mathbf{K}^y[\mathbf{y}]$

Size: $|\mathbf{x}| + |\mathbf{y}| \cdot \mathrm{poly}(\lambda)$



Application: Garbling with preprocessing





Offline

Garble(U)

$\to \hat{U}, (\mathbf{K}^C, \mathbf{K}^x, \mathbf{K}^y)$

$\xrightarrow{\hat{U}}$

Online (C known)

Size: $\mathrm{poly}(\lambda)$

Compress$(\mathbf{K}^C[C])$ $\longrightarrow$

Decompress $\to \mathbf{L}^C$

Transferring $\mathbf{K}^x[\mathbf{x}], \mathbf{K}^y[\mathbf{y}]$

Size: $|\mathbf{x}| + |\mathbf{y}| \cdot \mathrm{poly}(\lambda)$

Main Tool: Batch-Select

<u>Sender</u>

Message vectors

$\mathbf{m}_1, \mathbf{m}_2 \in \mathbb{Z}_p^w$

$\mathrm{Enc}_1(\mathbf{m}_1) \to \mathrm{st}_1, \mathrm{ct}_1$

$\mathrm{Enc}_2(\mathbf{m}_2) \to \mathrm{st}_2, \mathrm{ct}_2$

Main Tool: Batch-Select

Sender

Message vectors

$\mathbf{m}_1, \mathbf{m}_2 \in \mathbb{Z}_p^w$

$$\mathrm{Enc}_1(\mathbf{m}_1) \to \mathrm{st}_1, \mathrm{ct}_1$$

$$\mathrm{Enc}_2(\mathbf{m}_2) \to \mathrm{st}_2, \mathrm{ct}_2$$

"Selection" vector

$\mathbf{x} \in \mathbb{Z}_p^w$

$$\mathrm{KeyGen}(\mathrm{st}_1, \mathrm{st}_2, \mathbf{x}) \to \mathrm{sk}_{\mathbf{x}}$$

Main Tool: Batch-Select

Message vectors

$\mathbf{m}_1, \mathbf{m}_2 \in \mathbb{Z}_p^w$

"Selection" vector

$\mathbf{x} \in \mathbb{Z}_p^w$

<u>Sender</u>

$\mathrm{Enc}_1(\mathbf{m}_1) \to \mathrm{st}_1, \mathrm{ct}_1$

$\mathrm{Enc}_2(\mathbf{m}_2) \to \mathrm{st}_2, \mathrm{ct}_2$

$\mathrm{KeyGen}(\mathrm{st}_1, \mathrm{st}_2, \mathbf{x}) \to \mathrm{sk}_{\mathbf{x}}$

<u>Receiver</u>

$\mathrm{Dec}(\mathrm{sk}_{\mathbf{x}}, \mathrm{ct}_1, \mathrm{ct}_2, \mathbf{x})$

$\to \mathbf{m}_1 \odot \mathbf{x} \ + \ \mathbf{m}_2$

$$= \begin{pmatrix} \mathbf{m}_1[1] \cdot x_1 + \mathbf{m}_2[1] \\ \vdots \\ \mathbf{m}_w[w] \cdot x_w + \mathbf{m}_2[w] \end{pmatrix}$$

Main Tool: Batch-Select

Message vectors

$\mathbf{m}_1, \mathbf{m}_2 \in \mathbb{Z}_p^w$

"Selection" vector

$\mathbf{x} \in \mathbb{Z}_p^w$

<u>Sender</u>

$\mathrm{Enc}_1(\mathbf{m}_1) \to \mathrm{st}_1, \mathrm{ct}_1$

$\mathrm{Enc}_2(\mathbf{m}_2) \to \mathrm{st}_2, \mathrm{ct}_2$

$\mathrm{KeyGen}(\mathrm{st}_1, \mathrm{st}_2, \mathbf{x}) \to \mathrm{sk}_{\mathbf{x}}$

<u>Receiver</u>

$\mathrm{Dec}(\mathrm{sk}_{\mathbf{x}}, \mathrm{ct}_1, \mathrm{ct}_2, \mathbf{x})$

$$\to \mathbf{m}_1 \odot \mathbf{x} + \mathbf{m}_2 = \begin{pmatrix} \mathbf{m}_1[1] \cdot x_1 + \mathbf{m}_2[1] \\ \vdots \\ \mathbf{m}_w[w] \cdot x_w + \mathbf{m}_2[w] \end{pmatrix}$$

Succinctness: $|\mathrm{sk}_{\mathbf{x}}| = \mathrm{poly}(\lambda)$

Main Tool: Batch-Select

Sender | **Receiver**

Message vectors

$\mathbf{m}_1, \mathbf{m}_2 \in \mathbb{Z}_p^w$

$\mathrm{Enc}_1(\mathbf{m}_1) \to \mathrm{st}_1, \mathrm{ct}_1$

$\mathrm{Enc}_2(\mathbf{m}_2) \to \mathrm{st}_2, \mathrm{ct}_2$

$$\mathrm{Dec}(\mathrm{sk}_{\mathbf{x}}, \mathrm{ct}_1, \mathrm{ct}_2, \mathbf{x})$$

$$\to \mathbf{m}_1 \odot \mathbf{x} + \mathbf{m}_2$$

$$= \begin{pmatrix} \mathbf{m}_1[1] \cdot x_1 + \mathbf{m}_2[1] \\ \vdots \\ \mathbf{m}_w[w] \cdot x_w + \mathbf{m}_2[w] \end{pmatrix}$$

"Selection" vector

$\mathbf{x} \in \mathbb{Z}_p^w$

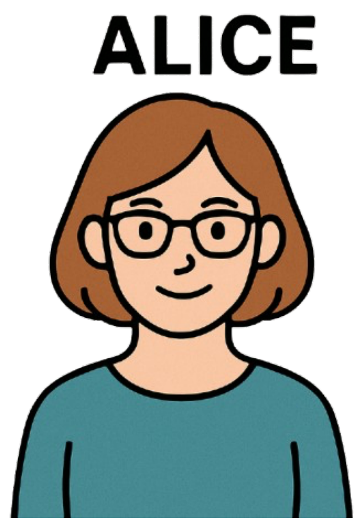
$\mathrm{KeyGen}(\mathrm{st}_1, \mathrm{st}_2, \mathbf{x}) \to \mathrm{sk}_{\mathbf{x}}$

Simulation Security:

For all $\mathbf{m}_1, \mathbf{m}_2, \mathbf{x}$:

$$(\mathrm{sk}_{\mathbf{x}}, \mathrm{ct}_1, \mathrm{ct}_2) \approx \mathrm{Sim}(\mathbf{m}_1 \odot \mathbf{x} + \mathbf{m}_2, \mathbf{x})$$

