Permutation-Based Hash Chains with Application to Password Hashing

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FSE 2025

17 March 2025

Introduction

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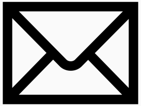

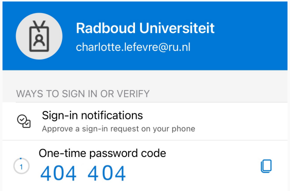

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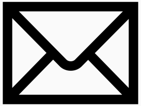

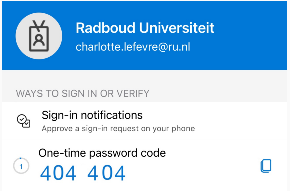

- Some use time-based one-time password schemes

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where:

  - HMAC may be $\mathrm{HMAC\text{-}SHA\text{-}256}$ or $\mathrm{HMAC\text{-}SHA\text{-}512}$
  - ctr is a counter based on the current time (typically changes every 30s)
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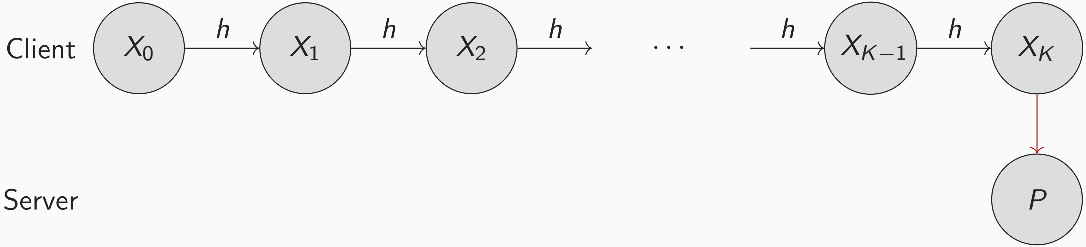
- **Downside:** Server must securely store κ

Hash Chains and S/Key [Lam81, Hal95] (RFC 1760)



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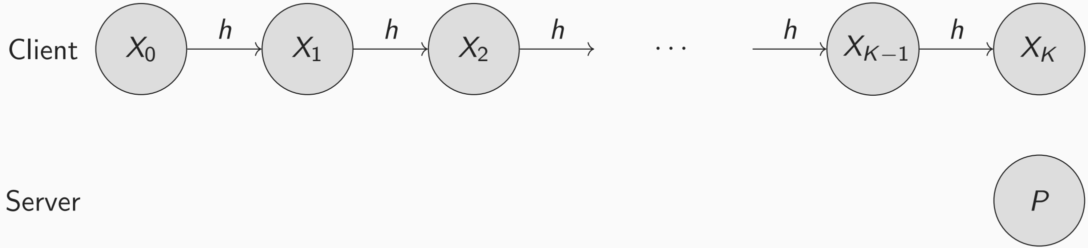

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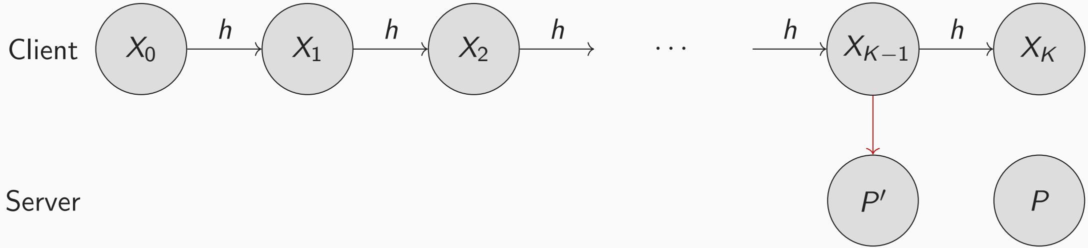

