Radbound University

To Pad or Not to Pad? Padding-Free Arithmetization-Oriented Sponges

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Radboud University (The Netherlands)
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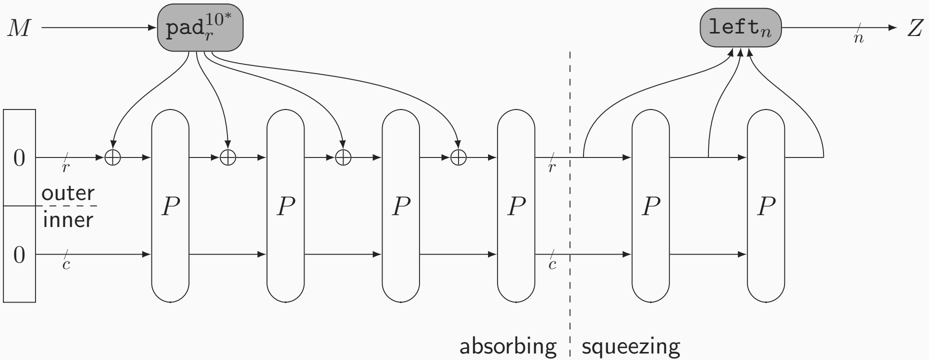
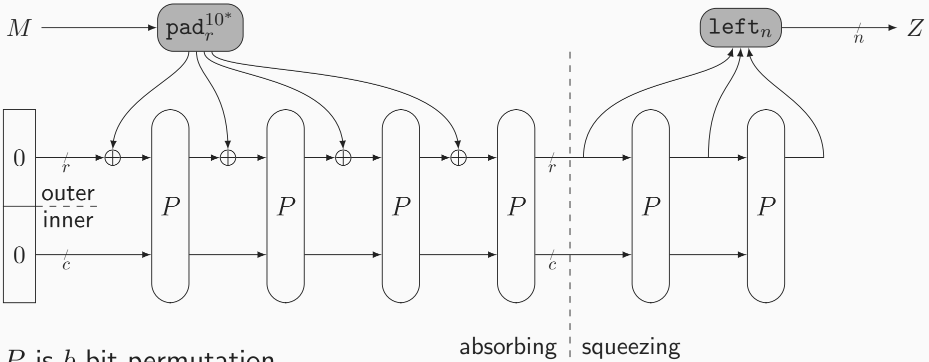
March 17, 2025

Sponge Construction [BDPV07]





- P is b-bit permutation

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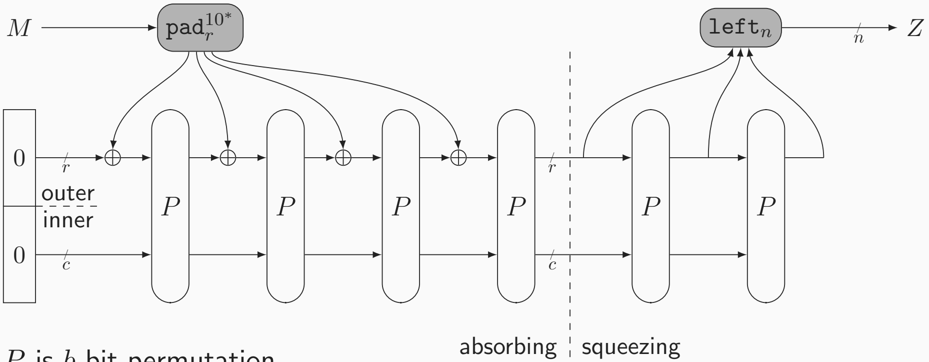

- P is b-bit permutation
  - r is the rate
  - c is the capacity
  - $b = r + c$

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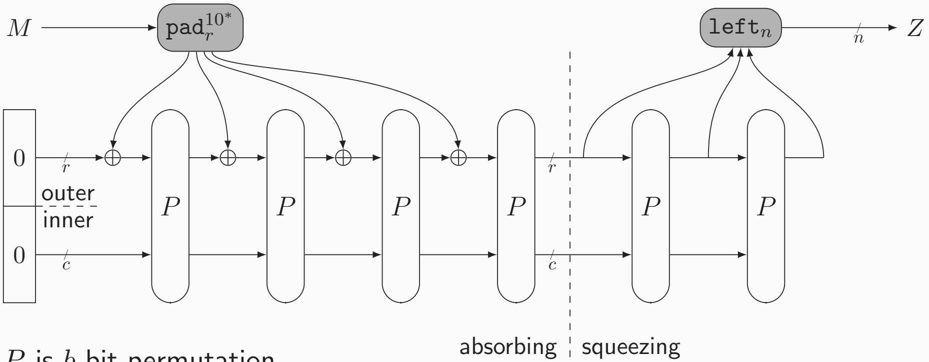
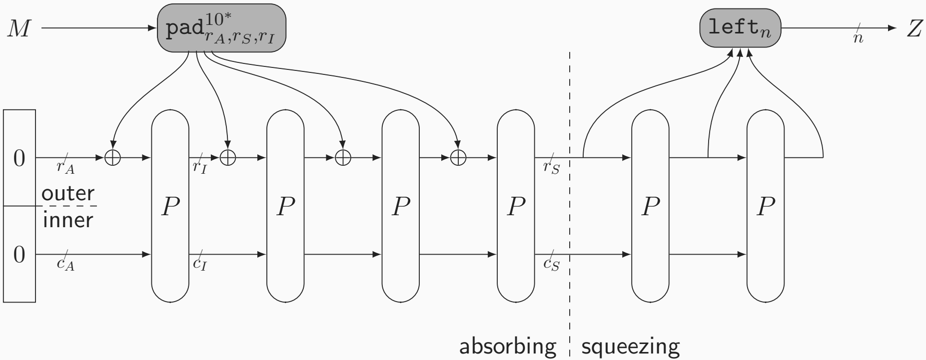
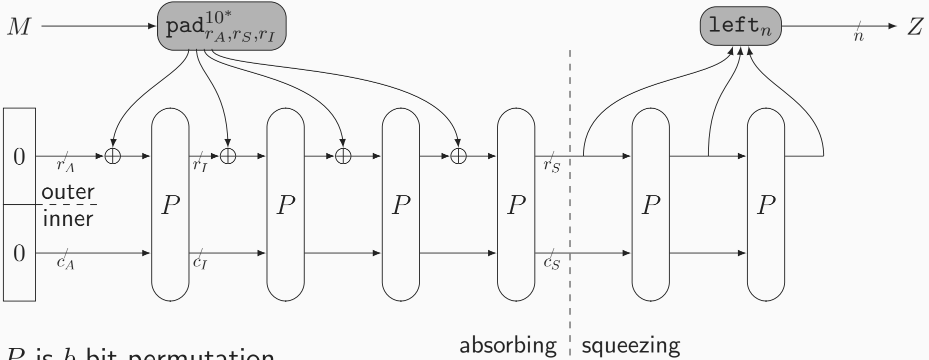