Compressing Labels using Batch-Select



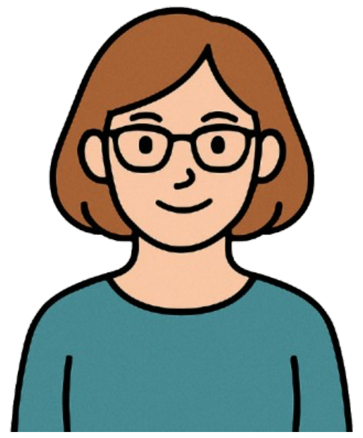
GC input keys

$\mathbf{K[0]}, \mathbf{K[1]} \in \mathbb{Z}_p^w$

$\mathrm{Enc}_1(\mathbf{K[1]} - \mathbf{K[0]}) \rightarrow \mathrm{st}_1, \mathrm{ct}_1$

$\mathrm{Enc}_2(\mathbf{K[0]}) \rightarrow \mathrm{st}_2, \mathrm{ct}_2$

Compressing Labels using Batch-Select



GC input keys

$\mathbf{K}[\mathbf{0}], \mathbf{K}[\mathbf{1}] \in \mathbb{Z}_p^w$

$\mathrm{Enc}_1(\mathbf{K}[\mathbf{1}] - \mathbf{K}[\mathbf{0}]) \rightarrow \mathrm{st}_1, \mathrm{ct}_1$

$\mathrm{Enc}_2(\mathbf{K}[\mathbf{0}]) \rightarrow \mathrm{st}_2, \mathrm{ct}_2$

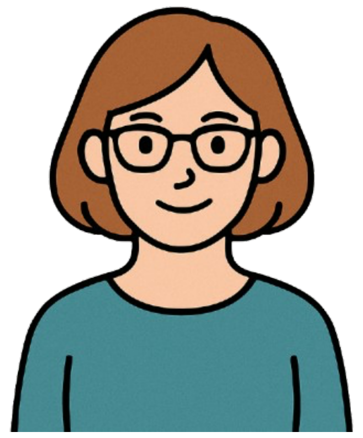
Input

$\mathbf{x} \in \{0,1\}^w$

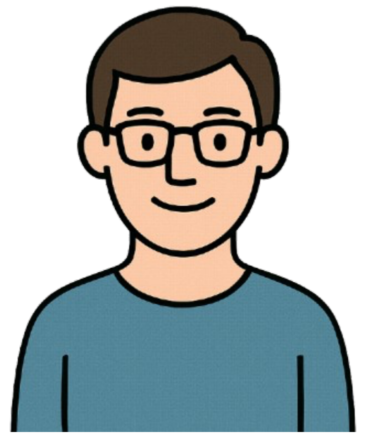
$\mathrm{KeyGen}(\mathrm{st}_1, \mathrm{st}_2, \mathbf{x}) \rightarrow \mathrm{sk}_{\mathbf{x}}$

Compressing Labels using Batch-Select

ALICE



BOB



GC input keys

$\mathbf{K}[0], \mathbf{K}[1] \in \mathbb{Z}_p^w$

Input

$\mathbf{x} \in \{0,1\}^w$

$\mathrm{Enc}_1(\mathbf{K}[1] - \mathbf{K}[0]) \to \mathrm{st}_1, \mathrm{ct}_1$

$\mathrm{Enc}_2(\mathbf{K}[0]) \to \mathrm{st}_2, \mathrm{ct}_2$

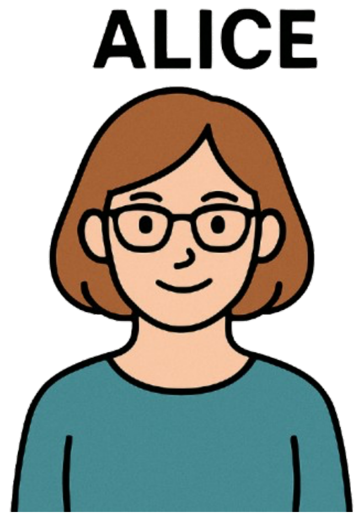
$\mathrm{KeyGen}(\mathrm{st}_1, \mathrm{st}_2, \mathbf{x}) \to \mathrm{sk}_{\mathbf{x}}$

$\mathrm{Dec}(\mathrm{sk}_{\mathbf{x}}, \mathrm{ct}_1, \mathrm{ct}_2, \mathbf{x})$

$$\to (\mathbf{K}[1] - \mathbf{K}[0]) \odot \mathbf{x} + \mathbf{K}[0]$$

$$= \begin{pmatrix} \mathbf{K}[x_1] \\ \vdots \\ \mathbf{K}[x_w] \end{pmatrix} = \mathbf{K}[\mathbf{x}]$$

Compressing Labels using Batch-Select



GC input keys

$\mathbf{K}[0], \mathbf{K}[1] \in \mathbb{Z}_p^w$

$\mathsf{Enc}_1(\mathbf{K}[1] - \mathbf{K}[0]) \rightarrow \mathsf{st}_1, \mathsf{ct}_1$

$\mathsf{Enc}_2(\mathbf{K}[0]) \rightarrow \mathsf{st}_2, \mathsf{ct}_2$

offline communication

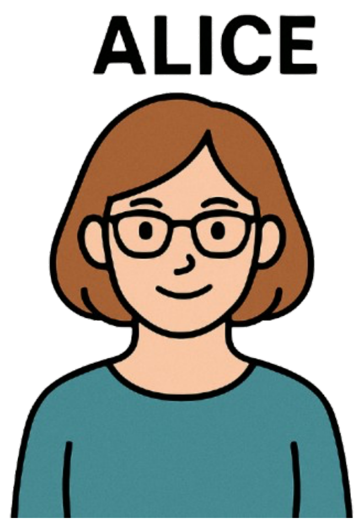
Input

$\mathbf{x} \in \{0,1\}^w$

$\mathsf{KeyGen}(\mathsf{st}_1, \mathsf{st}_2, \mathbf{x}) \rightarrow \mathsf{sk}_{\mathbf{x}}$

online communication

Compressing Labels using Batch-Select



GC input keys

$\mathbf{K}[0], \mathbf{K}[1] \in \mathbb{Z}_p^w$

$\mathrm{Enc}_1(\mathbf{K}[1] - \mathbf{K}[0]) \rightarrow \mathrm{st}_1, \mathrm{ct}_1$

$\mathrm{Enc}_2(\mathbf{K}[0]) \rightarrow \mathrm{st}_2, \mathrm{ct}_2$

offline communication

Input

$\mathbf{x} \in \{0,1\}^w$

$\mathrm{KeyGen}(\mathrm{st}_1, \mathrm{st}_2, \mathbf{x}) \rightarrow \mathrm{sk}_{\mathbf{x}}$