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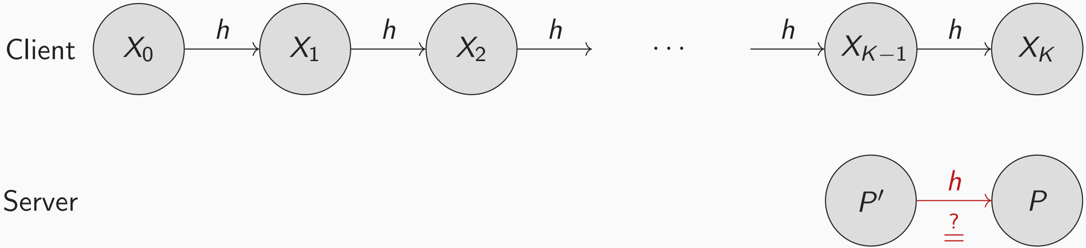

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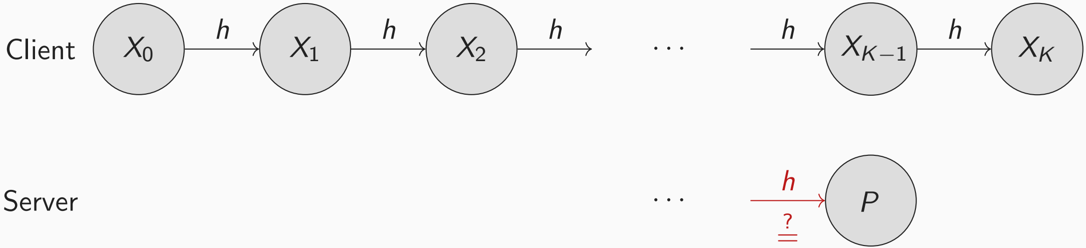

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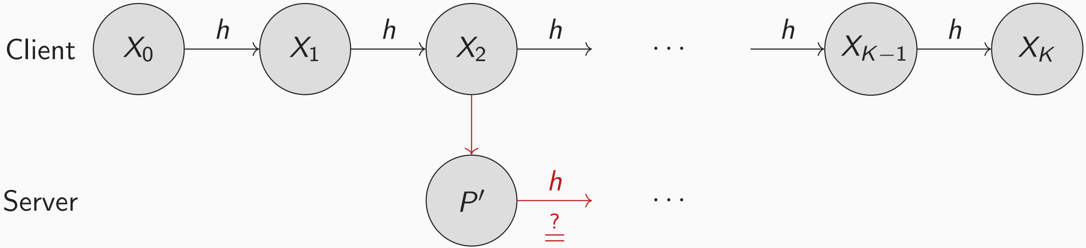

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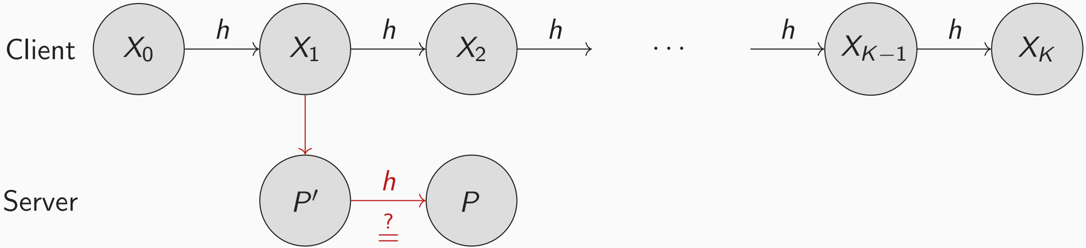

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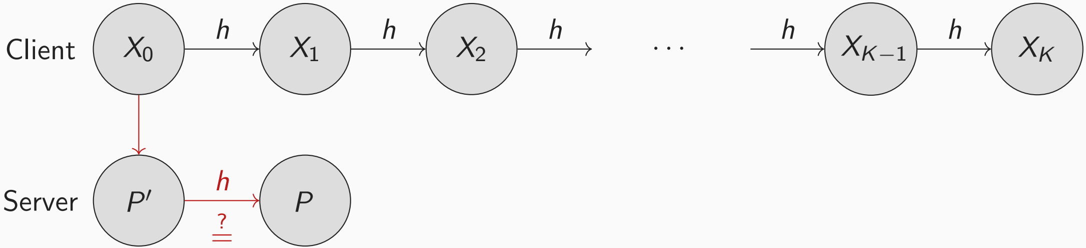