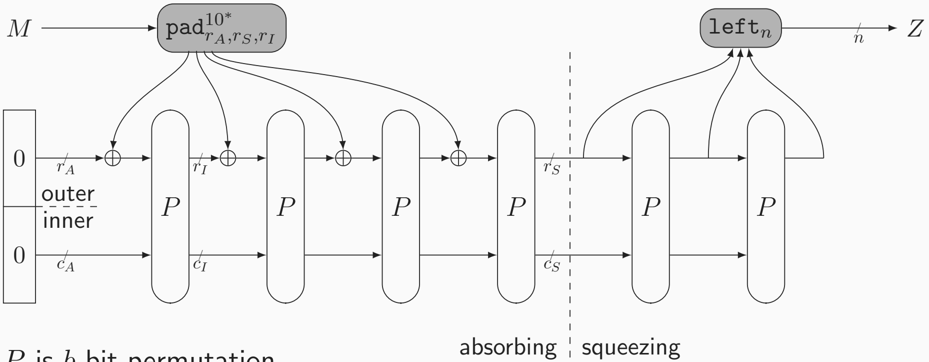
- P is b-bit permutation
  - r is the rate
  - c is the capacity
  - $b = r + c$
- **Security**: generically behaves like $\mathtt{RO}$ up to $\mathcal{O}\left(2^{c/2}\right)$ queries [BDPV08]

Generalized Sponge Construction [NO14]

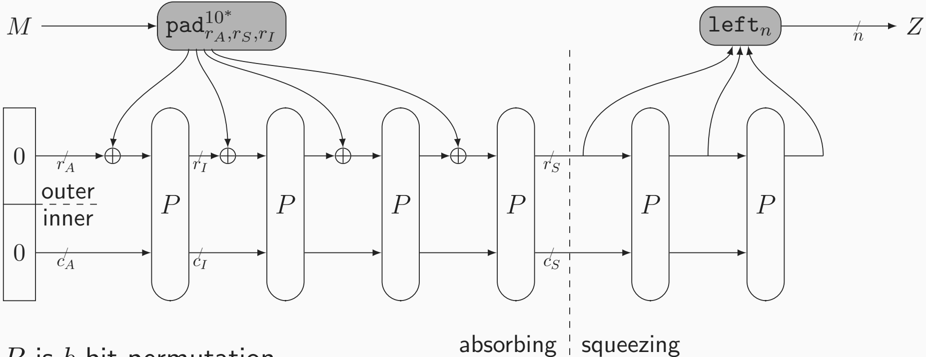




- P is b-bit permutation

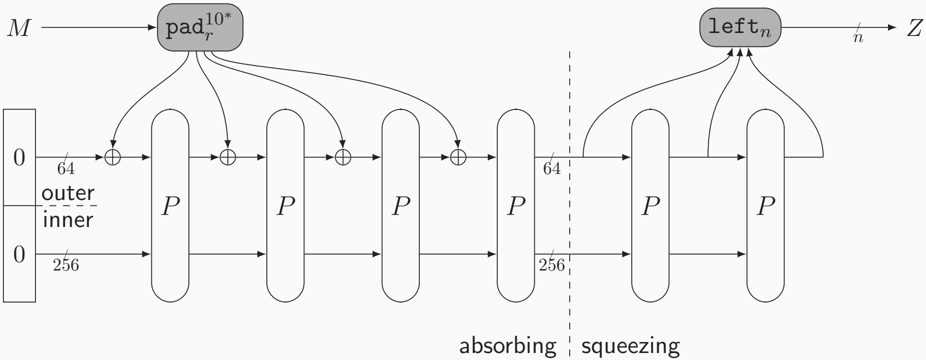


- P is b-bit permutation
  - r_A, r_I, r_S are the rates and usually $r_A \simeq r_I + c_I/2$
  - c_A, c_I, c_S are the capacities
  - $b = r_A + c_A = r_I + c_I = r_S + c_S$