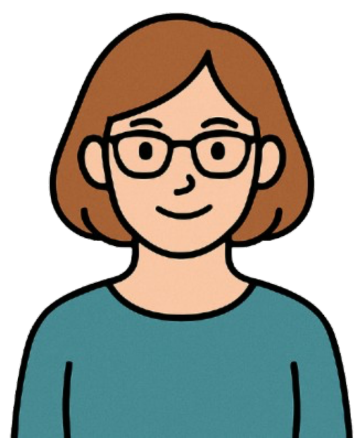
$|\mathcal{R}_q|$

online communication

(Assuming our construction from RingLWE)

Compressing Labels using Batch-Select

ALICE



GC input keys

$\mathbf{K}[0], \mathbf{K}[1] \in \mathbb{Z}_p^w$

$\mathrm{Enc}_1(\mathbf{K}[1] - \mathbf{K}[0]) \to \mathrm{st}_1, \mathrm{ct}_1$

$\mathrm{Enc}_2(\mathbf{K}[0]) \to \mathrm{st}_2, \mathrm{ct}_2$

offline communication

Input

$\mathbf{x} \in \{0,1\}^w$

$\mathrm{KeyGen}(\mathrm{st}_1, \mathrm{st}_2, \mathbf{x}) \to \mathrm{sk}_{\mathbf{x}}$

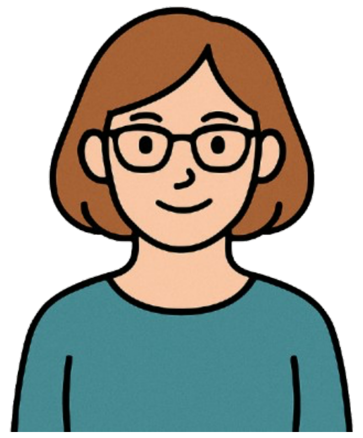
$|\mathscr{R}_q|$

($+|\mathbf{x}|$ if input is secret)

online communication

(Assuming our construction from RingLWE)

Compressing Labels using Batch-Select



GC input keys

$\mathbf{K}[0], \mathbf{K}[1] \in \mathbb{Z}_p^w$

$\mathrm{Enc}_1(\mathbf{K}[1] - \mathbf{K}[0]) \to \mathrm{st}_1, \mathrm{ct}_1$

$\mathrm{Enc}_2(\mathbf{K}[0]) \to \mathrm{st}_2, \mathrm{ct}_2$

$O(w \log w \cdot |\mathbb{Z}_p|)$

offline communication

Input

$\mathbf{x} \in \{0,1\}^w$

$\mathrm{KeyGen}(\mathrm{st}_1, \mathrm{st}_2, \mathbf{x}) \to \mathrm{sk}_{\mathbf{x}}$

$|\mathcal{R}_q|$

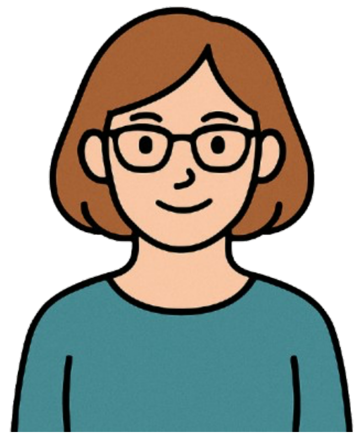
(+ $|\mathbf{x}|$ if input is secret)

online communication

(Assuming our construction from RingLWE)

Compressing Labels using Batch-Select

ALICE



GC input keys

$\mathbf{K}[0], \mathbf{K}[1] \in \mathbb{Z}_p^w$

$\mathrm{Enc}_1(\mathbf{K}[1] - \mathbf{K}[0]) \to \mathrm{st}_1, \mathrm{ct}_1$

$\mathrm{Enc}_2(\mathbf{K}[0]) \to \mathrm{st}_2, \mathrm{ct}_2$

Can be optimized to be essentially free!

offline communication

Input

$\mathbf{x} \in \{0,1\}^w$

$\mathrm{KeyGen}(\mathrm{st}_1, \mathrm{st}_2, \mathbf{x}) \to \mathrm{sk}_{\mathbf{x}}$

$|\mathscr{R}_q|$

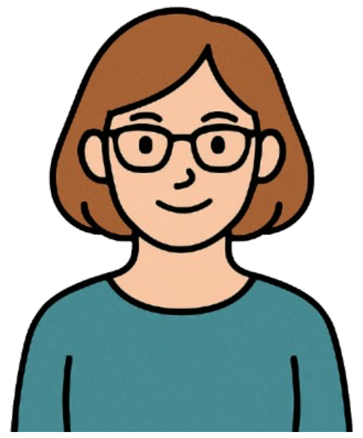
(+ $|\mathbf{x}|$ if input is secret)

online communication

(Assuming our construction from RingLWE)

Optimization 1: Free-XOR keys

ALICE



GC input keys

$\mathbf{K}[0]$

$\mathbf{K}[1] = \mathbf{K}[0] + \Delta \cdot \mathbf{1}_w$

$\mathrm{Enc}_1(\mathbf{K}[\mathbf{1}] - \mathbf{K}[\mathbf{0}]) \to \mathrm{st}_1, \mathrm{ct}_1$

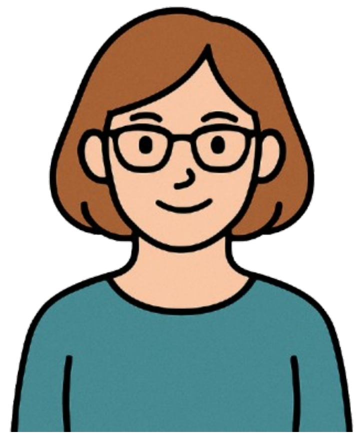
$\mathrm{Enc}_2(\mathbf{K}[\mathbf{0}]) \to \mathrm{st}_2, \mathrm{ct}_2$

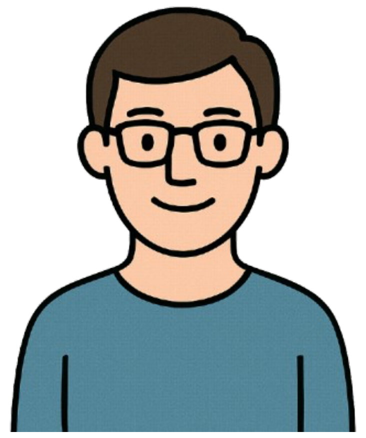
Input

$\mathbf{x} \in \{0,1\}^w$

$\mathrm{KeyGen}(\mathrm{st}_1, \mathrm{st}_2, \mathbf{x}) \to \mathrm{sk}_{\mathbf{x}}$