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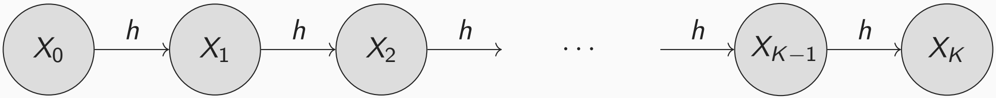
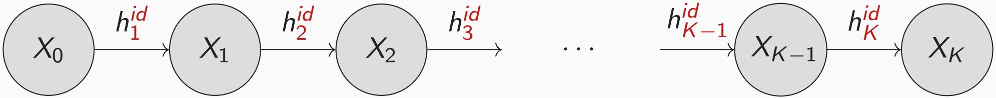
Hash Chains and S/Key (cont'd)



Weaknesses

- Not time-based: increases attack window if logins are scarce
- Iterating a hash function weakens its security $\Longrightarrow$ security degradation by factor K

T/Key [KMB17]



- Domain separation and salt incorporation: $h_k^{id}(\cdot) := h(\langle \mathrm{ctr}_k \rangle_t \parallel id \parallel \cdot)$:
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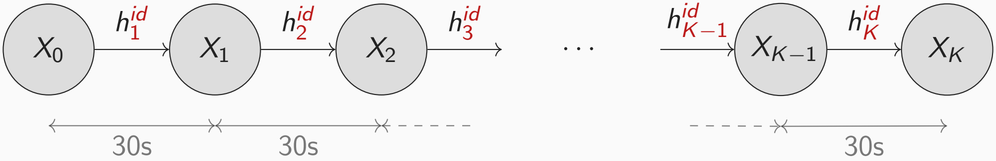

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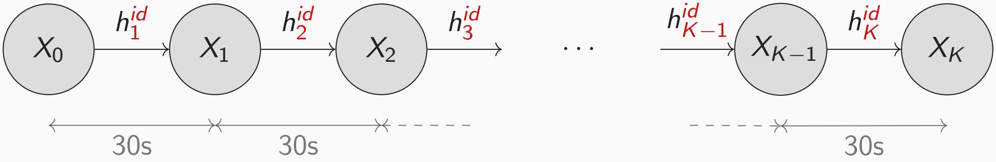

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- Every point on the chain is valid for a limited amount of time
- Suggestion by the designers:

$s = 80$ $t = 32$ $K = 2^{21}$ $n = 130$ 30s time frames

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Assuming that h is a random oracle, T/Key is secure up to bound [KMB17]

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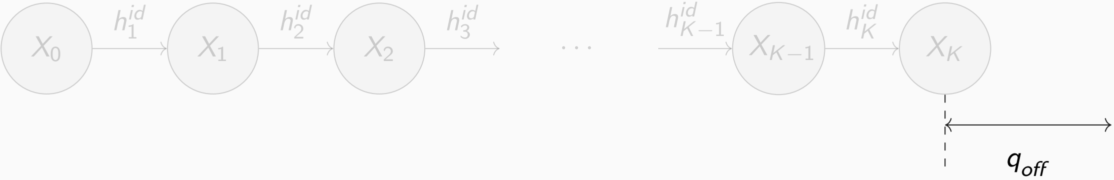

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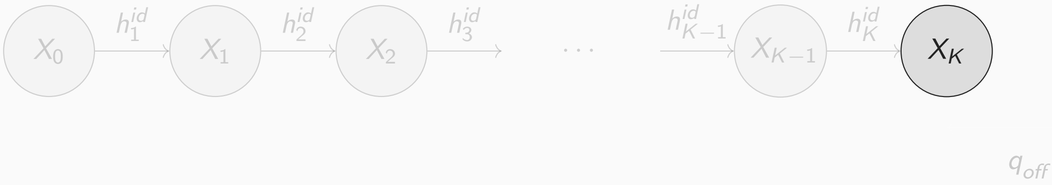