- P is b-bit permutation
  - r_A, r_I, r_S are the rates and usually $r_A \simeq r_I + c_I/2$
  - c_A, c_I, c_S are the capacities
  - $b = r_A + c_A = r_I + c_I = r_S + c_S$
- **Security**: behaves like RO up to $\mathcal{O}\left(2^{\min\{c_I/2, c_S/2\}}\right)$ queries [NO14]

A Concrete Example



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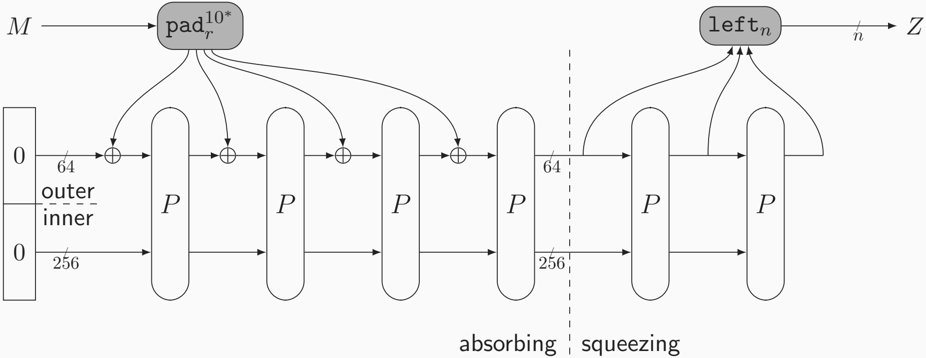
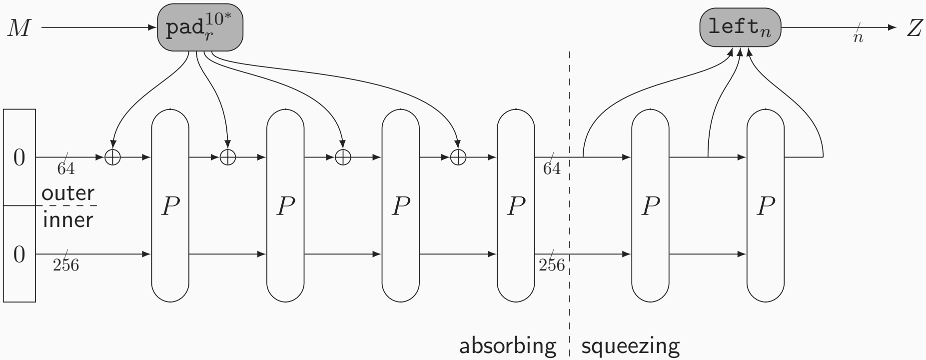


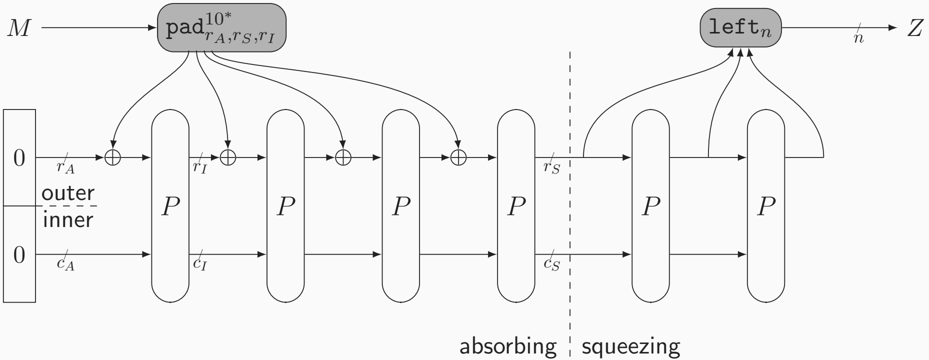
- Ascon-Hash256: $b = 320, r = 64, c = 256$

A Concrete Example



- Ascon-Hash256: $b = 320, r = 64, c = 256$
- Padding cost: small overhead

A Reinforced Concrete Example [GKL+22]



- Security level: 128

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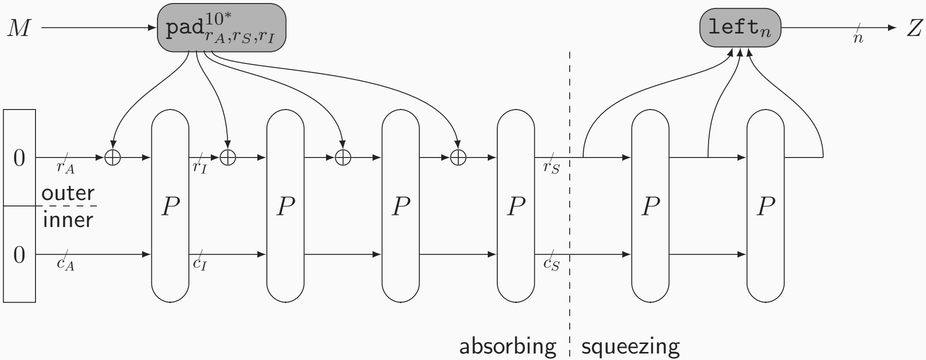