Optimization 1: Free-XOR keys





GC input keys

$\mathbf{K}[0]$

$\mathbf{K}[1] = \mathbf{K}[0] + \Delta \cdot \mathbf{1}_w$

$\mathrm{Enc}_1(\Delta \cdot \mathbf{1}_w) \to \mathrm{st}_1, \mathrm{ct}_1$

$\mathrm{Enc}_2(\mathbf{K}[0]) \to \mathrm{st}_2, \mathrm{ct}_2$

$\mathrm{Dec}(\mathrm{sk}_{\mathbf{x}}, \mathrm{ct}_1, \mathrm{ct}_2, \mathbf{x})$

$\to (\Delta \cdot \mathbf{1}_w) \odot \mathbf{x} \; + \; \mathbf{K}[0]$

$= \mathbf{K}[\mathbf{x}]$

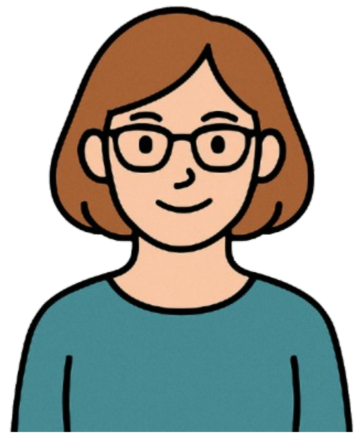
Input

$\mathbf{x} \in \{0,1\}^w$

$\mathrm{KeyGen}(\mathrm{st}_1, \mathrm{st}_2, \mathbf{x}) \to \mathrm{sk}_{\mathbf{x}}$

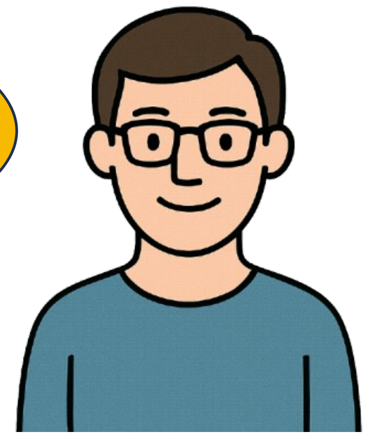
Optimization 1: Free-XOR keys

ALICE



Reusable!

BOB



GC input keys

$\mathbf{K}[0]$

$\mathbf{K}[1] = \mathbf{K}[0] + \Delta \cdot \mathbf{1}_w$

$\mathrm{Enc}_1(\Delta \cdot \mathbf{1}_w) \rightarrow \mathrm{st}_1, \mathrm{ct}_1$

$\mathrm{Enc}_2(\mathbf{K}[0]) \rightarrow \mathrm{st}_2, \mathrm{ct}_2$

$\mathrm{Dec}(\mathrm{sk}_{\mathbf{x}}, \mathrm{ct}_1, \mathrm{ct}_2, \mathbf{x})$

$\rightarrow (\Delta \cdot \mathbf{1}_w) \odot \mathbf{x} \; + \; \mathbf{K}[0]$

$= \mathbf{K}[\mathbf{x}]$

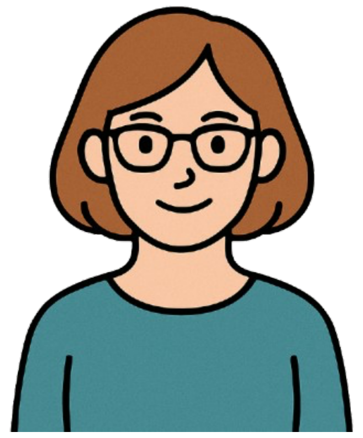
Input

$\mathbf{x} \in \{0,1\}^w$

$\mathrm{KeyGen}(\mathrm{st}_1, \mathrm{st}_2, \mathbf{x}) \rightarrow \mathrm{sk}_{\mathbf{x}}$

Optimization 2: Free encryption of random keys

ALICE



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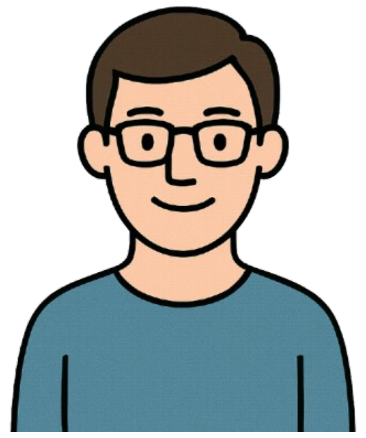


GC input keys

$\mathbf{K}[0] \leftarrow \$$

$\mathbf{K}[1] = \mathbf{K}[0] + \Delta \cdot \mathbf{1}_w$

$\mathrm{Enc}_1(\Delta \cdot \mathbf{1}_w) \to \mathrm{st}_1, \mathrm{ct}_1$

$\mathrm{Enc}_2(\mathbf{K}[0]) \to \mathrm{st}_2, \mathrm{ct}_2$

$\mathrm{Dec}(\mathrm{sk}_{\mathbf{x}}, \mathrm{ct}_1, \mathrm{ct}_2, \mathbf{x})$

$\to (\Delta \cdot \mathbf{1}_w) \odot \mathbf{x} \; + \; \mathbf{K}[0]$

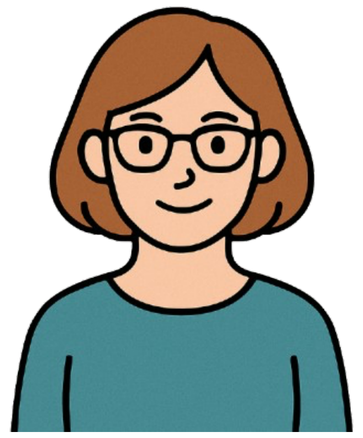
$= \mathbf{K}[\mathbf{x}]$

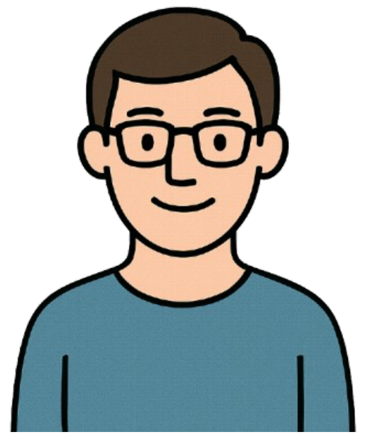
Input

$\mathbf{x} \in \{0,1\}^w$

$\mathrm{KeyGen}(\mathrm{st}_1, \mathrm{st}_2, \mathbf{x}) \to \mathrm{sk}_{\mathbf{x}}$