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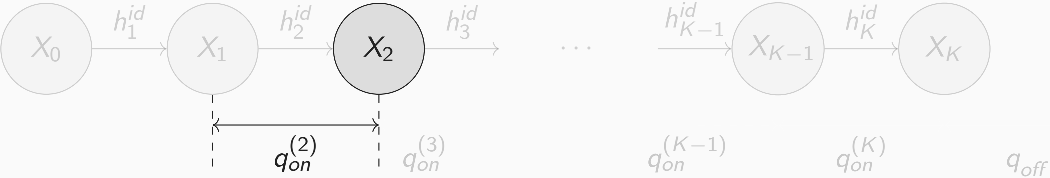

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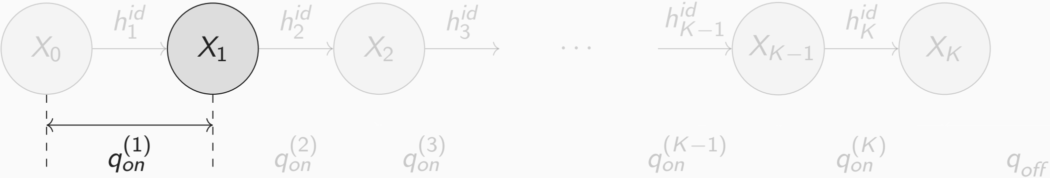

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- Security advantage is denoted by $\mathbf{Adv}_h^{\mathrm{T/Key}}(q_{off}, q_{on}, M)$, where
  - $q_{on} = \sum_k q_{on}^{(k)}$ (we assume $q_{on} \ll 2^{100}$)
  - M denotes the number of users

- Bound from T/Key designers:

$$\mathbf{Adv}_{\mathcal{RO}}^{\mathrm{T/Key}}(q_{\mathit{off}}, q_{\mathit{on}}, 1) = \mathcal{O}\left(\frac{q_{\mathit{off}} + q_{\mathit{on}}}{2^n}\right)$$

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- Assuming $q_{\mathit{off}} \ll 2^{128}$, $q_{\mathit{on}} \ll 2^{100}$, and $M \ll 2^{52}$, password size can be reduced from 130 to 100

Security of T/Key with a Hash Function Construction

- Let $\mathcal{H}$ be a hash function construction:

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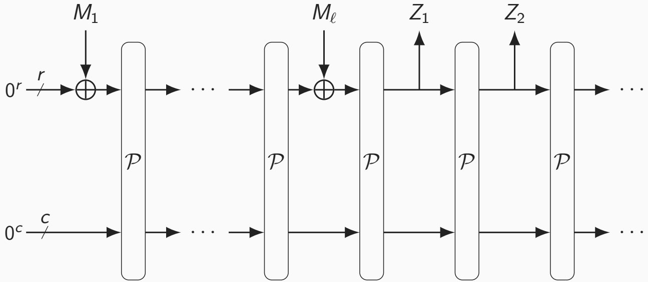
A dedicated analysis will likely give a better bound

T/Key with the Sponge Construction

The Sponge Construction [BDPV07]



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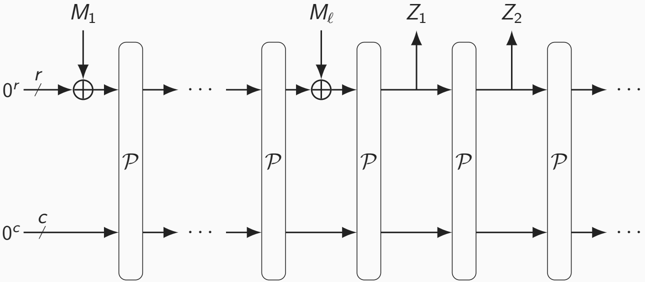

- Permutation $\mathcal{P}$ of size $b = r + c$
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- The sponge construction has a tight indifferentiability bound [BDPV08]:

$$\mathbf{Adv}^{\mathrm{iff}}_{\mathrm{Sponge}}(q) \leq \frac{q(q+1)}{2^c}$$

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Random X_0

- Assume that $s + t = 2r$ and $n + 1 = 2r$

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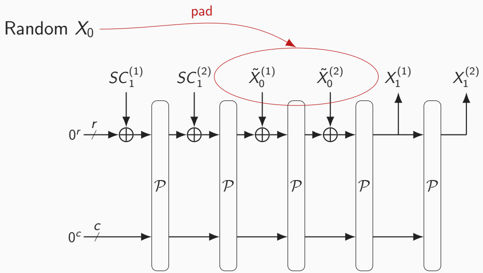

