

Malleable SNARKs and Their Applications

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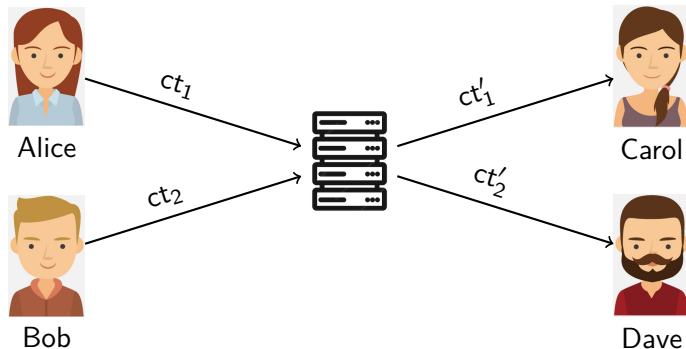
³Aarhus University

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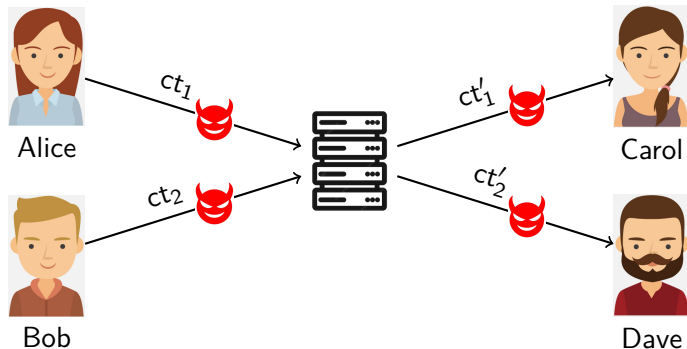
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2025-05-08

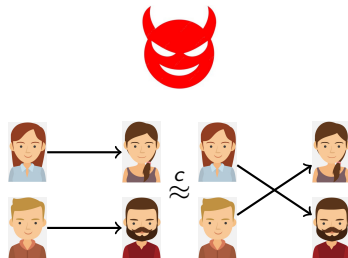
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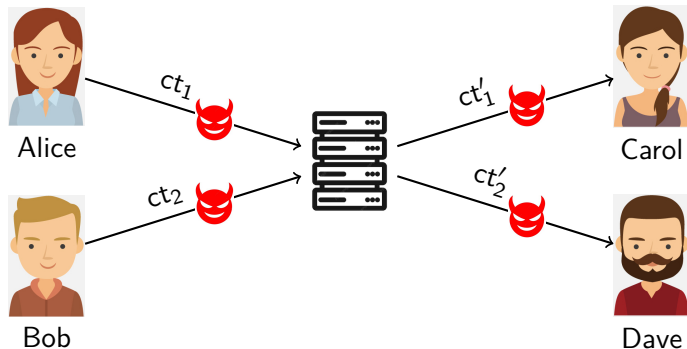
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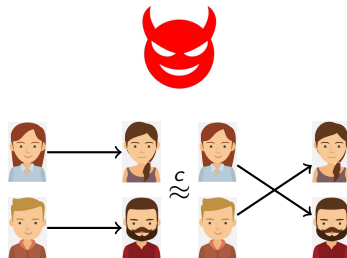
Security:




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Solution:  rerandomizes ciphertexts (+ anonymity of the encryption scheme)

IND-RCCA secure rerandomizable PKE

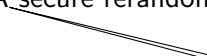
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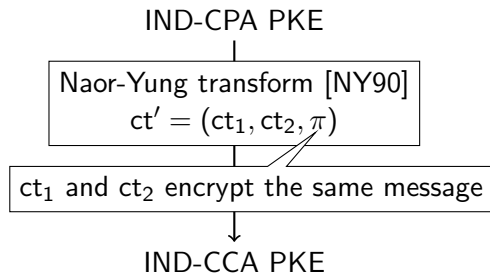