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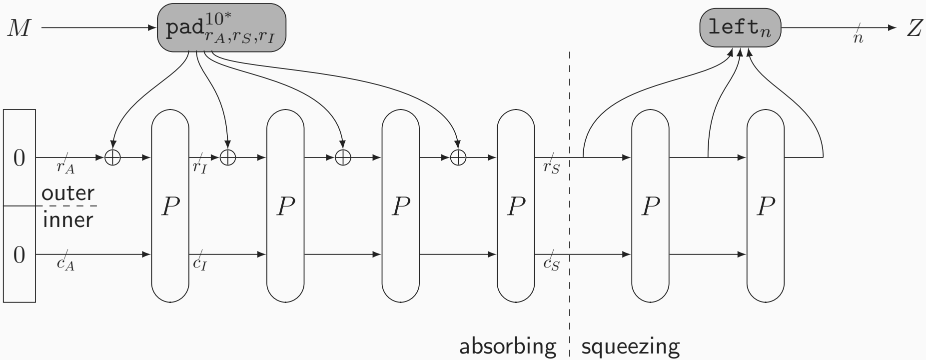
- $p \simeq 2^{256}$

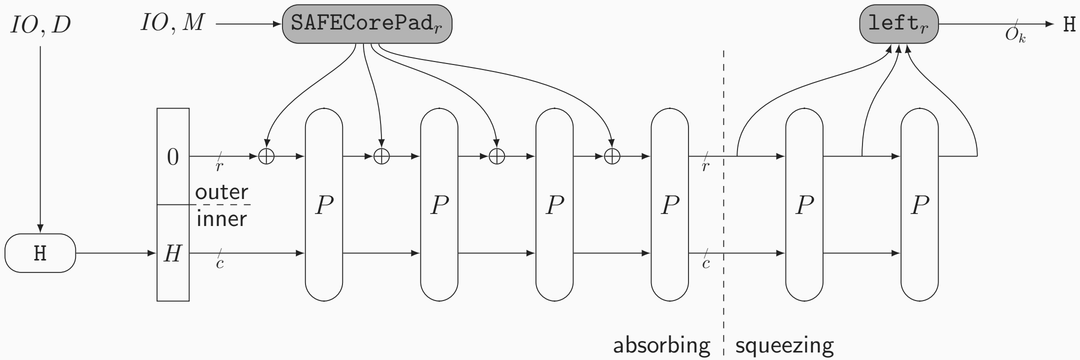
A Reinforced Concrete Example [GKL+22]

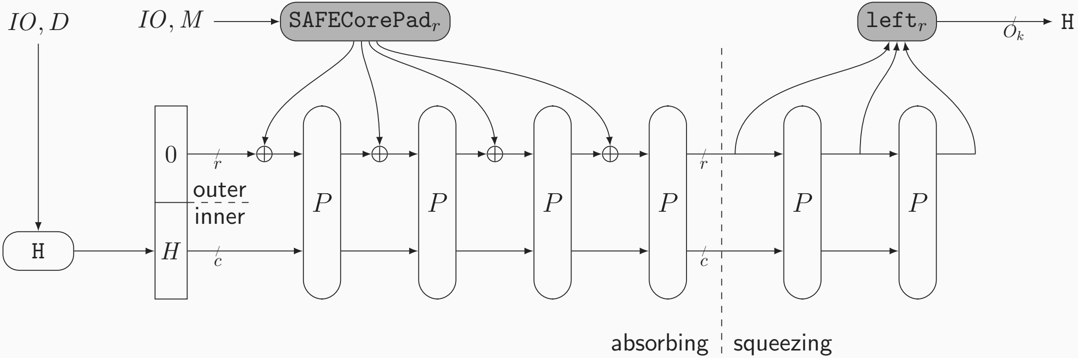


- Security level: 128
- $p \simeq 2^{256}$
- $b = 3$, $r = 2$, and $c = 1$

Padding may cost 256 bits!

The Rise Of SAFE [AKQ22]: A Restricted Setting Solution





- **Security**: behaves like RO up to $\mathcal{O}\left(2^{c/2}\right)$ queries [KMM23]

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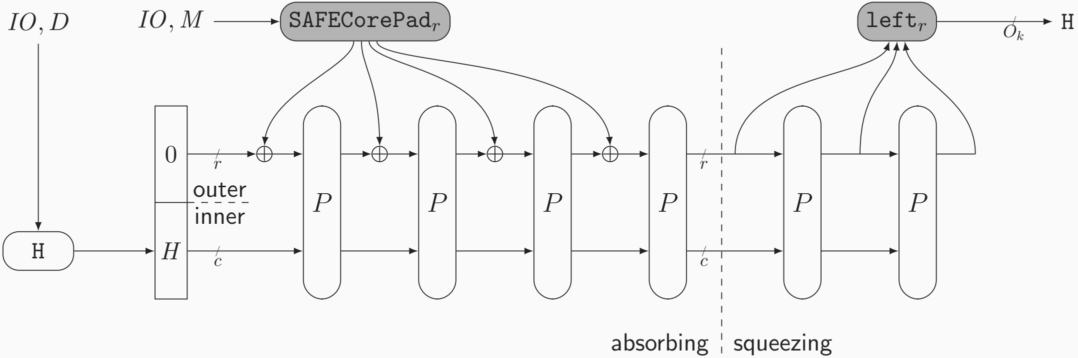
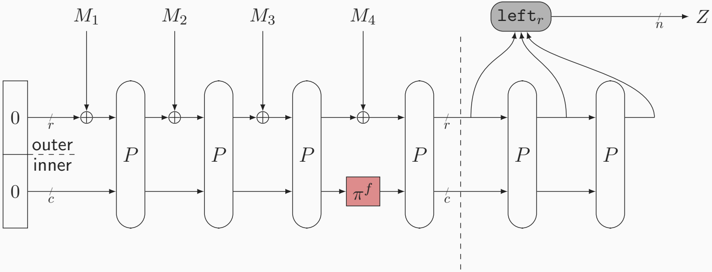
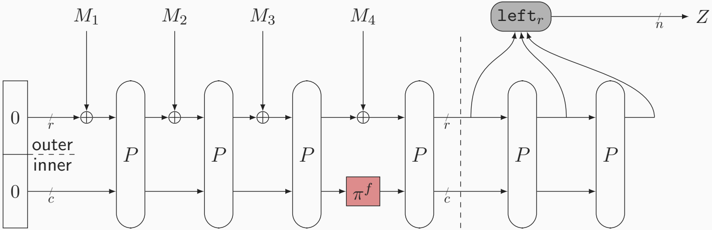