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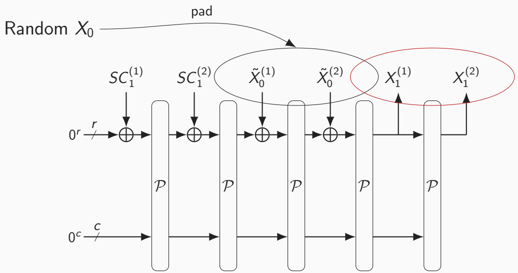

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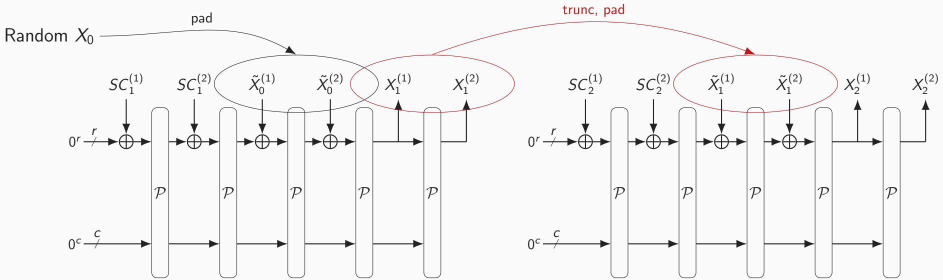

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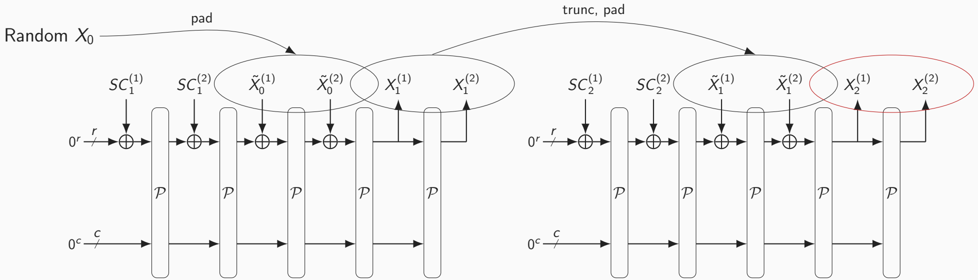

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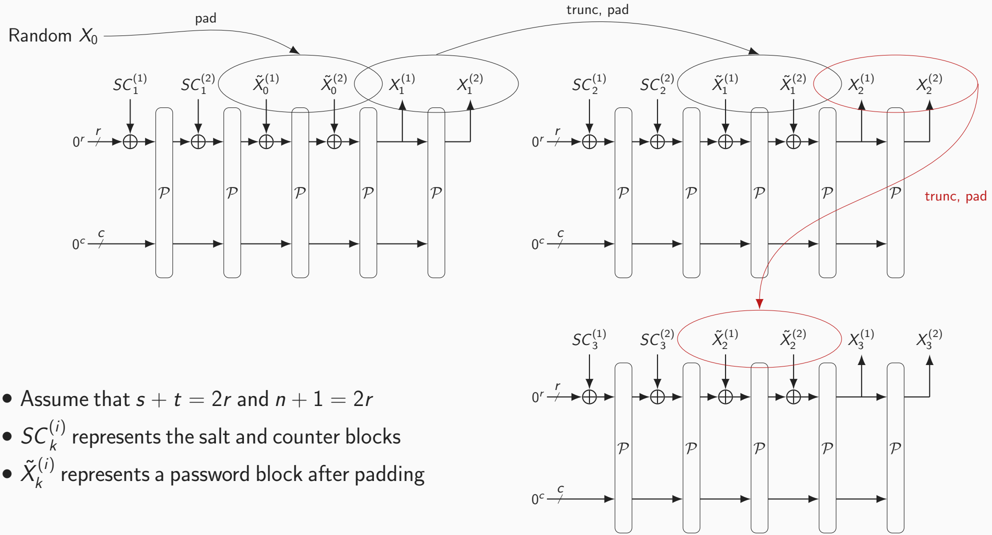