Dec oracle returns \diamond if ciphertext decrypts to m_0^* or m_1^*

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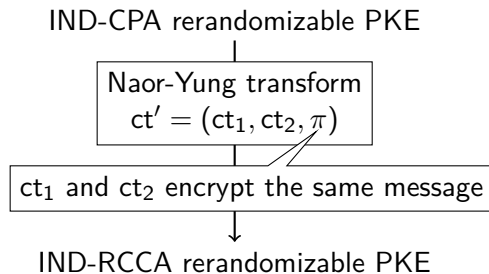
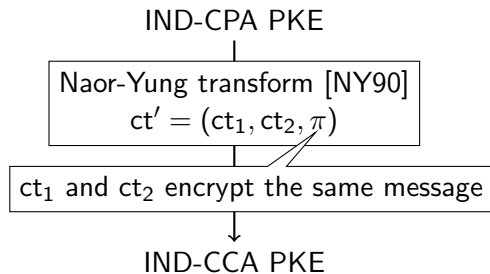
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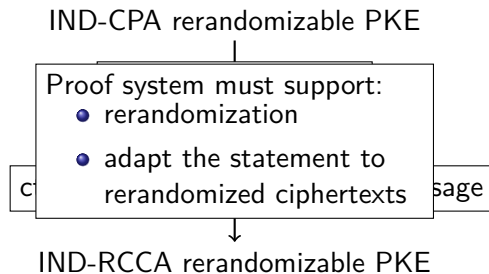
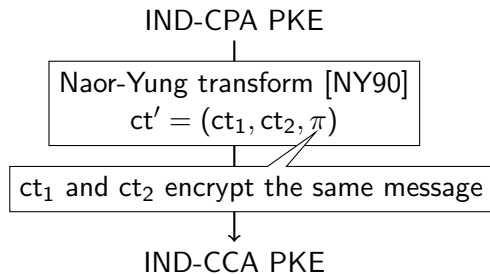
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Malleable NIZKs [CKLM12]

- NP relation \mathcal{R}
- Allowed transformations \mathcal{T}

such that

$$\forall (x, w) \in \mathcal{R}, (T_x, T_w) \in \mathcal{T} \rightarrow (T_x(x), T_w(w)) \in \mathcal{R}$$

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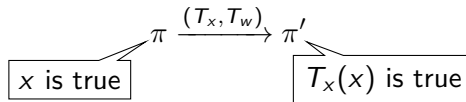
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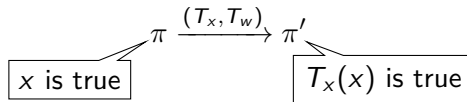
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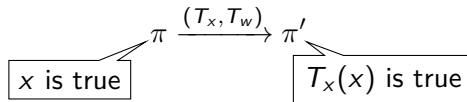
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Simulation-soundness: Takes into account that simulated statements can be modified with \mathcal{T}

Instantiations of malleable NIZKs

Groth-Sahai proofs [CKLM12]:

- ✓ Very efficient
- ✗ Only for a specific \mathcal{R}
(pairing-product equations)
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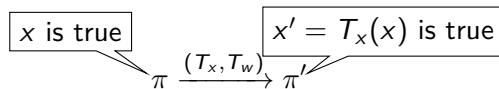
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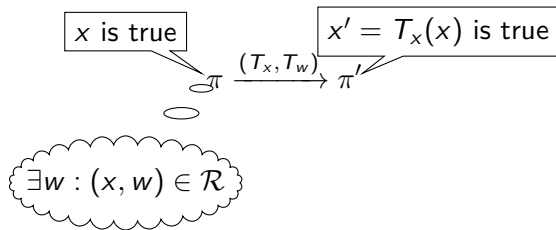
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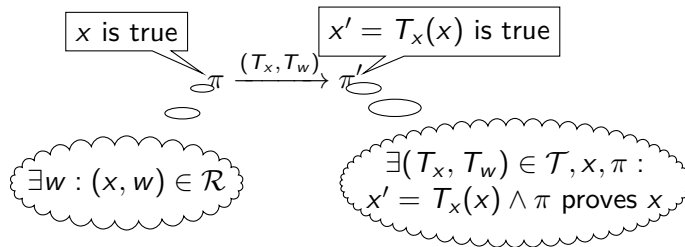
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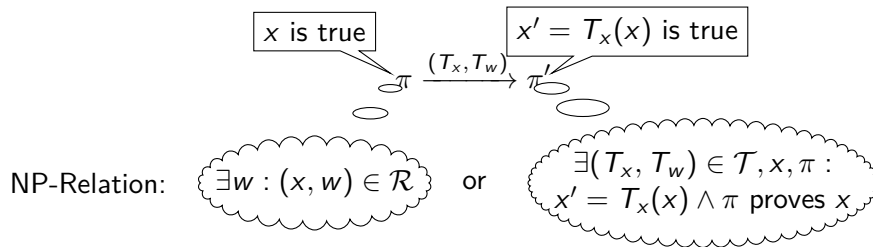
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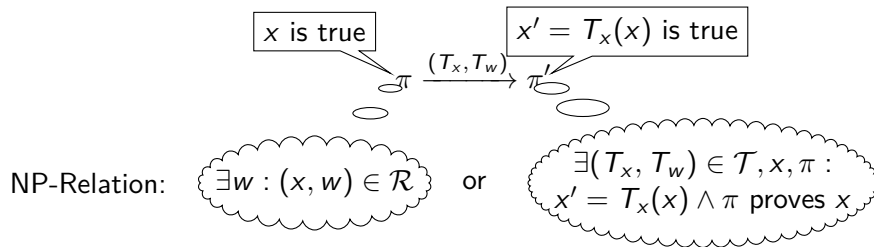
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Recursive usage of SNARKs

- succinctness avoids blow-up of proof size/verification time
- ✓ zero-knowledge ✓ derivation private (by zero-knowledge of the SNARK)
- also used for incrementally verifiable computation (IVC), proof carrying data (PCD), blockchains (to compress proofs)

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Main technical challenge

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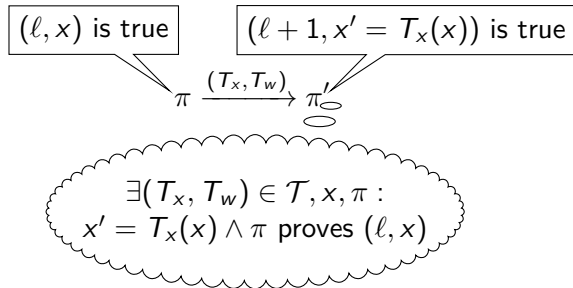
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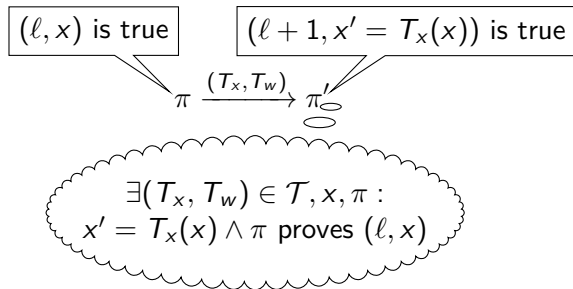
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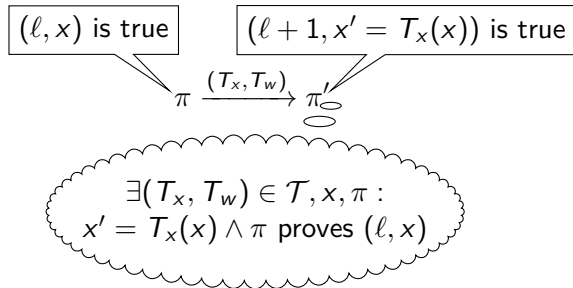
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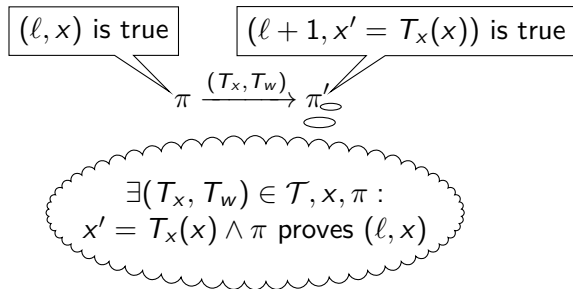
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- ✗ Only bounded number of recursions