- **Security**: behaves like RO up to $\mathcal{O}(2^{c/2})$ queries [KMM23]
- Inflexible message sizes

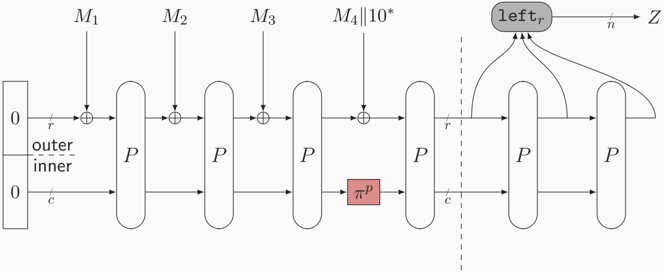
- Message length is a multiple of r

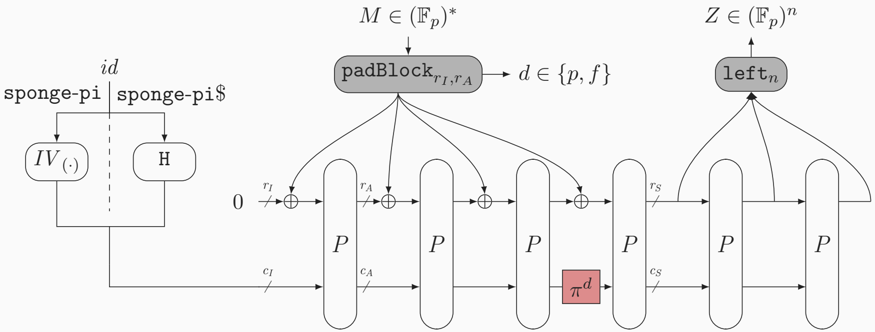


- Message length is a multiple of r

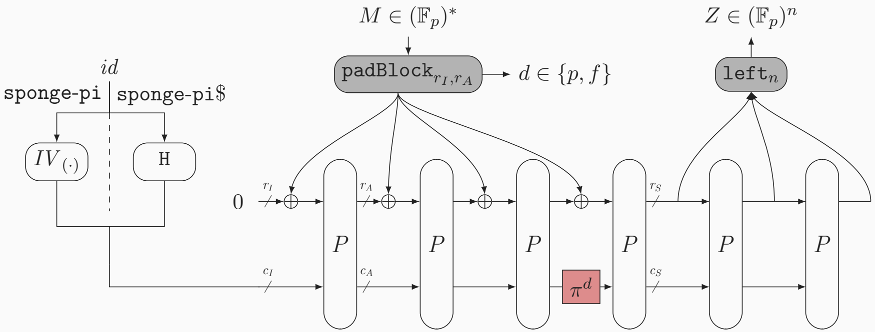


- Message length is not a multiple of r

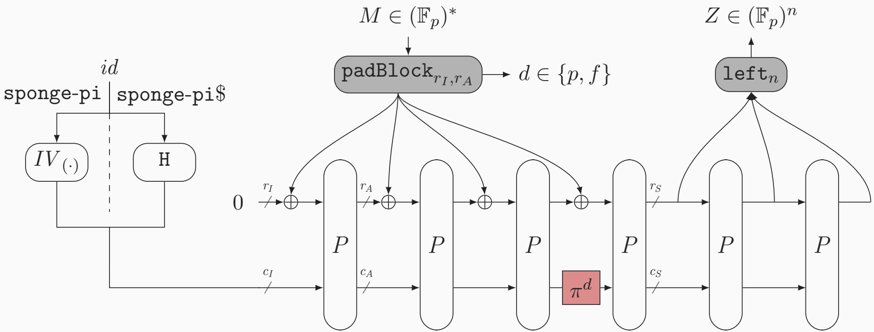




id
sponge-pi
sponge-pi$
$IV_{(\cdot)}$
H
$M \in (\mathbb{F}_p)^*$
$\texttt{padBlock}_{r_I, r_A}$
$d \in \{p, f\}$
$Z \in (\mathbb{F}_p)^n$
$\texttt{left}_n$
0
r_I
r_A
r_S
c_I
c_A
c_S
P
π^d