Optimization 2: Free encryption of random keys





GC input keys

$\mathbf{K}[0] \leftarrow \$$

$\mathbf{K}[1] = \mathbf{K}[0] + \Delta \cdot \mathbf{1}_w$

$\mathrm{Enc}_1(\Delta \cdot \mathbf{1}_w) \rightarrow \mathrm{st}_1, \mathrm{ct}_1$

$\mathrm{ct}_2 \leftarrow \mathrm{RO}(\mathrm{seed})$

$\mathbf{K}[0], \mathrm{st}_2 \leftarrow \mathrm{Dec}(\mathrm{ct}_2)$

$\mathrm{Dec}(\mathrm{sk}_{\mathbf{x}}, \mathrm{ct}_1, \mathrm{ct}_2, \mathbf{x})$

$\rightarrow (\Delta \cdot \mathbf{1}_w) \odot \mathbf{x} + \mathbf{K}[0]$

$= \mathbf{K}[\mathbf{x}]$

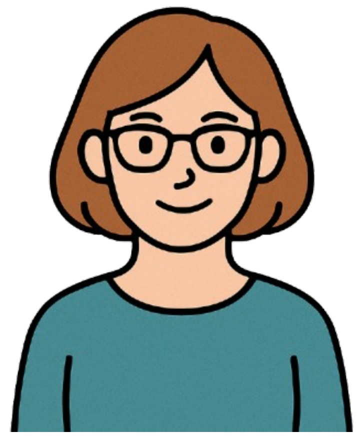
Input

$\mathbf{x} \in \{0,1\}^w$

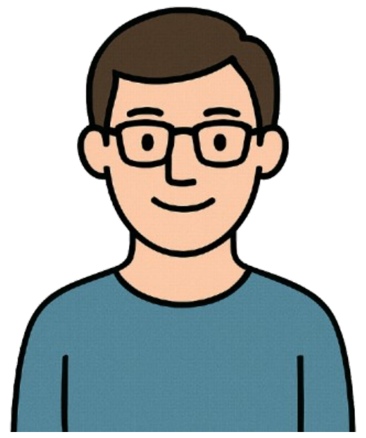
$\mathrm{KeyGen}(\mathrm{st}_1, \mathrm{st}_2, \mathbf{x}) \rightarrow \mathrm{sk}_{\mathbf{x}}$

Optimization 2: Free encryption of random keys

ALICE



BOB



GC input keys

$\mathbf{K}[0] \leftarrow \$$

$\mathbf{K}[1] = \mathbf{K}[0] + \Delta \cdot \mathbf{1}_w$

Input

$\mathbf{x} \in \{0,1\}^w$

$\mathrm{Enc}_1(\Delta \cdot \mathbf{1}_w) \rightarrow \mathrm{st}_1, \mathrm{ct}_1$

$\mathrm{ct}_2 \leftarrow \mathrm{RO}(\mathrm{seed})$

$\mathbf{K}[0], \mathrm{st}_2 \leftarrow \mathrm{Dec}(\mathrm{ct}_2)$

$\mathrm{KeyGen}(\mathrm{st}_1, \mathrm{st}_2, \mathbf{x}) \rightarrow \mathrm{sk}_\mathbf{x}$

Compressible to $|\mathrm{seed}| = \lambda$ bits!

$\mathrm{Dec}(\mathrm{sk}_\mathbf{x}, \mathrm{ct}_1, \mathrm{ct}_2, \mathbf{x})$

$\rightarrow (\Delta \cdot \mathbf{1}_w) \odot \mathbf{x} + \mathbf{K}[0]$

$= \mathbf{K}[\mathbf{x}]$

Constructing Batch-Select from RingLWE

LHE

[AIK11,AIKW13,BLLL23,...]

$$\mathbf{m}_1 \cdot x + \mathbf{m}_2$$

public scalar

succinct

$\mathbf{x}$-dependent key

Batch-Select

$$\mathbf{m}_1 \odot \mathbf{x} + \mathbf{m}_2$$

public vector

succinct

$\mathbf{x}$-dependent key

Constructing Batch-Select from RingLWE

LHE [AIK11,AIKW13,BLLL23,...]		LEnc [DKLLMR23]		Batch-Select
$\mathbf{m}_1 \cdot x + \mathbf{m}_2$	$+$	$\mathbf{m}_1 \odot \mathbf{x}$	$\Rightarrow$	$\mathbf{m}_1 \odot \mathbf{x} + \mathbf{m}_2$
public scalar				**public** vector
succinct				**succinct**
$\mathbf{x}$-dependent key				$\mathbf{x}$-dependent key

Constructing Batch-Select from RingLWE

LHE

[AIK11,AIKW13,BLLL23,…]

$$\mathbf{m}_1 \cdot x + \mathbf{m}_2$$

public scalar

succinct

$\mathbf{x}$-dependent key

$+$

LEnc

[DKLLMR23]