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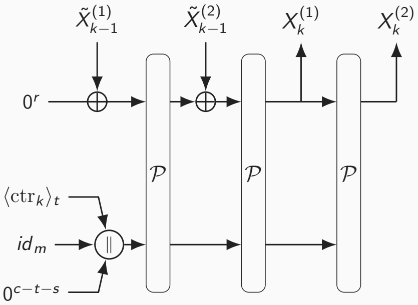
T/Key with the Sponge Construction: Optimization



- Salt and counter are part of the initial state
- Requires that $s + t \leq c$
- Our security bound holds both for the sponge and this optimization

T/Key with the Sponge Construction: Security

Using generic composition: (assuming $q_{off} \geq q_{on}$)

$$\mathbf{Adv}^{\mathrm{T/Key}}_{\mathrm{Sponge}}(q_{off}, q_{on}, 1) = \mathcal{O}\Bigg(\underbrace{\frac{q_{off}}{2^{n+s}} + \frac{q_{on}}{2^n}}_{\mathbf{Adv}^{\mathrm{T/Key}}_{\mathcal{RO}}} + \underbrace{\frac{q_{off}^2}{2^c}}_{\mathbf{Adv}^{\mathrm{iff}}_{\mathrm{Sponge}}}\Bigg)$$

$\Longrightarrow$ Minimum permutation size: $b = 256$

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We derive:

$$\mathbf{Adv}^{\mathrm{T/Key}}_{\mathrm{Sponge}}(q_{off}, q_{on}, 1) = \mathcal{O}\left(\frac{Kq_{off}}{2^{n}} + \frac{Kq_{off}}{2^{c}} + \min\left(\frac{q_{on}}{2^{n-r}}; \frac{q_{off}\, q_{on}}{2^{c}}\right)\right)$$

$\Longrightarrow$ Password size increased to $n \geq 149$ ...

... but permutation size can be lowered to $b = 150$

$\Longrightarrow$ Could be instantiated with, e.g., Spongent permutation [BKL+11] ($b = 176, c = 150, r = 26$)

T/Key with the Sponge Construction: Proof Idea

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- Preimage resistance of the sponge [LM22]:

$$\mathbf{Adv}^{\mathrm{epre}}_{\mathrm{Sponge}}(q) = \mathcal{O}\left(\frac{q}{2^n} + \min\left(\frac{q}{2^{n-r}}; \frac{q(q+1)}{2^c}\right)\right)$$

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$$\mathbf{Adv}^{\mathrm{T/Key}}_{\mathrm{Sponge}}(q_{off}, q_{on}, 1) = \mathcal{O}\left(\frac{Kq_{off}}{2^n} + \frac{Kq_{off}}{2^c} + \min\left(\frac{q_{on}}{2^{n-r}}; \frac{q_{off}\, q_{on}}{2^c}\right)\right)$$

Building blocks:

- Preimage resistance of the sponge [LM22]:

$$\mathbf{Adv}^{\mathrm{epre}}_{\mathrm{Sponge}}(q) = \mathcal{O}\left(\frac{q}{2^n} + \min\left(\frac{q}{2^{n-r}}; \frac{q(q+1)}{2^c}\right)\right)$$

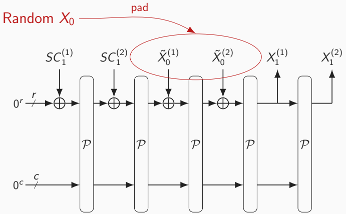
- PRF security of the outer keyed sponge with key size n [ADMV15, NY16, Men18]: (assuming $n \leq b$)

$$\mathbf{Adv}^{\mathrm{prf}}_{\mathrm{OKS}}(M, N) = \mathcal{O}\left(\frac{NM}{2^c} + \frac{N}{2^n}\right)$$

where M denotes the online complexity and N the offline complexity

T/Key with the Sponge Construction: Proof Idea (cont'd)

Simplified idea: the game can be decomposed into K different games



Game 1: Adversary wins if it finds a preimage of X_1

- Offline phase: $q_{off} + \sum_{k=2}^{K} q_{on}^{(k)}$ queries
- Online phase: $q_{on}^{(1)}$ queries