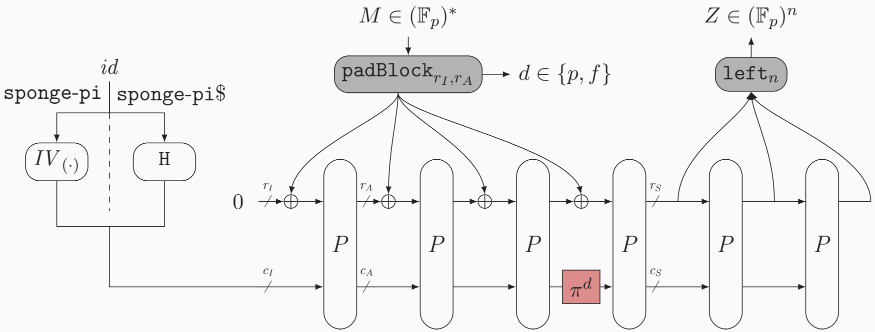


- sponge-pi: User-determined IVs



- sponge-pi: User-determined IVs
- sponge-pi$: Randomly determined IVs

Our Constructions: sponge-pi And sponge-pi$



- sponge-pi: User-determined IVs
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Same, but different

sponge-pi and sponge-pi$: Simplified Security Comparison

- Simplified setting: $c_I = c_A = c_S = c$
- Q_P: Queries to P
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sponge	$\dfrac{Q_P^2}{p^c} + \dfrac{\mu Q_P}{p^c}$
`sponge-pi`	$\dfrac{7Q_P^2}{p^c} + \dfrac{7\mu Q_P}{2p^c}$
`sponge-pi$`	$\dfrac{7Q_P^2}{p^c} + \dfrac{22 Q_P Q_H}{p^c}$

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Where does the 7 come from?

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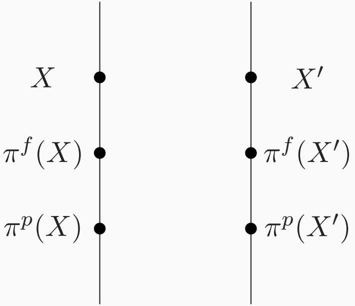

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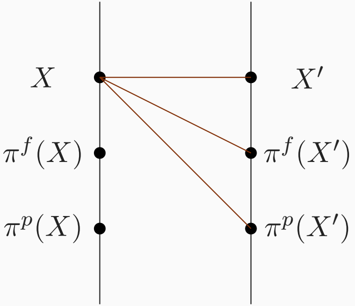

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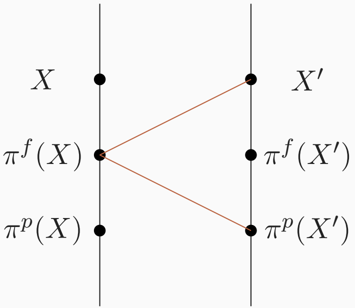

- Horizontal lines are equivalent

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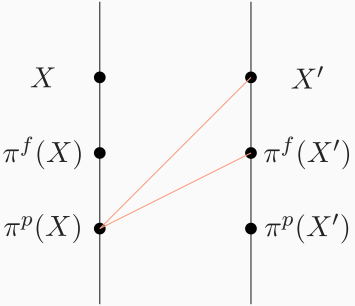

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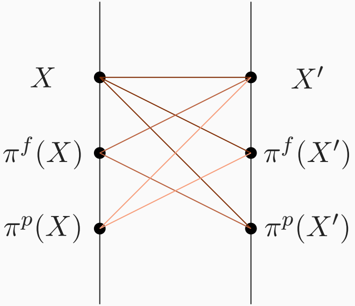

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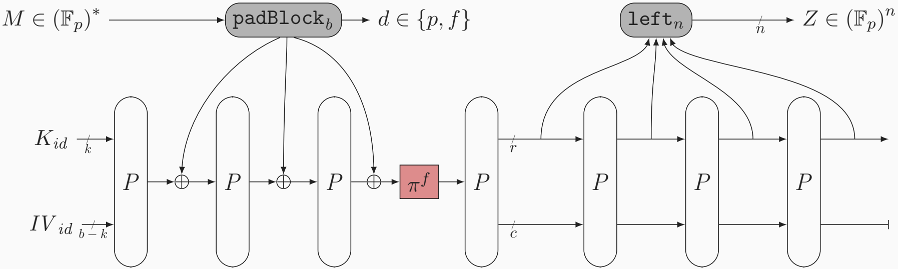
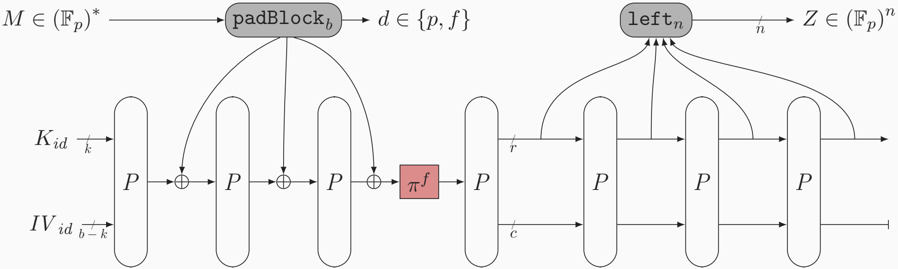
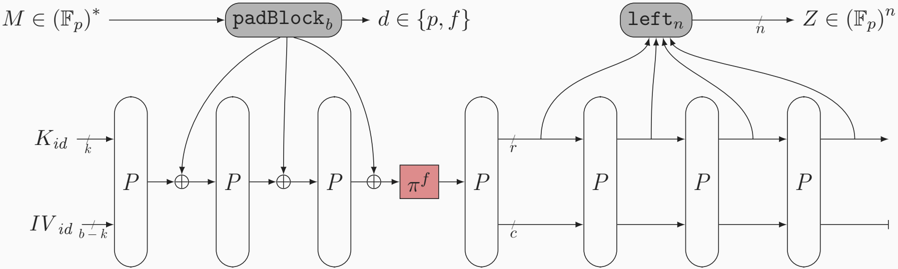