Non-solution: One-way permutation

OWP $f : X \rightarrow X$

Counter ℓ is replaced by $\xi \in X$

- Initially: $\xi \leftarrow X$
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We need a variant of OWF that is secure if the adversary chooses the value x to invert.

Adversarial one-way functions (AOWFs)

Stateful 2-stage adversary $(\mathcal{A}_1, \mathcal{A}_2)$

1st attempt: \mathcal{A}_1 outputs y
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- f is only sequentially computable
- n depends on the runtime of \mathcal{A}_1 ($\text{Time}_{\mathcal{A}_1} < n \cdot \text{Time}_{f(\cdot)}$)

there is no trivial attack!

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- For our candidate this can be achieved by inputting additional random bits in the hash function

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Requires assumption specific to the underlying HE scheme

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- Good candidates for SNARKs with fast extraction



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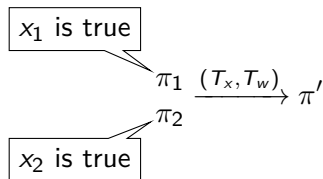
Public-key cryptosystems provably secure against chosen ciphertext attacks.

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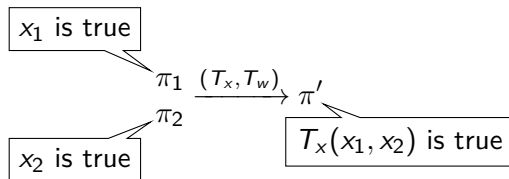
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Alice, Bob, and other faces, Server: freepik.com
Matryoshka doll: holz-leute.de

Transformations with arity ≥ 2

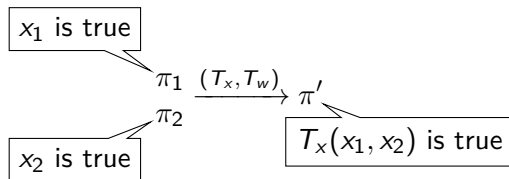


Transformations with arity ≥ 2



Counter-based approach:
$$\begin{pmatrix} \ell_1, \pi_1 \\ \ell_2, \pi_2 \end{pmatrix} \xrightarrow{(T_x, T_w)} (\max\{\ell_1, \ell_2\} + 1, \pi')$$

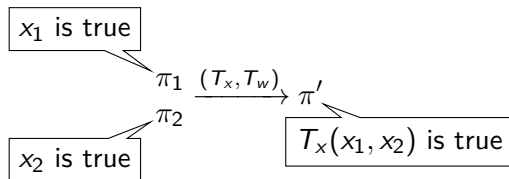
Transformations with arity ≥ 2



Counter-based approach: $(\ell_1, \pi_1) \xrightarrow{(T_x, T_w)} (\max\{\ell_1, \ell_2\} + 1, \pi')$
 (ℓ_2, π_2)

✗ Only for constant-depth

Transformations with arity ≥ 2



Counter-based approach: $(\ell_1, \pi_1) \xrightarrow{(T_x, T_w)} (\max\{\ell_1, \ell_2\} + 1, \pi')$
 (ℓ_2, π_2)

✗ Only for constant-depth

AOWF-based approach:

✓ Unbounded depth

- Needs higher arity variant of AOWFs (works for hash functions)
- Statements and proofs must be input to the AOWF
- Extractor must cache extracted SNARKs