$$\mathbf{m}_1 \odot \mathbf{x}$$

non-succinct

$\mathbf{x}$-dependent key

$\Rightarrow$

Batch-Select

$$\mathbf{m}_1 \odot \mathbf{x} + \mathbf{m}_2$$

public vector

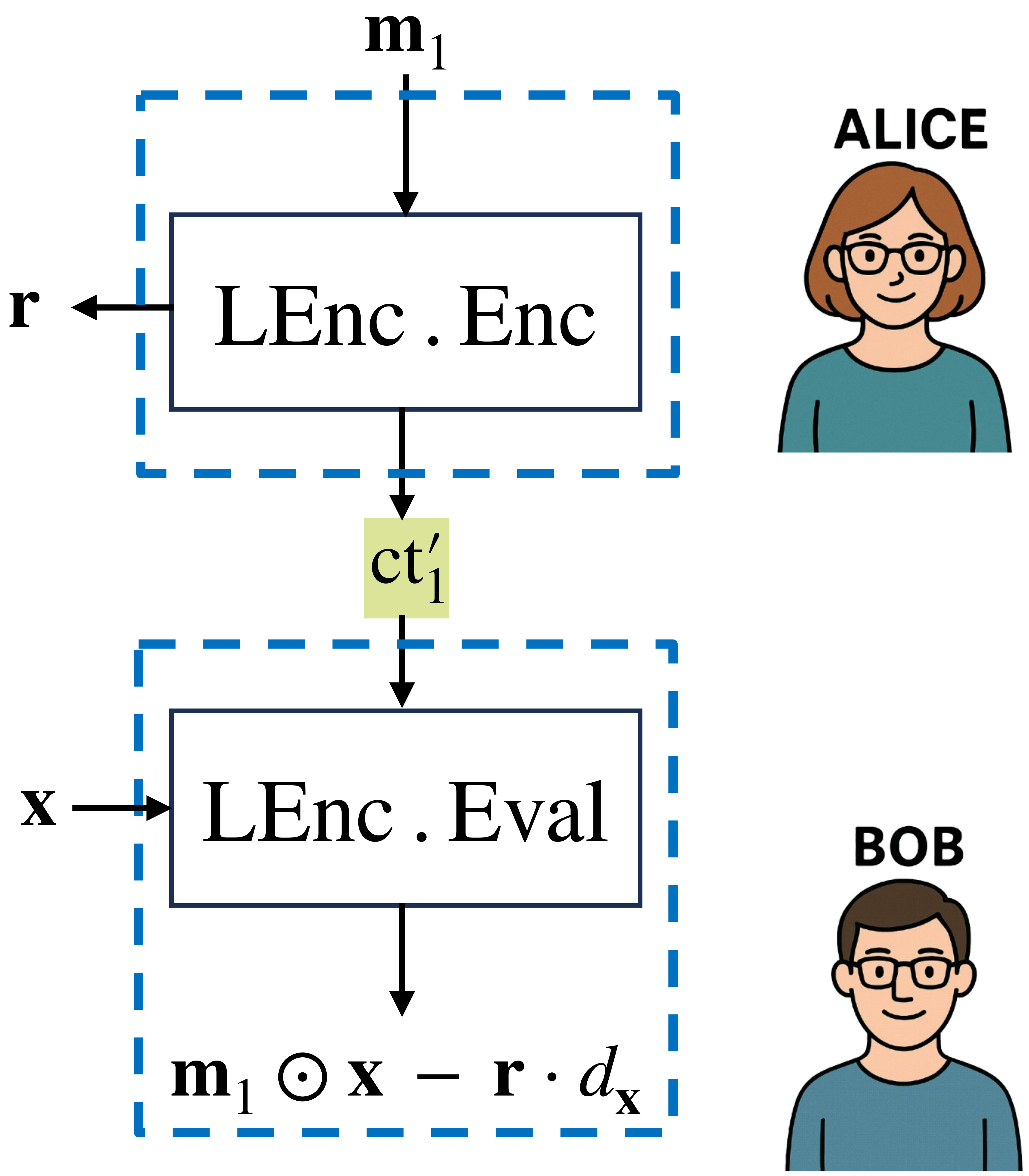
succinct

$\mathbf{x}$-dependent key

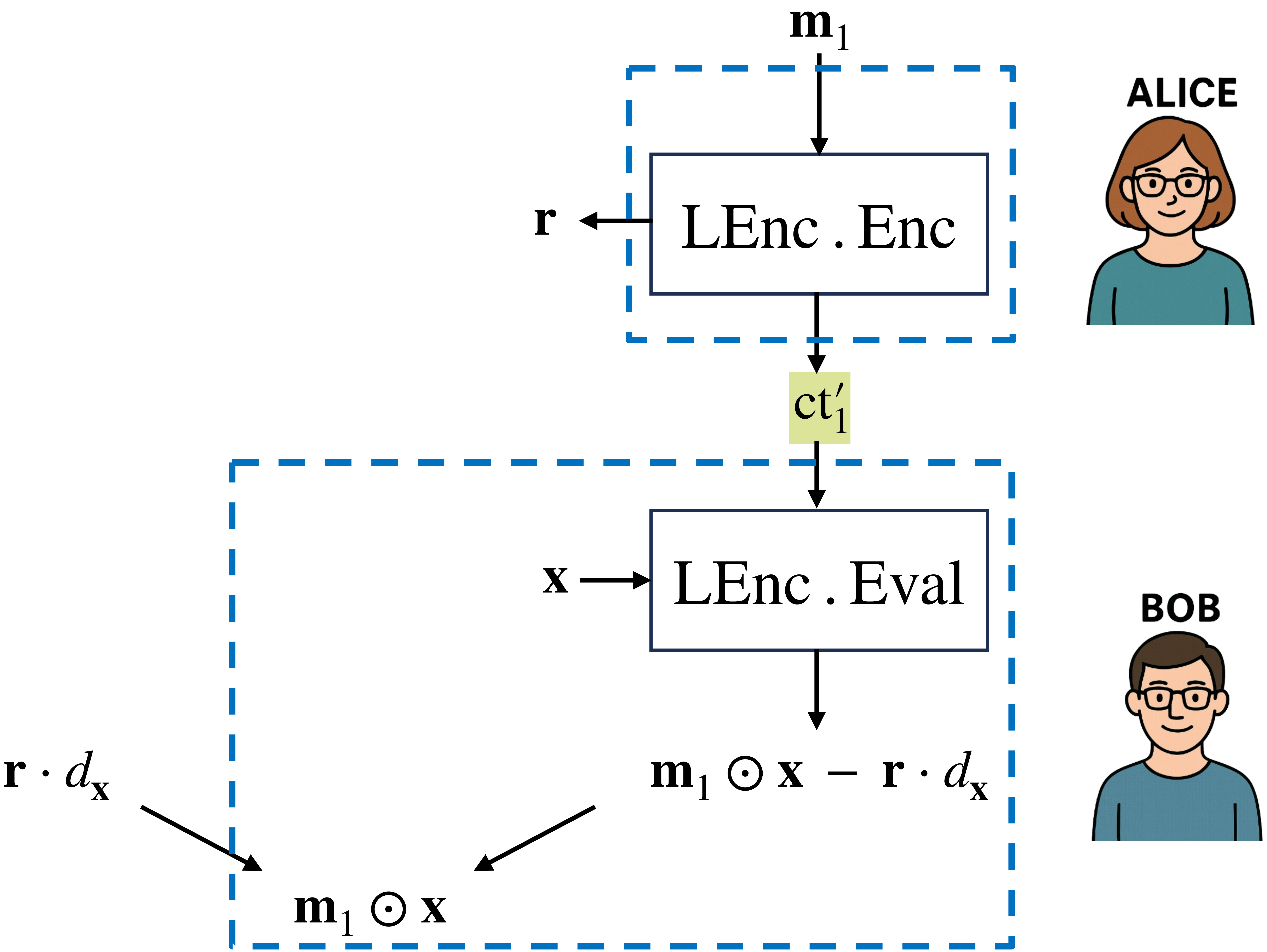
Constructing Batch-Select from RingLWE

LHE [AIK11,AIKW13,BLLL23,...]		LEnc [DKLLMR23]		Batch-Select
$\mathbf{m}_1 \cdot x + \mathbf{m}_2$	$+$	$\mathbf{m}_1 \odot \mathbf{x}$	$\Rightarrow$	$\mathbf{m}_1 \odot \mathbf{x} + \mathbf{m}_2$
public scalar				**public** vector
succinct		**non-succinct**		**succinct**
$\mathbf{x}$-dependent key		$\mathbf{x}$-dependent key		$\mathbf{x}$-dependent key
		$\mathrm{sk}_{\mathbf{x}} = \mathbf{r} \cdot d_{\mathbf{x}}$		