$M \in (\mathbb{F}_p)^*$
padBlock$_b$
$d \in \{p, f\}$
left$_n$
n
$Z \in (\mathbb{F}_p)^n$
K_{id}
k
IV_{id}
$b-k$
P
P
P
π^f
P
r
c
P
P
P

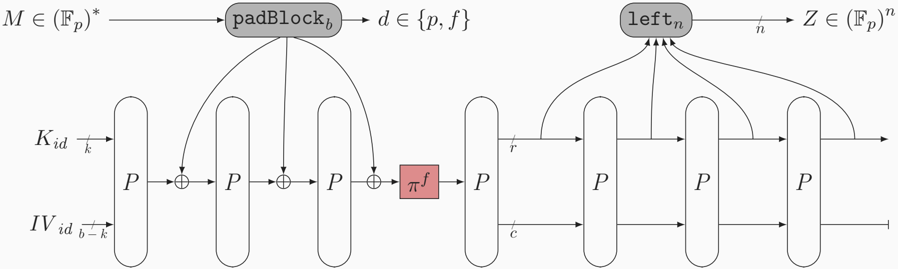


- P is b-bit permutation

Our Constructions: fks-pi



- P is b-bit permutation
  - k is the key size
  - c is the capacity
  - $b = r + c$



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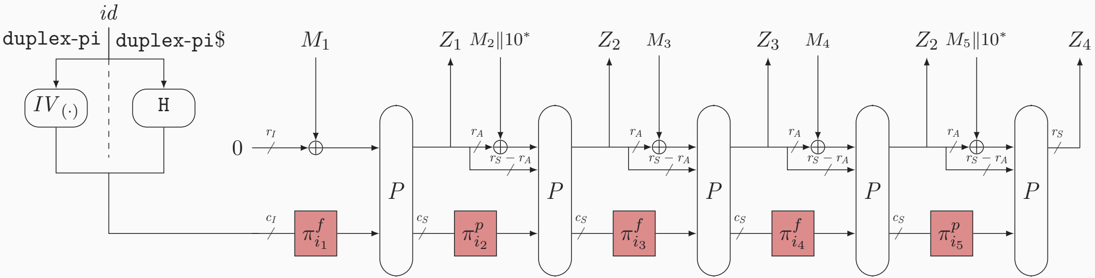
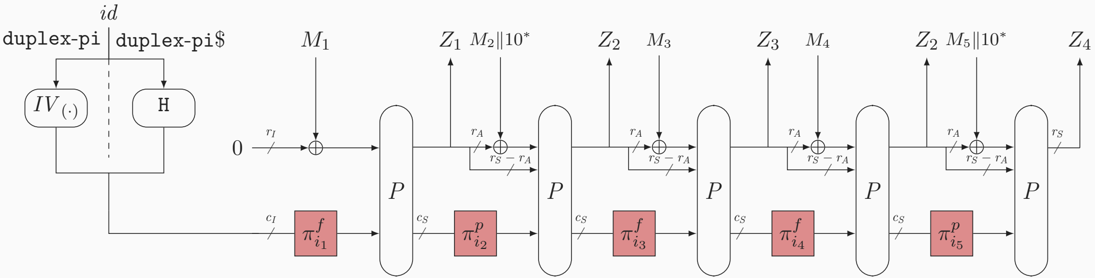
  - k is the key size
  - c is the capacity
  - $b = r + c$
- **Security**: behaves like RO up to $\mathcal{O}\big(\min\{p^k, p^b/\sigma, p^c\}\big)$, where σ is the online complexity