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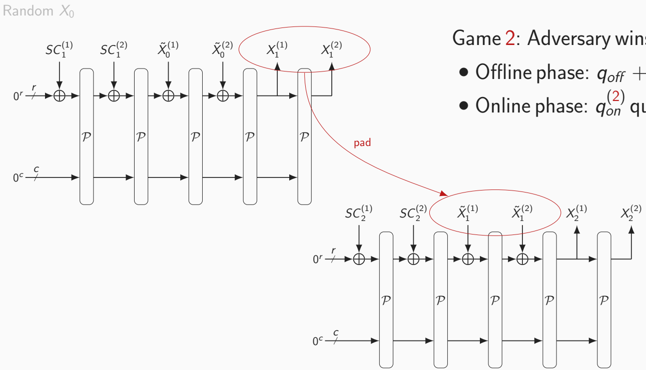
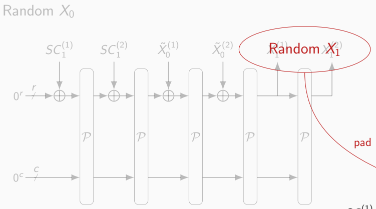


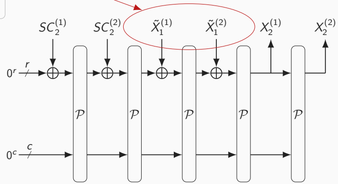
Game 2: Adversary wins if it finds a preimage of X_2

- Offline phase: $q_{off} + \sum_{k=3}^{K} q_{on}^{(k)}$ queries
- Online phase: $q_{on}^{(2)}$ queries

T/Key with the Sponge Construction: Proof Idea (cont'd)

Simplified idea: the game can be decomposed into K different games





Game 2: Adversary wins if it finds a preimage of X_2

- Offline phase: $q_{off} + \sum_{k=3}^{K} q_{on}^{(k)}$ queries
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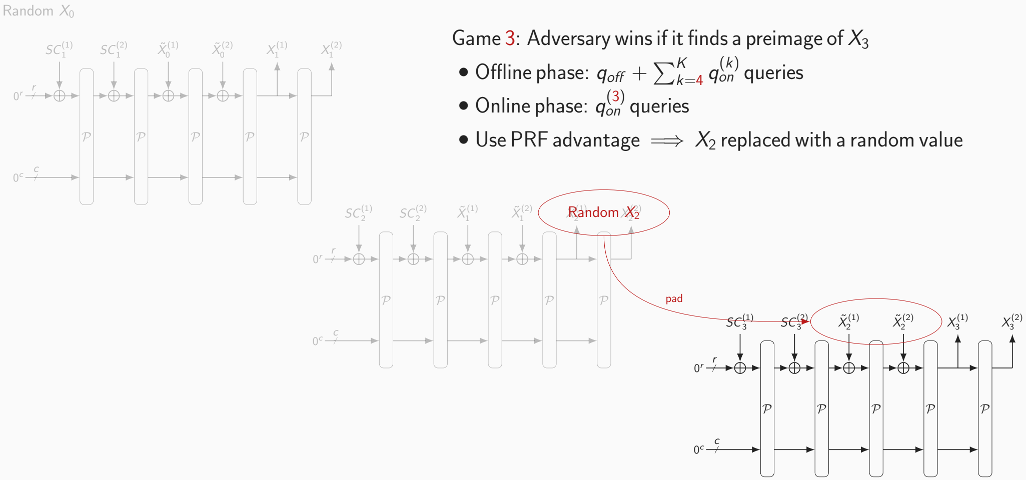
- Use PRF advantage $\Longrightarrow$ X_1 replaced with a random value