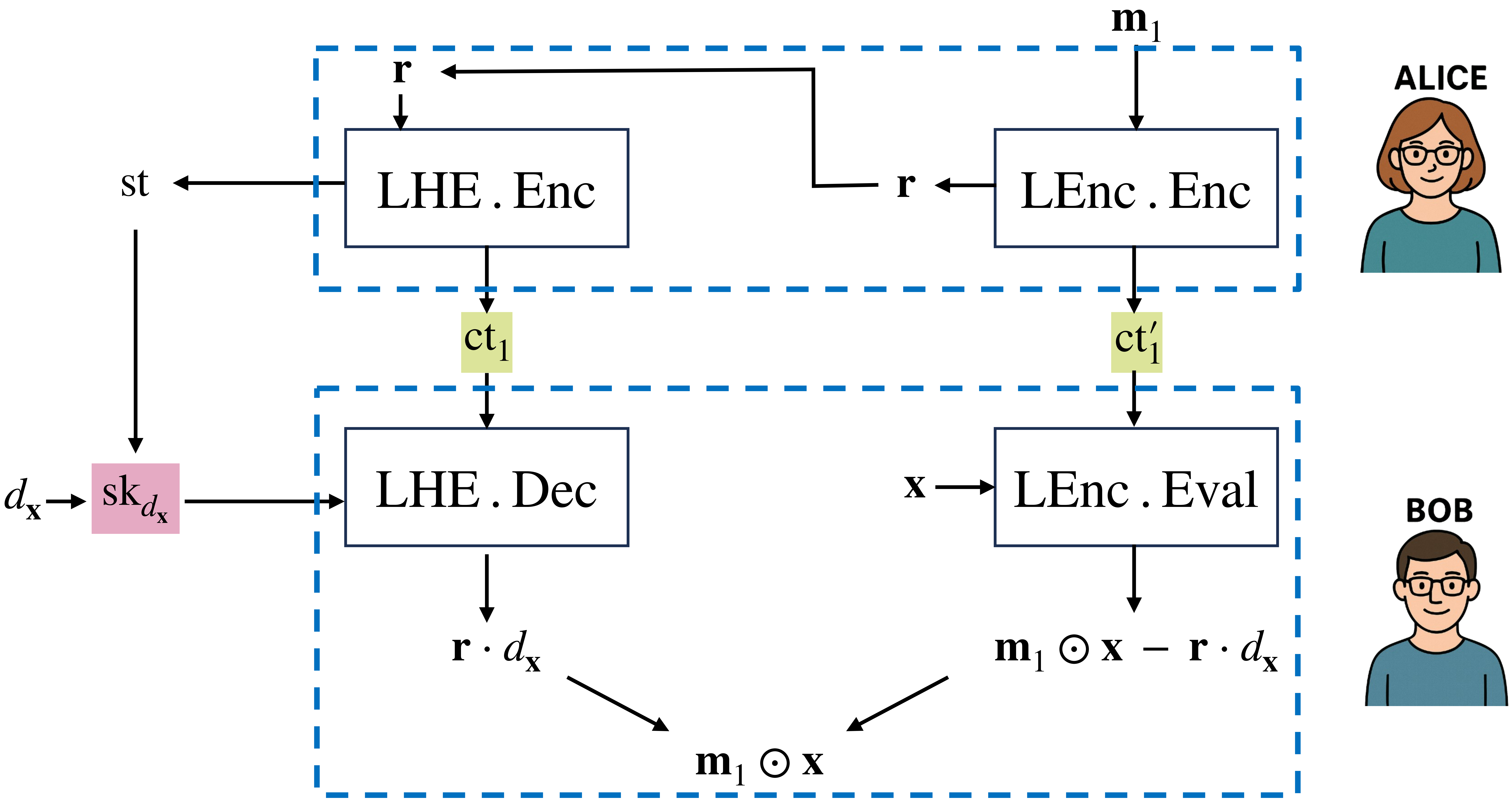
Constructing Batch-Select from RingLWE



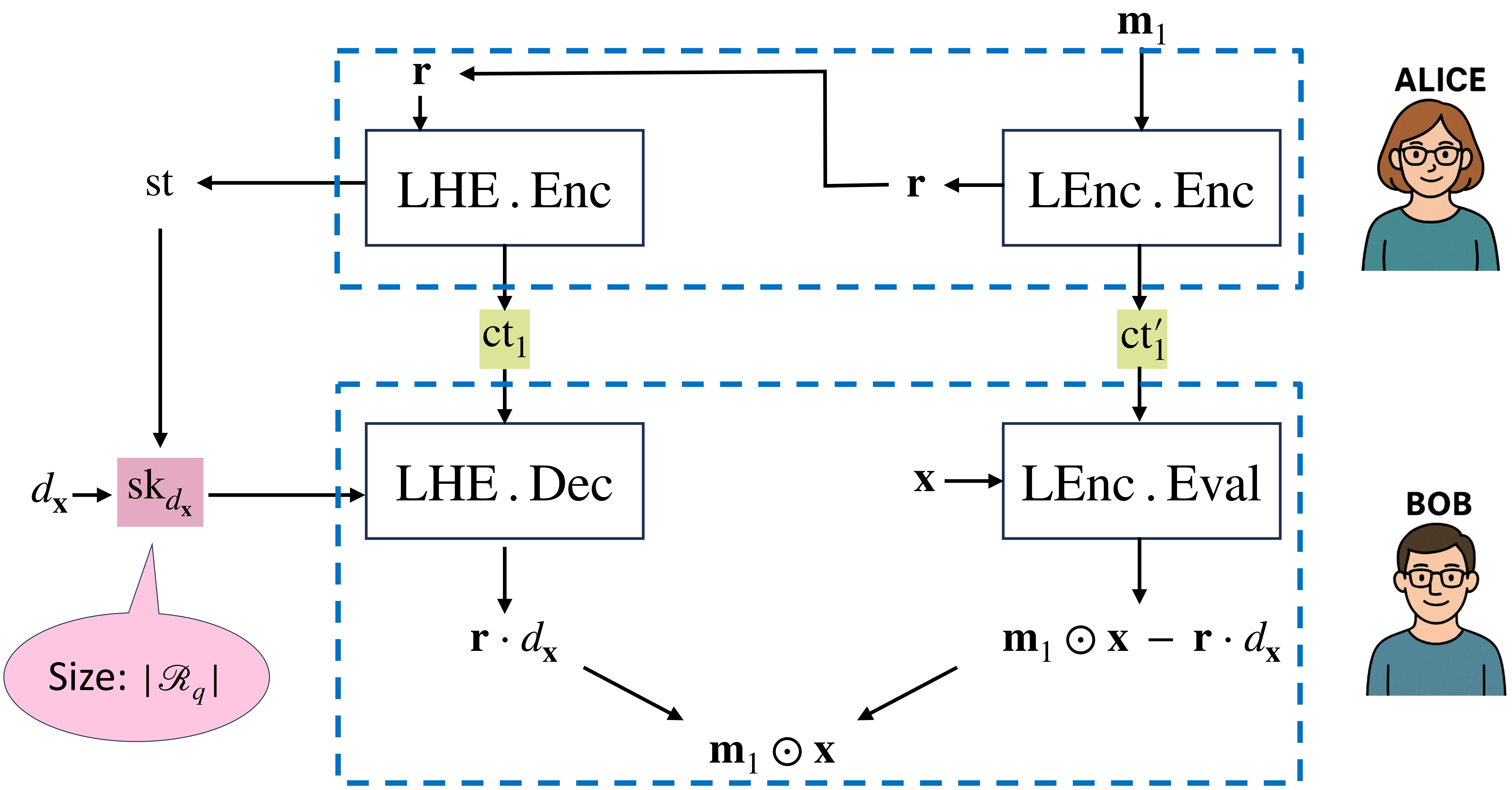
Constructing Batch-Select from RingLWE



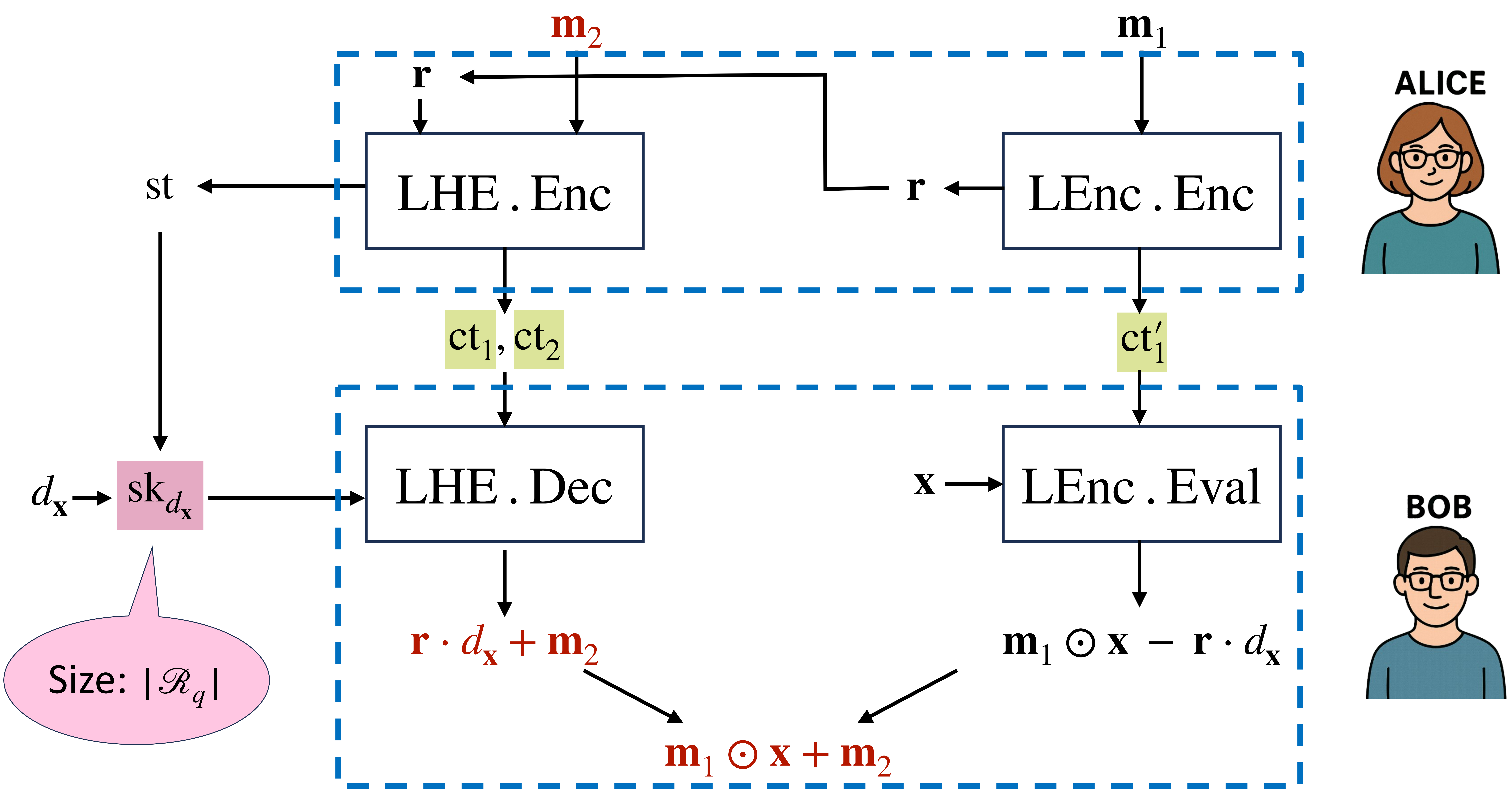
Constructing Batch-Select from RingLWE



Constructing Batch-Select from RingLWE



Constructing Batch-Select from RingLWE



Open Questions

- *Practical* label compression from other assumptions?
- Can succinct garbling *without offline phase* be made practical?

Open Questions

- *Practical* label compression from other assumptions?
- Can succinct garbling *without offline phase* be made practical?

Thank you!

https://ia.cr/2024/2048

References

- **[AIKW13]**: Applebaum, B., Ishai, Y., Kushilevitz, E., & Waters, B. (2013, August). **Encoding Functions with Constant Online Rate or How to Compress Garbled Circuits Keys**. In *Annual Cryptology Conference* (pp. 166-184). Berlin, Heidelberg: Springer Berlin Heidelberg.

- **[GS18]**: Garg, S., & Srinivasan, A. (2018). **Adaptively secure garbling with near optimal online complexity**. In Advances in Cryptology–EUROCRYPT 2018: 37th Annual International Conference on the Theory and Applications of Cryptographic Techniques, Tel Aviv, Israel, April 29-May 3, 2018 Proceedings, Part II 37 (pp. 535-565). Springer International Publishing.

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- **[DKLLMR23]**: Döttling, N., Kolonelos, D., Lai, R. W., Lin, C., Malavolta, G., & Rahimi, A. (2023, April). **Efficient laconic cryptography from learning with errors**. In *Annual International Conference on the Theory and Applications of Cryptographic Techniques* (pp. 417-446). Cham: Springer Nature Switzerland.

- Images generated with ChatGPT