Our Constructions: duplex-pi and duplex-pi$

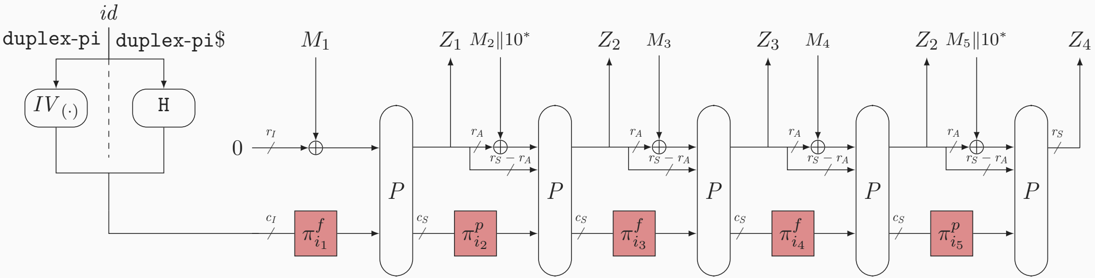


- ξ: Number of different NCPs

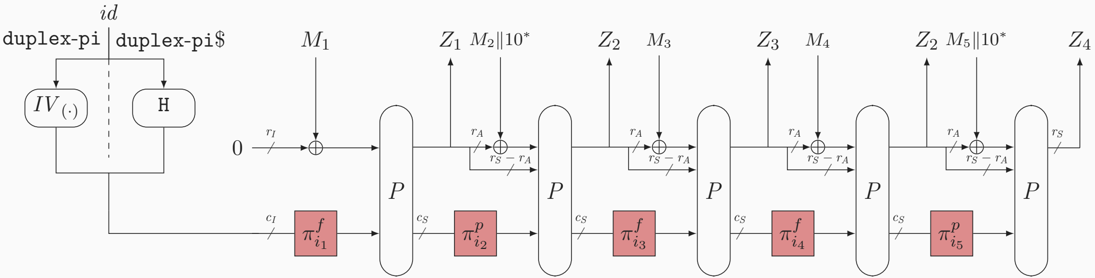
Our Constructions: duplex-pi and duplex-pi\$



- ξ: Number of different NCPs
- `duplex-pi`: User-determined IVs



- ξ: Number of different NCPs
- duplex-pi: User-determined IVs
- duplex-pi$: Randomly determined IVs



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Same, but different

- Simplified setting: $c_I = c_A = c_S = c$
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duplex-pi and duplex-pi$: Simplified Security Comparison

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FKD	$\frac{Q_P^2}{p^c} + \frac{2\nu_{r,c}^{2Q_P} Q_P}{p^c}$
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Where does the ξ^2 come from?

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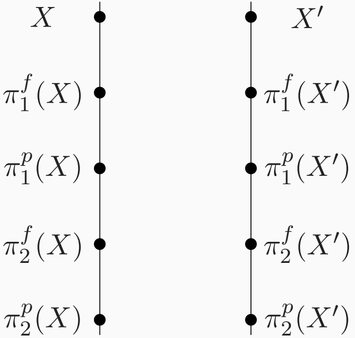

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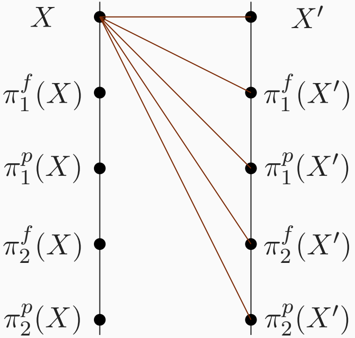

duplex-pi and duplex-pi\$: Simplified Security Comparison

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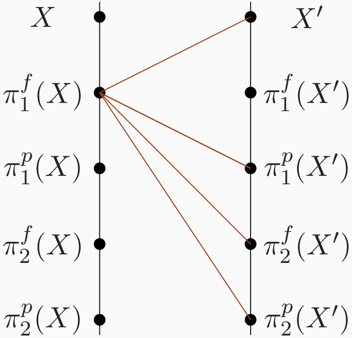

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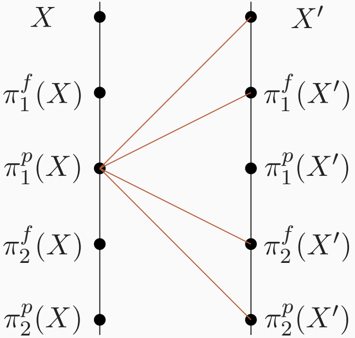
Where does the ξ^2 come from?



- Horizontal lines are equivalent

duplex-pi and duplex-pi\$: Simplified Security Comparison

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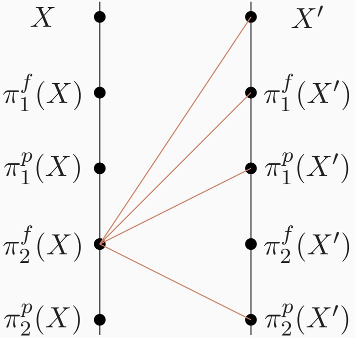

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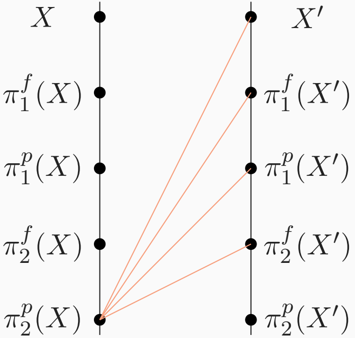

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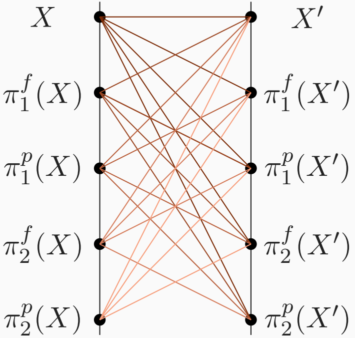

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- Total: $\underbrace{\xi^2}_{\text{pairs}} - \underbrace{(\xi - 1)}_{\text{horizontal}}$

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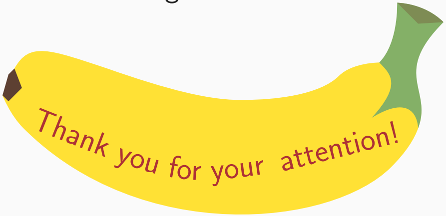

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