Simplified idea: the game can be decomposed into K different games

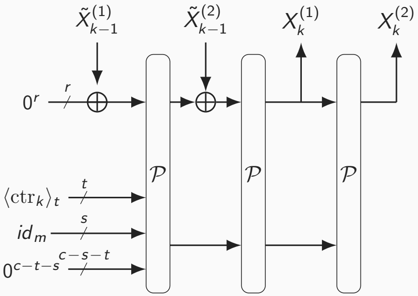


Game 3: Adversary wins if it finds a preimage of X_3

- Offline phase: $q_{off} + \sum_{k=4}^{K} q_{on}^{(k)}$ queries
- Online phase: $q_{on}^{(3)}$ queries
- Use PRF advantage $\implies$ X_2 replaced with a random value

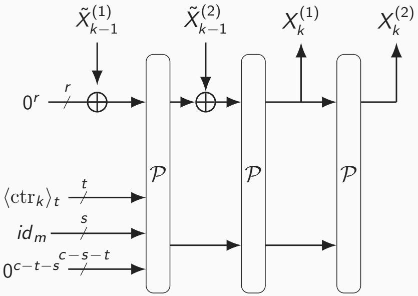
T/Key with a Truncated Permutation

T/Key with a Truncated Permutation



- Using a sponge?

T/Key with a Truncated Permutation



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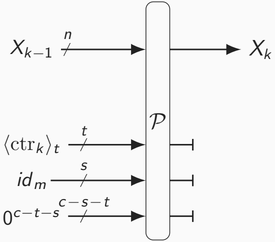
- In most cases, the permutation is large enough to absorb everything at once

T/Key with a Truncated Permutation



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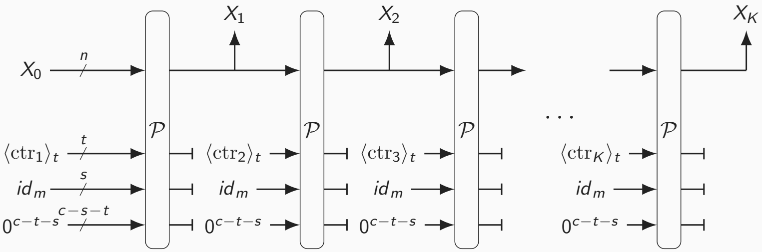
T/Key with a Truncated Permutation



- Using a sponge?
- In most cases, the permutation is large enough to absorb everything at once
- Construction behaves as truncated permutation
- Hash chain becomes truncated permutation chain

Security of T/Key with a Truncated Permutation

- Using generic composition with indifferentiability:

$$\mathbf{Adv}^{\mathrm{T/Key}}_{\mathrm{TruncP}}(q_{off}, q_{on}, 1) = \mathcal{O}\Bigg(\underbrace{\frac{q_{off}}{2^{n+s}} + \frac{q_{on}}{2^{n}}}_{\mathbf{Adv}^{\mathrm{T/Key}}_{\mathcal{RO}}} + \underbrace{\frac{q_{off}^{3/2}}{2^{\frac{2b-n}{2}}} + \frac{q_{off}}{2^{b-(n+t)}}}_{\mathbf{Adv}^{\mathrm{iff}}_{\mathrm{TruncP}}}\Bigg)$$

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- We derive:

$$\mathbf{Adv}^{\mathrm{T/Key}}_{\mathrm{TruncP}}(q_{off}, q_{on}, M) = \mathcal{O}\left(\mathbf{Adv}^{\mathrm{T/Key}}_{\mathcal{RO}}(q_{off}, q_{on}, M) + \frac{KM \cdot q_{off}}{2^b} + \max\left(\frac{KM}{2^n}; 1\right) \cdot \frac{q_{on}}{2^c} \right)$$

- The bound is tight

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- The bound is tight
- Assume we want password sizes of $n = 100$, $q_{off} \ll 2^{128}$, $q_{on} \ll 2^{100}$:
  - Generic composition indicates we need $b \geq 260$
  - Our bound indicates we need $b \geq 200$ as long as $M \ll 2^{40}$

Conclusion

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We analyzed the security of hash chain based password systems:

- **Refined model** that distinguishes offline vs. online complexity
- **Security proofs** with a random oracle, sponge, and truncated permutation:
  - Shows that truncated permutations work for most use cases
  - With truncated permutation, password size can be lowered to $n = 100$
  - Improved understanding of the preimage resistance of cascaded sponge evaluations
- Results hold in the random oracle/permutation model

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